



# **Inflation dynamics in Tunisia: a smooth transition autoregressive approach**

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# **Inflation Dynamics in Tunisia**

## **a Smooth Transition Autoregressive Approach**

### **Abstract**

The fact that inflation is still on the rise, despite measures undertaken by the Tunisian central bank, prompts questions as to whether or not inflation dynamics has changed, exhibiting higher levels of persistence and volatility. This paper employs the smooth transition autoregressive model (STAR) to analyze Tunisian inflation dynamics on monthly data over the last three decades. We distinguish three periods based on monetary reforms. The non-linearity tests suggest that the ESTAR specification describes better the behavior of inflation. Our results suggest changes in persistence and important shifts in volatility, which confirm the effectiveness of the monetary reforms to a certain extent given the past political instability and the democratic transition in Tunisia.

**Keywords:** Inflation, persistence, volatility, Smooth Transition Autoregressive, Tunisia.

## 1. Introduction

Inflation's impact on the economy is somehow controversial; it depends on the inflation level, the phase of the business cycle in which the economy is and the central bank's actions. Generally speaking, inflation redistributes wealth in a distorted way; punishes some while favoring others. But one thing is certain inflation deteriorates the purchasing power which feeds social tension. The latter is an important fact for the Tunisian economy cradle of the Arab spring revolutions, since it still suffers from its main drivers; high unemployment rates and low real wages. In this context, understanding inflation's dynamics through the evolution of its persistence and volatility becomes interesting, especially in light of monetary reforms undertaken by the Tunisian's central bank. Until 2005 the central bank of Tunisia perused several objectives at once in a discretionary way and adopted price stability as its main goal only in 2006. But it didn't really pursue a proactive monetary policy aimed at anticipating inflation until 2011, with the establishment of a corridor to its main interest rate. In the same year political turbulence hit the country; the Tunisian economy experienced an inflation that kept rising to unprecedeted levels in the last two decades. But before we focus on our case let us briefly overview the economic literature.

Inflation tends to be persistent when the rate of change of price level shows a tendency to stay constant in the absence of shocks, Fuhrer (2010). It is generally accepted that less persistence is associated with less volatility (Cogley & Sargent (2002) and Amano (2007)). In the early eighties Gordon, King and Modigliani (1982) introduce the concept of sacrifice ratio, which means that lowering inflation requires losing output. But for Gordon (1982) inflation inherit persistence not only from real activity but also from its own past by including lags in an accelerationist Phillips curve. According to Cecchetti and Debelle (2006) and Mishkin (2007) the more inflation is persistent the more it is costly for the central bank to stabilize it. Economic literatures suggest that attaching primary importance to price stability can lead to a lower level of inflation persistence (Gerlach and Tillman (2012) and Walsh (2009)) among others. In this paper we use Smooth Transition Autoregressive approach (STAR), we test for nonlinearity to analyze the evolution of the Tunisian inflation's dynamics in terms of persistence and volatility. The Smooth Transition Autoregressive Models (STAR) was developed by Teräsvirta (1994). But if the term "smooth transition" was first introduced by Bacon and Watts (1971), explaining fluctuations with econometric nonlinear modeling goes back to Kaldor (1940).

No work have been done on inflation persistence using STAR models for the case of Tunisia to our knowledge, but an attempt to tackle the issue in a nonlinear context was made by Fiti and al (2015) using an evolutionary spectral approach. The authors found a stable inflation regime with a high degree of inertia since the adoption of price stability as an ultimate goal by the central bank. The lack of studies on inflation dynamics for Tunisia in a non-linear context, especially during the coexistence of important monetary reforms with unprecedented political turmoil, motivates us to fill this gap. Our contribution is an attempt to analyze the evolution of both volatility and persistence following each major monetary reform in the last three decades and their effectiveness in the presence of political turmoil and democratic transition. The paper is organized as follows. Section 2 presents the model. Section 3 details the results, while Section 4 contains the concluding remarks.

## 1. The model

Smooth transition models STR are state-dependent, nonlinear time series models, where the variable varies between two extreme regimes. In the case of smooth transition autoregressive model STAR, predetermined variables are lags of the dependent variable and regimes are endogenously determined.

$$\mathbf{y}_t = \sum_{j=0}^{m-1} \mathbf{1}_j(s_t; c, \gamma) \mathbf{Z}'_t \boldsymbol{\delta}_j + \mathbf{X}_t \boldsymbol{\alpha} + \boldsymbol{\epsilon}_t \quad (1)$$

$\mathbf{1}_j(\cdot)$  is a  $(0,1)$  regime indicator depending on the observed variable  $s_t$ ,  $c$  is one or more thresholds,  $\gamma > 0$  is the slope parameter of threshold,  $Z$  denotes the variables with varying coefficients across the  $m$  regimes and  $X$  are the variables with regime invariant coefficients. Restricting ourselves to  $m = 2$ , as using the fact that  $\mathbf{1}_j(\cdot) = 1$  for exactly one  $j$ , equation (1) can be rewritten as:

$$\begin{aligned} \mathbf{y}_t &= \mathbf{1}_0(s_t; c, \gamma) \mathbf{Z}'_t \boldsymbol{\delta}_0 + \mathbf{1}_1(s_t; c, \gamma) \mathbf{Z}'_t \boldsymbol{\delta}_1 + \mathbf{X}_t \boldsymbol{\alpha} + \boldsymbol{\epsilon}_t \\ &= (1 - \mathbf{1}_1(s_t; c, \gamma)) \mathbf{Z}'_t \boldsymbol{\delta}_0 + \mathbf{1}_1(s_t; c, \gamma) \mathbf{Z}'_t \boldsymbol{\delta}_1 + \mathbf{X}_t \boldsymbol{\alpha} + \boldsymbol{\epsilon}_t \end{aligned}$$

To construct the two-regime STAR model, the indicator function must be replaced with a continuous transition function  $G$  that returns values between 0 to 1. Then we have

$$y_t = (1 - G(s_t; c, \gamma))Z'_t\delta_0 + G(s_t; c, \gamma)Z'_t\delta_1 + X_t\alpha + \epsilon_t \quad (2)$$

where  $\mathbf{G}$  has different properties as  $s \rightarrow -\infty$ ,  $s \rightarrow +\infty$  and  $s = c$ , depending on the specific functional form. The key modeling choices in, are the choice of the threshold variable  $s$  and the selection of a transition function  $\mathbf{G}$ . For a given  $s$  and  $\mathbf{G}$ , we may estimate the regression parameters  $(\boldsymbol{\delta}_0, \boldsymbol{\delta}_1, \alpha)$  and the threshold values and slope  $(c, \gamma)$  with nonlinear least squares. Additionally, given a list of candidate variables for  $s$ , we can select a threshold variable using model selection techniques. Smooth transition autoregressive models was initially introduced by Bacon and Watts (1971) and later popularized by Teräsvirta (1994, 1998). Common transition function choices are given by: Logistic LSTAR, Normal NSTAR, Exponential ESTAR and Logistic, second-order L2STR. Choosing the right transition function is based on Teräsvirta (1994) linearity tests on the first-order Taylor approximation and will be the subject of the following section.

## 2. Empirical results

Data is retrieved from the international monetary fund and the Tunisian national institute of statistics. We start by a sample covering the period 1990M01 to 2020M03 for the Tunisian consumer price index monthly frequency, year-on-year evolution and test for unit root with the breakpoint unit root test following Perron (1989).

