Modelling Cryptocurrency High-Low Prices using Fractional Cointegrating VAR

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Abstract

This paper empirically provides support for fractional cointegration of high and low cryptocurrency price series, using particularly, Bitcoin, Ethereum, Litecoin and Ripple; synchronized at different high time frequencies. The difference of high and low price gives the price range, and the range-based estimator of volatility is more efficient than the return-based estimator of realized volatility. A more general fractional cointegration technique applied is the Fractional Cointegrating Vector Autoregressive framework. The results show that high and low cryptocurrency prices are actually cointegrated in both stationary and non-stationary levels; that is, the range of high-low price. It is therefore quite interesting to note that the fractional cointegration approach presents a lower measure of the persistence for the range compared to the fractional integration approach, and the results are insensitive to different time frequencies. The main finding in this work serves as an alternative volatility estimation method in cryptocurrency and other assets' price modelling and forecasting.

Keywords: Fractional cointegration; Cryptocurrency; Fractional integration; FCVAR; Price range

JEL Classifications: C22

1. INTRODUCTION

Financial analysts and traders are recently interested in the high, low and price range of intra-day commodity prices. The price range series (high-low price difference) is a more efficient returns series, used in the estimation and analysis of equilibrium (stationary) and volatility (risk) than the close-close or close/open log-return series (Parkinson 1980; Cheung et al. 2009; Yaya and Gil-Alana, 2018). Furthermore, extant analyses of realized volatility in asset prices; such as stocks, exchange rates and commodity prices; frequently use daily prices to obtain transformed series, which are subsequently used as proxy variables in measuring volatility. For the daily time frequency, log-transformed price differences are applicable but not sufficient whenever several prices are recorded at different intraday time periods, either minutely or hourly. For example, while opening and closing prices are linked with daily frequency series, the resulting transformed volatility series hides the intraday variability, thereby leading to loss of some important information (Haniff and Pok, 2010; Degiannakis and Floros, 2013; among others). Thus, different intraday high and low asset prices, are used as alternatives to close and open prices; such that the range (intraday high price minus intraday low price), an unbiased measure of realized volatility, could serve as a reference value for investors in bid and ask orders (Alizadeh, Brandt and Diebold, 2002; Barunik and Dvorakova, 2015; Xiong, Li and Bao, 2017; among others).

The concept of high and low prices was introduced by Cheung (2007), given the assumption that the underlying data generating trends in highs and lows are the same, and the plausibility of their differences converging over time, even when prices seem to move apart one from the other. Some researchers empirically showed the robustness of the range-based volatility estimator to microstructure noise (for example, the bid-ask bounce), and its preference over the traditional volatility estimator, based on closing/opening prices, since it overcomes the limitation of the latter (see Brandt and Diebold, 2006 and Shu and Zhang, 2006). Daily highs and lows (used as stop-loss bandwidths) provide information relating to liquidity and price discovery. Caporin et al. (2013) opined that low prices correspond to bid quotes and high prices to ask quotes; though high/low prices are subjected to unexpected public announcements or other shocks.

The range can be expressed as the cointegrating relation between the daily high and low prices with fixed cointegrating vector (1, -1). Daily high and low prices are also useful in measuring price dispersion from the mean price, particularly, for different trading time frequencies. The dispersion shows the degree of uncertainty, that is, the risk associated with a particular asset (cryptocurrency, in this case). Parkinson (1980) showed the variance estimator that is based on close/open returns to be less efficient compared to the range-based volatility estimator, which is considered in this work. The range-based volatility estimator is statistically efficient and robust against microstructure noise, as it is less contaminated by measurement error, and explains even volatility of volatility other than autocorrelation of volatility, as other measurement indicators induce (Alizadeh et al., 2002).Thus, range-based method of analysis serves as alternative method to the traditional log-returns modelling of volatility in financial econometrics.

The contributions of this paper can be discussed in the following. First, several studies have investigated the long- and short-run dynamics of the high and low prices of stocks (see Cheung, 2007; Cheung et al., 2009; Cheung et al. 2010; Caporin et al., 2013; Barunik and Dvorakova, 2015; Maciel, 2018; Afzal and Sibbertsen, 2019; among others), exchange rate (He and Wan, 2009) and oil prices (see He et al., 2010; Yaya and Gil-Alana, 2018; among others). However, to our knowledge, no single study has been done to investigate the long-run and short-run dynamics of the high and low prices of cryptocurrency.

Second, this study was motivated by the work of Barunik and Dvorakova (2015), which conducted cointegration analyses on daily high and low stock prices, in some world stock market indices. Their results showed that the differences between high and low prices (that is, ranges) exhibited long memory features and were found to be non-stationary, which contracts the belief that volatility series might be non-stationary. Yaya and Gil-Alana (2018); using selected commodity prices, such as crude oil, natural gas, gold and silver, that were available in daily, hourly and minutely frequencies; conducted fractional integration and fractional cointegration analyses. The authors found that the two price measurements were fractionally cointegrated, and the range series of highs and lows exhibited stationary and non-stationary persistence, that changed substantially across time frequencies. Following Cheung (2007)'s suggestion of modelling the cointegration relationship with Vector Error Correction Mechanism (VECM), Cheung et al. (2009) and He and Wan (2009) empirically analysed the daily high and low stock prices.

As a deviation from the stance in extant literature on long-term relationships among variables, which was usually considered along the I(0) or I(1) characteristics, Caporin et al. (2013) and Barunik and Dvorakova (2015) adopted Fractional Vector Error Correction Mechanism (FVECM), which is considered a more flexible approach that allows for fractional order of integration. Afzal and Sibbertsen (2019) showed the existence of long-run relationships between high and low stock prices in six Asian countries, using the fractional cointegration in a VECM framework. Their study revealed the consistent outperformance of FVECM over the heterogeneous Autoregressive (AR) model and Autoregressive Fractionally Integrated Moving Average (ARFIMA) model. Following this recent development, the analysis in this study was derived from the latest method presented in Johansen and Nielsen (2012, 2014) that allows for multiple series in the cointegrating framework, in Vector Autoregressive (VAR) system.

Against the above background, this work contributes to the empirical literature on cryptocurrency price modelling, investigating whether high and low prices at different time frequencies are cointegrated, using four notable currencies: the Bitcoin, Ethereum, Litecoin and Ripple. The range that is the residual series from cointegration of high and low price series is found to be stationary and non-stationary. Thus, it deduced the possibility of obtaining non-stationary range-based volatility estimator, other than logged transformed returns series obtained from close/open asset prices. We find that the Fractional Cointegrated Vector Autoregressive (FCVAR) approach detected persistence of the range series often in stationary range than when fractional integration of the range series was obtained. Our results are insensitive to various time frequencies considered.

Following from the introductory part, the rest of the paper is thus structured: Section 2 gives the overview of the econometric method, involving fractional cointegration framework, using a modified approach -the FCVAR modelling. Section 3 presents details on empirical analysis covering data description, as well as presentation and discussion of empirical results obtained, while the paper is concluded in Section 4.

2. ECONOMETRIC APPROACH

Here, we adopt the fractional cointegration framework to ascertain the plausibility of cointegration between the high and low prices of cryptocurrencies. As consistent with the cointegration framework and in a simple bivariate case; two series, x_{1t} and x_{2t} (where $t = 1,...,T$) are cointegrated whenever the order of integration, d for the individual series are the same (Engle and Granger, 1987), that is, x_{1t}, $x_{2t} \approx I(d)$. Consequently, this implies that a linear combination, $x_{1t} - \beta x_{2t}$, which exists between both series, is integrated of a smaller order, $d - b$, where b > 0. This is a linear model with intercept, α and slope, β with residual series, u_t. A major focus of this paper is when the parameters d and b do not take on integer values only, that is, a deviation from the assumption, $d = b = 1$; a rather restrictive assumption that has usually been adopted in most empirical studies. Engle and

Granger (1987) proposed a two-step methodology. First, the order of integration of each of the series is tested, using the conventional Augmented Dickey-Fuller (ADF) test (Dickey and Fuller, 1979). Second, upon confirming that the series are $I(1)$, the null of no unit root in the residuals of the cointegrating regression in equation (1) is tested.

$$x_{1t} = \alpha + \beta x_{2t} + u_t \tag{1}$$

A non-rejection of the null of an $I(0)$ process would imply that both series x_{1t} and x_{2t} are cointegrated. Alternatively, for any two series to be cointegrated, upon being individually integrated of order 1, their paired differences and/or log differences is expected to be integrated of order zero, i.e., $I(0)$.

