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Abstract

We analyse the time evolution of the empirical cross-sectional distribution of firms profit and growth rates. In particular, we analyse the conditional properties of the empirical distributions depending on the size of the firms and business cycle phase. In order to do so, we employ the Laplace distribution as a benchmark, further considering the Subbotin and Asymmetric Exponential Power (AEP hereafter) distributions, to capture the potential asymmetry and leptokurtosis of the empirical distribution. Our results show that the profit rates of large firms are characterised by an asymmetric Laplace distribution with parameters largely independent of the business cycle phase. Small firms, instead, are characterised by the AEP distribution, which accounts for the conditional dependence of distribution on the phase of the business cycle. We observe that the largest firms are more robust to downturns compared to the small firms, given their invariant distributional characteristics during crisis periods.

JEL codes. C16 · L10 · D21 · E10 ·

Keywords: Profit rates · Growth rates · Firm size · Business cycle · Laplace distribution · Asymmetric Exponential Power distribution

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1. Introduction

Historically, Gibrat (1931) was the first scholar to propose a stochastic process in order to model the growth of firms based exclusively on general probabilistic concepts. His basic hypothesis states that the logarithmic growth rate of a firm size is independent of its level and it is Normally distributed. The Normal distribution assumption can be justified on the premise of the Central Limit Theorem (CLT hereafter). The logarithmic growth rate of a firm in a given time period (one year, for instance) can be decomposed as a sum of a large number of shocks hitting the firm at a higher frequency (e.g. daily). Within this time decomposition, the emergence of the Normal distribution of growth rates is a natural consequence of the CLT, assuming that the shocks are independent and identically distributed. Under these assumptions, the distribution of firms' size is Lognormal. From an economic perspective, the Gibrat's hypotheses are compatible with an ensemble of *independent firms*, experiencing, possibly, a common trend and idiosyncratic destinies. The Gibrat's statistical approach has been generalised in order to account for other economic phenomena, such as the entry and exit of firms in a market, the turbulence and the learning of firms, leading Sutton to call for the existence of a Gibrat's legacy (Sutton, 1997).

Challenging the Gibrat's hypothesis of Normality, many authors (Amaral et al., 1997; Bottazzi et al., 2001; Bottazzi and Secchi, 2003, 2006; Bottazzi and Secchi, 2011; Buldyrev et al., 2007; Alfarano and Milakovic, 2008; Riccaboni et al., 2011) have empirically shown that firms' growth rates follow a Laplace distribution rather than a Normal distribution.[1] Starting from the basic assumption of iid shocks leading to a Gaussian distribution, the empirical identification of the Laplace distribution can be alternatively interpreted as the imprint of a systemic dependence among the shocks hitting all firms. In order to account for the "Laplacian" deviations from the Gaussian hypothesis, one must replace the assumption of iid shocks by perturbations characterised by some degree of system-wide correlation due to systemic economic interactions among firms. The Laplace distribution of cross-sectional firms growth rates, thus, can be thought as the macroscopic evidence of the existence of complex interactions among firms. Some models have been proposed in order to account for the emergence of the Laplace distribution. Bottazzi and Secchi (2006) show that the Laplace distribution stems from a competitive context in which firms are able to seize new growth opportunities proportional to opportunities already taken. Under the resource-based view of the firm (Penrose and Penrose, 2009), Coad and Planck (2012) consider a mechanism of employment growth in a hierarchy, leading to an exponential distribution of firm size and a Laplace distribution of growth rates.

Recently, some authors (Alfarano et al., 2012; Mundt et al., 2016) proposed a new focus to analyse firms dynamics from the Gibrat's perspective beyond the growth rates of firm size. They claim that a more informative quantity to account for the dynamics of the ensemble of firms in a competitive environment is to consider profit rates instead of growth rates as the key measure of firm performance. This change of focus allows to rely on the

[1] Moreover, some authors (Axtell, 2001; Gaffeo et al., 2003) show that the distribution of firms' size follows a power law rather than a lognormal distribution.

general principle of the tendency for equalisation of profit rates based on the idea of classical competition. In this respect, Alfarano and Milakovic (2008) introduced a theoretical framework for the profit rate distribution by considering as the intellectual base Adam Smith's notion of classical competition (Smith, 1776), which describes a negative feedback mechanism: capital seeks out those sectors in which profit rates are higher than the economy-wide average, essentially attracting labour, raising output, reducing prices and eventually profit rates. Capital, thus, leaves the sector giving rise to an increase of prices and profit rates for those firms that remain in the industry. The entire process tends to equalise profit rates across sectors and firms. The idea of classical competition can be framed in terms of a statistical equilibrium model for the profit rate distribution, which leads to an Exponential Power or Subbotin distribution (Subbotin, 1923). Such theoretical framework has been empirically tested in several contributions (Alfarano et al., 2012; Erlingsson et al., 2013; Mundt et al., 2016), showing that the profit rate distribution can be described by a Laplace distribution, whose first and second moment are very stable over time, much more than the corresponding moments of the growth rate distribution. Interestingly, it has been shown that such stability emerges when one restricts the analysis to firms that survived for sufficiently long time (more than 25 years) (Alfarano et al., 2012). The entry and exit dynamics of firms is, therefore, excluded by construction from the analysis. In this regard, Mundt and Oh (2019) show that the Laplace distribution is not flexible enough to describe the profit rate distribution when entry and exit dynamics of firms is included. They observe an empirical profit rate distribution that exhibits a higher degree of leptokurtosis and a significant asymmetry when compared to a symmetric Laplace distribution. Hence, Mundt and Oh (2019) generalise the model proposed by Alfarano et al. (2012) in order to include changes in the nature of the competitive environment and the strength of competitive pressure between entering/existing and incumbent firms. Their model shows that these features can be accounted by the AEP distribution, proposed by Bottazzi and Secchi (2011). The AEP generalises the Subottin distribution in order to include a given degree of asymmetry.

To shed more light on this strand of literature, we study a large dataset of 35.910 Spanish long-lived firms, analysing the recent financial crisis and its business cycle phases: the period of the real estate bubble (1998-2007), the subsequent crisis (2008-2013) and the period of economic recovery (2014-2016). The large dataset at our disposal allows for an extensive analysis of the Laplacian hypothesis of profit rate distribution and its stability over time. The contribution of our paper is threefold. First, following Alfarano et al. (2012) and Mundt and Oh (2019), we examine whether the empirical profit and growth rate distributions of Spanish firms are described by the Laplace, Subbotin or AEP distribution. Compared Mundt and Oh (2019), our analysis is not limited to profit rates but also include the comparison to growth rates. Second, we analyse how the empirical distribution changes according to the different firm size and the phases of the business cycle. Finally, our analysis allows to understand whether the astonishing stability of the profit rate cross-sectional distribution is an intrinsic characteristic of surviving firms or other conditionalities should be considered. Understanding the cross-sectional distribution of growth and profit rates during the different phases of the business cycle can help us to shed more light on macroeconomic fluctuations

(Higson et al., 2002, 2004; Gabaix, 2011; Bottazzi et al., 2019). Indeed, Haltiwanger (1997) stated that "it is becoming increasingly apparent that changes in the key macroaggregates at cyclical and secular frequencies are best understood by tracking the evolution of the cross-sectional distribution of activity and changes at the micro level." The availability of micro-data has allowed scholars to study how the microeconomic adjustment behaviour of firms affects the aggregate dynamics of the economy. For example, Higson et al. (2002) show that fastest growers and declining firms seem to be indifferent to recessions, in the same line as Geroski and Gregg (1997). De Veirman and Levin (2011) analyse trends and cycles in the volatility of U.S. companies observing that firm-specific volatility is not an important driver of the business cycle. Holly et al. (2013) underline that changes in the density of firm growth are a relevant factor to analyse the evolution of the business cycle. Bachmann and Bayer (2014) propose a heterogeneous-firm business cycle model that is able to replicate the procyclical behaviour of the empirical cross-sectional dispersion of firm-level investment rates.

The paper is structured as follows. After providing a summary in the introduction, we give a description of our data in Sec. (2). The employed methodology for the empirical analysis is described in Sec. (3). The results of the empirical analysis are shown in Sec. (4), distinguishing between symmetric and asymmetric distributions. Finally, Sec. (5) summarises the main findings of the paper.

2. Data

The dataset is sourced from the System of Analysis of Iberian Balance Sheets (SABI, 2020) and it offers information over the balance sheet of 2.000.000 Spanish firms from 1985 to 2016. Thus, we can examine the evolution of the distribution of growth and profit rates during different phases of the business cycle. As stated in the introduction, our empirical analysis focuses on long-lived firms. We filter a total of 35.910 firms that have been present in the market for the whole period.[2] Our dataset allows to generalise the previous findings on the distributional properties of the profit rates, since we extend the number of firms in more than two orders of magnitude, from few hundreds to several thousands, whose sizes span five orders of magnitude. In order to compare our results to the previous literature, we consider four groups of firms according to their sales in 2016. These groups include the 200, 1.000, 10.000 largest firms and the entire sample.[3]

As starting point, we consider the 200 largest firms due to two main reasons. First, we take as intellectual base the Gabaix's granular hypothesis (Gabaix, 2011). His seminal paper rests on the idea that the idiosyncratic shocks to the largest firms account for a significant fraction of the GDP fluctuations. Following Gabaix (2011), one third of aggregate fluctuations in US GDP growth can be explained by the idiosyncratic shocks of the 100 largest firms.

[2]All firms from the financial sector (Standard Industrial Classification (SIC) codes 6000-6799) have been excluded since their total assets are on average about one order of magnitude larger than firms included in all the other sectors. This is due to the different nature of the banking/financial sector, where total assets can be increased due to the financial intermediation activity. The average ROA for banks turns out to be one order of magnitude larger than for firms in other sectors.

[3]Fig. (13), in the Appendix, shows the sum of sales as a function of GDP for each group of firms.

Blanco-Arroyo et al. (2018) and Blanco-Arroyo et al. (2019) show that the Spanish economy is also characterised by granular fluctuations, since the granular residual of the 100 largest firms accounts approximately for 45% of GDP variations. The second reason is related to the fact that we employ the AEP distribution to characterise the profit and growth rate distribution. By means of numerical simulations, Bottazzi and Secchi (2011) state that "the bias of the maximum likelihood estimators, being very small, can be safely ignored at least for samples with more than 100 observations". Therefore, we start the empirical analysis considering the largest 200 firms to ensure the reliability of the estimated parameters.

