Baggage fees in airlines: Is this a good idea?

Botteon Costa, Raone and Ferman, Bruno and Monte, Daniel

Sao Paulo School of Economics -FGV, Sao Paulo School of Economics -FGV, Sao Paulo School of Economics -FGV

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Raone B. Costa Bruno Ferman Daniel Monte

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Abstract

A recurrent policy question in the airline industry is whether baggage should be priced independently from airline tickets. We show that this policy has ambiguous welfare implications, depending on the cost of carrying luggage and on the market power of the firm. The intuition is simple: there is a trade-off between over-consumption caused by the non-existence of a baggage price against under-consumption caused by firm markups in the case of a separate price for baggages. The commonly used argument that the price of travelling without luggage might drop under a two price-system does not hold in our model.

Keywords: baggage fees, screening, two part tariff, welfare

JEL Codes: D82, D83, L15, M3

1 Introduction

In the last decade, it became common for airline companies to charge a separate fee for ancillary services, such as for baggage handling. This began in 2008, when U.S. airlines spearheaded the efforts, but quickly become an industrial trend worldwide. In 2008, baggage fees revenues amounted to US$ 0.5 billion in U.S. companies, while by the end of 2016 this number had quickly grown to more than US$ 4.0 billion[1] - and now amounts to more than 10% of the revenues of the majority of large airline carriers.[2] Understandably, this change generated political attention and cries for regulation. In 2011 U.S. Senator Mary Landreau attempted to banish baggage fees (Halsey III, 2011) and recently there's been a discussion about forcing airlines to disclose baggage fees to consumers at the point of sale (Henry, 2017), which is due to concerns about baggage fees salience. However, there is still limited evidence on the welfare effect of this change.

We construct a simple model to investigate this issue. In our model, consumers optimally choose whether to consume two goods - passenger travel and baggage travel - under the restriction that the

*Costa: Sao Paulo School of Economics -FGV. E-mail: raone.botteon@fgv.br. Ferman: Sao Paulo School of Economics -FGV. E-mail: bruno.ferman@fgv.br. Monte: Sao Paulo School of Economics -FGV. E-mail: daniel.monte@fgv.br.

[1] `https://www.bts.gov/content/baggage-fees-airline-2017`

[2] `https://bit.ly/2BN2NsX`

second good is only available after the purchase of the first. The model is solved for two distinct scenarios: in the first, the monopolist firm is obliged to set the price of baggage fees to zero, and in the second it may optimally choose prices using a two-part tariff. The results show that allowing for two prices is not always welfare enhancing, and depends on the marginal cost of conveying baggage. The intuition is simple: we have a trade-off between over-consumption and under-consumption of baggages in the different scenarios. When the airline can only charge a single price, some consumers with low valuation for baggages might opt to embark their luggage anyway (as the price for doing so is zero) leading to over-consumption. With two prices this does not happen, but due to the firm's monopolist power, it will set its prices in such a way that some consumers with baggage valuation higher than its marginal cost will still opt to withdraw from the market.

After this initial analysis, we extend the model to a price competition duopoly similar to the Bertrand model, which we see as an approximation of the perfect competition case. This allows us to document the intuitive result that firms' market power is also important. In a competitive setting, allowing for two distinct prices is always welfare enhancing. Finally, we study the issue of allocation. We show that while a monopolist firm is at least neutral towards the change from a single price to two distinct prices, the effects on consumer utility are uncertain. In our main model, consumers are only differentiated by their preferences over baggage handling, and that makes them at best indifferent towards this change as the monopolist firm is able to fully extract consumer surplus on the passenger market. However, once the model is extended to allow for heterogeneous preferences over both goods, we show that consumers might also be favorable to the change.

The literature on the impact of baggage fees on social welfare is still limited. Allon et al. (2011) studies the problem using a different modelling approach. In their setting, consumers choose not whether to buy baggage transportation, but whether to exert an effort to avoid having to travel with baggage. They conclude that moving from one to two prices for airlines services is unequivocally good for society, even in a monopoly setting. This result contrasts with our main finding, which was that the change from a single to two distinct prices is not always socially optimal. The difference in results is due to different modelling techniques: in their setting, there is not the possibility of under-consumption of baggage as this is undesired by both consumers and the firms. As such, the introduction of the explicit baggage fee in their setting leads naturally to the optimal effort level by the consumer, essentially solving the problem. In our model, this is not the case, as consumers derive utility directly from baggage travel. In this sense our model contributes to the discussion by analyzing the case in which consumers effectively want to embark their baggage and see that as an consumption good.

Research generally agrees that explicit baggage fees are positive for firm metrics. We have evidence that baggage fees have a much lower elasticity than regular fares, which tends to increase firms market power (Scotti and Dresner, 2015). These fees also tend to increase airlines stock price (Barone et al., 2012) and the likelihood of on-time departure performance (Nicolae et al., 2017). Moreover, contrafactual exercises on the potential regulation of such fees indicates that banning baggage fees would have little to no effect on total travel prices (Agarwal et al., 2014). The main criticism for allowing dual prices comes from salience issues, as there is evidence that the opacity of baggage prices is relevant for revenues in this market (Bradley and Feldman, 2016) and that this feature might hurt consumers utility in behavioral ways (Coy and Chiang, 2012). Our model connects to this strand of research by showing that those improvements on firm's metrics might not be good for society even without bounded rationality and salience issues.

In methodological terms, as our model can be understood as a two-part tariff model in which consumers are restricted to only being allowed to purchase one of the goods after the consumption of the other, it connects to the larger literature on price discrimination and bundling. Two good examples of such are Armstrong and Vickers (2010) and Armstrong (2006). Our research also connects to the literature of multi-dimensional screening, as in our final extension we deal with two distinct sources of consumer private information. A good review on the subject can be found in Armstrong and Rochet (1999).

The rest of the paper is organized as follows. Section 2 describes our main model. Section 3 briefly discusses some interesting extensions and section 4 provides a conclusion, with an eye on potential applications of the model to different problems.

2 The Model

2.1 Setup

Consider an economy with 2 goods: airplane passenger travel and luggage travel. The goods are assumed to be offered in discrete quantities in which each consumer may opt to consume either one unit or no units of the good (i.e. they choose to travel or not). There is a monopolist firm that offers both goods and a continuum of measure one of consumers. Let q_1 denote consumption of airplane travel and q_2 denote consumption of luggage travel.

Each consumer i have to choose between three travel options. He can either choose to travel heavy (i.e: travel with a dispatched baggage), travel light (i.e. to travel without any dispatched baggage) or not to travel at all. If the consumer decides to travel, he gets utility $\overline{u} > 0$, which is assumed to be constant for all consumers by simplicity.[3] If he decides to embark his baggage, he gets utility δ_i, which is a random variable drawn from an uniform $[0, 1]$. The utility of not travelling is normalized to 0. Note that this structures entails that consumers can purchase q_2 only if they also decided to purchase q_1.

The firm has two distinct marginal costs: c_1 is the marginal cost of transporting passengers and c_2 the marginal cost of transporting baggage. We assume that both c_1 and c_2 are greater than 0 and, for simplicity, we assume that there no fixed costs.[4]

[3]This will be relaxed in the extensions provided in the next section.

[4]This is, of course, a simplification, but we believe it captures the main aspects of the industry costs. Passenger travel is obviously costly due to spacing issues inside the airplane. Luggage travel is also costly, as it affects the weight of the

There are two distinct prices: one for the consumption of q_1 (called p_1) and one for consumption of q_2 (p_2). Still, we analyze two distinct cases. In the first case, named "single price case", firms are restricted to set $p_2 = 0$. In the second case (the "dual price case") they may set both prices as they see fit.

In all cases, prices and consumer utilities are assumed to be measured in the same unit. Hence, the decision to consume one unit of passenger travel adds $\overline{u}$ units to the consumer's utility, but subtract p_1 units.

2.2 Demand

Consumer demand comes from standard utility maximization. In the single price case (i.e. $p_2 = 0$) we have the following demand functions:

$$q_{1i}(p_1) := 1_{\{\overline{u} \geq p_1\} \cup \{\overline{u}+\delta_i \geq p_1\}} \quad \text{and} \quad q_{2i}(p_1) := 1_{\{\overline{u}+\delta_i \geq p_1\}}. \tag{1}$$

Equation 1 tells us that consumers will opt to travel (and to embark their luggage) when the the utility of doing so is at least as high as the costs associated to it. For the case of the first good, this may happen by two reasons. Either travelling is good enough to be consumed on its own (that is, $\overline{u} \geq p_1$) or it might be a cost worth paying to travel heavy (which implies that, $\overline{u} + \delta_i \geq p_1$). Note that in this setting, due to the presence of a single price for both goods, the consumer will never prefer to travel light over travelling heavy.[5] Hence, we may simplify demand to $q_{1i}(p_1) = q_{2i}(p_1) := 1_{\{\overline{u}+\delta_i \geq p_1\}}$

In our second case, demand will be given by:

$$q_{1i}(p_1, p_2) := 1_{\{\overline{u} \geq p_1\} \cup \{\overline{u}+\delta_1 \geq p_1+p_2\}} \quad \text{and} \quad q_{2i}(p_1, p_2) := 1_{\{\overline{u}+\delta_i \geq p_1+p_2\} \cap \{\delta_i > p_2\}}. \tag{2}$$

Now, because of the two price structure, we cannot simplify demand for the first good like we did before, as the restrictions are not nested anymore. Moreover, demand for the second good now requires two conditions. Passengers will only opt to travel heavy when this option is better than his other two options: not travelling (which requires $\overline{u} + \delta_i \geq p_1 + p_2$) and travelling light (which requires $\delta_i \geq p_2$).

2.3 First case: Single price

In this scenario, the firm is restricted to set $p_2 = 0$. Hence, it will choose p_1 to solve the following problem:

$$\text{Max } \Pi_1(p_1) := \mathbb{E}\{(p_1 - c_1)q_{1i}(p) - c_2 q_{2i}(p)\}, \tag{3}$$

The subscript 1 on the above equation denotes that this is the profit function of our first case. Proposition 1 stated below characterize the solution to this case.

airplane, which has impact on fuel costs (Stromberg, 2015).