Table 1 Breakpoint unit root test

	Akaike information criterion				Hannan-Quinn information criterion			
	Intercept		Trend and intercept		Intercept		Trend and intercept	
	Intercept	Intercept	Trend intercept	Trend	Intercept	intercept	Trend intercept	Trend
Break								
91M07	91M07	91M03	92M01		91M07	91M07	91M03	92M01
Test stat								
-16.106	-16.221	-16.216	-15.980		-16.106	-16.221	-16.216	-15.980
P-value								
< 0.01	< 0.01	< 0.01	< 0.01		< 0.01	< 0.01	< 0.01	< 0.01
Critical values								
1%	-4.949	-5.348	-5.719	-5.067	-4.949	-5.348	-5.719	-5.067
5%	-4.444	-4.860	-5.176	-4.525	-4.444	-4.860	-5.176	-4.525
10%	-4.194	-4.607	-4.894	-4.261	-4.194	-4.607	-4.894	-4.261

The inflation series is stationary in first difference; in fact all the p-value < 0.01 with both Akaike and Hannan-Quinn information criterion, for more detail see Table A1 in the appendix. The Break Selection is based on the minimization of Dickey-Fuller t-statistic. Now we exclude the break dates shown in table 1, and determine the optimal lag length for the period 1993M4 to 2020M3.

Table 2 VAR Lag Order Selection Criteria

Lag	LogL	LR	FPE	AIC	SC	HQ
13	-71.51437	65.62272*	0.099266*	0.527866*	0.691232*	0.593073*

\* indicates lag order selected by the criterion

Table 2 indicates that 13 is the optimal lag which minimizes the log likelihood (LogL) following the sequential modified LR test statistic (LR), the final prediction error (FPE), the Akaike information criterion (AIC), the Schwarz information criterion (SC) and the Hannan-Quinn information criterion (HQ). Now we proceed by including these lags in our estimation.

Estimating STAR model requires determining the value of the constant and the delay parameters. According to Teräsvirta (1998) the delay parameter is determined by the smallest *p-value* of LM statistic.

Table 3 Linearity Tests

Null Hypothesis	F-statistic	d.f	p-value
H04: b1=b2=b3=b4=0	1.555562	(39, 297)	0.0229
H03: b1=b2=b3=0	1.555562	(39, 297)	0.0229
H02: b1=b2=0	1.487646	(26, 310)	0.0627
H01: b1=0	1.393612	(13, 323)	0.1604
Terasvirta Sequential Tests			
H3: b3=0	1.614696	(13, 297)	0.0801
H2: b2=0   b3=0	1.550787	(13, 310)	0.0984
H1: b1=0   b2=b3=0	1.393612	(13, 323)	0.1604
Escribano-Jorda Tests			
H0L: b2=b4=0	1.224651	(26, 284)	0.2125
H0E: b1=b3=0	1.108525	(26, 284)	0.3299

The H0i test uses the i-th order Taylor expansion ( $b_j=0$  for all  $j>i$ ) and all tests are based on the third-order Taylor expansion ( $b_4=0$ ). The linearity test is rejected for both the LSTAR hypothesis ( $(\varphi_4 = 0)$  is rejected) and ESTR (hypothesis ( $\varphi_3 = 0$ ) is rejected) models. But, the Escribano-Jorda Tests suggests the ESTAR model with nonzero threshold ( $\Pr(H0L) < \Pr(H0E)$  with  $\Pr(H0L) \geq 0.05$ )

In order to analyze inflation's dynamic we divide the sample in three periods. The first one from 1993M04 to 2006M3, a month before Tunisian's central bank declares that price stability is its first priority; the second period starts in 2006M4 and ends 2010M12 just before the Tunisian central bank adopt a proactive monetary policy aimed at anticipating inflation. The third period range from 2011M1 to 2020M03 and starts with the establishment of a corridor by the Tunisian central bank to its main interest rate. Also the third period coincides with the political turmoil which triggered the democratic transition in Tunisia in particular and the Arab spring in general.

The three periods do not contain structural breaks as shown by the CUSUM test (see Figures 1, 3 and 5 in the appendix) of Brown, Durbin, and Evans (1975). The smooth transition estimation is carried with HAC (Newey West) covariance method using observed Hessian to overcome serial correlation and heteroskedasticity. The choice of the transition lag and the fluctuations in each regime are based on the sum of squared residuals. The persistence level is determined as the sum of the threshold lag's coefficients in both regimes.

Table 4 Smooth Threshold Regression for the period 1993M04 2006M03

Variable	Coefficient	Std.Err	t-Statistic	Prob	Coefficient	Std.Err	t-Statistic	Prob
Threshold Variables (linear part)					Threshold Variables (nonlinear part)			
INF(-1)	3.928602	1.683864	2.333087	0.0212	-2.861322	1.675837	-1.707399	0.0902
INF(-2)	-4.250093	2.582365	-1.645814	0.1023	4.127673	2.574851	1.603073	0.1114
INF(-3)	4.200745	1.843138	2.279126	0.0243	-4.122777	1.859444	-2.217209	0.0284
INF(-4)	-3.812364	1.243834	-3.065011	0.0027	3.911003	1.244366	3.142968	0.0021
INF(-5)	2.314489	1.471865	1.572487	0.1183	-2.606548	1.481612	-1.759265	0.0809
INF(-6)	-3.878271	1.700140	-2.281148	0.0242	3.877584	1.671818	2.319382	0.0220
INF(-7)	-0.160853	1.203904	-0.133610	0.8939	0.383510	1.228607	0.312150	0.7554
INF(-8)	7.956861	5.262577	1.511970	0.1330	-8.139965	5.270703	-1.544379	0.1250
INF(-9)	-3.178675	1.501754	-2.116642	0.0362	3.427017	1.483066	2.310766	0.0225
INF(-10)	-5.149208	3.296567	-1.561991	0.1208	5.052754	3.322141	1.520933	0.1308
INF(-11)	3.309294	1.382077	2.394436	0.0181	-3.548145	1.420534	-2.497755	0.0138
INF(-12)	-4.322209	1.921903	-2.248921	0.0262	4.113741	1.944879	2.115166	0.0364
INF(-13)	4.358613	2.008320	2.170278	0.0318	-3.988340	2.023175	-1.971327	0.0509
cons	0.175521	0.085166	2.060916	0.0414	$R^2$	0.962297	Mean dep var	3.384615
$\gamma$	8.313155	3.489337	2.382446	0.0187	$\bar{R}^2$	0.953984	S.D. dep var	1.31596
$c$	5.196624	0.097033	53.55534	0.0000	SER	0.282291	AIC	0.474366
					$\sum \sigma^2$	10.12037	SC	1.041326
					Log L	-8.000553	HQC	0.704641
							DW stat	2.069509
					F-stat	115.7653	Prob (F-stat)	0.000000

Table 4 show that the threshold variable  $c$  chosen for the period 1993M04 2006M03 is the inflation's third lag. In this first period the slope  $\gamma$  or speed of transition between both regimes is 8,31, for more details see Table A3 in the appendix. Persistence in this period is 0,08 and the transition occurs when inflation is over 5,2%. It is worth noticing that 90,38% of the data in this period are below the threshold and fit into the first regime, while 9,62% are in the second regime. Both regimes have a significant difference in fluctuations; the sum of squared residuals in the first one is 9,59 and 0,53 in the second one (see Table A4 in the appendix for more details).

The second period range from 2006M04 to 2010M12, it start with the adoption of price stability as main goal by the Tunisian central bank and ends before its pursuing of a proactive monetary policy aimed at anticipating inflation.

