Financial Shocks to Banks, RD Investment, and Recessions

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Abstract

In some classes of macroeconomic models with financial frictions, an adverse financial shock successfully explains a drop in GDP, but simultaneously induces a stock price boom. The latter theoretical result is not consistent with data from actual financial crises. This study develops a simple macroeconomic model featuring a banking sector, financial frictions, and R&D-led endogenous growth to examine the impacts of an adverse financial shock to banks on firms' R&D investments and equity prices. Both the analytical and numerical investigations show that a shock that hinders the banks' financial intermediary function can be a key to generating both a prolonged recession and a drop in the firms' equity prices.

JEL classification: E32; E44; G01; O31; O41
Keywords: Banks; Endogenous growth; Financial frictions; Financial shocks; Quality-ladder growth model

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1 Introduction

Before the 2008-2009 financial crisis, macroeconomic models with financial frictions were major workhorses in business cycle studies. However, most of them focused only on the role of financial frictions in propagating and amplifying shocks originating in firms' productivity, households' preferences, or government policies. After the crisis, some studies shed light on the shocks on agents' financial constraints itself as a key influence on business cycles. Examples of such studies include those by Jermann and Quadrini (2012), Kahn and Thomas (2013), Buera and Moll (2015), Shi (2015), Kiyotaki and Moore (2019), and so on. Shocks to financial constraints are referred to as financial shocks, of which there are two main classes: a credit crunch that affects agents' borrowing capacity (Jermann and Quadrini, 2012; Kahn and Thomas, 2013; Buera and Moll, 2015) and a liquidity shortage that affects agents' ability to issue and resell equity (Shi, 2015; Kiyotaki and Moore, 2019). These studies show that adverse financial shocks successfully replicate the fall in GDP, aggregate consumption, investment, and employment.

Despite their successful explanation of realistic co-movements among major macroeconomic variables, some authors criticize these models. In particular, Shi (2015) points out that an adverse financial shock in such models, be it a credit crunch or a liquidity shortage, induces a stock price boom. Obviously, this theoretical prediction is not consistent with observations during recessions; rather, the opposite is true. The intuition behind this counterfactual prediction is straightforward. The common feature in these models is that the agents' net worth functions as collateral when they need the external funds. Then, after the negative financial shock to the agents, whether it decreases their borrowing capacity or issuing equity, the value of their net worth increases. As Shi (2015) notes, this problem is important and must be addressed because in actual economies, a fall in equity prices is thought to be the prime transmission channel of a financial shock to the aggregate economy. How can we resolve this problem? This study presents a simple theory to explain both prolonged recessions and stock price declines.

Recently, Guerron-Quintana and Jinnai (2020) provided an elegant solution to this problem. They examine the effects of the liquidity shock to investors in the modified model of Shi (2015) such that capital accumulation is associated with a learning-by-doing externality. Their numerical analysis shows that connecting the financial shock and endogenous growth can resolve the problem of the counterfactual stock price movement. This study then accounts for the following two factors explicitly. First, the event triggering the financial crisis is often a negative shock to the banking sector, like the bankruptcy of Lehman Brothers. Second, long-term investments such as innovation expenditures decreased significantly during this period (OEIO, 2012). Since even a short-term decline in these activities can have detrimental consequences in the long run, it is important to explicitly incorporate R&D activities into the model.

Against this background, I embed the banking sector as in Gertler and Karadi (2015), Gertler and Kiyotaki (2015), and Gertler et.al. (2020) into the quality-ladder growth model developed by Grossman and Helpman (1991). Households make deposits, entrant firms issue equities to conduct R&D activities, and banks intermediate financial funds between them. This study inherits the following two key features of banks in these studies. First, although households can purchase equities directly, banks are more efficient in doing so. Second, each bank has an incentive to divert its assets for personal use. Owing to this potential of moral hazard, the banks' capacity to collect deposits is limited and they face an upper bound of their leverage ratio. In the equilibrium, both households and banks purchase the equities owing to this financial friction. Within this framework, I first analytically characterize the balanced growth equilibrium in which all real variables continue to grow, led by quality-upgrading innovations. Then, I consider an adverse financial shock as the event hindering the banks' ability to intermediate funds and examine its effects on R&D investments and equity prices. The results show that such a shock generates both a prolonged downward shift in the real GDP and a sharp decline in equity prices. This result thus claims that if a financial shock hits the financial intermediary's ability to collect funds, then it can resolve the problem of the counterfactual equity price response.

The mechanism generating this result is simple and explained as follows. After the shock occurs, banks become less able to finance their equity investment by external funds. This induces less efficient allocation in equity purchases because households must purchase the equities directly, but they are less efficient in doing so. Then, demand for equities overall decreases and the equity price falls. In turn, the drop in the equity price makes R&D activities less profitable for the entrant firms. Then, their R&D investment; that is, their employment of labor for R&D activities drops. Although this decrease in investment is a transitory phenomenon, it has a long-lasting negative effect on the level of real economic variables like the real GDP. Note that this simultaneous drop in the quantities and equity prices never occurs when we consider only a negative shock to the entrant firms' R&D technology. In this case, innovation slows down, but equity prices rise.

The persistent downward shift in the real GDP and the equity price drop are consistent with the recent observation after the 2008-2009 global financial crisis. Although Guerron-Quintana and Jinnai (2020) obtain similar results, the mechanism differs from this study in significant ways. In their model, the liquidity shock makes investors more cash-strapped, and hence disturbs physical capital accumulation. A learning-by-doing externality then amplifies and sustains this negative effect to generate a persistent economic downturn. In addition, they consider the liquidity shock to investors as the financial shock, whereas this study focuses on the credit crunch to the banks. In the sense that this study pursues business cycle implications in an R&D-based endogenous growth model, this study is also related to the literature linking business cycles to economic growth, such as studies by Comin and Gertler (2006), Kobayashi and Shirai (2018), Bianchi et al. (2019), Guerron-Quintana and Jinnai (2019), and Ikeda and Kurozumi (2019). These studies build on quantitative DSGE models and explore the impacts of several economic shocks on the economy. On the other hand, the goal of this study is to develop a tractable macroeconomic model and examine the relationships among financial shocks, R&D investments, and firms' equity prices. One strength of the model proposed here is its tractability, which allows us to easily characterize the equilibrium and conduct the comparative statics. The tractability can provide insight into the inner workings of the model when considering the effects of financial shocks.

The rest of this paper is organized as follows. Section 2 sets up the model. Section 3 analytically characterizes the equilibrium without shocks and provides the comparative statics. Section 4 presents the numerical results of a transitory financial shock to the banks. Section 5 concludes.

2 Model

Time is discrete and extends from zero to infinity ($t = 0, 1, 2, \ldots$). The supply side is a discrete-time version of a quality-ladder growth model developed by Grossman and Helpman (1991). The economy has a single final good used for consumption. The final good is a composite of a continuum of differentiated intermediate goods indexed by $\omega \in [0, 1]$. Following the literature on the quality-ladder growth models, I choose the expenditure for the final good as the numeraire; that is, I normalize the expenditure to unity. There is one primary factor, labor, which is used for production of intermediate goods and R&D activities. The households supply labor and earn wages. They save in the form of deposits at banks and direct claims on equalities, but for the latter, they are less efficient in doing so relative to banks. The banks specialize in making loans and thus intermediate funds between households and firms. The banks' behavior is based on Gertler and Kiyotaki (2015).

