Heterogeneity, Decentralized Trade, and the Long-run Real Effects of Inflation

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Abstract

Real effects of long-run inflation are studied in a standard matching model. The sign and degree of the output-inflation correlation depend on the cause of inflation and, more specifically, on how the underlying policy assigns money among agents. The correlation may be negative and weak and may be positive and strong. Although inflation increases inequality in wealth, whether it increases inequality or the social difference in welfare depends on the underlying policy. The strong and positive output effect can be compatible with a steep and positively sloped Phillips curve. Policy may be chosen in a way that inflation improves both output and ex ante welfare and there is some base for such a policy to be adopted by policy makers.

JEL Classification Number: E31, E40, E50

Key Words: Heterogeneity, Phillips Curve, Decentralized Trade, Inequality

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1 Introduction

An influential view on long-run inflation is that it has little real effects as long as it can be kept below some level. This view sees superneutrality as a good approximation to reality. After all, empirical studies overwhelmingly support that the Phillips curve is vertical or nearly vertical in the long run. Superneutrality, however, is questionable for at least two reasons. First, evidence from other aggregates is not that clearly cut. One aggregate of great importance is output; evidence on the long-run output-inflation correlation is much mixed. In particular, the correlation may be positive in some countries, including the U.S., when inflation is low; e.g., Ahmed and Rogers [1], Bullard and Keating [5], and Rapach [20]. Second, it is not new that inflation would have redistribution effects by different channels. A widespread narrative says that inflation hurts poor people more than rich. As documented by Easterly and Fischer [9], poor people are more concerned about inflation. Recently, a popular opinion points to the contributing role of monetary policy for the growing inequality of wealth and income. The opinion seems to reach central bankers; e.g., Bernanke [3], Bullard [4], and Constâncio [6].

Here we study the long-run real effects of inflation in the model of Trejos and Wright [25] and Shi [23] with general individual money holdings. This basic model of the New Monetarist economics resembles much of the Bewley model, the workhorse model to study inequality; it is attractive because of the following consideration. In the Bewley model, agents accumulate wealth in spot markets to self-insure against idiosyncratic shocks, shocks that affect the individual ability to earn labor incomes. But the labor-income earning process in reality is rather decentralized, which may affect the individual self-insuring capacity and its reaction to policies. Having agents trade by pairs, the Trejos-Wright-Shi model permits us to explore potential consequences of a decentralized income earning process against a simple background.

We begin with an abstract program that repeatedly transfers money to agents; a transfer is either regressive or progressive and designed to meet a given inflation target. The main findings are (a) both regressive and progressive transfers stretch the distribution of wealth (i.e., increase inequality) with respect to the zero-transfer benchmark; (b) stretching the distribution has a significant and positive effect on output, as long as overall incentives to produce are maintained; (c) only a regressive transfer can maintain overall incentives to uphold its redistribution effect; and (d) if decentralized trade is replaced with centralized trade on spot markets as in the Bewley model, both regressive and progressive transfers have much limited effects on the distribution and output. Finding (b) is in line with a key finding by Jin and Zhu [13] for one-shot transfers in the same model. One shot transfers alter the distribution but barely alter incentives to produce. Repeated transfers alter both and findings (a), (c) and (d) are all related to incentives.

The key to understand incentives is the individual consumption-production risk; although each agent ultimately cares about his consumption stream, he needs to produce to support his consumption and randomness of production matters because the marginal disutility of working is not constant. A regressive transfer enhances the individual consumption-production risk as if having a concave transformation of the value function of nominal wealth, a transformation that maintains overall incentives to produce; a progressive transfer does the opposite and dilutes overall incentives. The progressive transfer paradoxically stretches the distribution because of a general equilibrium effect due to the reduction in the risk—agents dramatically raise their expenditures. The regressive transfer actually discourages agents to spend but the magnitude is much less dramatic, which may be understood on the base that absent any transfer, the risk already sufficiently restrains spending. Centralized trade eliminates the constraint that how much an agent as a seller can adjust up his wealth is subject to the wealth of his trading partner and, hence, greatly reduces the risk which, in turn, limits the degrees of alteration by a transfer on the distribution and aggregate output.

Next we show that financing interests of government bonds by inflation resembles a regressive transfer and we extend the model with bonds in two directions. One extension requires a seller, interpreted as a worker, to pay a cost before he can get employed in the decentralized market. Inflation influences employment mainly through redistributing potential buyers (employers) for the seller, which slightly undermines the seller's participation incentive because the seller prefers working for a buyer with average wealth to a randomly-assigned buyer; so, in particular, inflation slightly raises unemployment. But working hours still increase with inflation because the channel for inflation to affect employment has little to do with either incentives of employed sellers or the redistribution effect of inflation on aggregate output.

Another extension introduces a bond-based money-transfer program which can be regressive, progressive, or hybrid—progressive among poor agents and regressive among rich. Here we ask whether there is a base for agents in the zero-inflation steady state to select an output-increasing inflation policy. We find that mild inflation generated by a hybrid policy can increase output not at the cost of average welfare. In terms of the individual response, the poor side of the society favors a insurance-providing progressive policy; the rich side favors a regressive output-increasing policy; and the poor side is much more sensitive to which policy may be selected. An output-increasing hybrid policy may gain a ground because it reduces rich agents' disfavoring degree to a policy that insures poor agents.

With flexible prices, inflation tends to reduce output because it undercuts people's incentives to obtain money in most familiar models; e.g., the cash-in-advance model, the shopping-time model, and the search-matching models of Lagos and Wright [16] and Shi [24]. A prominent channel for inflation to affect inequality explored by the literature is the consumption-tax channel—poor agents are hit harder by inflation because they (endogenously) rely more on money in payments; see Erosa and Ventura [10] for a leading study. Our contribution is to demonstrate by an off-the-shelf model that with flexible prices, inflation need not undercut the overall incentives to obtain money; it is practically easy to assign more of injected money to the rich even when money is the unique payment method; and inflation policy can be far more important for the poor than the rich.

We describe the model with an abstract transfer program in section 2 and report findings of quantitative analysis in section 3. The model with nominal bonds and its extensions are studied in section 4. We discuss the related literature and some further extensions of our work in section 5.

2 The model with direct money transfer

Time is discrete, dated as $t \geq 0$. There is a unit mass of infinitely lived agents. There exists a durable and intrinsically useless object, called money. Money is indivisible and, without loss of generality, let its smallest unit be 1; the initial money stock is M; there is a finite but arbitrarily large upper bound B on the individual money holdings; and the initial distribution of money π_0 is public information.

Each period t consists of two stages, 1 and 2. At stage 1, the government transfers money to agents in form of lotteries; for an agent holding m units of money at the start of the period, a lottery is a probability measure on the set $\{0, ..., B-m\}$ such that the measure of x is the probability for the agent to receive x units of money from the government. Following Wallace [26], we characterize a transfer policy by a pair of parameters $(C_0, C) \in \mathbb{R} \times \mathbb{R}_+$: the lottery specified by the policy for the agent holding m has the mean equal to $\min\{\max\{0, C_0 + C \cdot m\}, B - m\}$ and has the minimal variance.[1]

At stage 2, each agent has the equal chance to be a buyer or a seller. Following the type realization, each seller is randomly matched with a buyer. In each pairwise meeting, the seller can produce a good only consumed by the buyer. The good is divisible and perishes at the end of the period. By exerting l units of the labor input, each seller can produce $y = l$ units of goods. A trading outcome in the meeting is a lottery on the feasible transfers of goods and money. Without loss of generality, we represent a generic trading outcome by a pair (y, μ), meaning that the seller transfers $y \geq 0$ units of goods with probability one (it is never optimal for agents to randomize on the transfer of goods) and the buyer pays $d \in \{0, \ldots, \min(m^b, B - m^s)\}$ units of money with probability $\mu(d)$. If the seller exerts l units of the labor input, his disutility is

$$c(l) = l^{1+1/\eta} / (1 + 1/\eta), \ \eta > 0. \tag{1}$$

If the buyer consumes y units of goods produced by the seller, his period utility is

$$u(y) = \left[(y + \omega)^{1-\sigma} - \omega^{1-\sigma}\right] / (1 - \sigma), \ \sigma > 0, \tag{2}$$

where ω is a small positive number. The trading outcome is determined by the weighted

[1]The minimal variance is obtained when the support of the lottery is the two integers neighboring the mean of the lottery if the mean is not an integer and the support is the mean otherwise. If money is divisible and $B = \infty$, then the agent receives exactly $\max\{0, C_0 + C \cdot m\}$ amount of money. All lotteries used in this paper are also adopted by Jin and Zhu [13] to mitigate indivisibility of money.

egalitarian solution of Kalai [14],[2] where the buyer's share of surplus is θ. Each agent can observe his meeting partner's money holdings.

At the end of date t, each unit of money independently disintegrates with the probability $\delta_t = 1 - M_t/M_t^+$, where M_t and M_t^+ are the stocks of money before and after the stage-1 transfer at period t, respectively; this disintegration turns the money stock back to M_t and implies $M_t = M$, all t.[3] Each agent maximizes his expected utility with a discount factor $\beta \in (0, 1)$.

To describe equilibrium conditions at period t, let $v_{t+1}(m)$ be the value for an agent holding m units of money at the start of $t+1$ and $\pi_t(m)$ be the proportion of agents who hold m units of money at the start of t. Given the distribution π_t, the proportion of agents who hold m units of money right following the stage-1 money transfer is

$$\hat{\pi}_t(m) = \sum_{m'} \lambda_t(m, m') \pi_t(m'), \tag{3}$$

where $\lambda_t(m, m')$ is the proportion of agents with m' units of money receiving $m - m'$ units of transferred money that is fully determined by the transfer policy (C_0, C) and is described in the appendix. Given the value function v_{t+1}, the value function for an agent holding m units of money right prior to the disintegration of money at the end of period t is

$$\tilde{v}_t(m) = \beta \sum_{m' \leq m} \binom{m}{m'} (1 - \delta_t)^{m'} \delta_t^{m-m'} v_{t+1}(m'), \tag{4}$$

where δ_t is the disintegration probability given $M_t^+ = \sum m\hat{\pi}_t(m)$. Given the value function $\tilde{v}_t$, the trading outcome when a buyer holding m^b meets a seller holding m^s at stage 2 is

$$\left(y_t\left(m^b, m^s\right), \mu_t\left(m^b, m^s\right)\right) = \arg\max_{(y,\mu)} S_t^b\left(y, \mu, m^b\right) \tag{5}$$

subject to

$$\theta S_t^s(y, \mu, m^s) = (1 - \theta) S_t^b\left(y, \mu, m^b\right), \tag{6}$$

where

$$S_t^b\left(y, \mu, m^b\right) = u(y) + \sum_d \mu(d) \left[\tilde{v}_t\left(m^b - d\right) - \tilde{v}_t\left(m^b\right)\right] \tag{7}$$

is the buyer's surplus from trading (y, μ) and

$$S_t^s(y, \mu, m^s) = -c(y) + \sum_d \mu(d) \left[\tilde{v}_t(m^s + d) - \tilde{v}_t(m^s)\right] \tag{8}$$

[2]This bargaining protocol is applied to matching models of money by the recent literature because it preserves concavity of value functions.