Table 5 Smooth Threshold Regression for the period 2006M04 2010M12

Variable	Coefficient	Std.Err	t-Statistic	Prob	Coefficient	Std.Err	t-Statistic	Prob
Threshold Variables (linear part)					Threshold Variables (nonlinear part)			
INF(-1)	0.730690	0.303880	2.404539	0.0231	0.457803	0.403364	0.129644	0.8978
INF(-2)	-0.450919	0.370601	-1.216725	0.2339	0.457803	0.433377	1.056363	0.2998
INF(-3)	1.240503	0.618853	2.004518	0.0548	-0.920417	0.702578	-1.310057	0.2008
INF(-4)	-3.451624	1.087028	-3.175286	0.0036	3.413596	1.115247	3.060844	0.0048
INF(-5)	2.946144	1.369874	2.150667	0.0403	-2.796056	1.477822	-1.892011	0.0689
INF(-6)	-0.751476	0.650038	-1.156048	0.2574	0.562351	0.822060	0.684076	0.4996
INF(-7)	-0.059495	0.624194	-0.095315	0.9247	0.296277	0.735414	0.402871	0.6901
INF(-8)	0.130810	0.635012	0.205997	0.8383	-0.245557	0.711652	-0.345052	0.7326
INF(-9)	-1.649768	0.860907	-1.916314	0.0656	1.697657	0.897483	1.891576	0.0689
INF(-10)	0.828284	0.601957	1.375985	0.1797	-0.987055	0.718803	-1.373192	0.1806
INF(-11)	1.996249	0.785713	2.540686	0.0169	-2.004355	0.800000	-2.505444	0.0183
INF(-12)	0.886884	0.754724	1.175110	0.2498	-1.366542	0.868333	-1.573753	0.1268
INF(-13)	-1.178297	0.671542	-1.754614	0.0903	1.669150	0.693877	2.405543	0.0230
					$R^2$	0.915680	Mean dep var	3.514035
cons	-0.207008	0.372188	-0.556192	0.5825	$\bar{R}^2$	0.831360	S.D. dep var	0.641271
					SER	0.263343	AIC	0.475977
$\gamma$	13.93176	6.553033	2.126001	0.0425	$\Sigma\sigma^2$	1.941785	SC	1.515425
					Log L	15.43464	HQC	0.879942
$c$	3.136070	0.028666	109.3988	0.0000			DW stat	2.314850
					F-stat	10.85959	Prob (F-stat)	0.000000

Table 5 show that the threshold variable  $c$  chosen for the period 2006M04 2010M12 is the inflation's eleventh lag. In this second period the slope  $\gamma$  or speed of transition between both regimes is 13,93, which is faster than the first period. The transition occurs when inflation is over 3,14%, for more details see Table A6 in the appendix. Also the Persistence in this period

is lower and equal  $-0,01$ . Only 19,30% of the data in this period are below the threshold and fit into the first regime, while 80,70% are in the second regime. Both regimes have different levels of fluctuations; the sum of squared residuals in the first one is 0,4 and 1,54 in the second one, for more details see Table A7 in the appendix. Inflation's volatility is lower compared to the first period. The monetary reform adopted by central bank of Tunisia in this period reduced considerably inflation's volatility and slightly its persistence

The third period is from 2011M01 to 2020M03; starts with the establishment of a proactive monetary policy, aimed at anticipating inflation, but also coincides with the start of political turbulence in the country. The latter complicated the task for the central bank of Tunisia.

Table 6 Smooth Threshold Regression for the period 2011M01 2020M03

Variable	Coefficient	Std.Err	t-Statistic	Prob	Coefficient	Std.Err	t-Statistic	Prob
Threshold Variables (linear part)					Threshold Variables (nonlinear part)			
INF(-1)	1.355280	0.244990	5.531989	0.0000	-0.289809	0.279346	-1.037454	0.3026
INF(-2)	0.709472	0.442220	1.604343	0.1125	-1.163965	0.446016	-2.609693	0.0108
INF(-3)	-0.839762	0.180385	-4.655394	0.0000	1.190887	0.257420	4.626247	0.0000
INF(-4)	-0.198091	0.226395	-0.874978	0.3841	0.293176	0.272770	1.074810	0.2856
INF(-5)	0.189572	0.197403	0.960326	0.3397	-0.366886	0.253995	-1.444463	0.1524
INF(-6)	-0.205179	0.129916	-1.579320	0.1181	0.740350	0.180921	4.092126	0.0001
INF(-7)	1.048104	0.305191	3.434259	0.0009	-1.438010	0.331096	-4.343182	0.0000
INF(-8)	-0.456757	0.338252	-1.350346	0.1806	0.385996	0.382407	1.009387	0.3158
INF(-9)	-1.585783	0.588092	-2.696487	0.0085	1.753246	0.591694	2.963096	0.0040
INF(-10)	-0.122236	0.158829	-0.769607	0.4437	-0.010123	0.224393	-0.045114	0.9641
INF(-11)	0.679650	0.168295	4.038445	0.0001	-0.509959	0.252981	-2.015801	0.0471
INF(-12)	0.237653	0.257585	0.922623	0.3589	-0.907071	0.323823	-2.801133	0.0064
INF(-13)	0.220338	0.084765	2.599398	0.0111	0.236145	0.133781	1.765157	0.0813
					$R^2$	0.976433	Mean dep var	5.047748
cons	0.285698	0.117760	2.426106	0.0175	$\bar{R}^2$	0.968386	S.D. dep var	1.321284
$\gamma$	15.01815	3.784429	3.968407	0.0002	SER	0.234928	AIC	0.160635
$c$	5.135680	0.019375	265.0729	0.0000	$\Sigma\sigma^2$	4.525669	SC	0.868530
					Log L	20.08477	HQC	0.447807
							DW stat	1.949856
					F-stat	121.3390	Prob (F-stat)	0.000000

Table 6 show that the threshold variable  $c$  chosen for the period 2011M01 2020M03 is the inflation's ninth lag. In this last period the slope  $\gamma$  or speed of transition between both regimes is 15,02, (see Table A9 in the appendix for more details), which is the fastest of all three periods. The transition occurs when inflation is over 5,14% higher than the second period. Also, 56,76% of the data in this period are below the threshold and fit into the first regime, while 43,24% are in the second regime. Both regimes have different levels of volatility; the

sum of squared residuals in the first one is 2,59 and 1,94 in the second one (see Table A10 in the appendix for more details). Volatility is slightly higher than its level in the second period but persistence appears to be the highest in all three periods and equal 0,17. It seems that the proactive monetary policy adopted by the central bank of Tunisia helped slow down the expected rise in inflation's volatility given the unstable political context that accompanied the democratic transition.

### **3. Conclusion**

We estimated a smooth transition autoregressive model STAR using monthly data to analyze inflation's dynamics in Tunisia over three periods, through changes in its persistence and volatility. The first period from 1990M03 to 2006M3, just before Tunisian's central bank declares that price stability is its first priority; the second one starts in 2006M4 and ends in 2010M12 at the beginning of the political turbulence and the Arab spring. The third and last period range from 2011M1 to 2020M03. We found evidence of non-linearity, in fact the non-linearity tests suggested that the ESTAR specification describes better the behavior of inflation in Tunisia. Our main findings are high level of volatility in the first period but a significant drop in the second period, with the adoption of price stability by the Tunisian central bank as its primary goal. We also found a slight fall in Persistence. In the third period the Tunisian central bank started pursuing a proactive monetary policy aimed at anticipating inflation that kept inflation volatility close to its level in the second period. Unfortunately for the Tunisian economy, 2011 was also the beginning of inevitable political turmoil which complicated the task of the conduct of the monetary policy. Inflation reached unprecedented levels and according to our findings, the evolution of its dynamic, in a non-linear perspective, suggests that both regimes now have close levels of volatility. However, the depreciation of exchange rate, the multiple wage increases and the escalating public debt contribute to maintain inflation at high level.

## References

Amano, R. (2007) “Inflation Persistence and Monetary Policy: A Simple Result” *Economics Letters* 94, 26-31.