As a contrast to the rather restrictive assumption of integer-valued integration/cointegration parameters, the cointegrating framework in equation (1), as in extant studieshas been extended to include cases where the order of integration/cointegration could take on fractional values (see Cheung and Lai, 1993; Robinson and Yajima, 2002; Gil-Alana, 2003; Robinson and Hualde, 2003; Robinson and Marinucci, 2003; Hualde and Robinson, 2007; Gil-Alana and Hualde, 2009; among others). Moving from the single equation framework, the concept of cointegration is further extended to the VAR framework, with integer-valued cointegration (the CVAR) (Johansen, 1995) and fractional cointegration (FCVAR) (Johansen and Nielsen, 2012; 2014) frameworks. Put differently, while the CVAR model framework assumes unity as the order of integration, the FCVAR model allows for fractional or real number valued order of integration among the series. The FCVAR model is equivalent to the CVAR model whenever the integration parameter, d is equal to unity, and thus, the latter is said to be nested in the former.

The FCVAR, and implicitly, the CVAR, model estimations, proceed in two steps. First, the univariate orders of integration; $d_{x_{1t}}$ and $d_{x_{2t}}$, for each series, x_{1t} and x_{2t}, respectively; are obtained using the log-periodogram-type estimator (GPH) of Geweke and Porter-Hudak (1983) and Robinson (1995a), and the Exact Local Whittle estimator (ELW) (Robinson, 1995b). Both GPH and ELW methods of estimating fractional unit root parameters are semi-parametric approaches (relying on time series frequency domain approach), which both use periodogram ordinates. The fractionally integrated series x_{1t}, is mathematically expressed in (2) and expanded using the Binomial expansion in (3),

$$(1 - L)^d x_{1t} = u_t, \qquad t = 1, 2, \ldots, T \tag{2}$$

$$x_{1t} = d\, x_{1t-1} - \frac{d\,(d-1)}{2} x_{1t-2} + \frac{d\,(d-2)\,(d-3)}{6} x_{1t-3} + \ldots + u_t \tag{3}$$

where $L$is the backward shift operator, such that $Lx_{1t} = x_{1t-1}$, and d is the fractional integrated order.

Second, the CVAR and FCVAR models are estimated, premised on the satisfaction of the first step. Given that the FCVAR model structure draws from the conventional CVAR model, our discussion would commence from the CVAR model structure, which also provides the basis for its fractional variant, the FCVAR. Given a $p-$dimensional $I(1)$ series, x_{1t}, $t = 1, 2, \ldots, T$, the CVAR model specification is given in equation 4

$$\begin{aligned} \Delta x_{1t} &= \alpha\beta' x_{1t-1} + \sum_{i=1}^{k} \Gamma_i \Delta x_{1t-i} + \varepsilon_t \\ &= \alpha\beta' L x_{1t} + \sum_{i=1}^{k} \Gamma_i \Delta L^i x_{1t} + \varepsilon_t \end{aligned} \tag{4}$$

The FCVAR model, which is subsequently derived by substituting Δ^b and $L_b = 1 - \Delta^b$ for the difference operator, Δ and the lag operator, L, respectively, in equation (4); is given in equation (5):

$$\Delta^b x_{1t} = \alpha\beta' L_b x_{1t} + \sum_{i=1}^{k} \Gamma_i \Delta^b L_b{}^i x_{1t} + \varepsilon_t \tag{5}$$

while applying the same to $x_{1t} = \Delta^{d-b} y_t$, we obtain equation (6)

$$\Delta^d y_t = \alpha\beta' L_b \Delta^{d-b} y_t + \sum_{i=1}^{k} \Gamma_i \Delta^b L_b{}^i y_t + \varepsilon_t \tag{6}$$

where Δ^d denotes the fractional operator, b denotes the cointegrating factor and L_b represents the fractional lag operator; α and β, in this model are $p \times r$ matrices of long-run parameters, such that $0 \le r \le p$ and r represents the cointegrating rank; $d-b$ (such that $b > 0$) is the degree of fractional cointegration, that is, the degree of the fractional integration order of the long run equilibrium, $\beta' x_t$. The elements of $\beta' x_t$ give the cointegrating relationships in the system, where k determines the number of long-run equilibria, i.e. the cointegration or co-fractional rank and $\Gamma = \Gamma_1, \ldots, \Gamma_k$ governs the short-run dynamics; ε_t is a $p-$vector of the error term, which is independently and identically distributed with mean and covariance 0and Ω, respectively.

The model in (6) is the restricted constant case (see Johansen and Nielsen, 2012), while the unrestricted constant term case is given in Dolatabadi et al. (2016) as

$$\Delta^d y_t = \alpha L_b \left(\beta' y_t + \rho'\right) \Delta^{d-b} + \sum_{i=1}^{k} \Gamma_i \Delta^b L_b{}^i y_t + \xi + \varepsilon_t \tag{7}$$

where ρ represents the restricted constant in this case (restricted to the form $\alpha\rho'$), that is, the mean level of the long run equilibria, and ξ is the unrestricted constant term.

In estimating the FCVAR model framework, we proceed in four basic steps. First, we empirically obtain the optimal lag length; second, we obtain the cointegration rank; third, using the results in the first and second steps, we test for fractional cointegration; and finally, we compare FCVAR and CVAR models, using the likelihood ratio (LR) test. The test assumes $d = b = 1$ under the null hypothesis, such that a rejection of the null would imply preference of the FCVAR model over the conventional CVAR model; otherwise, the latter is preferred over the former. For a detailed description of the estimation procedure and Matlab codes, see Nielsen and Popiel (2018).

3. EMPIRICAL ANALYSIS

3.1 Nature of Data, Sources and Description

The time series data used for the analysis are the high and low prices of four cryptocurrencies, obtained from ForexTime (FXTM) Global Ltd trading platform (www.forextime.com). These datasets were collected and analysed at the following time frequencies: 1 minute (M1), 5 minutes (M5), 15 minutes (M15), 30 minutes (M30), one hour (H1), four hours (H4), daily (D1), weekly (W) and monthly (MN). The sampled cryptocurrencies are: Bitcoin (BTCUSD), Ethereum (ETHUSD), Litecoin (LTCUSD) and Ripple (RPLUSD), which were all priced in US dollars. Those intraday prices were synchronized minutely (M1, M5, M15 and M30) and hourly (H1 and H4), with both start and stop dates/times given, while for daily (D1), weekly (W) and monthly (MN) synchronized prices, the mid-night time (00:00) are recorded as opening times. Table 1 presents the data description, sample start and stop dates, as well as the sample sizes. In all, the high-frequency datasets have very large samples, while monthly frequency datasets have the smallest sample sizes, particularly, the case of RPLUSD with the sample size of 22.

PUT TABLE 1 AROUND HERE

3.2 Empirical Results and discussion

We first considered unit root testing of cryptocurrency prices, since in theory, it is expected that prices are non-stationary. We applied only ADF unit root test, with the three regression specifications: no intercept, constant only (C) and constant with the trend (C&T) for robustness. Results, presented in Table 2, showed that except for Ethereum (H4) for high price (H) and low price (L) series, Litecoin (D1) for only high price (H), Ripple (WK) for high price (H) and Ripple (MN) for low price (L) that are detected by the unit root tests to be stationary, all other series, with different time frequencies are non-stationary. First differences of these series sternly rejected the hypothesis of a unit root in all, for both high and low price series implying that cryptocurrency prices are I(d = 1) series. By computing the range and carrying out unit root test on it, we found rejections of unit root hypotheses in the range except in the cases for Ethereum (D1, WK & MN). Range series may be non-stationary (Cheung, 2007; Barunik and Dvorakova, 2015 and Yaya and Gil-Alana, 2018) as in the case of Ethereum. Evidences of nonstationarity in Ethereum price series and stationarity in few cryptocurrencyprices are enough justification for alternative unit root tests that would be robust to aberrant observations, outliers and occasional jumps.

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The ADF unit root test plays between I(d=1)/I(d=0) hypotheses, which is too restrictive, as some *d*s might have been over or under differenced during the computation of the ADF unit root regression, leading to failure of the test to detect unit root correctly. This weakness was noted in Diebold and Rudebusch (1991) and Hassler and Wolters (1994) and since then, fractional alternatives to unit root testing are gaining popularities. Earlier discussed in the methodology are the two semi-parametric estimation approaches to fractional integration parameter (the ELW and GPH). These are presented for bandwidths of two periodogram ordinates, $T^{0.5}$ and $T^{0.6}$ (Tdenotes the sample size), for high, low and range prices. The results for ELW estimates are given in Table 3, while those of GPH estimates are given in Table 4. The essence of $T^{0.5}$ and $T^{0.6}$ is to obtain two comparable estimates; although, other periodogram ordinates were considered, their results varied widely from those reported in this paper. In Table 3, we have the results based on ELW estimates for d_H, d_L and d_R, as well as the standard error of d_R [that is, $s.e.(d_R)$]. For the two ordinates, estimates of d_H and d_L are non-stationary $I(1)$ in most cases except in the case of Ripple (WK, MN), where these estimates are non-stationary but mean-reverting. Estimates of d_R are all significant, with quite low standard errors. These estimates are found in both stationary $(0 < d < 0.5)$ and non-stationary $(0.5 < d_R < 1)$ ranges. The fact that each d_R estimate is smaller than any of d_H and d_L estimates implies cointegration of intraday high and low cryptocurrency prices. The persistence in price series of the four cryptocurrencies varies substantially across time series frequencies, with no regular pattern observed from high-frequency to low-frequency series. The results of the fractional unit root, based on GPH method are given in Table 4, indicating similar results as with those presented in Table 3. Here, the range series are also found to be stationary for some time frequencies and non-stationary for other time frequencies. In cases where these are significant, these values are less than d_H and d_L, implying possible cointegration of high and low cryptocurrency prices.