[5] As we assume for simplicity that $\delta \geq 0$, we get that $\overline{u} \geq p_1$ implies $\overline{u} + \delta_i \geq p_1$

Proposition 1 (Single Price Characterization) *The single price problem is characterized by the following function:*

$$p_1^* = \begin{cases} \overline{u} \text{ if } c_1 + c_2 \leq \overline{u} - 1, \\ \dfrac{1 + \overline{u} + c_1 + c_2}{2} \text{ if } \overline{u} - 1 \leq c_1 + c_2 \leq \overline{u} + 1, \\ \infty \text{ if } c_1 + c_2 \geq \overline{u} + 1. \end{cases}$$

Proposition 1 tells us that the single price case divides the plane (c_1, c_2) into three different regions. In the region defined by $c_1 + c_2 \leq \overline{u} - 1$, which we name **L**, costs are low enough so that the firm finds it optimal to bring all consumers to the market, as the potential cost of losing a fraction of consumers is higher than the benefit of an increased price for the fraction that remains. In order to do that, the firm sets $p_1^* = \overline{u}$, eliminating consumer surplus for the consumption of q_1. In this case, firms earn $\Pi_1(\overline{u}) = \overline{u} - c_1 - c_2$ and all consumers opt to travel heavy.

In the region $c_1 + c_2 \geq \overline{u} + 1$, which we define as **H**, the opposite happens: costs are so high that the firm finds it optimal to shut down the market entirely. In this case, profits are obviously null and consumers will always choose not to travel.

In between those two extremes, which we name **M**, the firm find it optimal to only attend consumers that have a sufficiently high value of δ_i. To see this point more clearly, let λ_i be a random variable that assumes value 0 if consumer i opts to not travel, 1 if he decides to travel light and 2 if he decides to travel heavy. We then have that in region **M**:

$$\Pr(\lambda_i = 2) := \Pr(\delta_i \geq p_1^* - \overline{u}) = \left(\frac{1 + \overline{u} - c_1 - c_2}{2} \right). \tag{4}$$

As with a single price the cost of travelling heavy and travelling light are the same, consumers will never opt for the latter option in this case, so $\Pr(\lambda_i = 1) = 0$ and $\Pr(\lambda_i = 0) = 1 - \Pr(\lambda_i = 2)$.

To finish our analysis, profits in this intermediate case will me given by

$$\Pi_1(p_1^*) = \left(\frac{1 + \overline{u} - c_1 - c_2}{2} \right)^2. \tag{5}$$

Table 1 summarizes our findings for the single price case and figure 1 exemplify the regions for the case $\overline{u} = 3$.

2.4 Second case: Two distinct prices

In this scenario, firms are free to set p_2 at its optimal value, so the firm problem turns into:

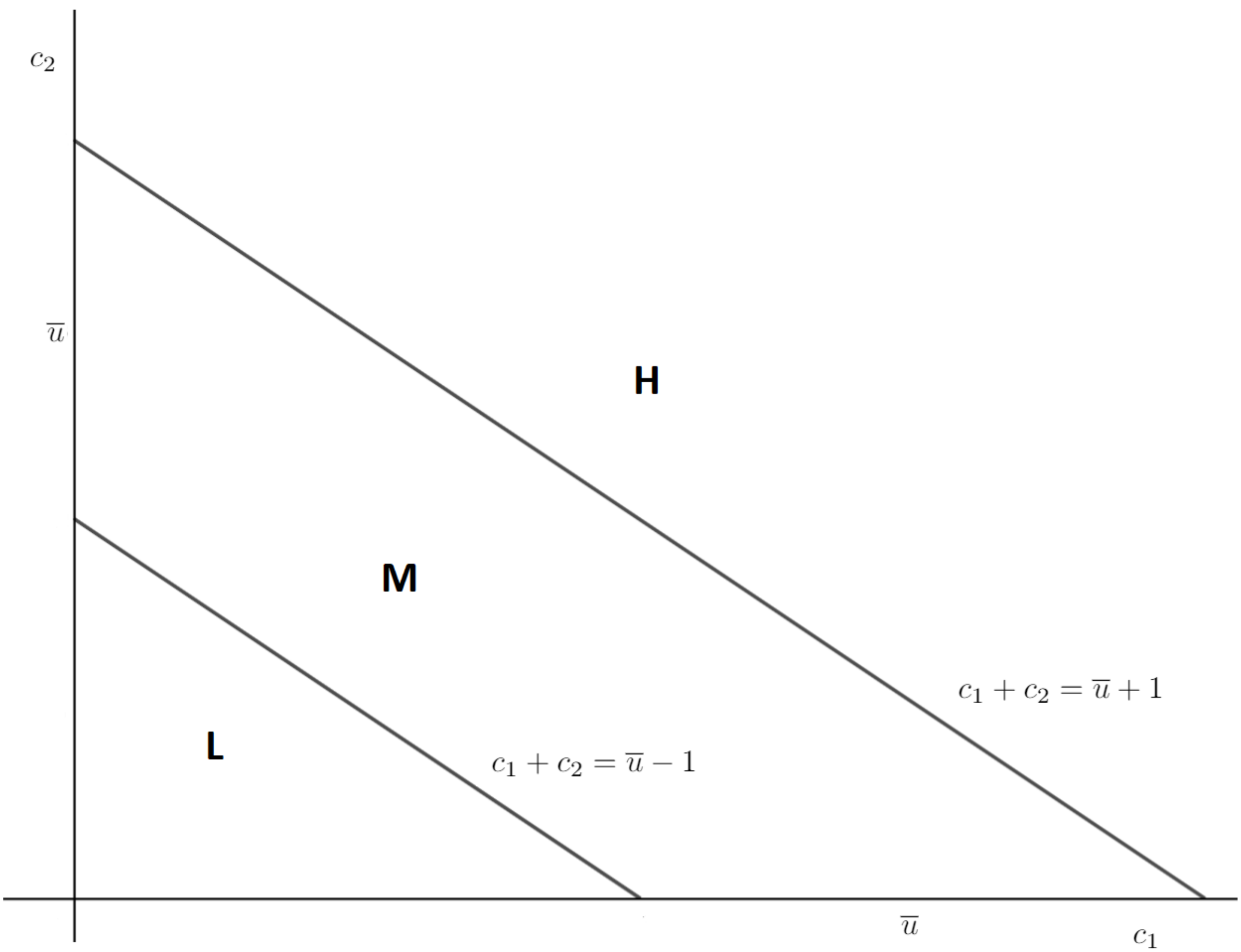
$$\text{Max } \Pi_2(p_1, p_2) := \mathbb{E}\{(p_1 - c_1)q_{1i}(p) + (p_2 - c_2)q_{2i}(p)\}. \tag{6}$$

The subscript 2 on the above equation denotes that this is the profit function of our second case. Proposition 2 stated below characterize the solution to this case.

Table 1: Single Price Setting Resume

	L	**M**	**H**
p_1^*	$\overline{u}$	$\left(\dfrac{1+\overline{u}+c_1+c_2}{2}\right)$	∞
Π_1	$\overline{u}-c_1-c_2$	$\left(\dfrac{1+\overline{u}-c_1-c_2}{2}\right)^2$	0
$\Pr(\lambda_i = 1)$	0	0	0
$\Pr(\lambda_i = 2)$	1	$\left(\dfrac{1+\overline{u}-c_1-c_2}{2}\right)$	0

Figure 1: Optimal Choice for the single price scenario with $\overline{u} = 3$



Proposition 2 (Two-Price Characterization) *The two-price problem is characterized by the following function:*

$$(p_1^*, p_2^*) = \begin{cases} \left(\overline{u}, \dfrac{1+c_2}{2}\right) \text{ if } c_1 \leq \overline{u} \text{ and } c_2 \leq 1, \\ (\overline{u}, \infty) \text{ if } c_1 \leq \overline{u} \text{ and } c_2 > 1, \\ \left(\dfrac{1+\overline{u}+c_1+c_2}{2}, 0\right) \text{ if } c_1 > \overline{u} \text{ and } c_1 + c_2 \leq \overline{u}+1, \\ (\infty, \infty) \text{ if } c_1 > \overline{u} \text{ and } c_1 + c_2 > \overline{u}+1. \end{cases}$$

Proposition 2 tells us that the two price case divides the plane (c_1, c_2) into four different regions. We will name those regions after the first letters of the alphabet, to differentiate from the letters used in the single price case.

If $c_1 < \overline{u}$ then firm behavior is effectively different from the single price case. In this case, the firm knows that passenger travel is a profitable endeavor by itself and can set p_1 to the value that maximizes profits in that activity, namely $\overline{u}$. Then, the firm evaluates whether of not it wants to price baggage travel in a way that attracts consumers. If $c_2 \leq 1$ it finds profitable to do so, and sets $p_2 = \frac{1+c_2}{2}$. In that case, it earns profits of $\Pi_2(\overline{u}, \frac{1+c_2}{2}) = (\overline{u} - c_1) + [\frac{(1-c_2)}{2}]^2$, a fraction $1 - \frac{(1-c_2)}{2}$ of passengers travel light and $\frac{1-c_2}{2}$ of passenger travel heavy. No passenger opts not to travel. We name this region as **A**. On the other hand, if $c_2 > 1$ then it is more profitable to shut down the baggage market. In that case, firm profit is given by $\Pi_2(\overline{u}, \infty) = (\overline{u} - c_1)$ and all consumers travel light. We name this region **B**. Note that in the single price case the firm could not act in this way because it lacked capacity to separate its consumers in the two markets (i.e. it was impossible for the firm to force a consumer to travel light in that environment).