Bacon, D.W and D.G. Watts (1971) “Estimating the Transition between Two Intersecting Straight Lines” *Biometrika* 58, 525-534.

Brown, R.L. Durbin, J and J.M. Evans (1975) “Techniques for testing the constancy of regression relationships over time” *Journal of the Royal Statistical Society B*-37, 149–163.

Cecchetti, S.G and G. Debelle (2006) “Inflation Persistence: Does It Change?” *Economic Policy* April, 311-352.

Cogley, T and T.J. Sargent (2002) “Evolving Post-World War II U.S. Inflation Dynamics” *NBER Macroeconomics Annual*, 16, 331-388.

Führer, J.C. (2010) “Inflation Persistence” in *Handbook of Monetary Economics* by B.M. Friedman and M. Woodford, edition 1, volume 3, chapter 9, pages 423-486 Elsevier.

Ftiti, Z. Guesmi, K. Nguyen, D and F. Teulon (2015) “Modelling inflation shifts and persistence in Tunisia: perspectives from an evolutionary spectral approach” *Applied Economics* 47:57, 6200-6210.

Gerlach, S and P. Tillmann (2012) “Inflation targeting and inflation persistence in Asia-Pacific” *Journal of Asian Economics* 23, 360–373.

Gordon, R.J. King, S.R and F. Modigliani (1982) “The output cost of disinflation in traditional and vector autoregressive models” *Brookings Papers on Economic Activity* 1982(1): 205–244.

Gordon, R.J. (1982) “Price inertia and policy ineffectiveness in the United States, 1890–1980” *Journal of Political Economy* 90(6): 1087–1117.

Kaldor, N. (1940) “A model of the trade cycle” *Economic Journal* 50, pp. 78 – 92.

Mishkin, S.F. (2007) “Inflation dynamics” *NBER Working Paper Series* 13147.

Perron, P. (1989) “The great crash, the oil price shock and the unit root hypothesis” *Econometrica* 57: 1361–401.

Teräsvirta, T. (1998) “Modelling Economic Relationships with Smooth Transition Regressions” In *Handbook of Applied Economic Statistics* by A. Ullah and D.E.A. Giles, (Eds.), (507-552).

Teräsvirta, T. (1994) “Specification, Estimation, and Evaluation of Smooth Transition Autoregressive Models” *Journal of the American Statistical Association* 89, 208-218.

Walsh, C.E. (2009) “Inflation Targeting: What Have We Learned?” *International Finance*, Wiley Blackwell vol. 12(2), pages 195-233, August.

## Appendix

Table A1 Breakpoint unit root test Summary Statistics

$R^2$	0.0678	0.0748	0.0505	0.0346	0.0678	0.0748	0.0505	0.0346
$\bar{R}^2$	0.0600	0.0644	0.0371	0.0265	0.0600	0.0644	0.0371	0.0265
SER	0.3569	0.3561	0.3612	0.3632	0.3569	0.3561	0.3612	0.3632
$\Sigma \sigma^2$	45.472	45.131	46.317	47.090	45.472	45.131	46.317	47.090
Log L	-138.28	-136.92	-141.60	-144.59	-138.28	-136.92	-141.60	-144.59
F-stat	8.6559	7.1933	3.7741	4.2690	8.6559	7.1933	3.7741	4.2690
Prob F	0.0000	0.0000	0.0024	0.0056	0.0000	0.0000	0.0024	0.0056
M.d.v	-0.0017	-0.0017	-0.0017	-0.0017	-0.0017	-0.0017	-0.0017	-0.0017
S.D.d.v	0.3681	0.3681	0.3681	0.3681	0.3681	0.3681	0.3681	0.3681
AIC	0.7883	0.7863	0.8178	0.8232	0.7883	0.7863	0.8178	0.8232
SC	0.8313	0.8401	0.8824	0.8663	0.8313	0.8401	0.8824	0.8663
HQC	0.8054	0.8077	0.8435	0.8404	0.8054	0.8077	0.8435	0.8404
DW stat	1.9290	1.9294	1.9837	1.9815	1.9290	1.9294	1.9837	1.9815