As the first step, we compute the logarithmic growth rate for each firm i defined as:

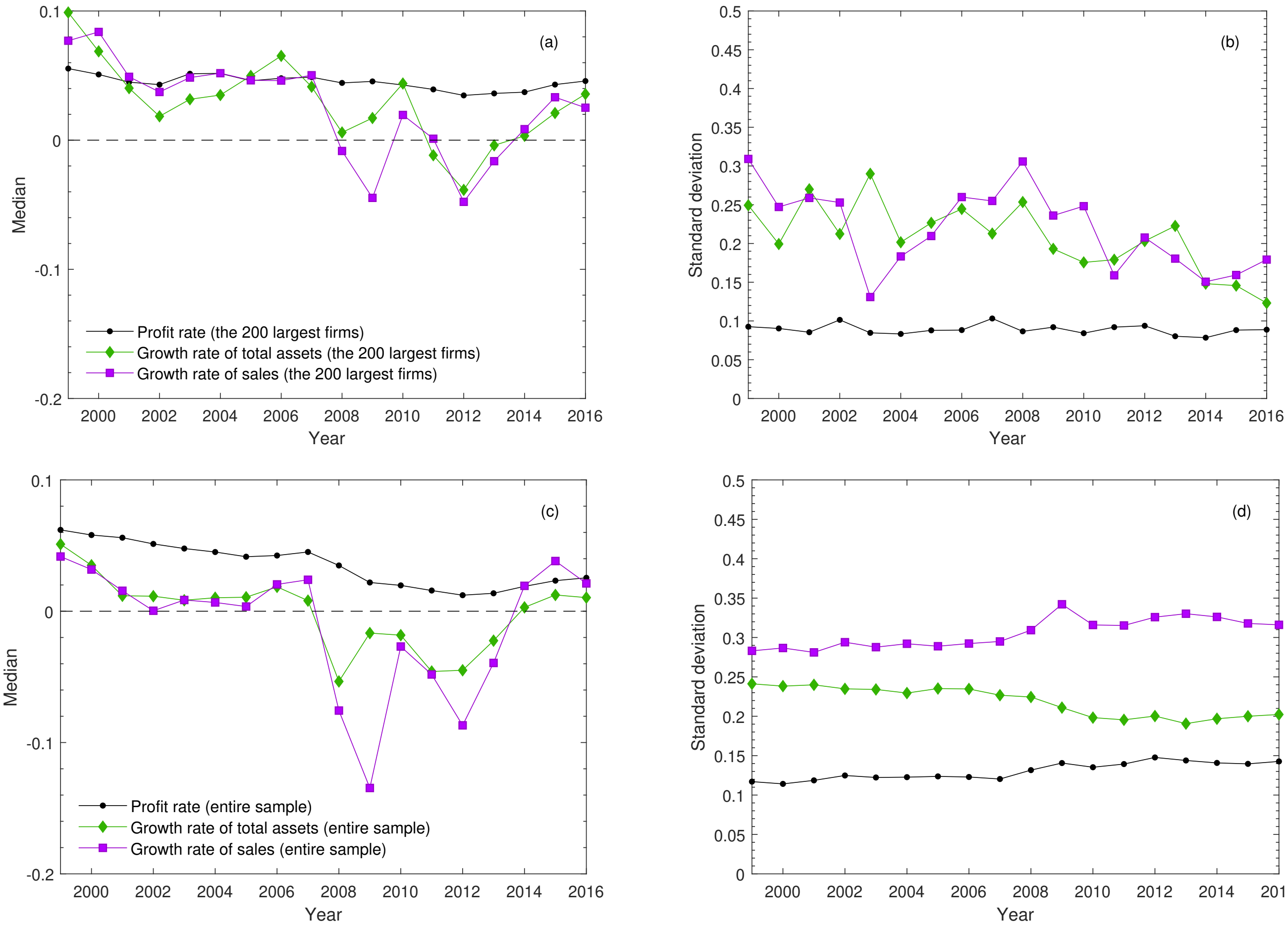
$$\tilde{g}_i(t) = \ln(S_i(t)) - \ln(S_i(t-1)), \tag{1}$$

where t denotes the year and $S_i(t)$ the firm size, whose proxy is the value of total assets or sales (Axtell, 2001; Stanley et al., 1996).

The variable chosen as a proxy for profit rate is the return on assets (ROA), which is defined as earnings before interest and taxes ($EBIT$) divided by total assets (TA) of firm i at time t,

$$ROA_i(t) = \frac{EBIT_i(t)}{TA_i(t)}. \tag{2}$$

Figure. 1: The evolution of the cross-sectional median and standard deviation of growth ($\tilde{g}$) and profit rates for the 200 largest long-lived firms and the entire sample at our disposal (base year 2016).



A visual inspection to Fig. (1) (a and b) shows that the median of profit rates for the largest 200 long-lived firms exhibit a considerable stability over time compared to the the median growth rates of total assets and sales, which instead exhibits a much higher volatility. The time evolution of the median of profit and growth rates is also reported by Alfarano et al. (2012) using a sample of publicly traded US companies, observing similar results. Our results are also in line with Mundt et al., 2014, who find that the median of profit rates is much more stable than the median of growth rates in more than 40 countries using a dataset of publicly traded companies. Moreover, we confirm the results reported by Coad et al. (2013), who observe a much higher stability of the profit rate cross-sectional average when companies survive more than 11 years. We observe that the median of profit rates exhibits a higher stability compared to the median of growth rates even when considering the entire sample of long-lived firms. However, it shows higher fluctuations with respect to the sample composed by the 200 largest firms, due to the impact of the smaller firms. In both cases reported in Fig. (1), the first two moments of the profit rate distribution are more stable than those of growth rates.

Under a Gaussian hypothesis for the distribution of profit and growth rates, the analysis of the first two moments would be a sufficient statistics. However, an extensive literature in industrial dynamics (see e.g. Sutton (1997)) shows that the empirical distribution of relevant measures of firm performance exhibits significant deviations from the Normality assumption. We, therefore, have to characterise of the entire distribution of profit and growth rates. Following the literature (Bottazzi and Secchi, 2006), we consider the normalized logarithmic size:

$$s_i(t) = \ln\left(S_i(t)\right) - N^{-1} \sum_{i=1}^{N} \ln\left(S_i(t)\right). \tag{3}$$

where N is the number of considered firms in the sample, namely 200, 1000, 10000 and the entire sample. We define the annual growth rate of a firm i as:

$$g_i(t) = s_i(t+1) - s_i(t), \tag{4}$$

where t denotes time and s_i denotes normalised logarithm of firm size. Profit rates are not manipulated and simply remain in their raw form.

3. Methodology

Alfarano and Milakovic (2008) introduce a theoretical framework to analyse the distribution of profit rates by considering as an intellectual base the Adam Smith's notion of classical competition (Smith, 1776). It describes a negative feedback mechanism in the reallocation of capital in perpetual search for profitability, leading to a *tendency* for the equalisation of profit rates among competitive economic activities. In the empirical data, however, the complete elimination of profit rates differentials is never achieved.[4] Alfarano et al. (2012), thus, express the outcome of classical competition in terms of a statistical equilibrium model, considering that the complexity of the competitive interactions among firms leads to a non-degenerate distribution of profit rates. In particular, firms disperse their profit rates, denoted as x, around a measure of central tendency, denoted as m, which represents the economy-wide profit rate. The tendency for equalization of profit rates can be encoded as a moment constrain on the dispersion of their distribution measured by the standardized α-th moment:

$$\sigma^\alpha = E\left[|x-m|^\alpha\right]. \tag{5}$$

In order to obtain the profit rate distribution, Alfarano and Milakovic (2008) employ the Maximum Entropy Principle (MEP), which establishes a unique connection between a set of given moment constraints and a probability distribution. The MEP yields the combinatorially most likely distribution maximising the multiplicity of feasible assignments given the moment constrains (see Jaynes, 1978). The result of MEP for the moment constraint in Eq. (5) is an Exponential Power or Subbotin distribution, defined as

[4] A perfect elimination of profit rate differentials would lead to a Dirac's delta distribution.

$$f(x;m,\sigma,\alpha)=\frac{1}{2\sigma\alpha^{\frac{1}{\alpha}}\Gamma(1+\frac{1}{\alpha})}\exp\left(-\frac{1}{\alpha}\left|\frac{x-m}{\sigma}\right|^{\alpha}\right). \tag{6}$$

This symmetric distribution is characterized by three parameters: a location parameter m, a scale parameter $\sigma>0$ and a shape parameter $\alpha>0$. Depending on the value of the shape parameter, we have three different cases: (i) a platykurtic distribution for $\alpha>2$, (ii) a leptokurtic distribution for $\alpha<2$, and (iii) a Gaussian distribution for the edge case $\alpha=2$. In particular, the Subbotin distribution reduces to the Laplace distribution when $\alpha=1$. The distribution in Eq. (6) has been widely employed in the literature of industrial dynamics (Bottazzi and Secchi, 2003; Bottazzi and Secchi, 2006; Coad and Planck, 2012; Alfarano et al., 2012; Erlingsson et al., 2013; Mundt et al., 2016) to characterise the empirical distribution of profit and growth rates of firm size, essentially because it interpolates between the Gaussian and the Laplace distribution. Following the growth rate literature (Stanley et al. (1996), Bottazzi and Secchi, 2006 and Coad and Planck, 2012), we consider the Laplace distribution as the benchmark to compare the estimation results.

