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26 July 2020

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Abstract

Using two types of overlapping generations (OLG) model, we show that involuntary unemployment is in a Nash equilibrium of a game with a firm and consumers, and we can achieve full-employment by fiscal policy financed by seignorage not tax. Once we achieve it, it is maintained without government expenditure. Also we show that a fall in the nominal wage rate may not decrease involuntary unemployment.

1. Introduction

This paper is a sequel of the previous paper (Tanaka (2020)) in which we have analyzed the existence of involuntary unemployment under perfect competition. In this paper using very simple models we study the existence of involuntary unemployment from the game theoretic view point, and examine whether involuntary unemployment is in a Nash equilibrium of a game with a firm and consumers. Also we consider realization of full-employment by fiscal policy. Involuntary unemployment in this paper is a situation where workers are willing to work at the market wage or just below but are prevented by factors beyond their control, mainly, deficiency of aggregate demand[1].

We use two types of overlapping generations (OLG) model simplifying and extending the model in Otaki (2007), Otaki (2009), Otaki (2011) and Otaki (2015). He assumes indivisibility (or inelasticity) of individual labor supply. If labor supply is indivisible, it may be 1 or 0[2]. The first model, which is a two generations OLG model with pay-as-you-go pension system, used in the next section is the following economy.

1. There are one firm and one good. One unit of the good is produced by one unit of labor.

2. There are two younger (working) consumers who may be employed or not employed by the firm. An unemployed consumer consumes the good by borrowing money from an employed consumer against pension.

3. There are two older (retired) consumers who may be employed or not employed by the firm when they are young.

4. There is a pay-as-you-go pension system for older consumers.

5. There may exist government expenditure.

Using this model we will show the existence of involuntary unemployment in a Nash equilibrium, and realization of full-employment by fiscal policy financed by seignorage. Once we achieve it, it is maintained without government expenditure.

The second model, which is a three generations OLG model with childhood consumers without pension, used in Section 3 is the following economy.

1. There are one firm and one good. One unit of the good is produced by one unit of labor.

2. There are two younger (working) consumers who may be employed or not employed by the firm.

3. There are two older (retired) consumers who may be employed or not employed by the firm when they are young.

[1] About the studies of involuntary unemployment without wage rigidity please see, for example, Umada (1997) and Lavoie (2001) .

[2] About indivisibility of labor supply please see Hansen (1985).

4. There are two childhood consumers who consume the good by borrowing money from consumers of the younger generation. Their consumptions are constant.

5. There may exist government expenditure.

Using this model we will show the existence of involuntary unemployment in a Nash equilibrium, and realization of full-employment by fiscal policy financed by seignorage. Once we achieve it, it is maintained without government expenditure. We also show that a fall in the nominal wage rate may not decrease involuntary unemployment.

In both models younger consumers split their income in half between consumption in Period 1 and consumption in Period 2. Taxes (for unemployment benefit and pay-as-you-go pension) and money (lending or borrowing) are evaluated by the good.

In Appendix we will examine the effect of a fall in the nominal wage rate in a three generations model with pay-as-you-go pension.

2. Two generations model with pay-as-you-go pension

We consider the following model.

1. The model is a two generations OLG model. Consumers live over two periods, Period 1 and Period 2. In Period 1 (younger period, working period) they can work and earn the wages. They buy and consume the good by wages. In Period 2 (older period, retired period) they buy and consume the good by their savings and the pay-as-you-go pension. The good can be consumed by any unit, that is, a consumer can consume, for example, 0.1 units of the good.

2. There is one good and one firm. The firm produces one unit of the good by one unit of labor. Though there is only one firm, it behaves competitively. Let w be the nominal wage rate and p be the price of the good. Then, $p = w$, and the real wage rate is one.

3. There are two younger consumers. If a consumer is employed, he supplies one unit of labor, receives the wage, consumes the good, pays tax for pay-as-you-go pension for consumers in the older generation, lends money to an unemployed younger consumer (if involuntary unemployment exists) and saves the remaining income.

   If a consumer is not employed, his income is zero. However, he can receive pay-as-you-go pension after retirement, so he consumes the good in his Period 1 by borrowing money from an employed younger consumer against pension.

   They determine their consumption and labor supply at the beginning of their Period 1.

4. There are two older consumers. If a consumer was employed in his Period 1, he consumes the good by his savings and the pay-as-you-go pension. If he is not employed, he consumes by only the pay-as-you-go pension, and repays his debt in Period 1.

