Measuring Energy Efficiency: An Application of Stochastic Frontier Production Function Analysis to Power Sector in Kerala

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Abstract

Traditionally, there are two basically reciprocal energy efficiency Indicators: one, in terms of energy intensity, that is, energy use per unit of activity output, and the other, in terms of energy productivity, that is, activity output per unit of energy use. The enquiry that has proceeded from the problems associated with this method of a single energy input factor in terms of productivity has led to multi-factor productivity analysis. We have here two approaches: parametric and non-parametric. Parametric approach famously includes two methods: the erstwhile popular total factor energy productivity analysis and the currently fanciful stochastic frontier production function analysis; The non-parametric approach is popularly represented by data envelopment analysis. The present paper is an attempt to measure efficiency in electrical energy consumption in Kerala, India. We apply the parametric method of stochastic frontier production function analysis on a panel data of the Kerala power sector with three sectors (Primary, Secondary and Tertiary) for the period from 1970-71 to 2016-17. For a comparative purpose, we also have a regression with a pooled data stochastic frontier. The results indicate that the sector-wise technical efficiency estimates of the Kerala power sector are independent of time, which can significantly refer to a technically stagnant situation in energy efficiency. The implication of the time-varying decay model, even though statistically insignificant, of a falling trend in the technical efficiency of all the three sectors also is a hot matter of serious concerns.

1. Introduction

Traditionally, there are two basically reciprocal energy efficiency Indicators: one, in terms of energy intensity, that is, energy use per unit of activity output, and the other, in terms of energy productivity, that is, activity output per unit of energy use. As a general concept, "energy efficiency refers to using less energy to produce the same amount of services or useful output. For example, in the industrial sector, energy efficiency can be measured by the amount of energy required to produce a tonne of product." (Patterson, 1996: 377). Thus Patterson defines energy efficiency broadly by the simple ratio of the useful output of a process in terms of any good produced that is enumerated in market process, to energy input into that process (ibid.).

Energy efficiency research in general has opened up three avenues of enquiry, namely, the measurement of energy productivity, the identification of impact elements and the energy efficiency assessment. The traditional interest in energy efficiency has centred on a single energy input factor in terms of productivity that has become famous through the index method proposed by Patterson (1996). In this case, energy intensity is obtained by dividing energy consumption by GDP, which implies the quantum of energy consumption that must be input in order to increase one unit of GDP. The enquiry that has proceeded from the problems associated with this method has led to identifying the effect source of variation, in terms of some decomposition analysis. Analyzed in terms of energy intensity changes, the index falls under two major decomposition methods, namely, Structural Decomposition Analysis (SDA) and Index Decomposition Analysis (IDA).

SDA has both inputs and outputs as its theoretical foundation, and is hence also known as equilibrium analysis. There are two approaches here: input-output method and neo-classical production function method.

The stringent assumptions associated with these approaches have made them practically unattractive for policy-orientated empirical exercises. Moreover, the prime significance of energy consumption reduction through energy use efficiency improvements following the 1973 oil crisis has essentially required complete evaluation of energy consumption patterns and identifying the driving factors of changes in energy consumption, creating a demand for effective tools to decompose aggregate indicators.

This need led to the development of the Index Decomposition Analysis (IDA) in the late 1970s in the United States (Myers and Nakamura 1978) and in the United Kingdom (Bossanyi 1979). These pioneering studies then spurred a number of different decomposition methods, most of which were derived from the index number theory, initially developed in economics to study the respective contributions of price and quantity effects to final aggregate consumption. A variant of factor decomposition analysis, IDA takes energy as a single factor of production, and explores various effects on energy intensity changes, by decomposing these changes into pure intensity changes effect and industrial structure changes effect. The first component (pure intensity changes effect) implies that when the industrial structure remains unchanged, the energy intensity change may be taken as the result of energy use efficiency changes in some sector, and the second implies that given the fixed energy efficiencies of various industries and their different energy intensity levels, the total energy intensity changes effect may be taken as the result of the dynamic changes of the yield of each industry.

IDA, as applied to time series data of a specific period, involves results which are very sensitive to the choice of the base period during the study period. In terms of the selection of base period, the approach usually considers Laspeyres Index of fixed weights and Divisia Index of variable weights.

Divisia index decomposition approach has become very popular these days in the context of analysis of energy intensity changes (see Ang and Zhang (2000), and Ang (2004) for a survey of index decomposition analysis in this field). There are two common Divisia index decomposition methods: Arithmetic mean (AMDI) and Logarithmic Mean Divisia index (LMDI). The AMDI method was first used by Gale Boyd, John McDonald, M. Ross and D. A. Hansont in 1987, for "separating the changing composition of the US manufacturing production from energy efficiency improvements" using Divisia index approach (as the title shows). This was followed by a number of studies, some attempts being directed towards modifying the index. These efforts were finally culminated in Ang and Choi (1997), who used logarithmic mean function as weights for aggregation with the attractive property that the decomposition leaves no residuals at all. Ang et al. (1998) called this model "Logarithmic Mean Divisia index (LMDI)".

Finally, a new energy efficiency estimation method, criticizing the single factor energy efficiency method, has come up utilizing a multi-variate structure. We have here two approaches: parametric and non-parametric. Parametric approach famously includes two methods: the erstwhile popular total factor energy productivity analysis and the currently fanciful stochastic frontier production function analysis; The non-parametric approach is popularly represented by data envelopment analysis (Charnes, A., W.W. Cooper and E. Rhodes (1978); Banker, R.D., A. Chames and W.W. Cooper (1984); Coelli, T.J., Rao D.S.P., O'Donnell C.J. and Battese G.E. (2005); Cooper, W.W., Seiford L.M. and Tone K. (2006).

The present paper is an attempt to measure efficiency in electrical energy consumption in Kerala, India, using the parametric version of the multi-factor productivity analysis, viz., the stochastic frontier production function method on a panel data of the Kerala power sector with three sectors (Primary, Secondary and Tertiary) for the period from 1970-71 to 2016-17. The paper is structured in six parts. The next section discusses the theoretical framework of frontier production function in general; section 3 continues the discussion with frontier approach and introduces both the deterministic and stochastic frontiers. A detailed presentation of the panel data stochastic frontier model that we utilize in our empirical exercise for the Kerala power sector also follows in the same section. Part four discusses the regression results from the empirical study. For a comparative purpose, we also present the regression results from a pooled data stochastic frontier approach in section five. The last section concludes the paper.