[3]The disintegration is introduced by Deviatov and Wallace [8] and also adopted by Jin and Zhu [13] for the purpose to define the individual state by the normalized individual holdings of money, i.e., the ratio of the individual money holdings to the stock of money. If money is divisible, then the disintegration turns m units of money right after the transfer into $m \times (1 - \delta_t)$ units of money. Also, notice that if $B = \infty$, then $M_t^+ = C_0 + CM$ so δ_t does not depend on how money is distributed at the start of t.

is the seller's. Given stage-2 meeting outcomes and the distribution $\hat{\pi}_t$, the value for an agent holding m right prior to the stage-2 meetings is

$$\hat{v}_t(m) = \tilde{v}_t(m) + 0.5 \sum_{m'} \hat{\pi}_t(m') \left[S_t^b(y_t(m,m'), \mu_t(m,m'), m) + S_t^s(y_t(m',m), \mu_t(m',m), m)\right]; \tag{9}$$

the proportion of agents who hold m right prior to date-t disintegration of money is

$$\tilde{\pi}_t(m) = 0.5 \sum_{m^b, m^s} \left[\hat{\lambda}_t^b\left(m, m^b, m^s\right) + \hat{\lambda}_t^s\left(m, m^b, m^s\right)\right] \hat{\pi}_t\left(m^b\right) \hat{\pi}_t(m^s), \tag{10}$$

where $\hat{\lambda}_t^b(m, m^b, m^s)$ and $\hat{\lambda}_t^s(m, m^b, m^s)$ are the proportions of buyers with m^b and the proportion of sellers with m^s, respectively, ending up with m after those buyers meeting those sellers that are fully determined by the payment lottery $\mu(m^b, m^s)$ and are described in the appendix. Finally, the value for an agent holding m at the start of t is

$$v_t(m) = \sum_{m'} \lambda_t(m', m) \hat{v}_t(m'); \tag{11}$$

the proportion of agents who hold m at the start of $t+1$ is

$$\pi_{t+1}(m) = \sum_{m' \geq m} \binom{m'}{m} (1-\delta_t)^m \delta_t^{m'-m} \tilde{\pi}_t(m'). \tag{12}$$

Notice that (3), (10), and (12) determine the law of motion from the distributions π_t to π_{t+1}; and (4), (9), and (11) determine the recursive relationship between the value functions v_t and v_{t+1}.

Definition 1 *Given (π_0, C_0, C), a sequence $\{v_t, \pi_{t+1}\}_{t=0}^{\infty}$ is an* equilibrium *if it satisfies (3)-(12) all t. A pair (v, π) is a* steady state *if $\{v_t, \pi_{t+1}\}_{t=0}^{\infty}$ with $v_t = v$ and $\pi_t = \pi$ for all t is an equilibrium.*

In an equilibrium, *aggregate output* at period t is

$$Y_t = 0.5 \sum_{m^b, m^s} \hat{\pi}_t\left(m^b\right) \hat{\pi}_t(m^s) y_t\left(m^b, m^s\right), \tag{13}$$

the average payment is

$$D_t = \sum_{m^b, m^s} \hat{\pi}_t\left(m^b\right) \hat{\pi}_t(m^s) d_t\left(m^b, m^s\right),$$

the average price is

$$P_t = \sum_{m^b, m^s} \hat{\pi}_t\left(m^b\right) \hat{\pi}_t(m^s) p_t\left(m^b, m^s\right),$$

where $d_t(m^b, m^s) = \sum d\mu_t(d; m^b, m^s)$ and $p_t(m^b, m^s) = d_t(m^b, m^s)/y_t(m^b, m^s)$. We define

$$\varphi_{t+1} = \left(M_t^+/M\right) P_{t+1}/P_t - 1$$

as *the inflation rate*, where $(M_t^+/M)P_{t+1}$ is the average price at $t+1$ if there were no disintegration at the end of t.

Because our interest is in long-run real effects of inflation, below we mainly compare steady states corresponding to different transfer policies with a steady state corresponding to the benchmark no-transfer policy by quantitative analysis. Given a set of policy and non-policy parameter values, we compute a steady state (v, π) such that the value function v is strictly increasing and concave—a value function is *concave* if its linear interpolation is concave. We cannot prove that this sort of steady state is unique for given parameter values. We experiment with many different sets of parameter parameter values; for each set, we start from many different initial conditions but our algorithm always converges to the same steady state. So we refer to the steady state found by our algorithm as *the* steady state corresponding to the set of parameter values. In our quantitative analysis, we fix non-policy parameter values and vary policy parameter values; the *benchmark* policy is the no-transfer zero-inflation policy.

We follow Lagos and Wright [16] to pin down the value of the buyer's surplus θ by markup. The period-t *average markup* in an equilibrium is

$$\sum_{m^b, m^s} \hat{\pi}_t\left(m^b\right) \hat{\pi}_t\left(m^s\right) V_t\left(m^b, m^s\right) / c\left(y_t\left(m^b, m^s\right)\right); \tag{14}$$

where $V_t(m^b, m^s) = \sum_d \mu_t(d; m^b, m^s)[v_t(m^s+d) - v_t(m^s)]$ and $V_t(m^b, m^s)/c(y_t(m^b, m^s))$ is the (expected) markup in a meeting between a buyer with m^b and a seller with with m^s at period t.[4] We identify the value of θ by which the average markup in the benchmark steady state meets a target; we apply this θ when policy deviates from the benchmark. An obvious alternative is to identify a different value θ by which the average markup meets the same target for a different policy; we discuss this alternative in section 3.

We choose B sufficiently large to mitigate effects of bounding one's nominal wealth and M sufficiently large to mitigate effects of indivisibility of money. Through experiments, we find that $B = 150$ and $M = 30$ serve the purposes well. We let the annual discount rate be 4% so that $\beta = 1/(1 + 0.04/F)$ when agents meet F rounds in the decentralized market per year. All results presented in the paper use $(\sigma, \eta, \omega) = (1, 1, 10^{-4})$ (see (1) and (2)) and $F = 4$; we discuss different parameter values in section 3. We choose 1.39 as the target of the average markup in the benchmark steady state, a target that is at the high end of markup values estimated by empirical studies.[5] Given $(\sigma, \eta, \omega) = (1, 1, 10^{-4})$ and $F = 4$, the average markup reaches 1.39 at $\theta = 0.98$ in the benchmark steady state.

[4]The seller's surplus can be written as $(V/Q) \cdot Q - C(Q)$, where $V = V_t(m^b, m^s)$, $Q = c(y_t(m^b, m^s))$, and $C(Q) = Q$; that is, the seller exchanges his present utility loss due to production Q with his future utility gain due to the monetary payment V under the price V/Q. Treating the seller's surplus as his profit, V/Q is the conventional price-marginal cost markup.

[5]This target is suggested by Lagos, Rocheteau and Wright [15] and also adopted by Jin and Zhu [13].

		C_0	C	φ	ΔY	ΔY_π	D	Σ	Gini
decentralized	benchmark	0	0	0	0	0	0.21	1.479	0.117
	regressive	-0.01	0.01	1%	4.94%	4.32%	0.17	1.564	0.183
		-0.02	0.02	2%	11.53%	10.88%	0.15	1.575	0.232
		-0.03	0.03	3%	18.97%	20.66%	0.14	1.562	0.277
	progressive	0.3	0	1%	-2.76%	2.38%	0.76	0.783	0.163
		0.6	0	2%	-4.59%	5.60%	1.34	0.658	0.205
		0.9	0	3%	-6.40%	9.20%	1.92	0.599	0.240
centralized	benchmark	0	0	0	0	0	4.98	0.382	0.267
	regressive	-0.01	0.01	1%	0.17%	-0.003%	4.93	0.381	0.268
		-0.02	0.02	2%	0.33%	-0.002%	4.87	0.379	0.268
		-0.03	0.03	3%	0.49%	0.001%	4.82	0.378	0.268
	progressive	0.3	0	1%	-3.29%	-0.12%	6.71	0.428	0.285
		0.6	0	2%	-5.86%	-0.23%	8.15	0.440	0.303
		0.9	0	3%	-8.03%	-0.38%	9.56	0.421	0.323

Table 1: Steady states under various policy parameters.

3 Real effects of inflation

A government's transfer policy or, simply, a transfer is *regressive* if $C_0 < 0$ and *progressive* if $C_0 > 0$. In this section, we illustrate that and explain why by examples (a) regressive transfers imply a much different inflation-output correlation from progressive transfers and (b) decentralized trade is critical to a positive and significant output-inflation correlation. In examples we use three inflation targets, 1%, 2%, and 3%; we choose transfer parameters such that each pair (C, C_0) leads to a corresponding φ around an inflation target. For regressive transfers, we fix $C_0/C = -1$ (regressiveness requires $C > 0$) and set $C_0 = -0.01$, -0.02, and -0.03, corresponding to $\varphi = 1\%$, 2%, and 3%; for progressive transfers, we fix $C = 0$ (so each transfer is a lump-sum transfer) and set $C_0 = 0.3$, 0.6, and 0.9, corresponding to $\varphi = 1\%$, 2%, and 3%. By these examples, we also illustrate and explain why (a') regressive and progressive transfers have a similar effect on the distribution of wealth and (b') decentralized trade is critical to those effects of transfers on the distributions.