2.1. Case 1: A game without government expenditure

Assume that there exists no government expenditure. Let us consider the following pair of strategies of the firm and the younger consumers at a steady state. The strategies of the consumers are derived from their utility maximization over two periods. Suppose that the firm employs one younger consumer, and the other younger consumer is not employed. Let $0 < t \leq 0.25$.

1. Firm: employs one younger consumer and produces one unit of the good.
2. Employed younger consumer: supplies one unit of labor, receives wage w, consumes $0.5 - \frac{t}{2}$ units of the good, pays $2t$ units of tax for pay-as-you-go pension for the older generation, lends $\frac{t}{2}$ units of money to the unemployed younger consumer, and saves the rest of the income $(0.5 - 2t)$.
3. Unemployed younger consumer: consumes $\frac{t}{2}$ units of the good by borrowing money from the employed younger consumer against pension in the next period.

Since we consider a steady state, one older generation consumer consumes $0.5 - \frac{t}{2}$ units of the good by his savings including lending to an unemployed consumer $(0.5 - \frac{3}{2}t)$ and the pay-as-you-go pension (t). The other older generation consumer consumes $\frac{t}{2}$ units of the good by the pay-as-you go pension (t) repaying the debt $(\frac{t}{2})$. Their total consumption is 0.5. The total demand for the good is one unit which is equal to the supply.

Is this pair of strategies in a Nash equilibrium?

1. Firm: given the consumption behavior of two younger consumers, employment of one unit of labor and production of une unit of the good are optimal.
2. Employed younger consumer: given the behavior of the firm and the unemployed consumer, supply of one unit of labor and $0.5 - \frac{t}{2}$ units of consumption are optimal.
3. Unemployed younger consumer: given the behavior of the firm and the employed consumer, $\frac{t}{2}$ units of consumption is optimal.

In this case there exists one unit of involuntary unemployment.

2.2. Case 2: A game with government expenditure in Case 1

Now we consider 0.5 units of the government expenditure which is spent on the good produced by the firm. The state is not a steady state. The government expenditure should be financed by *seignorage* not by tax. Suppose that two consumers are employed by the firm. Let $0 < t \leq 0.25$ again. Consider the following pair of strategies.

1. Firm: employs two younger consumers and produces two units of the good.
2. Employed younger consumer 1: supplies one unit of labor, receives wage w, consumes 0.5 units of the good, pays t units of tax for pay-as-you-go pension for the older generation, and saves the rest of the income $(0.5 - t)$.

3. Employed younger consumer 2: supplies one unit of labor, receives wage w, consumes 0.5 units of the good, pays t units of tax for pay-as-you-go pension for the older generation, and saves the rest of the income $(0.5 - t)$.

One older generation consumer consumes $0.5 - \frac{t}{2}$ units of the good. The other older generation consumer consumes $\frac{t}{2}$ units of the good. Their total consumption is 0.5. The demand by younger consumers is one, and the government's demand is 0.5. Therefore, the total demand for the good is two units which is equal to the supply. Similarly to the previous case, we can verify that this pair of strategies are optimal for the firm and for each younger consumer given strategies of other players. This case means that we can achieve *full-employment* by the fiscal policy.

Without government expenditure, above pair of strategies can not be in a Nash equilibrium because consumption of the older generation consumers is insufficient. The multiplier of the government expenditure is two.

Full employment steady state In the next period above pair of strategies without government expenditure maintains *full-employment* because the total consumption of the older generation is one unit of the good in the new steady state, 0.5 for each older consumer.

3. Three generations model with childhood period

Next we consider the following model.

1. The model is a three generations OLG model. Consumers live over three periods, Periods 0, 1 and 2. In Period 1 they can work and earn the wages. They buy and consume the good by wages. In Period 2 they buy and consume the good by their savings. In Period 0 (childhood period) they consume the good by borrowing money from consumers of the previous generation. They must repay the debts in the next period. However, if a consumer is not employed, he can not repay the debt. Then, he receives unemployment benefit which is covered by tax paid by an employed consumer of the same generation. The good can be consumed by any unit.

2. There is one good and one firm. The firm produces one unit of the good by one unit of labor. Though there is only one firm, it behaves competitively. Let w be the nominal wage rate and p be the price of the good. Then, $p = w$, and the real wage rate is one.

3. There are two younger consumers. If a consumer is employed, he supplies one unit of labor, receives the wage, consumes the good, pays tax for unemployment benefit for an unemployed consumer in the same generation, and saves the remaining income. If he is not employed, his income is zero, and then his consumption is zero. His debt due to consumption in Period 0 is repaid by unemployment benefit.