Figure 1 Stability diagnosis for the first period

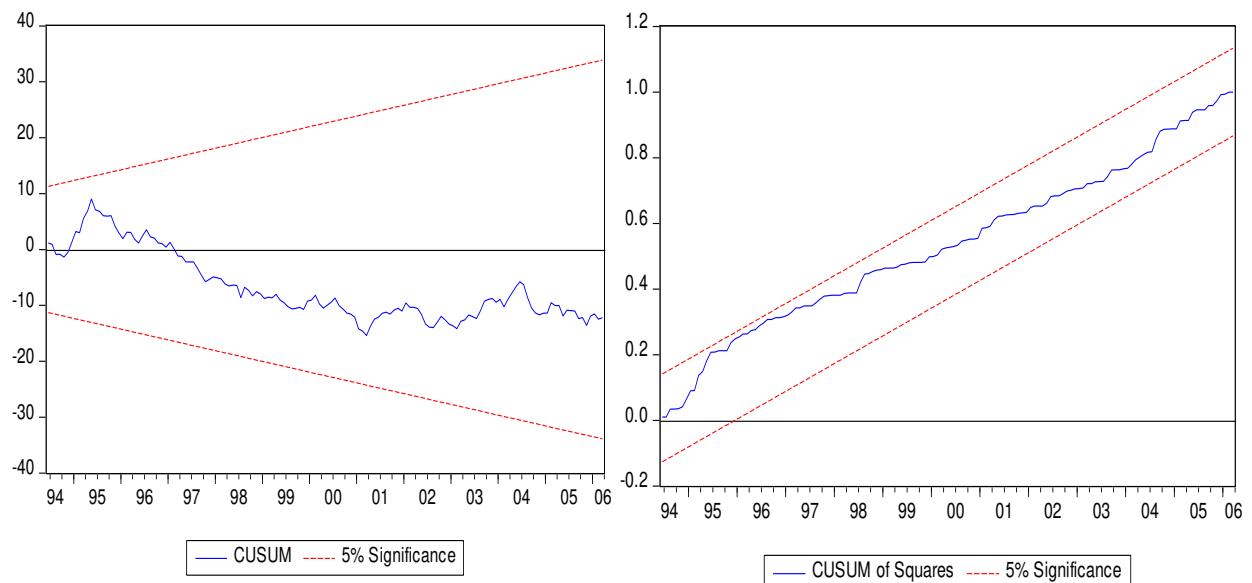


Figure 2 Threshold weight function with kernel density for the first period

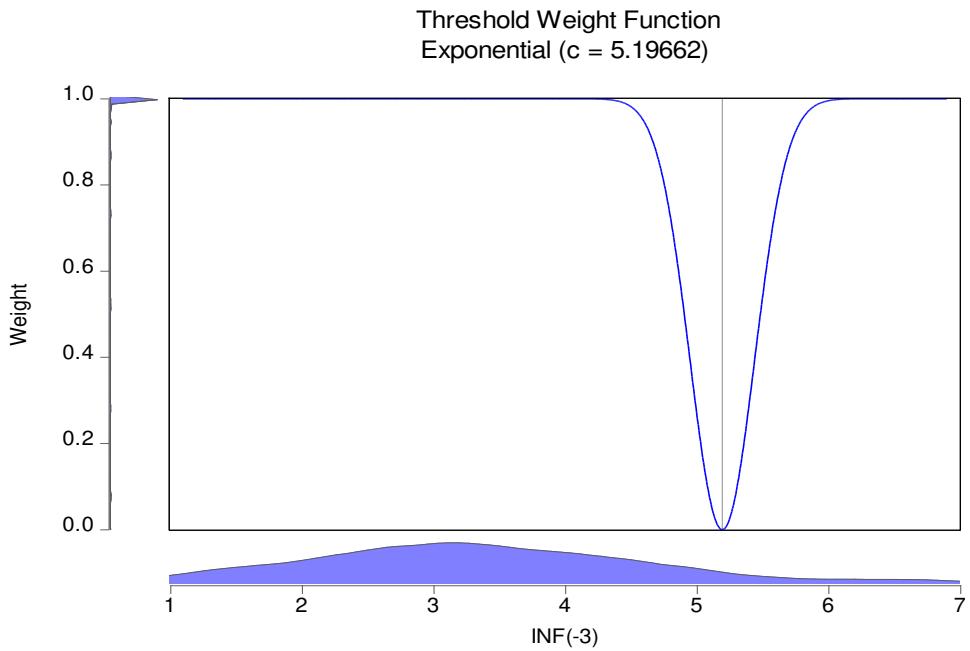


Table A4 Actual fitted residual for the first period

obs	Actual	Fitted	Residual	Residual Plot			
1993M04	3.20000	3.64714	-0.44714	*	.	.	
1993M05	3.00000	2.78010	0.21990	.	.	*	.
1993M06	3.60000	3.24890	0.35110	.	.	.	*
1993M07	4.20000	4.28535	-0.08535	.	*	.	.
1993M08	3.90000	4.07645	-0.17645	.	*	.	.
1993M09	4.00000	3.92797	0.07203	.	*	.	.
1993M10	4.50000	4.00085	0.49915	.	.	.	*
1993M11	4.40000	4.37895	0.02105	.	*	.	.
1993M12	4.20000	4.22698	-0.02698	.	*	.	.
1994M01	4.40000	4.35370	0.04630	.	*	.	.
1994M02	4.50000	4.47378	0.02622	.	*	.	.
1994M03	4.30000	4.56326	-0.26326	*		.	.
1994M04	5.10000	4.69465	0.40535	.	.	.	*
1994M05	5.40000	5.20684	0.19316	.		*	.
1994M06	4.90000	4.93989	-0.03989	.	*	.	.
1994M07	4.80000	4.72696	0.07304	.	*	.	.
1994M08	4.40000	4.43044	-0.03044	.	*	.	.
1994M09	4.60000	4.54233	0.05767	.	*	.	.
1994M10	4.30000	4.58508	-0.28508	*		.	.
1994M11	4.70000	4.34324	0.35676	.	.	.	*
1994M12	5.30000	5.11516	0.18484	.		*	.
1995M01	5.50000	5.28180	0.21820	.		*	.
1995M02	5.30000	5.45001	-0.15001	.	*		.

1995M03	6.00000	6.00734	-0.00734	.	*	.
1995M04	6.20000	6.14354	0.05646	.	*	.
1995M05	6.60000	6.60447	-0.00447	.	*	.
1995M06	6.40000	6.67178	-0.27178	*		.
1995M07	6.70000	6.42795	0.27205	.		*
1995M08	6.80000	6.68092	0.11908	.	*	.
1995M09	6.80000	6.62701	0.17299	.	*	.
1995M10	6.90000	6.72680	0.17320	.	*	.
1995M11	6.20000	6.43868	-0.23868	*		.
1995M12	5.60000	5.82255	-0.22255	.	*	.
1996M01	5.10000	5.33888	-0.23888	*		.
1996M02	5.20000	4.96556	0.23444	.		*
1996M03	4.60000	4.44186	0.15814	.	*	.
1996M04	3.80000	3.80321	-0.00321	.	*	.
1996M05	3.40000	3.42887	-0.02887	.	*	.
1996M06	3.80000	3.59137	0.20863	.		*
1996M07	3.90000	3.56089	0.33911	.		*
1996M08	3.10000	3.58901	-0.48901	*	.	.
1996M09	2.90000	2.99420	-0.09420	.	*	.
1996M10	2.90000	2.97146	-0.07146	.	*	.
1996M11	3.20000	3.15794	0.04206	.	*	.
1996M12	3.00000	3.23823	-0.23823	*		.
1997M01	3.40000	3.08158	0.31842	.		*
1997M02	3.30000	3.47951	-0.17951	.	*	.
1997M03	3.30000	3.63262	-0.33262	*		.
1997M04	3.80000	3.69706	0.10294	.	*	.
1997M05	3.80000	3.84050	-0.04050	.	*	.
1997M06	3.70000	3.49165	0.20835	.		*
1997M07	3.80000	3.69100	0.10900	.	*	.
1997M08	4.10000	4.31112	-0.21112	.		.
1997M09	3.80000	4.03660	-0.23660	*		.
1997M10	3.40000	3.66851	-0.26851	*		.
1997M11	3.60000	3.37378	0.22622	.		*
1997M12	3.90000	3.58874	0.31126	.		*
1998M01	3.70000	3.70432	-0.00432	.	*	.
1998M02	3.60000	3.67900	-0.07900	.	*	.
1998M03	3.50000	3.62410	-0.12410	.	*	.
1998M04	3.30000	3.29399	0.00601	.	*	.
1998M05	3.30000	3.21725	0.08275	.	*	.
1998M06	3.30000	3.29357	0.00643	.	*	.
1998M07	2.50000	3.12770	-0.62770	*	.	.
1998M08	2.90000	2.31835	0.58165	.		.
1998M09	3.10000	3.24041	-0.14041	.	*	.
1998M10	2.80000	3.18464	-0.38464	*	.	.
1998M11	2.80000	2.49902	0.30098	.		*
1998M12	2.80000	2.90516	-0.10516	.	*	.
1999M01	2.70000	2.92186	-0.22186	*		.
1999M02	2.70000	2.53870	0.16130	.	*	.
1999M03	2.80000	2.90556	-0.10556	.	*	.
1999M04	3.00000	2.78198	0.21802	.	*	.

1999M05	2.70000	2.97236	-0.27236	*	.	
1999M06	2.70000	2.92179	-0.22179	.	*	.
1999M07	2.80000	2.96148	-0.16148	.	*	.
1999M08	2.50000	2.61464	-0.11464	.	*	.
1999M09	2.40000	2.39052	0.00948	.	*	.
1999M10	2.60000	2.54982	0.05018	.	*	.
1999M11	2.50000	2.68698	-0.18698	.	*	.
1999M12	2.90000	2.43448	0.46552	.	.	*
2000M01	3.10000	3.08271	0.01729	.	*	.
2000M02	3.40000	3.15610	0.24390	.	.	*
2000M03	2.90000	3.29000	-0.39000	*	.	.
2000M04	2.70000	2.92824	-0.22824	*	.	.
2000M05	3.10000	2.81764	0.28236	.	.	*
2000M06	3.20000	3.01121	0.18879	.	.	*
2000M07	3.30000	3.13379	0.16621	.	.	*
2000M08	3.00000	3.43618	-0.43618	*	.	.
2000M09	3.00000	3.21092	-0.21092	.	*	.
2000M10	2.80000	2.93861	-0.13861	.	*	.
2000M11	2.70000	2.78374	-0.08374	.	*	.
2000M12	2.30000	2.44702	-0.14702	.	*	.
2001M01	1.40000	2.08665	-0.68665	*	.	.
2001M02	1.20000	1.40127	-0.20127	.	*	.
2001M03	1.10000	1.47958	-0.37958	*	.	.
2001M04	1.50000	1.14819	0.35181	.	.	*
2001M05	1.60000	1.28986	0.31014	.	.	*
2001M06	1.70000	1.72989	-0.02989	.	*	.
2001M07	2.10000	1.87215	0.22785	.	.	*
2001M08	2.40000	2.33029	0.06971	.	*	.
2001M09	2.40000	2.46967	-0.06967	.	*	.
2001M10	2.60000	2.33952	0.26048	.	.	*
2001M11	2.90000	2.77340	0.12660	.	*	.
2001M12	3.00000	3.18660	-0.18660	.	*	.
2002M01	3.80000	3.38452	0.41548	.	.	*
2002M02	3.80000	4.05836	-0.25836	*	.	.
2002M03	3.80000	3.79939	0.00061	.	*	.
2002M04	3.70000	3.67434	0.02566	.	*	.
2002M05	3.40000	3.80094	-0.40094	*	.	.
2002M06	2.70000	3.18116	-0.48116	*	.	.
2002M07	2.10000	2.36143	-0.26143	*	.	.
2002M08	1.90000	2.07410	-0.17410	.	*	.
2002M09	2.00000	1.79186	0.20814	.	.	*
2002M10	2.20000	1.97061	0.22939	.	.	*
2002M11	1.90000	2.18989	-0.28989	*	.	.
2002M12	1.60000	1.85968	-0.25968	*	.	.
2003M01	1.30000	1.40499	-0.10499	.	*	.
2003M02	1.10000	1.31474	-0.21474	.	*	.
2003M03	1.40000	0.97086	0.42914	.	.	*
2003M04	1.50000	1.41993	0.08007	.	*	.
2003M05	2.00000	1.75970	0.24030	.	.	*
2003M06	2.40000	2.49145	-0.09145	.	*	.

