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Abstract. We propose a scenario for computing the real gross domestic product in a macroeconomic framework. The scenario is designed upon simple assumptions while ensuring basic algebraic relationships between the four usual macroeconomic accounts.

1. Introduction

In computational and algebraical points of view, a macroeconomic framework is regarded as a mechanism which links the following five macroeconomic sectors according to their respective accounting identities.

- Real sector: consists of variables at constant prices (1) and variables at current prices (2), (3).

$$q_t = cg_t + cp_t + ig_t + ip_t + xs_t - ms_t, \tag{1}$$

where

q_t	real gross domestic product (GDP);
cg_t	real government consumption;
cp_t	real private consumption;
ig_t	real government investment;
ip_t	real private investment;
xs_t	real exports of goods and non-factor services;
ms_t	real imports of goods and non-factor services.

$$Y_t^d = Y_t + e_t \left(Y_t^f + Y^{tr} \right), \tag{2}$$

$$Y_t = C_t^g + C_t^p + I_t^g + I_t^p + X_t - Z_t, \tag{3}$$

where

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Y_t^d	gross national disposable income (GNDI);
Y_t^{tr}	net transfer - foreign currency;
Y_t^f	net income - foreign currency;
e_t	period average exchange rate;
Y_t	nominal GDP;
C_t^g	nominal government consumption;
C_t^p	nominal private consumption;
I_t^g	nominal government investment / capital expenditure;
I_t^p	nominal private investment;
X_t	nominal exports of goods and non-factor services;
Z_t	nominal imports of goods and non-factor services.

- Public sector:

$$(T_t - C_t^g) - I_t^g + e_t \Delta F_t^g + \Delta D_t^g + \Delta B_t + \varepsilon_t^g = 0, \tag{4}$$

where

T_t	disposable income attributable to government;
ΔF_t^g	change in net foreign borrowing by the government - foreign currency;
ΔD_t^g	change in borrowing (net) from domestic banks by the government;
ΔB_t	change (net) in government borrowing from the private sector;
ε_t^p	financial external gap [5].

- External sector:

$$X_t - Z_t + e_t \left(Y_t^f + Y_t^{tr} + \Delta F_t^g + \Delta F_t^p \right) + \varepsilon_t^g = e_t \Delta R_t, \tag{5}$$

where

ΔF_t^p	change in net foreign borrowing by the private sector - foreign currency;
ΔR_t	change (net) in international reserves.

- Monetary sector:

$$M_t^d = (1/v_t)\, Y_t, \tag{6}$$

$$\Delta M_t^s = e_t \Delta R_t + \Delta D_t^g + \Delta D_t^p + \varepsilon_t^m, \tag{7}$$

$$M_t^d = M_t^s = M_t, \tag{8}$$

$$\varepsilon_t^m = (E_t - e_t)\, \Delta R_t + R_{t-1} \Delta E_t, \tag{9}$$

where

ΔM_t^s	change in money supply;
ΔM_t^d	change in money demand;
E_t	exchange rate at the end of period t;
v_t	velocity of money;
ΔD_t^p	change in borrowing (net) from domestic banks by the private sector;
ε_t^m	valuation adjustment.

By (2), formula (5) can be written in terms of the national economy, namely

$$S_t - I_t + e_t \left(\Delta F_t^g + \Delta F_t^p \right) + \varepsilon_t^g = e_t \Delta R_t, \tag{10}$$

where $S_t := Y_t^d - (C_t^g + C_t^p)$ denotes the national saving, and $I_t := I_t^g + I_t^p$ is the gross investment. (10) can be disaggregated as

$$(S_t^g - I_t^g) + (S_t^p - I_t^p) + e_t \left(\Delta F_t^g + \Delta F_t^p \right) + \varepsilon_t^g = e_t \Delta R_t, \tag{11}$$

where $S_t^g := T_t - C_t^g$ is the government saving, and $S_t^p := \left(Y_t^d - T\right) - C_t^p$ is the private saving ($Y_t^d - T_t$ denotes the private disposable income). By (4) we have

$$(S_t^g - I_t^g) + e_t\Delta F_t^g + \Delta D_t^g + \Delta B_t + (S_t^p - I_t^p) + e_t\Delta F_t^p - \Delta D_t^g - \Delta B_t + \varepsilon_t^g = e_t\Delta R_t. \quad (12)$$

We deduce the formula for the private sector:

$$S_t^p - I_t^p + e_t\Delta F_t^p - \Delta D_t^g - \Delta B_t - e_t\Delta R_t = 0, \quad (13)$$

or

$$\left(Y_t^d - T_t - C_t^p\right) - I_t^p + e_t\Delta F_t^p + \Delta D_t^p - \Delta B_t - \Delta M_t + \varepsilon_t^m = 0. \quad (14)$$

Let us consider a macroeconomic framework as a system of equations. Some variables in the system are considered exogenous or targeted (instrumental variables), and extra variables interlinking can be added (the so-called behavioral functions). Models for solving such system include the World Bank's RMSM [1, 4, 6] and the IMF's financial programming [6, 7]. In this note, we propose a scenario for computing the values of the real GDP and its components. Value of q_t is usually obtained from an exogenous growth rate in the RMSM [1], while IMF programs compute the growth rate from the simultaneous assessment of structural changes and the external environment [7, p. 6]. Our system consists of equations (1)–(9), and most of the assumptions for determining the solution are borrowed from the previous mentioned models.