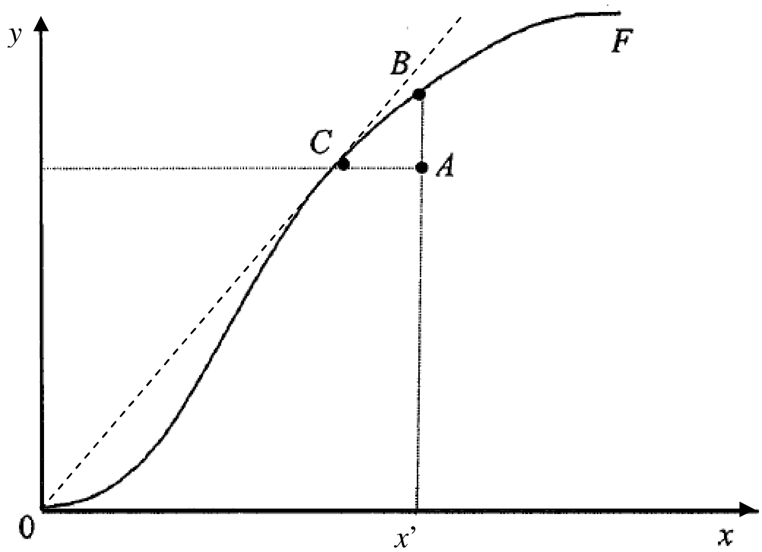
2. Frontier Production Function

A production function in microeconomic theory is defined as yielding maximum output (y) from a specified set of inputs (x), given the existing technology, and is given as

$$y = f(x; \beta), \qquad (1)$$

where β represents the production parameters. The function is assumed to be single-valued continuous one, with continuous first- and second-order partial derivatives. "The production function differs from the technology in that it presupposes technical efficiency and states the *maximum* output obtainable from every possible input combination." (Henderson and Quandt 1971; 54). Thus, the production function determines a production frontier, points on which represent technically efficient input combinations. Points such as B and C in Fig. 1 are thus technically efficient, but point A is not. The technical efficiency of the firm at point A with an input level of x' is given by $x\text{'}A \,/\, x\text{'}B$, where the denominator is the 'frontier output' and the numerator, the actual output of the firm, both associated with that input level; that is, the distance between the points A (actual output) and B (frontier output) represents its technical inefficiency at that input level.

Fig. 1: Technical Efficiency with a Frontier Production Function



It was the seminal paper of Farrell (1957) that stimulated econometric modeling of production functions as frontiers. According to him, the overall efficiency (now called economic efficiency) of a production unit is composed of two components, viz., technical efficiency and price efficiency (now called allocative efficiency); the former refers to the capability of the unit to produce maximum output from a given bundle of inputs, and the latter to the capability of the unit to utilize the inputs in an optimum proportion subject to the given input prices. In this chapter, we are considering the technical efficiency only (represented in Fig. 1 by points B and C).

However, there is a difference between the two efficient points B and C. We know that a ray through the origin as in Fig. 1 has a slope equal to y/x (that is, output/input) and is thus a measure of productivity. The ray from the origin has the maximum slope when it is at tangent to the production frontier and the point of tangency thus defines the point of maximum possible productivity. In Fig. 1, the point C represents optimum productivity, in addition to technical efficiency. Note that in this paper, we consider only technical efficiency.

Remember the efficiency of a production unit is measured in relation to an efficient production function (representing an efficient firm), which is in fact unknown and must be estimated using the sample data. For estimation, Farrell suggested (i) a parametric frontier function, such as the Cobb-Douglas production function, estimated from the data in such a way that no actual data point should lie to the right or above it, or (ii) a non-parametric piecewise-linear convex isoquant, estimated from the data in such a way that no actual data point should lie to the left or below it. Farrell used his models with agricultural data for the 48 states of the US.

5.3 Frontier Production Function Analysis

There are two types of production frontiers: (i) deterministic and (ii) stochastic frontiers.

Deterministic frontiers

The econometric model of the deterministic production frontier is obtained from the above equation (1) by adding an inefficiency term to the right side frontier and indexing the model for each of the n firms under study, as follows:

$$y_i = f(x_i;\, \beta)\, \exp(-u_i), \quad i = 1, 2, \ldots, n \qquad (2)$$

where y_i is the actual production level of the ith firm in the sample;

$f(x_i;\, \beta)$ models the frontier, represented by a suitable functional form, such as Cobb-Douglas or Translog, of the of inputs x_i and production parameters β of the ith firm;

u_i is a non-negative random variable representing the technical inefficiency of the ith firm;

n is the number of firms in the cross-sectional sample of the industry, and

exp represents exponential.

Remember that the technical efficiency of a firm is defined in terms of the ratio of the actual level of production of the firm to its frontier output. In the case of the above deterministic frontier model, the actual output for the ith firm is given by $f(x_i; \beta) \exp(-u_i)$, and the frontier output is $f(x_i; \beta)$ such that the technical efficiency of the ith firm (TE_i) is given by

$$
\begin{aligned}
TE_i &= \text{actual output/frontier output} \\
&= f(x_i; \beta) \exp(-u_i) / f(x_i; \beta) \\
&= \exp(-u_i). \qquad (3)
\end{aligned}
$$

Using appropriate estimation methods, we can have the frontier parameter estimates, which, along with the given sample input levels of individual firms, will yield the corresponding frontier output estimates; a comparison of the actual level of output with this will reveal the technical efficiency of each of the firms in the sample. It was Aigner and Chu (1968) who first estimated such a model by considering Cobb-Douglas production frontier and using linear programming technique. Taking natural log of (2), we obtain the technical inefficiency of the ith firm as the difference between the log of its actual and frontier output levels. Aigner and Chu (1968) sought to minimize the sum of the inefficiency subject to the constraint that u_i is non-negative; they also suggested quadratic programming as another solution method. The first econometric estimation came with Afriat (1972), who assumed gamma distribution for the u_i random variables and used the maximum likelihood method. Then Richmond (1974) followed, using a modified least squares method, known as *modified* (or *corrected*) ordinary least squares (MOLS or COLS), making the estimates unbiased and consistent. Schmidt (1976) assumed exponential and half-normal distributions for the random variable and estimated the model by the maximum likelihood method.