In this paper, we complement the distributional analysis based on the symmetric distribution of Eq. (6) by using the AEP distribution. Mundt and Oh (2019), generalising the result given by Alfarano and Milakovic (2008), provide an economic foundation for the AEP distribution within a statistical equilibrium approach that includes structural differences between the right and left part of the distribution. In particular, they show that the former reflects the activity of incumbent firms while the latter represents the activity of entering/existing companies characterised by low/negative profit rates. Instead of a symmetric behaviour around the measure of central tendency, defined by the Eq. (5), which implies the emergence of a symmetric distribution, they define two different conditional measures of dispersion around m: $\sigma_l=[E|x-m|^{\alpha_l}]^{\frac{1}{\alpha_l}}$ for $x<m$ and $\sigma_r=[E|x-m|^{\alpha_r}]^{\frac{1}{\alpha_r}}$ for $x>m$, where l and r refer to the left and right part of the distribution, respectively. Using the MEP, the probability distribution for the variable x based on the two moment constraints is the following:

$$f_{AEP}(x;\mathbf{p})=\frac{1}{C}\exp\left[-\left(\frac{1}{\alpha_l}\left|\frac{x-m}{\sigma_l}\right|^{\alpha_l}\theta(m-x)+\frac{1}{\alpha_r}\left|\frac{x-m}{\sigma_r}\right|^{\alpha_r}\theta(x-m)\right)\right], \tag{7}$$

where $\mathbf{p}=(\alpha_l,\alpha_r,\sigma_l,\sigma_r,m)$, $\theta(x)$ is the Heaviside function[5] and $C=\sigma_l\alpha_l^{1/\alpha_l}\Gamma(1+1/\alpha_l)+\sigma_r\alpha_r^{1/\alpha_r}\Gamma(1+1/\alpha_r)$ is the normalization constant with $\Gamma(\cdot)$ the Gamma function. Eq. (7) is a five-parameters family of distributions that is characterized by the location parameter, m, which is the mode of the distribution, two shape parameters, α_l and α_r, describing the density in the lower and upper tail respectively, and two scale parameters, σ_l and σ_r, connected with the distribution width below and above m. The Laplace distribution is nested in the AEP when $\alpha_l=\alpha_r=1$ and $\sigma_l=\sigma_r=\sigma$. Note that the parameter m in the Laplace distribution represents the mean, the median and the mode of the distribution. Those three measures of central tendency, however, might not coincide

[5]The function $\theta(x)$ is equal to 1 for $x>0$, and 0 for $x<0$.

in the AEP distribution. In this case, m represents the mode of the AEP distribution.

4. Empirical results

In this section, we report the main results of our empirical analysis. In Sec. (4.1), we analyse the empirical probability density of profit and growth rates by testing the goodness of fit of the Laplace distribution against the Subbotin distribution. In Sec. (4.2), we examine the distributional properties of profit and growth rates testing the Laplace distribution against the AEP distribution.

4.1. Symmetric case

We estimate the main parameters of the Subbotin distribution for the largest 200 long-lived firms, using the the maximum likelihood estimation method.[6] We observe that the Laplace distribution provides a relatively poor fit for the profit rate distribution, since, at the 5% significance level, we reject the null hypothesis of $\alpha = 1$ in 11 out of 19 years (see Fig. (2)). However, as it has been underlined by Bottazzi and Secchi (2006), Bottazzi et al. (2014) and Mundt et al. (2016), the presence of outliers can significantly affect the estimation of the shape parameter α. Fig. (6) shows the presence of some large negative and positive values in several years.[7] Therefore, to avoid the effect of the outliers in the estimation of the parameters α and σ, we delete in each year the most positive and negative observation.[8] We show in the inset of Fig. (2) that the Laplace distribution cannot be rejected at the 5% significance level with the exception of 2009. Growth rates of total assets and sales, instead, show a more leptokurtic distribution compared to profit rates distribution with a shape parameter significantly smaller than unity for all years, even when deleting the most positive and negative values.[9] Looking at estimators of the scale parameter, we confirm the astonishing stability in the magnitude of profit rate fluctuations. Interestingly, we cannot reject the hypothesis that the scale parameter is constant along the entire period regardless of the phase of the business cycle. This is not the case for the scale parameter of the distributions of growth rates of sales and total assets whose time evolution shows persistent fluctuations, with periods significantly above or below the long term median value (see Sec. (4.1.1)).

[6] In the case of the parameter m, the likelihood function is non-analytical and, therefore, the ML estimator does not have the typical asymptotic properties. To avoid convergence and consistency problems, we estimate m as the mode of the distribution and then, we apply the ML to estimate the parameters α and σ conditionally on the value of m (Bottazzi and Secchi, 2006). Using the mean or the median to estimate m, the results are not significantly different when we estimate the parameters of the AEP distribution (material upon request). We opt for the mode for consistency within the estimation of the AEP parameters.

[7] The presence of outliers is also observed for growth rates of total assets (Fig. (8)) and sales (Fig. (14))

[8] From now on, we always delete the extreme positive and negative observations in each year when estimating the parameters of the Subbotin as well as the AEP distribution.

[9] In the case of growth rates of total assets, the Laplace distribution can be rejected in the majority of cases.

Figure. 2: Estimates of the shape and scale parameter of the Subbotin distribution for profit rates. Error bars show two standard errors. The results refer to the 200 largest long-lived firms according to their sales in 2016. The dashed line in the scale parameter figure represents the median of the estimates.

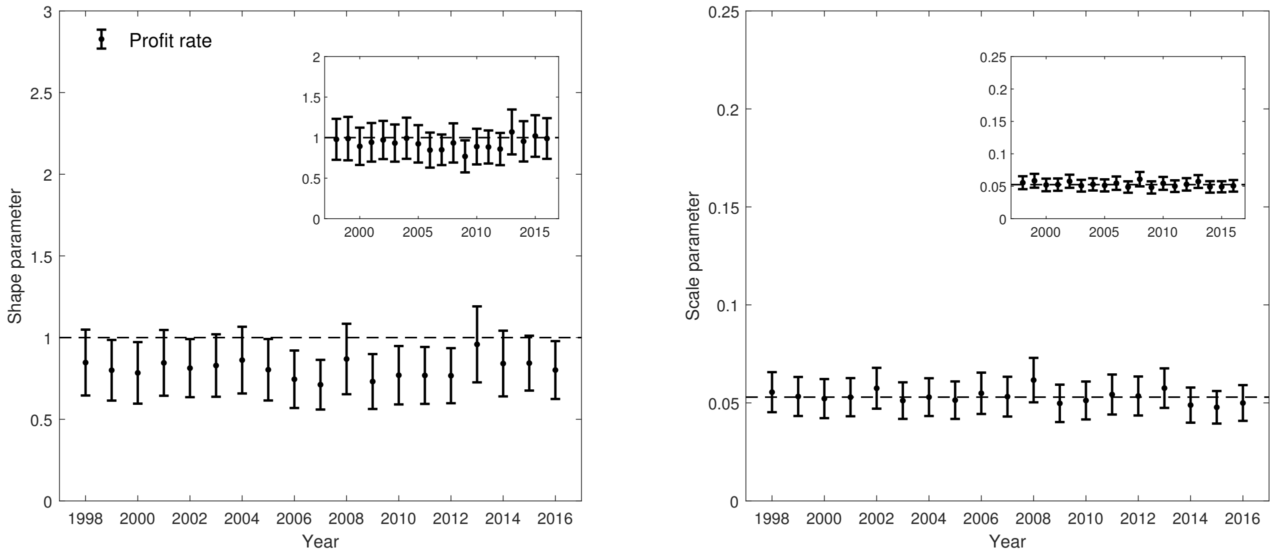
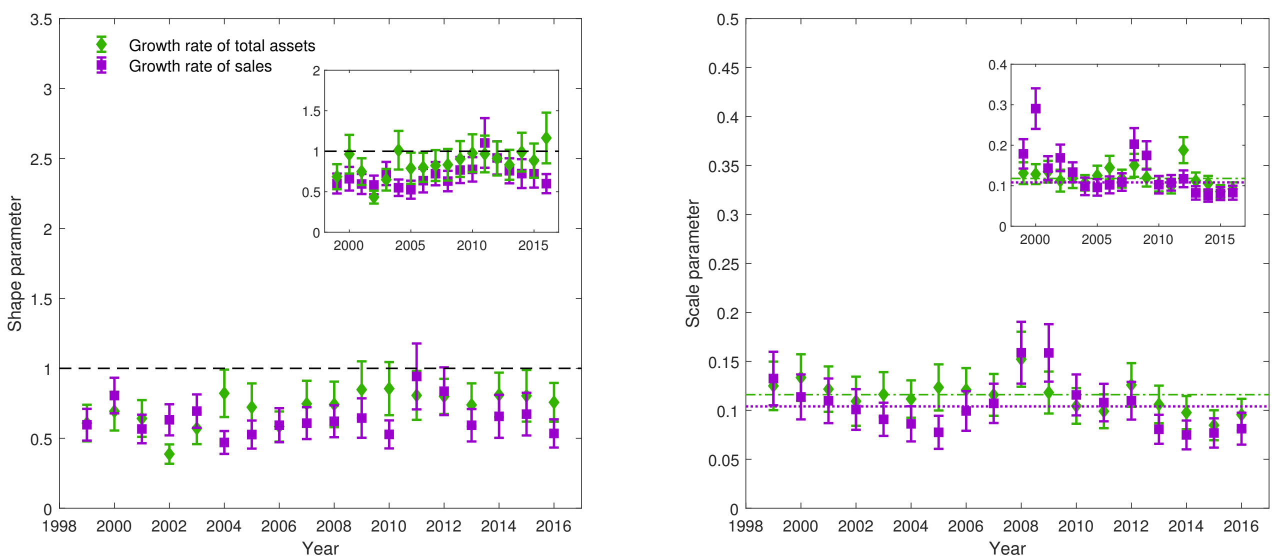


Figure. 3: Estimates of the shape and scale parameter of the Subbotin distribution for growth rates of total assets and sales. Error bars show two standard errors. The results refer to the 200 largest long-lived firms according to their sales in 2016. The dotted line (sales) and dashed line with dots (total assets) in the scale parameter figure represent the median of the estimates.



To go beyond a visual inspection, we employ the likelihood ratio test (LRT hereafter) to assess the performance of the Laplace distribution in describing the data, obtaining similar results (see Table 9 in the Appendix) as compared to the simpler inspection of the estimates of α and σ in Figs. (2) and (3). The Laplace distribution does not provide a good performance in describing the probability distribution of profit rates, unless deleting the highest and lowest values in each year. In this case, the results of the LRT, reported in Table 1, support the previous findings, since we can only reject the null hypothesis for the profit rate distribution in 2009 (p-value $= 0.04$). When comparing the results of the LRT to Fig. (3), we observe virtually identical results for the distribution of growth rates of total assets and sales.