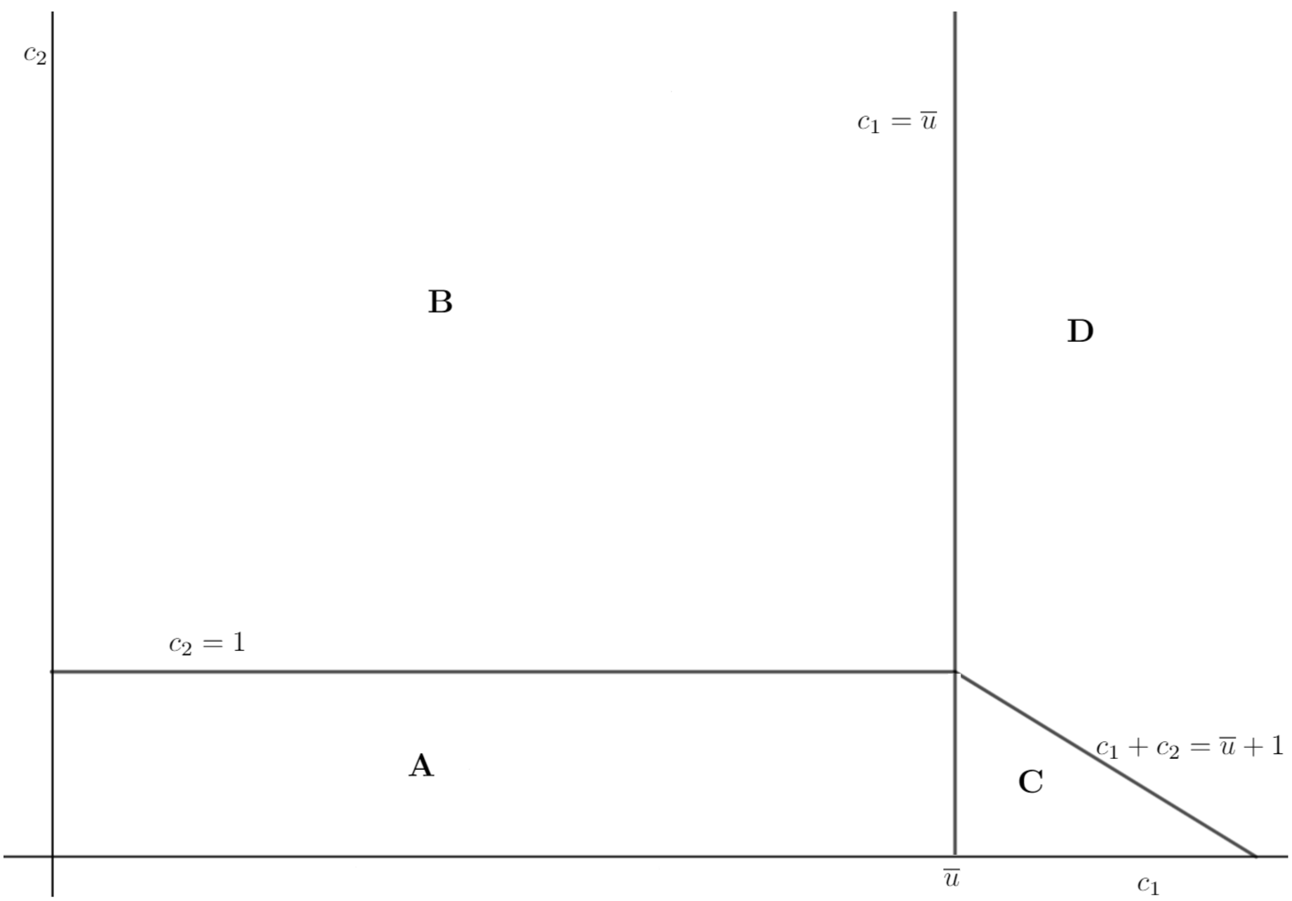
If $c_1 \geq \overline{u}$ the firm's choices in the two price case are effectively the same as in the single price case. In this case, passenger travel is never profitable enough to be sold on its own, so the firm is only interested in providing the consumption of baggage travel. As the model forces consumers to buy q_1 in order to buy q_2, the firm must offer q_1 at a loss to some consumers in order to earn profits from the sale of q_2. This strips away from the firm the possibility to differentiate light travellers from heavy travellers, making all its consumers heavy travellers, just as in the single price case. The solution is then equal to that case, that is, firms differentiate between consumers if $c_1 + c_2 \leq \overline{u} + 1$ and shot down the market if $c_1 + c_2 \geq \overline{u} + 1$. We name the first region as **C** and the second as **D**. Both profits and passenger behavior are obviously equal to the respective single price case in those scenarios.

Table 2 summarizes our findings for the two price case and figure 2 exemplify the regions for the case $\overline{u} = 3$.

Table 2: Two Price Setting Resume

	A	**B**	**C**	**D**
p_1^*, p_2^*	$(\overline{u}, (1+c_2)/2)$	$(\overline{u}, \infty)$	$((1+\overline{u}+c_1+c_2)/2, 0)$	(∞, ∞)
Π_2	$(\overline{u}-c_1)+[(1-c_2)/2]^2$	$\overline{u}-c_1$	$[(1+\overline{u}-c_1-c_2)/2]^2$	0
$\Pr(\lambda_i = 1)$	$1-(1-c_2)/2$	1	0	0
$\Pr(\lambda_i = 2)$	$(1-c_2)/2$	0	$(1+\overline{u}-c_1-c_2)/2$	0

Figure 2: Optimal Choice for the two distinct prices scenario with $\overline{u} = 3$



2.5 Efficiency and Welfare Analysis

Both scenarios are inefficient in different contexts. The single price case suffers from over-consumption issues, that is, consumers with private benefit for luggage travel inferior to its marginal costs (i.e: $\delta_i < c_2$) might still opt to consume as the firm cannot prevent the consumption of q_2 without also dampening consumption of q_1 with only a single price for both goods[6]. This is obviously not an issue in the two-price analysis. However the two price scenario has a different source of inefficiency. With two prices, the firm can better separate its consumers between those that are profitable to serve in both goods and those that are not. By doing this, the effective markup of the firm increases, and in doing so we also increase its monopoly inefficiency. Thus, the two price case suffers from under-consumption issues, that is, consumers with private benefit for luggage travel superior to its marginal costs might be priced away from the market due to the monopoly power of the firm. This leads to a natural question: which scenario is the best for society in each region of the plane (c_1, c_2)? Naturally, the firm should be at least indifferent when moving from a single price to dual price scenario as it can always set $p_2 = 0$ optimally if needed, but what about consumers and overall welfare? This is the question we tackle in this section.

To begin our analysis, let us define our aggregate welfare function and a bit of notation. Let $k \in \{1, 2\}$ denote the different price setting that we are dealing, with $k = 1$ the single price case and $k = 2$ for the two price case. Let $r \in \{\mathbf{A}, \mathbf{B}, \mathbf{C}, \mathbf{D}\} \cap \{\mathbf{L}, \mathbf{M}, \mathbf{H}\} := R_2 \cap R_1$ denote the region on the plane (c_1, c_2) that we are analyzing. Finally, let U_{kri} denote consumer i utility in region r of case k.

Using the previous section results and a bit of algebra, we have that aggregate utility in each case and region are given by:

$$\int_0^1 U_{1ri} \mathrm{d}i = \begin{cases} \dfrac{1}{2} & \text{if } r = \mathbf{L}, \\ \dfrac{(1 + \overline{u} - c_1 - c_2)^2}{8} & \text{if } r = \mathbf{M}, \\ 0 & \text{if } r = \mathbf{H}. \end{cases} \tag{7}$$

$$\int_0^1 U_{2ri} \mathrm{d}i = \begin{cases} \dfrac{(1 - c_2)^2}{8} & \text{if } r = \mathbf{A}, \\ 0 & \text{if } r = \mathbf{B}, \\ \dfrac{(1 + \overline{u} - c_1 - c_2)^2}{8} & \text{if } r = \mathbf{C}, \\ 0 & \text{if } r = \mathbf{D}, \end{cases} \tag{8}$$

Let Π_{kr} denote aggregate profits for region r and case k. As we have no reason to differentiate consumers, we define the aggregate welfare function is a natural way as

[6] It is easy to see that this happens to at least some consumers in regions $\mathbf{L}$ and $\mathbf{M}$, as the choice to consume q_2 in this case does not depend solely on p_2 and δ_i.

$$W_{kr} := \Pi_{kr} + \int_0^1 U_{kri} \mathrm{d}i. \tag{9}$$

Define the correspondence $g : R_1 \to R_2$ as $g(\mathbf{L}) \mapsto \{\mathbf{A}, \mathbf{B}\}$, $g(\mathbf{M}) \mapsto \{\mathbf{A}, \mathbf{B}, \mathbf{C}\}$ and $g(\mathbf{H}) \mapsto \{\mathbf{B}, \mathbf{D}\}$. We interpret g as the correspondence that connects each region in the single price case to a possible counterpart in the two price case.[7] Now, define the map $f : x \to g(x)$. Using this notation, we are interested in the values of $\Delta W_{r,f(r)} := W_{2,f(r)} - W_{1,r}$ and $\Delta U_{r,f(r)} := \int_0^1 U_{2f(r)i} - U_{1ri} \mathrm{d}i$ that we interpret as variations in, respectively, aggregate welfare and aggregate consumer utility from moving to the dual price case from the single price case.

The following proposition, which is our main result, shows that the change from a single price to two prices is not always good for aggregate welfare as defined above. It also shows that the consumers are never better off with this change.

Proposition 3 (Welfare Analysis is Uncertain) *If $r = \mathbf{L}$, then $\Delta W_{r,f(r)} < 0$ if and only if $c_2 < 1/3$. If $r = \mathbf{M}$ then $\Delta W_{r,f(r)} < 0$ if and only if $c_2 < (\overline{u} - c_1)/2 - 1/3$. Finally, if $r = \mathbf{H}$ then $\Delta W_{r,f(r)} \geq 0$. Furthermore $\Delta U_{r,f(r)} \leq 0$ in all cases.*

As regions **L** and **M** are the ones in which we have an active market in the single price case, a corollary of Proposition 3 is that whenever we have an active market in the single price case such that $c_1 < \overline{u} - 2/3$,[8] then there is a positive value c_2^* such that, if $c_2 \leq c_2^*$, then the change of a single price to a dual price case is negative to overall welfare. This is the case when the inefficiencies generated by the under-consumption of baggages in the dual price case are larger than the ones generated by the over-consumption of the single price case. Therefore, the value of c_2 is an important variable for a social planner to consider when promoting this change.