Note that the random variable in this model, assumed to be non-negative, stands to capture both the statistical noise *and* the inefficiency of the firm, and this is the major limitation of this model; all the deviations from the frontier is taken to indicate the effect of inefficiency. Another problem is that it does not satisfy the regularity condition of maximum likelihood (ML) method that the dependent variable be distributed independent of the parameter vector.

Attempts to solve these problems of the deterministic frontier method led to the development of the stochastic frontier approach.

Stochastic frontiers

Introduced by Aigner, Lovell and Schmidt (1977) and Meeusen and van den Broeck (1977) independently, the stochastic frontier approach to efficiency analysis defines the frontier property in a stochastic, rather than a deterministic, sense and seeks to decompose the random error term into two components, one for the random noise and the other for technical efficiency. This effectively helps us estimate technical efficiency directly. For detailed reviews of literature, see Forsund, Lovell and Schmidt (1980), Schmidt and Sickles (1984), Schmidt (1985), Bauer (1990), Seiford and Thrall (1990), Lovell (1993), Greene (1993), Ali and Seiford (1993) and Kumbhakar and Lovell (2000).

Since our data set contains information on three sectors (primary, secondary and tertiary) over a period of time that defines a panel data, we discuss first the features of panel data stochastic frontier and then the pooled data stochastic frontier.

Panel Data Stochastic Frontier

As earlier, we start with a frontier production function, but this time in a panel framework:

$$y_{it} = f(x_{it}; \beta), \; i = 1, 2, \ldots, n; \; t = 1, 2, \ldots, T, \qquad (4)$$

where $f(x_{it}; \beta)$ is the frontier production level of the ith firm at time t in the sample. As stated above, the random disturbance term in this model has two components, one having a strictly nonnegative distribution, representing technical efficiency, and the other representing the usual idiosyncratic error having a symmetric distribution. These two components we introduce in (4) as follows.

Note that the basic assumption of the (stochastic) frontier production function is that each firm is subject to some degree of inefficiency and hence potentially produces less than the frontier output. Thus we modify (4) as

$$y_{it} = f(x_{it};\ \beta)\varepsilon_{it},\ \ i = 1, 2, \dots, n;\ \ t = 1, 2, \dots, T, \tag{5}$$

where ε_{it}, lying in the interval (0;1] represents the degree of technical efficiency of firm i at time t. Since the actual output is strictly positive, ($y_{it} > 0$), the degree of technical efficiency also is assumed to be strictly positive ($\varepsilon_{it} > 0$). When $\varepsilon_{it} = 1$, there is no inefficiency and the firm produces its optimal output, determined by the frontier function $f(x_{it};\ \beta)$. On the other hand, when $\varepsilon_{it} < 1$, the firm produces less, depending upon the degree of inefficiency.

Now we modify (5) by adding the usual noise term (as the output is subject to random shocks, v_{it}),

$$y_{it} = f(x_{it};\ \beta)\varepsilon_{it}\exp(v_{it}),\ \ i = 1, 2, \dots, n;\ \ t = 1, 2, \dots, T. \tag{6}$$

Taking the natural log of (6), we get

$$\ln(y_{it}) = \ln f(x_{it};\ \beta) + \ln(\varepsilon_{it}) + v_{it},\ \ i = 1, 2, \dots, n;\ \ t = 1, 2, \dots, T. \tag{7}$$

If we define inefficiency term $-u_{it} = \ln(\varepsilon_{it})$, we can rewrite the above equation as

$$\ln(y_{it}) = \ln f(x_{it};\ \beta) + v_{it} - u_{it},\ \ i = 1, 2, \dots, n;\ \ t = 1, 2, \dots, T. \tag{8}$$

Note that we are subtracting u_{it} from $\ln f(x_{it};\ \beta)$; hence, if we restrict $u_{it} \geq 0$, we will get

$0 < \varepsilon_{it} \leq 1$, as required above.

The above equation is estimated under different specifications of the u_{it} term. In general, there are two models: (i) time-invariant inefficiency model and (ii) time-varying decay model; the former is the simplest specification.

In the time-invariant inefficiency specification, the inefficiency term u_{it} is assumed to be a time-invariant truncated normal random variable, truncated at zero with mean μ and variance σ^2. Note that the time-invariant model implies $u_{it} = u_i$, and hence we have the following assumptions:

$$u_i \sim \text{iid } N^+(\mu\,;\sigma_u^2), \text{ and } v_{it} \sim \text{iid } N(0;\, \sigma_v^2),$$

where u_i and v_{it} are distributed independently of each other and of the covariates in the model.

In the time-varying decay model, the inefficiency term is specified as

$$u_{it} = \exp\{-\eta(t - T_i)\}\ u_i, \qquad (9)$$

where

η = the decay parameter,

T_i = the last period in the ith panel, and

$u_i \sim$ iid $N^+(\mu\,;\sigma_u^2)$, and $v_{it} \sim$ iid $N(0;\, \sigma_v^2)$, both distributed independently of each other and of the covariates in the model ($\sim$ iid = independently and identically distributed as; N^+ = truncated (at zero) normal distribution; and N = normal distribution).

With the above specification (9), the time-varying decay model functions as follows:

when $\eta > 0$, the degree of inefficiency decreases over time;

when η < 0, the degree of inefficiency increases over time.

Note that since $t = T_i$ in the last period, the last period for firm i is assumed to contain the base level of its inefficiency, and hence, when $\eta > 0$, the degree of inefficiency decays toward the base level and when $\eta < 0$, it increases to the base level.