2.1 Firms

Final good firms: The reduced form of the production function for this good is

$$Y_t = Z_t \exp\left\{\int_0^1 \ln\left[(\lambda)^{K_t(\omega)} x_t(\omega)\right] d\omega\right\},$$

where Y_t is the output of the final good, $x_t(\omega)$ is the demand for variety ω, $K_t(\omega)(= 1, 2, \ldots)$ represents the highest quality of variety ω in period t, and $\lambda > 1$ represents the size of the quality improvement achieved by one innovation. Without loss of generality, I assume the initial condition $K_0(\omega) = 1$ for all ω. Then, $K_t(\omega) - 1$ is the number of occurrences of quality-upgrading innovations for ω before period t.

The term Z_t is the exogenous technology level, which captures the technological progress via factors besides R&D activities. I assume that it grows at a constant rate of $g_Z > 0$ and its initial value is normalized to unity. Then, $Z_t = (1 + g_Z)^t$. Let $p_t(\omega)$ denote the price of variety ω. The expenditure for the final good is the numeraire: $P_t Y_t = 1$, where P_t is the price of the final good. Profit maximization yields the demand for variety ω:

$$x_t(\omega) = 1/p_t(\omega),$$

and the following zero profit condition:

$$P_t = \frac{1}{Z_t} \exp\left\{\int_0^1 \ln\left[\frac{p_t(\omega)}{(\lambda)^{K_t(\omega)}}\right] d\omega\right\}. \tag{1}$$

Intermediate goods firms: To produce $x_t(\omega)$ units requires $x_t(\omega)$ units of labor. The unit cost of production is thus the wage rate, denoted by w_t. Each variety has several potential suppliers who can produce the intermediate good with a quality of less than $K_t(\omega)$. Thus, the leader firm charges the following limit price, which is equal to the marginal cost of the firm with the second-highest quality:

$$\forall \omega \in [0, 1], \quad p_t(\omega) = \lambda w_t. \tag{2}$$

Then, it sells $x_t(\omega) = 1/(\lambda w_t)$ units of the good and earns the monopoly profit $\pi_t(\omega) = \pi \equiv 1 - 1/\lambda$ for all $\omega \in [0, 1]$.

Equity price: Let q_t denote the end-of-period equity price of the leader firm. Here, "end-of-period" has two meanings. First, q_t is *ex-dividend*; that is, q_t is evaluated after the dividend in period t has been paid. Second, q_t is evaluated after the next innovation did not occur.[1] Let R^e_{t+1} denote the one-period gross rate of return from holding the equity from the end of period t to $t+1$. Then, q_t and R^e_{t+1} must satisfy

$$q_t = \frac{\pi + (1 - I_{t+1})q_{t+1}}{R^e_{t+1}}, \tag{3}$$

where $I_{t+1} \in [0, 1]$ denotes the probability that an innovation by potential entrants succeeds in period $t + 1$ and the current leader loses its market power. I_t is determined endogenously from the resource constraint in this economy. As in the literature on quality-ladder growth, I_t is i.i.d. across varieties. Then, from the law of large numbers, it is equal to the ex-post measure of varieties in which innovation occurs.

R&D investment and firm entry: I next address the R&D activities of potential entrants. If each entrant hires κI_t units of labor in period t, then it can succeed in innovation with probability I_t, where $\kappa > 1$ is the labor requirement to obtain 100% success in innovations. If the innovation succeeds, then the entrant becomes the new leader firm for one variety from period $t + 1$. Since then, the new leader now faces the idiosyncratic risk of the next innovation and other aggregate risks. The expected benefit of innovation in period t is therefore given by $I_t \times E_t \left\{(R^e_{t+1})^{-1}[\pi + (1 - I_{t+1})q_{t+1}]\right\} = I_t q_t$, where $E_t(\cdot)$ is the expectation operator conditioned on the information available in period t. Then, the free entry condition of R&D activities for a variety is

$$q_t \leq w_t \kappa, \tag{4}$$

the equality of which holds if entrants conduct R&D.

2.2 Households

There is a continuum of homogeneous households of measure one. Let C_t and L_t denote the representative household's consumption and labor supply, respectively. Moreover, let $S^h_t \in [0, 1]$ be the measure of the intermediate goods firms, the equities of which the representative household holds directly at the end of

[1] The results obtained in this study do not change if the stock price is defined at the beginning of a period. Let $\tilde{q}_t$ denote the stock price evaluated at the beginning of period t. Then, $\tilde{q}_t$ and R^e_{t+1} must satisfy

$$\tilde{q}_t = \pi + (1 - I_t)\frac{\tilde{q}_{t+1}}{R^e_{t+1}}$$

Because of $\tilde{q}_{t+1}/R^e_{t+1} = q_t$, the above equation implies $R^e_t q_{t-1} = \pi + (1 - I_t)q_t$, which is essentially the same as equation (3).

period t. His/her life-time utility function is given by

$$E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \Big[\ln C_t + \zeta \ln(1 - L_t) - \Gamma(S_t^h) \Big] \right\}, \tag{5}$$

where $\beta \in (0, 1)$ is the discount factor and $\zeta > 0$ is the weight of the utility from leisure. In equation (5), function Γ represents the disutility from the household's direct equity holding. Following Gertler et.al. (2020), I introduce this disutility function to simply capture the household's lower efficiency in handling investments compared to banks. In this study, I assume that Γ satisfies

$$\Gamma'(S^h) > 0, \Gamma''(S^h) > 0 \text{ for } S^h > 0, \Gamma'(0) \geq 0.$$

Let S_{t-1}^h and D_{t-1} denote the measure of firms that the household holds directly and his/her deposits at the end of period $t-1$, respectively. Because of the diversified equity investments, the total values of equity held by the household change from $q_{t-1} S_{t-1}^h$ to $q_t(1 - I_t) S_{t-1}^h$. The gross interest income from holding equities is therefore given by $\pi + q_t(1 - I_t) S_{t-1}^h = R_t^e q_{t-1} S_{t-1}^h$. In addition, he/she also obtains the gross interest income from deposits, $R_t^d D_{t-1}$, where R_t^d is the gross interest rate of their deposits. Therefore, the budget constraint in period t is given by

$$R_t^d D_{t-1} + R_t^e q_{t-1} S_{t-1}^h + w_t L_t - T_t^h = P_t C_t + D_t + q_t S_t^h, \tag{6}$$

where T_t^h is the net transfer from the government in the form of taxes or subsidies depending on its sign.

The representative household chooses $\{C_t, L_t, S_t^h, D_t\}_{t=0}^{\infty}$ to maximize (5) subject to (6). The conditions for utility maximization are

$$\begin{aligned}
&\frac{\zeta}{1 - L_t} = \frac{w_t}{P_t C_t}, \\
&\frac{1}{P_t C_t} = \beta E_t \left(\frac{1}{P_{t+1} C_{t+1}} R_{t+1}^d \right), \\
&\frac{\Gamma'(S_t^h)}{q_t} + \frac{1}{P_t C_t} = \beta E_t \left(R_{t+1}^e \frac{1}{P_{t+1} C_{t+1}} \right).
\end{aligned}$$

Since the market equilibrium of the final good implies $P_t C_t = P_t Y_t (= 1)$, the conditions above reduce to

$$L_t = 1 - \frac{\zeta}{w_t}, \tag{7}$$

$$E_t \left(R_{t+1}^d \right) = \frac{1}{\beta}, \tag{8}$$

$$E_t \left(R_{t+1}^e - R_{t+1}^d \right) = \frac{\Gamma'(S_t^h)}{\beta q_t}. \tag{9}$$

2.3 Banks

Let S_t^b denote the measure of the intermediate goods firms of which equities the banks purchase. Because the total measure of intermediate goods firms is unity,

$$S_t^h + S_t^b = 1.$$

The aggregate net income of the banks in period t is therefore $R^e_t q_{t-1} S^b_{t-1} - R^d_t D_{t-1}$. With the i.i.d. probability of $1-\delta \in (0,1)$, a bank exits and upon exit the government collects its revenue. Since the exit probability is i.i.d. across banks, the $1-\delta$ share of this income is transferred to the government:

$$T^b_t = (1-\delta)(R^e_t q_{t-1} S^b_{t-1} - R^d_t D_{t-1}),$$

where T^b_t is the transfer from the exiting banks to the government. As in Gertler and Kiyotaki (2015), I assume the following inequality:

Assumption 1. $\delta < \beta$.