Contrast between regressive and progressive transfers

The upper part of Table 1 reports the relative change in aggregate output $\Delta Y = Y'/Y - 1$, i.e.,

$$\Delta Y = 0.5 \sum_{m^b, m^s} \left[\hat{\pi}'\left(m^b\right)\hat{\pi}'\left(m^s\right)y'\left(m^b, m^s\right) - \hat{\pi}\left(m^b\right)\hat{\pi}\left(m^s\right)y\left(m^b, m^s\right)\right]/Y$$

for each of the three regressive transfers and three progressive transfers in our exercise. Throughout, we remove the time subscription from an object X_t in an equilibrium

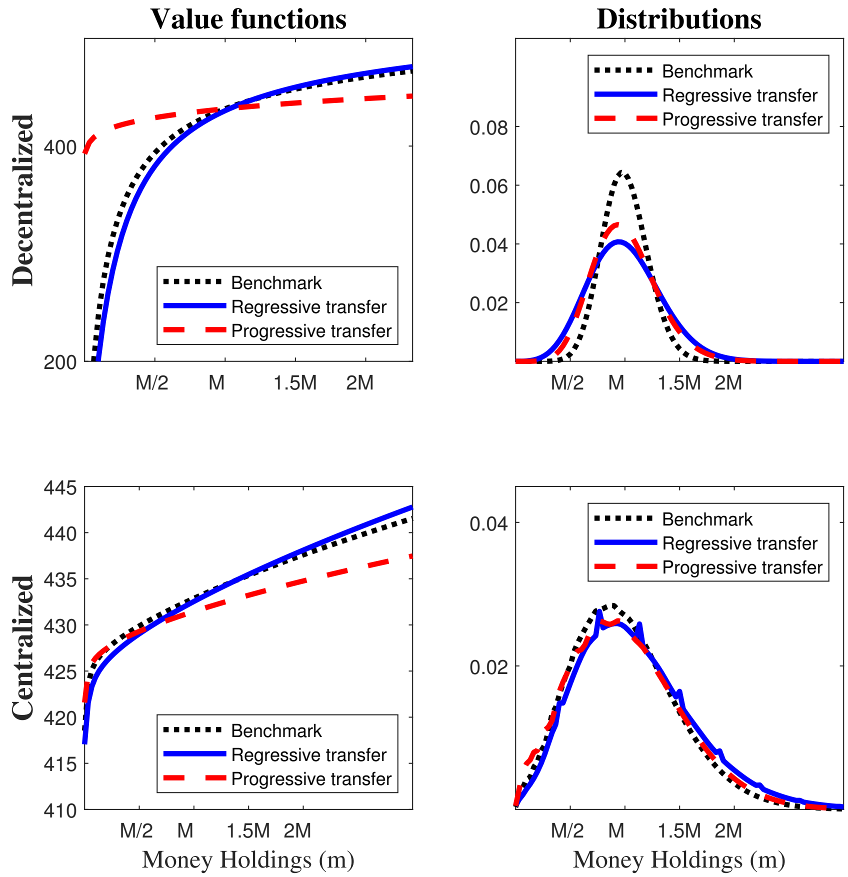

Figure 1: Steady-state value functions and distributions under $(C, C_0) = (0, 0)$ (benchmark), $(C, C_0) = (0.01, -0.01)$ (regressive transfer), and $(C, C_0) = (0, 0.3)$ (progressive transfer), respectively. Upper: decentralized trade; lower: centralized trade.

to represent that object in a steady state; and when expressing a relative change of the object from the zero-inflation benchmark to another steady state, we use X and X' to represent the object's values in the benchmark and the other steady state, respectively. We can see that regressive transfers give rise to a significant and positive output-inflation correlation and progressive transfers do the opposite. To understand why different transfers lead to different output-inflation correlations, we first analyze how a transfer affects the value function and distribution relatively to the benchmark; next we explain the relative change in aggregate output caused by the transfer by how it affects the value function and distribution.

The upper part of Figure 1 displays steady-state value functions and distributions for $(C, C_0) = (0.01, -0.01)$ ($\varphi = 1\%$), $(C, C_0) = (0, 0.3)$ ($\varphi = 1\%$), and $(C, C_0) = (0, 0)$

($\varphi = 0$).[6]

How each transfer reshapes the benchmark value function may be related to the individual consumption-production risk. The risk stems from idiosyncratic shocks underlying heterogeneity of wealth in the model; curvature of the value function indicates directly how much an agent is averse to variation in wealth and indirectly how much the risk is experienced by the agent. Qualitatively, a regressive transfer enhances the individual risk (as it offers more to an agent when he is rich than when he is poor), acting on the benchmark value function as if making a concave transformation; a progressive transfer does the opposite. Quantitatively, we measure the experienced consumption-production risk in the steady state (v, π) corresponding to a given policy by

$$\Sigma = \sum_{m} \pi(m) \varsigma(m), \tag{15}$$

referred to as the *indirect risk aversion*, where $\varsigma(m)$ is the relative risk aversion at m for a smooth approximation of the function v. The value of Σ is 1.479 at the benchmark, moving up to 1.564 for $(C, C_0) = (0.01, -0.01)$ and down to 0.783 for $(C, C_0) = (0, 0.3)$,[7] as reported in Table 1; those numbers indicate that there is a substantial risk at the benchmark and that a transfer may change the risk by a great magnitude. The substantial benchmark risk permits a risk-reducing force to act more evidently than a risk-enhancing force (the progressive transfer in the upper part of Figure 1 seems to reshape the value function more).

The direction that the regressive transfer reshapes the benchmark distribution in Figure 1 is anticipated but, why does the regressive transfer reshape it in the same direction? This is again related to the individual consumption-production risk. A transfer has an *assignment effect* on the distribution—it disperses the distribution if $C_0 < 0$ and squeezes if $C_0 > 0$; it also has a general-equilibrium *expenditure effect* on the distribution—it disperses the distribution if agents tend to spend more and squeezes if less. A progressive transfer encourages agents to increase their payments by reducing the risk; a regressive transfer does the opposite. The average payment is 0.21 at the benchmark, moving up to 0.76 for $(C, C_0) = (0, 0.3)$ and down to 0.17 for $(C, C_0) = (0.01, -0.01)$, as reported in Table 1. A dramatic change in payments due to a progressive transfer may be attributed to a great reduction in the risk and, it easily allows the expenditure effect to be the dominant factor. A far limited change in payments due to a regressive transfer may be attributed to the fact that payments are already on a low level at the benchmark and, it renders the dominant role to the assignment effect.

[6] The upper part of Figure 1 does not display the value functions over a small neighborhood of zero. Over this neighborhood, the increments of the value functions for $(C, C_0) = (0.01, -0.01)$ and $(C, C_0) = (0, 0.3)$ are close to 400.

[7] By definition, both the change in the value function and the change in the distribution caused by a transfer contribute to the change in the indirect risk aversion. But for each steady state, the standard deviation of $\varsigma(m)$ is no greater than 0.11; that is, the change in the indirect risk aversion is mostly contributed by and, hence, gives a good indication about the change in the value function.

Now we can explain why a transfer may cause a certain relative change in aggregate output. To begin with, let the *redistribution effect* of the transfer on aggregate output be defined by

$$\Delta Y_\pi = 0.5 \sum_{m^b,m^s} \left[\hat{\pi}'\left(m^b\right)\hat{\pi}'\left(m^s\right) - \hat{\pi}\left(m^b\right)\hat{\pi}\left(m^s\right)\right] y\left(m^b, m^s\right)/Y,$$

which contributes to ΔY solely by reshaping the distribution (although the distribution $\hat{\pi}$ is not the same as the distribution π, the two distributions are altered by the transfer similarly). As reported in Table 1, regressive and progressive transfers all have positive and significant redistribution effects. This is in line with a finding in Jin and Zhu [13] for shot-one transfers; that is, a transfer can increase aggregate output by stretching the distribution, provided that overall incentives to produce are maintained.

For a one-shot transfer, the redistribution effect contributes to almost all the change in aggregate output; but this is not the case for repeated transfers because repeated transfers affect the value function, which determines incentives to produce or, simply, meeting output. Indeed, a progressive transfer here undercuts incentives in most meetings because its way of reshaping the value function lowers the incremental values over most money holdings; a regressive transfer may maintain and even further enhance incentives because its way of reshaping the value function in general raises the incremental values of money. The part in ΔY complementary to ΔY_π, i.e.,

$$\Delta Y_y = 0.5 \sum_{m^b,m^s} \hat{\pi}'\left(m^b\right)\hat{\pi}'\left(m^s\right)\left[y'\left(m^b, m^s\right) - y\left(m^b, m^s\right)\right]/Y,$$

is actually the weighted change in incentives, with weights assigned by the distribution $\hat{\pi}'$.[8]

Put together, each transfer has a positive redistribution effect but only a regressive transfer can maintain incentives to uphold this effect. By this reasoning, it is the risk-enhancing force C_0 instead of the inflating force C in a regressive transfer that induces a positive ΔY_π and assures a positive ΔY_y. In the Table-1 exercise, the ratio of C_0 to C is fixed among regressive transfers so that the risk-enhancing force increases with inflation, explaining the positive output-inflation correlation.[9] On the other hand, whether ΔY_y of a progressive transfer dominates ΔY_π, as all other numbers reported in Table 1, can only be told by computation; but given the degree that the benchmark value function is reshaped by the progressive transfer in Figure 1, it is of no surprise that the transfer has a dominating ΔY_y and a larger C_0 undercuts incentives further more, leading to a more negative ΔY.

[8] Recall that the change in $\hat{\pi}$ affects the value function $\tilde{v}$ by affecting the disintegration probability. So strictly speaking, the change in meeting output is contributed by the change in v and the change in $\hat{\pi}$. But the change in v is the dominant factor because the change in $\hat{\pi}$ is mainly to shift the whole function $\tilde{v}$ down but the change in v affects the incremental values of $\tilde{v}$.

[9] Notably, it is feasible to increase the risk-enhancing force C_0 but decrease inflation with regressive transfers. For those transfers, our analysis indicates a negative output-inflation correlation. As a simple example, fix $C = 0.01$ and decrease C_0 from -0.01 to -0.02 and -0.03. As C_0 falls, inflation moves down from 1% to 0.97% and 0.93%, ΔY_π up from 4.32% to 5.09% and 6.21%, and ΔY up from 4.94% to 5.93% and 7.10%.

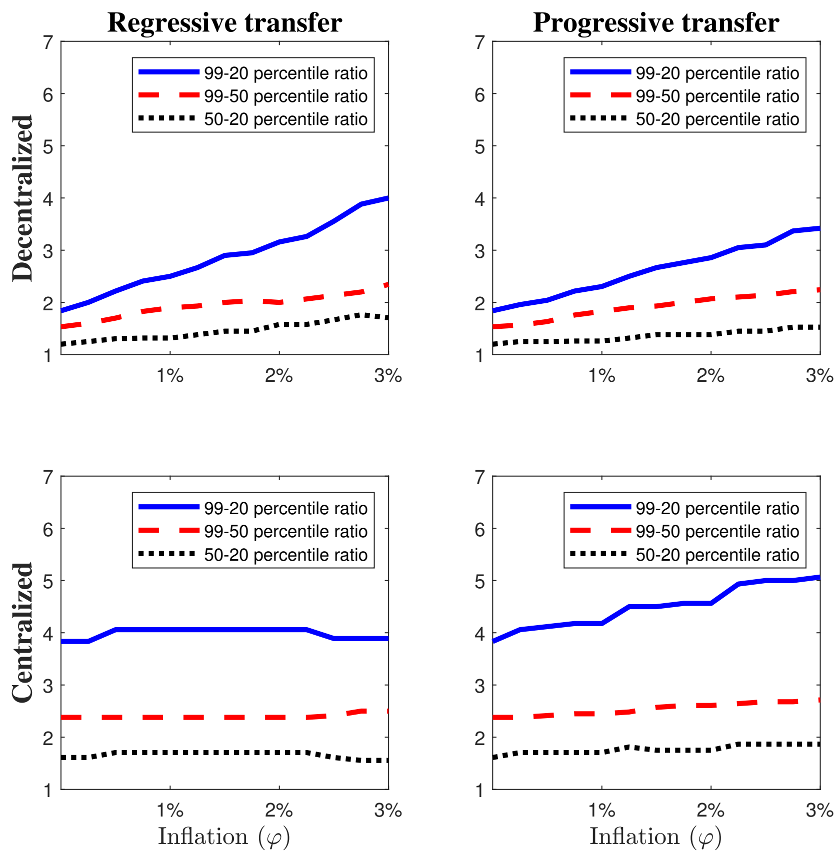

Figure 2: Selected percentile ratios of the wealth distribution under different inflation rates generated by regressive or progressive transfers. Upper: decentralized trade; lower: centralized trade.