Also note that when $\eta = 0$, the time-varying decay model reduces to the time-invariant model.

4 Panel Data Stochastic Frontier: Regression Results

For estimating the panel data stochastic frontier of the power sector in Kerala, we consider three sectors as above (Primary, Secondary and Tertiary) for the period from 1970-71 to 2016-17. Because of the data unavailability for estimating a usual production function in terms of factors of production, we propose the following relationship:

> Sectoral energy consumption = *f*(Sectoral number of consumers; Sectoral GSDP at constant 2011-12 prices); all variables in log.

Note that unlike the usual frontier function with factors of production, we have a frontier function with activity factors.

Below we give the regression results for the time-invariant inefficiency model:

Table 1:

Panel Data Stochastic Frontier Results

for Time-invariant Inefficiency Model

```
Time-invariant inefficiency model               Number of obs      =       141
Group variable: sect                            Number of groups   =         3

                                                Obs per group:
                                                            min =        47
                                                            avg =        47
                                                            max =        47

                                                Wald chi2(2)       =    567.43
Log likelihood  = -50.598226                    Prob > chi2        =    0.0000
```

e	Coef.	Std. Err.	z	P>|z|	[95% Conf.	Interval]
y	.360787	.0533456	6.76	0.000	.2562314	.4653425
n	.441173	.054168	8.14	0.000	.3350056	.5473404
_cons	-2.118838	.481568	-4.40	0.000	-3.062694	-1.174982
/mu	.2731814	4.993982	0.05	0.956	-9.514844	10.06121
/lnsigma2	1.466197	2.134616	0.69	0.492	-2.717574	5.649968
/lgtgamma	3.696187	2.191414	1.69	0.092	-.5989048	7.99128
sigma2	4.332726	9.248708			.0660347	284.2824
gamma	.975783	.0517842			.3545943	.9996617
sigma_u2	4.227801	9.248752			-13.89942	22.35502
sigma_v2	.1049254	.0126262			.0801786	.1296723

Remember that we have used all the variables in log in the model specification; hence, the estimated coefficients are to be taken as elasticity measures. The estimates are highly significant; and energy consumption appears highly inelastic with respect to real GSDP and number of consumers, which signify positive implication for energy efficiency in general!

In the third (bottom) panel of the results, we have the variance estimates of the error components. Thus, sigma_v2 is the estimate of the variance of the usual idiosyncratic error component, σ_v^2, and sigma_u2 is that of the inefficiency component, σ_u^2. The first estimate reported, sigma2, is the estimate of the total error variance in terms of the sum of the above two, $\sigma_S^2 = \sigma_v^2 + \sigma_u^2$. The second one, gamma, gives the estimate of the ratio of the variance of the inefficiency component to the total error variance estimate, $\gamma = \sigma_u^2 / \sigma_S^2$.

The estimates given in the intermediate panel are;

/mu is the estimate of μ, the mean of the inefficiency term ($u_i \sim$ iid $N^+(\mu ; \sigma_u^2)$).

/lgtgamma is the estimate of the logit of γ; logit of γ is used to parameterize the optimization, as γ must be between 0 and 1.

/lnsigma2 is the estimate of $\ln(\sigma_S^2)$; $\ln(\sigma_S^2)$ is used to parameterize the optimization, as σ_S^2 must be positive.

Below we report some summary indicators of the panel time-invariant technical efficiency measures:

```
summarize tecefpnl
```

Variable	Obs	Mean	Std. Dev.	Min	Max
tecefpnl	141	.3657113	.3863919	.0389499	.9062921

Table 2 below reports the results for the time-varying decay inefficiency model:

Table 2:

Panel Data Stochastic Frontier Results

for Time-varying Decay Inefficiency Model

Time-varying decay inefficiency model		Number of obs =	141
Group variable: sect		Number of groups =	3
Time variable: year		Obs per group:	
		min =	47
		avg =	47
		max =	47
		Wald chi2(2) =	286.71
Log likelihood = -50.588669		Prob > chi2 =	0.0000

e	Coef.	Std. Err.	z	P>|z|	[95% Conf.	Interval]
y	.3592522	.0542185	6.63	0.000	.2529858	.4655186
n	.447967	.0730351	6.13	0.000	.3048209	.5911132
_cons	-2.169179	.6060753	-3.58	0.000	-3.357065	-.9812934
/mu	.334731	4.869846	0.07	0.945	-9.209993	9.879455
/eta	-.0002099	.0015121	-0.14	0.890	-.0031736	.0027538
/lnsigma2	1.46332	2.104342	0.70	0.487	-2.661115	5.587755
/lgtgamma	3.693502	2.160548	1.71	0.087	-.5410951	7.928098
sigma2	4.320278	9.091343			.0698703	267.1352
gamma	.9757195	.0511855			.3679329	.9996397
sigma_u2	4.215379	9.091384			-13.60341	22.03417
sigma_v2	.1048985	.012623			.0801579	.1296391

We know that if $\eta = 0$, the time-varying decay model reduces to the time-invariant model. In the above result, we find that the estimate of η is insignificant (zero); and the other estimates are not much different from the estimates of the time-invariant model. That means the time-varying decay model reduces to the time-invariant model. Its implication that the sector-wise technical efficiency estimates of the Kerala power sector are independent of time, that they remain constant over time, is highly significant in that it may refer to a technically stagnant situation in energy efficiency.

Below we report some summary indicators of the panel time-varying decay technical efficiency measures:

```
summarize tecefpnltv
```

Variable	Obs	Mean	Std. Dev.	Min	Max
tecefpnltv	141	.3632659	.3859875	.0379863	.9039742

Next we turn to the pooled data stochastic frontier model, just for comparative purpose.