Consider a bank that is not hit by the exit shock but survives in period $t+1$. Let n_t denote this bank's income in period t. This bank finances equity purchases with this income and newly issued deposits:

$$n_t + d_t = q_t s^b_t, \tag{10}$$

where d_t is the newly issued deposits and s^b_t is the measure of firms that the bank purchases. Note that n_t corresponds to the bank's net worth on the balance sheet (10). Then, n_{t+1} is determined by

$$n_{t+1} = R^e_{t+1} q_t s^b_t - R^d_{t+1} d_t. \tag{11}$$

The bank's objective function at the end of period t is $\tilde{V}_t$, where

$$\tilde{V}_t \equiv E_t \left\{ \sum_{j=1}^{\infty} \beta^j (1-\delta)\delta^{j-1} n_{t+j} \right\}.$$

In the above equation, $(1-\delta)\delta^{j-1}$ is the conditional probability of exit in period $t+j$, given that the bank does not exit in period t.[2] Then, given the bank's net worth n_t, I formulate its problem recursively as

$$\begin{aligned} V_t(n_t) &= \max_{s^b_t, d_t} \tilde{V}_t \\ &= \max_{s^b_t, d_t} E_t \left\{ \beta \left[(1-\delta) n_{t+1} + \delta V_{t+1}(n_{t+1}) \right] \right\}, \end{aligned}$$

where $V_t(n_t)$ is the value function. The constraints are (10), (11), and the following incentive constraint:

$$\tilde{V}_t \geq \theta_t q_t s^b_t. \tag{12}$$

This constraint comes from the potential moral hazard problem. After buying equities, the bank decides whether to operate these assets. The bank has two options. One is holding the assets, receiving dividends, and then meeting its deposit obligations in period $t+1$. The other is selling the assets secretly to obtain the funds for personal use. To remain undetected, the bank can sell only up to the fraction θ_t of the

[2] As in the canonical DSGE literature, I first define the stochastic discount factor applied to n_{t+j} as $\beta^j \frac{P_t C_t}{P_{t+j} C_{t+j}}$, but it is always equal to β^j in this model.

assets. Inequality (12) is the constraint that the bank has no incentive to divert; thus, the bank's leverage ratio is limited:

$$\frac{q_t s_t^b}{n_t} - 1 \leq \frac{1}{\theta_t}\frac{\widetilde{V}_t}{n_t} - 1.$$

Given the bank's net worth n_t, the larger θ_t is, the less the banks can leverage their investment by additional funds from collecting deposits. In this constraint, θ_t is an exogenous random variable that I call a financial shock. I assume that it follows

$$\ln(\theta_{t+1}/\theta) = \rho \ln(\theta_t/\theta) + \varepsilon_{t+1},$$

where θ is the baseline value of θ_t, ε_t is an i.i.d. shock, and $\rho \in (0,1)$ is the parameter specifying the persistence of shocks.

To solve the problem, I use the guess and verify method. I guess the value function $V_t(n_t)$ as a linear function of n_t: $V_t(n_t) = \psi_t n_t$, where ψ_t remains to be solved. Using equation (10), I rewrite equation (11) as

$$n_{t+1} = R^d_{t+1} n_t + (R^e_{t+1} - R^d_{t+1}) q_t s_t^b.$$

I rewrite the Bellman equation as

$$\begin{aligned}
\psi_t n_t &= E_t \left\{ \beta(1-\delta+\delta\psi_{t+1}) \max_{s_t^b} n_{t+1} \right\} \\
&= E_t \left\{ \beta(1-\delta+\delta\psi_{t+1}) \max_{s_t^b} \left[R^d_{t+1} n_t + \left(R^e_{t+1} - R^d_{t+1} \right) q_t s_t^b \right] \right\},
\end{aligned}$$

subject to $\psi_t n_t \geq \theta_t q_t s_t^b$. Then, I find that as long as $R^e_{t+1} - R^d_{t+1} > 0$, the bank invests as much as it can:

$$q_t s_t^b = \frac{\psi_t n_t}{\theta_t}. \tag{13}$$

Substituting this result into the Bellman equation,

$$\begin{aligned}
\psi_t &= E_t \left\{ \beta(1-\delta+\delta\psi_{t+1}) \left[R^d_{t+1} + \left(R^e_{t+1} - R^d_{t+1} \right) \frac{\psi_t}{\theta_t} \right] \right\} \\
\Leftrightarrow & \left\{ 1 - \frac{\beta}{\theta_t} E_t \left[(1-\delta+\delta\psi_{t+1}) \left(R^e_{t+1} - R^d_{t+1} \right) \right] \right\} \psi_t = \beta E_t (1-\delta+\delta\psi_{t+1}).
\end{aligned} \tag{14}$$

Here, I assume

Assumption 2. $\theta_t > \beta E_t \left[(1-\delta+\delta\psi_{t+1}) \left(R^e_{t+1} - R^d_{t+1} \right) \right]$.

Note that the incentive constraint is

$$q_t s_t^b \leq \frac{E_t(1-\delta+\delta\psi_{t+1})}{\theta_t - \beta E_t \left[(1-\delta+\delta\psi_{t+1}) \left(R^e_{t+1} - R^d_{t+1} \right) \right]} n_t.$$

As long as Assumption 2 is satisfied, the right-hand-side is positive, and it is therefore surely optimal for the bank to make this constraint binding.

Next, consider a bank that newly enters the market in period t. I assume that the mass of such banks is $1-\delta$. Let e_t denote its initial net worth, which is fully subsidized by the government. I assume that e_t is equal to $\mu \in (0,1) \times 100\%$ of the aggregate net worth in the previous period:

$$e_t = \mu N_{t-1},$$

where N_t is the aggregate net worth of banks. The government budget constraint is

$$\underbrace{(1-\delta)(R_t^e q_{t-1} S_{t-1}^b - R_t^d D_{t-1})}_{=T_t^b} + T_t^h = (1-\delta)\mu N_{t-1}.$$

The net transfer to the household T_t^h is determined from this government budget constraint. The following assumption is useful to obtain the uniqueness of the equilibrium:

Assumption 3. $\mu < \frac{\beta-\delta}{\beta(1-\delta)} (< 1)$.

This assumption means that the government's subsidies to the new banks are not so large. The new bank's behavior is given by (10), (11), and (13), with n_t replaced by e_t. Equation (14) is then implied for the new bank as well. Since each bank's choice s_t^b is linear with respect to its state variable n_t, the banks' variables are easily aggregated over all banks. The aggregate net worth of the banks N_t is the sum of the incumbent banks' net worth and the newly entrants' net worth:

$$N_t = \delta(R_t^e q_{t-1} S_{t-1}^b - R_t^d D_{t-1}) + (1-\delta)\mu N_{t-1}.$$

Then, S_t^b must satisfy

$$q_t S_t^b = \frac{\psi_t N_t}{\theta_t}. \tag{15}$$

The leverage ratio of the banks in this economy is thus $\psi_t/\theta_t - 1$.

