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Abstract

This paper investigates the effects of monetary policy on income inequality in a Schumpeterian growth model with endogenous human capital accumulation and household heterogeneity. The source of heterogeneity arises from both unequal distributions of (tangible) wealth and (intangible) human capital. We find that inflation unambiguously lowers economic growth rate, whereas its impact on the income inequality is quite diverse, depending on the relative dispersions of human capital and wealth, and the response of the relative interest-wage income share to inflation. Inflation may increase income inequality when the dispersion of human capital dominates (is dominated by) that of wealth, and the relative interest-wage income share is decreasing (increasing) in inflation rate. One interesting scenario in our analysis is that the model can generate a non-monotonic U-shaped relationship between income inequality and inflation. Moreover, our quantitative example shows that this U-shaped relationship is likely to occur in a reasonable range of parameter configuration and the threshold level of inflation is consistent with the current empirical findings using the U.S. data.

JEL classification: D31, O30; O40; E41.
Keywords: Income Inequality; Inflation; Endogenous economic growth; Human capital.

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1 Introduction

It is widely acknowledged that people in the economy reap unequal benefits of economic growth (e.g., Kuznets (1955), Alesina and Rodrik (1994), and Piketty (2014)). An important question centers on how the policy instruments, which aim to promote economic growth, are able to affect the distribution of personal income? Do these policies exacerbate or mitigate income inequality? One strand of recent literature has shown a sizable impact of monetary policy on the long-run economic growth (see, for example, Chu and Cozzi (2014) and Arawatari, Hori and Mino (2018)). A few other studies have also started to investigate its micro-economic implications on the well-being of the population from different income groups, focusing on the issue of its impact on income inequality (e.g, Chu, Cozzi, Fan, Furukawa and Liao (2019a) and Zheng (2020)).

At the same time, an increasing number of empirical studies have documented a high correlation between education inequality (human capital inequality) and income inequality. On the one hand, Castelló and Doménech (2002), Checchi (2004), and Castelló-Climent (2010) reported that the human capital inequality is significantly correlated with income inequality in a wider cross-country perspective. On the other hand, empirical findings such as Rodríguez-Pose and Tselios (2009) and Hasanov and Izraeli (2011) indicated that high levels of inequality in educational attainment is associated with higher income inequality across regions within the E.U. and the U.S. economy. Surprisingly, to the best of our knowledge, very few existing theoretical models have taken human capital heterogeneity into account.[1] In this regard, our study intends to fill this void by incorporating human capital into a scale-invariant Schumpeterian model with cash constraint on R&D.

This study follows the previous research (i.e., Chu and Cozzi (2014), Huang, Chang and Ji (2015), He and Zou (2016) and Zheng, Huang and Yang (2019)) to model money demand by imposing the cash-in-advance (CIA) constraint on entrepreneurs' R&D activities.[2] It enables us to investigate the effects of inflation on economic growth and income inequality. We model households' heterogeneity via unequal levels of wealth endowment and human capital. The different wealth endowment gives rise to an unequal distribution of households' interest income, in part causing the income inequality. In the presence of human capital heterogeneity, the households can choose different levels of education, devote themselves into either high-skilled or low-skilled labor, and consequently receive different amounts of wage income. Accordingly, by taking into

[1] One notable exception is Jin (2009) who built an AK endogenous growth model and considered an exogenous skill heterogeneity. We hereby complement Jin (2009) by considering the link between money and R&D, the endogenous human capital accumulation, and the fact that R&D is a skill-intensive sector which requires a huge amount of investment in human capital.

[2] This approach is supported by many empirical findings. For example, early empirical literature such as Himmelberg and Petersen (1994) reported a strong relationship between R&D and cash flows in the U.S. firms. More recently, Brown, Martinsson and Petersen (2012) found that the feature of R&D's susceptibility to financing constraint and binding liquidity forces firms to hold enormous amounts of cash for R&D smoothing, and Bakker (2013) documented the existence of cash-requirement for R&D investment even for the giant companies like Apple, Google, Facebook, and Amazon.

account two dimensions of heterogeneity, our model can explore the monetary effects on the interaction of households' interest income and wage income inequality, and how both inequalities contribute to total income inequality.

In contrast to the roughly consistent empirical findings on inflation and growth,[3] the nexus between inflation and income inequality is rather inconclusive. Early studies by Edwards (1997), Al-Marhubi (1997), and Albanesi (2007) found a positive linkage between inflation rate and income inequality across countries, while Cutler, Katz, Card and Hall (1991), Jäntti (1994) and Mocan (1999) confirmed the progressive effect of inflation on income distribution in the U.S. over the past decades. Interestingly, some recent empirical studies argue that the relationship between inflation and income inequality found in previous studies tends to depend on the level of inflation. They found positive correlations at high inflation levels (using the data from developing countries) and negative correlations at low inflation levels (applying the data from multiple developed countries). Of importance, most of the previous studies did not explicitly control for the non-linearity in their regression models (Galli and van der Hoeven (2001) and Monnin (2014)). Galli and van der Hoeven (2001) and Auda (2010) reported a U-shaped relationship between inflation and income inequality, and Monnin (2014) either found a U-shaped relationship in OECD countries.[4] Besides, Bulíř (2001) and Balcilar, Chang, Gupta and Miller (2018) also found U-shaped results from the U.S. data. Our model is able to generate predictions that are supported by the above empirical findings.

Our results show that a higher nominal interest rate (inflation) unambiguously lowers economic growth rate, while its impact on income inequality is diverse. In particular, we find that a rise in the nominal interest rate (inflation) exacerbates income inequality when the relative dispersion of human capital to wealth is greater (smaller) than the relative *interest-wage* income share conditional on this relative income share being decreasing (increasing) in the nominal interest rate. Intuitively, on one hand, a rise in the nominal interest rate dampens economic growth rate and in turn real interest rate via increasing the cost of R&D activities due to the CIA constraint; on the other hand, it also generates an ambiguous change on the wage income-to-wealth ratio. The two effects jointly determine the movement of the relative income share. If the former effect dominates, the relative income share is decreasing, leading to an increase (decrease) in wage income (interest income) share. Suppose the economy is endowed with a sufficiently high dispersion of human capital to wealth, an increasing weight (i.e., wage income share) on a larger dispersion of human capital raises the income inequality. If the latter effect is negative and dominates, it leads conversely to an increasing (decreasing) interest income (wage income)

[3] Vaona (2012) and Barro (2013) showed a monotonically decreasing relationship between inflation rate and economic growth rate. Some other studies such as López-Villavicencio and Mignon (2011) and Kremer, Bick and Nautz (2013) showed that when the inflation rate is low, an insignificant negative linkage or a positive linkage between inflation rate and economic growth rate can be found, while when the inflation rate is high, a robust and significantly negative linkage holds.

[4] Chu, Cozzi, Fan, Furukawa and Liao (2019a) found an inverted-U impact of inflation on income inequality from their cross-country regressions.

share. Suppose now the economy is endowed with a sufficiently low dispersion of human capital to wealth, a declining (wage income) share of a lower dispersed human capital still raises income inequality.

Finally, we arrive at an interesting scenario that a U-shaped relation between income inequality and nominal interest rate (inflation) might emerge. If we hold everything else the same but only switch the sign in the above endowment condition to let the relative dispersion of human capital to wealth be smaller (greater) than the relative income share, we find that this endowment condition is now subject to change, depending on the status quo nominal interest rate. In particular, there will be a threshold level of nominal interest rate below and above which income inequality would decrease and increase in response. A rise in nominal interest rate, henceforth, may lead to monotonically decreasing income inequality if the interest rate never reaches the threshold within its feasible range; otherwise, a U-shaped relationship would emerge if the nominal interest rate is able to exceed the threshold thorough its feasible range. The prediction of a U-shaped relationship is consistent with the aforementioned empirical findings. Moreover, by calibrating our model to the U.S. economy, we also numerically show that our model can deliver this U-shaped relationship under reasonable parameter configuration whereby the threshold nominal interest rate (inflation) coincides with the empirical estimates.

Our study is related to the strand of literature investigating income disparity in the R&D-based growth models, such as Chou and Talmain (1996), Zweimüller (2000), Foellmi and Zweimüller (2006), García-Peñalosa and Wen (2008), Chu and Cozzi (2018), Jones and Kim (2018), and Aghion, Akcigit, Bergeaud, Blundell and Hémous (2019). These studies mainly focus on the relationship between income (wealth) inequality and innovation-driven growth, whilst our paper tries to answer the question of how the monetary policy can affect income inequality by incorporating both wealth and human capital inequalities. In addition, our paper is closely related to Chu, Cozzi, Fan, Furukawa and Liao (2019a), who explored the impact of inflation on innovation, growth and income inequality.[5] By building up a random-quality improvement Schumpeterian growth model, they investigated the monetary effect on innovation and income inequality and found an inverted-U effect in a general case with positive entry cost. While they focus exclusively on wealth heterogeneity in explaining the nexus of inflation and income inequality, the heterogeneity of human capital is omitted. The present study thus complements their paper by incorporating human capital heterogeneity, which enables us to generate mixed results that reconcile with the empirical inconsistency.

The rest of this study proceeds as follows. The basic model is spelled out in Section 2. Section 3 solves the model and explores the growth effects of monetary policy. Section 4 characterizes the

[5] Zheng (2020) and Zheng, Mishra and Yang (2020) also asked a similar question of how inflation affects income inequality in a Schumpeterian and variety-expansion growth model, respectively, in which firms suffer from costly pricing adjustment. However, both studies does not consider the human capital inequality. More importantly, both studies shows that inflation affects linearly income inequality, which fails to account for the diverse empirical findings on inflation and income inequality.

human capital and wealth distribution, and Section 5 investigates the effect of monetary policy on income inequality. Section 6 provides numerical analysis and the final section concludes.

2 The model

We construct a monetary Schumpeterian growth model based on Grossman and Helpman (1991) that features endogenous human capital accumulation and heterogeneous households. We introduce money demand via a CIA constraint on R&D as in Chu and Cozzi (2014). Moreover, we model household heterogeneity by assuming that households have different levels of wealth endowments (including financial assets and cash holdings) and human capital endowments, in order to generate an endogenous income distribution.

