Long Memory and Correlation Structures of Select Stock Returns Using Novel Wavelet and Fractal Connectivity Networks

BHANDARI, AVISHEK

Institute of Management Technology Hyderabad

1 June 2020

Long memory and correlation structures of select stock returns using novel wavelet and fractal connectivity networks

Avishek Bhandari[1]

Abstract

This study investigates the long range dependence and correlation structures of some select stock markets. Using novel wavelet methods of long range dependence, we show presence of long memory in the stock returns of some emerging economies and the lack of it in developed markets of Europe and the United States. Moreover, we conduct a wavelet based fractal connectivity analysis, which is the first application in economics and financial studies, to segregate markets into fractally similar groups and find that developed markets have similar fractal structures. Similarly stock returns of emerging markets exhibiting long-memory tend to follow similar fractal structures. Furthermore, network analyses of fractal connectivity support our findings on market efficiency which has theoretical roots in both fractal and adaptive market hypothesis.

Keywords: Long memory, Fractal connectivity, Wavelets, Hurst, Complex networks.

JEL Classification: C13, C14, C22, C32, G15

1.1 Introduction

Long memory processes, also known as long-range dependent process, are ubiquitous in financial and economic time-series. This study seeks to understand the long memory behaviour of global equity returns using novel methods from wavelet analysis, where long-range dependence and long-run correlation structure of major global equity returns are analysed within the framework of wavelet log-scale analysis and the recently introduced fractal connectivity matrix generated via the implementation of multivariate wavelet long memory estimators of Achard and Gannaz (2016), making it the first application of fractal connectivity analysis to studies in economics and finance. The genesis of the long memory experiment is due to Hurst (1951) who analysed the flow of Nile River. Mandelbrot and Van Ness (1968), using the idea of Hurst exponent, employed the idea of long-memory processes in conjunction with fractional Brownian motion and

[1] Institute of Management Technology Hyderabad. Email: bavisek@imthyderabad.edu.in

related stochastic processes. However, in the field of time series analysis, Granger and Joyeux (1980) and Hosking (1981) were among the first to integrate long memory processes with existing time series methods. The majority of research focusing on estimation of long memory relies on the traditional rescaled range (R/S) approaches of Mandelbrot and Wallis (1968) and its modified version developed by Lo (1991). The spectral domain approach proposed by Geweke and Porter-Hudak (1983) to estimate the long memory parameter has been used by many researchers too.

This paper investigates long memory among select global equity markets using estimators from the wavelet domain. Studies investigating long memory in global financial markets based on wavelet based long memory methods are relatively few as compared to traditional time and spectral domain estimators of long memory. Furthermore, empirical studies based on log-scale wavelet domain estimator of long-range dependence are practically nonexistent. Moreover, to the best of our knowledge, this is the first empirical study of long-run correlation structure generated by fractal connectivity matrix in the domain of economics and finance.The dearth of studies concerning wavelet based analysis of long memory and long run fractal connectivity based correlation structures of global equity markets necessitates an exploration based on these methods.