Table 1: P-values of the likelihood ratio test for profit and growth rates of total assets and sales. The null hypothesis is the Laplace distribution, while the alternative hypothesis is the Subbotin distribution. The results refer to the 200 largest long-lived firms, according to their sales in 2016, when deleting the extreme positive and negative value.

LRT	1998	1999	2000	2001	2002	2003	2004	2005	2006	2007
Profit rate	0.87	0.93	0.39	0.64	0.81	0.58	0.95	0.53	0.20	0.14
Total assets	-	**0.00**	0.77	**0.01**	**0.00**	**0.00**	0.57	**0.05**	**0.05**	0.09
Sales	-	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**

LRT	2008	2009	2010	2011	2012	2013	2014	2015	2016
Profit rate	0.61	**0.04**	0.35	0.29	0.19	0.61	0.72	0.88	0.93
Total assets	0.11	0.42	0.84	0.77	0.42	0.08	0.86	0.29	0.15
Sales	**0.00**	**0.01**	**0.01**	0.48	0.35	**0.01**	**0.01**	**0.01**	**0.00**

4.1.1. Distributional properties conditional on size and business cycle phase

When increasing the sample, Fig. (4) shows that the distribution of profit and growth rates exhibits a shape parameter α significantly smaller than 1 most of the years. The data indicate that the distribution of growth rates of firm size roughly retains its shape parameter across the different samples (see Table 2), excluding the sample with the largest firms (see Table 2). The profit rate distribution, instead, exhibit a clear tendency to become more leptokurtic, while the scale parameter is virtually independent of the size of the considered firms, showing an astonishing stability. For the growth rate distribution, we observe a slight increase in the estimate of the scale parameter with size (see Table 3). Such effect is compatible with the inverse power law scaling of the volatility of growth rates as a function of firm size (see for instance Bottazzi et al., 2019).

Table 2: Median of the estimates of the shape parameter reported in Fig. (4).

Nº firms	Profit rate	TA	Sales
200	0.94	0.88	0.70
1000	0.83	0.79	0.67
10000	0.74	0.79	0.66
Entire sample	0.66	0.72	0.58

Table 3: Median of the estimates of the scale parameter reported in Fig. (5).

Nº firms	Profit rate	TA	Sales
200	0.055	0.123	0.105
1000	0.054	0.122	0.112
10000	0.053	0.119	0.117
Entire sample	0.055	0.130	0.145

Overall, our results shows that the distribution of profit rates is well described by the Laplace distribution, when we limit the analysis to the case of large long-lived firms. We observe, instead, systematic deviations from the Laplace benchmark when we include smaller firms in the sample, i.e. the smaller the firm we include the fatter the tails of the distribution of profit rate.

Figure. 4: Estimates of the shape parameter of the Subbotin distribution of profit rates, growth of total assets and sales. Error bars show two standard errors. Results refer to the largest long-lived firms of our sample according to their sales in 2016.

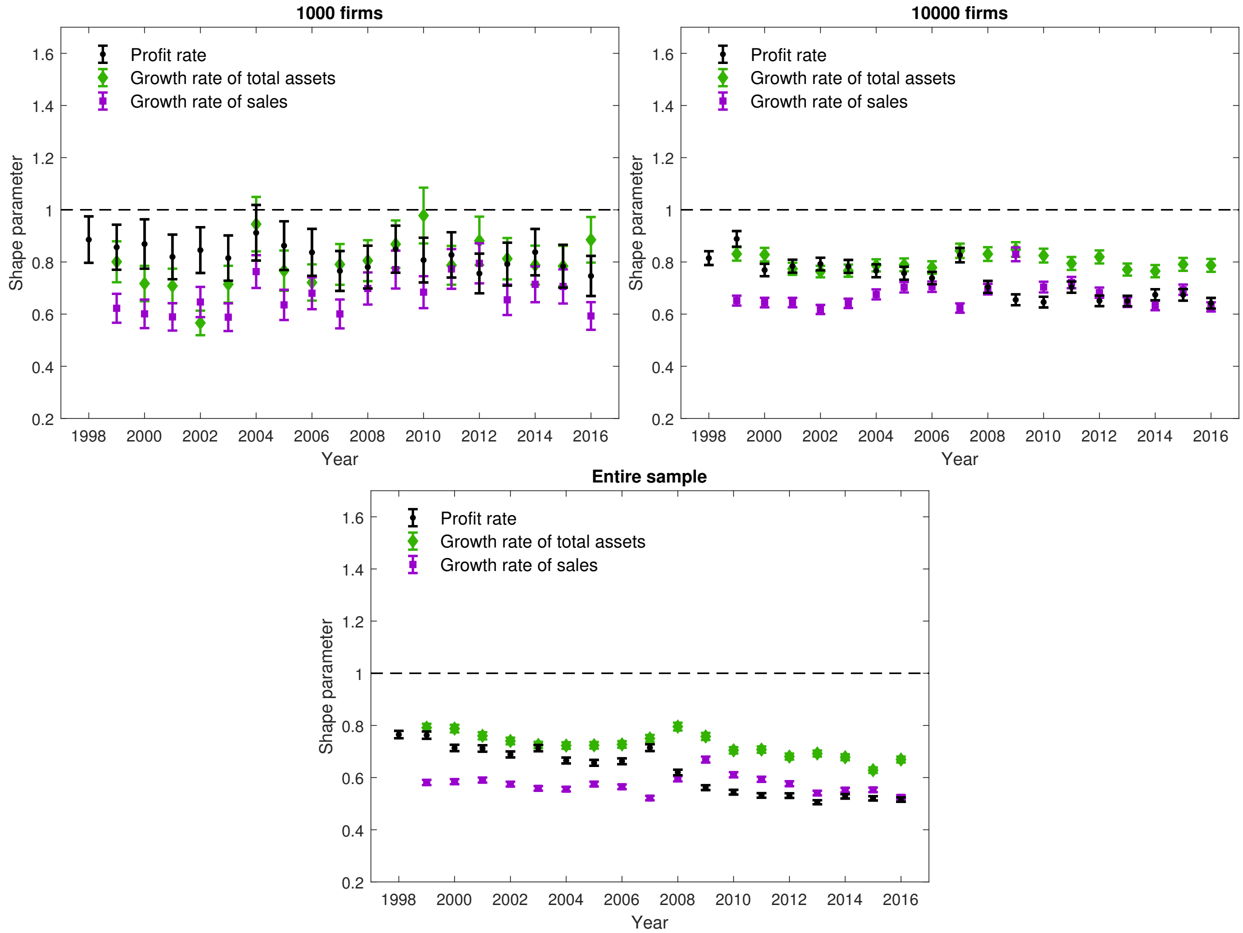
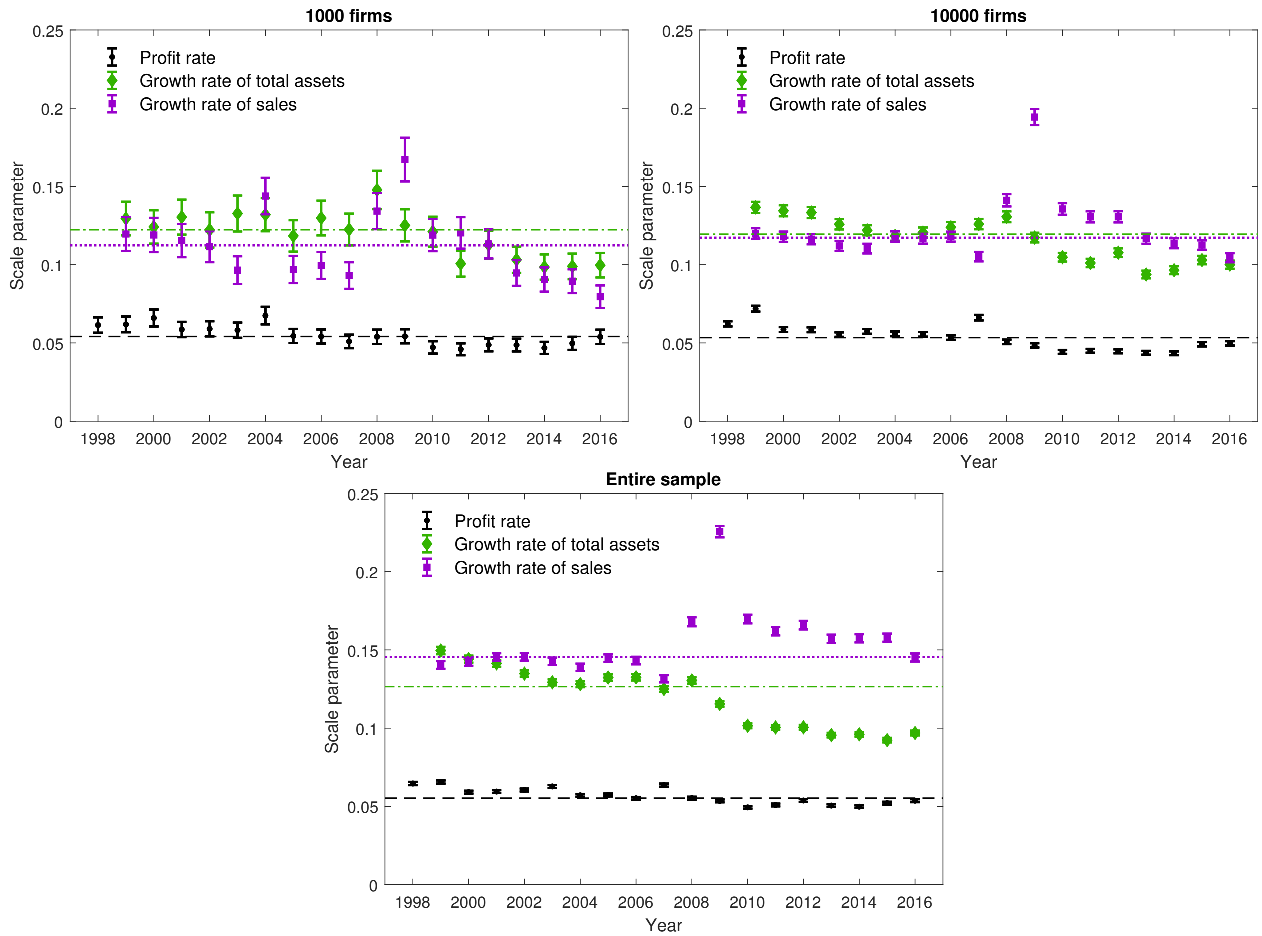


Figure. 5: Estimates of the scale parameter of the Subbotin distribution for profit rates, growth of total assets and sales. Error bars show two standard errors. Results refer to the largest long-lived firms of our sample according to their sales in 2016. The dashed line (profit), dotted line (sales) and dashed line with dots (total assets) in the scale parameter figure represent the median of the estimates.