Here, I summarize the timing of events during a period.

1. Aggregate financial shocks are realized. The households determine their labor supply, the final- and intermediate goods firms produce the goods, and the intermediate goods firms pay dividends to the equity owners.

2. The outcomes of R&D are realized. By the law of large numbers, the fraction I_t of the leader firms are leapfrogged and the stock price of these firms becomes zero. Since the households and banks have diversified equity investments, their total values of equity change from $q_{t-1}S_{t-1}^{h(b)}$ to $q_t(1-I_t)S_{t-1}^{h(b)}$. Their gross interest income from holding equities is $\pi + q_t(1-I_t)S_{t-1}^{h(b)} = R_t^e q_{t-1} S_{t-1}^{h(b)}$. In this stage, the households also obtain the gross interest income from their deposits, $R_t^d D_{t-1}$.

3. Each bank exits in this stage with i.i.d. probability of $1-\delta \in (0,1)$. Upon exit, the government recovers the revenues of such banks, T_t^b. A mass $1-\delta$ of new banks enters the financial market with their initial net worth fully subsidized by the government. The households pay or obtain the transfer of T_t^h from the government.

4. The asset markets open. The households consume the final good and determine their portfolios, $q_t S_t^h$ and D_t, respectively, and the banks buy the equities $q_t S_t^b$.

2.4 Market-clearing conditions

The market-clearing conditions close the model. The market-clearing condition for the final good is $Y_t = C_t = 1/P_t$. The labor market clears as

$$L_t = \frac{1}{\lambda w_t} + \kappa I_t. \tag{16}$$

The market-clearing condition of equities is $S_t^h + S_t^b = 1$. Finally, the deposits D_t must satisfy

$$D_t + N_t = q_t S_t^b. \tag{17}$$

From these market-clearing conditions together with the agents' behavior, the household's budget constraint (6) is automatically satisfied from Walras' law.

3 Equilibrium

In this section, I analytically characterize the equilibrium in the case of no aggregate risks by assuming $\varepsilon_t = 0$ (that is, $\theta_t = \theta$) for all t. I drop the expectation operator $E_t(\cdot)$. From equation (8), $R_{t+1}^d = 1/\beta$ for all t. Equation (9) then implies

$$R_{t+1}^e = \frac{1}{\beta} + \frac{\Gamma'(S_t^h)}{\beta q_t}. \tag{18}$$

3.1 Equilibrium conditions

Substituting (18) into (3) and (14), the dynamics of q_t and ψ_t are, respectively,

$$q_t + \Gamma'(S_t^h) = \beta \left[\pi + (1 - I_{t+1}) q_{t+1}\right], \tag{19}$$

$$\psi_t = (1 - \delta + \delta \psi_{t+1}) \left(1 + \frac{\psi_t}{\theta} \frac{\Gamma'(S_t^h)}{q_t}\right). \tag{20}$$

The banks' aggregate net worth in period $t+1$ is

$$N_{t+1} = \delta \left(R_{t+1}^e q_t S_t^b - R_{t+1}^d D_t\right) + (1 - \delta) \mu N_t.$$

Substituting equations (15), (17), and (18) into the above equation, the dynamics of the banks' aggregate net worth is

$$N_{t+1} = \left[\frac{\delta}{\beta} \left(1 + \frac{\psi_t}{\theta} \frac{\Gamma'(S_t^h)}{q_t}\right) + (1 - \delta) \mu\right] N_t. \tag{21}$$

Note that N_t is a state variable and thus its initial value N_0 is historically predetermined, whereas q_t and ψ_t are forward-looking variables, so their initial values of q_0 and ψ_0 are determined endogenously. The dynamic system (19)–(21) includes S_t^h and I_{t+1}. From $S_t^h + S_t^b = 1$ and equation (15), S_t^h is

$$S_t^h = 1 - \frac{\psi_t N_t}{\theta q_t}. \tag{22}$$

I focus on the case of $I_t > 0$, which implies $w_t = q_t/\kappa$ from (4). From the condition of the household's labor supply (7) and the labor market equilibrium (16),

$$I_{t+1} = \frac{1}{\kappa} - \frac{1+\lambda\zeta}{\lambda}\frac{1}{q_{t+1}}. \tag{23}$$

Thus, the equilibrium conditions in the deterministic economy are given by the dynamic system of (19)–(23).

3.2 Balanced growth equilibrium

As in the endogenous growth literature, I define the balanced growth equilibrium as the equilibrium with the variables in their real terms growing over time at the same rate. In this type of equilibrium for this model, the endogenous variables $(q_t, \psi_t, N_t, I_t, S_t^h)$ in (19)–(23) become stationary. I omit the time subscript here.

Equation (21) with $N_t = N_{t+1}$ implies

$$1 + \frac{\psi}{\theta}\frac{\Gamma'(S^h)}{q} = B^*, \tag{24}$$

where

$$B^* \equiv \frac{\beta[1-(1-\delta)\mu]}{\delta}.$$

Note that B^* depends only on the exogenous parameters and Assumption 3 ensures $B^* > 1$. Substituting this into (20) and imposing $\psi_t = \psi_{t+1}$ yields the stationary value of ψ:

$$\psi^* = \frac{(1-\delta)B^*}{1-\delta B^*} > 0.$$

Hereafter, a superscript asterisk over a variable represents its stationary value. Substituting the obtained ψ^* back into equation (24), I obtain the relationship between the equity price q and the households' equity purchases S_t^h:

$$q = \frac{\delta\psi^*}{[\beta - \delta - \beta(1-\delta)\mu]\,\theta}\Gamma'(S^h), \tag{25}$$

where the sign of the denominator is positive from Assumption 3. Since the disutility function of the households' direct equity purchases is strictly convex, this equation shows a positive relationship between q and S^h. Equation (25) represents their relationship from the banks' perspective. When S^h becomes larger, households become more reluctant to hold equities directly unless their rate of return becomes sufficiently higher. Indeed, $R^e - R^d = \Gamma'(S^h)/(\beta q)$ experiences upward pressure. This upward pressure in turn has a positive impact on the banks' aggregate net worth, and hence they want to purchase more of these equities. In the stationary equilibrium where banks' net worth is constant, such an increase in their equity demand puts upward pressure on the equity price. As equation (24) shows, the upward pressure on S^h is offset by a rise in the equity price such that $R^e - R^d$ remains constant.

The economy has the other relationship between q and S^h. By imposing $q_t = q_{t+1}$ in (19), substituting (23) into the resulting equation, and using $\pi = 1 - 1/\lambda$, one can find the following negative relationship between q and S^h:

$$q = \frac{\beta(1+\zeta) - \Gamma'(S^h)}{1 - \beta(1 - 1/\kappa)}, \tag{26}$$

where the sign of the denominator is positive. Equation (26) indicates the relationship between q and S^h from the households' perspective. The intuition is straightforward. The increase in S^h makes the households less willing to hold equities unless their rate of return becomes sufficiently higher. Therefore, this unwillingness depresses the unit cost of the equity purchase q.