1.2 Literature Review

The presence of long memory in squared daily returns of S&P 500 index is evident in the works of Ding et al. (1993) where significant autocorrelation for lags up to ten years were present. Similarly, Lobato and Savin (1998) also demonstrated the presence of long memory in the squared returns of the S&P 500 dataset spanning three decades. Furthermore, Lobato and Velasco (2000) using a frequency domain tapering procedure in a multi stage semi-parametric method unearthed the presence of long memory in stock returns and volatility of returns. The presence of long memory in the returns of Brazilian equity market is documented in Assaf and Cavalcante (2005). Barkoulas et al. (2000), while investigating the long memory properties of the Athens stock exchange, find evidence of long-range persistence in the returns of the Athens stock market. Moreover, the forecast performance of a long memory incorporated model significantly outdid forecasts generated from a regular random walk model. Similarly, Panas (2001), using a spectral measure of fractality along with the Levy index, found nonlinearities in Greek equity returns and unearthed the existence of long memory, thereby rejecting the weak-form efficiency of the Greek equity market. Henry (2002), using a mixture of semi-parametric and spectral estimators, found evidence of long memory in the returns of South Korean stock market. Moreover, some evidence of weak long memory was unearthed in the markets of Germany and Taiwan. The empirical investigations documented the presence of long-range dependence in four emerging eastern European markets, thereby rejecting evidence in favour of the efficient market hypothesis. Similar analysis using wavelet based methods to detect long memory in the returns of the Dow Jones Industrial average (DJI) were employed by Elder and Serletis (2007) where no evidence of long memory was detected, thereby supporting results from a vast number of studies that reject the presence of long memory in the developed markets of the U.S. However, the presence of long memory in the equity returns of some developed markets of Europe, the U.S., and Japan is documented in Ozdemir (2007). Furthermore, Ozun and Cifter (2007), also using a wavelet based estimator of long memory, found some evidence of long-range dependence in the returns of the Istanbul Stock Index, thereby rejecting the weak form efficiency of Istanbul share prices. Similarly, evidence of long memory in the equity markets of G7 countries is documented in Bilal and Nadhem (2009). On the other hand, Mariani et al (2010), using detrended fluctuation analysis and truncated Levy flight method, found evidence of long memory in several eastern European markets. However, among the countries that are part of the Organisation for Economic Co-operation and Development (OECD), long-memory, as investigated by Tolvi (2003), was only evidenced in the smaller equity markets of Denmark and Finland. Jefferis and Thupayagale (2008), using a long-memory variant of the GARCH model, investigated long memory behaviour of some select African equity markets and found evidence supporting the presence of long-memory in the developing markets of Botswana and Zimbabwe. The presence of long memory in the developing markets of Central and Eastern European countries (CEE) is documented in the studies of Jagric et al. (2006) and Kasman et al. (2009), where the presence of long memory in equity returns is specifically limited to the developing markets of Hungary, Czech, Slovenia and Croatia. Kristoufek and Vosvrda (2012) constructed a measure of efficiency by measuring the distance between an efficient case and a vector containing long memory and other measures of fractality. Long memory is evidenced in many developing and emerging markets whereas all developed markets show signs of efficiency, with the Japanese NIKKIEI leading all other developed markets in terms of efficiency. Cont (2005) attempted to identify economic intuition and mechanisms behind the existence of fractality and long memory in returns and returns volatility. The possible economic factors underlying the existence of long memory in volatility are, i) heterogeneous investment horizons of market agents, ii) evolutionary trading models that employ genetic algorithms, iii) market fluctuations arising out of investors' sudden switch between several trading strategies, and iv) the inactivity of investors, operating at certain time periods and market regimes, based on trading strategies or behavioral aspects. Vuorenmaa (2005) investigated the time-varying long memory of Nokia Oyj returns using the wavelet OLS method and found significantly strong long memory during the dot-com bubble period. Ozun and Ciftr (2007), demonstrating the superiority of wavelet OLS method as compared to the spectral long memory estimator of Geweke and Porter-Hudak (1983), found significant long memory in the returns of Istanbul stock exchange. Similarly, DiSario et al. (2008), on investigating the volatility structure of S&P 500 returns using the wavelet OLS method, found evidence of long-memory in the S&P 500 returns volatility. In the same vein as the aforementioned studies, Tan et al. (2012) while examining the fractal structure of emerging economies using wavelet OLS method demonstrated significant long memory in the returns of larger firms as compared to smaller firms. Likewise, Tan et al. (2014), using the wavelet estimator of Jensen (1999) and detrended fluctuation analysis, examined long memory behavior of equity returns and volatility of ten markets from both developing and developed economies. On the other hand, Power and Turvey (2010) investigated long memory structure of fourteen commodity futures using the Hurst estimator of Veitch and Abry (1999) and demonstrated long-range dependence in all commodities. Boubaker and Peguin-Feissolle (2013) proposed semiparametric wavelet base long memory estimators and demonstrated its superiority, with respect to several non-wavelet estimators, using simulation experiments. Pascoal and Monteiro (2014), while investigating the predictability of the Portuguese stock returns using wavelet estimators of long memory, fractal dimension and the Holder exponent, found no evidence of long memory in the PSI20 returns, thereby confirming the efficiency of the Portuguese equity market. More recently, Tiwari et al. (2019) examined the efficiency of oil prices using several methods of long memory including wavelet and the periodogram approach and found future contracts of Brent oil to be less efficient than WTI oil. However, markets are in a constant stage of development which can influence efficiency and predictability

Our study, however, implements the wavelet based approaches of Abry and Veitch (1998) and Abry et al. (2003) to graphically examine the Hurst exponents of select equity returns using a log-scale wavelet plot. We then proceed to examine the long run correlation structure of both developed and emerging markets using fractal connectivity approach of Achard and Gannaz (2016). Finally, we conduct a network analysis to support the findings from fractal connectivity based experiment. The novelty of our approach stems from the application of such measures to empirical studies in economics and finance, which we believe is the first such exercise pertaining to applications in economics and finance.

1.3 Data

The empirical data consists of some select developed stock markets of France (CAC40), Germany (DAX), U.S. (S&P500), Great Britain (FTSE100), Switzerland (SMI), and the Eurozone (STOXX50). The data for emerging economies constitute the select stock markets of India (BSE30), Brazil (Brazil), Indonesia (JKSE), Pakistan (KSE100), China (SSE), and Malaysia (KLSE). The period of study ranges from 01-07-1997 to 20-01-2014 consisting of 4096 dyadic length observations making it suitable for various wavelet methods. Returns of all the stock indices are calculated by taking the first order logarithmic differences.