5 Pooled Data Stochastic Frontier: Regression Results

We start with our earlier model

$$\ln(y_{it}) = \ln f(x_{it}; \beta) + v_{it} - u_{it}, \ \ i = 1, 2, \dots, n; \ \ t = 1, 2, \dots, T. \qquad (8)$$

where v_{it} is the idiosyncratic error and u_{it} is a time-varying panel-level effect. If the panel-level effect is insignificant, we get the pooled data model. There are three different models depending upon the distributional specification of the inefficiency term; in all these models, the idiosyncratic noise term is assumed to be independently distributed as normal, N(0; σ_v^2). The three models are:

(i) Exponential model, in which the inefficiency component is independently exponentially distributed with variance σ_u^2;

(ii) Half-normal model, with the inefficiency component independently and half-normally distributed, $N^+(0; \sigma_u^2)$;

(iii) Truncated-normal model, with the inefficiency component independently and truncated-normally distributed with truncation point at 0, $N^+(\mu; \sigma_u^2)$.

Table 3 presents the pooled data stochastic frontier model estimation results for the Kerala power sector with three sectors (Primary, Secondary and Tertiary) for the period from 1970-71 to 2016-17, for the same relationship as above:

Sectoral energy consumption = *f*(Sectoral GDP at constant 2011-12 prices; Sectoral number of consumers); all variables in log.

Table 5.3:

Pooled Data Stochastic Frontier Results for Half-Normal Model

```
Stoc. frontier normal/half-normal model          Number of obs     =        141
                                                 Wald chi2(2)      =   4.77e+09
Log likelihood = -187.47731                      Prob > chi2       =     0.0000
```

e	Coef.	Std. Err.	z	P>|z|	[95% Conf.	Interval]
y	.4933377	.0000116	4.2e+04	0.000	.4933149	.4933605
n	.0416695	6.33e-06	6583.47	0.000	.0416571	.0416819
_cons	.5266126	.000133	3959.43	0.000	.5263519	.5268732
/lnsig2v	-32.74632	206.774	-0.16	0.874	-438.0159	372.5233
/lnsig2u	1.20767	.1190983	10.14	0.000	.9742415	1.441098
sigma_v	7.75e-08	8.01e-06			7.69e-96	7.81e+80
sigma_u	1.82912	.1089225			1.627623	2.055562
sigma2	3.34568	.3984646			2.564703	4.126656
lambda	2.36e+07	.1089225			2.36e+07	2.36e+07

```
LR test of sigma_u=0: chibar2(01) = 91.27              Prob >= chibar2 = 0.000
```

As in the earlier model (Table 1), the estimates are highly significant; and energy consumption appears highly inelastic with respect to real GSDP and number of consumers, which signify positive implication for energy efficiency in general!

In the bottom panel, sigma_v and sigma_u, represent the estimates of the standard deviations of the two error components, v and u, respectively. The next term, sigma2, is the estimate of the total error variance, $\sigma_S^2 = \sigma_v^2 + \sigma_u^2$, and lambda represents the estimate of the ratio of the standard deviation of the inefficiency term to that of the idiosyncratic term, $\lambda = \sigma_u / \sigma_v$.

In the intermediate panel, we have

/lnsig2v and /lnsig2u, to represent the estimates of $\ln \sigma_v^2$ and $\ln \sigma_u^2$ respectively.

Note that at the bottom of the output (last line), the result of a test that there is no technical inefficiency term in the model is given, with the null hypothesis H_0: $\sigma_u^2 = 0$, against the alternative hypotheses H_1: $\sigma_u^2 > 0$. If we fail to reject the null hypothesis, the stochastic frontier model reduces to an OLS model with normal errors. For our half-normal model, we have the results that the likelihood ratio statistic (LR) = 91.27 with a p-value of 0.000. Thus we reject the null hypothesis; the stochastic frontier model is valid.

Below we report some summary indicators of the pooled data half-normal model technical efficiency measures:

```
summarize tecef
```

Variable	Obs	Mean	Std. Dev.	Min	Max
tecef	141	.5052589	.3596574	.0117641	.9999996

Next we turn to the exponential model results (Table 4).

Table 4:

Pooled Data Stochastic Frontier Results for Exponential Model

Stoc. frontier normal/exponential model	Number of obs	=	141
	Wald chi2(2)	=	32.88
Log likelihood = -233.06765	Prob > chi2	=	0.0000

e	Coef.	Std. Err.	z	P>|z|	[95% Conf.	Interval]
y	.1535739	.1451959	1.06	0.290	-.1310049	.4381527
n	.2886048	.073699	3.92	0.000	.1441574	.4330522
_cons	1.475255	1.787001	0.83	0.409	-2.027202	4.977713
/lnsig2v	.4010961	.2491265	1.61	0.107	-.0871828	.8893749
/lnsig2u	-2.24892	3.260228	-0.69	0.490	-8.638849	4.141009
sigma_v	1.222072	.1522253			.957345	1.560003
sigma_u	.3248279	.5295064			.0133075	7.928823
sigma2	1.598974	.1919779			1.222704	1.975244
lambda	.2658009	.6648633			-1.037307	1.568909

LR test of sigma_u=0: chibar2(01) = 0.09 Prob >= chibar2 = 0.380

Note that for our exponential model, the results of the likelihood ratio test shows that the statistic (LR) = 0.09 with a p-value of 0.380. Thus we fail to reject the null hypothesis; the stochastic frontier model reduces to an OLS model with normal errors.

Though we have tried to estimate the truncated normal model, the estimation process has failed to converge.