PUT TABLE 3 AROUND HERE

PUT TABLE 4 AROUND HERE

Range series, being a stationary series is expected to possess long memory, then we conducted Qu (2011) test of spurious long memory test on the range series. The test is based on Robinson (1995b) ELW estimation. In the testing framework, the null of stationary long memory for range series was tested against the alternative of range series having regime changes or smoothly varying trends. Results obtained are given in Table 5 across all cryptocurrencies and different time frequencies. For periodogram points $T^{0.5}$, we found more rejections of null of long memory, that is, evidences of regime changing or smoothly varying trends in the estimates of long memory, even though some of the estimates are quite above 0.5. Based on periodogram points $T^{0.6}$, fewer evidences of spurious long memory were detected.

PUT TABLE 5 AROUND HERE

We also carried out test of homogeneity of fractional orders on the paired series for cointegration, that is, testing the null, $H_0: d_H = d_L$. This test isgiven in Robinson and Yajima (2002), as a semi-parametric technique.[1] In Table 6, the results are given for the two periodogram points $T^{0.5}$ and $T^{0.6}$, and for each of these periodogram numbers, the test indicated significant evidence of equality of fractional orders at 5% level.

PUT TABLE 6 AROUND HERE

The results of test of fractional cointegration by Johansen and Nielsen (2012) is tabulated in Table 7. This test is based on a rank test for unrestricted constant model. A grid-search was first conducted for the optimal k value for the cointegration rank test, using minimum information criteria, having set $k = 4$ in each case. For each optimal k value that was chosen, an appropriate rank is obtained from the cointegration rank test. In all the cases considered, the null hypotheses of rank zero were rejected, based on the Likelihood Ratio (LR) test statistics, while null hypotheses of rank 1 could not be further rejected against rank 2, since LR tests are not available for paired series. The results of the cointegration rank test actually confirmed the presence of fractional cointegration between the paired series. Thus, only one cointegrating vector is included in the estimated FCVAR model. The values obtained for d and b in the cointegration rank test in Table 7 were used in the estimation of the FCVAR model. We considered two FCVAR model specifications: the unrestricted and restricted constant models. The unrestricted constant case results is presented in Table 8,

[1] A more recent application is found in Yaya, Gil-Alana and Olubusoye (2017), Yaya and Gil-Alana (2018) and Caporale and Gil-Alana (2019).

while results for the restricted constant case is given in Table 9. In each result table, the third column presents estimates of d, that is, the joint fractional order of the bivariate series (high and low price series); the fourth column is the cointegrating factor b, while the fifth column shows the coefficients of long-run equilibria. The FCVAR model is estimated for $d \neq b$, while for $d = b$, the FCVAR model reduces to CVAR model (Johansen, 1995), such that the range order of integration is zero ($I(d-b)=0$), which is not the case, since the results of ELW and GPH in Tables 3 and 4 imply the rejection of CVAR specifications, given that the range series are not $I(0)$ series. In the unrestricted case in Table 8, the estimates of the cointegration vector reported in the fifth column are very close to the vector $(1, -1, c)$, where "c" is the constant. This implies an imposition of a $(1, -1)$ restriction for the cointegrating vector without a constant, which is the range. For the restricted case $(1, -1)$, presented in Table 9, the results are found not differing from those relating to the unrestricted constant case in Table 8. In each case in Tables 8 and 9, we observed quite strong cointegration, as estimates of*b*are quite above 0.5 in almost all the cases, implying strong cointegration, while estimates of d are around unity.[2] In the case of Ripple (MN), cointegration was not found, as there was no convergence of the initial values of d and b during estimation. Expectedly, the fractional integration results in the third and fourth tables, and cointegration rank in Table 7 have earlier signalled such possibility of no cointegration in the pair for Ripple (MN), possibly due to the small sample size.

PUT TABLE 7 AROUND HERE

PUT TABLE 8 AROUND HERE

PUT TABLE 9 AROUND HERE

[2] Weak cointegration is when $0 < b < 0.5$, while $0.5 < b < d$ indicates strong cointegration (Nielsen and Popiel, 2018). Johansen and Nielsen (2012) provides the asymptotic distributions of each case.

So far, cointegration has been established based on the results of fractional integration techniques (ELW and GPH), with reports presented in Tables 3 and 4 for high, low and range prices. The fractional persistence estimates for price range are presented in Table 10 for ELW and GPH estimates. These are quite around the long memory range, above 0.5 in a number of cases. This same table also reports corresponding estimates of fractional cointegration order, $\hat{d}-\hat{b}$, reported in Table 8 for the unrestricted constant case and Table 9 for the restricted constant case. These estimates are quite lower than those reported by ELW and GPH estimators.

PUT TABLE 10 AROUND HERE

Due to high frequency datasets used in this work, we investigated further the constancy of the cointegration relation between high and low cryptocurrency prices over time by testingthe cointegrating series (the range) for stationary three regime Self-Exciting Threshold Autoregression (SETAR). The results obtained, though not reported showed support for time varying cointegration for high and low cryptocurrency prices at all time frequencies except at monthly frequency where cointegration is constant. The time variation is actually expected due to highly volatile long time series involved. The FCVAR framework is built on constancy of cointegration as we applied it in this work.

4. Concluding remarks

This work contributes to the literature on modelling of cryptocurrency prices, having considered Bitcoin, Ethereum, Litecoin and Ripple at different high time frequencies. For each currency, high and low prices, and their difference - the range, were considered. The range is considered an efficient and robust estimator of volatility, and is the resulting equilibrium series in a cointegrating system. Also, the range-based volatility measurement relies on high/low price movements, rather than close/open price variations often applied in financial time series modelling. Lastly, the persistence of ranges is allowed to fall into a non-stationary range, unlike log-returns series that is stationary and unpredictable.These make the new approach appealing in financial econometrics.

Various time frequencies in minutes, hours, days, weeks and monthsare considered for each cryptocurrency price series. We find that the range-based volatility estimated with the FCVAR framework as an error correction mechanism is stationary and long-range dependent in most of the cases in the four cryptocurrencies, while based on fractional integration techniques, the range-based volatility is a mix of stationary and non-stationary series. These results are insensitive to different time frequencies. Though, our results showed evidences to support time varying cointegration in high frequency datasets, extension of FCVAR framework is yet to be develop to handle this extension.

The fact that range series are either stationary or non-stationary persistent allows the predictability of the variance to be embedded in a mean model of high and low asset cryptocurrency prices. One can easily obtain forecasts of future extreme prices, based on past values of cryptocurrency. The evidence contained in this paper is quite relevant and serves as a guide to traders, since many trading strategies employed are based on daily ranges.

One of the economic implications of the foregoing findings is that traders, investors and policy analysts can forecast future extreme prices of the analysed crypto-currencies, using their past values to aid their decision making. Besides, cointegration among these crypto-currencies may imply limited arbitrage opportunities for investors or traders across their markets. Also, participants should realise that, although efficiency of the markets seems unstable (as shown by unstable stationarity results) there is long-run relationship among the crypto-currencies analysed (as shown by robust cointegration results). Thus, hedge funds strategies involving portfolio mix with crypto-currencies should recognise their identified properties, especially their long-run behaviours and relationships. Portfolio allocation and diversification strategies may not combine assets which reflect short-term behaviour with crypto-currencies which exhibit long-term behaviour and relationship. This is based on the idea that mis-pricing and over-hedging can occur in the absence of cointegration

The data that support the findings of this study are openly available at ForexTime (FXTM) Global Ltd trading platform (www.forextime.com).