Following Holly et al. (2013), in order to analyse the relation of the estimates for the profit rate distribution with the business cycle, we report in Table 4 the Pearson correlation coefficients between the time series of GDP growth rates with those of the estimates of m, α and σ. Regarding the parameter m, we observe a general tendency in which the correlation increases as we include smaller firms in the sample. Interestingly, the parameter σ and α of the profit rate distribution for large firms are essentially independent of the phase of the business cycle, where the only dependence is through m, which confirm the stability of the parameters over time. Such independence is instead lost as soon as we include small firms in the sample. Large firms, then, show more resilience to the business cycle, while small firms are much more dependent on the phase of the economy.

Table 4: Pearson correlation coefficient between the time series of the estimates of m, α and σ with the time series of GDP growth rates.

	m & GDP			α & GDP			σ & GDP		
Nº of Firms	Profit rates	TA	Sales	Profit rates	TA	Sales	Profit rates	TA	Sales
200	0.51**	0.27	0.48**	0.07	-0.38	-0.76***	0.00	0.38	-0.14
1000	0.36	0.64***	0.58**	0.37	-0.51**	-0.68***	0.69***	0.46**	-0.25
10000	0.59***	0.81***	0.59***	0.79***	-0.21	-0.50**	0.78***	0.70***	-0.59***
Entire sample	0.63***	0.93***	0.92***	0.83***	0.40	-0.36	0.74***	0.76***	-0.76***

Our results are in line with the literature since, in the case of growth rates, Dosi and Nelson (2010), Bottazzi and Secchi (2011), Erlingsson et al. (2013) and Mundt et al. (2016) show that the growth rate distribution is more leptokurtic than the Laplace distribution. We clearly show that the Laplace distribution nicely accounts for the profit rate distribution just in the case of large and long-lived firms, with the scale parameter almost invariant over time.

4.2. Asymmetric case

Results of the dependence for σ and α could give rise to misleading findings since we do not know which part of the distribution (right or left) is affected by the business cycle. Using the AEP helps us to understand the dynamics of the firms activity in terms of the GDP. The parameters $\mathbf{p}$=(α_l, α_r, σ_l, σ_r) of the AEP distribution of profit and growth rates are estimated with the maximum likelihood method using the software SUBBOTOOLS created by Bottazzi (2004), conditional on the value of m estimated with the mode of the distribution.[10] The estimation of the slope and scale parameters are shown in Fig. (7) and Fig. (9) for the 200 largest long-lived firms.

Recall that a given AEP distribution turns out to be a symmetric Laplace as long as $\alpha_l = \alpha_r = 1$ and $\sigma_l = \sigma_r$. Considering the sample of large firms, the shape parameters of the distribution of profit rates fluctuate around the condition $\alpha_l = \alpha_r = 1$ without any systematic pattern, confirmed also by the absence of significant correlations with the growth rate of GDP (see Tables 7 and 8). The scale parameters, instead, show a significant difference most of the years, favouring the right scale parameter, i.e. $\sigma_r > \sigma_l$. Such gap widens during the housing bubble and the subsequent banking crisis, while it shows a tendency to close during the years of the economic recovery. The use of the AEP distribution makes apparent the not satisfactory fit of the symmetric Laplace benchmark for large and long-lived companies. A more appropriate model for the profit rate distribution of large firms is an asymmetric Laplace distribution[11] with the mode correlated to the business cycle.[12]

[10]For the symmetric distribution the estimates are essentially independent of the chosen estimator for m, namely median, mean or mode. For the AEP distribution, using the mean or the median bias significantly the results (see Bottazzi and Secchi (2011)).

[11]The asymmetric Laplace distribution is defined by a nested AEP distribution in which $\alpha_l = \alpha_r = 1$ with time-variant σ_l and σ_r.

[12]We support these results with the LRT in Sec. (6.2) of the Appendix, in which (i) the symmetric Laplace distribution is rejected most of the years while (ii) the asymmetric Laplace is not rejected in 12 out of 19 years, compared to the AEP distribution.

Figure. 6: Probability density function (PDF) of profit rates along with the AEP (dotted line) and Laplace (dashed line) distribution. The results refer to the 200 largest long-lived firms according to their sales in 2016.

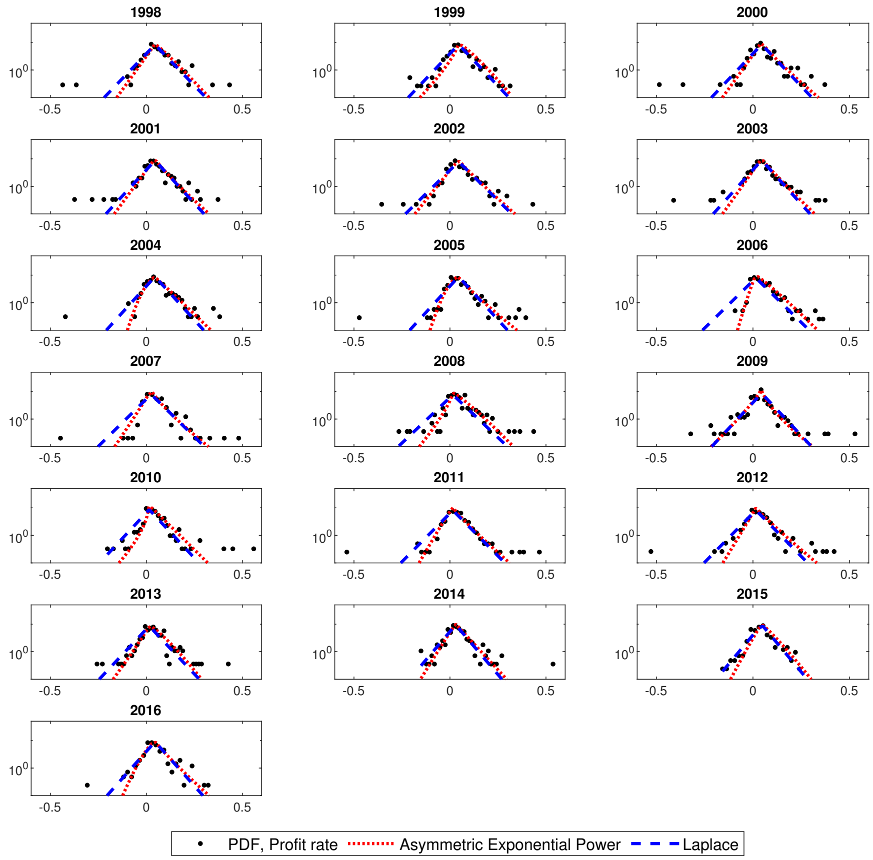
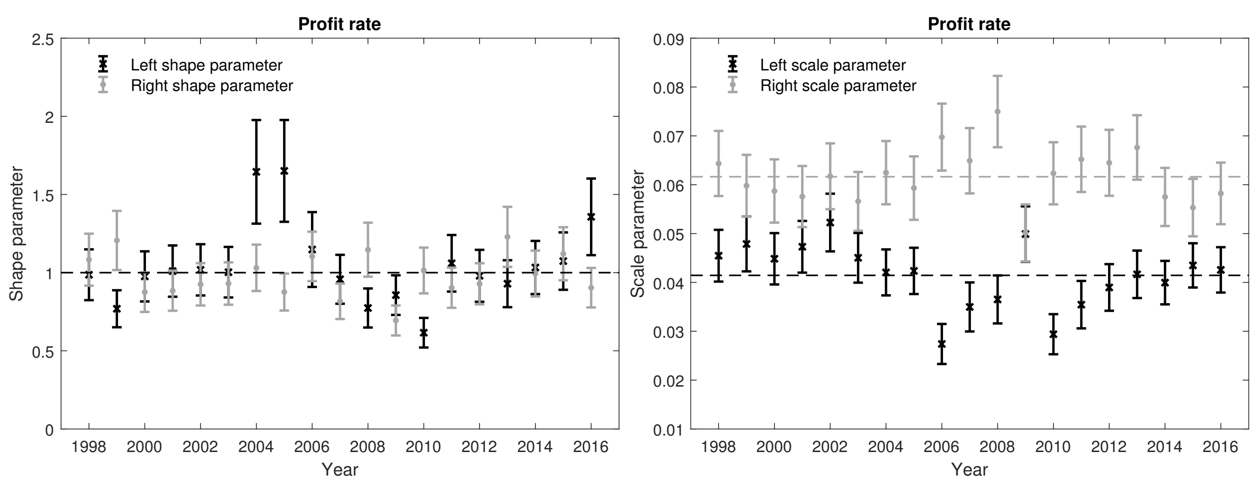


Figure. 7: Estimates of the two shape parameters (α_l and α_r) and two scale parameters (σ_l and σ_r) for the profit rates distribution. The results refer to the 200 largest long-lived firms according to their sales in 2016, removing the two extreme values in each year. Gray and black dashed lines refer to the mean of the estimates of σ_l and σ_r, respectively.



The distribution of growth rates of total assets and sales of the largest long-lived firms (see Fig. (9)) are characterised by a strong deviation from the Laplace distribution and a high level of volatility, which is in line with the literature (see for instance,Fu et al., 2005, Bottazzi and Secchi, 2006 and Dosi and Nelson, 2010)

Figure. 8: Probability density function (PDF) of growth rates of total assets along with the AEP (dotted line) and Laplace (dashed line) distributions. The results refer to the 200 largest long-lived firms according to their sales in 2016.