In the last column of Table 1, we report the Gini coefficient for the distribution of wealth in each of the seven steady states. In Figure 2 (the upper row), we present three different ratios of percentiles of the wealth distribution, where "*a*-*b* percentile ratio" is the ratio between wealth levels at the ath percentile and at the bth percentile. Apparently, the model in its current form cannot generate the spread of wealth observed in the data (which is discussed more in section 5). The model does tell that the spread of wealth is quite responsive to inflation. Moreover, inflation in the model generates a trend of the spread of wealth that resembles a key feature observed in data; that is, the very top gains significantly with respect to the mid class and to the bottom while the mid class does not gain as much with respect to the bottom. Furthermore, the model offers a novel lesson that the distribution of wealth may not be a sufficient indicator for how unequal a society is. Indeed, the difference in the expected lifetime utility at the two ends of the wealth distribution can either increase or decrease as the distribution becomes more dispersed.

Role of decentralized trade

To see how decentralized trade contributes to output-inflation correlations, we replace pairwise meetings with a centralized market where agents take the price of money ϕ_t as given. That is, a trading outcome for an agent carrying m into the market is (y, μ), meaning that the agent receives y units of goods from the market and pays to the market $d \in \{0, \ldots, m\}$ units of money with probability $\mu(d)$ when he is a buyer, that he surrenders y units of goods to the market and receives from the market $d \in \{0, \ldots, B - m\}$ units of money with probability $\mu(d)$ when he is a seller, and that the mean of the distribution μ is y/ϕ_t. All other aspects of the basic model are unchanged.

Given the constraint of ϕ_t imposed on trading outcomes, equilibrium conditions at period t are again described by the value function v_{t+1} and the distribution π_t. As above, π_t and (C, C_0) fully determine the distribution $\hat{\pi}_t$; for an agent carrying m into the market, the surplus $S_t^b(y, \mu, m)$ from a trading outcome (y, μ) when he is a buyer and the surplus $S_t^s(y, \mu, m)$ when he is a seller are fully determined by v_{t+1} and $\hat{\pi}_t$. The agent's trading outcome is

$$\left(y_t^a(m), \mu_t^a(.; m)\right) = \arg\max_{(y,\mu)} S_t^a(y, \mu, m), a \in \{b, s\}. \tag{16}$$

Market clearing requires

$$\sum_m \hat{\pi}_t(m) \sum_d \mu_t^b(d; m) = \sum_m \hat{\pi}_t(m) \sum_d \mu_t^s(d; m). \tag{17}$$

Given (π_0, C_0, C), a sequence $\{v_t, \pi_{t+1}, \phi_t\}_{t=0}^{\infty}$ is an *equilibrium* if it satisfies the recursive relation between the value functions v_t and v_{t+1}, the law of motion from the distributions π_t to π_{t+1}, and the market clearing condition (17), all t. A tuple (v, π, ϕ) is a *steady state* if $\{v_t, \pi_{t+1}, \phi_t\}_{t=0}^{\infty}$ with $(v_t, \pi_{t+1}, \phi_t) = (v, \pi, \phi)$ all t is an equilibrium. Details of equilibrium conditions are given in the appendix. Now in an equilibrium, aggregate output at period t is $Y_t = 0.5 \sum_m \hat{\pi}_t(m) y_t^s(m)$ and average payment is $D_t = \sum_m \hat{\pi}_t(m) \sum_d \mu_t^b(d; m)$.

The lower part of Table 1 displays output-inflation correlations for the same values of (C, C_0) used in the upper (now $\Delta Y = 0.5 \sum_m [\hat{\pi}'(m) y^{s\prime}(m) - \hat{\pi}(m) y^s(m)]/Y$ for a transfer). Compared to the correlations in the upper part of the table, the positive output-inflation correlation with regressive transfers is much weakened and the negative correlation with progressive transfers is somewhat strengthened. To reveal the force coming with centralized trade, we first analyze how a transfer affects the value function and distribution relatively to the zero-inflation benchmark under centralized trade.

The lower part of Figure 1 displays the steady-state value functions and distributions when $(C, C_0) = (0.01, -0.01)$, $(C, C_0) = (0, 0.3)$, and $(C, C_0) = (0, 0)$ with centralized trade. Those functions and distributions differ from those in the upper part of the figure in four apparent ways. First, value functions are much less concave (this may be confirmed by values of Σ as reported in Table 1). Second, distributions are much more dispersed (e.g., the standard deviation of the benchmark distribution is 14.33 under centralized trade and 6.23 under decentralized trade). Third, the distribution is squeezed (relatively to the benchmark) by the progressive transfer, shifted to the right by the regressive transfer, and is reshaped by either transfer with a much lesser degree; the lesser degree can also be seen from the Ginis and percentile ratios (the last column of Table 1 and the bottom row of Figure 2). Lastly, the value function is reshaped (relatively to the benchmark) on the poor side with a much lesser degree by the progressive transfer.

Centralized trade make these differences because trading by pairs itself is a source of idiosyncratic shocks or, the market provides a sort of insurance absent with decentralized trade. To elaborate, trade being centralized or not, each agent faces an idiosyncratic shock that determines his type at each period. But with the competitive market, y units of goods is exchanged for y/ϕ_t units of expected monetary payments; with pairwise trade, in contrast, how much money y units of goods may be exchanged for depends on the wealth of his trading partner. Intuitively, the insurance is more relevant for a poor agent. For, the agent can adjust up his wealth as a seller on the market according to the price ϕ_t by his desire; but trading by pairs constrains the adjustment by the wealth of his trading partner.

The indirect risk aversion Σ reported in Table 1 does indicate that the market-provided insurance greatly reduces the individual consumption-production risk, explaining the first difference indicated above. With this insurance, agents tend to spend much more ($D = 4.98$ at the benchmark) and so the distribution becomes much more dispersed, explaining the second difference. Given that the average payments are already high, there is little room for the progressive transfer to generate the expenditure effect on the distribution that may dominate the assignment effect but there may be some room for the regressive transfer to generate the expenditure effect that may somewhat offset the assignment effect, explaining the third difference. Finally, the market-provided insurance weakens the insurance role of and, hence, the reshaping force on the benchmark value function from the progressive transfer, explaining the fourth difference.

By their ways of reshaping the value function, a regressive transfer maintains incentives to produce and a progressive transfer undercuts (as they do under decentralized trade). By their ways of reshaping the distribution, a regressive transfer should have an ambiguous and rather small redistribution effect and a progressive transfer should have a negative and not large redistribution effect (now, for a transfer, the redistribution effect on aggregate output of a transfer is $\Delta Y_\pi = 0.5 \sum_m [\hat{\pi}'(m) - \hat{\pi}(m)] y^s(m)/Y$ and $\Delta Y_y = 0.5 \sum_m \hat{\pi}(m)[y'(m) - y(m)]/Y$ complementary to ΔY_π in ΔY). In summary, under centralized trade, the positive output-inflation correlation for regressive transfers is weakened because the redistribution effects are weakened; the negative output-inflation correlation for progressive transfers is strengthened because the redistribution effects become negative.

Different parameter values

In Table-1 exercises, we fix values of the meeting frequency F, the risk aversion coefficient σ in the utility function u, the labor supply elasticity η in the disutility function c, the constant term ω in u, and the buyer's surplus weight θ to demonstrate how two sorts of transfer policies affect output and distribution of wealth under different market structures. The mechanism behind those correlations is built on three properties of the model: (a) there is a substantial consumption-production risk at the benchmark; (b) the risk can be significantly reduced by a progressive transfer; and (c) the risk is significantly reduced if decentralized trade is replaced with centralized trade. Intuitively, those three properties have little to do with F and are robust as long as values of σ, η and ω do not sufficiently flatten the function u or c; moreover, the value of θ would be a factor only if it reduces the benchmark risk.

To confirm, we experiment gradually increasing F from 4 to 365, σ from 0.5 to 2, η from 0.5 to 2, and ω from 10^{-6} to 10^{-2},[10] and θ from 1 to 0.9; the patterns in Table 1 remain valid in our experiments. Notably, when θ moves down from 1 to 0.97, the average benchmark markup moves up from 1 to 1.61; over this range, the value of Σ is decreasing in θ but even at $\theta = 1$, Σ reaches 1.084. The main reason for the monotonic relationship between Σ and θ may be understood as follows: everything else equal, a smaller θ results in a higher consumption-production risk by making the distribution of an agent's consumption by meeting different sellers and the distribution of his production by meeting different buyers more diverse.[11]

Two more comments on θ are in order. First, we fix θ in Table-1 exercises when policy deviates from the benchmark. Alternatively, we may identify a different value of θ by which the average markup meets the markup target for a different policy;

[10] If we interpret ω as home production so that ω/Y is the home production to GDP ratio as in Jin and Zhu [13], then the range of ω covers the estimated home production to GDP ratios.

[11] The monotonic relationship is not global. Indeed, there is no risk at all when $\theta = 0$ (as money is valueless in any equilibrium) and, perceivably, there is less variation in consumption and production as θ approaches to zero.

but the values of θ turn out pretty close to 0.98 (e.g., if $\varphi = 3\%$ then $\theta = 0.988$ for the regressive transfer and $\theta = 0.982$ for the progressive transfer) and, hence, the correlations in Table 1 are not affected. Second, our analysis above does not exclude that ΔY_y may be dominated by ΔY_π for some progressive transfers. Actually this may be the case when θ is sufficiently close to 1 and inflation is sufficiently close to zero: θ matters because everything else equal, a larger value of θ tends to undercut meeting output less for a same progressive transfer; φ matters because $\Delta Y_y/\Delta\varphi$ and $\Delta Y_\pi/\Delta\varphi$ are not constant in φ.