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Table 1: Data Description and Sample

Commodity	Frequency	Sample range		Sample size
		Start Date	Stop Date	T
Bitcoin [BTCUSD]	1min (M1)	2018.08.18 - 05:03	2019.11.14 - 12:45	41,860
	5 mins (M5)	2018.10.17 - 08:30	2019.11.14 - 12.45	17,398
	15 mins (M15)	2018.09.28 - 11:45	2019.11.14 - 12:45	6,836
	30 mins (M30)	2018.08.31 - 00:30	2019.11.14 - 12:30	7,751
	1 hour (H1)	2018.04.17 - 04:00	2019.11.14 - 12:00	8,290
	4 hour (H4)	2017.05.24 - 20:00	2019.11.14 - 12:00	3,996
	Daily (D1)	2012.08.11 - 00:00	2019.11.14 - 00:00	2,415
	Weekly (W)	2010.07.11 - 00:00	2019.11.10 - 00:00	484
	Monthly (MN)	2010.07.01 - 00:00	2019.11.01 - 00:00	111
Ethereum [ETHUSD]	1min (M1)	2019.11.12 - 07:05	2019.11.14 - 12:47	2,048
	5 mins (M5)	2019.11.04 - 21:40	2019.11.14 - 12:50	2,049
	15 mins (M15)	2019.10.16 - 00:15	2019.11.14 - 12:45	2,048
	30 mins (M30)	2019.09.16 - 20:00	2019.11.14 - 12:30	2,048
	1 hour (H1)	2018.07.03 - 13:00	2019.11.14 - 12:00	6,940
	4 hour (H4)	2018.07.23 - 00:00	2019.11.14 - 12:00	2,048
	Daily (D1)	2016.05.02 - 00:00	2019.11.14 - 00:00	1,056
	Weekly (W)	2016.05.01 - 00:00	2019.11.10 - 00:00	183
	Monthly (MN)	2016.05.01 - 00:00	2019.11.01 - 00:00	43
Litecoin [LTCUSD]	1min (M1)	2019.11.12 - 22:03	2019.11.14 - 12:56	2,050
	5 mins (M5)	2019.11.05 - 07:45	2019.11.14 - 12:55	2,048
	15 mins (M15)	2019.10.16 - 03:15	2019.11.14 - 12:45	2,048
	30 mins (M30)	2019.09.16 - 19:30	2019.11.14 - 12:30	2,048
	1 hour (H1)	2018.07.03 - 14:00	2019.11.14 - 12:00	6,941
	4 hour (H4)	2018.07.23 - 00:00	2019.11.14 - 12:00	2,048
	Daily (D1)	2012.08.18 - 00:00	2019.11.14 - 00:00	2,409
	Weekly (W)	2011.10.23 - 00:00	2019.11.10 - 00:00	408
	Monthly (MN)	2011.10.01 - 00:00	2019.11.01 - 00:00	95
Ripple [RPLUSD]	1min (M1)	2019.11.12 - 23:05	2019.11.14 - 13:02	2,051
	5 mins (M5)	2019.11.08 - 18:40	2019.11.14 - 13:00	1,025
	15 mins (M15)	2019.10.15 - 01:45	2019.11.14 - 13:00	2,048
	30 mins (M30)	2019.09.13 - 01:30	2019.11.14 - 13:00	2,048
	1 hour (H1)	2018.06.27 - 20:00	2019.11.14 - 13:00	4,294
	4 hour (H4)	2018.02.08 - 04:00	2019.11.14 - 12:00	2,747
	Daily (D1)	2018.02.08 - 00:00	2019.11.14 - 00:00	459
	Weekly (W)	2018.02.04 - 00:00	2019.11.10 - 00:00	93
	Monthly (MN)	2018.02.01 - 00:00	2019.11.01 - 00:00	22

Note: M1, M5, M15 and M30 denote intraday currency prices at every 1, 5, 15 and 30 minutes, respectively; H1 and H4 indicate prices at every 1 and 4 hours, respectively; while D1, WK and MN represent daily, weekly and monthly currency prices, respectively.

Table 2: ADF test for level series, first differences and range of high and low cryptocurrency prices

Crypto.	Frequency	High Price Series						Low Price Series						Range Price Series		
		Level			*First Difference*			*Level*			*First Difference*			*Level*		
		None	**C**	**C & T**	**None**	**C**	**C & T**	**None**	**C**	**C & T**	**None**	**C**	**C & T**	**None**	**C**	**C & T**
Bitcoin	M1	0.4404	-1.0340	-1.5137	**-200.806**	**-200.806**	**-200.804**	0.4337	-1.0428	-1.5266	**-199.893**	**-199.893**	**-199.890**	**-11.262**	**-15.686**	**-15.827**
	M5	0.1230	-0.7674	-2.3035	**-130.155**	**-130.153**	**-130.160**	0.1115	-0.7884	-2.3138	**-131.937**	**-131.935**	**-131.942**	**-11.699**	**-16.704**	**-18.018**
	M15	0.1168	-1.0677	-1.8292	**-82.238**	**-82.234**	**-82.234**	0.1318	-1.0446	-1.8108	**-82.505**	**-82.501**	**-82.501**	**-7.353**	**-20.973**	**-21.771**
	M30	-0.0448	-1.1758	-1.9178	**-86.247**	**-86.243**	**-86.243**	-0.0551	-1.2056	-1.9458	**-88.206**	**-88.201**	**-88.202**	**-6.787**	**-13.742**	**-14.722**
	H1	-0.4090	-1.5371	-1.7538	**-89.514**	**-89.508**	**-89.507**	-0.4264	-1.5999	-1.8172	**-89.963**	**-89.958**	**-89.957**	**-3.666**	**-6.384**	**-6.492**
	H4	-0.2624	-1.8858	-1.8317	**-61.786**	**-61.783**	**-61.779**	-0.3991	-2.1263	-2.1144	**-63.548**	**-63.545**	**-63.539**	**-3.078**	**-4.299**	**-4.325**
	D1	-0.9373	-1.5483	-2.8154	**-10.974**	**-10.992**	**-11.002**	-0.3633	-1.0051	-2.3211	**-11.546**	**-11.576**	**-11.599**	**-2.714**	**-3.300**	**-3.872**
	WK	-0.8415	-1.3994	-2.6645	**-8.788**	**-8.809**	**-8.822**	-0.1779	-0.7828	-2.1589	**-25.309**	**-25.322**	**-25.333**	**-3.193**	**-3.591**	**-4.822**
	MN	-1.5632	-2.1307	**-3.5182**	**-7.556**	**-7.561**	**-7.552**	0.4203	-0.2395	-1.7938	**-9.004**	**-9.067**	**-9.140**	**-2.232**	-2.640	**-3.906**
Etherum	M1	-0.1773	-2.1153	-2.0536	**-27.638**	**-27.632**	**-27.634**	-0.1617	-2.2547	-2.2471	**-27.956**	**-27.950**	**-27.949**	**-4.864**	**-19.087**	**-19.085**
	M5	-0.1515	-2.7491	-3.1786	**-40.411**	**-40.401**	**-40.396**	-0.1562	-2.7863	-3.1607	**-43.819**	**-43.809**	**-43.801**	**-4.116**	**-17.691**	**-18.522**
	M15	0.1410	-1.6407	-2.3250	**-43.678**	**-43.668**	**-43.658**	0.1239	-1.7441	-2.5064	**-47.045**	**-47.034**	**-47.023**	**-3.672**	**-11.800**	**-11.863**
	M30	-0.2729	-2.0222	-2.0034	**-25.160**	**-25.154**	**-25.150**	-0.2687	-2.1607	-2.1415	**-44.123**	**-44.113**	**-44.104**	**-4.629**	**-14.756**	**-20.692**
	H1	**-2.2086**	-2.7775	-2.3001	**-60.332**	**-60.349**	**-60.376**	**-2.1601**	-2.8052	-2.3602	**-60.442**	**-60.457**	**-60.482**	**-3.702**	**-6.792**	**-7.031**
	H4	**-2.5609**	**-3.7181**	-3.3568	**-44.058**	**-44.088**	**-44.173**	**-2.4156**	**-3.6528**	-3.3240	**-45.534**	**-45.559**	**-45.636**	**-3.301**	**-8.360**	**-8.466**
	D1	-1.6492	-2.4020	-2.4359	**-6.437**	**-6.436**	**-6.446**	-1.4918	-2.2331	-2.2546	**-7.100**	**-7.099**	**-7.108**	**-2.102**	-2.617	-2.600
	WK	-1.1067	-1.7426	-1.6236	**-13.090**	**-13.056**	**-13.051**	-1.0544	-1.7706	-1.6590	**-13.025**	**-12.993**	**-12.983**	**-1.967**	-2.492	-2.449
	MN	-1.3355	-2.1036	-1.9799	**-4.560**	**-4.506**	**-4.511**	-1.0800	-1.9086	-1.8078	**-6.304**	**-6.236**	**-5.950**	-1.704	-2.319	-2.212
Litecoin	M1	-1.3992	0.2680	-1.1886	**-43.095**	**-43.128**	**-43.168**	-1.2365	-0.0240	-1.4965	**-46.186**	**-46.211**	**-46.241**	**-4.213**	**-12.677**	**-13.171**
	M5	-0.5860	-1.8865	-3.2207	**-41.944**	**-41.940**	**-41.944**	-0.5560	-2.0709	-3.3116	**-45.035**	**-45.030**	**-45.030**	**-3.555**	**-13.242**	**-13.781**
	M15	0.2464	-1.3682	-1.9185	**-42.806**	**-42.798**	**-42.789**	0.2250	-1.4901	-2.1400	**-45.220**	**-45.211**	**-45.201**	**-3.257**	**-12.612**	**-12.648**
	M30	-0.8323	-1.9422	-1.7191	**-40.936**	**-40.934**	**-40.945**	-0.8148	-2.0500	-1.8448	**-33.729**	**-33.729**	**-33.742**	**-4.741**	**-14.813**	**-20.268**
	H1	-0.8335	-1.6201	-1.6457	**-60.444**	**-60.441**	**-60.437**	-0.8308	-1.6393	-1.6677	**-64.442**	**-64.439**	**-64.435**	**-3.142**	**-5.817**	**-5.859**
	H4	-0.8500	-1.4470	-1.6700	**-44.939**	**-44.931**	**-44.921**	-0.8577	-1.5295	-1.7795	**-48.579**	**-48.569**	**-48.559**	**-4.216**	**-7.429**	**-7.675**
	D1	**-2.3642**	**-2.8718**	**-3.5909**	**-18.461**	**-18.459**	**-18.455**	-1.6309	-2.1098	-2.7502	**-12.853**	**-12.855**	**-12.852**	**-4.636**	**-5.112**	**-5.636**
	WK	**-2.1155**	-2.6127	-3.2678	**-15.961**	**-15.944**	**-15.924**	-1.5396	-2.0521	-2.7817	**-6.652**	**-6.652**	**-6.644**	**-3.994**	**-4.437**	**-4.850**
	MN	**-2.0353**	-2.5068	-3.0508	**-10.185**	**-10.134**	**-10.080**	-0.9142	-1.4770	-2.2829	**-8.774**	**-8.753**	**-8.708**	**-3.850**	**-4.303**	**-4.710**
Ripple	M1	-0.9759	-0.2107	-1.4111	**-46.333**	**-46.343**	**-46.386**	-0.8479	-0.4871	-1.7126	**-37.624**	**-37.631**	**-37.668**	**-3.457**	**-11.005**	**-14.274**
	M5	-0.7856	-0.9328	-2.6645	**-29.218**	**-29.222**	**-29.250**	-0.6664	-1.4643	-3.1437	**-25.711**	**-25.712**	**-25.748**	**-3.271**	**-11.682**	**-11.708**
	M15	-0.7102	-1.6936	-1.9810	**-33.686**	**-33.689**	**-33.690**	-0.6397	-2.1380	-2.3876	**-32.593**	**-32.592**	**-32.590**	**-3.755**	**-23.173**	**-23.343**
	M30	-0.0252	-2.2800	-2.1519	**-41.820**	**-41.811**	**-41.814**	-0.0013	-2.1981	-2.0782	**-24.848**	**-24.843**	**-24.852**	**-3.266**	**-11.384**	**-21.339**
	H1	-1.0422	-2.3070	-2.9908	**-13.739**	**-13.747**	**-13.745**	-1.1291	-1.8021	-2.1935	**-50.510**	**-50.512**	**-50.507**	**-5.222**	**-8.377**	**-9.667**
	H4	-1.5127	-2.0884	-2.7384	**-52.477**	**-52.478**	**-52.471**	-1.5220	-2.0700	-2.6600	**-51.444**	**-51.446**	**-51.439**	**-4.467**	**-10.062**	**-11.991**
	D1	-1.5230	-2.1814	-2.9014	**-22.084**	**-22.099**	**-22.098**	-1.5366	-2.1357	-2.7393	**-20.554**	**-20.557**	**-20.545**	**-4.183**	**-6.510**	**-7.712**
	WK	**-2.3504**	**-3.1108**	-3.3930	**-11.770**	**-11.857**	**-11.929**	-1.6600	-2.3116	-2.8196	**-6.249**	**-6.318**	**-6.396**	**-2.438**	**-6.360**	**-8.105**
	MN	**-2.4533**	-2.6343	-2.7906	**-4.682**	**-5.049**	**-5.370**	**-2.2290**	**-3.7541**	**-4.2256**	**-6.341**	**-6.303**	**-6.167**	-1.503	-2.805	**-4.269**