Table 5: Technical Efficiency Estimates: Primary Sector

Year	Pooled Half-Normal	Panel Time-Invariant	Panel Time-Varying	Year	Pooled Half-Normal	Panel Time-Invariant	Panel Time-Varying
1970-71	0.0118	0.0389	0.0392	1994-95	0.0722	0.0389	0.0386
1971-72	0.0216	0.0389	0.0392	1995-96	0.0871	0.0389	0.0385
1972-73	0.0241	0.0389	0.0391	1996-97	0.0925	0.0389	0.0385
1973-74	0.0299	0.0389	0.0391	1997-98	0.0996	0.0389	0.0385
1974-75	0.0321	0.0389	0.0391	1998-99	0.1046	0.0389	0.0385
1975-76	0.0363	0.0389	0.0391	1999-00	0.1258	0.0389	0.0384
1976-77	0.0337	0.0389	0.0390	2000-01	0.1217	0.0389	0.0384
1977-78	0.0279	0.0389	0.0390	2001-02	0.0963	0.0389	0.0384
1978-79	0.0307	0.0389	0.0390	2002-03	0.0355	0.0389	0.0384
1979-80	0.0295	0.0389	0.0390	2003-04	0.0411	0.0389	0.0383
1980-81	0.0322	0.0389	0.0389	2004-05	0.0340	0.0389	0.0383
1981-82	0.0382	0.0389	0.0389	2005-06	0.0331	0.0389	0.0383
1982-83	0.0377	0.0389	0.0389	2006-07	0.0401	0.0389	0.0382
1983-84	0.0389	0.0389	0.0389	2007-08	0.0421	0.0389	0.0382
1984-85	0.0336	0.0389	0.0388	2008-09	0.0398	0.0389	0.0382
1985-86	0.0374	0.0389	0.0388	2009-10	0.0461	0.0389	0.0382
1986-87	0.0552	0.0389	0.0388	2010-11	0.0433	0.0389	0.0381
1987-88	0.0644	0.0389	0.0387	2011-12	0.0526	0.0389	0.0381
1988-89	0.0715	0.0389	0.0387	2012-13	0.0574	0.0389	0.0381
1989-90	0.0740	0.0389	0.0387	2013-14	0.0585	0.0389	0.0381
1990-91	0.0637	0.0389	0.0387	2014-15	0.0549	0.0389	0.0380
1991-92	0.0669	0.0389	0.0386	2015-16	0.0555	0.0389	0.0380
1992-93	0.0723	0.0389	0.0386	2016-17	0.0628	0.0389	0.0380
1993-94	0.0755	0.0389	0.0386				

Tables 5, 6, and 7 provide the technical efficiency estimates for the three sectors, primary, secondary and tertiary respectively, for the study period from 1970-71 to 2016-17 derived from the three models estimated, viz., (i) panel data stochastic frontier time invariant model, (ii) panel data stochastic frontier time-varying model, and (iii) pooled data half-normal model.

Table 6: Technical Efficiency Estimates: Secondary Sector

Year	Pooled Half-Normal	Panel Time-Invariant	Panel Time-Varying	Year	Pooled Half-Normal	Panel Time-Invariant	Panel Time-Varying
1970-71	0.7334	0.9063	0.9040	1994-95	0.9391	0.9063	0.9035
1971-72	0.6943	0.9063	0.9040	1995-96	0.9462	0.9063	0.9035
1972-73	0.7237	0.9063	0.9039	1996-97	0.6492	0.9063	0.9035
1973-74	0.7467	0.9063	0.9039	1997-98	0.7201	0.9063	0.9035
1974-75	0.7536	0.9063	0.9039	1998-99	0.9011	0.9063	0.9034
1975-76	0.7800	0.9063	0.9039	1999-00	0.9133	0.9063	0.9034
1976-77	0.8075	0.9063	0.9039	2000-01	1.0000	0.9063	0.9034
1977-78	0.8838	0.9063	0.9038	2001-02	0.8583	0.9063	0.9034
1978-79	0.8736	0.9063	0.9038	2002-03	0.7899	0.9063	0.9034
1979-80	0.7851	0.9063	0.9038	2003-04	0.7070	0.9063	0.9033
1980-81	0.8547	0.9063	0.9038	2004-05	0.7507	0.9063	0.9033
1981-82	0.8092	0.9063	0.9038	2005-06	0.7578	0.9063	0.9033
1982-83	0.8925	0.9063	0.9037	2006-07	0.7834	0.9063	0.9033
1983-84	0.7083	0.9063	0.9037	2007-08	0.7740	0.9063	0.9033
1984-85	0.8753	0.9063	0.9037	2008-09	0.7512	0.9063	0.9033
1985-86	0.9279	0.9063	0.9037	2009-10	0.8166	0.9063	0.9032
1986-87	0.8376	0.9063	0.9037	2010-11	0.7843	0.9063	0.9032
1987-88	0.7339	0.9063	0.9037	2011-12	0.6835	0.9063	0.9032
1988-89	0.8514	0.9063	0.9036	2012-13	0.6863	0.9063	0.9032
1989-90	0.9658	0.9063	0.9036	2013-14	0.6908	0.9063	0.9032
1990-91	0.9596	0.9063	0.9036	2014-15	0.6976	0.9063	0.9031
1991-92	0.9862	0.9063	0.9036	2015-16	0.6665	0.9063	0.9031
1992-93	0.8758	0.9063	0.9036	2016-17	0.6608	0.9063	0.9031
1993-94	0.8772	0.9063	0.9035				

Table 7: Technical Efficiency Estimates: Tertiary Sector

Year	Pooled Half-Normal	Panel Time-Invariant	Panel Time-Varying	Year	Pooled Half-Normal	Panel Time-Invariant	Panel Time-Varying
1970-71	0.2662	0.1519	0.1490	1994-95	0.6574	0.1519	0.1476
1971-72	0.2515	0.1519	0.1490	1995-96	0.6718	0.1519	0.1476
1972-73	0.1278	0.1519	0.1489	1996-97	0.7510	0.1519	0.1475
1973-74	0.2647	0.1519	0.1489	1997-98	0.7907	0.1519	0.1474
1974-75	0.1448	0.1519	0.1488	1998-99	0.8629	0.1519	0.1474
1975-76	0.1626	0.1519	0.1487	1999-00	0.8052	0.1519	0.1473
1976-77	0.3927	0.1519	0.1487	2000-01	0.8129	0.1519	0.1473
1977-78	0.8080	0.1519	0.1486	2001-02	0.6496	0.1519	0.1472
1978-79	1.0000	0.1519	0.1486	2002-03	0.7036	0.1519	0.1471
1979-80	0.9315	0.1519	0.1485	2003-04	0.7010	0.1519	0.1471
1980-81	0.8203	0.1519	0.1484	2004-05	0.6351	0.1519	0.1470
1981-82	0.8963	0.1519	0.1484	2005-06	0.7382	0.1519	0.1470
1982-83	0.5175	0.1519	0.1483	2006-07	0.8131	0.1519	0.1469
1983-84	0.3876	0.1519	0.1483	2007-08	0.8557	0.1519	0.1468
1984-85	0.3681	0.1519	0.1482	2008-09	0.7869	0.1519	0.1468
1985-86	0.4218	0.1519	0.1481	2009-10	0.7970	0.1519	0.1467
1986-87	0.4542	0.1519	0.1481	2010-11	0.8171	0.1519	0.1467
1987-88	0.4651	0.1519	0.1480	2011-12	0.8874	0.1519	0.1466
1988-89	0.5399	0.1519	0.1480	2012-13	0.8903	0.1519	0.1465
1989-90	0.4859	0.1519	0.1479	2013-14	1.0000	0.1519	0.1465
1990-91	0.6204	0.1519	0.1478	2014-15	0.9615	0.1519	0.1464
1991-92	0.6533	0.1519	0.1478	2015-16	0.9757	0.1519	0.1464
1992-93	0.7161	0.1519	0.1477	2016-17	0.9822	0.1519	0.1463
1993-94	0.5990	0.1519	0.1477				