They determine their consumption and labor supply at the beginning of their Period 1.

4. There are two older consumers. If a consumer was employed in his Period 1, he consumes the good by his savings. If he is not employed, his consumption is zero.

5. There are two childhood consumers. Their consumptions are constant.

3.1. Case 3: A game without government expenditure

Assume that there exists no government expenditure. Let us consider the following pair of strategies of the firm and the younger consumers at a steady state. The strategies of the consumers are derived from their utility maximization over two periods (Periods 1 and 2). Consumption of each consumer in the childhood period is constant. Suppose that the firm employs one younger consumer, and the other younger consumer is not employed. Let $0 < \sigma \leq 0.5$. The total consumption of two childhood consumers is 2σ, σ for each consumer.

1. Firm: employs one younger consumer and produces one unit of the good.

2. Employed younger consumer: supplies one unit of labor, receives wage w, consumes $0.5-\sigma$ units of the good, repays σ units of debt, pays σ units of tax for unemployment benefit, and saves the rest of his income $(0.5 - \sigma)$.

3. Unemployed younger consumer: consumption is zero. He repays the debt by unemployment benefit.

Since we consider a steady state, one older generation consumer consumes $0.5 - \sigma$ units of the good by his savings. Consumption of the other older generation consumer is zero. Their total consumption is $0.5 - \sigma$. The childhood consumers consume 2σ units of the good in total. The total demand for the good is one unit which is equal to the supply.

Is this pair of strategies in a Nash equilibrium?

1. Firm: given the consumption behavior of two younger consumers, employment of one unit of labor and production of une unit of the good are optimal.

2. Employed younger consumer: given the behavior of the firm and the unemployed consumer, supply of one unit of labor and $0.5 - \sigma$ units of consumption are optimal.

3. Unemployed younger consumer: given the behavior of the firm and the employed consumer, zero consumption is optimal.

In this case there exists one unit of involuntary unemployment.

Fall in the nominal wage rate Proposition 2.1 in Otaki (2016) says

> Suppose that the nominal wage sags. Then, as far as its indirect effects on the aggregate demand are negligible, this only results in causing a proportionate fall in the price level. In other words, a fall in the nominal wage never rescues workers who are involuntarily unemployed.

However, *indirect effects* on aggregate demand due to a fall in the nominal wage rate may exist. We use the following notation.

- M: The total savings of older consumers.
  It is equal to the total demand of older consumers. It is an asset.
- D: Debt of a younger consumer due to his consumption in the childhood period.
- R: Unemployment benefit, $R = D$.
- $\hat{D}$: Consumption in the childhood period of a next generation consumer.

Since in our model an employed younger consumer splits his income in half between consumption in Period 1 and consumption in Period 2, the total demand of younger consumers is

$$\frac{1}{2}[w - D - R] = \frac{1}{2}w - D,$$

and the total demand for the good is

$$\frac{1}{2}w - D + M + 2\hat{D}.$$

We can think that the real values of w and $\hat{D}$ are invariant to price changes. On the other hand, the nominal values of M and D do not change. Thus, a fall in the nominal wage rate increases (decreases) total demand for the good if

$$M - D > 0 (< 0).$$

In our mode, since $m = 0.5 - \sigma$ and $D = \sigma$, this is equivalent to

$$0.5 - 2\sigma > 0 (< 0).$$

Therefore, if $\sigma > 0.25$, a fall in the nominal wage rate decreases the total demand for the good, and increases involuntary unemployment.

The discussion here is from the different perspectives of the real balance effect for which the argument was fought by Pigou (1943) and Kalecki (1944).

3.2. Case 4: A game with government expenditure in Case 3

Now we consider 0.5 units of the government expenditure which is spent on the good produced by the firm. The state is not a steady state. The government expenditure should be financed by *seignorage* not by tax. Suppose that two consumers are employed by the firm. Consider the following pair of strategies.

1. Firm: employs two younger consumers and produces two units of the good.

2. Employed younger consumer 1: supplies one unit of labor, receives wage w, consumes $0.5 - \frac{\sigma}{2}$ units of the good, repays σ units of debt, and saves the rest of his income $(0.5 - \frac{\sigma}{2})$.

3. Employed younger consumer 2: supplies one unit of labor, receives wage w, consumes $0.5 - \frac{\sigma}{2}$ units of the good, repays σ units of debt, and saves the rest of his income $(0.5 - \frac{\sigma}{2})$.