Note: The figures are t-statistics of the ADF test, with bold, denoting statistical significance at both 1% and 5% levels. “C” denotes constant, “T” denotes time trend in the unit root test regression model. M1, M5, M15 and M30 denote intraday currency prices at every 1, 5, 15 and 30 minutes, respectively; H1 and H4 indicate prices at every 1 and 4 hours, respectively; while D1, WK and MN represent daily, weekly and monthly currency prices, respectively.

Table 3: Estimates of Exact Local Whittle (ELW) of fractional integration parameter d for high (d_H), low prices (d_L) and their differences, the range (d_R) for two bandwidths m for the number of periodogram ordinates ($m = T^{0.5}$ and $m = T^{0.6}$)

		Bandwidths			**ELW$_{m=T^{0.5}}$**				**ELW$_{m=T^{0.6}}$**			
Crypto.	**Series Frequency**	**T**	**$T^{0.5}$**	**$T^{0.6}$**	**d_H**	**d_L**	**d_R**	**s.e.(d_R)**	**d_H**	**d_L**	**d_R**	**s.e.(d_R)**
Bitcoin	M1	41860	204	593	1.029	1.029	**0.380**	0.035	1.009	1.008	**0.421**	0.021
	M5	17398	131	350	1.005	1.004	**0.458**	0.044	1.008	1.008	**0.422**	0.027
	M15	6836	82	199	1.002	1.003	**0.438**	0.055	1.009	1.010	**0.365**	0.035
	M30	7751	88	215	0.983	0.981	**0.539**	0.053	0.997	0.998	**0.381**	0.034
	H1	8290	91	224	1.028	1.025	**0.786**	0.052	1.024	1.021	**0.669**	0.033
	H4	3996	63	144	1.044	1.005	**0.669**	0.063	1.033	1.004	**0.553**	0.042
	D1	2415	49	107	0.947	0.976	**0.642**	0.071	0.916	0.917	**0.853**	0.048
	WK	484	22	40	0.931	1.015	**0.601**	0.107	0.991	1.047	**0.761**	0.079
	MN	111	10	16	0.709	0.707	**0.586**	0.158	0.815	0.899	**0.586**	0.125
Ethereum	M1	2,048	45	97	0.892	0.895	**0.198**	0.075	0.920	0.892	**0.246**	0.051
	M5	2,049	45	97	1.019	1.019	**0.630**	0.075	1.008	1.009	**0.519**	0.051
	M15	2,048	45	97	1.121	1.108	**0.337**	0.075	1.034	1.059	**0.321**	0.051
	M30	2,048	45	97	1.121	1.124	**0.258**	0.075	1.056	1.002	**0.314**	0.051
	H1	6,940	83	201	1.007	1.007	**0.633**	0.055	1.009	1.007	**0.613**	0.035
	H4	2,048	45	97	0.934	0.938	**0.471**	0.075	1.023	1.023	**0.527**	0.051
	D1	1,056	32	65	0.840	0.868	**0.591**	0.088	1.007	0.963	**0.945**	0.062
	WK	183	13	22	0.848	0.922	**0.636**	0.139	1.007	0.883	**0.947**	0.107
	MN	43	6	9	1.013	1.278	**0.691**	0.204	0.995	0.947	**0.955**	0.167
Litecoin	M1	2,050	45	97	1.016	1.017	**0.625**	0.075	1.010	1.010	**0.535**	0.051
	M5	2,048	45	97	1.023	1.011	**0.489**	0.075	0.959	0.943	**0.384**	0.051
	M15	2,048	45	97	1.137	1.112	**0.350**	0.075	1.069	1.099	**0.328**	0.051
	M30	2,048	45	97	1.044	1.057	**0.270**	0.075	1.021	0.986	**0.284**	0.051
	H1	6,941	83	201	0.983	0.983	**0.562**	0.055	0.999	0.996	**0.590**	0.035
	H4	2,048	45	97	0.939	0.946	**0.470**	0.075	1.011	1.015	**0.490**	0.051
	D1	2,409	49	106	0.890	0.941	**0.589**	0.071	0.746	0.756	**0.580**	0.049
	WK	408	20	36	0.945	1.138	**0.592**	0.112	0.882	1.019	**0.623**	0.083
	MN	95	9	15	0.760	0.832	**0.537**	0.167	0.888	1.225	**0.552**	0.129
Ripple	M1	2,051	45	97	1.016	1.016	**0.664**	0.075	1.009	1.009	**0.603**	0.051
	M5	1,025	32	64	1.051	1.050	**0.768**	0.088	1.023	1.023	**0.589**	0.063
	M15	2,048	45	97	0.900	0.886	**0.235**	0.075	0.929	0.934	**0.227**	0.051
	M30	2,048	45	97	1.064	1.067	**0.247**	0.075	1.045	1.007	**0.269**	0.051
	H1	4,294	65	151	1.040	1.045	**0.586**	0.062	0.959	0.970	**0.439**	0.041
	H4	2,747	52	115	0.918	0.921	**0.634**	0.069	1.105	1.116	**0.559**	0.047
	D1	459	21	39	0.745	0.754	**0.447**	0.109	0.912	0.929	**0.436**	0.080
	WK	93	9	15	0.590	0.527	**0.551**	0.167	0.697	0.738	**0.472**	0.129
	MN	22	4	6	0.411	0.814	0.064	0.250	0.619	0.634	0.327	0.204