1.4 Methodology

Long memory process is associated with a slow power law decay of the autocorrelation function of a stationary process *x*. The covariance function $\gamma_x(k)$ of the long memory process *x* takes the following form,

$$\gamma_x(k) \sim \mathrm{c}_\gamma\, k^{-(2-2H)}, \quad k \to +\infty \tag{1.1}$$

where c_γ is a positive constant and $H \in (0, 0.5)$. The Hurst parameter H is used to measure the presence of long memory. The spectrum $\Gamma_x(\nu)$ of the long memory process *x* is given by,

$$\Gamma_x(\nu) \sim c_f\, |\nu|^{1-2H}, \quad \nu \to 0 \tag{1.2}$$

where ν is the frequency, $c_f = \pi^{-1} c_\gamma \Lambda(2H-1)\sin(\pi - \pi H)$, and the Gamma function is given by Λ. This mathematical structure of long memory processes is the reason for its inclusion in a class of stochastic processes which have the $1/|\nu|^{\alpha}$ form. The property of long memory also finds some close association with the phenomenon of scale invariance, self-similarity and fractals. Hence, statistically a self- similar process like fractional Brownian motion (FBM) is closely related to long memory phenomenon.

Let γ_0 be an arbitrary reference frequency selected by the choice of ψ_0, the mother wavelet. The amount of energy in the signal during scaled time $2^j k$ and scaled frequency $2^{-j}\nu_0$ is measured by the squared absolute value of the detail wavelet coefficient $|d_x(j,k)|^2$. A wavelet based spectral estimator of Abry et al. (1993) is constructed by taking a time average of $|d_x(j,k)|^2$ at a given scale, and is given by,

$$\hat{\Gamma}_x(2^{-j}\nu_0) = \frac{1}{n_j}\sum_k |d_x(j,k)|^2 \tag{1.3}$$

where n_j is the "number of wavelet coefficients" at level j, and $n_j = 2^{-j}n$, where n is the data length. Therefore , $\hat{\Gamma}_x(\nu)$ captures the amount of energy that lies within a given bandwidth and around some frequency ν. Hence, $\hat{\Gamma}_x(\nu)$ can be regarded as an estimator for the spectrum $\Gamma_x(\nu)$ of x. The wavelet based estimator of the Hurst exponent $\hat{H}$ is designed by performing a simple linear regression of $\log_2(\hat{\Gamma}_x(2^{-j}\nu_0))$ on j, i.e.,

$$\log_2(\hat{\Gamma}_x(2^{-j}\nu_0)) = \log_2\left(\frac{1}{n_j}\sum_k |d_x(j,k)|^2\right) = (2\hat{H}-1)j + \hat{c} \tag{1.4}$$

where $\hat{c}$ estimates $\log_2(c_f \int |\nu|^{(1-2H)} |\Psi_0(\nu)|^2 d\nu)$, where Ψ_0 is the Fourier transform of the mother wavelet ψ_0. A weighted least square estimator is constructed by performing a WLS fit between the wavelet scales j_1 and j_2 which gives the estimator of the "*Hurst exponent*", H. The estimators of multivariate long memory and the related "*fractal connectivity matrix*", based on the above univariate estimator is given in Achard et al. (2008) and Achard and Gannaz (2016).

1.5 Empirical results

1.5.1 Wavelet based log-scale alignment approach

The presence of long memory in the volatility of select equity returns, as given by the absolute value of equity returns, is investigated by applying the wavelet based estimator of the Hurst exponent developed by Abry and Veitch (1998) and Abry et al. (2003).