2.1 Households

The economy is populated by a unit continuum of households indexed by $s \in [0, 1]$. Households share the same preferences over consumption $c_t(s)$ and leisure $q_t(s)$ but differ in wealth and human capital. The lifetime utility function for each household s is given by

$$U(s) = \int_0^\infty e^{-\rho t} \left[\ln c_t(s) + \phi \ln q_t(s)\right] dt, \tag{1}$$

where $\rho > 0$ represents the discount rate, and $\phi > 0$ determines the intensity of the leisure preference relative to consumption. The budget constraint of each household s (expressed in units of final goods) is given by

$$\dot{a}_t(s) + \dot{m}_t(s) = r_t a_t(s) + w_{l,t} l_t(s) + w_{h,t} h_t(s) - \pi_t m_t(s) + i_t b_t(s) + \tau_t - c_t(s), \tag{2}$$

where $a_t(s)$ is the real value of financial assets, and r_t is the real interest rate. Each household provides raw labor $l_t(s)$ and earns the real wage rate $w_{l,t}$. $h_t(s)$ is human capital supplied for production and R&D, where the wage rate is $w_{h,t}$. $m_t(s)$ is the real value of cash holdings by household s and π_t is the inflation rate reflecting the cost of holding money. $b_t(s)$ is the amount of cash borrowed from household s by entrepreneurs for R&D, and i_t is the nominal interest rate. τ_t is the amount of lump-sum transfer that each household receives identically from the government. Therefore, the CIA constraint faced by each household is given by[6]

$$b_t(s) \leq m_t(s). \tag{3}$$

At any point in time t, household s owns an amount of human capital stock denoted by $h_t(s)$. We follow Chu, Cozzi and Liao (2013) to assume that households combine their remaining time endowment $1 - q_t(s)$ with their human capital $h_t(s)$ for work $l_t(s)$ and education $e_t(s)$ subject to

$$h_t(s)\left[1 - q_t(s)\right] = l_t(s) + e_t(s). \tag{4}$$

The law of motion of human capital stock for each household is

$$\dot{h}_t(s) = \gamma e_t(s), \tag{5}$$

where $\gamma > 0$ is a productivity parameter. The optimization problem is to maximize the discounted lifetime utility in (1) subject to the budget constraint (2) and the CIA constraint, together with the time allocation constraint (4) and the human capital accumulation technology (5). Solving this problem gives rise to the optimal condition for labor supply[7]

$$w_{l,t} q_t(s) h_t(s) = \phi c_t(s), \tag{6}$$

and the familiar Euler equation

$$\frac{\dot{c}_t(s)}{c_t(s)} = r_t - \rho. \tag{7}$$

We see from (7) that all households have the same growth rate of real consumption such that $\dot{c}_t(s)/c_t(s) = \dot{c}_t/c_t$, where $c_t \equiv \int_0^1 c_t(s)ds$ is the aggregate consumption by all households. Moreover, using the optimality condition for real money balance $m_t(s)$ and $b_t(s)$, the no-arbitrage condition between financial assets and money is given by $i_t = r_t + \pi_t$, which is the Fisher equation. Finally, we derive the following equilibrium condition that equates the returns on assets and human capital such that

$$r_t = \gamma \frac{w_{h,t}}{w_{l,t}} + \frac{\dot{w}_{l,t}}{w_{l,t}} + \frac{\gamma[l_t(s) + e_t(s)]}{h_t(s)}. \tag{8}$$

[6]In addition to capturing the empirical evidence of R&D-cash flow sensitivity, households' financial motives for money holding also include the following reason. In an economy where the money demand is modeled via a CIA constraint on consumption, the distribution of consumption expenditure across households is identical to that of money holdings. This contradicts with the empirical facts documented by Ragot (2014) who found that the distribution of money (M1) across households in Italy and U.S. is more similar to the distribution of financial assets than to that of consumption expenditure.

[7]Detailed derivations are available in Appendix A.1.

2.2 Final goods

Final goods are used for consumption and produced by a mass of identical perfectly competitive firms using intermediate goods as the only production factor. The production function adopts a standard Cobb-Douglas form given by

$$y_t = \exp\left(\int_0^1 \ln x_t(j)dj\right), \tag{9}$$

where $x_t(j)$ denotes the intermediate good $j \in [0, 1]$. From profit maximization, we obtain the conditional demand function for $x_t(j)$ such that

$$x_t(j) = y_t / p_t(j), \tag{10}$$

where $p_t(j)$ is the price of $x_t(j)$.

2.3 Intermediate goods

The differentiated intermediate goods in each industry j is produced by a monopolistic leader who holds a patent on the latest innovation. The leader's products would not be replaced until a new entrant who has a more advanced innovation comes into the market. The production factors for intermediate goods are raw labor l_t and human capital $h_{x,t}$, with the subscript x specifying the human capital devoted to the production sector. The production technology for the current leaders in industry j is

$$x_t(j) = z^{n_t(j)} \left[h_{x,t}(j)\right]^{\alpha} \left[l_t(j)\right]^{1-\alpha}, \tag{11}$$

where the parameter $z > 1$ measures the step size of each quality improvement, and $n_t(j)$ denotes the number of innovations between time 0 and t. The marginal cost of production for the current leader in industry j is therefore given by

$$MC_t(j) = \frac{1}{z^{n_t(j)}} \left[\frac{w_{h,t}(j)}{\alpha}\right]^{\alpha} \left[\frac{w_{l,t}(j)}{1-\alpha}\right]^{1-\alpha}. \tag{12}$$

In each industry of differentiated intermediate goods, the current and previous leaders engage in Bertrand competition. Following Grossman and Helpman (1991), we assume that the markup, which determines the price charged by the current monopolist over the marginal cost, equals to the step size z. Therefore, the amount of monopolistic profit is

$$\Pi_t(j) = \left(\frac{z-1}{z}\right) y_t, \tag{13}$$

where we have used (10). Finally, production factor expenditures for $h_{x,t}(j)$ and $l_t(j)$ are given by

$$w_{h,t}h_{x,t}(j) = \frac{\alpha y_t}{z}, \tag{14}$$

$$w_{l,t}l_t(j) = \frac{(1-\alpha)y_t}{z}. \tag{15}$$

2.4 Innovations and R&D

The net present value of a monopolist owning the top-to-line technology in the industry j is denoted as $v_t(j)$. Equation (13) implies a symmetric equilibrium that $\Pi_t(j) = \Pi_t$ and $v_t(j) = v_t$.[8] Denote by λ_t the *aggregate-level* Poisson arrival rate of innovations, the familiar no-arbitrage condition for the asset value v_t is then given by

$$r_t v_t = \Pi_t + \dot{v}_t - \lambda_t v_t \tag{16}$$

In equilibrium, the return on the asset $r_t v_t$ equals to the sum of flow profits Π_t, the capital gain $\dot{v}_t$, and the potential losses $\lambda_t v_t$ when an entrant succeeds in innovation and thereby replaces the current leader.

There is a unit continuum of R&D firms indexed by $k \in [0,1]$ that employ human capital $h_{r,t}(k)$ for innovation. To capture firms' cash requirement on innovative activity, we follow the existing literature such as Chu and Cozzi (2014) and Zheng, Huang and Yang (2019) to assume that each entrepreneur borrows the amount $b_t(k)$ of money from the households such that $b_t(k) = w_{h,t}h_{r,t}(k)$. The total amount of money required to finance entrepreneurs' innovative activities is $b_t = \int_0^1 b_t(k)dk$. Each household lends the amount $b_t(s) = \theta_{b,t}(s)b_t$ of cash to the innovating firms, where $\theta_{b,t}(s) \equiv b_t(s)/b_t$ denotes the share of bonds owned by household s. The free-entry in R&D sector determines the zero-expected-profit condition for firm k such that

$$v_t\lambda_t(k) = (1+i_t)w_{h,t}h_{r,t}(k). \tag{17}$$

In addition, we follow Chu, Cozzi and Liao (2013) and Chu, Ning and Zhu (2019b) to assume that the *firm-level* arrival rate per unit of time is $\lambda_t(k) = \varphi h_{r,t}(k)/h_t$. The aggregate arrival rate of innovation is therefore given by

$$\lambda_t = \int_0^1 \lambda_t(k)dk = \frac{\varphi h_{r,t}}{h_t} = \varphi\Gamma_t, \tag{18}$$

where $h_{r,t} = \int_0^1 h_{r,t}(k)dk$ is the aggregate human capital demand in the R&D sector, and $\Gamma_t = h_{r,t}/h_t$ denotes the fraction of human capital devoted to R&D sector with respect to the total

[8]See Cozzi, Giordani and Zamparelli (2007) for the justification for a symmetric equilibrium in this type of Schumpeterian growth model.

human capital stock. The assumption is consistent with the empirical findings in Laincz and Peretto (2006) that it is the fraction of resources devoted to R&D sector that determines the aggregate level of the arrival rate of innovation rather than the absolute level of aggregate input. Thus $1 - \Gamma_t = h_{x,t}/h_t$ is the fraction of human capital devoted to manufacturing, where $h_{x,t} = \int_0^1 h_{x,t}(j)dj$ is the aggregate human capital devoted to manufacturing production.

2.5 Monetary authority

Denote the nominal money supply and its growth rate by M_t and ϵ_t, respectively. The real money balance is given by $m_t = M_t/p_{yt}$, where p_{yt} is the price of final goods. Taking the log derivative of $m_t = M_t/p_{yt}$ with respect to time yields $\pi_t = \epsilon_t - \dot{m}_t/m_t$, where $\pi_t \equiv \dot{p}_{yt}/p_{yt}$ is the inflation rate as previously defined. We assume that the monetary policy instrument is the growth rate of money supply ϵ_t, which is exogenously set by the monetary authority. It follows that the inflation rate π_t is endogenously determined according to the Fisher equation. Substituting the above expression into Fisher equation (i.e., $i_t = \pi_t + r_t$) and using the fact that $\dot{m}_t/m_t = \dot{y}_t/y_t$ holds at all times (see equation (25) in the following section for the proof) yields a one-to-one relationship between the nominal interest rate and the nominal money supply, such that

$$i_t = \epsilon_t + \rho. \tag{19}$$

This result implies that i_t is effectively equivalent to ϵ_t, a policy instrument chosen by the monetary authority. Therefore, for analytical convenience, we will use i_t to represent the exogenous policy instrument throughout the rest of this study. Furthermore, we assume the government runs a balanced budget $\tau_t = \int_0^1 \dot{m}_t(s)ds + \int_0^1 \pi_t m_t(s)ds$ by collecting the aggregate seigniorage revenue $\int_0^1 \dot{m}_t(s)ds + \int_0^1 \pi_t m_t(s)ds$ and fully rebating it back to each household in a uniform lump-sum fashion.