From (25) and (26), the stationary value of the equity price, denoted by q^*, is

$$q^* = \frac{\beta(1+\zeta)\delta\psi^*}{[1 - \beta(1 - 1/\kappa)]\delta\psi^* + [\beta - \delta - \beta(1-\delta)\mu]\theta}. \tag{27}$$

Then, the households' equity holdings S^{h*} are determined accordingly. With $\Gamma'(0)$ adequately small, S^{h*} is positive. I define γ as $\gamma \equiv \Gamma'(1)$. From (25) and (26), one can easily obtain

$$S^{h*} < 1 \Leftrightarrow \gamma > \frac{q^*}{\delta\psi^*}[\beta - \delta - \beta(1-\delta)\mu]\theta.$$

As long as this inequality is satisfied, equity holdings are diversified between banks and households.

After obtaining q^*, the rate of innovation I^* is determined as:

$$I^* = \frac{1}{\kappa} - \frac{1 + \lambda\zeta}{\lambda q^*},$$

which is positive if and only if [3]

$$q^* > \frac{\kappa(1+\lambda\zeta)}{\lambda}.$$

Then, all real variables, such as consumption C_t, real wage w_t/P_t, real equity price q_t/P_t, and so on, grow at the same rate as $1/P_t$. Using (1) and (2),

$$1/P_t = \frac{(1+g_Z)^t}{w_t}(\lambda)^{\int_0^1 (K_t(\omega)-1)d\omega}. \tag{28}$$

Since the wage rate becomes constant and the law of large numbers implies $\int_0^1 (K_t(\omega) - K_{t-1}(\omega))d\omega = I^*$, it follows that $\ln P_{t-1} - \ln P_t = \ln(1+g_Z) + I^* \ln\lambda$. Then, I obtain the balanced growth rate as

$$g^* = g_Z + I^* \ln\lambda,$$

where $g^* \simeq \ln(1+g^*)$ and $g_Z \simeq \ln(1+g_Z)$ are used. Finally, we have to check that Assumption 2 is satisfied in the obtained balanced growth equilibrium. In this non-stochastic economy, this assumption is rewritten as

$$\theta > (1 - \delta + \delta\psi_{t+1})\frac{\Gamma'(S_t^h)}{q_t}.$$

Since $\Gamma'(S^{h*})/q^* = \theta(B-1)/\psi^*$ holds from (24) and $1 - \delta + \delta\psi^* = \psi^*/B$ holds from (20), I rewrite the inequality above as $\theta > \theta(B-1)/B$, which is necessarily satisfied.

[3] Since $\kappa > 1$, $I^* < 1$ is guaranteed without any restrictions.

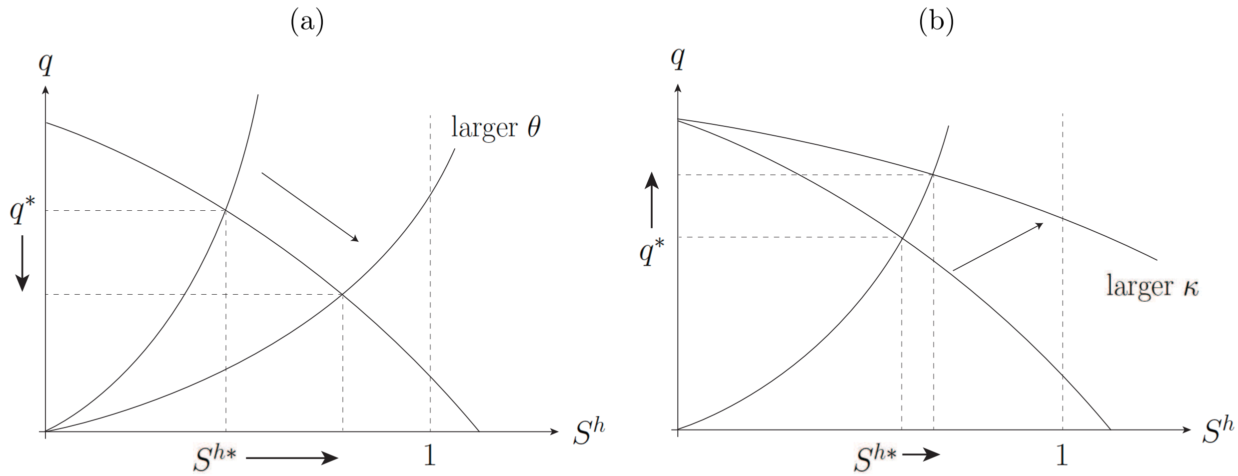

Figure 1: Comparative statics of the balanced growth equilibrium

Proposition 1. *There exists a unique balanced growth equilibrium with a positive growth rate and diversification of equity holdings if*

$$\frac{\kappa(1+\lambda\zeta)}{\lambda} < q^* < \frac{\delta\psi^*\gamma}{[\beta-\delta-\beta(1-\delta)\mu]\theta}.$$

Hereafter, I focus on the case in which q^* satisfies this inequality.

3.3 Comparative statics

The model is tractable enough to conduct a comparative statics analysis of the balanced growth equilibrium, from which one can gain insight on inner workings of the model. In panels (a) and (b) of Figure 1, the upward- and downward-sloping curves represent equations (25) and (26), respectively. Generally, a larger θ_t means a decrease in the banks' leverage ratio; that is, the banks' borrowing capacity decreases. Since θ does not affect ψ^* in the balanced growth equilibrium here, the variation of θ directly corresponds the difference in the banks' leverage ratios. In the economy where θ is large, the banks cannot finance their equity investment by external funds. As panel (a) of this figure shows, this leads households to purchase more equities directly. Since the households are reluctant to do so because of their psychic utility costs, the unit cost of equities must fall. Consequently, the equity price q^* is low in the economy with a large θ. This in turn makes the R&D activities less profitable for the potential entrants. Accordingly, the rate of innovation in the balanced growth equilibrium I^* also becomes low.

Proposition 2. *In the balanced growth equilibrium, a larger θ results in both a lower growth rate and a lower equity price.*

Proof. See the Appendix. □

Note that such a simultaneous decline never occurs when the R&D technology just changed. In the economy where κ is large, it becomes more costly for a potential entrant to conduct R&D activities. Then, through the entrants' free entry condition, the benefit of R&D must be high. As panel (b) shows, this induces upward shifts in the curve representing equation (26). Thus, in this case, the innovation rate becomes low, but the equity price becomes high.

The banks' total values are given by $\psi^* N^*$, where θ does not affect ψ^*. From (22), the banks' total net worth N^* is

$$N^* = \frac{\theta q^*(1 - S^{h*})}{\psi^*}.$$

With an increase in θ, the term $q^*(1 - S^{h*})$ decreases simply because the banks' asset holdings decrease. Simultaneously, an increase in θ has the direct effect of increasing N^*. In the Appendix, the following lemma is shown to hold.

Proposition 3. *A larger θ leads to a lower net worth for the banks if and only if $f(S^{h*}) < S^{h*}/S^{b*}$; that is,*

$$\frac{dN^*}{d\theta} \gtreqless 0 \Leftrightarrow f(S^{h*}) \gtreqless S^{h*}/S^{b*},$$

where function $f(S^{h*})$ is given by $f(S^{h*}) \equiv S^{h*}\Gamma''(S^{h*})/\Gamma'(S^{h*}) > 0$.

Proof. See the Appendix. □

The term S^{h*}/S^{b*} is the ratio of households' to banks' direct equity purchases. Function $f(S^{h*})$ represents the elasticity of the marginal disutility for the households' direct equity holdings. For example, if function $\Gamma(S^h)$ is

$$\Gamma(S^h) = \gamma\frac{(S^h)^{1+\eta}}{1+\eta}, \ \eta > 0, \tag{29}$$

then $f(S^h) = \eta > 0$ for all S^h.