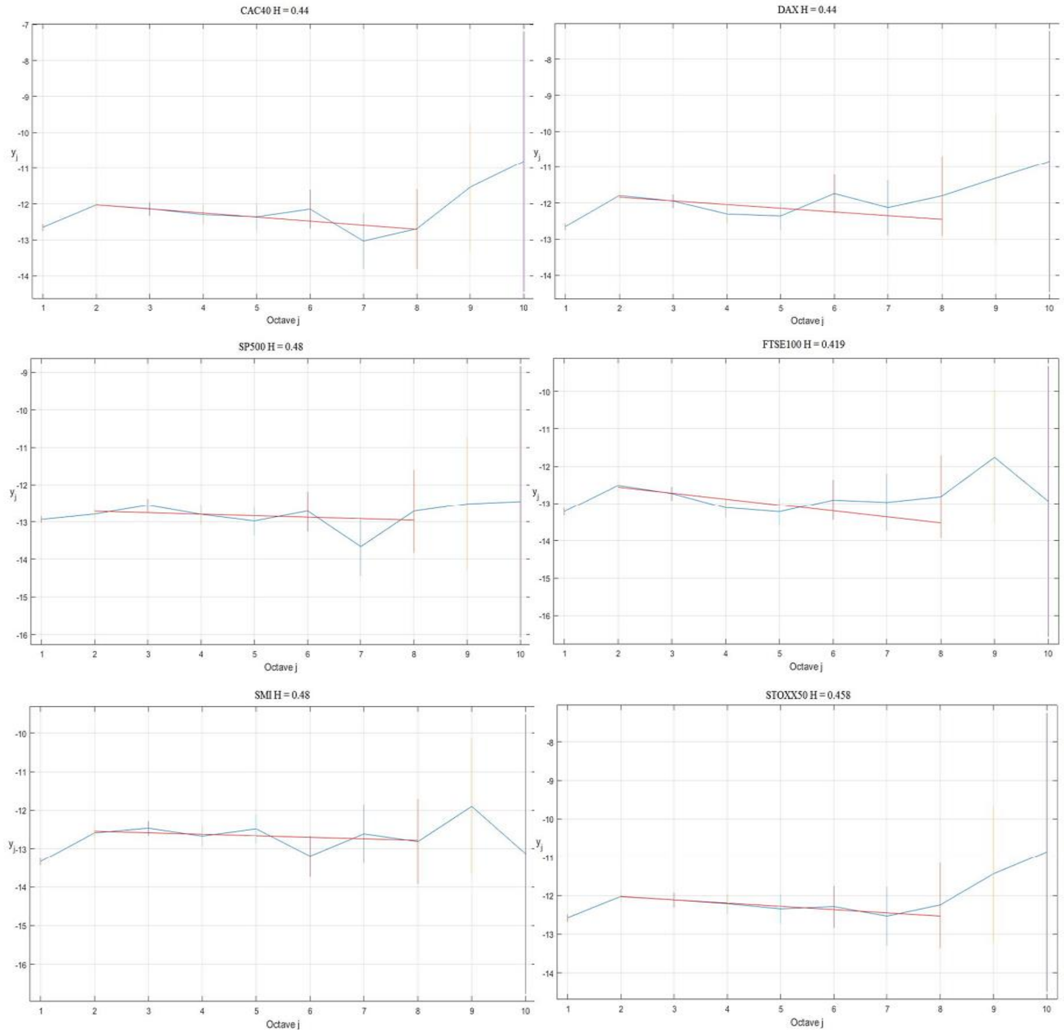
This method enables one to graphically analyze long memory from the log-log plot of the wavelet regression which contains additional information about the fractal nature of equity returns. The logscale diagram is a plot of wavelet variance at each scale against the wavelet scale. Formally, the plot of the logarithm of $v_j = \frac{1}{n_j}\sum_{k=1}^{n_j} |d_X(j,k)|^2$ against the wavelet scale j gives the *logscale* diagram. Here n_j is the "number of wavelet coefficients" at scale j and $d_X(j,k)$ is the wavelet details of the process $X(t)$. The visualization of the *logscale* diagram can help one detect regions of long-range dependence via the help of an *alignment region* in the graph. The range of wavelet scales where log (v_j) falls on a straight line is known as the *alignment region* (Abry et al. 2003) and perfect alignment, mostly at higher scales, normally constitutes long memory. In the logscale plot, perfect alignment requires the red straight line to cross (or touch) the vertical lines depicting the confidence band in an upward sloping manner. If the alignment region includes the largest scales in the logscale plot, then the returns exhibit long-range dependence. Furthermore, the value of the self-similar parameter[2] α should lie in the interval (0, 1). Correspondingly, the value of the Hurst exponent H should lie in the interval (0.5, 1) for the data to exhibit long memory. Figure 1.1 gives the logscale diagram[3] of the equity returns of select developed markets. It can be observed from the figure that straight line slopes downward and the corresponding Hurst exponents for all six developed markets of Europe and the U.S. lie within the interval (0, 0.5) indicating short-memory.

---

[2] α is also known as the scaling exponent of self-similarity. The Hurst parameter H and α are related by the expression: H=(1+ α)/2

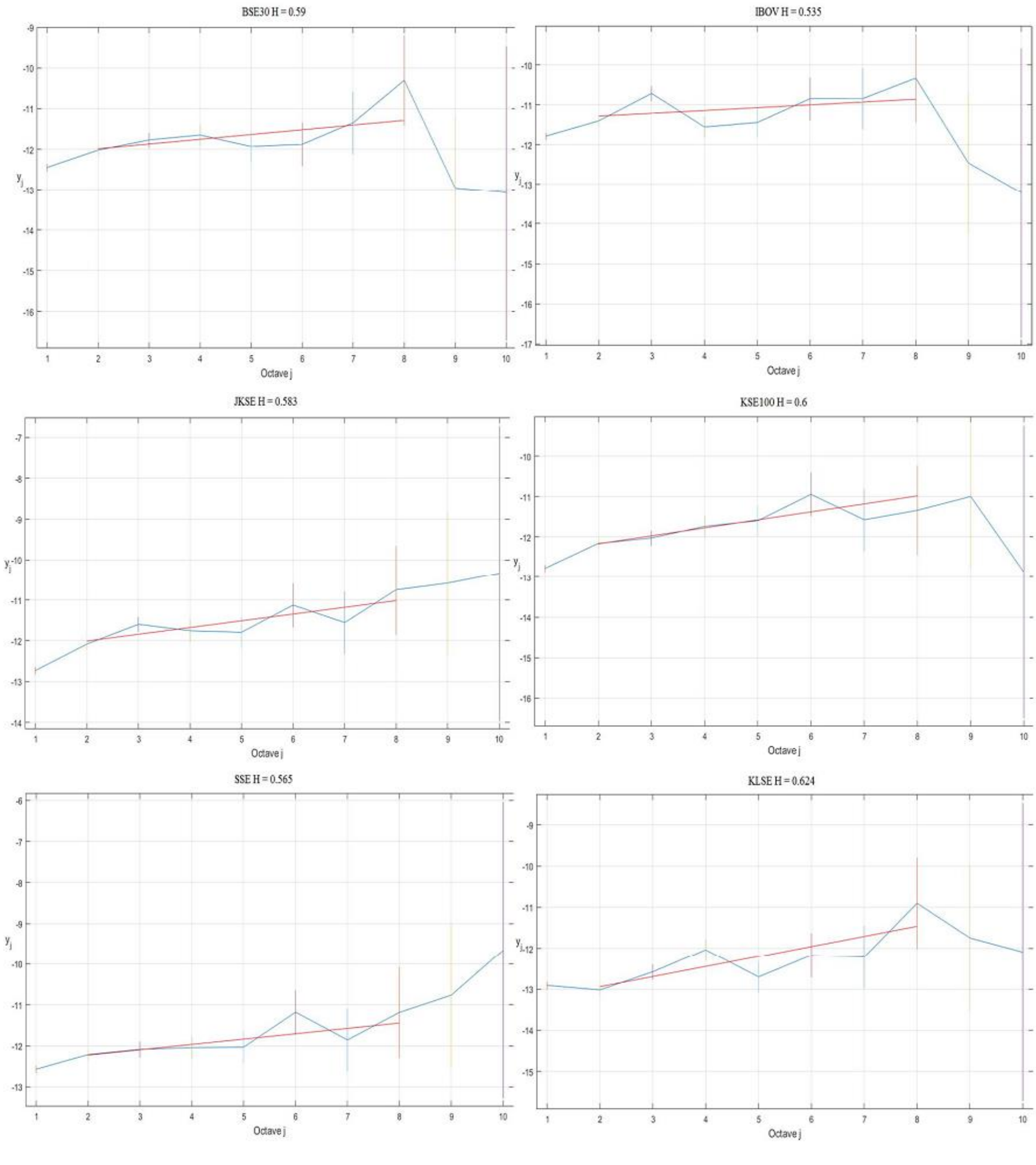
[3] After repeated simulations, the optimal lower cut-off scale is taken to be 2 and the highest scale is taken to be 8.