3 Equilibrium allocations and the growth effect of monetary policy

This section first characterizes the decentralized equilibrium, then shows that the economy exhibits a unique and stable balanced growth path (BGP) equilibrium, and finally explores the impacts of monetary policy on economic growth.

The decentralized equilibrium in the economy is defined as follows.

Definition 1. *A competitive equilibrium is a set of sequences of prices* $[p_t(j), w_{l,t}, w_{h,t}, r_t, v_t]$, *and a set of sequences of allocations* $[c_t(s), a_t(s), b_t(s), m_t(s), h_t(s), e_t(s), l_t(s), y_t, x_t(j), l_t(j), h_{x,t}(j), h_{r,t}(k)]$ *given the monetary policy* $[i_t]$ *such that (i) heterogeneous households* $s \in [0,1]$ *maximize their utility; (ii) all competitive final-goods firms, monopolistic leaders, and competitive R&D firms maximize their profits; (iii) the R&D entrepreneurs finance their wage payments through borrowing such that* $\int_0^1 b_t(s)ds = b_t =$

$w_{h,t}h_{r,t}$; (iv) final goods market clears: $\int_0^1 c_t(s)ds = c_t = y_t$; (v) raw labor market clears: $\int_0^1 l_t(s)ds = l_t = \int_0^1 l_t(j)dj$; (vi) human capital market clears: $\int_0^1 h_t(s)ds = h_t = h_{r,t} + h_{x,t}$; and (vii) asset market clears: $\int_0^1 a_t(s)ds = a_t = v_t$.

Next, we proceed to solve for the BGP equilibrium. Prior to that, we first establish the following lemma:

Lemma 1. *Holding a constant nominal interest rate i, the economy jumps to a unique steady state where $\Gamma_t = \Gamma$ (the share of human capital for R&D to total human capital stock), $l_t/h_t = l/h$ (the ratio of raw labor to human capital) and $e_t/h_t = e/h$ (the ratio of education to human capital) are stationary over time.*

Proof. See Appendix A.2. □

To derive the BGP, we first substitute (11) into (10) to yield the aggregate production function given by

$$y_t = Z_t(1-\Gamma_t)^{\alpha} h_t^{\alpha} l_t^{1-\alpha}, \tag{20}$$

where Z_t is the aggregate technology defined as

$$Z_t = \exp\left(\int_0^1 n_t(j)dj \ln z\right) = \exp\left(\int_0^t \lambda_v dv \ln z\right). \tag{21}$$

The second equality of (21) is obtained by applying the law of large numbers. We further take log derivative of (20) and use the result of stationary Γ and together with the fact that $\dot{l}_t/l_t = \dot{h}_t/h_t$ in Lemma 1 to obtain the output growth rate denoted by $g_{y,t}$,

$$g_{y,t} \equiv \dot{y}_t/y_t = g_{z,t} + g_{h,t}, \tag{22}$$

where $g_{z,t} \equiv \dot{Z}_t/Z_t$ and $g_{h,t} \equiv \dot{h}_t/h_t$, which are stationary on the BGP. It is explicit that the economy exhibits a two-engine growth: one is driven by R&D and the other by human capital accumulation. To obtain g_z and g_h, we further differentiate the log of (21) with respect to t and get the steady state technology growth rate such that

$$g_z \equiv \frac{\dot{Z}_t}{Z_t} = \lambda \ln z = \varphi\Gamma \ln z. \tag{23}$$

By plugging both g_z and e_t/h_t from (5) into (22) and using the steady state values of Γ and e/h from Lemma 1, we obtain the balanced growth rate of output g_y such that

$$g_y = \varphi\Gamma \ln z + \gamma(e/h). \tag{24}$$

Equation (24) shows that the steady state growth of output depends on Γ, the fraction of total human capital devoted to R&D, and the fraction of time devoted to education rather than to work e/h. To understand the effect of monetary policy on output growth, we investigate how monetary policy influences Γ and e/h in the following analysis.

Moreover, we use the conditions from equations (3), (14), (17), the stationary Γ, and final-goods and asset market clearing conditions to obtain the balanced growth path as shown below such that the equality holds at all times according to Lemma 1:

$$g_y = g_a = g_b = g_m = g_c, \tag{25}$$

where g_a, g_b, g_m and g_c are the balanced growth of a_t, b_t, m_t and c_t, respectively.[9]

3.1 Equilibrium Allocations and Growth Effect

In this section, we investigate the effect of monetary policy (nominal interest rate targeting) on the determinants of R&D growth, Γ and human capital growth, e/h. To derive Γ, we first substitute Euler equation (7) into (16), using the balanced growth conditions $g_y = g_v$ and (18) to obtain $v_t = \Pi_t/(\rho + \varphi\Gamma)$. We then substitute this expression and (13) into the left-hand-side of R&D free-entry condition (17), and replace $w_{h,t}h_{r,t}(k)$ in the right-hand-side of (17) with equation (14) expressed as a function of Γ, such that

$$\frac{(z-1)\lambda}{(\rho+\lambda)(1+i)} = \frac{\alpha\Gamma}{1-\Gamma} \Leftrightarrow \Gamma = \frac{(z-1)\varphi - \alpha\rho(1+i)}{(z-1)\varphi + \varphi\alpha(1+i)}. \tag{26}$$

It is easy to verify that Γ is decreasing in i. The intuition is straightforward. A higher nominal interest rate i increases the borrowing cost for firms to engage in R&D activities relative to manufacturing, the skilled labor is henceforth reallocated from R&D sector to manufacturing sector, resulting in a smaller fraction of human capital utilized in R&D sector and a diminish in the technology growth according to (23). In addition, by applying the BGP conditions, we eventually have[10]

$$\frac{w_h}{w_l} = \frac{\alpha\rho}{\gamma[\alpha + (1-\alpha)(1-\Gamma)]}, \tag{27}$$

$$\frac{l}{h} = \frac{\rho(1-\alpha)(1-\Gamma)}{\gamma[\alpha + (1-\alpha)(1-\Gamma)]}, \tag{28}$$

$$\frac{e}{h} = 1 - \frac{\rho(1-\Gamma)[\phi z + (1-\alpha)]}{\gamma[\alpha + (1-\alpha)(1-\Gamma)]}. \tag{29}$$

From the above expressions, we find that a rise in nominal interest rate i raises l/h, but decrease w_h/w_l and e/h. The intuition behind is that a higher nominal nominal interest rate i, as illustrated above, reallocates high-skilled labor from R&D sector to the manufacturing sector, which increases the marginal product of low-skilled labor as shown in (11). Consequently, it

[9] See Appendix A.2 for the proof.

[10] See Appendix A.2 for the derivations.

increases the relative demand for low-skilled labor and thus results in higher employment of the low-skilled relative to the high-skilled labor l/h. The increase in the relative demand for low-skilled labor pushes down the relative wage of the high-skilled to the low-skilled labor w_h/w_l. Henceforth, a diminish in wage gap between the higher and lower-skilled labors discourages the investment on education, leading to a smaller e/h and lower human capital growth as shown in (5).

By plugging (28) and (29) into expression (11), we can easily verify that the economic growth rate g_y is unambiguously decreasing in i. Accordingly, we establish the following proposition:

Proposition 1. *The growth rates of the aggregate technology, aggregate human capital, and aggregate economy are all decreasing in the nominal interest rate.*

Proof. Proven in the text. □

This result is standard and consistent with the theoretical predictions in Chu and Cozzi (2014), Huang, Yang and Cheng (2017) and Chu, Ning and Zhu (2019b), and also agrees with empirical studies such as Vaona (2012) and Barro (2013).

4 Human capital and wealth distribution

In this section, we discuss the properties of the distribution of human capital and wealth (including the financial assets and bonds issued by firms) and the effects of monetary policy (nominal interest rate targeting) on them.

4.1 Human capital distribution

We denote $\theta_{h,t}(s) \equiv h_t(s)/h_t$ the relative stock of human capital for household s at time t where $\theta_{h,0}(s) \equiv h_0(s)/h_0$ is exogenously given at time 0. The following lemma pertaining to the distribution of human capital greatly simplifies our analysis for income inequality in the next section:

Lemma 2. *For each household s, the share of human capital stock is constant over time and exogenously determined at time 0 such that $\theta_{h,t}(s) = \theta_{h,0}(s)$.*

Proof. Proven in Appendix A.3. □

In the economy, since all households are competitive in both skilled and unskilled labor markets, thus a change in the nominal interest rate that affects the skilled/unskilled wage rate generates a homogeneous effect on households' choices between accumulating human capital and providing raw labor. Moreover, the identical human capital accumulation productivity γ and depreciating rate δ across households jointly determine that all households accumulate human capital at the same rate. It further implies that the relative share of human capital for each household is invariant over time and equals to its initial steady state value. This result also means that the distribution of human capital is neutral to the monetary policy.

4.2 Wealth distribution

Define $d_t(s) \equiv a_t(s) + b_t(s)$ and $d_t \equiv a_t + b_t$ as household s' (tangible) wealth and aggregate wealth at time t, respectively. We further define $\theta_{d,t}(s) \equiv d_t(s)/d_t$ as the relative wealth of household s at time t where $\theta_{d,0}(s) \equiv d_0(s)/d_0$ is exogenously given at time 0. We then obtain a convenient property of the distribution of wealth similar to the human capital distribution in the following lemma:

Lemma 3. *For each household s, its asset share is constant over time and exogenously determined at time* 0 *such that* $\theta_{d,t}(s) = \theta_{d,0}(s)$ *for all t.*

Proof. See Appendix A.4. □

This property is similar to García-Peñalosa and Turnovsky (2006, 2007) and Chu and Cozzi (2018). It implies that the distribution of wealth is neutral to the monetary policy. According to Lemma 2 and 3, the initial human capital distribution and the wealth distribution do not change over time in equilibrium. Therefore, the human capital distribution is determined by the initial endowment of human capital stock, and the wealth distribution is pinned down by the initial financial asset holdings and cash holdings in the economy. However, as we will show in the next section, monetary policy affects the relative share of the two heterogeneities, causing the income distribution to be endogenously determined.