2003M07	2.70000	2.89498	-0.19498	.*   . .
2003M08	3.30000	2.81755	0.48245	.   . *
2003M09	3.90000	3.29444	0.60556	.   . *
2003M10	4.10000	3.90295	0.19705	.   *.
2003M11	4.30000	4.15701	0.14299	.   *.
2003M12	4.40000	4.57398	-0.17398	.*   .
2004M01	4.70000	4.54832	0.15168	.   *.
2004M02	4.30000	4.68791	-0.38791	*.   .
2004M03	4.50000	4.13737	0.36263	.   . *
2004M04	4.80000	4.82396	-0.02396	.   *.
2004M05	4.90000	4.58254	0.31746	.   . *
2004M06	5.00000	4.67801	0.32199	.   . *
2004M07	4.70000	4.56885	0.13115	.   *.
2004M08	3.70000	3.87552	-0.17552	.*   .
2004M09	2.60000	2.61794	-0.01794	.   *.
2004M10	2.00000	1.80297	0.19703	.   *.
2004M11	1.50000	1.71030	-0.21030	.*   .
2004M12	1.20000	1.19039	0.00961	.   *.
2005M01	1.10000	1.19759	-0.09759	.   *.
2005M02	1.90000	1.40886	0.49114	.   . *
2005M03	1.90000	1.96422	-0.06422	.   *.
2005M04	1.80000	1.81700	-0.01700	.   *.
2005M05	1.20000	1.76550	-0.56550	*.   .
2005M06	1.40000	1.15535	0.24465	.   . *
2005M07	1.60000	1.52115	0.07885	.   *.
2005M08	2.00000	1.97942	0.02058	.   *.
2005M09	2.20000	2.53997	-0.33997	.*   .
2005M10	2.70000	2.60991	0.09009	.   *.
2005M11	2.80000	3.25176	-0.45176	*.   .
2005M12	3.50000	3.00262	0.49738	.   . *
2006M01	3.70000	3.52747	0.17253	.   *.
2006M02	3.10000	3.31841	-0.21841	.*   .
2006M03	3.10000	3.05192	0.04808	.   *.