4 Numerical analysis of transitory financial shocks

The comparative statics results show the long-run performance of the economies that have different baseline θ values. In this section, I examine how a transitory shock to θ_t influences the economy.

4.1 Calibration

Hereafter, I specify the disutility function Γ as equation (29). There are 10 structural parameters in the model. Table 1 reports the results of calibration. A period in the model corresponds to one quarter of a year. I set the discount factor at $\beta = 0.99$, which is standard in the literature. I set the banks' survival probability at $\delta = 0.93$, as in Gertler et.al. (2020). I set the degree of quality improvement at $\lambda = 1.15$.

From the analytical result in Proposition 3, I expect that different values of η will have different impacts on the banks, as would a transitory change in θ_t. As in Table 1, I set $S^{h*} = S^{b*} = 0.5$ for simplicity. Proposition 3 leads to the conjecture that whether or not η exceeds one is critical. Then, I consider three cases: a low value ($\eta = 0.8$), an intermediate value ($\eta = 1$), and a large value ($\eta = 1.2$). In the Appendix, it is shown that this variation in η induces only a variation in γ.

I set the other parameters such that some variables in the balanced growth equilibrium achieve their target values. The Appendix provides the calibration details. I set the growth rate along the balanced

Table 1: Parameters

Parameter	Value	Source/Target
β	0.99	Exogenously chosen
δ	0.93	Exogenously chosen
λ	1.15	Exogenously chosen
η	(i)0.8, (ii)1, (iii)1.2	Exogenously chosen
γ	(i)0.037, (ii)0.042, (iii)0.049	$S^{h*} = 0.5$
ζ	2.18	$L^* = 0.3$
κ	1.38	$\kappa I^*/L^* = 0.07$
g_Z	0.0028	$g^* = 1.02^{1/4} - 1$
μ	0.206	$R^e - R^d = 1.02^{1/4} - 1$
θ	0.302	$q^* S^{b*}/N^* = 10$

Table 2: Balanced growth rate

Variable	Value	Description
I^*	0.015	Innovation rate
$I^* \ln \lambda$	0.0021 ($\simeq$ 0.85% per year)	Growth rate by R&D
g_Z	0.0028 ($\simeq$ 1.15% per year)	Growth rate by other factors

growth path at $g^* = 1.02^{1/4} - 1 \simeq 0.005$. I set aggregate hours of work at $L^* = 0.3$ and the employment share of R&D activities at 7%. Thus, $\kappa I^* = 0.021$. I set the target value of S^{h*} at 0.5, and the spread at $R^e - R^d = 1.02^{1/4} - 1$. Finally, I set the banks' leverage at $q^* S^{b*}/N^* = 10$, as Gertler and Kiyotaki (2015) and Gertler et.al. (2020) also use this value. Table 2 reports the decomposition of the balanced growth rate.

4.2 Impulse responses

Suppose that the economy is on the balanced growth path in period 0. In period 1, θ_1 unanticipatedly increases by 50% relative to its baseline value θ. Then, the economy experiences no other shocks and θ_t gradually recovers to its baseline according to $\ln(\theta_t/\theta) = \rho \ln(\theta_{t-1}/\theta)$. Following the existing studies, I set the persistence of financial shocks at $\rho = 0.9$. I replace θ with θ_t in the dynamic system (19)–(23) and log-linearize this system around $(q^*, \psi^*, N^*, S^{h*}, I^*, \theta)$. I then compute the impulse response functions of these and other key variables. The Appendix provides the log-linear approximation of the dynamic system.

Figure 2 illustrates the results of the transitory adverse financial shock. The horizontal axis is the number of quarters. I plot the percentage deviations in levels of the variables from those without shocks. The first panel shows the financial shock. The second to fifth panels display the impulse response functions of the equity price q_t, R&D investment I_t, and the banks' value $\psi_t N_t$, respectively. The directions of the transitory changes are the same as the long-run changes obtained from the comparative statics. In particular, as expected, the ranking between η and S^{h*}/S^{b*} is critical to determine the response in the

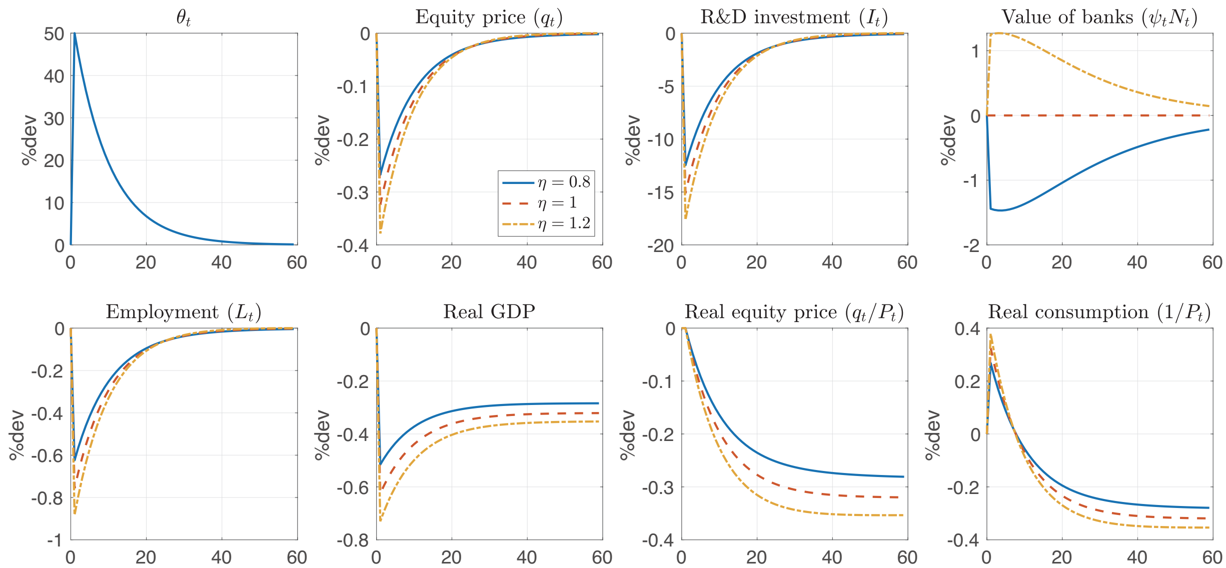

Figure 2: Impulse response functions (horizontal axis: quarter; vertical axis: percentage deviation from the trends without shocks)

banks' values.

In the second row, the four panels show the responses of major macroeconomic variables. In this model, the real GDP in period t is

$$GDP_t \equiv \frac{w_t L_t + \pi}{P_t},$$

where the numerator is the sum of wage income and income gain.[4] Equation (28) provides the price of the final good P_t. In this model, the term $\int_0^1 (K_t(\omega) - 1) d\omega$ is the average number of innovations before period t. By the law of large numbers, it is given by $\sum_{t=0}^{s-1} I_s$. Then, it follows that

$$1/P_t = \frac{(1 + g_Z)^t}{w_t} (\lambda)^{\sum_{t=0}^{s-1} I_s}.$$

This clearly shows that even though the contraction of R&D activities is transitory, its impact on the real variables is cumulative, and hence their drops are persistent.

5 Conclusion

In some macroeconomic models with financial frictions, an adverse financial shock successfully explains a drop in GDP, but it is often associated with a stock price boom. The latter prediction is at odds with the observations in real recessions. This study develops a simple macroeconomic model featuring banks, financial frictions, and firms' R&D activities to tackle this problem. Both the analytical and numerical investigations show that a shock hindering the banks' financial intermediary function is a key to generate both a prolonged recession and a drop in the firms' equity prices.