5 Monetary Policy and Income Inequality

In this section, we first derive the income distribution and then analyze the effect of monetary policy on income inequality.

5.1 Income Distribution

The amount of before-transfer income earned by each household s is the sum of the wealth income and wage income such that $I_t(s) = r_t d_t(s) + w_{l,t} l_t(s) + w_{h,t} h_t(s)$. The average income of all households is therefore given by $I_t = r_t d_t + w_{l,t} l_t + w_{h,t} h_t$. Combining both equations, together with (28), (27) and (29), yields the share of income earned by each household s

$$\theta_{I,t}(s) \equiv \frac{I_t(s)}{I_t} = \frac{r_t \theta_{d,t}(s) + \Psi_t \theta_{h,t}(s)}{r_t + \Psi_t}, \tag{30}$$

where $\Psi_t \equiv (w_{l,t}l_t + w_{h,t}h_t)/d_t$ is the ratio of wage income to total wealth. It can be rewritten as

$$\Psi_t = (w_{l,t}l_t/w_{h,t}h_t + 1) \cdot (w_{h,t}h_t/d_t), \tag{31}$$

where

$$\frac{w_{l,t}l_t}{w_{h,t}h_t} = \frac{(1-\alpha)(1-\Gamma)}{\alpha},$$

from (27), and

$$\frac{w_{h,t}h_t}{d_t} = \frac{w_{h,t}h_t}{a_t + b_t} = \frac{\varphi}{1 + i + \Gamma\varphi},$$

is derived by combining $w_{h,t}h_t/a_t = \varphi/(1+i)$ from (17), with (18) and $b_t = w_{h,t}h_t\Gamma$ from the bond market-clearing condition. Moreover, recall from the Euler equation $r = \rho + g_y$, by substituting the above results into (30), we can show that the distribution of income at time t has a mean of one and the following variance

$$\sigma_{I,t}^2 = \sigma_I^2 = \left(\frac{\rho + g_y}{\rho + g_y + \Psi}\right)^2 \sigma_d^2 + \left(\frac{\Psi}{\rho + g_y + \Psi}\right)^2 \sigma_h^2, \tag{32}$$

where we assume a zero covariance (correlation) $\sigma_{d,h} = 0$ as the benchmark and discuss the case of non-zero covariance in the quantitative part.

Equation (32) shows that the income inequality (measured in variance) σ_I^2 can be decomposed into the variance of total wealth σ_d^2 and of human capital σ_h^2, multiplied by their individual weight (the square of interest income share $\left[(\rho + g_y)/(\rho + g_y + \Psi)\right]^2$ and the square of wage income share $\left[\Psi/(\rho + g_y + \Psi)\right]^2$, respectively). Lemma 2 and 3 have shown that the distributions of human capital and wealth are invariant, and are neutral to the change of nominal interest rate. Thus, the impact of i on σ_I^2 can be boiled down to the impact of i on the relative interest-wage income share $(\rho + g_y)/\Psi$. Recall that Proposition 1 has shown that economic growth is decreasing in i. It follows immediately that $(\rho + g_y)$ (which is r, according to equation (7)) is monotonically decreasing in i. This real-interest-rate effect, as identified by Chu and Cozzi (2018), results in more interest income (including asset income and bond income) losses to the rich than to the poor, and thus reduces the relative interest-wage income share $(\rho + g_y)/\Psi$.

We now proceed to investigate the impact of i on the ratio of wage income to wealth Ψ. For ease of illustration, we decompose Ψ into two parts: the wage income ratio of the unskilled to skilled labor $w_{l,t}l_t/w_{h,t}h_t$ and the ratio of skilled wage income to wealth $w_{h,t}h_t/(a_t + b_t)$ (from (31)). There are three channels through which i may affect Ψ. First, a rise in i increases the unskilled-skilled employment ratio l/h and also the wage ratio w_l/w_h between the two groups (from (28) and (27)), leading to an unambiguous rise in their product $w_{l,t}l_t/w_{h,t}h_t$ and in turn Ψ. Second, increasing i would reduce the fraction of skilled labor devoted into R&D and thus the amount of money (bonds) b_t needed for R&D activity, which tends to increase $w_{h,t}h_t/(a_t + b_t)$

and Ψ. Third, a higher i would also increase the asset value a by increasing the unit cost of R&D $(1+i)$ via free entry condition, resulting in a lower $w_{h,t}h_t/(a_t+b_t)$ and Ψ. The overall effect of i on Ψ is ambiguous and contingent on the parameter values (see Proposition 2 below for more detailed discussion and the proof is relegated to Appendix A.5).

Finally, given the relative dispersions of wealth and human capital, we are now able to investigate the monetary effect of i on income inequality. Our results are summarized in the following proposition.

Proposition 2. *Following an increase in the nominal interest rate i from $i = i_0$ for all i in the reasonable range $[0, \hat{i}]$:*[11]

1. *income inequality σ_I^2 will increase for all level of $i > i_0 = 0$ if (i) the relative variance of human capital distribution to wealth distribution is greater (lower) than the relative interest-wage income share at $i = i_0$, i.e., $\sigma_h^2/\sigma_d^2 > (<)[(\rho+g_y)/\Psi]|_{i=i_0}$; and (ii) this relative share is decreasing (increasing) in i, i.e., $d[(\rho+g_y)/\Psi]/di < (>)0$;*

2. *σ_I^2 is a monotonically decreasing or U-shaped function of i if (i) $\sigma_h^2/\sigma_d^2 < (>)(\rho+g_y)/\Psi$, and (ii) $d[(\rho+g_y)/\Psi]/di < (>)0$.*

Proof. See Appendix A.5. □

To understand Proposition 2, we start with the driving forces for $(\rho+g_y)/\Psi$. Recall that $r = \rho + g_y$, a rise in i increases inflation and hence the opportunity cost of engaging in R&D, and thus decreases the return to R&D activities r by reducing g_y. This negative real-interest-rate effect directly contributes to a decline in the relative interest-wage income share $(\rho+g_y)/\Psi$. However, a higher i may either increase or decrease the wage income-to-wealth ratio Ψ through the aforementioned three channels. If a higher i increases Ψ, it decreases $(\rho+g_y)/\Psi$. If a rise in i reduces Ψ, but at a lower rate than the decrease of g_y, the overall effect on $(\rho+g_y)/\Psi$ remains decreasing in i. By contrast, if i decreases Ψ at a higher rate than the decrease of g_y, the overall effect on $(\rho+g_y)/\Psi$ can be increasing in i.

We then turn to discuss the results in the two arguments of Proposition 2. Suppose that $(\rho+g_y)/\Psi$ is decreasing in i, an increase in i unambiguously increases the wage income share $\Psi/(\rho+g_y+\Psi)$ (the weight on σ_h^2) but decreases the interest income share $(\rho+g_y)/(\rho+g_y+\Psi)$ (the weight on σ_d^2) according to equation (32). We assume that the initial steady state of the economy i_0 is endowed with a sufficiently high dispersion of human capital, i.e., $\sigma_h^2/\sigma_d^2 > (\rho+g_y)/\Psi$. Since $(\rho+g_y)/\Psi$ is decreasing in i, the condition $\sigma_h^2/\sigma_d^2 > (\rho+g_y)/\Psi$ never switches sign within the range $[0,\hat{i}]$. As i increases, an increasing share of a larger dispersion σ_h^2 and a decreasing share of a lower dispersion σ_d^2 jointly induces an amplification of income inequality.

[11] The upper bound of the nominal interest rate $\hat{i}$ ensures that $\Gamma \geq 0$ (the fraction of total human capital devoted to R&D), see Appendix A.5 for more details.

By contrast, suppose that $(\rho + g_y)/\Psi$ is increasing in i, in this case, increasing i, instead, decreases the wage income share $\Psi/(\rho + g_y + \Psi)$ while increases its counterpart $(\rho + g_y)/(\rho + g_y + \Psi)$. Assume now that the initial steady state of the economy i_0 is endowed with a sufficiently low dispersion of human capital, i.e., $\sigma_h^2/\sigma_d^2 < (\rho + g_y)/\Psi$. Since $(\rho + g_y)/\Psi$ is increasing in i, the condition $\sigma_h^2/\sigma_d^2 < (\rho + g_y)/\Psi$ never switches sign within the range $[0, \hat{i}]$. As i increases, a decreasing share of a lower dispersion σ_h^2 along with an increasing share of a larger σ_d^2 again induces an exacerbation of income inequality. The above two cases illustrate the first argument of Proposition (2) with σ_I being monotonically increasing in i.

More interesting scenarios would emerge when we switch the sign of the endowment condition in (i) to become $\sigma_h^2/\sigma_d^2 > (<)[(\rho + g_y)/\Psi]|_{i=i_0}$ as shown in the second argument of Proposition 2 while holding everything else the same as the above analysis. Either a monotonically decreasing or a U-shaped relationship between σ_I and i could arise, depending on the feasible range of nominal interest rate $[0, \hat{i}]$.

In particular, along with $(\rho + g_y)/\Psi$ being decreasing in i, if the economy is initially endowed with a sufficiently low dispersion of human capital, i.e., $\sigma_h^2/\sigma_d^2 < (\rho + g_y)/\Psi$, increasing i from i_0 first triggers an increasing share $\Psi/(\rho + g_y + \Psi)$ of a lower σ_h^2 and a decreasing share $(\rho + g_y)/(\rho + g_y + \Psi)$ of a larger σ_h^2, which decreases income inequality σ_I. However, as i continues to increase, it may reach a threshold i^* such that the sign of the above condition will reverse to become $\sigma_h^2/\sigma_d^2 > (\rho + g_y)/\Psi$. From then on, increasing i from i^* increases income inequality just as the case depicted in the first argument. If i never reaches the threshold i^* in its feasible range $[0, \hat{i}]$, then $\sigma_h^2/\sigma_d^2 < (\rho + g_y)/\Psi$ remains, which ensures a decreasing relationship between σ_I and i. While if i is able to exceed the threshold i^* thorough the range $[0, \hat{i}]$, an interesting U-shaped relationship between σ_I and i will arise.