Fig. 2 provides a visual representation of these tables and brings out the patterns and the trends of the efficiency estimates. As the theory has already suggested, the panel data stochastic frontier time invariant model yields a constant estimate for each of the three sectors, and the panel data stochastic frontier time-varying decay model presents smoothly falling estimates over the time; note that the latter model is statistically not different from the former one such that their mean values are very close to each other (as Table 8 shows). The mean technical efficiency estimates for the three sectors derived from these two models are:

primary sector = 0.039; secondary sector = 0.906; and tertiary sector = 0.152. While the secondary sector performance goes well with the general expectation, the tertiary sector presents poor results, contrary to the expectation, and the primary sector remains as always the worst performer.

To be more precise, we have already seen that the time-varying decay model reduces to the time-invariant model of the Kerala power sector. Its implication that the sector-wise technical efficiency estimates of the Kerala power sector are independent of time, that they remain constant over time, is highly significant in that it may refer to a technically stagnant situation in energy efficiency. It goes without saying that this has immense policy implications. If we take the time-varying decay model into confidence, there is, though insignificant, a falling trend in the technical efficiency of all the three sectors (Fig. 5.1, third column).

The pooled data stochastic frontier half-normal model, which we use only for a comparative purpose, on the other hand, shows fluctuations in the estimates of all the three sectors. Both the primary and the tertiary sector estimates trend upwards over time through oscillations, whereas the secondary sector estimates show very high fluctuations, without any particular trend. It should be noted that a sharp fall in 2002-03 marks the primary sector estimates and a steep rise in 1977-78, followed by a fall around 1982-83, marks the tertiary sector estimates.

Fig. 2: Technical Efficiency Estimates (Sector- and Model-wise)