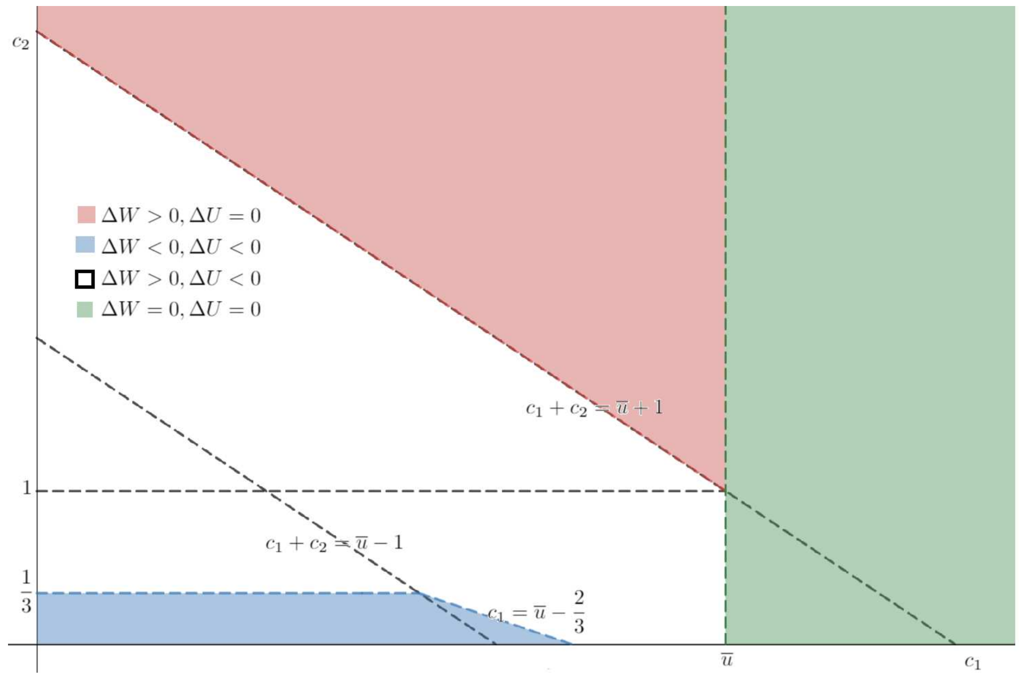
Proposition 3 also shows that consumers are never better off by the change from the single to the dual price case. There is however a region in which the move from a single to two prices in our model generates a Pareto-improvement. This is the region $\mathbf{H} \cap \mathbf{B}$, as in this region we would have no market in the single price case and a full (and profitable) market for passengers in the dual price case. Consumers are not better off with this change because we simplified away the demand for passengers using $\overline{u}$ to determine the utility of passenger travel, so our monopolist firm is able to fully extract consumer surplus in this case. This suggests that if we relax either the assumption of the single value of $\overline{u}$ as the utility for all consumers of the monopoly of the firm then it would be possible for consumers to also be better off by the change in airline pricing depending on the parameters. This is indeed the case, as will be shown in the next section.

The chart below summarizes our conclusions. The blue area denotes the region in which aggregate

[7] For example, when we are in region **H** of the single price case, we can be in two different regions of the two price case, region **B** if $c_1 \leq \overline{u}$ or region **D** otherwise, thus $g(\mathbf{H}) \mapsto \{\mathbf{B}, \mathbf{D}\}$

[8] To understand this inequality note that the equation $c_1 = \overline{u} - 2/3$ determines the point in which the value of c_2 that generates $\Delta W_{r,f(r)} < 0$ when $r = \mathbf{M}$ is lower than 0

Figure 3: Welfare and utility comparison in all regions



welfare is reduced when we move from a single to the dual price scenario, that is, the region in which the under-consumption problem of the dual price case is more relevant than the over-consumption of the single price case. In the white region this change is positive for aggregate welfare, but negative to consumers. In red we have the Pareto-improvement region and finally in green the region where the change provokes no economic effect.

3 Extensions

In this section we attempt to briefly extent our main model in two ways. In the first we solve the model under a price competition duopoly to understand the effect of market power in the analysis. Since we are considering a Bertrand duopoly, we can use these results as an approximation of a competitive equilibrium. In the second we differentiate consumer preference over passenger travel into two types, in order to clarify the allocative issues raised by the model in the previous analysis. The results shows that market power is crucial in the analysis, as in a perfectly competitive environment the two price case always generates at least the same aggregate welfare as the single price case. Moreover, consumers might benefit from this change when we allow for different valuations of the passenger travel, as in this case the firm will not be able to fully extract the surplus generate by the change.

3.1 Duopoly

In this section we assume that two firms exists in our previous model. The firms have equal costs and compete through prices to provide both goods. That simple change have profound effects in our model results. By allowing the presence of an extra firm, our model turns into a two part tariff competition in a homogeneous market duopoly, which is thoroughly analyzed by Krina and Nikolaos (2015). In this context we can prove the following proposition.

Proposition 4 (Bertrand Duopoly Characterization) *Assume that two firms compete though prices into q_1 and q_2 with the same costs, given by c_1 and c_2. Then the single price case is characterized by $p_1^1 = p_1^2 = (c_1 + c_2)$ and the dual price case by $(p_1^1, p_2^1) = (p_1^2, p_2^2) = (c_1, c_2)$ where p_i^j denotes the optimal price for firm j in good i. Moreover, both firms make 0 profits in both cases.*

Proof. In the single price case we have all the elements of the standard Bertrand model, which is well known to have its equilibrium characterized by the price being equal the marginal cost and profits begin null. As in this case all consumers that opt to purchase q_1 will also trivially want to purchase q_2 the result follows. In the dual price case the proof is a simple application of the first proposition of Krina and Nikolaos (2015) .

Using Proposition 4 it is straightforward to conclude that in the symmetric duopoly case both $\Delta W_{r,f(r)}$ and $\Delta U_{r,f(r)}$ are strictly positive whenever $\overline{u} > c_1$ and $c_2 > 0$ and neutral otherwise for all possible r. The intuition is that in this case we have no under-consumption issues, as the firms have no markup. As the problem of over-consumption in the single price case is still intact, the change from one to two distinct prices is at least neutral to overall welfare and consumer utility. Therefore, we conclude that a measure of market power is critical for the result of Proposition 3.

3.2 Heterogeneous preferences over passenger travel

In this section we relax the assumption that every consumers gets the same utility value $\overline{u}$ for the purchase of q_1. Specifically, we assume that consumers are now of two distinct types: L and H. While type L consumers gets utility u_L for the purchase of q_1, type H consumers get u_H, with $0 < u_L < u_H$. Moreover, we assume that the probability of consumer i being of type L is given by γ and that u_i is independent of δ_i. This effectively turns our model into a multidimensional screening one, as consumers have now two dimensions of private information. Those problems are notoriously hard to solve and have few conclusive results (see Armstrong and Rochet (1999) for a good review). As such, we do not attempt to fully present the solution, but rather to show its properties. The following proposition categorize the firm's optimal behavioral in the single price world.

Proposition 5 (Single-Price Heterogeneous Consumers Characterization) *Assume that the utility of passenger travel has two possible values u_L and u_H, with $0 < u_L < u_H$, $\Pr[u_i = u_L] = \gamma$ and $u_i \delta_i$.*

Then, we can reduce the firms price choice p_1^* of the single price case to one of the options below:

1) $p_1^* = u_L$, *with* $\Pi_1(p_1^*) = u_L - c_1 - c_2$.
2) $p_1^* = \dfrac{1+\gamma(u_L+c_1+c_2)}{2\gamma}$, *with* $\Pi_1(p_1^*) = \dfrac{1}{\gamma}\left(\dfrac{1+\gamma(u_L-c_1-c_2)}{2}\right)^2$. *This option is only available under the restrictions* $p_1^* \leq u_H$ *and* $u_L < p_1^* < 1+u_L$.
3) $p_1^* = \dfrac{1+\alpha_1+c_1+c_2}{2}$, *with* $\Pi_1(p_1^*) = \left(\dfrac{1+\alpha_1-c_1-c_2}{2}\right)^2$ *and* $\alpha_1 := \gamma u_L + (1-\gamma)u_H$. *This option is only available under the restrictions* $u_H < p_1^* < u_H + 1$ *and* $u_L < p_1^* < 1+u_L$.
4) $p_1^* = u_H$, *with* $\Pi_1(p_1^*) = (1-\gamma)(u_H - c_1 - c_2)$. *This option is only available under the restriction* $p_1^* \geq 1+u_L$.
5) $p_1^* = \dfrac{1+u_H+c_1+c_2}{2}$, *with* $\Pi_1(p_1^*) = (1-\gamma)\left(\dfrac{1+u_H-c_1-c_2}{2}\right)^2$. *This option is only available under the restrictions* $u_H < p_1^* < 1+u_H$ *and* $p_1^* \geq 1+u_L$.
6) $p_1^* = \infty$, *with* $\Pi_1(p_1^*) = 0$.

Proposition 5 tells us that firms optimal policy in the single price case now depends on 4 parameters: $(c_1+c_2), \gamma, u_L$ and u_H. As this case requires $p_2 = 0$, the firm can still only differentiate consumers that will choose not to travel from those that will choose to travel heavy. The only difference is that as we have two types of consumers now, the firm has more effective options in order to do this split. If option 1 is found to be the best, then the firm will effectively allow for all consumers to travel heavy. Under option 2, the firm will segregate between type L consumers, making that only those with high enough values of δ_i will decide travel heavy, while allowing for all type H to be inside the market. Under option 3 the firm segregates among the two consumers types. Option 4 is the case when type L are excluded from the market and all type H will opt to travel heavy, while option 5 is the case when type L are excluded and type H are segregated. Finally, option 6 is the case when the firm wants to shut down the market. We now need to do the same analysis for the two price case. This is done in the proposition below.

Proposition 6 (Two-Price Heterogeneous Consumers Characterization) *Assume that the utility of passenger travel has two possible values u_L and u_H, with $0 < u_L < u_H$, $\Pr[u_i = u_L] = \gamma$ and $u_i\delta_i$. Then, we can reduce the firms price choice (p_1^*, p_2^*) of the two price case to one of the options below:*

1) $(p_1^*, p_2^*) = \left(u_L, \dfrac{1+c_2}{2}\right)$, *with* $\Pi_1(p_1^*, p_2^*) = (u_L - c_1) + \left(\dfrac{1-c_2}{2}\right)$
2) $(p_1^*, p_2^*) = (u_L, \infty)$, *with* $\Pi_1(p_1^*, p_2^*) = (u_L - c_1)$
3) $(p_1^*, p_2^*) = \left(\dfrac{(1-\gamma)-\gamma^2(u_L+c_1)-\gamma(u_L-c_1)}{2\gamma(1-\gamma)}, \dfrac{c_2}{2} + \dfrac{\gamma}{1-\gamma}u_L\right)$, *with*
$\Pi_1(p_1^*, p_2^*) = \dfrac{1}{4}\left[\left(\dfrac{1}{\gamma} - \dfrac{\gamma+1}{1-\gamma}u_L - c_1\right)(1+\gamma\alpha_2) + \left(\dfrac{2\gamma}{1-\gamma}u_L - c_2\right)(1 - c_2 + \gamma(u_L - c_1))\right]$ *and* $\alpha_2 := (3u_L - c_1 - c_2)$. *This option is only available under the restrictions* $u_L < p_1^* < u_H$ *and* $u_L < p_1^* + p_2^* <$

$1 + u_L$.

4) $(p_1^*, p_2^*) = \left(u_H, \dfrac{1+c_2}{2}\right)$, *with* $\Pi_1(p_1^*, p_2^*) = (1-\gamma)\left[(u_H - c_1) + \left(\dfrac{1-c_2}{2}^2\right)\right]$. *This option is only available under the restrictions* $u_L < p_1^* \leq u_H$ *and* $p_1^* + p_2^* \geq 1 + u_L$.