On the contrary, along with $(\rho + g_y)/\Psi$ being increasing in i, if the economy is endowed with a sufficiently high dispersion of human capital, i.e., $\sigma_h^2/\sigma_d^2 > (\rho + g_y)/\Psi$, raising i from i_0 triggers a decrease (an increase) in wage income (interest income) share of a higher σ_h^2 (lower σ_d^2), leading to a reduction in σ_I. Once i crosses the threshold i^* from below, i becomes positively related to σ_I. Again, if i never reaches the threshold i^* within the feasible range $[0, \hat{i}]$, this leads to a decreasing relationship.

Our finding of a monotone relationship in the first argument of Proposition 2 is consistent with Jin (2009), whose result depends solely on the relative dispersions of capital and skill. Our major finding about a U-shape result is also found in Chu, Cozzi, Fan, Furukawa and Liao (2019a) where a U-shaped result holds under a more restricted assumption with zero entry cost. Our quantitative analysis will show that a U-shaped relationship is likely to occur in a reasonable range of parameter configuration and more importantly, it is supported by recent empirical findings.

6 Quantitative analysis

In this section, we calibrate the model to match the empirical moments in the U.S. economy and numerically evaluate the effects of nominal interest rate on economic growth and income inequality. Towards that end we first assign steady state values to the following structural parameters $\{\rho, z, \alpha, \gamma, \varphi, \phi\}$.

We set the discount rate ρ to a conventional value 0.04 as in Chu, Cozzi and Liao (2013) and Chu, Cozzi, Furukawa and Liao (2017). We then calibrate the step size of innovation z by considering an innovation arrival rate of $\lambda = 4.4\%$ which lies in an intermediate range of estimation,[12] and total factor productivity (TFP) growth rate 0.5% for the U.S. economy from 1990-2016 according to the Conference Board Total Economy Database. As for the factor intensity of human capital to output α, we set it to a conventional value of 0.33 as in Mankiw, Romer and Weil (1992) and Chu, Ning and Zhu (2019b). Next, we pin down the R&D productivity φ by the expression of technological growth from (23), and matching it to the TFP growth 0.5%. We then fit the rest of the parameter values by using the long-run average market nominal interest rate i up to 8%, and the above calibrated parameters to obtain a value of 0.3 for φ.

In addition, we jointly calibrate the human capital productivity γ and the leisure preference parameter ϕ. First, the growth rate of human capital is obtained by taking the difference between the average U.S. long-run economic growth rate 2% and the technological growth rate 0.5% using equation (24), which is 1.5%. We further match it to (4) and yield the first equation for our calibration. Next, we substitute the standard moment of working time $q = 2/3$ into (4) and replace the ratio of unskilled to skilled labor l/h and the ratio of education to human capital e/h with their steady state expressions from (28) and (29), respectively, and obtain our second equation for calibration. Using the two equations alongside the R&D intensity among skilled labor Γ from (26) jointly determine the values of γ and ϕ which are 0.12 and 1.93,[13] respectively.

Finally, for the measures of inequality indicators σ_a and σ_h, we use the standard deviations.[14] We find standard deviations of the U.S. income and wealth in a sample from the National Longitudinal Survey of Youth (NLSY) data covering the period 1994-2012 (Maroto, 2017).[15] We first

[12]Various studies have considered different values for the arrival rate of innovations. Lanjouw (1998) reported the probability of obsolescence to be situated within 7% to 12% and Chu and Cozzi (2018) considered a calibrated value of 12.5%. Acemoglu and Akcigit (2012) considered a relatively high value of 33%. Whereas Caballero and Jaffe (2002) and Laitner and Stolyarov (2013) documented a mean rate of creative destruction around 4% and 3.5%, respectively. Here, we set an intermediate value of 4.4% which is within the above range of estimations.

[13]Our calibrated value for ϕ is well consistent with many findings in the literature. Azariadis, Chen, Lu and Wang (2013) reported a leisure preference around 1.4, Chu and Cozzi (2014)'s finding varied in the range $[2.17, 2.22]$ and Cazzavillan and Olszewski (2011)'s calibration showed a value of 2.13.

[14]We do not report the Gini coefficient because the expression for income inequality in equation (32) would involve the procedure for Gini coefficient decomposition from income sources. The decomposition method has been studied extensively in the literature and is rather nontrivial. Most importantly, we lack critical data information available for decomposition. One example is the lack of "rank correlation" data for constructing Pseudo Gini coefficient. See Shorrocks (1982) and Lerman and Yitzhaki (1985) for an in-depth analysis.

[15]Maroto (2017) used NLSY79 cohort, a stratified sample of 12686 respondents between 14 and 22 years old when

calibrate the standard deviation of human capital by inserting the standard deviation values based on NLSY data into equation (32). We then follow Turnovsky and García-Peñalosa (2008) to normalize the standard deviation of wealth σ_d to 1 and express σ_h as 0.293 for our benchmark case. Table 1 summarizes our benchmark parameter values.

Table 1: Parameter values and targeted moments

		Targeted moments			
	λ	q	g_z	g	
	4.4%	2/3	0.5%	2%	
		Parameters			
ρ	z	α	φ	γ	ϕ
0.04	1.12	0.33	0.3	0.12	1.93

6.1 Inflation and growth

Figure 1 displays the quantitative relationship between nominal interest rate and economic growth, starting from the benchmark case with 2% economic growth rate and 8% nominal interest rate using the U.S. data. We find that there is a negative relationship between growth and nominal interest rate in a fairly large range of i from 0 to 50% (i.e., corresponding to π in the range $[-6\%, 44\%]$). An increase in the nominal interest rate decreases economic growth rate, which confirms our model prediction in Proposition 1.

As pointed above, our model features a two-engine growth. A higher nominal interest rate i increases the firms' borrowing cost for R&D activities, and in turn deters technology growth; while at the same time, a shift of skill labor to the manufacturing sector increases the marginal product of low-skilled labor, which reduces the wage gap between skilled and unskilled labor, discouraging education and thus the human capital growth. The model prediction is in line with several theoretical findings such as Chu and Cozzi (2014), Huang, Yang and Cheng (2017) and Chu, Ning and Zhu (2019b).

6.2 Inflation and inequality (benchmark U.S. economy)

Figure 2a-2c plot relationships between inflation and inequality under various dispersion ratios of human capital and wealth, with the nominal interest rate varying within the reasonable range from 0 and 0.5.

Figure 2a presents our benchmark result with $\sigma_h = 0.293$ and shows that the relationship between inflation and income inequality follows a non-monotonic U-shape. It is explicit that the income inequality σ_I decreases in nominal interest rate i for a lower level of i and starts to

first surveyed in 1979, and relied on seven survey waves in years from 1994 till 2012. A total of 4761 individuals were included in his sample.

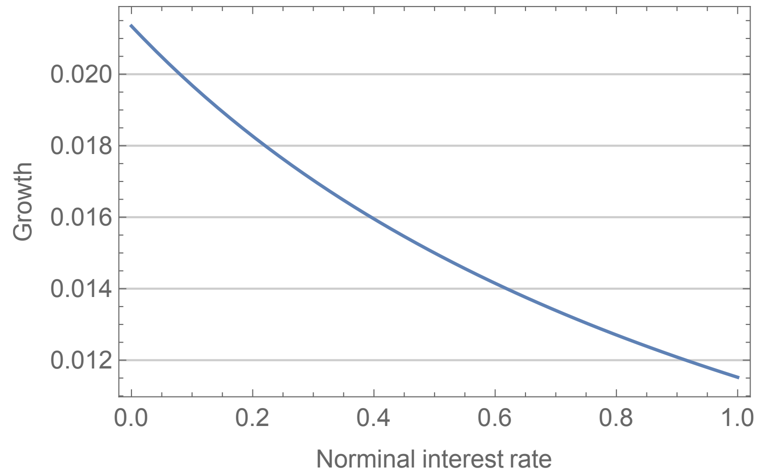

Figure 1: Growth rate and nominal interest rate.

increase in it at a higher level of i. This U-shape result is consistent with the more recent empirical estimates using non-linear regressions with U.S. data (see for example Galli and van der Hoeven (2001) and Balcilar, Chang, Gupta and Miller (2018)). Moreover, our threshold inflation rate of 6% (i.e., corresponds to a threshold nominal interest rate around $i = 0.12$) also coincides the range of the estimates between 5% and 6% found in Bulíř (2001), Galli and van der Hoeven (2001), and Balcilar, Chang, Gupta and Miller (2018).

It is worth noting that the majority of earlier empirical studies regarding developed countries find negative relationships between inflation and income inequality (whereas the U-shaped pattern was found to appear in the U.S. after 1995, according to Monnin (2014)).[16] We therefore adjust σ_h from our benchmark 0.293 up to 0.44 by calibrating it to match a sample of historical data from the Panel Study of Income Dynamics (PSID) as in Shea (1995), covering 1981-1987. Interestingly, Figure 2b shows that the previously shown U-shape result is replaced by a monotonically negative relationship, which is well in line with the earlier empirical findings for the U.S. case. Moreover, for purpose of comparison, we also adjust σ_h^2 from our benchmark 0.293 down to 0.161 using more recent survey data collected in 2013 according to Rauscher (2016).[17] Figure 2c shows that the inequality and inflation are positively related.

The main cause for the above results that display various patterns subject to different periods is mainly attributed to the pattern of human capital heterogeneity across time. Empirical evidence generally suggests that inequality in human capital continues to decline over time in the U.S. (see for example Ziesemer (2016) who covered a time period of 1950-2010). This de-

[16] Most of the studies on the inequality-inflation linkage in developed countries point out negative relationships between the two, with a few exceptions of positive relationships, though they are not robust. See Galli and van der Hoeven (2001) and Monnin (2014).