Figure 1.1 Logscale diagrams of stock returns from developed markets



The absence of long memory in the returns of developed markets is in confirmation with results from a vast majority of literature that rejects long memory in developed financial markets. Figure 1.2 gives the logscale diagram of the equity returns of some select emerging markets. It can be noticed that the Hurst exponents of emerging markets' equity returns lie within the interval (0.5, 1).

Figure 1.2 Logscale diagrams of stock returns from emerging markets



It can be observed from the above figure that the upward sloping alignment of the straight red line includes all higher scales, i.e. scales five up to eight, indicating the presence of "long-range dependence". However, among the six emerging markets, equity returns of India (BSE 30), Pakistan (KSE 30) and Malaysia (KLSE) exhibit relatively stronger long-memory.

1.5.2 The fractal connectivity approach

The dynamic evolution of long memory structure can differ depending on various stages of market development, where a movement towards sophisticated state from an under-developed market implies efficiency of financial markets in terms of correctly priced assets. Hence, varying developmental stages of emerging economies correspond to different stages of market efficiency based on the evolution or backward evolution of long-memory (M. Hull and McGroarty, 2013). This phenomenon is also consistent with the self-correcting mechanism propounded by Grossman and Stiglitz (1980). Moreover, markets are in a constant stage of development where investors, with their inherent behavioral biases, adapt to changing scenarios in financial markets. This idea led to the development of an alternative notion of efficiency as ingrained in the efficient market hypothesis in the form of adaptive market hypothesis (AMH) of Lo (2004), where evolutionary perspective and behavioral aspects of investors are taken into consideration. Contrary to the EMH, which is based on utility maximizing rational individuals, behavioral biases of investors could distort utility optimizing decisions. Moreover, investors adapt to changing market conditions where individuals who cannot adapt to changing market conditions are cast out. Therefore, as compared to the EMH, the AMH incorporates *"[...] considerably more complex market dynamics, with cycles as well as trends, and panics, manias, bubbles, crashes, and other phenomena that are routinely witnessed in natural market ecologies*." (Lo, 2004, p.24).

However, stock returns of both developed and emerging economies can exhibit similar or dissimilar fractal structures as opposed to the homogenous notion of efficiency as propounded in the EMH. Therefore, we attempt to investigate the underlying fractal structures based on the long-range correlation matrix, also referred to as fractal connectivity which offers, i) an efficient way to estimate long-memory, ii) evaluate a correlation structure, iii) and analyze similar fractal structure among markets. The aforementioned fractal connectivity matrix estimated via multivariate wavelet based estimator aids in inspecting correlation structure at long-range frequencies, thereby enabling one to identify markets with similar fractal structures. Furthermore, the fractal connectivity matrix of stock returns exhibiting long-memory is clustered based on the hierarchical algorithm in an attempt to classify ‘fractally’ similar market groups.