4 The model with nominal bonds

In this section, we focus on policies generating positive output-inflation correlations but not directly implemented by the section-2 transfer program. To this end, we replace that transfer program with discount nominal bonds as follows. At stage 1 of period t, the government issues nominal bonds on a competitive market; each unit of bonds automatically turns into one unit of money at the start of period $t+1$. Each agent chooses a probability measure $\hat{\mu}$ (a lottery) defined on the set $\Xi = \{\zeta = (\zeta_1, \zeta_2) : \zeta_1, \zeta_2 \in \mathbb{Z}_+, 1 \leq \zeta_1 + \zeta_2 \leq B\}$ that satisfies

$$\sum_{\zeta=(\zeta_1,\zeta_2)} \hat{\mu}(\zeta) \cdot \left[\zeta_1 + \zeta_2 (1+i_t)^{-1}\right] \leq m, \tag{18}$$

where m is the amount of money carried by the agent into the market, i_t is the nominal interest rate at t (i.e., $(1+i_t)^{-1}$ is the price of bonds) set by the government who stands to meet any demand on bonds, and $\hat{\mu}(\zeta)$ is the probability for the agent to leave the bond market with the portfolio $\zeta = (\zeta_1, \zeta_2)$ consisting of ζ_1 units of money and ζ_2 units of bonds. At stage 2, agents are matched in pairs as in the section-2 model. In each meeting, each agent can observe his meeting partner's portfolio, but bonds are illiquid and money is the unique payment method. After the meeting, the bonds mature and the money stock is

$$M_t^+ = M_t + L_t \left[1 - (1+i_t)^{-1}\right],$$

where L_t is the stock of bonds. The interest payments $L_t \left[1 - (1+i_t)^{-1}\right]$ are financed by inflation; analogous to the section-2 model, each unit of nominal assets (money and bonds) disintegrates with the probability that restores the nominal stock back to $M_t = M$ at the end of t.

The equilibrium conditions are described by a sequence $\{v_t, \hat{\pi}_t, \pi_{t+1}\}_{t=0}^{\infty}$, where v_t and π_t are the same as in the section-2 model and $\hat{\pi}_t$ is the distribution of portfolios right before pairwise meetings at period t. (We need $\hat{\pi}_t$ as a construct independent from (v_t, π_t) to deal with that the individual portfolio choice is endogenous.) Now the value for an agent to hold the portfolio ζ at the end of pairwise meetings is

$$\tilde{v}_t(\zeta) = \beta \sum_{m' \leq \zeta_1+\zeta_2} \binom{\zeta_1+\zeta_2}{m'} (1-\delta_t)^{m'} \delta_t^{\zeta_1+\zeta_2-m'} v_{t+1}(m'), \tag{19}$$

annual nominal interest	1%	2%	4%	8%	10%
annual inflation	0.97%	1.93%	3.87%	7.73%	9.67%
output change	1.00%	2.13%	4.75%	11.06%	14.53%
annual real interest	0.03%	0.07%	0.13%	0.27%	0.33%
wealth Gini	0.135	0.152	0.181	0.230	0.250

Table 2: Output-inflation correlation and Fisher effect.

where δ_t is the disintegration probability given by $M_t^+ = \sum_\zeta (\zeta_1 + \zeta_2)\hat{\pi}(\zeta)$; the trading outcome $(y_t(\zeta^b, \zeta^s), \mu_t(\zeta^b, \zeta^s))$ when a buyer holding ζ^b meets a seller holding ζ^s at stage 2 is determined by (5) with ζ^b substituting for m^b and ζ^s substituting for m^s; the value for an agent holding ζ right prior to stage-2 meetings is

$$\hat{v}_t(\zeta) = \tilde{v}_t(\zeta) + 0.5 \sum_{\zeta'} \hat{\pi}_t(\zeta') \left[S_t^b\left(y_t(\zeta, \zeta'), \mu_t(\zeta, \zeta'), \zeta\right) + S_t^s\left(y_t(\zeta', \zeta), \mu_t(\zeta', \zeta), \zeta\right) \right]; \tag{20}$$

and the proportion of agents who hold ζ right prior to date-t disintegration of money is

$$\tilde{\pi}_t(\zeta) = 0.5 \sum_{\zeta'} \left[\hat{\lambda}_t^b\left(\zeta, \zeta^b, \zeta^s\right) + \hat{\lambda}_t^s\left(\zeta, \zeta^b, \zeta^s\right) \right] \hat{\pi}_t\left(\zeta^b\right) \hat{\pi}_t\left(\zeta^s\right), \tag{21}$$

where $\hat{\lambda}_t^b(\zeta, \zeta^b, \zeta^s)$ and $\hat{\lambda}_t^s(\zeta, \zeta^b, \zeta^s)$ are analogous to $\hat{\lambda}_t^b(m, m^b, m^s)$ and $\hat{\lambda}_t^s(m, m^b, m^s)$ in (10). The portfolio choice problem for an agent holding m can be expressed as

$$v_t(m) = \max_{\hat{\mu}} \sum_{\zeta} \hat{\mu}(\zeta) \hat{v}_t(\zeta) \tag{22}$$

subject to (18); letting $\hat{\mu}_t(.; m)$ be the $\hat{\mu}$ that solves the problem, then the proportion of agents holding ζ prior to pairwise meetings is

$$\hat{\pi}_t(\zeta) = \sum_m \hat{\mu}_t(\zeta; m)\pi_t(m). \tag{23}$$

The proportion of agents who hold m at the start of $t+1$ is

$$\pi_{t+1}(m) = \sum_{\zeta_1+\zeta_2 \geq m} \binom{\zeta_1 + \zeta_2}{m} (1-\delta_t)^m \delta_t^{\zeta_1+\zeta_2-m} \tilde{\pi}_t(\zeta). \tag{24}$$

Definition 2 *Given* π_0 *and* $\{i_t\}_{t=0}^\infty$, *a sequence* $\{v_t, \hat{\pi}_t, \pi_{t+1}\}_{t=0}^\infty$ *is an* equilibrium *if it satisfies* (19)-(24) *all* t. *If* $i_t = i$ *all* t, *a tuple* $(v, \hat{\pi}, \pi)$ *is a* steady state *if* $\{v_t, \hat{\pi}_t, \pi_{t+1}\}_{t=0}^\infty$ *with* $(v_t, \hat{\pi}_t, \pi_{t+1}) = (v, \hat{\pi}, \pi)$ *all* t *is an equilibrium.*

Quantitative analysis in this section follows the same procedure and adopt the same parameter values as in section 2. Again the benchmark policy is the one with zero inflation. Given that interests are all financed by inflation, the nominal interest rate is zero at the benchmark and inflation increases as the nominal interest rate increases. Table 2 displays a positive output-inflation correlation similar to the one in the upper part of Table 1 (note that a period is a quarter so annual nominal interest and annual inflation are $4i$ and 4φ, respectively). To make sense of this correlation, notice that the expected interest payments for an agent who enters the bond market with m units of money are

$$[m - g(m)]\, i = -g(m)\, i + im,$$

where $g(m)$ is the expected amount of money implied by the agent's portfolio choice. If $g(m)/m$ is decreasing in m, bonds serve as a regressive transfer. While $g(m)$ is increasing in m, it has a narrow range: more than 99% of the agents choose $g(m) = 1$. Approximating $g(m)$ by a constant, raising i is equivalent to raising C and keeping C_0/C in the section-2 model. Ginis for wealth reported in Table 2 remain to be quite responsive to inflation. Table 2 also shows a violation of the Fisher effect; that is, inflation rises less than one-for-one with the nominal interest rate. By definition,

$$(\varphi - i)(1 + i) = [(L/M) - (1 + i)]\,.$$

So if inflation rises on a one-for-one base, then $L = (1+i)M$; that is, all agents should spend all money on bonds, which is obviously not the case.

One may note that government bonds are not all financed by inflation in reality and, naturally, may wonder how much this would affect the inflation-output correlation present in Table 2. As a straightforward experiment, we assume that the government can extract some taxes from the labor incomes in each pairwise transaction and let the tax rate τ be such that for some benchmark nominal interest rate, the tax revenues just cover interests of bonds, i.e., the nominal interest rate at the benchmark is also the real interest rate. When the nominal interest rate moves above the benchmark rate, inflation finances the part of interests not covered by the labor taxes. As an exercise, we choose 1% annual nominal interest rate at the benchmark; this requires $\tau = 0.491$ (notice that the government does not have any other tax revenues). By lowering the bonds price, the inflation-output correlation is quite similar the one in Table 2.

In what follows, we extend the model with nominal bonds (but without real taxes) in two directions, each of which is motivated by a specific issue discussed below.

Extension 1: costly market participation

In our model, a seller may be interpreted as a self-employed worker who sells his product to a buyer or a buyer may be interpreted as an employer who hires a seller to produce for her. Either way, a seller's employment status is not a choice variable while his working hours are.[12] But if sellers can choose their employment status, would it be possible that the employment rate moves little (i.e., the long-run Phillips curve is nearly vertical) when those who get employed tend to work much more on aggregates? To address this issue, we follow Rocheteau and Wright [21] to generate unemployment by a setup of costly market participation.