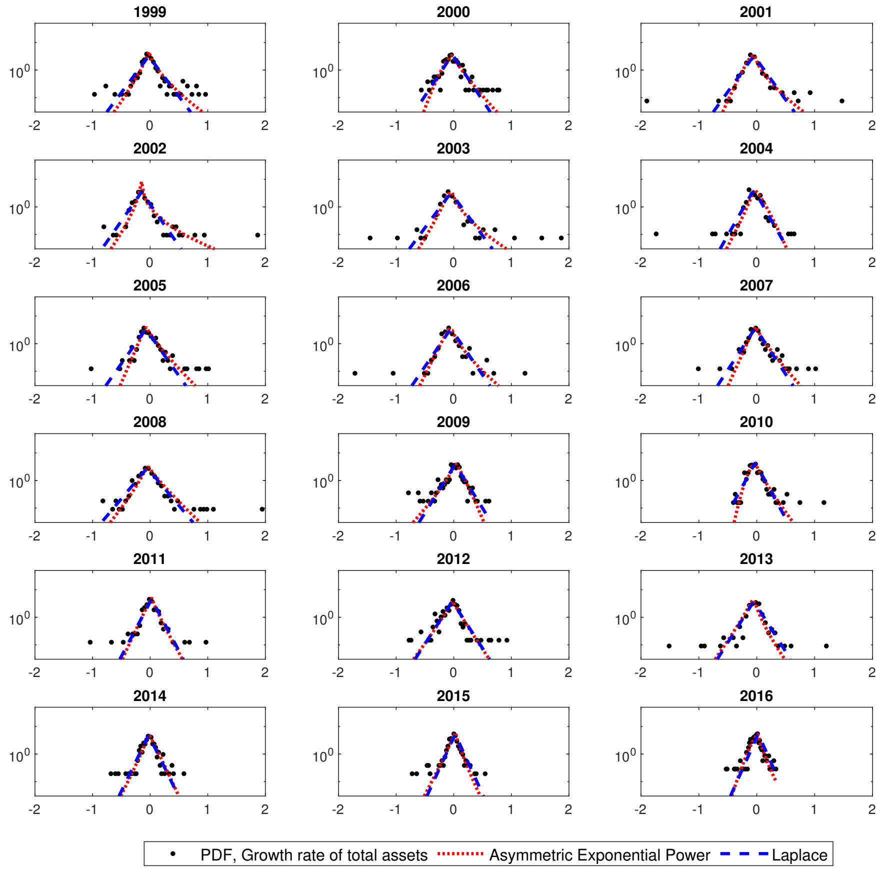
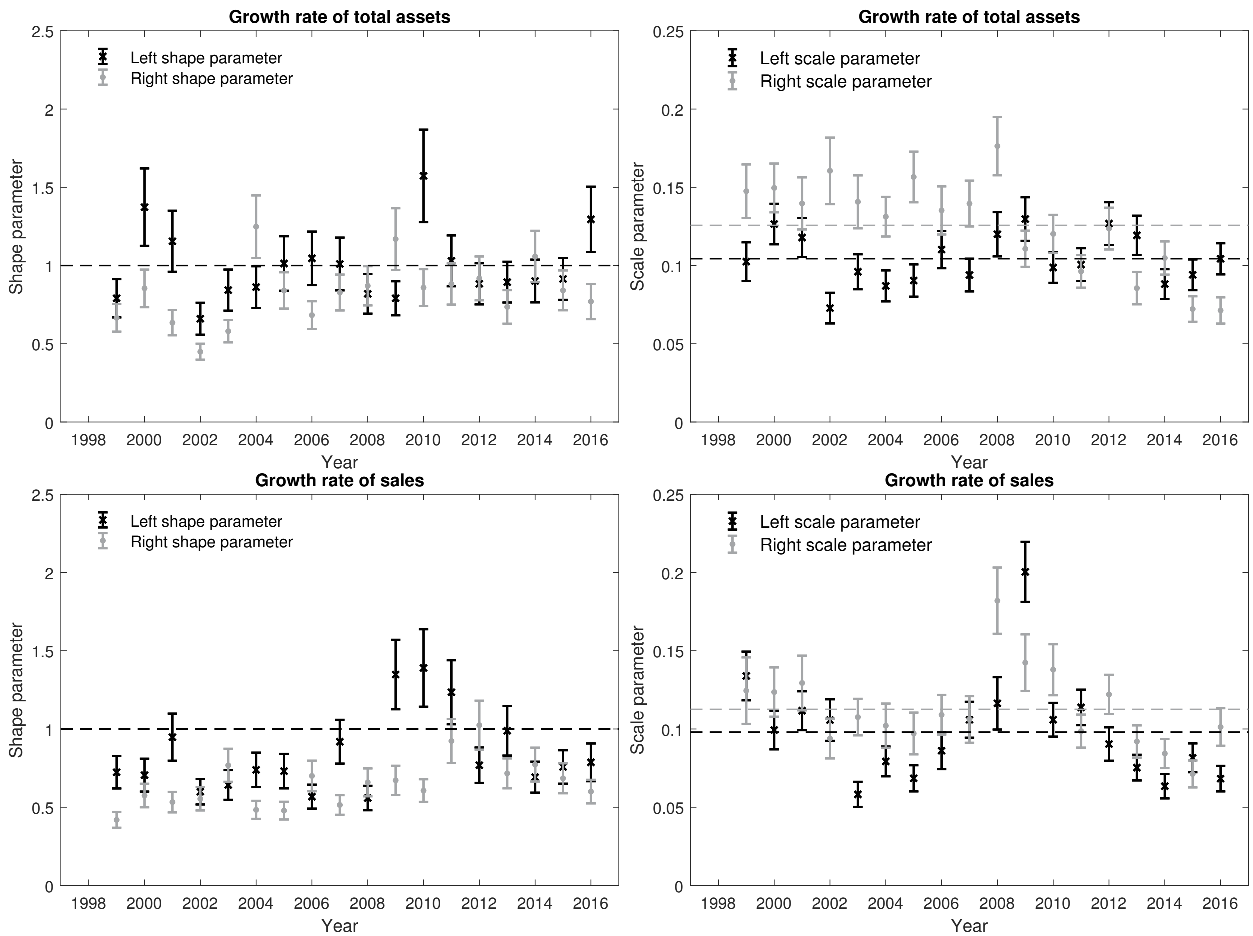


Figure. 9: Estimates of the two shape parameters (α_l and α_r) and two scale parameters (σ_l and σ_r) for growth rates of total assets and sales. Results refer to the 200 largest long-lived firms according to their sales in 2016. Gray and black dashed lines refer to the mean of the estimates of σ_l and σ_r, respectively.

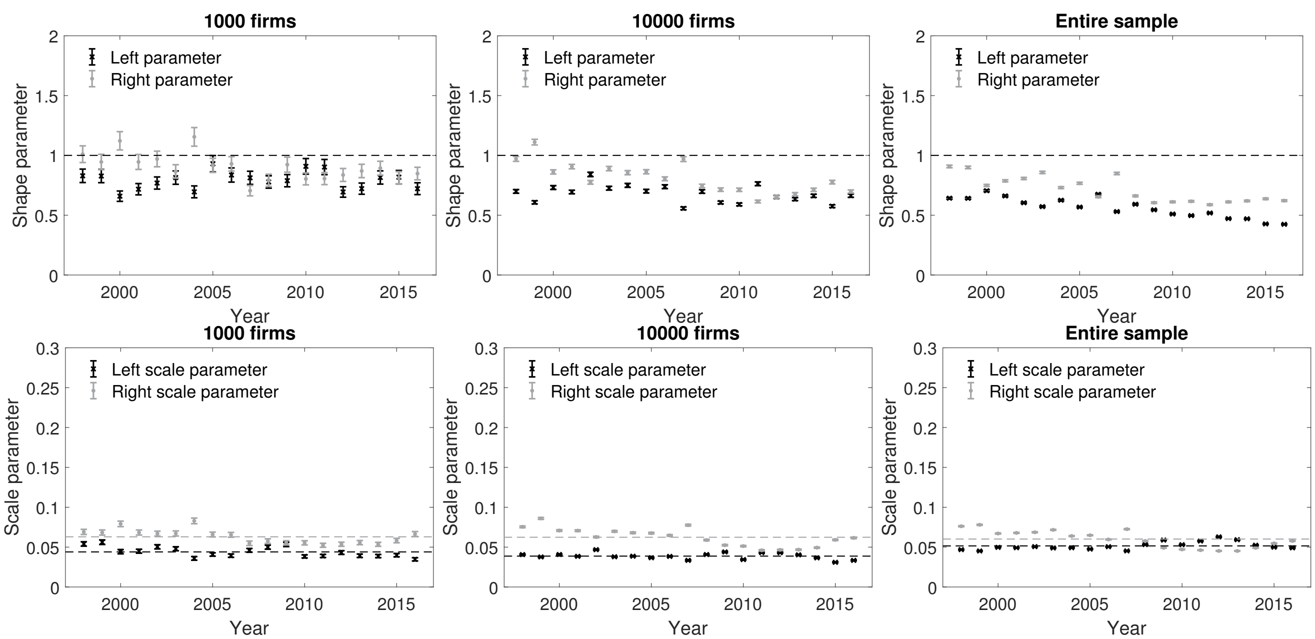


4.2.1. *Distributional properties conditional on size and business cycle phase*

In Figs. (10), (11) and (12), we report the estimates of the shape and scale parameters of the AEP computed for profit and growth rates of total assets and sales, conditional on size.