5) $(p_1^*, p_2^*) = (u_H, \infty)$, *with* $\Pi_1(p_1^*, p_2^*) = (1-\gamma)(u_H - c_1)$

6) $(p_1^*, p_2^*) = (p_1^S, 0)$, *where* p_1^S *is the optimal price of single price case as defined in proposition 5*

Proposition 6 tells us that firms optimal policy in the dual price case now depends on 5 parameters: c_1, c_2, γ, u_L and u_H. Now the firm can differentiate both types of consumers into the full three options. If options 1 or 2 is found to be the best, then the firm will effectively allow for all consumers to at least travel light, the difference among the options being if the market for baggage travel will be open (option 1) or not (option 2). This is analogous to what our firm did in regions **A** and **B** in the dual price case of our main model but now the firm is not fully extracting consumer surplus on the passenger travel market of the H type consumers. Under option 3 the firm segregates among the two consumers types, but type L consumers will decide whether to travel heavy or not to travel, and type H will decide whether to travel light or travel heavy[9]. Note that, as $p_1^* < u_H$ we once again have the result that the firm is not able to perfectly extract consumer surplus of the passenger travel market in this case. Under option 4 and 5 the firm will opt to only serve type H consumers. It will either allow for baggage travel (option 4) or not (option 5) depending on c_2. In both cases it will be able to fully extract consumer surplus as no type L is present. Finally, under option 6 the firm behave as it did on the single price case, which will happen either if costs are too high (and the firm prefers to set $p_1^* > u_H$ effectively behaving just like our firm did in regions **C** and **D** in the dual price case of our main model) or if the firm wants to segregate type L consumers among those that will travel heavy and not to travel using only p_1^* (that is, setting $u_L < p_1^* \leq u_H$) but allowing for all type H passengers to travel heavy (that is, setting $p_2^* = 0$).

The analysis of Propositions 5 and 6 suggests that the results of our main model about efficiency are still largely valid in this environment, as the issues of over and under consumption of baggage are unaltered. However, the allocative properties must be different now, as in some cases of proposition 6 the firm is not able to fully extract consumer surplus on the passenger market, which it always did in our main model. The propositions below confirms this intuition:

Proposition 7 (For high values of c_2 welfare change is positive) *Let* $U_i(c_2) := \int_0^1 U_{ij}(c_2)dj$ *denote aggregate utility understood as a function of* c_2, *with* $i = 1$ *denoting the single price case and* $i = 2$ *the dual price case. Let* $\Pi_i(c_2)$ *define optimal profits as a function of* c_2 *for each case, which is given as the maximal option in the list provided by propositions 5 and 6. Furthermore, define* $\Delta W(c_2) := \Delta\Pi(c_2) + \Delta U(c_2)$ *where* $\Delta\Pi(c_2) := \Pi_2(c_2) - \Pi_1(c_2)$ *and* $\Delta U(c_2) := U_2(c_2) - U_1(c_2)$. *Fix* c_1, γ, u_L, u_H. *Then, there exists* $\bar{c_2}(c_1, \gamma, u_L, u_H) \in R$ *such that* $\forall c_2 \geq \bar{c_2} \Rightarrow \Delta W(c_2) \geq 0$.

[9] As $u_L < p_1^* \leq u_H$ the option to travel light is never better than the option not to travel for type L consumers and always at least as good as this option for type H

Proposition 8 (It is possible for welfare to be negative) *Let U_i and Π_i defined as in proposition 7. Then there exists values of $(c_1, c_2, \gamma, u_L, u_H)$ such that $\Delta W > 0$ and $\Delta U > 0$. There also exists a different set of parameters that generates $\Delta W < 0$ and $\Delta U < 0$.*

Proposition 7 ensures us that for sufficiently high values of c_2 the change from the single to the dual price case is at least neutral for overall welfare. The last part of proposition 8 then guarantees that for small values of c_2 this change is not always beneficial to society, just like we had in our main model. The novelty is that the first part of proposition 7 ensures us that for some parameters, this change promotes gains not only for the firms, but also to aggregate consumer utility.

4 Conclusion

This paper models an airline company in two different scenarios. It it either restricted to a single price for the provision of two goods - passenger travel and baggage travel - or it might charge two different prices. We concluded that a change from the single price to dual price case leads to uncertain effects in overall welfare, which is a new result in the literature. Moreover, our model sheds light on the relevant factor for this analysis, that is, the magnitude of the marginal cost of baggage travel and the market power of the firm. The intuition for these results is that in markets with high market power (i.e: a monopoly) and low marginal cost of baggage travel, allowing for two distinct prices generates a problem of under-consumption of baggages due to the markup of the firms. Depending on the parameters, this problem might be higher than the more commonly thought problem of over-consumption of baggages generated by the single price scenario. In terms of allocative issues, we showed that the change from a single price to two distinct prices is always at least neutral for the firms and uncertain for the consumers, with the last result begin dependent on parameters and the capacity of the firm to extract consumer surplus.

Although specifically tailored to the airline sector, our model can be readily applied to other problems in which firms have two products and can restrict the consumption of one of them until the purchase of the other. A good example of such case is the mobile phone market. In this market, it is usual for the firm to have two distinct prices for its telephone and internet services, and consumers can usually only get internet access after purchasing a telephone plan. This paper' analysis suggests that this might not necessarily be optimal for overall welfare, so regulation might be desirable.

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5 Appendix

Proof of Proposition 1. Using equation 1 and the uniform $[0,1]$ assumption on δ_i the above problem can be rewritten as

$$\text{Max } \Pi_1(p) = \begin{cases} (p_1 - c_1 - c_2)(1 - p + \overline{u}) & \text{if } p_1 \in [\overline{u}, \overline{u}+1] \\ (p_1 - c_1 - c_2) & \text{if } p_1 \leq \overline{u} \\ 0 & \text{if } p_1 \geq \overline{u}+1 \end{cases} \tag{10}$$

Let us first deal with the scenario where the firms choose $p_1 \in [\overline{u}, \overline{u}+1]$. We will first solve for the unrestricted problem and then look for boundary issues. As the objective function is concave, the first order condition for the unrestricted problem completely determines the interior optimal. This is given by:

$$p_1^* = \frac{1 + \overline{u} + c_1 + c_2}{2} \tag{11}$$

Let us now look for conditions for the optimal to this problem to be indeed interior. For that we need $p_1^* \in [\overline{u}, \overline{u}+1]$. This is equivalent to $\overline{u} - 1 \leq c_1 + c_2 \leq \overline{u} + 1$. If the lower bound on this inequality is not met, the firm would have to settle for a corner solution with the lowest possible price[10]. That is, in this case firms settle $p_1^* = \overline{u}$. Notice that this scenario is exactly the (trivial) solution for the case when the firms are restricted to choose price on the range $p_1 \leq \overline{u}$, so we do not need to bother with that case. If the upper bound is not met, the opposite happens and firms settle the maximum possible price at $p_1^* = \overline{u} + 1$. Once again, this (one of) the solution(s) to the case when firms are restricted to choose prices on the range $p_1 \geq \overline{u} + 1$, so we do not need to bother with that case as well. We pick ∞ to represent the optimal price in this case just to clarify that the firm will opt to shut down the market in that scenario. ■

Proof of Proposition 2. We will split the problem in two cases in order to facilitate solution. In the first, firms will solve the problem under the restriction $p_1 \leq \overline{u}$. In the second, they solve under $p_1 > \overline{u}$. We then compare profits to see which case is the relevant firm choice for each pair (c_1, c_2). We start by proving the following two claims:

Claim 1: The second case is equivalent to the single price scenario.

Proof: Assume first firms choose $p_1 \geq \overline{u}$. Then, equation 2 collapses to $q_{1i}(p_1, p_2) = q_{2i}(p_1, p_2) = 1_{\{\overline{u}+\delta_1 \geq p_1 + p_2\}}$. Thus, we can rewrite equation 6 as:

$$\begin{cases} (p_1 - c_1 + p_2 - c_2)(1 - p_2 - p_1 + \overline{u}) & \text{if } \overline{u} \leq p_1 + p_2 \leq 1 + \overline{u} \\ 0 & \text{if } p_1 + p_2 \geq 1 + \overline{u} \\ (p_1 - c_1 + p_2 - c_2) & \text{if } p_1 + p_2 \leq \overline{u} \end{cases} \tag{12}$$

We first solve for the interior solution, then check borders. The first order conditions for the unrestricted problem is the same for both p_1 and p_2 and equals

$$1 - 2p_2^* - 2p_1^* + \overline{u} + c_1 + c_2 = 0 \tag{13}$$

Equation 13 defines p_1^* implicitly as a function of p_2^*. Note that $p_1^*(0) = \frac{1+\overline{u}+c_1+c_2}{2}$. This is the same as in the interior solution of the single price case. Note that, for the optimal solution for this problem to

[10] As the objective function is a negative square function of price

be indeed interior, we need $\overline{u} \leq p_2^* + p_1^* \leq 1 + \overline{u}$. This is equivalent to $\overline{u} - 1 \leq c_1 + c_2 \leq \overline{u} + 1$, which is precisely the same restriction that defines region $\mathbf{Y}$ in the single price problem. We can check trivially that the border solutions are also the same. Thus we conclude that when firms set $p_1 \geq \overline{u}$ the solution is equivalent to the case with a single price price.