[17] Rauscher (2016) used the Rosters and Transfers Module of the Panel Study of Income Dynamics which provided transfer information between parents and adult children from a total of 9107 families who took part in the 2013 survey.

clining trend is consistent with the three standard deviations we used for different time periods. The earlier period (1981 to 1987) is characterized by a higher value of σ_h as shown in Figure 2b, corresponding to the scenario where the relative variance of human capital to wealth is larger than the relative interest-wage income share (i.e., $\sigma_h^2/\sigma_d^2 > (\rho + g_y)/\Psi$). Together with the relative share being increasing in i, a negative relationship between σ_I and inflation appears for a plausible range of i before crossing the threshold $\tilde{i}$. Next, our benchmark period (1994 to 2012) is characterized by a smaller value of σ_h than the earlier period indicating that although $\sigma_h^2/\sigma_d^2 > (\rho + g_y)/\Psi$ holds at the benchmark $i = 0.08$, the relative share $(\rho + g_y)/\Psi$ increases in i and eventually the condition could be reversed to $\sigma_h^2/\sigma_d^2 < (\rho + g_y)/\Psi$ as i exceeds the threshold value $\tilde{i} = 0.12$, resulting in a U-shaped relationship between σ_I and inflation. Finally, the most recent data in 2013 gives the smallest value of σ_h as shown in Figure 2c, which delivers a positive relationship between σ_I and inflation. This result is not surprising provided that the condition $\sigma_h^2/\sigma_d^2 < (\rho + g_y)/\Psi$ being held at the initial level of i, and remains to hold as i increases. All the above results conform well to the two arguments in Proposition 2.

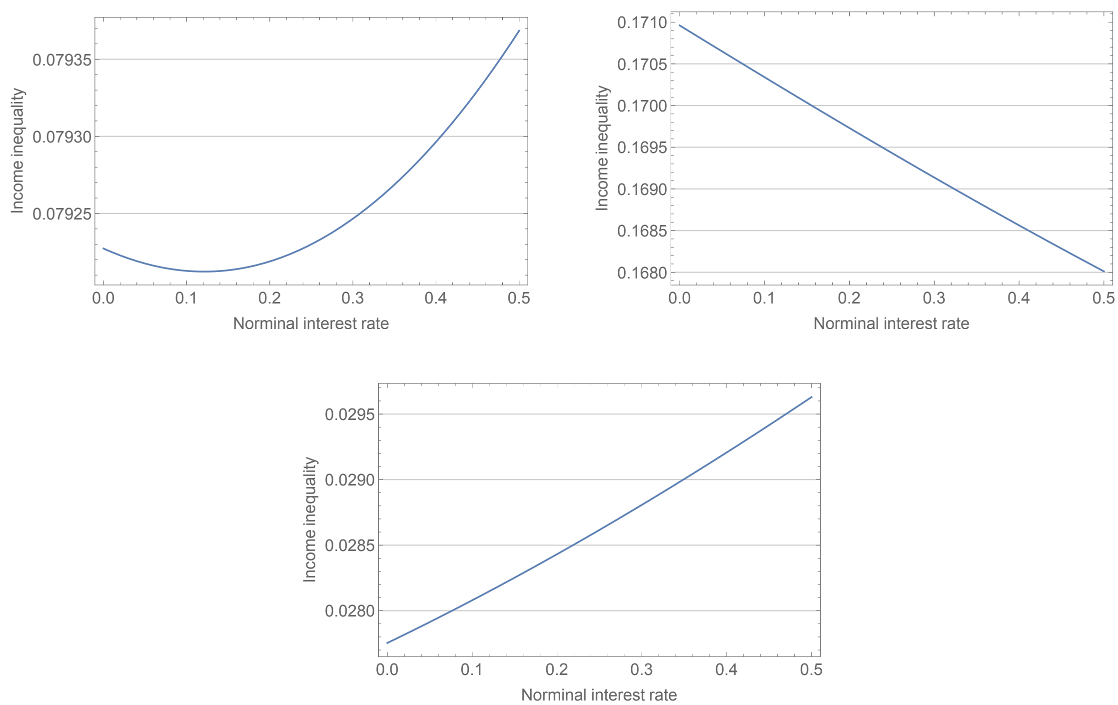

Figure 2: Income inequality and nominal interest rate: (a) $\sigma_h/\sigma_d = 0.293$; (b) $\sigma_h/\sigma_d = 0.44$; (c) $\sigma_h/\sigma_d = 0.161$.

6.3 The correlation (covariance) between wealth and human capital heterogeneities.

In this subsection, we relax our assumption about the independency between wealth and human capital dispersions by integrating their covariance in our sensitivity analysis.

In an empirical study, Pfeffer (2011) used samples from NLSY and PSID data covering years around 2005 to 2007, which contain observations from the U.S. families, and reported the correlation coefficient between family wealth and child education attainment, denoted as $\rho_{d,h}$, to be in the range $[0.288, 0.376]$. We take as our benchmark the average value (0.33) of a set of correlation coefficients documented in their study for $\rho_{d,h}$.[18] Figure 3a shows that under a correlation coefficient $\rho_{d,h} = 0.33$, the income inequality and nominal interest rate become positively correlated for the same set of benchmark parameter values as we used before.

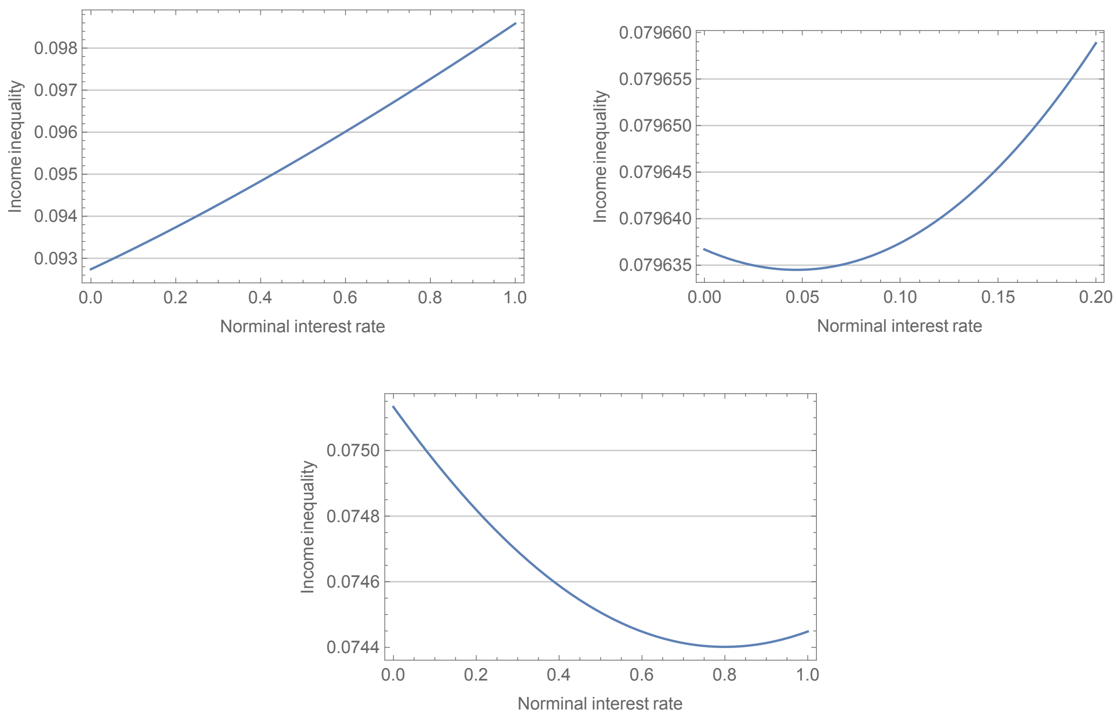

Figure 3: Income inequality and nominal interest rate with non-zero covariance: (a) $\rho_{d,h} = 0.33$; (b) $\rho_{d,h} = 0.01$; (c) $\rho_{d,h} = -0.1$.

In Figure 3b, by lowering $\rho_{d,h}$ to 0.01, we observe the emergence of a U-shaped relationship, while the threshold value of inflation lying at a negative level around -1% (with $i = 5\%$). Finally, we examine a negative correlation at $\rho_{d,h} = -0.1$, which may appear due to the wealth effect that children borne to rich families may have less incentive to achieve education attainment.[19] Figure

[18] García-Peñalosa and Turnovsky (2015) also set the correlation between capital and skills endowments to 0.33 in their numerical analysis.

[19] García-Peñalosa and Turnovsky (2012) argued the correlation between skill and capital endowments can be both

3c shows that with $\rho_{d,h} = -0.1$, a U-shaped result maintains but is accompanied by a much higher threshold level of inflation 74% (nominal interest rate 80%).

7 Conclusion

In this study, we explore the effects of monetary policy on income inequality in a Schumpeterian growth model with human capital accumulation where we consider both asset and human capital heterogeneities. We find that a higher nominal interest rate (inflation) unambiguously leads to a decrease in economic growth rate whereas its impact on income inequality depends on the relative dispersions of human capital and wealth and the response of the relative interest-wage income share to the nominal interest rate. Income inequality may increase (decrease) in inflation rate when the dispersion of human capital dominates (is dominated by) that of wealth and the relative interest-wage income share is decreasing in nominal interest rate. We show that our model can produce a U-shaped relationship between income inequality and inflation which is consistent with the current empirical findings. Moreover, our quantitative example shows that this U-shaped relationship occurs in the range of parameter configuration that matches the U.S. data and the threshold level of inflation which is around 6% is also consistent with the U.S. empirical facts. One future extension can be done by applying a more commonly used and precise measure of income inequality – the Gini coefficient. With more accurate estimations of the Gini coefficient decomposition, one can numerically evaluate the nexus between inflation and income inequality measured in the Gini coefficient to check the robustness of the U-shaped relationship.

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Appendix A

A.1 Derivations of the household optimality conditions.