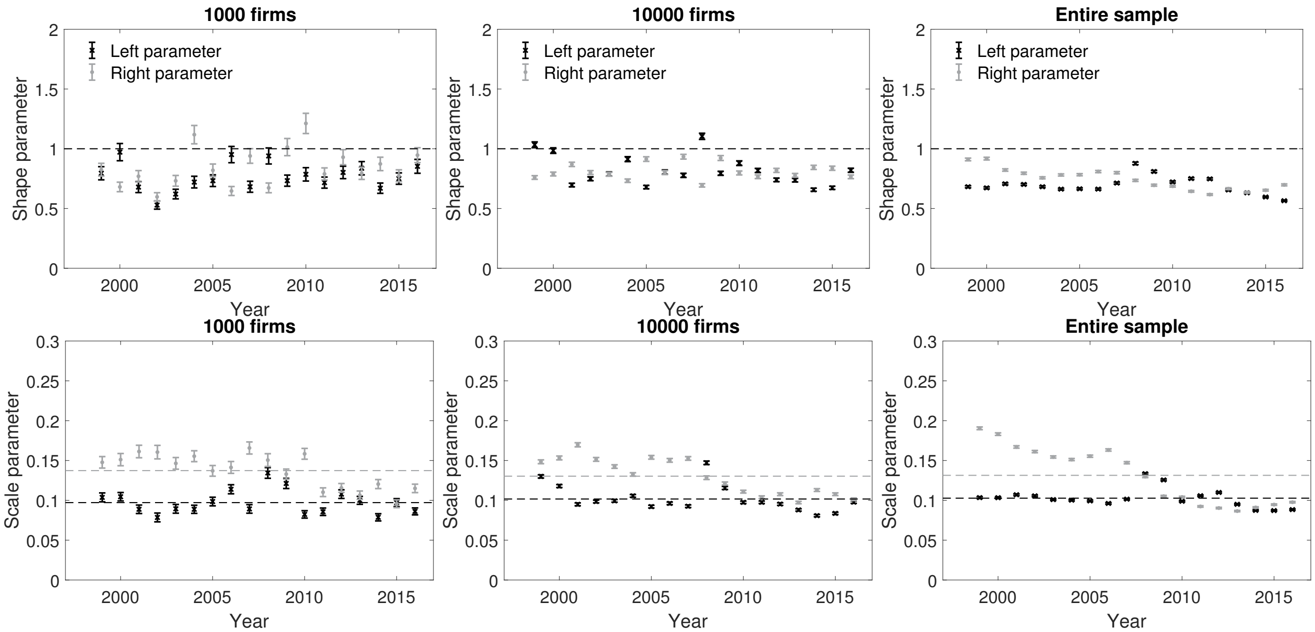
Regarding profit rates, we observe that α_l and α_r are significantly smaller than 1 most of the years. The shape of the distribution of profit rates depends on the size of the firms becoming fatter the smaller are the firms included in the sample. The scale parameter, instead, shows a remarkable stability as a function of the size, with the systematic tendency $\sigma_r > \sigma_l$. This condition changes when we consider the entire sample. The dispersion on the left side, measured by σ_l, is higher than the σ_r during the phase of the crisis. This change can be attributed to the effect of the business cycle on the profitability of small firms. As can be observed in Tables 7 and 8, the correlation between the AEP estimated and the GDP growth rates is stronger when including small firms. This result underlines the robustness of the profitability of large firms to the business cycle phase, while the small firms seem to be more affected by the adverse phase of the cycle.

Figure. 10: Estimates of the two shape parameters (α_l and α_r) and two scale parameters (σ_l and σ_r) of the AEP distribution for profit rates conditional on size. Error bars show two standard errors. Gray and black dashed lines refer to the mean of the estimates of σ_l and σ_r, respectively.



Focusing on growth rates of total assets, we always reject the Laplace distribution hypothesis, due to the differences in the scale parameters and shape parameters, i.e. $\alpha_l \neq \alpha_r$ and $\sigma_l \neq \sigma_r$. Moreover, the estimates of the shape parameters are different from 1 most of the years. Interestingly, the scale parameters show a similar behaviour to profit rates since the cross-sectional volatility is higher on the right side (σ_r) for the large firms but, when analysing the entire sample, we identify a remarkable decrease/increase of the cross sectional volatility on the right/left side during the crisis period.

Figure. 11: Estimates of the two shape parameters (α_l and α_r) and two scale parameters (σ_l and σ_r) of the AEP distribution for growth rates of total assets conditional on size. Error bars show two standard errors. Gray and black dashed lines refer to the mean of the estimates of σ_l and σ_r, respectively.



Finally, in relation to growth rates of sales, the Laplace distribution is also rejected since $\alpha_l \neq \alpha_r \neq 1$ and $\sigma_l \neq \sigma_r$. When including the smallest firms in the analysis, we observe a higher volatility on the left part of the distribution compared to the right one during the downturn, which is consistent with the results reported for profit rates and growth rates of total assets.[13] Thus, with Figs. (10), (11) and (12), we are able to underline the effect of the crisis on small firms by means of the scale parameters of profit and growth rates. On the other hand, in relation to the shape parameters, we observe a different dynamics between profit and growth rates. More specifically, profit rates tend to be more leptokurtic on both parts of the distribution when including smaller firms on the sample. However, the shape parameters of growth rates on the left part of the distribution become more platikurtic (i.e. we observe a slimming down of the left tail) during the crisis period, compared to the right tail. This particular behaviour has been already reported with the 200 largest long-lived firms (see Figs. (9) and (14)) in which we observe that, during the downturn, growth rates show a higher dispersion on the left part of the distribution with a slimming down of the left tail.[14]

[13] The main difference from profit rates and growth rates of total assets is found on the dispersion of the right part of the distribution of growth rates of sales given that it is quite constant regardless of the crisis, as can be observed in Fig. (12) and Table 8. This feature can be attributed to the natural volatility of sales in both parts of the distribution (Bottazzi et al., 2019).

[14] This feature can be clearly observed in Fig. (14) in 2009 and 2010.

Figure. 12: Estimates of the two shape parameters (α_l and α_r) and two scale parameters (σ_l and σ_r) of the AEP distribution for growth rates of sales conditional on size. Error bars show two standard errors. Gray and black dashed lines refer to the mean of the estimates of σ_l and σ_r, respectively.

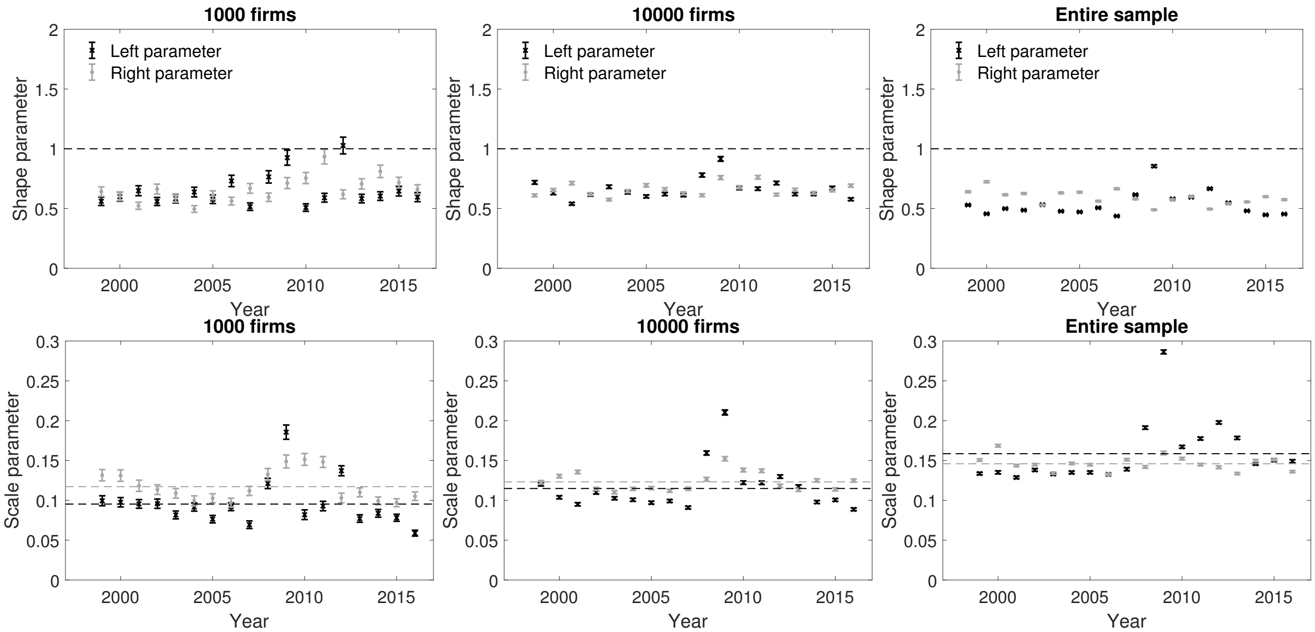


Table 5: Median of the estimates α_l and α_r reported in Figs. (10), (11) and (12).

	α_l			α_r		
Nº of firms	Profit rates	TA	Sales	Profit rates	TA	Sales
200	1.00	0.91	0.75	0.93	0.84	0.63
1000	0.81	0.74	0.60	0.89	0.81	0.65
10000	0.69	0.79	0.63	0.78	0.80	0.65
Entire sample	0.57	0.68	0.50	0.66	0.75	0.59

Table 6: Median of the estimates σ_l and σ_r reported in Figs. (10), (11) and (12).

	σ_l			σ_r		
Nº of firms	Profit rates	TA	Sales	Profit rates	TA	Sales
200	0.04	0.10	0.09	0.06	0.13	0.11
1000	0.04	0.09	0.09	0.07	0.14	0.11
10000	0.04	0.10	0.10	0.06	0.13	0.12
Entire sample	0.05	0.10	0.14	0.06	0.14	0.14

Table 7: Pearson correlation coefficient between the time series of the estimates of α_l and α_r with the time series of GDP growth rates.

	α_l & GDP			α_r & GDP		
Nº of Firms	Profit rates	TA	Sales	Profit rates	TA	Sales
200	0.35	0.05	-0.61***	0.07	-0.43	-0.68***
1000	-0.04	-0.04	-0.49**	0.4	-0.4	-0.61***
10000	0.33	0.16	-0.52**	0.77***	-0.02	-0.35
Entire sample	0.63***	-0.33	-0.72***	0.78***	0.83***	0.71***

Table 8: Pearson correlation coefficient between the time series of the estimates of σ_l and σ_r with the time series of GDP growth rates.

	σ_l & GDP			σ_r & GDP		
Nº of Firms	Profit rates	TA	Sales	Profit rates	TA	Sales
200	0.07	-0.38	-0.30	0.21	0.54**	-0.11
1000	0.12	-0.14	-0.46*	0.73***	0.58**	-0.35
10000	-0.26	0.11	-0.61***	0.86***	0.83***	-0.45*
Entire sample	-0.88****	-0.21	-0.81***	0.89***	0.87***	0.00

5. Conclusion

In this paper, we shed some light on the firm dynamics literature by analysing on what extent the Laplace distribution describes the Spanish long-lived firms distribution of profit and growth rate, against its alternative more general distributions, namely Subbotin and AEP. Moreover, compared to recent literature, we analyse the effect of the different phases of the business cycle and the firm size on the distributional characteristics of profit and growth rates.