Claim 2: The first case solution is characterized by

$$(p_1^*, p_2^*) = \begin{cases} \left(\overline{u}, \dfrac{1+c_2}{2}\right) \text{ if } c_1 \leq \overline{u} \text{ and } c_2 \leq 1 \\ (\overline{u}, \infty) \text{ if } c_1 \leq \overline{u} \text{ and } c_2 > 1 \end{cases}$$

Its profits are given by

$$\Pi_2(p_1^*, p_2^*) = \begin{cases} \overline{u} - c_1 + \left(\dfrac{1-c_2}{2}\right)^2 \text{ if } c_1 \leq \overline{u} \text{ and } c_2 \leq 1 \\ \overline{u} - c_1 \text{ if } c_1 \leq \overline{u} \text{ and } c_2 > 1 \end{cases}$$

Proof: Using equation 2, this case problem can be simplified to:

$$\Pi_2(p_1, p_2) = \begin{cases} (p_1 - c_1) + [(p_2 - c_2)(1 - p_2)] & \text{if } 0 \leq p_2 \leq 1 \\ p_1 - c_1 & \text{if } p_2 > 1 \\ p_1 - c_1 + p_2 - c_2 & \text{if } p_2 < 0 \end{cases} \tag{14}$$

Once again, we solve for interior solution and then deal with boundary issues. Let μ_1 and μ_2 denote, respectively, the multiplier associated with the restriction $p_1 \leq \overline{u}$ and the restriction $0 \leq p_1$. The Kuhn-Tucker conditions for this problem are given by:

$$1 - \mu_1 + \mu_2 = 0 \tag{15a}$$
$$(1 - p_2) + (p_2 - c_2)(-1) = 0 \tag{15b}$$
$$\mu_1(p_1 - \overline{u}) = 0 \tag{15c}$$
$$\mu_2(-p_1) = 0 \tag{15d}$$
$$\tag{15e}$$

By 15a we get that $\mu > 0$. Hence we must have $p_1 = \overline{u}$ in this case. Condition 15b then imply that the optimal internal price is given by

$$p_2^* = \frac{1 + c_2}{2} \tag{16}$$

We need $0 \leq p_2^* \leq 1$ for an internal solution to be valid. This is equivalent to $-1 \leq c_2 \leq 1$. The lower bound on this inequality is true by assumption. The upper bound is equivalent to $c_2 \leq 1$. When the upper bound is not met, firms settle for a corner solution with the highest possible price. If that is the case, consumers would always travel light and profits equal

$$\Pi_2(\overline{u}, \infty) = (\overline{u} - c_1)$$

This is (one of) the solution(s) for the case when the firms are restricted to choose price on the range $p_2 > 1$ so we do not worry about this sub-case. When the firm is restricted to choose $p_2 \leq 0$ it will obviously set $(p_1^*, p_2^*) = (\overline{u}, 0)$. This provides profits of $\overline{u} - c_1 - c_2$, which is inferior to setting $p_2 > 1$, so this sub-case never happens.

To finish analysis, we need to compare profits in both cases. We claim that the firm prefer to set $p_1 \leq \overline{u}$ if $c_1 \leq \overline{u}$. To prove this statement we will split the plane (c_1, c_2) in six regions and then compare the difference in profits between the first and the second case.

Region 1: $c_1 + c_2 \leq \overline{u} - 1$ *and* $c_2 \leq 1$.
First note that in this region, we always have $\overline{u} \geq c_1$. Then if the firm opts to set $p_1 \geq \overline{u}$, it will get profits equivalent to those earned in region **X** of the single price case. However, if it choose to set $p_1 = \overline{u}$ it will get $\Pi_2(\overline{u}, p_2^*) = \overline{u} - c_1 + \left(\frac{1-c_2}{2}\right)^2$. The difference in profits between those choices is then given by: $\overline{u} - c_1 + \left(\frac{1-c_2}{2}\right)^2 - (\overline{u} - c_1 - c_2) = \left(\frac{1-c_2}{2}\right)^2 + c_2$. This is greater than 0 as $c_2 > 0$. Hence in this region the firm will always prefer to put itself into the first case. We proceed in similar fashion for the other regions of the (c_1, c_2) plane.

Region 2: $\overline{u} - 1 \leq c_1 + c_2 \leq \overline{u} + 1$ *and* $c_2 \leq 1$.
In this region, the difference between profits is given by:

$$\overline{u} - c_1 + \left(\frac{1 - c_2}{2}\right)^2 - \left(\frac{1 + \overline{u} - c_1 - c_2}{2}\right)^2 \tag{17}$$

Simple algebra shows that when $\overline{u} \geq c_1$, the condition for equation 17 to be positive is $2 + 2c_2 - (\overline{u} - c_1) \geq 0$. By assumption we know that $c_1 + c_2 \geq \overline{u} - 1$ and $c_2 \leq 1$. This implies $\overline{u} - c_1 < 1 + c_2 \leq 2 < 2 + c_2$. Thus this condition is always met implying that the firm will once again always prefer to put itself in the first case in this region when $\overline{u} \geq c_1$. On the other hand, when $\overline{u} \leq c_1$, the condition for equation 17 to be positive is $2 + 2c_2 - (\overline{u} - c_1) \leq 0$. This is obviously never met, so in this case the firm prefer to put itself in the second case.

Region 3: $c_1 + c_2 \geq \overline{u} + 1$ *and* $c_2 \leq 1$.
First note that in this case we always have $\overline{u} \leq c_1$. The difference in profits is given by $\overline{u} - c_1 + \left(\frac{1-c_2}{2}\right)^2$. As in this region we assumed $c_1 + c_2 \geq \overline{u} + 1$ we get that $\overline{u} - c_1 \leq c_2 - 1$. As $c_2 \leq 1$ we also have $c_2^2 < 1$. Thus:

$$\begin{aligned}(\overline{u} - c_1) + \left(\frac{1-c_2}{2}\right)^2 &\leq (c_2 - 1) + \left(\frac{1-c_2}{2}\right)^2 \\ &= \frac{c_2^2 - 1}{2} < 0\end{aligned}$$

Thus in this region the firm will prefer to be in the second case.

Region 4: $c_1 + c_2 \leq \overline{u} - 1$ *and* $c_2 \geq 1$.
In this case we always have $\overline{u} \geq c_1$. The difference in profits is given by $\overline{u} - c_1 - (\overline{u} - c_1 - c_2) = c_2 > 0$. Hence, in this region the firm prefer to be in the first case.

Region 5: $\overline{u} - 1 \leq c_1 + c_2 \leq \overline{u} + 1$ *and* $c_2 \geq 1$
In this case we always have $\overline{u} \geq c_1$. The difference in profits is given by

$$\overline{u} - c_1 - \left(\frac{1 + \overline{u} - c_1 - c_2}{2}\right)^2 \tag{18}$$

To see the signal of equation 18 assume first that $(\overline{u} - c_1) \geq 1/4$. As we know by assumption that $c_1 + c_2 \geq \overline{u} - 1$ we get $\overline{u} - c_1 \leq c_2 + 1$. Thus we have:

$$\begin{aligned}(\overline{u} - c_1) - \left(\frac{1 + \overline{u} - c_1 - c_2}{2}\right)^2 &\geq (\overline{u} - c_1) - \left(\frac{1 + 1 + c_2 - c_2}{2}\right)^2 \\ &= (\overline{u} - c_1) - \frac{1}{4} \geq 0\end{aligned}$$

Now assume $(\overline{u} - c_1) < 1/4$. As $c_2 > 1$ and $c_1 + c_2 \leq \overline{u} + 1$ we have $0 \leq \overline{u} - c_1$. Define $\alpha := \overline{u} - c_1 \in [0, 1/4[$. By $c_1 + c_2 \leq \overline{u} + 1$ we get that $c_2 \in]1, 1 + \alpha]$. We then have that:

$$\begin{aligned}(\overline{u} - c_1) - \left(\frac{1 + \overline{u} - c_1 - c_2}{2}\right)^2 &= \alpha - \left(\frac{1 + \alpha - c_2}{2}\right)^2 \\ &\geq \alpha - \frac{\alpha^2}{4} \geq 0\end{aligned}$$

Hence in this region firms will always prefer to put itself in the first case.

Region 6: $c_1 + c_2 \geq \overline{u} + 1$ *and* $c_2 \geq 1$.

The difference in profits is given by $\overline{u} - c_1$. This is obviously positive whenever $\overline{u} \geq c_1$.

Collecting the results of the six regions proves our last claim and finishes the proof of the proposition. ■

Proof of Proposition 3.

We split the proof into three cases:

Case 1: $r = \mathbf{X}$: If $r = \mathbf{L}$, then $f(r) \in \{\mathbf{A}, \mathbf{B}\}$. If $f(r) = \mathbf{A}$, we can use the previous results for profits and utility to conclude that $\Delta U_{r,f(r)} = \frac{(1-c_2)^2}{8} - \frac{1}{2}$ and $\Delta W_{r,f(r)} = \frac{3c_2^2+2c_2-1}{8}$. As the definition of $\mathbf{A}$ ensures that $c_2 \leq 1$ we get that $\Delta U_{r,f(r)} < 0$ in this case. Moreover, it is straightforward to show that $\Delta W_{r,f(r)} \geq 0 \iff c_2 \geq 0$. If $f(r) = \mathbf{B}$ we have that $\Delta U_{r,f(r)} = -\frac{1}{2}$ and $\Delta W_{r,f(r)} = c_2 - \frac{1}{2}$. Trivially then $\Delta U_{r,f(r)} < 0$. As the restrictions on region $\mathbf{L} \cap \mathbf{B}$ force $c_2 > 1$ we get that $\Delta W_{r,f(r)}$ is always positive in this region. Taking these observations together prove the first part of the proposition.

Case 2: $r = \mathbf{M}$: If $r = \mathbf{M}$ then $f(r) \in \{\mathbf{A}, \mathbf{B}, \mathbf{C}\}$. If $f(r) = \mathbf{A}$, we get that $\Delta U_{r,f(r)} = \frac{(1-c_2)^2}{8} - \frac{(1+\overline{u}-c_1-c_2)^2}{8}$. This is clearly non-positive, as both $(1 - c_2)$ and $(\overline{u} - c_1)$ are non negative due to the restrictions of the case. Moreover, it is straightforward to show that

$$\Delta W_{r,f(r)} = (\overline{u} - c_1)\frac{3(c_1 + c_2 - \overline{u} + 1) + 3c_2 - 1}{8} \tag{19}$$

To see the sign of equation 19, note that in region $\mathbf{A} \cap \mathbf{M}$ we get that $\overline{u} - c_1 \geq 0$. Thus, the change in welfare in this region is proportional to $3(c_1 + c_2 - \overline{u} + 1) + 3c_2 - 1$. Rewriting this term, we conclude that $\Delta W_{r,f(r)} \geq 0 \iff c_2 \geq \frac{\overline{u}-c_1}{2} - \frac{1}{3}$.