[12] The seller may be unemployed if he hits the exogenous bound B on nominal wealth or his meeting partner does not have money (the mass of agents with wealth B or 0 is negligible). Or, he may be unemployed if we assume that some meetings are non single coincidence meetings.

annual nominal interest	1%	2%	4%	8%	10%
annual inflation	0.97%	1.93%	3.87%	7.73%	14.50%
unemployment rate	4.83%	4.85%	4.90%	5.01%	5.08%
output change	0.97%	2.07%	4.60%	10.67%	13.99%
wealth Gini	0.133	0.150	0.178	0.225	0.245

Table 3: Unemployment-inflation and output-inflation correlations

Specifically, after a seller learns his type at stage 2, he enters the decentralized market with probability ρ by paying the participation cost

$$k(\rho) = A\rho^{1+\alpha}, \tag{25}$$

where A and α are positive parameters; the seller is randomly matched with a buyer if the seller enters the market and is idle in the rest of stage 2 otherwise. The participation cost is a utility cost; it may include but need not be restricted to utility of leisure given up for one to travel to his workplace in reality. Now equilibrium conditions are described by the same sequence $\{v_t, \hat{\pi}_t, \pi_{t+1}\}_{t=0}^{\infty}$ as given above. For a seller with a portfolio $\zeta^s = (\zeta_1^s, \zeta_2^s)$, his optimal participation choice is

$$\rho_t\left(\overline{\zeta}^s\right) = \arg\max_{\rho} \left[\rho \sum_{\zeta^b} \hat{\pi}_t\left(\zeta^b\right) f_t^s\left(\zeta^b, \zeta^s\right)\right] - k(\rho),$$

where $\overline{\zeta}^s \equiv \zeta_1^s + \zeta_2^s$ and $f_t^s\left(\zeta^b, \zeta^s\right)$ is his surplus if he enters the market and meets a buyer with ζ^b; notice that the solution to the participation problem depends on wealth $\overline{\zeta}^s$ contained in the portfolio ζ^s but not on the composition of the portfolio and that as above, the meeting surplus $f^s\left(\zeta^b, \zeta^s\right)$ is completely pinned down by v_{t+1} and $\hat{\pi}_t$. Details of equilibrium conditions are given in the appendix. In an equilibrium the unemployment rate at period t is

$$U_t = 1 - \sum_{\zeta^s} \hat{\pi}_t\left(\zeta^s\right) \rho_t\left(\overline{\zeta}^s\right),$$

and aggregate output is

$$Y_t = 0.5 \sum_{\zeta^b, \zeta^s} \hat{\pi}_t\left(\zeta^b\right) \hat{\pi}_t\left(\zeta^s\right) \rho_t\left(\overline{\zeta}^s\right) y_t\left(\zeta^b, \zeta^s\right).$$

In quantitative analysis, we target unemployment rate around 4.8% and $\rho(0) = 1$ at the benchmark. Given non-policy parameter values specified in section 2, the pair (A, α) in (25) that meets the target is $(0.0369, 5)$, with which the participation costs are worth of 3% of consumption per period. Table 3 shows a positively-sloped but rather steep Phillips curve: as annual inflation rises from 0 to 10%, unemployment rate merely increases from 4.8% to 5.1%. Ginis for wealth are similar to those in Table 2.

To reconcile the Phillips curve and output-inflation correlation in Table 3, let the redistribution effect on aggregate output of a transfer be defined by

$$\Delta Y_\pi = 0.5 \sum_{\zeta^b, \zeta^s} \left[\hat{\pi}'\left(\zeta^b\right) \hat{\pi}'\left(\zeta^s\right) \widetilde{\rho}\left(\overline{\zeta}^s\right) - \hat{\pi}\left(\zeta^b\right) \hat{\pi}\left(\zeta^s\right) \rho\left(\overline{\zeta}^s\right)\right] y\left(\zeta^b, \zeta^s\right) / Y$$

and let the redistribution effect on unemployment of the transfer be defined by

$$\Delta U_\pi = \sum_{\zeta^s} \hat{\pi}(\zeta^s)\,\rho\left(\overline{\zeta}^s\right) - \sum_{\zeta^s} \hat{\pi}'(\zeta^s)\,\widetilde{\rho}\left(\overline{\zeta}^s\right),$$

where $\widetilde{\rho}\left(\overline{\zeta}^s\right)$ is the solution to the participation-decision problem under the distribution $\hat{\pi}'$ when the incentive component (i.e., the meeting surplus) in the problem is fixed at the benchmark value $f^s\left(\zeta^b,\zeta^s\right)$, that is,

$$\widetilde{\rho}\left(\overline{\zeta}^s\right)=\arg\max_{\rho}\left[\rho\sum_{\zeta^b} \hat{\pi}'\left(\zeta^b\right) f^s\left(\zeta^b,\zeta^s\right)\right] - k(\rho).$$

As above, for the transfer, let ΔY_y be the part complementary to ΔY_π in the relative change in aggregate output $\Delta Y = Y'/Y - 1$; let ΔU_y be the part complementary to ΔU_π in the change in the unemployment rate $\Delta U = U' - U$.

When annual inflation rises from zero to 5%, $(\Delta Y, \Delta Y_\pi) = (2.98\%, 2.61\%)$ and $(\Delta U, \Delta U_\pi) = (0.12\%, 0.10\%)$. Why does ΔU_π differ from ΔY_π? It helps to first see why ΔU_π is positive. When a seller enters the decentralized market, the mapping from the buyer's wealth to the seller's surplus is increasing and has a *concave* shape. So spreading the distribution lowers the expected surplus for the seller and hence, $\widetilde{\rho}(\overline{\zeta}^s)$ is less than $\rho(\overline{\zeta}^s)$. This is the main contributor to ΔU_π. Next, we note that the presence of $\widetilde{\rho}(\overline{\zeta}^s)$ and $\rho(\overline{\zeta}^s)$ in ΔY_π has little influence on ΔY_π because the difference between $\widetilde{\rho}$ and ρ is of secondary order compared to the difference between $\hat{\pi}'$ and $\hat{\pi}$: the weighted averages of $\left|\widetilde{\rho}(\overline{\zeta}^s) - \rho(\overline{\zeta}^s)\right|$ and $|\hat{\pi}'(\zeta) - \hat{\pi}(\zeta)|$ are 0.14% and 1.62%, respectively. This is easy to understand: the difference between $\widetilde{\rho}$ and ρ is a consequence of the difference between $\hat{\pi}'$ and $\hat{\pi}$ as the input to the participation problem.

Why does ΔU_y differ from ΔY_y? By definition,

$$\Delta Y_y = 0.5\sum_{\zeta^b,\zeta^s} \hat{\pi}'\left(\zeta^b\right)\hat{\pi}'\left(\zeta^s\right)\left[\rho'\left(\overline{\zeta}^s\right) y'\left(\zeta^b,\zeta^s\right) - \widetilde{\rho}\left(\overline{\zeta}^s\right) y\left(\zeta^b,\zeta^s\right)\right]/Y.$$

Recall that when a regressive transfer makes a concave transformation of the value function, the transforming effect largely falls on agents with below average wealth; those agents contribute the most of ΔY_y when they are employed sellers because the transformation gives them incentives to produce more for a fixed amount of payments. But those sellers contribute little to

$$\Delta U_y = \sum_{\zeta^s} \hat{\pi}'\left(\zeta^s\right)\left[\widetilde{\rho}\left(\overline{\zeta}^s\right) - \rho'\left(\overline{\zeta}^s\right)\right]$$

and the reason is simple: they are so eager to work that they tend to choose to participate in the market with probability close to one, regardless of which incentive component, benchmark or not, and which distribution component, benchmark or not, are applied to the participation problem; that is, both $\rho'(\overline{\zeta}^s)$ and $\widetilde{\rho}(\overline{\zeta}^s)$ are almost identical to unity.

In summary, inflation by altering the distribution of wealth has a negative effect on the individual seller's labor participation through an incentive channel (i.e., the expected surplus conditional on that the seller enters the market); the change in the labor participation, however, is of secondary order, compared with the change in the distribution of wealth itself, in affecting the distribution of matched portfolios, and, hence, in affecting aggregate output or working hours. On the other hand, while inflation induces employed sellers to work or produce more through another incentive channel (i.e., incremental values of money), the strengthened incentives do not translate into a higher aggregate participation rate because sellers who can be mostly attracted to participate have already been mostly employed. So even when the employment status is solely driven by the seller's or worker's participation decision, inflation leads employment and working hours to move in opposite directions and by much different magnitudes.

In the present extension, we let sellers pay participation costs because this setup directly deals with the question why the employment rate moves little when those who get employed tend to work much more on aggregate. Alternatively, we may let buyers pay the participation costs. Whoever pay the costs, the mechanism for inflation to affect aggregate output is the same and inflation affecting unemployment is essentially through affecting the participation rates. Recall that a seller prefers working for a buyer with average wealth to working for a rich employer or a poor employer by chance. Analogously, a buyer prefers a seller with average wealth so inflation again lowers the participation rates by spreading the distribution; thus the alternative setup leads to unemployment-inflation and output-inflation correlations similar to those in Table 3.[13]

Extension 2: bond-based money transfer program

Would agents in the benchmark steady state select an inflation policy that increases aggregate output above the benchmark? While this question pertains to the policy choice made by heterogeneous agents, it helps to first have a simple statistic that measures the average welfare of a policy; the statistic is the average expected discount utility or *ex ante welfare*

$$W = \sum \pi(m)v(m) \tag{26}$$

in the steady state $(v, \hat{\pi}, \pi)$ corresponding to the policy. Our quantitative analysis shows that the zero-inflation policy always strictly dominates a regressive policy in ex ante welfare. This finding is hardly surprising as regressive policies enhance the individual consumption-production risk. But the finding need not mean a negative answer to the question in concern. After all, ex ante welfare is an aggregate measurement; moreover, some inflation policies may mix regressive and progressive natures and could increase ex ante welfare and output at the same time. To proceed, we explicitly introduce such hybrid policies by attaching a money-transfer program to stage-1 purchasing

[13]Some research suggests that the positive correlation between inflation and unemployment may go beyond the nearly-vertical range; see Berentsen, Menzio and Wright [2]. We suspect that the participation cost alone may not be sufficient to induce a more elastic response of unemployment to inflation in our model.

of bonds as follows.

The government transfers money to agents in forms of lotteries as in section 2; what is new here is how much an agent receives depends on his purchasing of bonds. A transfer policy here is characterized by a pair of non negative parameters (K,λ): if the lottery chosen by an agent on the bond market is realized as some $\zeta = (\zeta_1, \zeta_2)$, then the policy assigns to the agent a lottery $\tilde{\mu}(;\zeta)$; the lottery $\tilde{\mu}(;\zeta)$ has the mean equal to $\min\{K(1+\lambda\zeta_2)^{-1}, B-\zeta_1-\zeta_2\}$ and has the minimal variance (see footnote 1). A transfer policy is *active* if $K > 0$ and *inactive* if $K = 0$. An active policy is apparently progressive and, if $\lambda > 0$, it further resembles a progressive tax on wealth. Now equilibrium conditions are described by the same sequence $\{v_t, \hat{\pi}_t, \pi_{t+1}\}_{t=0}^{\infty}$ as above while the objective function in (22) is modified as

$$v_t(m) = \max_{\hat{\mu}} \sum_{\zeta=(\zeta_1,\zeta_2)} \hat{\mu}(\zeta) \left[\sum_z \tilde{\mu}(z;\zeta)\, \hat{v}_t(\zeta_1 + z, \zeta_2) \right]. \tag{27}$$

Details of equilibrium conditions are given in the appendix.

For quantitative analysis, we begin with what we refer to as welfare-neutral output-inflation correlations. Specifically, for a given positive nominal interest rate we seek a transfer policy just progressive enough to offset the regressive nature of bonds in that the corresponding steady state delivers the same ax ante welfare as the benchmark steady state. Because a transfer policy involves two parameters, we fix λ and seek the suitable value of K, denoted $K(i)$, for each given $i > 0$. For a given λ, we refer to the policy $(i, K(i))$ as a *welfare-neutral policy*; denote by $\varphi(i)$ the inflation rate and by $Y(i)$ aggregate output in the corresponding steady state; let $\Delta Y(i) = Y(i)/Y - 1$, where Y as usual is the benchmark aggregate output; then the *welfare-neutral output-inflation correlation* is given by $\{(\varphi(i), \Delta Y(i))\}_i$.