We find evidence of systematic deviations of the profit rate distribution from the Laplace benchmark when small firms are included in the analysis. The empirical distribution becomes more leptokurtic without changing the scale parameters. Therefore, the Laplace benchmark turns out to be a reasonable approximation if we limit the sample to large and surviving firms. Relaxing the symmetric constraint, the use of the AEP distribution shows that, instead of a Laplace, the better approximation for firm profit rate distribution is an asymmetric Laplace. Interestingly, except for the location parameter, the shape and scale parameters do not depend on the business cycle phase. Small firms, instead, show a much higher dependence of their profit rates on the business cycle phase, signalling a marked difference with large firms. Taking into account these results, we underline the robustness of the large firms during the financial crisis in terms of profitability given (i) the significant larger dispersion of the right part of the distribution, compared to the left one, and (ii) the absence of relation between the time series of GDP growth rates and the time series of the estimates of σ_l, σ_r, α_l and α_r for the largest 200 long-lived firms' profit rates. However, this robustness is lost when including small firms in the sample since (i) we observe that the dispersion of the left part of the distribution is significantly larger than the right one during the years of the downturn, and (ii) the estimates of the entire sample show a remarkable relation with the GDP growth rates.

Finally, focusing on growth rates, we observe a similar tendency compared to profit rates given the effect of the crisis on small firms growth distribution ($\sigma_r < \sigma_l$). This result is supported by the stronger correlation between the time series of the estimate parameters and GDP growth rates when including small firms in the sample. Interestingly, we observe that profit and growth rates of total assets show a similar dynamics in terms of dispersion, while growth rates of total assets and sales are more similar regarding the shape of the distribution.

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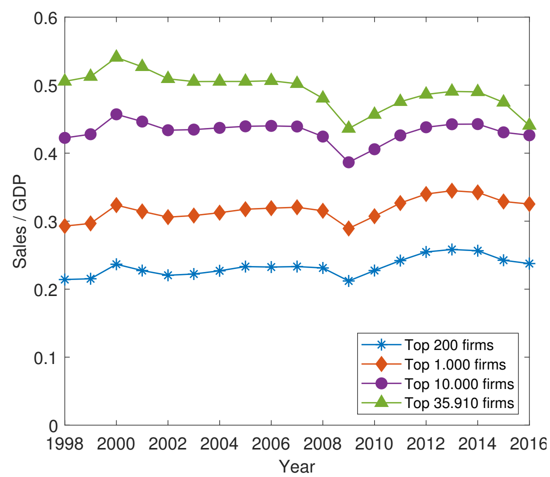
6. Appendix

6.1. Firms

Figure. 13: Sales as a function of GDP for the largest long-lived firms in our sample.



6.2. Likelihood ratio test for the 200 largest long-lived firms

Table 9 and Table 10 show the LRT in which we test the Laplace distribution compared to the AEP as alternative hypothesis. As can be observed, the null hypothesis of the Laplace distribution is rejected most of the years for profit rates and growth rates of total assets and sales. This result supports the outcome observed by Mundt and Oh (2019) since the AEP seems to characterise better the empirical density of profit rates.

Table 9: P-values of the likelihood ratio test for profit and growth rates of total assets and sales. The null hypothesis is the Laplace distribution, while the alternative hypothesis is the Subbotin distribution. The results refer to the 200 largest long-lived firms according to their sales in 2016.

LRT	1998	1999	2000	2001	2002	2003	2004	2005	2006	2007
Profit rate	0.17	0.05	**0.04**	0.16	0.06	0.11	0.21	0.06	**0.01**	**0.00**
Total assets	-	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**	0.05	**0.00**	**0.00**	**0.01**
Sales	-	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**

LRT	2008	2009	2010	2011	2012	2013	2014	2015	2016
Profit rate	0.27	**0.01**	**0.02**	**0.02**	**0.01**	0.73	0.15	0.08	**0.04**
Total assets	**0.01**	0.17	0.16	**0.04**	**0.00**	**0.00**	**0.02**	0.06	**0.00**
Sales	**0.00**	**0.00**	**0.00**	0.62	0.05	**0.00**	**0.00**	**0.00**	**0.00**

Table 10: P-values of the likelihood ratio test for profit rates and growth rates of total assets and sales. The null hypothesis is the Laplace distribution, while the alternative hypothesis is the AEP distribution. Results refer to the 200 largest long-lived firms, according to their sales in 2016.

LRT	1998	1999	2000	2001	2002	2003	2004	2005	2006	2007
Profit rate	**0.00**	**0.00**	**0.00**	**0.03**	**0.04**	**0.01**	**0.00**	**0.00**	**0.00**	**0.00**
Total assets	-	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**
Sales		**0.00**	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**

LRT	2008	2009	2010	2011	2012	2013	2014	2015	2016
Profit rate	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**
Total assets	**0.00**	**0.00**	**0.00**	0.22	0.39	**0.00**	**0.03**	**0.00**	**0.00**
Sales	**0.00**	**0.00**	**0.00**	**0.02**	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**

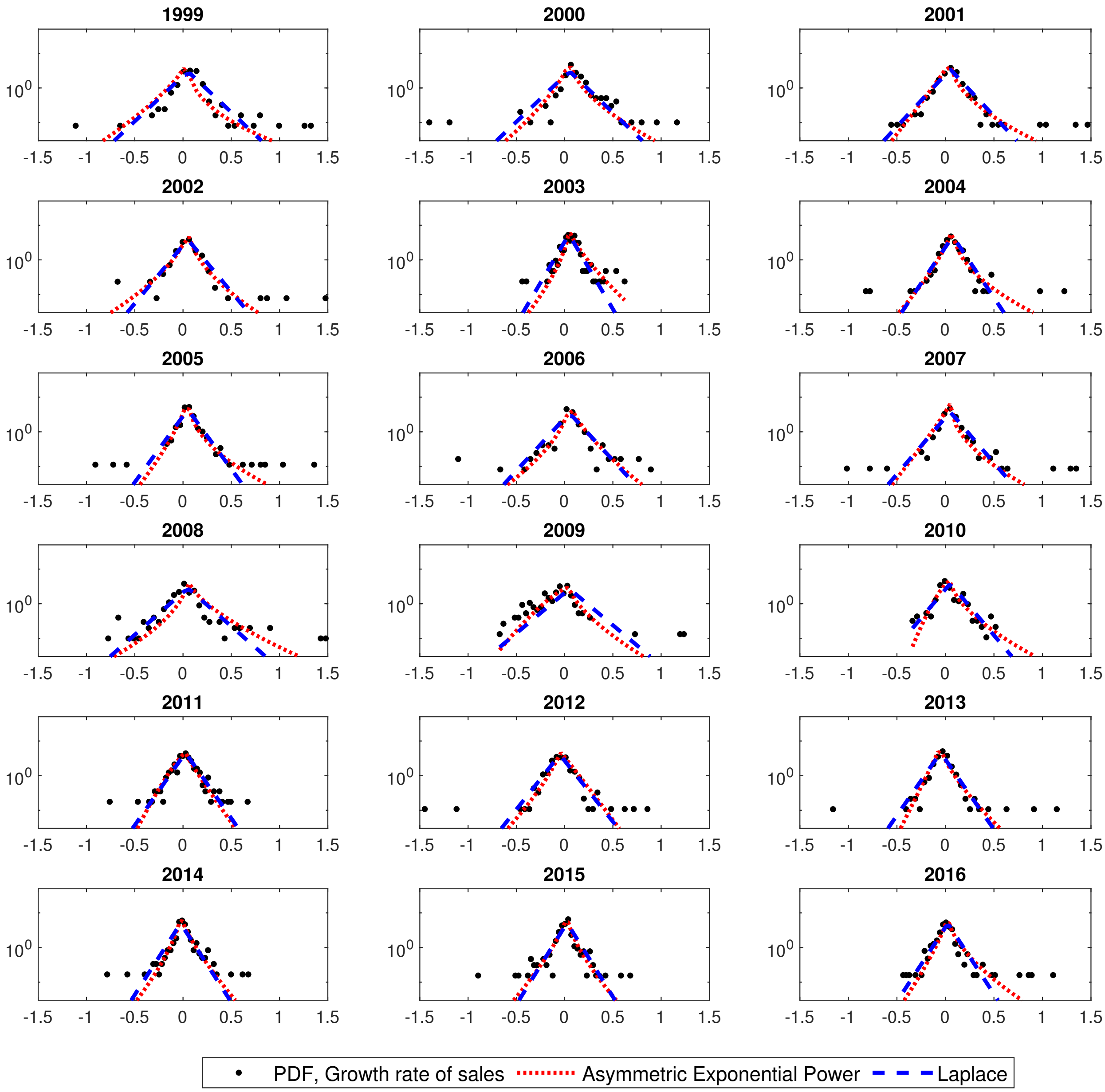
Table 11: P-values of the likelihood ratio test for profit rates and growth rates of total assets and sales. The null hypothesis is the asymmetric Laplace distribution, while the alternative hypothesis is the AEP distribution. Results refer to the 200 largest long-lived firms, according to their sales in 2016.

LRT	1998	1999	2000	2001	2002	2003	2004	2005	2006	2007
Profit rates	0.57	**0.01**	0.40	0.39	0.56	0.58	**0.01**	**0.00**	0.39	0.15
Total assets		**0.00**	**0.01**	**0.00**	**0.00**	**0.00**	**0.02**	0.14	**0.00**	0.10
Sales		**0.00**	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**

LRT	2008	2009	2010	2011	2012	2013	2014	2015	2016
Profit rates	**0.01**	**0.01**	**0.00**	0.29	0.58	0.07	0.81	0.43	**0.01**
Total assets	0.18	**0.01**	**0.00**	0.28	0.41	**0.03**	0.29	0.29	**0.00**
Sales	**0.00**	**0.00**	**0.00**	0.08	**0.03**	**0.00**	**0.01**	**0.00**	**0.00**

6.3. Probability density function of growth rates of sales

Figure. 14: Probability density function (PDF) of growth rates of sales along with the AEP (dotted line) and Laplace (dashed line) distribution. The results refer to the 200 largest long-lived firms according to their sales in 2016.



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