Figure 1.3 Fractal connectivity matrices of select stock returns

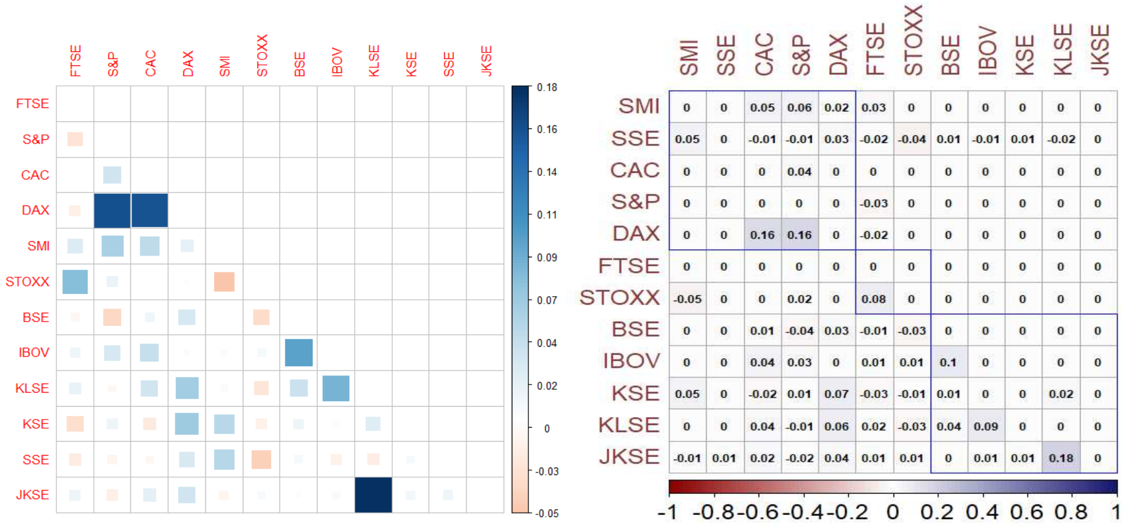
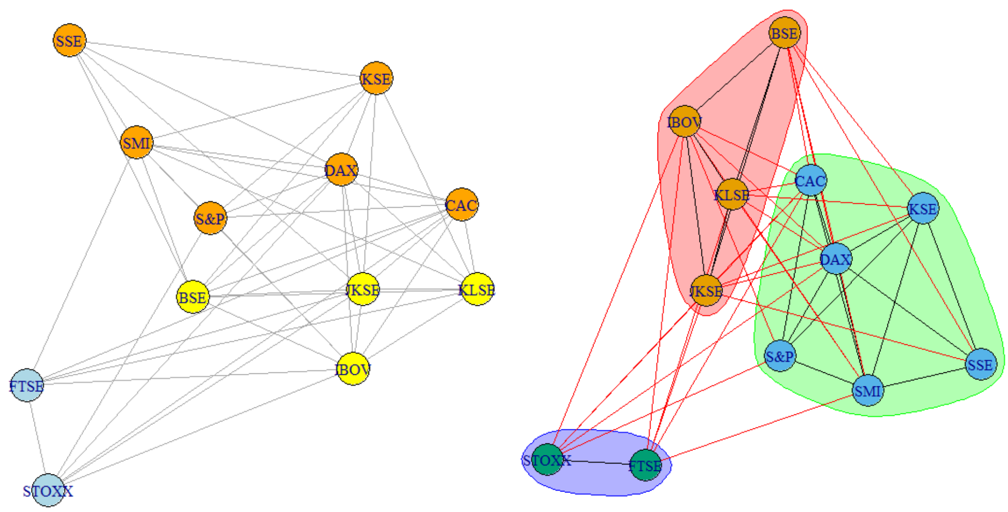


Figure 1.3 displays the fractal connectivity matrix, revealing long-run correlation structure among the developed stock markets of France (CAC40), Germany (DAX), U.S. (S&P500), Great Britain (FTSE100), Switzerland (SMI), and the Eurozone (STOXX50), and some select emerging markets of India (BSE30), Brazil (Brazil), Indonesia (JKSE), Pakistan (KSE100), China (SSE), and Malaysia (KLSE). The left panel of the above figure shows the fractal connectivity matrix whereas the right panel of the figure displays the hierarchical clustered version of the matrix. The color coded legend on the right axis of the figure on the left panel and below the bottom axis of the figure in the right panel, aids in categorizing the strength of long-range correlations. The corresponding abbreviated stock indices of both developed and emerging markets are displayed on the left and towards the top of the matrix. It is evident from the fractal connectivity matrix on the left panel that developed markets of Europe and the United States exhibit significant long-range correlation structures as indicated by larger number of elements in blue depicting positive correlations. However, a clearer picture of similar and differing fractal structures is evident from the clustered matrix. There are three clusters in the fractal connectivity matrix and they are depicted by three blue squares. It is evident from the market clusters that developed markets of SMI, SSE, CAC, S&P, and DAX exhibit

similar fractal structures. Moreover, the developed markets of United Kingdom (FTSE100) and the Eurozone (STOXX50) are clustered together revealing similar fractal structure among these markets. On the contrary, emerging markets of BSE, IBOV, KSE, KLSE, and JKSE revealing significant long-run correlation among them, are clustered together, thereby demonstrating different fractal structure as compared to developed markets. This supports our previous wavelet log-scale alignment analysis of long memory wherein developed stock market returns were mostly devoid of long range dependence whereas stock returns of emerging markets exhibited long memory. This is consistent with the view of M. Hull and McGroarty (2013) where market efficiency viewed in the lens of efficient market hypothesis (EMH), and thus long memory, corresponds to different stages of economic development. Contrary to the EMH lens of efficiency, where interpretation of information inherent in prices arises ideally due to simultaneous and uniform information arrival, it discards the phenomenon of heterogeneous information arrivals where investors may interpret information in different ways and at different times. This alternative way of deciphering information has its roots in Fractal market hypothesis (FMH) of Peters (1994).