Figure 3a displays the welfare-neutral output-inflation correlation for $\lambda = 1$. Along the path of inflation in Figure 3, Ginis for wealth range from 0.147 to 0.233, similar to those in Table 2. The correlation pattern fits well with the empirical finding in Bullard and Keating [5]; that is, inflation mildly expands output over a limited range and the expanding effect gradually phases out beyond this range. This pattern is representative in that not only the basic reverse U shape is preserved but the peaking point and magnitude do not move much for other values of λ.

To understand this pattern, we display in Figure 3b value functions with annual inflation at 0, 1%, and 3% along the correlation in Figure 3a. The main observation is the following. In partial equilibrium, a rise in i strengthens the regressive feature of the policy but, in general equilibrium, the rise in K to maintain ex ante welfare at the benchmark level effectively leads the whole policy $(i, K(i))$ to perform as a policy more progressive than the one prior to the rise in i. So when i becomes larger, the value function becomes more flattened and, consistent with our analysis in section 3, the distribution becomes more dispersed. In short, progressiveness of policy $(i, K(i))$ is increasing in i. Consequently, when i is small, a low degree of progressiveness allows the redistribution effect ΔY_π to dominate the incentive effect ΔY_y; this dominance is reversed by a high degree of progressiveness as i grows.

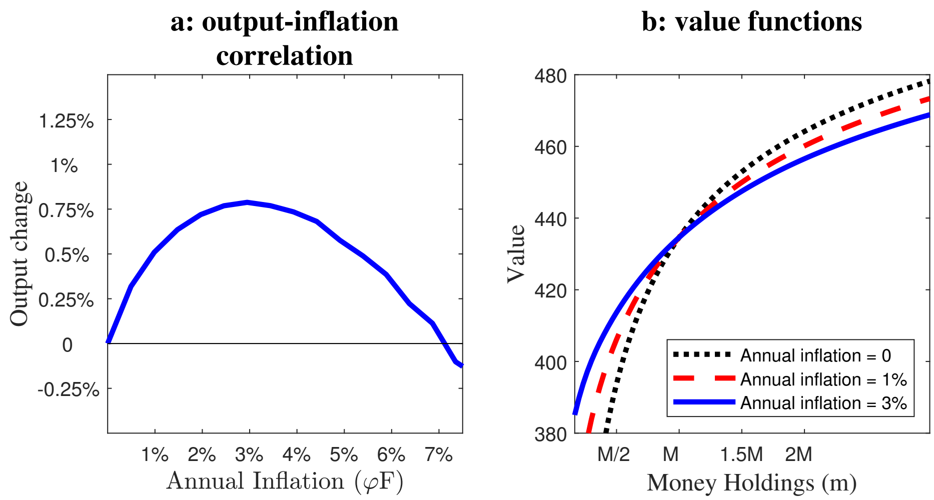

Figure 3: Output-inflation correlations and value functions associated with welfare-neutral bond-based money transfers.

For a fixed λ, we may generate many non welfare-neutral correlations by departing from the welfare-neutral output-inflation correlation. For example, given i, we can raise K above $K(i)$ to have inflation above $\varphi(i)$ and aggregate output below $Y(i)$; this gives rise to a correlation along which ex ante welfare is always above the benchmark value and by controlling the difference between K and $K(i)$, the correlation may resemble the shape of the welfare-neutral correlation. So, in particular, there is some room for both ex ante welfare and aggregate output to move above the benchmark values with inflation over some range. In terms of ex ante welfare, however, we find that an output-increasing inflation policy is always dominated by one with $(i, \lambda) = (0, 0)$ and a small $K > 0$, i.e., a lump-sum policy; this finding is in line with a finding of Molico [18], who shows that a lump-sum policy dominates the zero-inflation policy when the buyer's surplus weight is unity. Notably, the gain in ex ante welfare by a lump-sum policy seems limited—the highest ΔW achieved by a lump-sum policy is around 0.29%.[14]

Regardless of the implied gain or loss being limited or not, ex ante welfare need not be adequate to reflect an individual agent's evaluation of a policy. Taking the question raised at the start of this subsection seriously, we consider how each individual agent responds to a potential policy change. To this end, let $(v, \hat{\pi}, \pi)$ denote the benchmark steady state; let $(v', \hat{\pi}', \pi')$ denote the steady state corresponding to an alternative policy; and let $\{v'_t, \hat{\pi}'_t, \pi'_{t+1}\}_{t=0}^{\infty}$ denote the transitional equilibrium connecting the two steady states, i.e., it starts from the initial distribution $\pi'_0 = \pi$ and converges to $(v', \hat{\pi}', \pi')$ as t goes to ∞. For an agent holding m units of nominal wealth, $v(m)$ is his life-time welfare measured at date 0 if there is no policy change and $v'_0(m)$ is his

[14] The corresponding measurement in consumption units is around 1.31%. We choose to measure welfare gains in expected utility because it is not obvious how to convert the individual welfare change $\delta(m)$ defined by (28) below into a consumption-equivalent object.

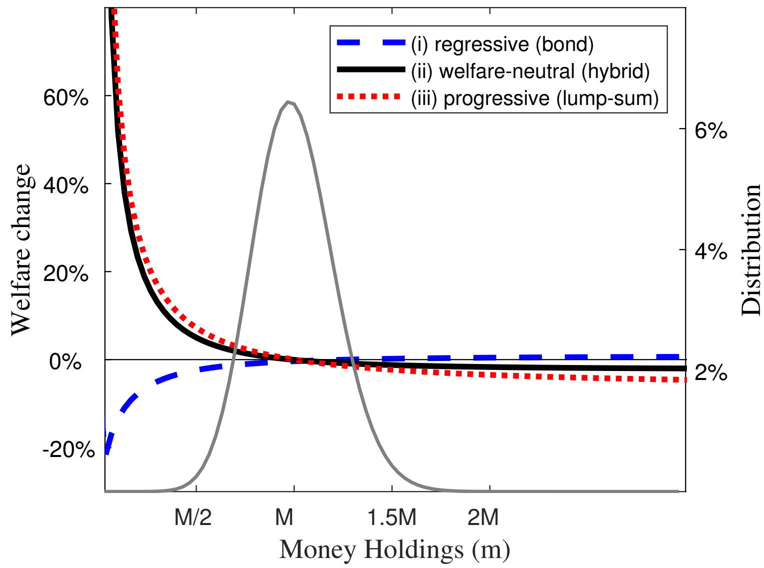

Figure 4: Changes in individual welfare ($\delta(m)$).

life-time welfare measured at date 0 if the alternative policy is adopted; the relative change in the agent's welfare

$$\delta(m) \equiv v_0'(m) / v(m) - 1 \tag{28}$$

gives a measurement regarding how sensitive the agent is to the policy change.

Figure 4 displays three δ functions for three alternative policies leading to inflation: a regressive policy $(K, \lambda, i) = (0, 0, 3\%/4)$; a welfare-neutral policy $(K, \lambda, i) = (K(3\%/4), 1, 3\%/4)$; and a progressive policy $(K, \lambda, i) = (0.174, 0, 0)$ giving the highest ex ante welfare. The figure has three important patterns that are representative for other policy parameter values. First, no inflation policy wins a majority support; for a regressive policy, we can always find a hybrid policy and a progressive policy that have more supporters; for a progressive policy, we can find a hybrid policy that has almost equal supporters. Second, agents in the middle of the wealth distribution are not sensitive to which policy is adopted; moving away from the middle, agents become more and more sensitive; but the change in the individual sensitivity is much more obvious as moving to the poor end. Third, poor agents are much more disfavoring a regressive policy than rich agents favoring the policy; and poor agents are much more favoring a progressive or hybrid policy than rich agents disfavoring the policy.

These responding patterns indicate no simple answer to our question in concern but they are suggestive in several ways. It may be too naive to only count how many people favor a policy but ignore how much some people disfavor the policy. Moreover, the demand for some insurance from the poor side of the society may be a dominant factor for the social choice even when each agent's wealth status is transient. Furthermore, insurance for poor agents served by a progressive policy can be nearly served by a hybrid policy and, the incremental improvement on poor agents due to a change from the hybrid policy to the progressive policy is not in the same degree as the incremental deterioration on rich agents. So a rationale for an output-increasing inflation policy may lie in that it reduces the disfavoring degree of rich agents compared with a policy insuring poor agents.

5 Discussion

While it has never been a mainstream proposition, that inflation may be expansionary can be at least dated back to Hume,

> ...[I]t is of no manner of consequence, with regard to the domestic happiness of a state, whether money be in a greater or less quantity. The good policy of the magistrate consists only in keeping it, if possible, still increasing; because, by that means he keeps alive a spirit of industry in the nation. [Hume [12, p 173]]

Hume, however, did not spell out why increasing the quantity of money may keep alive a spirit of industry. Modern economics does offer some answers. In the presence of capital, the negative incentive effect of inflation on output may be dominated by the Tobin effect; see Orphanides and Solow [19] for a survey, including the anti-Tobin effect that strengthens the negative incentive effect and why superneutrality in Sidrauski [22] may not be robust. Moreover, inflation may be expansionary when agents have nonstandard preferences; e.g., Graham and Snower [11]. Furthermore, it is well known that with nominal rigidity, inflation can raise output as in the New Keynesian model; e.g., Devereux and Yetman [7] and Levin and Yun [17]. In our model, the price is flexible and preferences are standard and, what kind of correlation would emerge depends on how inflation assigns wealth among agents, which may help reconcile the difference in empirical evidence.

It is not a mainstream proposition that monetary policy in general and inflation in specific would play a major role in shaping inequality in the long run, either. Nonetheless three stylized facts in the U.S. economy seem to draw a fair amount of attention from the literature: poor people conduct higher fractions of transactions by cash; poor people hold higher fractions of wealth in cash; and only a fraction of households hold financial accounts. Erosa and Ventura [10] formulate the first heterogeneous-agent model that endogenizes the first two facts. They assume that some agents are more productive than other and paying by some non-cash method is more costly than paying by cash. Inflation is effectively a regressive consumption tax. Motivated by the third fact, Williamson [27] assumes that some agents cannot receive transfer of money from the government. As such, inequality grows with inflation. Our study is complementary to theirs. In our model, inflation can easily be regressive to shift the distribution by a large degree when agents are ex ante identical, all transactions are paid by money, and the financial market and money-transfer program are free to access.