If $f(r) = \mathbf{B}$, we get that $\Delta W_{r,f(r)} = -\dfrac{(1 + \overline{u} - c_1 - c_2)^2}{8}$. As the definition of region $\mathbf{M}$ ensures that $c_1 + c_2 \leq \overline{u} + 1$ we get that $\Delta W_{r,f(r)} \leq 0$ in this case. Moreover, the previous results imply that

$$\Delta W_{r,f(r)} = (\overline{u} - c_1) - \frac{3}{8}(1 + \overline{u} - c_1 - c_2)^2 \tag{20}$$

The following claim guarantees that $\Delta W_{r,f(r)}$ is positive in this region:

Claim: For all $(c_1, c_2) \in \mathbf{M} \cap \mathbf{B}$ we have $\Delta W_{r,f(r)} \geq 0$

Proof: Assume first that $(\overline{u} - c_1) \geq \frac{3}{8}$. Pick any $x \in \mathbf{B} \cap \mathbf{M}$. As $x \in \mathbf{M}$, we know that $c_1 + c_2 \geq \overline{u} - 1$. This implies $\overline{u} - c_1 \leq c_2 + 1$. Thus we have:

$$\begin{aligned}(\overline{u} - c_1) - \frac{3}{8}(1 + \overline{u} - c_1 - c_2)^2 &\geq (\overline{u} - c_1) - \frac{3}{8}(1 + 1 + c_2 - c_2)^2 \\ &= (\overline{u} - c_1) - \frac{3}{8} \geq 0\end{aligned}$$

Now assume that $(\overline{u} - c_1) < \frac{3}{8}$. As $x \in \mathbf{B}$ we know that $c_2 > 1$. As $x \in \mathbf{M}$, we know that

$c_1 + c_2 \leq \overline{u} + 1$. Together, these conditions imply $0 < c_2 - 1 \leq \overline{u} - c_1$. By $c_1 + c_2 \leq \overline{u} + 1$ we get that $c_2 \in]1, 1 + \overline{u} - c_1]$. We then have that:

$$\begin{aligned}(\overline{u} - c_1) - \frac{3}{8}(1 + \overline{u} - c_1 - c_2)^2 &= \overline{u} - c_1 - \frac{3}{8}((\overline{u} - c_1) - (c_2 - 1))^2 \\ &> \overline{u} - c_1 - \frac{3(\overline{u} - c_1)^2}{8} \geq 0\end{aligned}$$

This finish the proof of the claim

Finally, if $f(r) = \mathbf{C}$ then trivially $\Delta U_{r,f(r)} = \Delta W_{r,f(r)} = 0$ as the single price and the dual price behavioral are the same in this case. Taking these observations together proves the second part of the proposition

Case 3: $r = \mathbf{Z}$: If $r = \mathbf{Z}$ then $f(r) \in \{\mathbf{B}, \mathbf{D}\}$. If $f(r) = \mathbf{B}$ then the previous results imply $\Delta U_{r,f(r)} = 0$. Moreover $\Delta W_{r,f(r)} = \overline{u} - c_1$ which is always positive by definition of region $\mathbf{B}$. If $f(r) = \mathbf{D}$ then trivially $\Delta U_{r,f(r)} = \Delta W_{r,f(r)} = 0$ as we have no market for both goods in all cases. This proves the last part of the proposition.

■

Proof of Proposition 5.

Let Φ_δ denotes the CDF of δ. As we are in the single price case the firm's problem can by written as Max $(p_1 - c_1 - c_2) \Pr[\delta_i + u_i \geq p_1]$. By $u_i \delta_i$, this is equivalent to Max $(p_1 - c_1 - c_2)[1 - (\Phi_\delta(p_1 - u_L)\gamma + \Phi_\delta(p_1 - u_H)(1 - \gamma))]$. As both $\Phi_\delta(p_1 - u_L)$ and $\Phi_\delta(p_1 - u_H)$ might assume value 0, 1 or an interior value we have 9 possible cases. We deal with each one in what follows:

Case 1: p_1 is chosen such that $\Phi_\delta(p_1 - u_L) = \Phi_\delta(p_1 - u_H) = 0$.

This case's restrictions are $p_1 \leq u_H$ and $p_1 \leq u_L$ which collapses to $p_1 \leq u_H$. Thus, the firm's problem can be rewritten as

$$\text{Max } p_1 - c_1 - c_2 \text{ s.t. } p_1 \leq u_L$$

The trivial solution is $p_1^* = u_L$, which earns profits of $\Pi_1(p_1^*) = u_L - c_1 - c_2$. As the firm may always choose to put its price under u_L, we have no parameter restriction for this case to happen.

Case 2: p_1 is chosen such that $\Phi_\delta(p_1 - u_L) \in]0, 1[$ and $\Phi_\delta(p_1 - u_H) = 0$.

This case's restrictions are $p_1 \leq u_H$ and $u_L < p_1 < 1 + u_L$. Thus, the firm's problem collapses to

$$\text{Max } (p_1 - c_1 - c_2)[1 - (p_1 - u_L)\gamma] \text{ s.t. } \begin{cases} p_1 \leq u_H \\ u_L < p_1 < 1 + u_L \end{cases}$$

The objective function is strictly concave, so the problem's first order conditions are sufficient in the interior of the restrictions. Those are given by

$$(p_1 - c_1 - c_2)(-\gamma) + (1 - p_1\gamma + u_L\gamma) = 0$$

Which implies

$$p_1^* = \frac{1 + \gamma(u_L + c_1 + c_2)}{2\gamma}$$

Profits are then given by

$$\Pi_1(p_1^*) = \frac{1}{\gamma}\left(\frac{1 + \gamma(u_L - c_1 - c_2)}{2}\right)^2$$

For the solution to be indeed interior we need to verify the problems restrictions.

Case 3: p_1 is chosen such that $\Phi_\delta(p_1 - u_L) \in]0, 1[$ and $\Phi_\delta(p_1 - u_H) \in]0, 1[$.

This case's restrictions are $u_H < p_1 < 1 + u_H$ and $u_L < p_1 < 1 + u_L$. Thus, the firm's problem collapses to

$$\text{Max } (p_1 - c_1 - c_2)[1 - (p_1 - u_L)\gamma] \text{ s.t. } \begin{cases} u_H < p_1 < 1 + u_H \\ u_L < p_1 < 1 + u_L \end{cases}$$

The objective function is strictly concave, so the problem's first order conditions are sufficient in the interior of the restrictions. Those are given by

$$(p_1 - c_1 - c_2) + [1 - \gamma(p_1 - u_L) - (1 - \gamma)(p_1 - u_H)] = 0$$

Which implies

$$p_1^* = \frac{1 + \gamma u_L + (1 - \gamma)u_H + c_1 + c_2}{2}$$

Profits are then given by

$$\Pi_1(p_1^*) = \left(\frac{1 + \gamma u_L + (1 - \gamma)u_H - c_1 - c_2}{2}\right)^2$$

For the solution to be indeed interior we need to verify the problems restrictions.

Case 4: p_1 is chosen such that $\Phi_\delta(p_1 - u_L) = 1$ and $\Phi_\delta(p_1 - u_H) \in]0, 1[$.

This case's restrictions are $u_H < p_1 < 1 + u_H$ and $p_1 \geq 1 + u_L$. Thus, the firm's problem collapses to

$$\text{Max } (p_1 - c_1 - c_2)[1 - (\gamma + (p_1 - u_H)(1 - \gamma)] \text{ s.t. } \begin{cases} u_H < p_1 < 1 + u_H \\ p_1 \geq 1 \end{cases}$$

The objective function is strictly concave, so the problem's first order conditions are sufficient in the interior of the restrictions. Those are given by

$$-(p_1 - c_1 - c_2)(1 - \gamma) + [1 - \gamma - (1 - \gamma)(p_1 - u_H)] = 0$$

Which implies

$$p_1^* = \frac{1 + u_H + c_1 + c_2}{2}$$

Profits are then given by

$$\Pi_1(p_1^*) = (1 - \gamma)\left(\frac{1 + u_H - c_1 - c_2}{2}\right)^2$$

For the solution to be indeed interior we need to verify the problems restrictions.

Case 5: p_1 is chosen such that $\Phi_\delta(p_1 - u_L) = 1$ and $\Phi_\delta(p_1 - u_H) = 0$.

This case's restrictions are $p_1 \leq u_H$ and $p_1 \geq 1 + u_L$. Thus, the firm's problem collapses to

$$\text{Max } (p_1 - c_1 - c_2)[1 - \gamma] \text{ s.t. } \begin{cases} p_1 \leq u_H \\ p_1 \geq 1 \end{cases}$$

The trivial solution is $p_1^* = u_H$, which earns profits of $\Pi_1(p_1^*) = (1 - \gamma)(u_H - c_1 - c_2)$. For this solution to satisfy the restrictions we need $u_H \geq 1 + u_L$.

Case 6: p_1 is chosen such that $\Phi_\delta(p_1 - u_L) = \Phi_\delta(p_1 - u_H) = 1$.

This case's restrictions are $p_1 \geq u_H + 1$ and $p_1 \geq 1 + u_L$, which collapses to $p_1 \geq 1 + u_H$. Thus, the firm's problem can be rewritten as

$$\text{Max } 0 \text{ s.t. } p_1 \geq u_H + 1$$

Any price that satisfy the restrictions is a valid solution. Profits are obviously given by $\Pi_1(p_1^*) = 0$. As the firm may always choose to put its price above $u_H + 1$, we have no parameter restriction for this case to happen.

Cases 7,8,9: p_1 is chosen such that $\begin{cases} \Phi_\delta(p_1 - u_L) = 0 \text{ and } \Phi_\delta(p_1 - u_H) \in \ ]0,1[\\ \Phi_\delta(p_1 - u_L) = 0 \text{ and } \Phi_\delta(p_1 - u_H) = 1 \\ \Phi_\delta(p_1 - u_L) \in \ ]0,1[\text{ and } \Phi_\delta(p_1 - u_H) = 1 \end{cases}$.

In the first two cases we have $\Phi_\delta(p_1 - u_L) = 0 \iff p_1 - u_L \leq 0 \to p_1 - u_H \leq 0 \to \Phi_\delta(p_1 - u_H) = 0$. In the third case we have $\Phi_\delta(p_1 - u_L) \in \ ]0,1[\iff 0 \leq p_1 - u_L \leq 1 \to p_1 - u_H \leq 1 \to \Phi_\delta(p_1 - u_H) < 1$. Hence the restrictions gives rise to a maximization in an empty set for all cases

Taking together the 9 cases observation the desired result follows from continuity of Φ_δ.

■

Proof of Proposition 6

During the proof of proposition 2 we stated that the dual price case turned effectively into the single price case whenever the optimal price p_1^* of the dual price case is higher than $\overline{u}$. We start this proof by showing that this result is indeed general, and can be applied in the two-type consumer case as well with only minor changes. This is shown in the lemma below.