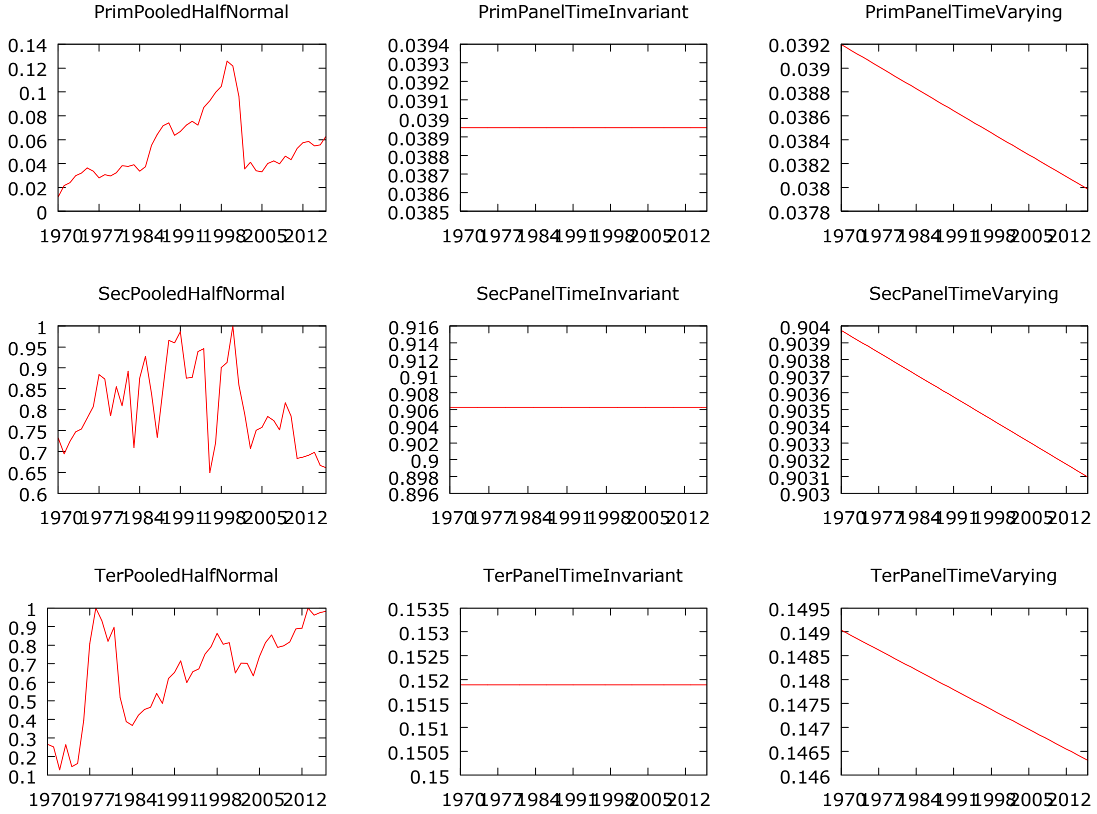


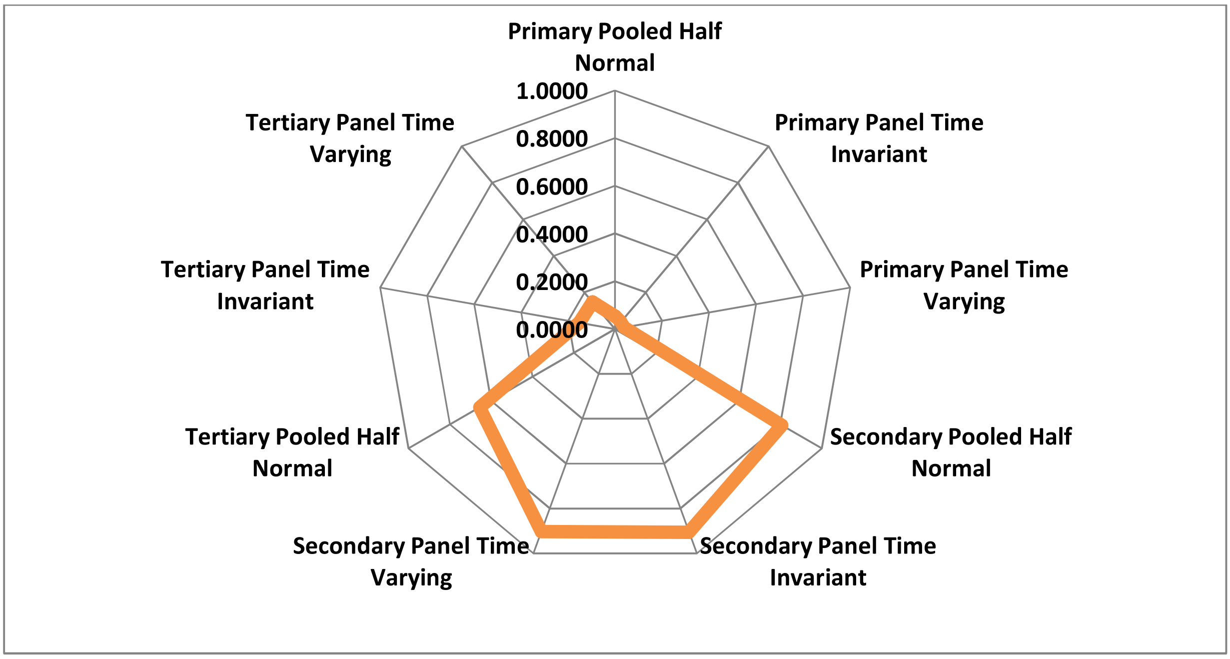
Table 8: Technical Efficiency Estimates: Summary Statistics

Sector	Model	Mean	Median	Minimum	Maximum	Std. Dev.	C.V.
Primary	Pooled Half Normal	0.0539	0.0433	0.0118	0.1258	0.0265	0.4919
	Panel Time Invariant	0.0390	0.0390	0.0390	0.0390	0	0
	Panel Time Varying	0.0386	0.0386	0.0380	0.0392	0.0004	0.0094
Secondary	Pooled Half Normal	0.8056	0.7852	0.6492	1	0.0964	0.1196
	Panel Time Invariant	0.9063	0.9063	0.9063	0.9063	0	0
	Panel Time Varying	0.9035	0.9035	0.9031	0.9040	0.0003	0.0003
Tertiary	Pooled Half Normal	0.6562	0.7036	0.1278	1	0.2452	0.3737
	Panel Time Invariant	0.1519	0.1519	0.1519	0.1519	0	0
	Panel Time Varying	0.1477	0.1477	0.1463	0.1490	0.0008	0.0055

Sector	Model	Skewness	Excess kurtosis	5% Percentile	95% Percentile	Inter-quartile range
Primary	Pooled Half Normal	0.9764	0.3304	0.0226	0.1149	0.0379
	Panel Time Invariant	undefined	undefined	0.03895	0.03895	0
	Panel Time Varying	0.0077	-1.2010	0.0380	0.0392	0.0006
Secondary	Pooled Half Normal	0.2721	-1.0031	0.6631	0.9781	0.1535
	Panel Time Invariant	undefined	undefined	0.90629	0.90629	0
	Panel Time Varying	-0.0030	-1.2011	0.9031	0.9040	0.0005
Tertiary	Pooled Half Normal	-0.5967	-0.6341	0.1519	0.9929	0.3552
	Panel Time Invariant	undefined	undefined	0.15189	0.15189	0
	Panel Time Varying	0.0031	-1.2011	0.1464	0.1490	0.0014

Table 8 reports the sector-wise summary statistics of the technical efficiency estimates for the three models under consideration. The pooled data stochastic frontier half-normal model stands apart from the other two models with much higher variation of the estimates, coming out of lower minimum and higher maximum values (the maximum being unity for secondary (in 2000-01) and tertiary sectors (1978-79 and 2013-14). Fig. 2 visualizes the sector-wise and model-wise mean values of these estimates. Further information is given in the appendix to this chapter.

Fig. 3: Mean Technical Efficiency Estimates (Sector- and Model-wise)



6. Conclusion

The present paper is an empirical exercise for the Kerala power sector in terms of the multi-factor productivity analysis, with the stochastic frontier production function method. We have started with a general theoretical framework of frontier production function in general; and then introduced both the deterministic and stochastic frontiers. In our empirical exercise for the Kerala power sector, we have utilized the panel data stochastic frontier model, and for a comparative purpose only, we have also estimated a pooled data stochastic frontier model.

The panel data stochastic frontier model comes in two variants – (i) time-invariant inefficiency model and (ii) time-varying decay model; the former being the simplest specification. The empirical results for the two models show that the differentiating characteristic of the second model is insignificant and it reduces to the time-invariant model, yielding constant efficiency estimates over time. The sector-wise difference among these estimates is very high; while the secondary sector performance goes well with the general expectation (with an efficiency of 0.906), the tertiary sector presents poor results (0.152), contrary to the expectation, and the primary sector remains as always the worst performer (0.039). That the sector-wise technical efficiency estimates of the Kerala power sector are independent of time can significantly refer to a technically stagnant situation in energy efficiency. The implication of the time-varying decay model, even though statistically insignificant, of a falling trend in the technical efficiency of all the three sectors also is a hot matter of serious concerns. It goes without saying that this has immense policy implications, and we need to go a long way.

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