The Hamiltonian equation is given by

$$\begin{aligned} H = e^{-\rho t}[\ln c_t(s) + \phi \ln q_t(s)] + \eta_t(s)[r_t a_t(s) + w_{l,t} l_t(s) + w_{h,t} h_t(s) + i_t b_t(s) + \tau_t(s) - \pi_t m_t(s) \\ - c_t(s)] + \nu_t(s)\,[m_t(s) - b_t(s)] + \omega_t(s)\gamma e_t(s), \end{aligned} \tag{A.1}$$

Then the first-order conditions for $c_t(s)$, $l_t(s)$, $e_t(s)$, $b_t(s)$, $a_t(s)$, $m_t(s)$, and $h_t(s)$ are respectively given by

$$\frac{e^{-\rho t}}{c_t(s)} = \eta_t(s), \tag{A.2}$$

$$\frac{\phi e^{-\rho t}}{q_t(s)} \frac{1}{h_t(s)} = \eta_t(s) w_{h,t}, \tag{A.3}$$

$$\frac{\phi e^{-\rho t}}{q_t(s)} \frac{1}{h_t(s)} = \omega_t(s)\gamma, \tag{A.4}$$

$$\eta_t(s) i_t = \nu_t(s), \tag{A.5}$$

$$\eta_t(s) r_t + \dot{\eta}_t(s) = 0, \tag{A.6}$$

$$-\pi_t \eta_t(s) + \nu_t(s) + \dot{\eta}_t(s) = 0, \tag{A.7}$$

$$-\eta_t(s) w_{h,t} + \frac{\phi e^{-\rho t}}{q_t(s)} \frac{l_t(s) + e_t(s)}{[h_t(s)]^2} + \dot{\omega}_t(s) = 0. \tag{A.8}$$

Substituting (A.5) into (A.3), and then using equation (A.2) is able to obtain (6). Then differentiating (A.2) with respect to t, and using (A.6) yields the Euler equation (7). In addition, substituting (A.5) into (A.7) to eliminate $\nu_t(s)$, and further making use of (A.6) yield the Fisher equation such that $i_t = r_t + \pi_t$. To derive equation (8), we build up the first equation by dividing (A.3) using (A.4)

$$\frac{\eta_t(s)}{\omega_t(s)} = \frac{\gamma}{w_{h,t}}. \tag{A.9}$$

Differentiating (A.9) with respect to t yields

$$\frac{\dot{\eta}_t(s)}{\eta_t(s)} + \frac{\dot{w}_{l,t}}{w_{h,t}} = \frac{\dot{\omega}_t(s)}{\omega_t(s)}. \tag{A.10}$$

Dividing (A.8) by $\omega_t(s)$, and thereafter substituting (A.3), (A.6) (A.9), and (A.10) yields (8).

A.2 Proof of Lemma 1

Proof for the stationarity of Γ

Suppose that a time path of $[i_t]_{t=0}^{\infty}$ is stationary. First, define a transformed variable $\Phi_t \equiv y_t / v_t$, and its law of motion is given by

$$\frac{\dot{\Phi}_t}{\Phi_t} = \frac{\dot{y}_t}{y_t} - \frac{\dot{v}_t}{v_t}. \tag{A.11}$$

To derive the law of motion for v_t, substituting (13) into (16) yields

$$\frac{\dot{v}_t}{v_t} = r_t + \varphi\Gamma - \left(\frac{z-1}{z}\right) \Phi_t. \tag{A.12}$$

Plugging (7) and (A.12) into (A.11) yields

$$\frac{\dot{\Phi}_t}{\Phi_t} = \left(\frac{z-1}{z}\right) \Phi_t - \varphi\Gamma - \rho. \tag{A.13}$$

We need to derive a relationship between Φ_t and Γ. Using the human capital income condition (14) and the zero-expected-profit condition of R&D (15), we yield an expression to relate Γ to Φ_t

$$\Gamma = 1 - \frac{\alpha(1+i)}{\varphi z}\Phi_t. \tag{A.14}$$

Substituting (A.14) into (A.13) eventually yields an autonomous dynamical equation for Φ_t such that

$$\frac{\dot{\Phi}_t}{\Phi_t} = \frac{z - 1 + \alpha(1+i)}{z}\Phi_t - \varphi - \rho. \tag{A.15}$$

Given that Φ_t is a control variable and the coefficient on Φ_t is positive in (A.15), so that the dynamics of Φ_t is characterized by saddle-point stability in this model such that Φ_t jumps immediately to its interior steady state value given by

$$\Phi = \frac{z(\varphi + \rho)}{z - 1 + \alpha(1+i)}. \tag{A.16}$$

Equations (A.14) and (A.16) imply that when Φ_t is stationary, Γ must be stationary.

Proof of the stationarity of l_t/h_t and e_t/h_t

Next, we turn to prove l_t/h_t and e_t/h_t are stationary. Combining (15) with (14) gives rise to the human capital wage rate relative to raw labor wage rate such that

$$\frac{w_{h,t}}{w_{l,t}} = \frac{\alpha}{(1-\alpha)(1-\Gamma)}\frac{l_t}{h_t}. \tag{A.17}$$

Differentiating (15) with respect to t yields the dynamic condition for real wage rate of raw labor such that

$$\frac{\dot{w}_{l,t}}{w_{l,t}} = \frac{\dot{y}_t}{y_t} - \frac{\dot{l}_t}{l_t} = r_t - \rho - \frac{\dot{l}_t}{l_t}, \tag{A.18}$$

where we have used the condition $c_t = y_t$ and the Euler equation (7).

In addition, from (4) and (8), we can infer that the leisure $q_t(s)$ is identical across households, which in turn implies that $c_t(s)/h_t(s)$ is constant across households according to (6). Thus, households share the same leisure endowment while they differ in the their allocations of time into receiving education or working. Making use of this attribute, substituting equations (14) and (A.17) into (6) yields

$$q_t = \frac{\phi z}{1-\alpha}\frac{l_t}{h_t}, \tag{A.19}$$

where q_t is either the aggregate leisure endowment or the average leisure endowment given the population in the economy is a measure of one. Substituting (A.17), (A.18), and (A.19) into (8) yields

$$\rho = \frac{\gamma\alpha}{1-\alpha}\frac{1}{1-\Gamma}\frac{l_t}{h_t} - \frac{\dot{l}_t}{l_t} + \gamma\left(1 - \frac{\phi z}{1-\alpha}\frac{l_t}{h_t}\right). \tag{A.20}$$

On the balanced-growth equilibrium, $\dot{l}_t/l_t$ must be stationary over time. Therefore, given other exogenous parameters and a constant Γ, $l_t/h_t = l/h$ must be constant over time as well. As a result, e/h must be stationary according to (5), where $e_t \equiv \int_0^1 e_t(s)ds$ is the aggregate raw labor devoted to education. Therefore, we can derive

$$\frac{\dot{l}_t}{l_t} = \frac{\dot{h}_t}{h_t} = \gamma\frac{e}{h} - \delta = \gamma - \delta - \gamma\left[\frac{\phi z}{1-\alpha} + 1\right]\frac{l}{h}. \tag{A.21}$$

Putting (A.21) back into (A.20) gives rise to the following expressions of l/h and e/h on the BGP, respectively,

$$\frac{l}{h} = \frac{\rho(1-\alpha)(1-\Gamma)}{\gamma[\alpha + (1-\alpha)(1-\Gamma)]}, \tag{A.22}$$

and

$$\frac{e}{h} = 1 - \frac{\rho(1-\Gamma)[\phi z + (1-\alpha)]}{\gamma[\alpha + (1-\alpha)(1-\Gamma)]}. \tag{A.23}$$

Derivation of equation (25)

Given the stationary Γ on the BGP, differentiating the log of (17) with respect to time yields

$$\frac{\dot{v}_t}{v_t} = \frac{\dot{w}_{h,t}}{w_{h,t}} + \frac{\dot{h}_{r,t}}{h_{r,t}} = \frac{\dot{w}_{h,t}}{w_{h,t}} + \frac{\dot{h}_{x,t}}{h_{x,t}} = \frac{\dot{y}_t}{y_t} \tag{A.24}$$

where the first equation applies the condition that $\lambda = \varphi\Gamma$ is constant over time, and the last equality applies (14). From the asset market clearing condition, we then have $\dot{a}_t/a_t = \dot{v}_t/v_t = \dot{y}_t/y_t$. The final goods market clearing condition $c_t = y_t$ implies that $\dot{a}_t/a_t = \dot{v}_t/v_t = \dot{y}_t/y_t = \dot{c}_t/c_t$. In equilibrium, households lend all their money to the entrepreneurs such that $b_t = m_t$, which leads to

$$\frac{\dot{b}_t}{b_t} = \frac{\dot{m}_t}{m_t}. \tag{A.25}$$

Finally, according to the bond market clearing condition $b_t = w_{h,t}h_{r,t}$, we have

$$\frac{\dot{b}_t}{b_t} = \frac{\dot{w}_{h,t}}{w_{h,t}} + \frac{\dot{h}_{r,t}}{h_{r,t}} = \frac{\dot{y}_t}{y_t}. \tag{A.26}$$

Therefore, we eventually have

$$\frac{\dot{y}_t}{y_t} = \frac{\dot{a}_t}{a_t} = \frac{\dot{b}_t}{b_t} = \frac{\dot{m}_t}{m_t} = \frac{\dot{c}_t}{c_t}. \tag{A.27}$$

A.3 Proof of lemma 2

We assume that the share has a general distribution function f_h with a mean of one and a variance of σ_h^2. Recall from (8) and (6) that $q_t(s)$ should be identical for all households and thereby $c_t(s)/h_t(s) = c_t/h_t$. In addition, from equation (7), we know that $\dot{c}_t(s)/c_t(s) = \dot{c}_t/c_t$, which in turn implies that the evolution of human capital for all households obeys the same rule such that $\dot{h}_t(s)/h_t(s) = \dot{h}_t/h_t$. The law of motion for human capital share is thereafter given by

$$\frac{\dot{\theta}_{h,t}(s)}{\theta_{h,t}(s)} = \frac{\dot{h}_t(s)}{h_t(s)} - \frac{\dot{h}_t}{h_t} = 0. \tag{A.28}$$

Thus the share of human capital for each household is constant over time.