Figure 1.4 Fractal connectivity networks of select stock returns



1.5.3 Complex networks of fractal connectivity

The final analysis of similar or dissimilar fractal structures is carried out via network analysis by computing the adjacency matrix of fractal connectivity matrix and plotting the undirected network graphs. The nodes represent the markets and the edges are due to the long run correlation structure present in the fractal connectivity matrix. As evident form the network plot on the left panel, and with the exception of KSE, markets are grouped together according to their fractal structure. The right panel of figure 1.4 depicts community formation based on similar fractal and long memory structures. Moreover, the node centrality of the developed network community shaded in lighter green is dominated by the developed market of Germany (DAX) with high degree distribution.

1.6 Conclusion

This study investigated the phenomenon of long memory among select global equity returns using novel methods from the wavelet domain. Some evidence of long-memory in equity returns of emerging markets of Malaysia, Taiwan, Pakistan, China and Indonesia are unearthed. However, the application of improved fractal estimators of Abry et al. (2003), aided by the log-scale diagram of wavelet based scaling estimates, detected significant long memory in the emerging markets of India, China and Indonesia. On the other hand, equity returns of developed markets from Europe and the U.S. did not exhibit long-range dependence, thus validating results from existing studies that reject long memory in developed markets. Moreover, we have established that markets differ according to their fractal structures and efficiency is dynamic in nature as reflected by stages of economic and market development, as opposed to the static nature of market efficiency espoused by the EMH where utility optimizing rational investors form the basis of efficient market theory. However, as demonstrated by both fractal connectivity analysis and complex networks of long-run correlations, markets exhibit varying levels of fractal structures and long range dependence. As economies gradually evolve towards developed state of market structure, the notion of efficiency should thus incorporate time-varying, behavioral, and evolutionary aspect of market development as propounded in the adaptive market hypothesis.

1.7 References

Abry, P., & Veitch, D. (1998). Wavelet analysis of long range dependent traffic. *IEEE Transactions on Information Theory*, 44(1).

Abry, P., Flandrin, P., Taqqu, M., & Veitch, D. (2003). Self-similarity and long-range dependence through the wavelet lens. In Theory and Applications of Long Range Dependence (Edited by P. Doukhan, G. Oppenheim and M. S. Taqqu). Birkhauser, Basel.

Achard, S., & Gannaz, I. (2016) Multivariate wavelet Whittle estimation in long-range dependence. *Journal of Time Series Analysis*, Vol 37, N. 4, pages 476-512. http://arxiv.org/abs/1412.0391.

Achard, S., Bassett, D.S., Meyer-Lindenberg, A., & Bullmore, E.T. (2008). Fractal connectivity of long-memory networks. *Phys. Rev. E Stat. Nonlinear Soft Matter Phys.*, 77 (3 Pt 2)

Assaf, A., & Cavalcante, J. (2002). Long-range Dependence in the Returns and Volatility of the Brazilian Stock Market. [Internet]. Available from: <http:// www.long-memory.com/volatility/CavalcanteAssaf2002.pdf>

Barkoulas, T.J., Baum, C.F., & Travlos, N. (2000). Long memory in the Greek stock market. *Applied Financial Economics*, Vol. 10, No. 2, pp. 177-84.

Bilal, T.M., & Nadhem, S. (2009). Long Memory in Stock Returns: Evidence of G7 Stocks Markets. *Research Journal of International Studies*, *9*, 36-46.

Boubaker, H. & Péguin-Feissolle, A. (2013). Estimating the Long-Memory Parameter in Nonstationary Processes Using Wavelets. *Computational Economics*, Springer; Society for Computational Economics, vol. 42(3), 291-306.

Cont R. (2005). Long range dependence in financial markets. Fractals in Engineering: *New Trends in Theory and Applications*, Pages: 159-179, ISBN: 1846280478.

DiSario, R., Saraoglu, H., McCarthy, J., & Li, H.C. (2008). An investigation of long memory in various measures of stock market volatility, using wavelets and aggregate series. *Journal of Economics and Finance, 32,136*-147.

Ding, Z., Granger, C.W.J., & Engle, R.F. (1993). A long memory property of stock returns and a new model. *Journal of Empirical Finance*, *1* (1),83–106.

Elder, J. and Serletis, A. (2007). On fractional integration dynamics in the US stock market. *Chaos, Solitons and Fractals, 34*: 777-781.

Geweke, J., & Porter-Hudak, S. (1983). The estimation and application of long memory time series models. *Journal of Time Series Analysis, 4* (4), 221-238.