Note, s.e.(d_R) is the standard error of d_R. In bold, evidence of significant long-range dependency, that is $0<d_R<1$ with $d_R<\min(d_H, d_L)$ for each result in the periodogram points $T^{0.5}$ and $T^{0.6}$ for each corresponding series.

Table 4: Estimates of Geweke and Porter-Hudak (GPH) of fractional integration parameter d for high (d_H), low prices (d_L) and their differences, the range (d_R) for two bandwidths m for the number of periodogram ordinates ($m = T^{0.5}$ and $m = T^{0.6}$)

		Bandwidths			$GPH_{m=T^{0.5}}$				$GPH_{m=T^{0.6}}$			
Crypto.	**Series Frequency**	**T**	$T^{0.5}$	$T^{0.6}$	d_H	d_L	d_R	s.e.(d_R)	d_H	d_L	d_R	s.e.(d_R)
Bitcoin	M1	41,860	204	593	1.077	1.078	**0.391**	0.047	1.021	1.020	**0.448**	0.027
	M5	17,398	131	350	1.040	1.040	**0.463**	0.060	1.059	1.058	**0.395**	0.035
	M15	6,836	82	199	0.943	0.942	**0.424**	0.078	0.936	0.939	**0.336**	0.048
	M30	7,751	88	215	0.988	0.988	**0.509**	0.075	1.008	1.006	**0.374**	0.046
	H1	8,290	91	224	1.059	1.058	**0.819**	0.073	1.045	1.043	**0.674**	0.045
	H4	3,996	63	144	0.978	0.939	**0.770**	0.090	1.012	0.983	**0.531**	0.057
	D1	2,415	49	107	0.923	0.971	**0.619**	0.104	0.927	0.929	**0.854**	0.067
	WK	484	22	40	0.905	1.016	**0.602**	0.170	0.970	1.022	**0.747**	0.118
	MN	111	10	16	0.779	0.684	**0.725**	0.293	0.867	0.947	**0.670**	0.212
Ethereum	M1	2,048	45	97	0.847	0.867	0.140	0.110	0.972	0.940	**0.188**	0.071
	M5	2,049	45	97	0.965	0.964	**0.676**	0.110	0.850	0.854	**0.508**	0.071
	M15	2,048	45	97	1.130	1.135	**0.273**	0.110	1.030	1.080	**0.275**	0.071
	M30	2,048	45	97	1.152	1.148	**0.387**	0.110	1.099	1.044	**0.364**	0.071
	H1	6,940	83	201	0.928	0.925	**0.579**	0.077	0.987	0.988	**0.611**	0.048
	H4	2,048	45	97	0.977	0.967	**0.458**	0.110	1.052	1.044	**0.536**	0.071
	D1	1,056	32	65	0.896	0.921	**0.638**	0.135	1.032	0.983	**0.976**	0.089
	WK	183	13	22	0.829	0.874	**0.639**	0.243	1.062	0.928	**1.021**	0.172
	MN	43	6	9	1.079	1.603	0.743	0.440	1.072	1.026	**1.169**	0.325
Litecoin	M1	2,050	45	97	0.884	0.890	**0.563**	0.110	0.747	0.749	**0.473**	0.071
	M5	2,048	45	97	1.071	1.064	**0.541**	0.110	0.945	0.938	**0.431**	0.071
	M15	2,048	45	97	1.175	1.154	**0.393**	0.110	1.090	1.128	**0.358**	0.071
	M30	2,048	45	97	1.140	1.130	**0.399**	0.110	1.102	1.052	**0.337**	0.071
	H1	6,941	83	201	0.997	0.996	**0.519**	0.077	1.045	1.048	**0.548**	0.048
	H4	2,048	45	97	1.046	1.054	**0.452**	0.110	1.064	1.088	**0.441**	0.071
	D1	2,409	49	106	0.867	0.924	**0.580**	0.104	0.726	0.739	**0.569**	0.067
	WK	408	20	36	0.887	1.064	**0.573**	0.181	0.844	0.983	**0.611**	0.126
	MN	95	9	15	0.944	1.043	0.575	0.317	0.882	1.172	**0.549**	0.222
Ripple	M1	2,051	45	97	0.708	0.716	**0.573**	0.112	0.630	0.635	**0.471**	0.079
	M5	1,025	32	64	0.990	0.983	**0.569**	0.142	0.772	0.778	**0.470**	0.091
	M15	2,048	45	97	1.000	0.924	0.176	0.110	0.966	0.951	**0.168**	0.071
	M30	2,048	45	97	1.161	1.138	**0.364**	0.110	1.106	1.049	**0.309**	0.071
	H1	4,294	65	151	1.043	1.049	**0.593**	0.089	0.968	0.963	**0.432**	0.055
	H4	2,747	52	115	0.887	0.885	**0.694**	0.101	1.087	1.081	**0.602**	0.064
	D1	459	21	39	0.781	0.803	0.339	0.176	0.891	0.886	**0.366**	0.120
	WK	93	9	15	0.623	0.572	0.616	0.317	0.814	0.927	**0.566**	0.223
	MN	22	4	6	0.516	0.926	0.032	0.638	0.732	0.719	0.249	0.462

Note, s.e.(d_R) is the standard error of d_R. In bold, evidence of significant long-range dependency, that is $0<d_R<1$ with $d_R<$ min (d_H, d_L) for each result in the periodogram points $T^{0.5}$ and $T^{0.6}$ for each corresponding series.

Table 5: Test of Spurious Long memory in Range series

		Bandwidths			Qu test $_{m = T^{0.5}}$	Qu test $_{m = T^{0.6}}$
Crypto.	**Series Frequency**	**T**	**$T^{0.5}$**	**$T^{0.6}$**	**d_{R1}**	**d_{R2}**
Bitcoin	M1	41,860	204	593	0.6468	0.7306
	M5	17,398	131	350	**1.2948**	1.1819
	M15	6,836	82	199	**1.6869**	1.1145
	M30	7,751	88	215	**1.2679**	**2.2212**
	H1	8,290	91	224	0.9904	**1.4940**
	H4	3,996	63	144	**1.4782**	**1.8498**
	D1	2,415	49	107	0.9912	0.7856
	WK	484	22	40	1.0160	0.5914
	MN	111	10	16	1.2806	0.4582
Ethereum	M1	2,048	45	97	**1.3407**	0.4485
	M5	2,049	45	97	1.0215	0.9492
	M15	2,048	45	97	1.1305	0.5798
	M30	2,048	45	97	**2.2462**	0.8292
	H1	6,940	83	201	0.7792	0.6051
	H4	2,048	45	97	1.1722	0.5077
	D1	1,056	32	65	1.0463	**2.2507**
	WK	183	13	22	1.0827	0.8120
	MN	43	6	9	0.9226	0.2739
Litecoin	M1	2,050	45	97	**1.4179**	0.5377
	M5	2,048	45	97	0.9809	0.7338
	M15	2,048	45	97	**1.3930**	0.6219
	M30	2,048	45	97	**2.7464**	**1.4619**
	H1	6,941	83	201	0.8631	0.8436
	H4	2,048	45	97	**2.0003**	0.7708
	D1	2,409	49	106	1.0351	0.8869
	WK	408	20	36	1.1162	0.7837
	MN	95	9	15	**1.2819**	0.5462
Ripple	M1	2,051	45	97	**1.3028**	0.7674
	M5	1,025	32	64	1.0996	0.5742
	M15	2,048	45	97	1.2401	0.4482
	M30	2,048	45	97	**1.8243**	0.8887
	H1	4,294	65	151	**1.7756**	0.7276
	H4	2,747	52	115	**1.5909**	**1.3013**
	D1	459	21	39	1.2451	0.8942
	WK	93	9	15	**1.4123**	0.7246
	MN	22	4	6	0.6437	0.3003

In bold, significant of Wald statistic at 5% level. For details of critical values, see Qu (2011).