Lemma 1 *Assume that u_i is a random variable with support in $[u_L, u_H]$. Let $\Phi_{u_i+\delta_i}$ denote the CDF of $u_i + \delta_i$. Then, if the optimal price for passenger travel p_1^* of the dual price case is such that $p_1^* > u_H$ we get that the dual price case problem is analogous to the single price one.*

Proof. We start with the single price analysis in this environment. Using equation 1 the single price case can be written as:

$$\text{Max } (p_1 - c_1 - c_2)(1 - \Phi_{u_i+\delta_i}(p_1))$$

The first order condition is given by

$$(p_1 - c_1 - c_2)(-\Phi_{u_i+\delta_i}(p_1)) + (1 - \Phi_{u_i+\delta_i}(p_1)) = 0$$

. We now move to the analysis of the dual price problem. using equation 2 and the fact that $p_1^* > u_H$ we can rewrite our maximization problem as:

$$\text{Max } (p_1 + p_2 - c_1 - c_2)(1 - \Phi_{u_i+\delta_i}(p_1 + p_2))$$

The first order condition for both prices gives us the same result, which is:

$$(p_1 + p_2 - c_1 - c_2)(-\Phi_{u_i+\delta_i}(p_1 + p_2)) + (1 - \Phi_{u_i+\delta_i}(p_1 + p_2)) = 0 \tag{21}$$

Equation 21 defines p_1^* implicitly as a function of p_2^*. Moreover, it collapses to the first order condition of the single price case when we set $p_2^* = 0$.

Lemma 1 allows us to focus on the cases where $p_1 \leq u_H$ to prove our desired result as long as we allow the firm the option to behave as it did in the single price case - which we did in option 6) of the proposition. To prove the remainder of the proposition, let us further divide our problem in two cases. In the first, the firms solve the problem under the additional restriction $p_1 \leq u_L$ and in the second they solve under $u_l < p_1 \leq u_H$. As in the previous proposition's proof, we assume Φ_δ denotes the CDF of δ_i

Case 1: Using equation 2 and $p_1 \leq u_L$ the firms problem collapses to

$$\text{Max } (p_1 - c_1) + (p_2 - c_2)(1 - \Phi_\delta(p_2))$$

This problem was already solved in claim 2 of the proof of proposition 2. The solution is given by

$$(p_1^*, p_2^*) = \begin{cases} \left(u_L, \dfrac{1+c_2}{2}\right) \text{ if } c_2 \leq 1 \\ (u_L,) \text{ if } c_2 > 1 \end{cases}$$

The profits associated are given by:

$$\Pi_2(p_1^*, p_2^*) = \begin{cases} (u_L - c_1) + \left(\dfrac{1-c_2}{2}\right)^2 \text{ if } c_2 \leq 1 \\ u_L - c_1 \text{ if } c_2 > 1 \end{cases}$$

Case 2: We now assume $u_l < p_1 \leq u_H$. Hence our problem becomes

$$\begin{aligned} \text{Max } & (p_1 - c_1)[1 - \gamma + \gamma(1 - \Phi_\delta(p_1 + p_2 - u_L))] + \\ & + (p_2 - c_2)[(1-\gamma)(1 - \Phi_\delta(p_2)) + \gamma(1 - \Phi_\delta(p_1 + p_2 - u_L))] \end{aligned}$$

We will further simplify this problem into the following sub-cases to facilitate solution:

Sub-Case 2.1: (p_1, p_2) are chosen such that $\Phi_\delta(p_2) \in]0,1[$ and $\Phi_\delta(p_1 + p_2 - u_L) \in]0,1[$.

This case's restrictions are $0 < p_2 < 1$, $u_L < p_1 + p_2 < 1 + u_L$ and $u_L < p_1 \leq u_H$. Thus, the firm's problem collapses to

$$\text{Max } (p_1 - c_1)[1 - \gamma(p_1 + p_2 - u_L)] + (p_2 - c_2)[1 - p_2 - \gamma(p_1 - u_L)] \text{ s.t. } \begin{cases} 0 < p_2 < 1 \\ u_L < p_1 + p_2 < u_L + 1 \\ u_L < p_1 \leq u_H \end{cases}$$

The objective function is strictly concave, so the problem's first order conditions are sufficient in the interior of the restrictions. Those are given by

$$\gamma(p_1 - c_1 + p_2 - c_2) = 1 - \gamma(p_1 + p_2 - u_L)$$
$$\gamma(p_1 - c_1) + p_2 - c_2 = 1 - p_2 - \gamma(p_1 - u_L)$$

Which implies

$$p_1^* = \frac{(1-\gamma) - \gamma^2(u_L + c_1) - \gamma(u_L - c_1)}{2\gamma(1-\gamma)} \qquad\qquad p_2^* = \frac{c_2}{2} + \frac{\gamma}{1-\gamma}u_L$$

Profits are then given by

$$\begin{aligned}\Pi_1(p_1^*, p_2^*) =& \frac{1}{4}\Bigg[\left(\frac{1}{\gamma} - \frac{\gamma+1}{1-\gamma}u_L - c_1\right)(1 + \gamma(3u_L - c_1 - c_2) \\ &+ \left(\frac{2\gamma}{1-\gamma}u_L - c_2\right)(1 - c_2 + \gamma(u_L - c_1))\Bigg]\end{aligned}$$

For the solution to be indeed interior we need to verify the problems restrictions However, the first restriction is already embedded in the last two, so we may as well ommit it.

Sub-Case 2.2: (p_1, p_2) are chosen such that $\Phi_\delta(p_2) \in]0,1[$ and $\Phi_\delta(p_1 + p_2 - u_L) = 1$.

This case's restrictions are $0 < p_2 < 1$, $p_1 + p_2 \geq 1 + u_L$ and $u_L < p_1 \leq u_H$. Thus, the firm's problem collapses to

$$\text{Max } (p_1 - c_1) + (p_2 - c_2)(1-\gamma)(1-p_2) \text{ s.t. } \begin{cases} 0 < p_2 < 1 \\ p_1 + p_2 \geq u_L + 1 \\ u_L < p_1 \leq u_H \end{cases}$$

The problem is separable into two distinct problem, the first begin a maximization with p_1 as the choice variable (first term of the sum) and the second begin a maximization with p_2 as the choice variable (second term). In the first problem the obvious solution is $p_1^* = u_H$. In the second, we have a concave problem, which first order condition generates $p_2^* = \frac{1+c_2}{2}$. Taking these together, profits are then given by:

Profits are then given by

$$\Pi_1(p_1^*, p_2^*) = (1-\gamma)\left[(u_H - c_1) + \left(\frac{1-c_2}{2}\right)^2\right]$$

For the restriction $p_1 + p_2 \geq 1 + u_L$ to be respected we need $u_H - u_L \geq p_2^* = \frac{1-c_2}{2}$.

Sub-Case 2.3: (p_1, p_2) are chosen such that $\Phi_\delta(p_2) = 1$ and $\Phi_\delta(p_1 + p_2 - u_L) = 1$.

This case's restrictions are $p_2 \geq 1$, $p_1 + p_2 \geq 1 + u_L$ and $u_L < p_1 \leq u_H$. Thus, the firm's problem collapses to Max $(p_1 - c_1)(1-\gamma)$ plus the restrictions. The obvious solution is to set $p_1^* = u_H$ and p_2^* to any value that is higher than 1 (to accommodate the restrictions). Profits are then given by $\Pi_2(p_1^*, p_2^*) = (u_H - c_1)(1-\gamma)$.

Any other Sub-Case:

In any other sub-case the restrictions either generates an empty set or the firms choice collapse to the single price problem. If $\Phi_\delta(p_2) = 0$, then the firm is restricted to set $p_2 \leq 0$. The obvious choice is then to set $p_2^* = 0$ as without the effect on demand, profit is a strictly positive function of the prices. As this is the forced choice of the single price case, any sub-case in which $\Phi_\delta(p_2) = 0$ makes the dual price problem to collapse to the single price one. If $\Phi_\delta(p_2) = 1$ and $\Phi_\delta(p_1 + p_2 - u_L) \in]0, 1[$ or if $\Phi_\delta(p_2) = 1$ and $\Phi_\delta(p_1 + p_2 - u_L) = 0$ we have empty sets as $p_2 \geq 1$ makes that $u_L < p_1 \to u_L + 1 < p_1 + 1 \leq p_1 + p_2 \to 1 < p_1 + p_2 - u_L \to \Phi_\delta(p_1 + p_2 - u_L) = 1$. Finally, if $\Phi_\delta(p_2) \in]0, 1[$ and $\Phi_\delta(p_1 + p_2 - u_L) = 0$ we have $u_L < p_1 \to 0 < p_1 + p_2 - u_L \to \Phi_\delta(p_1 + p_2 - u_L) > 0$, so this also generates an empty set.

Taking together all the cases analyzed the desired result follows from continuity of Φ_δ and Lemma 1.

■

Proof of Proposition 7.

Take $c_2 \to \infty$. Then the set of options for the firm in the single price case collapse to options 1), 4) and 6), as the restrictions on the other options will not be met[11]. The obvious choice is then option 6). For the dual price case, the options collapse to 2), 5) and 6), with option 6) being $\Pi_2(c_2) = 0$ for sure. These observations allows us to conclude that for a large enough value of c_2 we have $\Pi_1(c_2) = 0$ and $\Pi_2(c_2) = \text{Max } \{0, u_L - c_1, (u_H - c_1)(1 - \gamma)\}$. It is then straightforward to check that, for large values of c_2 we have $U_1(c_2) = 0$ and

$$U_2(c_2) = \begin{cases} (1-\gamma)(u_H - u_L) \text{ if } \Pi_2(c_2) = u_L - c_1 \\ 0 \text{ in any other case} \end{cases}$$

As $u_H > u_L$ we get the desired result. ■

Proof of Proposition 8.

The easiest way to prove this proposition is through counterexamples. Let $(c_1, c_2, \gamma, u_L, u_H) = (1; 0.25; 0.3; 1; 1.3)$. The optimal options for the firm in this case is to choose option 3) of both propositions 5 and 6. This leads to $\Pi_1 \approx 0.23$ and $\Pi_2 \approx 0.30$. Utilities is then $U_1 \approx 0.12$ and $U_2 \approx 0.84$. This yields $\Delta U \approx 0.71$ and $\Delta W =\approx 0.78$, which proves the first part of the proposition

Let $(c_1, c_2, \gamma, u_L, u_H) = (0.9; 0.6; 0.5; 2.5; 3)$. The optimal options for the firm in this case is to choose option 2) of proposition 5 and option 1 of proposition 6. This leads to $\Pi_1 = 1.125$ and $\Pi_2 = 1.64$. Utilities is then $U_1 = 1.25$ and $U_2 = 0.27$. This yields $\Delta U = -0.98$ and $\Delta W = -0.465$, which proves the last part of the proposition. ■

[11]Taking limits under the optimal price of options 2), 3) and 5) when $c_2 \to \infty$ gives us $p_1^* \to \infty$, which is not possible given the restrictions of the options