As explained above, our study is capable of offering the novel results because the individual consumption-production risk is much amplified by the decentralized labor earning process in our model. Our model above certainly does not include all realistic aspects that may affect this risk. Three missing aspects are of particular importance. The first aspect is persistency in the idiosyncratic shock. We may let the productivity of an agent as a seller be determined by an idiosyncratic shock and the shock follows, say, an AR(1) process. Such a setting ought to further increase the individual consumption-production risk.

The second is a social safety net. Within the current setting, we may interpret ω in the utility function (see (2)) as a universal-consumption subsidy and choose the level of ω equal to a pre-chosen fraction a of the average consumption (which, by definition, is equal to the average labor income if we convert the labor income into the consumption units) in the zero-inflation steady state. If $a = 25\%$, then $\omega = 0.22$ and the risk aversion Σ is 0.84, sufficient to maintain main patterns of the inflation influence on output and the distribution. A better picture of how the risk is affected may be obtained in the presence of some safety net and persistent idiosyncratic shocks.

The third aspect is intrinsic heterogeneity. We may add to the model a small class of agents who are more productive (as sellers) or more patient or both and, hence, rich overall. Likely, the addition of the rich class would increase the individual consumption-production risk for agents in the non-rich class because the non-rich class only occupies a share of wealth to insure their risks. This conjecture requires some careful check, together with whether the addition of intrinsic heterogeneity may plausibly improve the model's ability to match the degree of inequality observed in data.

Our final remark pertains to the decentralized labor earning process. What is nicely captured by the Trejos-Wright-Shi model about this process is that the wealth status of a buyer or employer restricts on earnings of a seller or employee. Our study is a first step to demonstrate that this feature of the labor-earning process matters a lot in one modeling environment. It is for the future research to sort out whether this feature remains to be a powerful factor in other modeling environments.

Appendix A: Complete description of equilibria

A.1 The basic model

Under a transfer policy (C, C_0) in section 2, the expected amount of money received by an agent holding m units of money is $x(m) = \min\{\max\{0, C_0 + C \cdot m\}, B - m\}$. Let $\lfloor x(m) \rfloor$ denote the largest integer no greater than $x(m)$; let $\lceil x(m) \rceil$ denote the the smallest integer no less than $x(m)$ but no greater than $B - m$. If $\lceil x(m) \rceil \neq \lfloor x(m) \rfloor$, then $\lambda_t(m', m)$ is defined by

$$\lambda(m + \lfloor x(m) \rfloor, m) = \lceil x(m) \rceil - x(m),$$
$$\lambda(m + \lceil x(m) \rceil, m) = m - \lfloor x(m) \rfloor;$$

and if $\lceil x(m) \rceil = \lfloor x(m) \rfloor$, then $\lambda_t(m', m)$ is defined by

$$\lambda(m + \lfloor x(m) \rfloor, m) = 1.$$

In a stage-2 meeting between a buyer with m^b and a seller with m^s, the equilibrium trading outcome $\mu(m^b, m^s)$ implies that

$$\hat{\lambda}_t^b(m^b - d, m^b, m^s) = \mu(d; m^b, m^s),$$
$$\hat{\lambda}_t^s(m^s + d, m^b, m^s) = \mu(d; m^b, m^s),$$

where $d \in \{0, 1, ..., \min\{B - m^s, m^b\}\}$.

A.2 The model with centralized trade

Consider the version of model with a centralized market in stage-2. Given the trading outcome $(y_t^a(m), \mu_t^a(.; m))$ (determined by (16)) and the distribution prior to the market $\hat{\pi}_t$, the value for an agent holding m right prior to stage-2 market is

$$\hat{v}_t(m) = \tilde{v}_t(m) + 0.5 \sum_{m'} \hat{\pi}_t(m') \left[S_t^b\left(y_t^b(m, m'), \mu_t^b(m, m'), m\right) + S_t^s\left(y_t^s(m', m), \mu_t^s(m', m), m\right) \right]; \tag{29}$$

the proportion of agents who hold m right prior to date-t disintegration of money is

$$\tilde{\pi}_t(m) = 0.5 \sum_{m'} \left[\hat{\lambda}_t^b(m, m') + \hat{\lambda}_t^s(m, m') \right] \hat{\pi}_t(m'), \tag{30}$$

where $\hat{\lambda}_t^b(m, m')$ and $\hat{\lambda}_t^s(m, m')$ are the proportion of buyers with m' and the proportion of sellers with m', respectively, leaving the market with m; they are given by

$$\hat{\lambda}_t^b(m^b - d^b, m^b) = \mu^b(d^b; m^b),$$
$$\hat{\lambda}_t^s(m^s + d^s, m^s) = \mu^s(d^s; m^s),$$

where $d^b \in \{0, 1, ...m^b\}$ and $d^s \in \{0, 1, ..., B - m^s\}$.

Given π_0, a sequence $\{v_t, \pi_{t+1}, \phi_t\}_{t=0}^{\infty}$ is an *equilibrium* if it satisfies (3), (4), (11), (12), (17), (29), and (30), all t. A tuple (v, π, ϕ) is a steady state if $\{v_t, \pi_{t+1}, \phi_t\}_{t=0}^{\infty}$ with $(v_t, \pi_{t+1}, \phi_t) = (v, \pi, \phi)$ all t is an equilibrium.

A.3 The model with costly market participation

Given the equilibrium trading outcome $(y_t(\zeta^b, \zeta^s), \mu_t(\zeta, \zeta^s)$ between a buyer holding ζ^b and a seller holding ζ^s, $f_t^b(\zeta^b, \zeta^s) = S_t^b(y_t(\zeta^b, \zeta^s), \mu_t(\zeta^b, \zeta^s), \zeta^b)$ and $f_t^s(\zeta^b, \zeta^s) = S_t^s(y_t(\zeta^b, \zeta^s), \mu_t(\zeta^b, \zeta^s), \zeta^s)$ are the surplus of the buyer and the surplus of the seller, respectively. Given seller's optimal participation choice $\rho_t(\bar{\zeta}^s)$. The value for an agent holding ζ right prior to stage-2 (before making the participation choice) is

$$\hat{v}_t(\zeta) = \tilde{v}_t(\zeta) + 0.5 \sum_{\zeta'} \hat{\pi}_t(\zeta') \left[\rho_t(\bar{\zeta}') f_t^b(\zeta, \zeta') + \rho_t(\bar{\zeta}) f_t^s(\zeta', \zeta)\right] - 0.5k\left(\rho_t(\bar{\zeta})\right); \quad (31)$$

and the proportion of agents who hold ζ right prior to date-t disintegration of money is

$$\tilde{\pi}_t(\zeta) = 0.5 \sum_{\zeta'} \left[\hat{\lambda}_t^b\left(\zeta, \zeta^b, \zeta^s\right) + \hat{\lambda}_t^s\left(\zeta, \zeta^b, \zeta^s\right)\right] \hat{\pi}_t\left(\zeta^b\right) \hat{\pi}_t\left(\zeta^s\right) \quad (32)$$
$$+ 0.5\hat{\pi}_t\left(\zeta^b\right) U_t + 0.5\hat{\pi}_t\left(\zeta^s\right)\left(1 - \rho_t(\bar{\zeta}^s)\right),$$

where $U_t = 1 - \sum_{\zeta^s} \hat{\pi}_t(\zeta^s) \rho_t\left(\overline{\zeta}^s\right)$ and $\hat{\lambda}_t^b(\zeta, \zeta^b, \zeta^s)$ and $\hat{\lambda}_t^s(\zeta, \zeta^b, \zeta^s)$ are analogous to $\hat{\lambda}_t^b(m, m^b, m^s)$ and $\hat{\lambda}_t^s(m, m^b, m^s)$ in (10).

Given π_0 and $\{i_t\}_{t=0}^{\infty}$, a sequence $\{v_t, \hat{\pi}_t, \pi_{t+1}\}_{t=0}^{\infty}$ is an *equilibrium* if it satisfies (19), (31), (32), (22)-(24) all t. If $i_t = i$ all t, a tuple $(v, \hat{\pi}, \pi)$ is a *steady state* if $\{v_t, \hat{\pi}_t, \pi_{t+1}\}_{t=0}^{\infty}$ with $(v_t, \hat{\pi}_t, \pi_{t+1}) = (v, \hat{\pi}, \pi)$ all t is an equilibrium.

A.4 The model with bond-based money transfer program

Under a bond-based transfer policy (K, λ), the expected amount of money transfer received by an agent with portfolio ζ is $\tilde{x}(\zeta) = \min\{K(1 + \lambda\zeta_2)^{-1}, B - \zeta_1 - \zeta_2\}$. Let

$\lfloor \tilde{x}(\zeta) \rfloor$ denote the largest integer no greater than $\tilde{x}(\zeta)$; let $\lceil \tilde{x}(\zeta) \rceil$ denote the the smallest integer no less than $\tilde{x}(\zeta)$ but no greater than $B - \zeta_1 - \zeta_2$. If $\lceil \tilde{x}(\zeta) \rceil \neq \lfloor \tilde{x}(\zeta) \rfloor$, then $\tilde{\mu}(;\zeta)$ is defined by

$$\tilde{\mu}\left(\lfloor \tilde{x}(\zeta) \rfloor, \zeta\right) = \lceil \tilde{x}(\zeta) \rceil - \tilde{x}(\zeta),$$
$$\tilde{\mu}\left(\lceil \tilde{x}(\zeta) \rceil, \zeta\right) = \tilde{x}(\zeta) - \lfloor \tilde{x}(\zeta) \rfloor;$$

and if $\lceil \tilde{x}(\zeta) \rceil = \lfloor \tilde{x}(\zeta) \rfloor$, then $\tilde{\mu}(;\zeta)$ is defined by

$$\tilde{\mu}\left(\lfloor \tilde{x}(\zeta) \rfloor, \zeta\right) = 1.$$

Let $\hat{\mu}_t(.;m)$ denote the $\hat{\mu}$ that solves the problem (27), then the proportion of agents holding ζ prior to pairwise meetings is

$$\hat{\pi}_t(\zeta) = \sum_{\zeta'} \left[\tilde{\mu}\left(\zeta_1 - \zeta_1', \zeta'\right) \sum_m \hat{\mu}_t(\zeta'; m) \pi_t\left(m\right) \right]. \tag{33}$$

Given (π_0, K, λ) and $\{i_t\}_{t=0}^{\infty}$, a sequence $\{v_t, \hat{\pi}_t, \pi_{t+1}\}_{t=0}^{\infty}$ is an *equilibrium* if it satisfies (19)-(21), (24), (27), and (33) all t. If $i_t = i$ all t, a tuple $(v, \hat{\pi}, \pi)$ is a *steady state* if $\{v_t, \hat{\pi}_t, \pi_{t+1}\}_{t=0}^{\infty}$ with $(v_t, \hat{\pi}_t, \pi_{t+1}) = (v, \hat{\pi}, \pi)$ all t is an equilibrium.

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