Table 6: Test of Homogeneity of fractional integration orders

		Bandwidths			**Test statistic**	
Crypto.	**Series Frequency**	**T**	**$T^{0.5}$**	**$T^{0.6}$**	**$m = T^{0.5}$**	**$m = T^{0.6}$**
Bitcoin	M1	41,860	205	593	NC	NC
	M5	17,398	132	350	**0.4784**	**0.1473**
	M15	6,836	83	200	**0.0253**	**0.0101**
	M30	7,751	88	216	**0.0143**	**0.0891**
	H1	8,290	91	224	**0.2147**	**0.2253**
	H4	3,996	63	145	**0.8664**	**0.8801**
	D1	2,415	49	107	**0.1752**	**0.1868**
	WK	484	22	41	**0.6560**	**0.6849**
	MN	111	11	17	**0.2168**	**0.5004**
Ethereum	M1	2,048	45	97	**0.0367**	**0.5021**
	M5	2,049	45	97	**0.2170**	**0.2033**
	M15	2,048	45	97	**0.1647**	**0.4483**
	M30	2,048	45	97	**0.0736**	**0.9420**
	H1	6,940	83	202	**0.0023**	**0.0655**
	H4	2,048	45	97	**0.0796**	**0.0039**
	D1	1,056	32	65	**0.4214**	**0.3740**
	WK	183	14	23	**0.5227**	**0.7794**
	MN	43	7	10	**0.6217**	**0.4845**
Litecoin	M1	2,050	45	97	**0.0115**	**0.0466**
	M5	2,048	45	97	**0.1343**	**0.3781**
	M15	2,048	45	97	**0.2416**	**0.5579**
	M30	2,048	45	97	**0.1110**	**0.6203**
	H1	6,941	83	202	**0.0670**	**0.4060**
	H4	2,048	45	97	**0.1106**	**0.0890**
	D1	2,409	49	107	**0.4759**	**0.7761**
	WK	408	20	37	**1.2772**	**1.2915**
	MN	95	10	15	**0.6858**	**1.6300**
Ripple	M1	2,051	45	97	NC	NC
	M5	1,025	32	64	NC	NC
	M15	2,048	45	97	**0.3383**	**0.0614**
	M30	2,048	45	97	**0.1916**	**0.4522**
	H1	4,294	66	151	**0.3963**	**0.2617**
	H4	2,747	52	116	**0.1119**	**0.0724**
	D1	459	21	40	**0.0418**	**0.1479**
	WK	93	10	15	**0.1948**	**0.0018**
	MN	22	5	6	**0.0130**	**0.1335**

In bold, evidence of equal orders of integration of the pair, high and low series, at the 5% level.The critical value of the test at this level of significance is 1.9599. “NC” implies computation error of the periodogram used in the estimation for those time points.

Table 7: Fractional Cointegration Rank test by Johansen and Nielsen (2012) based on Unrestricted constant model

			r = 0			**r = 1**			**r = 2**	
Crypto.	**Series Frequency**	**k_{max} (4)**	**d**	**b**	**LR**	**d**	**b**	**LR**	**d**	**b**
Bitcoin	M1	4	0.800	0.169	63.661	**1.026**	**0.710**	**0.213**	1.027	0.709
	M5	3	0.800	0.100	82.045	**1.009**	**0.732**	**0.134**	1.013	0.734
	M15	2	0.800	0.100	96.358	**1.003**	**0.777**	**0.072**	1.004	0.778
	M30	2	0.800	0.100	114.172	**0.991**	**0.630**	**5.665**	0.866	0.100
	H1	0	0.852	0.500	3121.669	**1.023**	**0.745**	**0.706**	1.026	0.748
	H4	4	0.816	0.100	16.176	**1.038**	**0.682**	**0.988**	1.065	0.734
	D1	4	0.950	0.500	1032.090	**1.168**	**0.768**	**17.773**	1.200	0.797
	WK	4	0.800	0.900	33.997	**0.800**	**0.374**	**0.671**	0.800	0.391
	MN	0	0.955	0.5444	28.573	**1.147**	**0.355**	**4.822**	1.300	0.804
Ethereum	M1	0	0.845	0.500	577.170	**1.000**	**0.691**	**0.003**	1.000	0.690
	M5	3	0.800	0.100	25.552	**0.882**	**0.397**	**0.197**	0.870	0.413
	M15	0	0.844	0.500	652.748	**1.002**	**0.743**	**0.001**	1.003	0.743
	M30	0	0.858	0.500	580.470	**1.010**	**0.737**	**0.017**	1.010	0.738
	H1	0	0.845	0.500	2280.866	**1.011**	**0.708**	**1.489**	1.017	0.713
	H4	0	0.824	0.500	876.696	**1.018**	**0.611**	**2.482**	1.039	0.636
	D1	4	0.925	0.156	28.420	**0.880**	**0.756**	**3.044**	0.894	0.746
	WK	1	0.800	0.704	47.722	**0.800**	**0.798**	**4.201**	0.800	0.753
	MN	3	0.800	0.100	32.194	**1.200**	**0.900**	**1.574**	1.200	0.900
Litecoin	M1	0	0.839	0.500	683.469	**1.000**	**0.660**	**0.006**	1.000	0.661
	M5	0	0.837	0.500	691.124	**1.000**	**0.646**	**0.001**	1.000	0.646
	M15	0	0.844	0.500	640.221	**1.004**	**0.695**	**0.024**	1.004	0.695
	M30	0	0.864	0.500	533.475	**1.009**	**0.791**	**0.125**	1.011	0.794
	H1	0	0.852	0.500	2465.397	**1.029**	**0.745**	**1.598**	1.034	0.749
	H4	0	0.847	0.500	935.619	**1.047**	**0.692**	**1.988**	1.059	0.703
	D1	4	0.800	0.415	14.756	**0.800**	**0.472**	**0.543**	0.800	0.480
	WK	4	0.800	0.365	19.006	**0.970**	**0.697**	**0.502**	0.927	0.726
	MN	0	0.846	0.498	88.865	**1.024**	**0.549**	**3.501**	1.200	0.766
Ripple	M1	0	0.838	0.500	672.517	**1.000**	**0.648**	**0.000**	1.000	0.647
	M5	0	0.841	0.500	312.015	**0.999**	**0.657**	**0.000**	0.999	0.657
	M15	0	0.840	0.500	714.377	**1.002**	**0.803**	**0.069**	1.003	0.805
	M30	0	0.857	0.500	569.725	**1.005**	**0.796**	**0.077**	1.005	0.796
	H1	0	0.875	0.500	865.633	**1.020**	**0.735**	**1.125**	1.026	0.740
	H4	0	0.878	0.500	973.424	**1.051**	**0.717**	**2.038**	1.064	0.734
	D1	0	0.883	0.500	229.826	**1.096**	**0.900**	**5.245**	1.136	0.900
	WK	0	0.800	0.481	55.939	**1.070**	**0.900**	**4.344**	1.200	0.900
	MN	0	0.800	0.483	13.247	0.800	0.900	**0.137**	0.800	0.900

Note, maximum k is set at 4 and this gives the order of the error correction mechanism in the FCVAR system. The LR is the Likelihood Ratio statistics, computed for rank r = 0 and 1. This is not available for rank 2 since we are not rejecting any more rank after rank 1 indicated in bold, with LR test statistics insignificant at 5% level.