2. The scenario

As for the IMF's model [6, Chap. 4], we assume that the following variables are exogenous:

$$Y_t^f, Y_t^{tr}, \Delta F_t^g, \Delta F_t^p.$$

We also follow Mikkelsen [5], and assume that ΔD_t^g is exogenous. In practice, most of the variables in the public accounting are predetermined, and in this case, we simply consider ε_t^g as a balancing item. We assume then that $T_t > 0$ and ΔB_t are known. Throughout the rest of this note, a bar placed over a variable means that the corresponding variable is exogenous [6, p. 74], or that its value was already computed somewhere during the process. Sign "$*$" denotes a desired value of the corresponding variable. e_t^* and E_t^* are written in such fashion.

- We choose the following simple behavioral function for the nominal private consumption

$$C_t^p = \beta\left(Y_t^d\right)^\alpha, \quad (15)$$

with $0 < \alpha < 1$ and $\beta > 0$. By definition,

$$C_t^p = (1 - \overline{\rho_t})\left(Y_t^d - \overline{T_t}\right), \quad (16)$$

where ρ_t, with $0 < \rho_t < 1$, denotes the private investment saving propensity. Identities (15) and (16) yield

$$\beta\left(Y_t^d\right)^\alpha - (1 - \overline{\rho_t})\left(Y_t^d - \overline{T_t}\right) = 0. \quad (17)$$

Let Φ_t denote the function defined on $[0, +\infty[$ as $\Phi_t(y) = \beta y^\alpha - (1 - \overline{\rho_t})\left(y - \overline{T_t}\right)$. We have $\Phi_t(0) = (1 - \overline{\rho_t})\overline{T_t} > 0$ and $\Phi_t(u) < 0$ for u sufficiently large. Φ_t is continuous on $[0, u]$, hence by the Intermediate Value Theorem, Φ_t has a zero on this interval. Moreover, Φ_t is strictly increasing on $\left[0, \left(\frac{1 - \overline{\rho_t}}{\alpha\beta}\right)^{\frac{1}{\alpha - 1}}\right]$, and strictly decreasing on

$\left[\left(\frac{1-\overline{\rho_t}}{\alpha\beta}\right)^{\frac{1}{\alpha-1}},0\right]$. Therefore, the zero of Φ_t is unique, and is located on the latter interval. We let $\overline{Y_t^d}$ denote the zero of Φ_t which is also the solution of (17).

- By (2), nominal GDP is

$$Y_t = \overline{Y_t^d} - e_t^*\left(\overline{Y_t^f} + \overline{Y_t^{tr}}\right).$$

- Government expenditure $G_t := C_t^g + I_t^g$ is either exogenous [5], or is determined by targeting the public deficit in percentage of the GDP δ_t^*, i.e.,

$$G_t = \overline{T_t} - \delta_t^*\overline{Y_t} \tag{18}$$

Likewise, nominal government investment is computed from a targeted ratio in percent of GDP κ_t^*:

$$I_t^g = \kappa_t^*\overline{Y_t}. \tag{19}$$

Value of C_t^g immediately follows:

$$C_t^g = \overline{G_t} - \overline{I_t^g}. \tag{20}$$

- In the RMSM setting, the real exports grow at an exogenous rate $\overline{g_t}$, i.e.,

$$xs_t = xs_{t-1}\left(1+\overline{g_t}\right), \tag{21}$$

and the nominal exports is obtained using an exogenous price index XPI_t [1, 4]:

$$X_t = \overline{xs_t}\,\overline{XPI_t}. \tag{22}$$

- Nominal imports and gross official reserves are linked with a policy determined target for gross official reserves in months of imports m_t^*:

$$e_t^*\left(R_t + \overline{L_t}\right) = Z_t\left(m_t^*/12\right), \tag{23}$$

where L_t is the official foreign liabilities which is set exogenous [5]. Formula (5) becomes

$$\overline{X_t} - Z_t + e_t^*\left(\overline{Y_t^f} + \overline{Y_t^{tr}} + \overline{\Delta F_t}\right) + \overline{\varepsilon_t^g} = Z_t\left(m_t^*/12\right) - e_t^*\overline{L_t} - e_t^* R_{t-1}, \tag{24}$$

where

$$\overline{\varepsilon_t^g} = \left(\overline{G_t} - \overline{T_t}\right) - \left(e_t^*\overline{\Delta F_t^g} + \overline{\Delta D_t^g} + \overline{B_t}\right). \tag{25}$$