[4] From the labor market equilibrium (16), the numerator is equivalent to $1 + q_t I_t$; that is, the sum of value added from intermediate goods production and R&D activities.

The model in this study is highly stylized because the purpose here is to propose a clear-cut solution to the problem. Therefore, developing a model more suitable for quantitative analysis and examining the effects of the same shock would be a promising extension. Nonetheless, the results in this study provide a benchmark. In addition, because of its analytical tractability, the model is open to extensions. Among others, it would be interesting to introduce stock price bubbles and examine a self-fulfilling financial shock after a bubble bursts. In this model, I follow the literature in that the financial shock is exogenous for the sake of comparison. Then, it would be natural to examine the outcome if the shock is self-fulfilling.

References

Bianchi, F., H. Kung, and G. Morales (2019) Growth, Slowdowns, and Recoveries, *Journal of Monetary Economics* 101, 47–63.

Buera, F. and B. Moll (2015) Aggregate Implications of a Credit Crunch: The Importance of Heterogeneity, *American Economic Journal: Macroeconomics* 7, 1–42.

Comin, D. and M. Gertler (2006) Medium-term Business Cycles, *American Economic Review* 96, 523–551.

Gertler, M. and P. Karadi (2015) A Model of Unconventional Monetary Policy, *Journal of Monetary Economics* 58, 17–34.

Gertler, M. and N. Kiyotaki (2015) Banking, Liquidity, and Bank Runs in an Infinite Horizon Economy, *American Economic Review* 105, pp. 2011–2043.

Gertler, M., N. Kiyotaki, and A. Prestipino (2020) A Macroeconomic Model of Financial Panics, *Review of Economic Studies*, 87, 240–288.

Grossman, G. and E. Helpman (1991) Quality Ladders in the Theory of Growth, *Review of Economic Studies* 58, 43–61.

Guerron-Quintana, P. A. and R. Jinnai (2019) Financial Frictions, Trends, and the Great Recession, *Quantitative Economics*, 10, 735–777.

Guerron-Quintana, P. A. and R. Jinnai (2020) On Liquidity Shocks and Asset Prices, *Journal of Money, Credit and Banking*, forthcoming.

Ikeda, D. and T. Kurozumi (2019) Slow Post-Financial Crisis Recovery and Monetary Policy, *American Economic Journal: Macroeconomics* 11, 82–112

Jermann, U. and V. Quadrini (2012) Macroeconomic Effects of Financial Shocks, *American Economic Review* 102, 238–271.

Kahn, T. and J. K. Thomas (2013) Credit Shocks and Aggregate Fluctuations in an Economy with Production Heterogeneity, *Journal of Political Economy* 121, 1055–1107.

Kiyotaki, N. and J. Moore (2019) Liquidity, Business Cycles, and Monetary Policy, *Journal of Political Economy*, 127, 2926–2946.

Kobayashi, K. and D. Shirai (2018) Debt-Ridden Borrowers and Economic Slowdown, *CIGS Working Paper Series* 18-003E, The Canon Institute for Global Studies.

OECD (2012) Innovation in the Crisis and Beyond, *in OECD Science, Technology and Industry Outlook.*

Shi, S. (2015) Liquidity, Assets and Business Cycles, *Journal of Monetary Economics* 70, 116–132.

Appendix to "Financial Shocks to Banks, R&D Investment, and Recessions"

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Appendix A Proof of Propositions 2 and 3

This section shows the comparative statics for the balanced growth equilibrium, which includes the proof of Propositions 2 and 3. The variables q^*, S^{h*}, I^*, and N^* are determined from

$$
\begin{aligned}
q^* &= \frac{\delta\psi^*}{\beta-\delta-\beta(1-\delta)\mu}\frac{\Gamma'(S^{h*})}{\theta},\\
q^* &= \frac{\beta(1+\zeta)-\Gamma'(S^{h*})}{1-\beta+\beta/\kappa},\\
I^* &= \frac{1}{\kappa}-\frac{1+\lambda\zeta}{\lambda q^*},\\
N^* &= \frac{\theta q^*(1-S^{h*})}{\psi^*}.
\end{aligned}
$$

Note that ψ^* does not depend on θ or κ. From these equations,

$$\frac{dq^*}{q^*} = \frac{S^{h*}\Gamma''}{\Gamma'}\frac{dS^{h*}}{S^{h*}} - \frac{d\theta}{\theta}, \tag{30}$$

$$\frac{dq^*}{q^*} = -a\frac{S^{h*}\Gamma''}{\Gamma'}\frac{dS^{h*}}{S^{h*}} + b\frac{d\kappa}{\kappa}, \tag{31}$$

$$\frac{dI^*}{I^*} = \frac{1+\lambda\zeta}{\lambda q^* I^*}\frac{dq^*}{q^*} - \frac{1}{\kappa I^*}\frac{d\kappa}{\kappa}, \tag{32}$$

$$\frac{dN^*}{N^*} = \frac{dq^*}{q^*} - \frac{S^{h*}}{1-S^{h*}}\frac{dS^{h*}}{S^{h*}} + \frac{d\theta}{\theta}, \tag{33}$$

where

$$
\begin{aligned}
a &\equiv \frac{\Gamma'}{\beta(1+\zeta)-\Gamma'} > 0,\\
b &\equiv \frac{\beta/\kappa}{1-\beta+\beta/\kappa} \in (0,1).
\end{aligned}
$$

The value of a is positive as long as $q^* > 0$. From (30) and (31),

$$\frac{dq^*}{q^*} = \frac{1}{1+a}\left(-a\frac{d\theta}{\theta} + b\frac{d\kappa}{\kappa}\right), \tag{34}$$

$$\frac{dS^{h*}}{S^{h*}} = \frac{1}{1+a}\frac{\Gamma'}{S^{h*}\Gamma''}\left(\frac{d\theta}{\theta} + b\frac{d\kappa}{\kappa}\right). \tag{35}$$

Then,

$$
\begin{aligned}
&\frac{dq^*/q^*}{d\theta/\theta} < 0, \quad \frac{dq^*/q^*}{d\kappa/\kappa} > 0,\\
&\frac{dS^{h*}/S^{h*}}{d\theta/\theta} > 0, \quad \frac{dS^{h*}/S^{h*}}{d\kappa/\kappa} > 0.
\end{aligned}
$$

Substituting (34) into (32) and using the fact that $\frac{1+\lambda\zeta}{\lambda q^* I^*} = \frac{1}{\kappa I^*} - 1 > 0$,

$$\begin{aligned}\frac{dI^*}{I^*} &= \frac{1}{1+a}\left(\frac{1}{\kappa I^*}-1\right)\left(-a\frac{d\theta}{\theta}+b\frac{d\kappa}{\kappa}\right)-\frac{1}{\kappa I^*}\frac{d\kappa}{\kappa} \\ &= \frac{-a}{1+a}\left(\frac{1}{\kappa I^*}-1\right)\frac{d\theta}{\theta}-\frac{1}{1+a}\left(\frac{1+a-b}{\kappa I^*}+b\right)\frac{d\kappa}{\kappa},\end{aligned}$$

which implies

$$\frac{dI^*/I^*}{d\theta/\theta} < 0, \quad \frac{dI^*/I^*}{d\kappa/\kappa} < 0.$$

Then, Proposition 2 holds.