Granger, C. W. J., & Joyeux, R. (1980). An introduction to long-memory time series models and fractional differencing. *Journal of Time Series Analysis, 1* (1), 15-29.

Grossman, S. J., & Stiglitz, J. E. (1980). On the impossibility of informationally efficient markets. The American Economic Review, 70 (3), 393-408.

Henry, O.T. (2002). Long memory in stock returns: some international evidence. *Applied Financial Economics*, 12: 725–729.

Hosking, J. R. M. (1981). Fractional differencing. *Biometrika, 68*(1), 165-176.

Hull, M., & McGroarty, F. (2013). Do emerging markets become more efficient as they develop? Long memory persistence in equity indices. *Emerging Markets Review*, 18, 45-61.

Hurst, H. (1951). Long term storage capacity of reservoirs. *Transaction of the American Society of Civil Engineer, 116*, 770–799.

Jagric T, Rodobnik B, Kolanovic M, Jagric V (2006). Modelling Some Properties of Stock Markets in Transition Economics. *Journal of Economics,54*(8), 816-829.

Jefferis, K., & Thupayagale, P. (2008). Long memory in southern Africa stock markets. *South African Journal of Economics*, 76 (3) ,384-398

Jensen M (1999). Using wavelets to obtain a consistent ordinary least Squares estimator of the fractional differencing parameter. *J Forecast 18*:17–32.

Kasman, A., & Kasman, S., & Torun, E. (2009). Dual Long Memory Property in Returns and Volatility: Evidence from the CEE Countries Stock Markets. *Emerging Markets Review, 10, 122*-139.

Kristoufek, L. and Vosvrda, M. (2012). Measuring capital market efficiency: Global and local correlations structure. *Physica A,* 392, 184–193.

Lo, A. W. (1991). Long-term memory in stock market prices. *Econometrica*, 59(5), 1279-1313.

Lo, A. W. (2004). The adaptive markets hypothesis: Market efficiency from an evolutionary perspective. *Journal of Portfolio Management, 30*(5), 15-29.

Lobato, I.N., & Savin, N.E. (1998). Real and spurious long-memory properties of stock-market data. *Journal of Business Economic Statistics*; 261–268.

Lobato IN., & Velasco C (2000) Long memory in stock market trading volume. *J Bus Econ Stat, 18*:410–426

Mandelbrot, B., & Wallis, J.R. (1968). Noah, Joseph, and operational hydrology. *Water Resour. Res.* 4:909–918

Mandelbrot, B., & Van Ness, J. W. (1968). Fractional Brownian motions, fractional noises and applications. *SIAM Review, 10*, 422–437

Mariani, M.C., Florescub, I., Beccar Varelaa, M., & Ncheuguim, E. (2010). Study of memory effects in international market indices. *Physica A: Statistical Mechanics and its Applications*, 389 (8), 1653-1664.

Ozdemir, Z.A. (2007). Linkages between international stock markets: A multivariate long memory Approach. *Physica A: Statistical Mechanics and its Applications*, *388*(12), 2461-2468

Ozun, A., & Cifter, A. (2007). Modeling Long-Term Memory Effect in Stock Prices: A Comparative Analysis with GPH Test and Daubechies Wavelets. *MPRA Paper 2481*, University Library of Munich, Germany.

Power, G.J., & Turvey, C.G. (2010) Long-range dependence in the volatility of commodity futures prices: wavelet-based evidence. *Physica A, 389*, 79-90

Panas, E. (2001) Estimating fractal dimension using stable distributions and exploring long memory through ARFIMA models in Athens Stock Exchange. *Applied Financial Economics, 11,* 395-402.

Pascoal, R., & Monteiro, A.M. (2014). Market Efficiency, Roughness and Long Memory in PSI20 Index Returns: Wavelet and Entropy Analysis. *Entropy, 16*, 2768–2788.

Peters, E. (1994). *Fractal Market Analysis – Applying Chaos Theory to Investment and Analysis.* New York: John Wiley & Sons, Inc.

Tan, P.P., Chin,C.W., & Galagedera, D.U.A. (2014). A wavelet-based evaluation of time-varying long memory of equity markets: A paradigm in crisis. *Physica A, 410*, 345-358

Tan, P.P., Galagedera, D.U.A., & Maharaj, E.A. (2012). A wavelet based investigation of long memory in stock returns, *Physica A 391*: 2330–2341.

Tiwari, A.K., Kumar, S., Pathak, R., & Roubaud, D. (2019). Testing the oil price efficiency using various measures of long-range dependence. *Energy Economics*, Vol. 84, p. 104547.

Tolvi, J. (2003). Long memory and outliers in stock market returns. *Applied Financial Economics, Vol. 13*(7) 495-502.

Veitch, D. and Abry, A. (1999). A Wavelet based joint estimator of the parameters of long-range dependence. *IEEE Transactions on Information Theory, 45* (3):878–897.

Vuorenmaa, T. (2005). A wavelet analysis of scaling laws and long-memory in stock market volatility, Bank of Finland Research Discussion Paper.