We obtain

$$\overline{Z_t} = \frac{\overline{X_t} + e_t^*\left(\overline{Y_t^f} + \overline{Y_t^{tr}} + \overline{\Delta F_t^p} + \overline{L_t} + R_{t-1}\right) + \overline{G_t} - \overline{T_t} - \Delta\overline{D_t^g} - \overline{B_t}}{1+\left(m_t^*/12\right)}, \tag{26}$$

and then

$$R_t = \frac{\overline{Z_t}\left(m_t^*/12\right) - e_t^*\overline{L_t}}{e_t^*}. \tag{27}$$

- By (16) and (13), formula for private investment is

$$I_t^p = \overline{\rho_t}\left(\overline{Y_t^d} - \overline{T_t}\right) + e_t^*\left(\overline{\Delta F_t^p} - \overline{\Delta R_t}\right) - \Delta\overline{D_t^g} - \overline{B_t}. \tag{28}$$

- Money is given by

$$M_t = (1/\overline{v_t})\,\overline{Y_t}, \tag{29}$$

assuming that v_t is predictable [7, p. 19]. Change in borrowing (net) from domestic banks by the private sector is then computed as residual:

$$\Delta D_t^p = \overline{M_t} - e_t^* \overline{\Delta R_t} - \overline{\Delta D_t^g} - \overline{\varepsilon_t^m}, \tag{30}$$

where

$$\overline{\varepsilon_t^m} = (E_t^* - e_t^*)\,\overline{\Delta R_t} + (E_t^* - E_{t-1})\,R_{t-1}. \tag{31}$$

- We follow Khondker & Raihan [3] for the real consumption functions and set

$$cg_t = cg_{t-1}\left(\frac{\overline{C_t^g}}{C_{t-1}^g}\right)(1+\overline{\pi_t}) \tag{32}$$

and

$$cp_t = cp_{t-1}\left(\frac{\overline{C_t^p}}{C_{t-1}^p}\right)(1+\overline{\pi_t}), \tag{33}$$

where $\overline{\pi_t}$ is the rate of inflation.
- We compute ip_t and ig_t by setting the government and private investment deflators equal to the GDP deflator [2]:

$$ig_t = \frac{\overline{I_t^g}}{\overline{Y_t}/q_t} = q_t \frac{\overline{I_t^g}}{\overline{Y_t}} \tag{34}$$

and

$$ip_t = \frac{\overline{I_t^p}}{\overline{Y_t}/q_t} = q_t \frac{\overline{I_t^p}}{\overline{Y_t}}. \tag{35}$$

- The real imports function is constructed according to the weighted elasticity approach of the RMSM [1]:

$$ms_t = (1-\overline{\eta_t})ms_{t-1} + \overline{\eta_t} ms_{t-1}\,(q_t/q_{t-1}). \tag{36}$$

Finally, real GDP is computed as

$$q_t = \frac{\overline{cg_t} + \overline{cp_t} + \overline{xs_t} - (1-\overline{\eta_t})ms_{t-1}}{1 - \frac{\overline{I_t^g}}{\overline{Y_t}} - \frac{\overline{I_t^p}}{\overline{Y_t}} + \overline{\eta_t}\frac{ms_{t-1}}{q_{t-1}}}. \tag{37}$$

ig_t, ip_t and ms_t are then computed according to (34), (35) and (36), respectively.

References

[1] D. Addison, The World Bank revised minimum standard model (RMSM): concepts and issues, *Policy Research Working Paper Series* **231**, The World Bank, 1989.

[2] M. Dyakonova, E. Naval and L. Savenko, Macroeconomic development model for the national economy (second extension), *Computer Science Journal of Moldova* **6**(1998), 205–219.

[3] B. H. Khondker and S. Raihan, Macroeconomic framework for the economy of Bangladesh, *MPRA Paper* **38476**, 2008.

[4] L. C. Jemio, A framework for the analysis of the macroeconomic effects of multi-funds in the macedonian pension system, *Fourth Bolivian Conference on Development Economics (BCDE2012)*, 2012.
[5] J. G. Mikkelsen, A model for financial programming, *IMF Working Paper*, WP/98/80, 1998.
[6] C. A. Mills and R. Nallari, *Analytical Approaches to Stabilization and Adjustment Programs*, The World Bank Washington, D. C., 1998.
[7] V. Yotzov, *Macroeconomic Models of the International Monetary Fund and the World Bank (Analysis of Theoretical Approaches and Evaluation of Their Effective Implementation in Bulgaria)*, Bulgarian National Bank, 2001.