A.4 Proof of lemma 3

Since household s at any time exhausts all her cash such that $b_t^h(s) = m_t^h(s)$ in equilibrium, thus, households' asset-accumulation function in (2) can be rewritten as

$$\dot{a}_t(s) + \dot{b}_t(s) = r_t[a_t(s) + b_t(s)] - c_t(s) \underbrace{\left[1 - \frac{1-\alpha}{z} - \frac{\alpha}{z(1-\Gamma)}\right]}_{\chi_1} + \tau_t, \tag{A.29}$$

where we have used the condition (6) and (A.17) to express households' wage income in terms of consumption such that $w_{h,t}h_t(s) + w_{l,t}l_t(s) = w_{h,t}h_t(s)[(1-\alpha)(1-\Gamma)/\alpha + 1] = \{(1-\alpha)/z + \alpha/[z(1-\Gamma)]\}c_t(s)$. Aggregating (A.29) for all s yields $\dot{a}_t + \dot{b}_t = r_t(a_t + b_t) - \chi_1 c_t + \tau_t$. Thus, the motion $\theta_{d,t}(s)$ is given by

$$\frac{\dot{\theta}_{d,t}(s)}{\theta_{d,t}(s)} = \frac{\dot{d}_t(s)}{d_t(s)} - \frac{\dot{d}_t}{d_t} = \frac{\chi_1 c_t - \tau_t}{d_t} - \frac{\chi_1 c_t(s) - \tau_t}{d_t(s)} \Leftrightarrow \dot{\theta}_{d,t}(s) = \underbrace{\frac{\chi_1 c_t - \tau_t}{d_t}}_{\chi_2} \theta_{d,t}(s) - \frac{\chi_1 c_t \theta_{c,t}(s) - \tau_t}{d_t}, \tag{A.30}$$

where $\chi_2 = \rho > 0$ is obtained by applying the fact that $\{a_t, b_t, c_t, \tau_t\}$ all grow at the same rate g in equilibrium from Proposition 1 such that

$$\chi_2 = \frac{\chi_1 c_t - \tau_t}{d_t} = r - \frac{\dot{d}_t}{d_t} = \rho > 0.$$

Since $\theta_{d,t}(s)$ is a state variable and the coefficient of $\theta_{d,t}(s)$ is positive, the only solution for the one-dimensional differential equation that describes the potential evolution of $\theta_{d,t}(s)$ given an initial $\theta_{d,0}(s)$, as presented in (A.30), is $\dot{\theta}_{d,t}(s) = 0$ for all $t > 0$. This can be achieved by letting consumption share $\theta_{c,t}(s)$ jump to its steady state value $\theta_{c,0}^h(s)$. Imposing $\dot{\theta}_{dt}(s) = 0$ on (A.30) yields

$$\theta_{c,0}(s) = 1 - \frac{\rho[1 - \theta_{d,0}(s)]}{c_t/d_t}, \tag{A.31}$$

where c_t/d_t can be derived by using $a_t = v_t = \alpha(1+i)c_t/[z\varphi(1-\Gamma)]$ in (14) and (17), and $b_t = \Gamma w_{h,t}h_t = \alpha\Gamma c_t/[z(1-\Gamma)]$ according to (15) and the bond market-clearing condition, that is

$$\frac{c_t}{d_t} = \frac{c_t}{a_t + b_t} = \frac{z\varphi(1-\Gamma)}{\alpha(1+i) + \alpha\Gamma\varphi}. \tag{A.32}$$

A.5 Proof of Proposition 2

To explore the effect of nominal interest rate i on σ_I^2, we rewrite equation (32) as

$$\sigma_I^2 = \left(1 - \frac{\Psi}{\rho + g_y + \Psi}\right)^2 \sigma_d^2 + \left(\frac{\Psi}{\rho + g_y + \Psi}\right)^2 \sigma_h^2,$$

and differentiate σ_I^2 with respect to i yielding

$$\begin{aligned}\frac{d\sigma_I^2}{di} &= -2\left(1-\frac{\Psi}{\rho+g_y+\Psi}\right)\sigma_d^2\cdot\frac{d\left(\frac{\Psi}{\rho+g_y+\Psi}\right)}{di}+2\left(\frac{\Psi}{\rho+g_y+\Psi}\right)\sigma_h^2\cdot\frac{d\left(\frac{\Psi}{\rho+g_y+\Psi}\right)}{di} \\ &= \frac{2\Psi}{\rho+g_y+\Psi}\cdot\frac{d\left(\frac{\Psi}{\rho+g_y+\Psi}\right)}{di}\cdot\left\{\sigma_h^2-\left(\frac{\rho+g_y}{\Psi}\right)\sigma_d^2\right\},\end{aligned} \tag{A.33}$$

where

$$\begin{aligned}\frac{d\left(\frac{\Psi}{\rho+g_y+\Psi}\right)}{di} &= \frac{d\Psi/di}{\rho+g_y+\Psi}-\frac{\Psi\cdot(dg_y/di+d\Psi/di)}{(\rho+g_y+\Psi)^2} \\ &= \frac{dg_y/di}{(\rho+g_y+\Psi)^2}\cdot\left\{(\rho+g_y)\cdot\frac{d\Psi/di}{dg_y/di}-\Psi\right\}.\end{aligned} \tag{A.34}$$

It then follows that

$$\frac{d\left(\frac{\Psi}{\rho+g_y+\Psi}\right)}{di}\gtrless 0\Leftrightarrow(\rho+g_y)\cdot\frac{d\Psi/di}{dg_y/di}\lesseqgtr\Psi\Leftrightarrow\frac{dg_y/di}{d\Psi/di}\gtrless\frac{\Psi}{\rho+g_y}, \tag{A.35}$$

where the inequality applies the condition $dg_y/di<0$. Differentiating $\Psi=[(1-\alpha)(1-\Gamma)/\alpha+1][\varphi/(1+i+\Gamma\varphi)]$ with respect to i yields

$$\frac{d\Psi}{di}=-\frac{\varphi\left[(-\alpha-\alpha i+i+\varphi+1)\Gamma'(i)-(1-\alpha)\Gamma(i)+1\right]}{\alpha[\varphi\Gamma(i)+i+1]^2}. \tag{A.36}$$

Combining dg_y/di derived from (24) with $d\Psi/di$, we can show

$$\frac{dg_y/di}{d\Psi/di}=-\frac{\overbrace{\ln z\cdot\Gamma'(i)}^{<0}+\overbrace{\frac{d(e/h)}{di}\frac{\gamma}{\varphi}}^{<0}}{\underbrace{\{1-\Gamma(1-\alpha)+\Gamma'(i)[\varphi+(1+i)(1-\alpha)]\}}_{\Theta}}\cdot\frac{\alpha}{[\varphi\Gamma(i)+i+1]^2}. \tag{A.37}$$

To examine value of Θ, we insert $\Gamma(i)$ into Θ such that

$$\Theta=\frac{\alpha\left\{-\rho\alpha^2(1+i)^2+\alpha(1+i)[\rho(1+i)+\varphi(i+2z-1)]+\varphi(z-1)(z-1-\rho-\varphi)\right\}}{\varphi(\alpha+\alpha i+z-1)^2},$$

and differentiate Θ with respect to i yields

$$\frac{d\Theta}{di}=\frac{2\alpha^2(z-1)(\rho+\varphi)[(1-\alpha)(1+i)+\varphi]}{\varphi(z-1+\alpha+i\alpha)^3}>0, \tag{A.38}$$

which means that Θ is monotonically increasing in i. We know that i is restricted to lie within the range $[0,\hat{i}]$, where $\hat{i}=[\varphi(z-1)]/(\alpha\rho)-1$ to ensure $\Gamma\geq 0$. Therefore, the maximum of Θ is

$$\Theta|_{i=\hat{i}}=\frac{\varphi(z-1)-\alpha\rho(\rho-z+1)}{(z-1)(\rho+\varphi)}=\frac{\varphi+\alpha\rho\left(1-\frac{\rho}{z-1}\right)}{\rho+\varphi}>0, \tag{A.39}$$

which holds since the conventional values of z and ρ generally satisfy $z-1>\rho$, and the minimum of Θ is

$$\Theta|_{i=0}=\frac{\alpha}{\varphi(\alpha+z-1)^2}\cdot\{\alpha[(2z-1)+\rho(1-\alpha)]+\varphi(z-1)(z-1-\rho-\varphi)\}\,, \tag{A.40}$$

which could be negative or positive. Therefore, equations (A.38), (A.39) and (A.40) together imply that $(dg_y/di)/(d\Psi/di)$ could be either positive or negative for all $i\in[0,\hat{\imath}]$.

Case 1. For a sufficiently large and positive $(dg_y/di)/(d\Psi/di)$, (A.35) means $d(\Psi/(\rho+g_y+\Psi))/di>0$, or equivalently $d((\rho+g_y)/\Psi)/di<0$. Thus from (A.33) we have

$$\frac{d\sigma_I^2}{di}\gtrless 0\Leftrightarrow\frac{\sigma_h^2}{\sigma_d^2}\gtrless\frac{\rho+g_y}{\Psi}. \tag{A.41}$$

Since $d(\Psi/(\rho+g_y+\Psi))/di>0$, which implies that $(\rho+g_y)/\Psi$ is a decreasing function of i, thus if $\sigma_h^2/\sigma_d^2>[(\rho+g_y)/\Psi]|_{i=0}$, then $d\sigma_I^2/di>0$ for $i\in[0,\hat{\imath}]$. It then predicts a monotonically increasing relationship between i and σ_I^2. If $\sigma_h^2/\sigma_d^2<[(\rho+g_y)/\Psi]|_{i=0}$, there may exist an i^* such that $\sigma_h^2/\sigma_d^2<[(\rho+g_y)/\Psi]$ for $0<i<i^*$, and $\sigma_h^2/\sigma_d^2>[(\rho+g_y)/\Psi]$ for $i^*<i<\hat{\imath}$, which together predict a U-shaped relationship between i and σ_I^2; alternatively, σ_I^2 may also be monotonically decreasing in i for $i\in[0,\hat{\imath}]$.

Case 2. For a negative or an insufficiently large positive $(dg_y/di)/(d\Psi/di)$, we have $d(\Psi/(\rho+g_y+\Psi))/di<0$, or equivalently $d((\rho+g_y)/\Psi)/di>0$. Then (A.33) implies

$$\frac{d\sigma_I^2}{di}\gtrless 0\Leftrightarrow\frac{\sigma_h^2}{\sigma_d^2}\lessgtr\frac{\rho+g_y}{\Psi}. \tag{A.42}$$

Since $d(\Psi/(\rho+g_y+\Psi))/di<0$ indicates that $(\rho+g_y)/\Psi$ is increasing in i, thus if $\sigma_h^2/\sigma_d^2<[(\rho+g_y)/\Psi]|_{i=0}$, then $d\sigma_I^2/di<0$ holds for $i\in[0,\hat{\imath}]$, which implies a monotonically increasing relationship between i and σ_I^2. If $\sigma_h^2/\sigma_d^2>[(\rho+g_y)/\Psi]|_{i=0}$, there may exist an i^{**} such that $d\sigma_I^2/di<0$ for $0<i<i^{**}$, and $d\sigma_I^2/di>0$ for $i^{**}<i<\hat{\imath}$, predicting a U-shaped effect of i on σ_I^2. Alternatively, σ_I^2 may also be monotonically decreasing in i for $i\in[0,\hat{\imath}]$.