One older generation consumer consumes $0.5 - \sigma$ units of the good by his savings. The consumption of the other older generation consumer is zero. Their total consumption is $0.5 - \sigma$. The total demand by younger consumers is $1 - \sigma$, The total demand by the childhood consumers is 2σ, and the government's demand is 0.5. Therefore, the total demand for the good is two units which is equal to the supply. Similarly to the previous case, we can verify that this pair of strategies are optimal for the firm and for each younger consumer given strategies of other players. This case means that we can achieve *full-employment* by the fiscal policy. The multiplier of the government expenditure is two.

Without government expenditure, above pair of strategies can not be in a Nash equilibrium because consumption of the older generation is insufficient.

Full employment steady state In the next period above pair of strategies without government expenditure maintains *full-employment* because the total consumption of the older generation is $1 - \sigma$ units of the good in the new steady state.

4. Concluding Remark

In this paper labor supply is indivisible. However, because the real wage rate is constant, even if labor supply depends on the real wage rate, it is constant. Under increasing or decreasing returns to scale, the real wage rate and labor supply may not be constant. Even in such cases we think the analysis can proceed in the similar way.

One drawback of this paper is that the production of the good is carried out solely by labor. I would like to consider a model that includes capital in future research.

A. Appendix: About a fall in the nominal wage rate in a three generations model with pay-as-you-go pension

If there is a pay-as-you-go pension in Case 3, the following pair of strategies is in a Nash equilibrium. t and σ mean the same thing as before. Let $0 < t \leq 0.25$ and $0 < \sigma + t \leq 0.5$.

1. Firm: employs one younger consumer and produces one unit of the good.

2. Employed younger consumer: supplies one unit of labor, receives wage w, consumes $0.5-\sigma-\frac{t}{2}$ units of the good, repays σ units of debt, pays σ units of tax for unemployment benefit, pays $2t$ units of tax for pay-as-you-go pension, lends $\frac{t}{2}$ units of money to the unemployed younger consumer and saves the rest of his income $(0.5-\sigma-2t)$.

3. Unemployed younger consumer: consumes $\frac{t}{2}$ units of the good by borrowing money from the employed younger consumer against pension in the next period. He repays the debt due to consumption in Period 0 by unemployment benefit.

One older generation consumer consumes $0.5-\sigma-\frac{t}{2}$ units of the good. The other older generation consumer consumes $\frac{t}{2}$ units of the good. Their total consumption is $0.5-\sigma$. The total demand of younger consumers is also $0.5-\sigma$. The total demand of childhood consumers is 2σ. Thus, the total demand for the good is one unit which is equal to the supply.

Let Φ be the pay-as-you-go pension for an older consumer, Ψ be the tax for the pension and $\hat{\Phi}$ be the pay-as-you-go pension for a younger consumer when he retires. Then,

$$\Psi = 2\Phi$$

in this case. The total demand for the good by younger consumers is

$$\frac{1}{2}[w-2D-2\Phi+\hat{\Phi}]+\frac{1}{2}\hat{\Phi}=\frac{1}{2}w-D-\Phi+\hat{\Phi}.$$

$\frac{1}{2}\hat{\Phi}$ is demand by an unemployed younger consumer. The total demand for the good is

$$\frac{1}{2}w-D-\Phi+\hat{\Phi}+M+2\hat{D}.$$

M includes the pay-as-you-go pension for older consumers 2Φ. Thus, the net savings of older consumers is $M-2\Phi$.

We can think that the real values of w, $\hat{D}$, Φ and $\hat{\Phi}$ are invariant to price changes. On the other hand, the nominal values of $M-2\Phi$ and D do not change. Hence, a fall in the nominal wage rate increases (decreases) total demand for the good if

$$M-2\Phi-D>0(<0). \tag{1}$$

In our model the consumption of an older consumer who was employed in Period 1 is $0.5-\sigma-\frac{t}{2}$ and that of an older consumer who is not employed is $\frac{t}{2}$. Thus, $M=0.5-\sigma$. Therefore, (1) is equivalent to

$$0.5-\sigma-2t-\sigma=0.5-2\sigma-2t>0(<0).$$

Hence, if $\sigma+t>0.25$, a fall in the nominal wage rate decreases the total demand for the good, and increases involuntary unemployment.

Acknowledgment This work was supported by the Japan Society for the Promotion of Science KAKENHI (Grant Number 18K01594).

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