Substituting (34) and (35) into (33) yields

$$\frac{dN^*}{N^*} = \frac{1}{1+a}\left(1-\frac{\Gamma'}{S^{h*}\Gamma''}\frac{S^{h*}}{1-S^{h*}}\right)\left(\frac{d\theta}{\theta}+b\frac{d\kappa}{\kappa}\right).$$

Then,

$$\frac{dN^*/N^*}{d\theta/\theta} \gtreqless 0 \Leftrightarrow \frac{S^{h*}\Gamma''}{\Gamma'} \gtreqless \frac{S^{h*}}{1-S^{h*}},$$

which shows Proposition 3.

Appendix B Calibration details

The following parameters are chosen exogenously: $\beta = 0.99$, $\delta = 0.93$, and $\lambda = 1.15$. Since the value of η affects the comparative statics, I consider three cases: a low value ($\eta = 0.8$), an intermediate value ($\eta = 1$), and a large value ($\eta = 1.2$). I set the aggregate hours of work in the balanced growth equilibrium at $L^* = 0.3$, and the employment share of R&D activities at 7%. Then,

$$L^*_{R\&D} \equiv \kappa I^* = 0.07L^* = 0.021.$$

The wage rate w^* is given by $w^* = 1/[\lambda(L^* - L^*_{R\&D})]$. The value of ζ is given by

$$\zeta = w^*(1-L^*).$$

I set the target value of S^{h*} at 0.5. I also assume that the spread is 2% per year: $R^e - R^d = 1.02^{1/4} - 1$. Then, κ, γ, q^*, and I^* are determined from

$$\begin{aligned}&R^e - R^d = \frac{\gamma(S^h)^\eta}{\beta q^*}, \\ &q^* + \gamma(S^h)^\eta = \beta\pi + \beta(1-I^*)q^*, \\ &q^* = w^*\kappa, \\ &L^*_{R\&D} = \kappa I^*.\end{aligned}$$

Here, note that the variation of η induces only the variation of γ. From the last two equations,

$$q^* I^* = \underbrace{w^* L^*_{R\&D}}_{\text{already found}}.$$

Table 3: Eigenvalues of matrix $\mathbf{J}$

(i)	3.6542	0.9570	1.0214	0.9000
(ii)	3.6578	0.9538	1.0249	0.9000
(iii)	3.6614	0.9510	1.0281	0.9000

Substituting this into the second equation yields

$$(1-\beta)q^* + \gamma(S^h)^\eta = \beta(\pi - w^* L^*_{R\&D}).$$

Then, q^* and γ are determined from

$$\begin{pmatrix} \beta(R^e - R^d) & -(S^h)^\eta \\ 1-\beta & (S^h)^\eta \end{pmatrix} \begin{pmatrix} q^* \\ \gamma \end{pmatrix} = \begin{pmatrix} 0 \\ \beta(\pi - L^*_{R\&D}) \end{pmatrix}.$$

This equation shows that η does not affect q^*. Accordingly, I^* and κ are also independent of η.

I choose the balanced growth rate g^* such that the growth rate is 2% per year: $1 + g^* = 1.02^{1/4}$. The rate of exogenous technological progress g_Z is determined from $g_Z = g^* - I^* \ln \lambda$. Following Gertler and Kiyotaki (2015) and Gertler et.al. (2020), I set the banks' leverage $q^* S^{b*}/N^*$ at 10. Since $q^* S^{b*}$ is already known, this determines the value of N^*. Furthermore, it follows that

$$\psi^*/\theta = 10.$$

In the balanced growth equilibrium, the following equations hold:

$$\underbrace{1 + \frac{\psi^*}{\theta}\frac{\Gamma'(S^{h*})}{q^*}}_{\text{already found}} = B^* = \frac{\beta[1-(1-\delta)\mu]}{\delta},$$

where the first equality comes from (24), and the second one comes from the definition of B^*. Then, μ is determined. Finally, ψ^* and θ are determined from

$$\psi^* = \frac{(1-\delta)B^*}{1-\delta B^*},$$

and $\theta = \psi^*/10$.

Appendix C Log-linear approximation

Let a hat over a variable indicate the log-deviation of the variable from its stationary value. For example, $\widehat{q}_t = \ln(q_t/q^*) \simeq (q_t - q^*)/q^*$. The log-linear approximation of the system (19)–(23) around

$(q^*, \psi^*, N^*, S^{h*}, I^*)$ is

$$\begin{aligned}
&q^*\widehat{q}_t + \eta\gamma(S^{h*})^\eta \widehat{S}^h_t = \beta(1-I^*)q^*\widehat{q}_{t+1} - \beta q^* I^* \widehat{I}_{t+1},\\
&\widehat{\psi}_t = \frac{\delta\psi^*}{1-\delta+\delta\psi^*}\widehat{\psi}_{t+1} + \widehat{B}_t,\\
&\widehat{B}_t = \frac{B^*-1}{B^*}(\widehat{\psi}_t + \eta\widehat{S}^h_t - \widehat{q}_t - \widehat{\theta}_t),\\
&\widehat{N}_{t+1} = \frac{\delta}{\beta}B\widehat{B}_t + \widehat{N}_t,\\
&\widehat{S}^h_t = \frac{1-S^{h*}}{S^{h*}}(\widehat{q}_t - \widehat{\psi}_t - \widehat{N}_t + \widehat{\theta}_t),\\
&\widehat{I}_{t+1} = \frac{1+\lambda\zeta}{\lambda q^* I^*}\widehat{q}_{t+1},
\end{aligned}$$

where $B_t = 1 + \frac{\psi_t \Gamma'(S^h_t)}{\theta_t q_t} > 1$. These equations provides the following autonomous dynamic system:

$$\begin{pmatrix} \widehat{q}_{t+1} \\ \widehat{\psi}_{t+1} \\ \widehat{N}_{t+1} \\ \widehat{\theta}_{t+1} \end{pmatrix} = \underbrace{\begin{pmatrix} \frac{q^*+X^*}{\beta q^*(1-1/\kappa)} & -\frac{X^*}{\beta q^*(1-1/\kappa)} & -\frac{X^*}{\beta q^*(1-1/\kappa)} & \frac{X^*}{\beta q^*(1-1/\kappa)} \\ \frac{H^*(1-\eta\alpha^*)}{\Psi^*} & \frac{1-H^*(1-\eta\alpha^*)}{\Psi^*} & \frac{H^*\eta\alpha^*}{\Psi^*} & \frac{H^*(1-\eta\alpha^*)}{\Psi^*} \\ -M^*(1-\eta\alpha^*) & M^*(1-\eta\alpha^*) & 1-M^*\eta\alpha^* & -M^*(1-\eta\alpha^*) \\ 0 & 0 & 0 & \rho \end{pmatrix}}_{\equiv \mathbf{J}} \begin{pmatrix} \widehat{q}_t \\ \widehat{\psi}_t \\ \widehat{N}_t \\ \widehat{\theta}_t \end{pmatrix},$$

where

$$\begin{aligned}
&\alpha^* \equiv (1-S^{h*})/S^{h*},\\
&X^* \equiv \gamma\eta(S^{h*})^\eta\alpha^*,\\
&H^* \equiv \frac{B^*-1}{B^*},\\
&M^* \equiv \frac{\delta(B^*-1)}{\beta},\\
&\Psi^* = \frac{1-\delta+\delta\psi^*}{\delta\psi^*}.
\end{aligned}$$

Table 3 reports the eigenvalues of matrix $\mathbf{J}$, where the italic numbers ((i), (ii),...) correspond to the calibration scenario. This table shows that in all three scenarios, the dynamic system has two eigenvalues with absolute values less than 1. Thus, the impulse response function of each variable is uniquely determined in all three cases because the system has two state variables (N_t and θ_t) and two jump variables (q_t and ψ_t).