Figure 3 Stability diagnosis for the second period

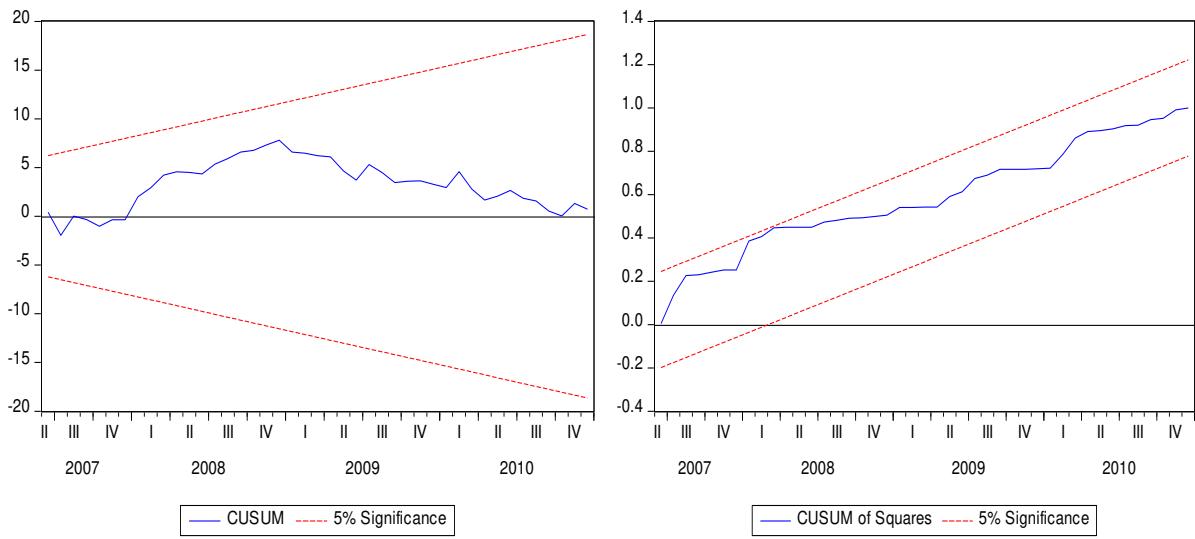


Figure 4 Threshold weight function with kernel density for the second period

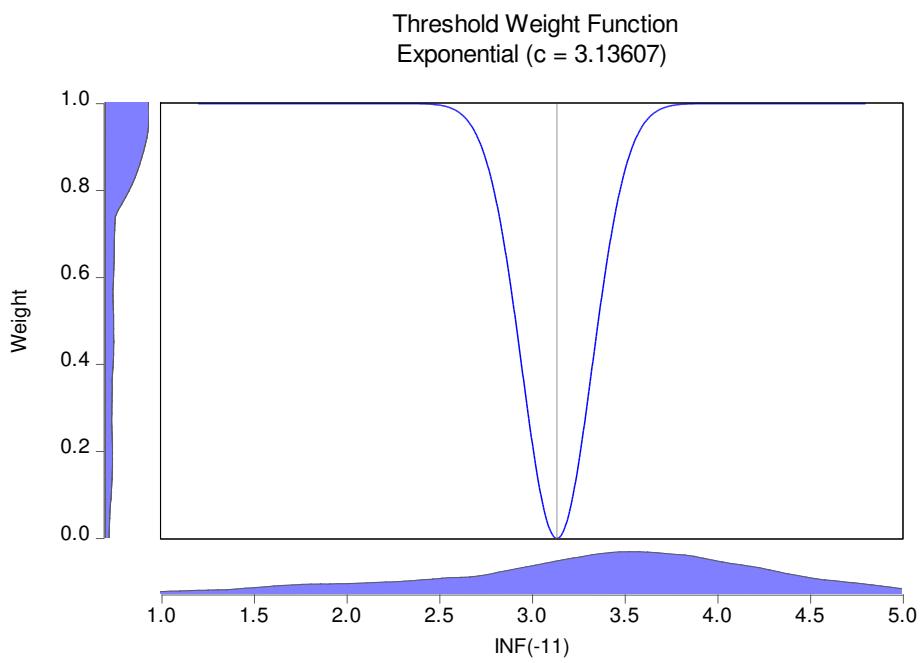


Table A7 Actual fitted residual for the second period

obs	Actual	Fitted	Residual	Residual Plot		
2006M04	3.40000	3.40769	-0.00769	.	*	.
2006M05	3.70000	3.84901	-0.14901	.	*	.
2006M06	3.40000	3.52673	-0.12673	.	*	.
2006M07	3.40000	3.40750	-0.00750	.	*	.
2006M08	2.90000	3.40065	-0.50065	*	.	.
2006M09	3.00000	2.78587	0.21413	.		*
2006M10	3.00000	3.02688	-0.02688	.	*	.
2006M11	3.10000	2.96331	0.13669	.		*
2006M12	2.60000	2.81338	-0.21338	*		.
2007M01	2.30000	2.24033	0.05967	.	*	.
2007M02	2.00000	1.99727	0.00273	.	*	.
2007M03	2.10000	2.26293	-0.16293	.		.
2007M04	2.00000	1.88923	0.11077	.	*	.
2007M05	2.70000	2.67072	0.02928	.	*	.
2007M06	3.30000	3.31754	-0.01754	.	*	.
2007M07	2.90000	2.85716	0.04284	.	*	.
2007M08	3.50000	3.52345	-0.02345	.	*	.
2007M09	3.70000	3.68466	0.01534	.	*	.
2007M10	3.40000	3.38929	0.01071	.	*	.
2007M11	3.70000	3.44677	0.25323	.		*
2007M12	3.90000	4.02436	-0.12436	.		.
2008M01	4.40000	4.18725	0.21275	.		*
2008M02	4.60000	4.48576	0.11424	.		*
2008M03	4.70000	4.52671	0.17329	.		*
2008M04	4.80000	4.78462	0.01538	.		*
2008M05	4.10000	4.12502	-0.02502	.		*
2008M06	3.60000	3.67720	-0.07720	.		.
2008M07	4.30000	4.09891	0.20109	.		*
2008M08	4.00000	4.09811	-0.09811	.		.
2008M09	4.20000	4.14543	0.05457	.		*
2008M10	4.60000	4.39256	0.20744	.		*
2008M11	4.40000	4.28628	0.11372	.		*
2008M12	4.40000	4.25114	0.14886	.		*
2009M01	3.80000	4.00034	-0.20034	.		.
2009M02	3.80000	3.86049	-0.06049	.		.
2009M03	3.90000	3.87140	0.02860	.		.
2009M04	3.70000	3.85346	-0.15346	.		.
2009M05	3.90000	4.10851	-0.20851	.		.
2009M06	3.90000	4.04504	-0.14504	.		.
2009M07	3.90000	3.52369	0.37631	.		*
2009M08	3.80000	3.88102	-0.08102	.		.
2009M09	3.30000	3.60036	-0.30036	*		.
2009M10	3.30000	3.17889	0.12111	.		*
2009M11	3.40000	3.43432	-0.03432	.		.
2009M12	3.30000	3.33872	-0.03872	.		.
2010M01	3.60000	3.50677	0.09323	.		*

2010M02	4.10000	3.46027	0.63973	.	.	*
2010M03	3.40000	3.81538	-0.41538	*	.	.
2010M04	3.10000	3.41760	-0.31760	*	.	.
2010M05	3.20000	3.15111	0.04889	.	*	.
2010M06	3.40000	3.16171	0.23829	.	*	.
2010M07	3.20000	3.31928	-0.11928	.	*	.
2010M08	3.30000	3.39298	-0.09298	.	*	.
2010M09	3.30000	3.29489	0.00511	.	*	.
2010M10	3.00000	3.11323	-0.11323	.	*	.
2010M11	3.30000	3.22074	0.07926	.	*	.
2010M12	3.30000	3.20604	0.09396	.	*	.