Table 8: FCVAR estimation results (no restriction)

Crypto.	Series Frequency	$\hat{d}$	$\hat{b}$	$\hat{\beta}$
Bitcoin	M1	1.026 (0.015)	0.710 (0.021)	[1, -1.001, 3.922]
	M5	1.009 (0.020)	0.732 (0.027)	[1, -1.001, -1.057]
	M15	1.003 (0.023)	0.777 (0.034)	[1, -1.003, 2.865]
	M30	0.991 (0.032)	0.630 (0.067)	[1, -1.005, 0.900]
	H1	1.023 (0.009)	0.745 (0.017)	[1, -1.012, 24.704]
	H4	1.038 (0.045)	0.682 (0.087)	[1, -1.055, 130.252]
	D1	1.168 (0.017)	0.768 (0.031)	[1, -1.124, 85.315]
	WK	0.800 (0.165)	0.374 (0.093)	[1, -0.993, -10.969]
	MN	1.300 (0.001)	0.803 (0.215)	[1, -1.783, 248.390]
Ethereum	M1	1.000 (0.017)	0.691 (0.045)	[1, -0.990,-1.892]
	M5	0.882 (0.079)	0.397 (0.068)	[1, -0.997, -0.689]
	M15	1.002 (0.017)	0.743 (0.036)	[1, -1.008, 0.772]
	M30	1.010 (0.017)	0.737 (0.037)	[1, -1.001, -1.073]
	H1	1.011 (0.010)	0.708 (0.019)	[1, -1.009, -0.430]
	H4	1.018 (0.018)	0.611 (0.039)	[1, -1.021, -0.683]
	D1	0.880 (0.104)	0.756 (0.087)	[1, -1.113, 3.963]
	WK	0.800 (0.124)	0.798 (0.113)	[1, -1.307, 12.864]
	MN	1.200 (0.213)	0.900 (0.179)	[1, -1.799, 12.130]
Litecoin	M1	1.000 (0.017)	0.660 (0.038)	[1, -0.998, -0.151]
	M5	1.000 (0.017)	0.646 (0.039)	[1, -0.993, -0.580]
	M15	1.004 (0.017)	0.695 (0.038)	[1, -1.015, 0.614]
	M30	1.009 (0.017)	0.791 (0.036)	[1, -1.012, 0.257]
	H1	1.029 (0.010)	0.745 (0.019)	[1, -1.018, 0.367]
	H4	1.047 (0.018)	0.692 (0.039)	[1, -1.036, 0.509]
	D1	0.800 (0.334)	0.472 (0.217)	[1, -1.092, 0.340]
	WK	0.970 (0.164)	0.697 (0.099)	[1, -1.267, 0.550]
	MN	1.024 (0.086)	0.549 (0.169)	[1, -1.908, 5.184]
Ripple	M1	1.000 (0.017)	0.648 (0.039)	[1, -1.000, 0.000]
	M5	0.999 (0.024)	0.657 (0.063)	[1, -0.989, -0.004]
	M15	1.002 (0.017)	0.803 (0.037)	[1, -1.024, 0.005]
	M30	1.005 (0.017)	0.796 (0.036)	[1, -1.022, 0.004]
	H1	1.020 (0.012)	0.735 (0.026)	[1, -1.031, 0.006]
	H4	1.051 (0.015)	0.717 (0.035)	[1, -1.045, 0.004]
	D1	1.096 (0.039)	0.900 (0.081)	[1, -1.112, 0.010]
	WK	1.070 (0.112)	0.900 (0.219)	[1, -1.268, 0.016]
	MN	**0.800 (0.181)**	**0.900 (0.000)**	**[1, -2.044, 0.157]**

In bold, evidence of no cointegration since b > d. In the third and fourth columns, standard errors of estimates are in parenthesis.

Table 9: FCVAR estimation results (With restriction)

Crypto.	Series Frequency	$\hat{d}$	$\hat{b}$	$\hat{\beta}$
Bitcoin	M1	1.024 (0.015)	0.710 (0.021)	[1, -1.001]
	M5	1.010 (0.020)	0.731 (0.027)	[1, -1.001]
	M15	1.003 (0.023)	0.777 (0.034)	[1, -1.003]
	M30	0.991 (0.032)	0.630 (0.067)	[1, -1.004]
	H1	1.023 (0.009)	0.740 (0.017)	[1, -1.009]
	H4	1.070 (0.053)	0.457 (0.128)	[1, -1.050]
	D1	1.168 (0.017)	0.760 (0.030)	[1, -1.123]
	WK	0.800 (0.165)	0.374 (0.094)	[1, -0.995]
	MN	1.300 (0.526)	0.685 (0.387)	[1, -1.789]
Ethereum	M1	1.000 (0.017)	0.684 (0.044)	[1, -1.001]
	M5	0.876 (0.088)	0.406 (0.075)	[1, -1.001]
	M15	1.002 (0.017)	0.745 (0.036)	[1, -1.003]
	M30	1.009 (0.017)	0.736 (0.038)	[1, -1.007]
	H1	1.011 (0.010)	0.707 (0.019)	[1, -1.010]
	H4	1.017 (0.018)	0.610 (0.039)	[1, -1.023]
	D1	0.948 (0.089)	0.694 (0.083)	[1, -1.108]
	WK	0.813 (0.111)	0.748 (0.121)	[1, -1.273]
	MN	1.200 (0.208)	0.900 (0.222)	[1, -1.774]
Litecoin	M1	1.000 (0.017)	0.658 (0.038)	[1, -1.001]
	M5	0.999 (0.017)	0.644 (0.039)	[1, -1.003]
	M15	1.003 (0.017)	0.699 (0.039)	[1, -1.005]
	M30	1.009 (0.017)	0.791 (0.036)	[1, -1.008]
	H1	1.029 (0.010)	0.741 (0.019)	[1, -1.014]
	H4	1.047 (0.018)	0.690 (0.039)	[1, -1.031]
	D1	0.800 (0.339)	0.471 (0.222)	[1, -1.090]
	WK	0.977 (0.169)	0.691 (0.102)	[1, -1.262]
	MN	1.022 (0.084)	0.506 (0.171)	[1, -1.876]
Ripple	M1	1.000 (0.017)	0.647 (0.039)	[1, -1.001]
	M5	0.999 (0.024)	0.648 (0.064)	[1, -1.002]
	M15	1.002 (0.017)	0.805 (0.037)	[1, -1.005]
	M30	1.004 (0.017)	0.801 (0.036)	[1, -1.008]
	H1	1.017 (0.012)	0.736 (0.027)	[1, -1.016]
	H4	1.051 (0.015)	0.706 (0.034)	[1, -1.040]
	D1	1.103 (0.040)	0.891 (0.074)	[1, -1.097]
	WK	1.089 (0.114)	0.900 (0.198)	[1, -1.239]
	MN	**0.800 (0.215)**	**0.900 (0.684)**	**[1, -1.625]**

In bold, evidence of no cointegration since b > d.In the third and fourth columns, standard errors of estimates are in parenthesis.

Table 10: Comparison of integration orders of range

		ELW		GPH		FCVAR	
Crypto.	**Series Frequency**	**m=$T^{0.5}$**	**m=$T^{0.6}$**	**m=$T^{0.5}$**	**m=$T^{0.6}$**	**NR**	**R**
Bitcoin	M1	0.380	0.421	0.391	0.448	0.316	0.314
	M5	0.458	0.422	0.463	0.395	0.277	0.279
	M15	0.438	0.365	0.424	0.336	0.226	0.226
	M30	0.539	0.381	0.509	0.374	0.361	0.361
	H1	0.786	0.669	0.819	0.674	0.278	0.283
	H4	0.669	0.553	0.770	0.531	0.356	0.613
	D1	0.642	0.853	0.619	0.854	0.400	0.408
	WK	0.601	0.761	0.602	0.747	0.426	0.426
	MN	0.586	0.586	0.725	0.670	0.497	0.615
Ethereum	M1	0.198	0.246	0.140	0.188	0.309	0.316
	M5	0.630	0.519	0.676	0.508	0.485	0.470
	M15	0.337	0.321	0.273	0.275	0.259	0.257
	M30	0.258	0.314	0.387	0.364	0.273	0.273
	H1	0.633	0.613	0.579	0.611	0.303	0.304
	H4	0.471	0.527	0.458	0.536	0.407	0.407
	D1	0.591	0.945	0.638	0.976	0.124	0.254
	WK	0.636	0.947	0.639	1.021	0.002	0.065
	MN	0.691	0.955	0.743	1.169	0.300	0.300
Litecoin	M1	0.625	0.535	0.563	0.473	0.340	0.342
	M5	0.489	0.384	0.541	0.431	0.354	0.355
	M15	0.350	0.328	0.393	0.358	0.309	0.304
	M30	0.270	0.284	0.399	0.337	0.218	0.218
	H1	0.562	0.590	0.519	0.548	0.284	0.288
	H4	0.470	0.490	0.452	0.441	0.355	0.357
	D1	0.589	0.580	0.580	0.569	0.328	0.329
	WK	0.592	0.623	0.573	0.611	0.273	0.286
	MN	0.537	0.552	0.575	0.549	0.475	0.516
Ripple	M1	0.664	0.603	0.573	0.471	0.352	0.353
	M5	0.768	0.589	0.569	0.470	0.342	0.351
	M15	0.235	0.227	0.176	0.168	0.199	0.197
	M30	0.247	0.269	0.364	0.309	0.209	0.203
	H1	0.586	0.439	0.593	0.432	0.285	0.281
	H4	0.634	0.559	0.694	0.602	0.334	0.345
	D1	0.447	0.436	0.339	0.366	0.196	0.212
	WK	0.551	0.472	0.616	0.566	0.170	0.189
	MN	0.064	0.327	0.032	0.249	-0.100	-0.100

Note “NR” denotes a model without restrictions on the cointegrating vector, and “R” denotes a model with restrictions.