Figure 5 Stability diagnosis for the third period

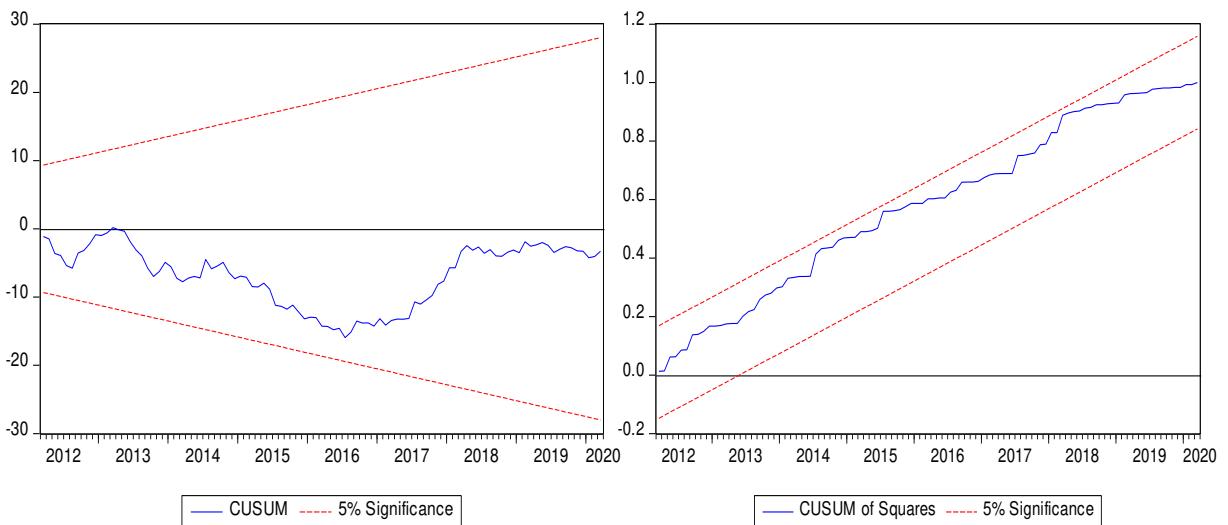


Figure A9 Threshold weight function with kernel density for the second period

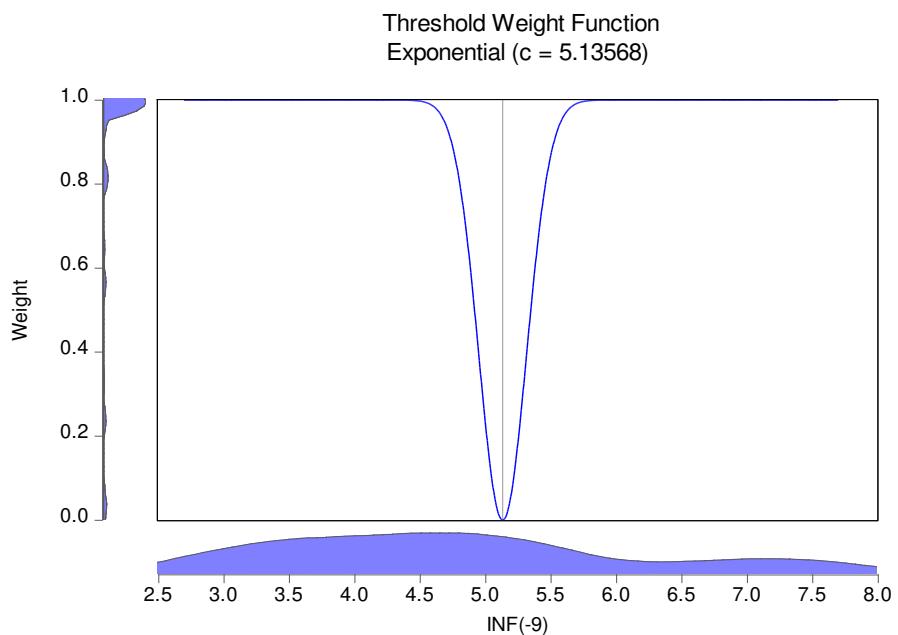


Table A10 Actual fitted residual for the third period

obs	Actual	Fitted	Residual	Residual Plot		
2011M01	3.20000	3.10571	0.09429	.	*	.
2011M02	2.70000	2.93323	-0.23323	*	.	.
2011M03	2.90000	3.16901	-0.26901	*	.	.
2011M04	3.00000	3.19170	-0.19170	*	.	.
2011M05	3.00000	3.17308	-0.17308	*	.	.
2011M06	3.00000	2.93689	0.06311	.	*	.
2011M07	3.00000	3.19684	-0.19684	*	.	.
2011M08	3.20000	2.87400	0.32600	.	.	*
2011M09	3.60000	3.33346	0.26654	.	.	*
2011M10	4.20000	3.91465	0.28535	.	.	*
2011M11	3.60000	3.98095	-0.38095	*	.	.
2011M12	3.30000	3.44101	-0.14101	*	.	.
2012M01	3.90000	3.57968	0.32032	.	.	*
2012M02	4.80000	4.54756	0.25244	.	.	*
2012M03	4.60000	4.75594	-0.15594	*	.	.
2012M04	4.80000	4.59769	0.20231	.	.	*
2012M05	4.50000	4.82373	-0.32373	*	.	.
2012M06	4.20000	4.39360	-0.19360	*	.	.
2012M07	4.40000	4.66393	-0.26393	*	.	.
2012M08	4.80000	4.98556	-0.18556	*	.	.
2012M09	4.90000	4.60563	0.29437	.	.	*
2012M10	4.30000	4.56592	-0.26592	*	.	.
2012M11	4.70000	4.35553	0.34447	.	.	*
2012M12	5.40000	5.09838	0.30162	.	.	*
2013M01	5.40000	5.35401	0.04599	.	*	.
2013M02	4.90000	4.80160	0.09840	.	*	.
2013M03	5.60000	5.10625	0.49375	.	.	*
2013M04	5.60000	5.48395	0.11605	.	*	.
2013M05	5.70000	5.63184	0.06816	.	*	.
2013M06	6.00000	6.04030	-0.04030	.	*	.
2013M07	5.80000	6.02759	-0.22759	*	.	.
2013M08	5.10000	5.37214	-0.27214	*	.	.
2013M09	4.60000	4.79463	-0.19463	*	.	.
2013M10	4.90000	4.73011	0.16989	.	.	*
2013M11	5.10000	5.15060	-0.05060	*	.	.
2013M12	5.20000	4.92032	0.27968	.	.	*
2014M01	4.80000	5.02578	-0.22578	*	.	.
2014M02	4.50000	4.86906	-0.36906	*	.	.
2014M03	3.80000	4.09216	-0.29216	*	.	.
2014M04	4.00000	3.98270	0.01730	.	*	.
2014M05	4.50000	4.51238	-0.01238	.	*	.
2014M06	4.40000	4.31972	0.08028	.	*	.
2014M07	5.00000	4.94057	0.05943	.	*	.
2014M08	5.00000	5.00080	-0.00080	.	*	.

2014M09	5.10000	5.18035	-0.08035	.	*	.	
2014M10	5.10000	4.97677	0.12323	.	.	*	
2014M11	4.80000	5.25764	-0.45764	*	.	.	
2014M12	4.40000	4.48111	-0.08111	.	*	.	
2015M01	5.00000	4.91415	0.08585	.	.	*	
2015M02	5.30000	5.34213	-0.04213	.	*	.	
2015M03	5.20000	5.51393	-0.31393	*	.	.	
2015M04	5.20000	5.29744	-0.09744	.	*	.	
2015M05	4.80000	4.87808	-0.07808	.	*	.	
2015M06	4.50000	4.55081	-0.05081	.	*	.	
2015M07	3.70000	3.65430	0.04570	.	*	.	
2015M08	3.80000	3.97072	-0.17072	.	*	.	
2015M09	3.90000	3.96543	-0.06543	.	*	.	
2015M10	4.20000	4.19544	0.00456	.	*	.	
2015M11	4.00000	4.04771	-0.04771	.	*	.	
2015M12	3.80000	3.72711	0.07289	.	*	.	
2016M01	3.40000	3.33633	0.06367	.	*	.	
2016M02	3.10000	3.21211	-0.11211	.	*	.	
2016M03	3.10000	3.29631	-0.19631	*	.	.	
2016M04	3.20000	3.21506	-0.01506	.	*	.	
2016M05	3.50000	3.32211	0.17789	.	.	*	
2016M06	3.60000	3.47441	0.12559	.	.	*	
2016M07	3.60000	3.86355	-0.26355	*	.	.	
2016M08	3.60000	3.45415	0.14585	.	.	*	
2016M09	4.10000	3.66792	0.43208	.	.	*	
2016M10	4.00000	4.00209	-0.00209	.	*	.	
2016M11	4.10000	4.01172	0.08828	.	*	.	
2016M12	4.20000	4.28317	-0.08317	.	*	.	
2017M01	4.70000	4.43879	0.26121	.	.	*	
2017M02	4.70000	4.91118	-0.21118	*	.	.	
2017M03	4.90000	4.87093	0.02907	.	*	.	
2017M04	5.00000	4.97363	0.02637	.	*	.	
2017M05	4.80000	4.93800	-0.13800	.	*	.	
2017M06	4.80000	4.83637	-0.03637	.	*	.	
2017M07	5.60000	5.16564	0.43436	.	.	*	
2017M08	5.80000	5.83463	-0.03463	.	*	.	
2017M09	5.50000	5.37087	0.12913	.	.	*	
2017M10	5.70000	5.64142	0.05858	.	*	.	
2017M11	6.10000	5.79328	0.30672	.	.	*	
2017M12	6.20000	6.22286	-0.02286	.	*	.	
2018M01	6.60000	6.52205	0.07795	.	*	.	
2018M02	6.80000	7.02080	-0.22080	*	.	.	
2018M03	7.20000	6.84998	0.35002	.	.	*	
2018M04	7.50000	7.19681	0.30319	.	.	*	
2018M05	7.50000	7.68675	-0.18675	*	.	.	
2018M06	7.70000	7.65790	0.04210	.	*	.	
2018M07	7.30000	7.53849	-0.23849	*	.	.	
2018M08	7.30000	7.13507	0.16493	.	.	*	
2018M09	7.40000	7.73155	-0.33155	*	.	.	
2018M10	7.50000	7.55789	-0.05789	.	*	.	

2018M11	7.40000	7.22127	0.17873	.	*	.
2018M12	7.50000	7.48516	0.01484	.	*	.
2019M01	7.10000	7.19884	-0.09884	.	*	.
2019M02	7.30000	6.90255	0.39745	.	.	*
2019M03	7.10000	7.29507	-0.19507	.	*	.
2019M04	6.90000	6.78070	0.11930	.	*	.
2019M05	7.00000	6.79721	0.20279	.	*	.
2019M06	6.80000	6.91479	-0.11479	.	*	.
2019M07	6.50000	6.64813	-0.14813	.	*	.
2019M08	6.60000	6.53126	0.06874	.	*	.
2019M09	6.70000	6.57220	0.12780	.	*	.
2019M10	6.40000	6.32953	0.07047	.	*	.
2019M11	6.30000	6.36813	-0.06813	.	*	.
2019M12	6.10000	6.12344	-0.02344	.	*	.
2020M01	5.90000	6.09292	-0.19292	.	*	.
2020M02	5.80000	5.76680	0.03320	.	*	.
2020M03	6.10000	5.90488	0.19512	.	*	.