Revisiting income convergence with DF-Fourier tests: old evidence with a new test

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Abstract

Motivated by the purpose to assess the income convergence hypothesis, a simple new Fourier-type unit root test of the Dickey-Fuller family is introduced and analysed. In spite of a few shortcomings that it shares with rival tests, the proposed test generally improves upon them in terms of power performance in small samples.

The empirical results that it produces for a recent and updated sample of data for 25 countries clearly contrast with previous evidence produced by the Fourier approach and, more generally, they also contradict a recent wave of optimism concerning income convergence, as they are mostly unfavourable to it.

Keywords: income convergence; unit root tests; structural breaks.
JEL codes: O47, C22, F43.

*Or *Most likely you go your way (and I'll go mine).*

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1 Introduction

According to the income convergence hypothesis, the diminishing marginal product of capital of the neoclassical growth model implies that, in the long-run, initial conditions — namely the physical and the human stocks of capital — should play no role in determining a country's *per capita* income. Therefore, in the long-run, independently of those initial conditions, the *per capita* incomes of different countries should converge to an identical level.

Adopting a time series framework, this paper contains two contributions to the empirical assessment of this hypothesis: a) a simple new unit root test, of the Dickey-Fuller family, Fourier-type variety, is proposed and analysed; b) it is applied to a sample of 25 countries whose data were recently released in an updated and improved version of the Maddison database (see Bolt et al., 2018).

Robustness to general non-linearities and, in particular, to breaks in level (and/or trend) is an attractive feature of some unit root tests, specially when a long span of time is involved[1]. Fourier-type unit root tests possess this property and, when combined with the Dickey-Fuller approach, they become also simple to implement. They encompass any type of non-linearity in the deterministic component of the series, they allow the presence of any kind of break and they are particularly suited to smooth breaks. Moreover, they do not require any knowledge about neither the nature nor about the number and the date(s) of the break(s). The breaks can be left unspecified and it is not even necessary to estimate them. An important source of leaks in power (see, e.g., Lee and Strazicich, 2001) is therefore avoided.

The test proposed in this paper adds to this flexibility some further benefits: besides particularly adapted to the income convergence testing problem and with improved power properties, two shortcomings of previous versions are also overcome. These concern the disregard for the endogeneity of the selection process for the frequency parameter and for the (pre-testing) nature of those versions, both liable to contribute to size distortion problems. As regards the power properties, they are improved mostly through the (*min*) form adopted for the test statistic. A narrowing of the length of the interval for the set of admissible values for the

[1] Greasley and Oxley (1997) and Li and Papell (1999) pioneered this approach in the income convergence testing literature and they have found evidence reestablishing some credit to the hypothesis.

frequency parameter also contributes to an improvement in power.

Throughout the paper, the version of the test that is privileged is the one that is better suited to the income convergence testing, the "no trend", "constant only" version. This is the version whose power and size properties in small samples is the subject of the Monte Carlo analysis. However, the distributions of the two other usual versions are also tabulated.

Compared to a close surrogate of the Enders and Lee (2012a) Dickey-Fuller Fourier-type test (FDF), the new test has better size and power properties. When breaks coexist with the unit root, the new *min*-test is the least affected. In terms of power, the new test clearly dominates the standard DF and the FDF tests, particularly when the sample size is relatively large (and provided there are really breaks in the DGP). However, it is not free from a few shortcomings, which it shares with its rivals: a) spurious rejections of the null hypothesis in some cases, suffering from a "converse Perron disease"; b) low power in some cases when the sample size is small. Note, however, that the general power performance of the new test is very good, its power frequently becoming (much) larger than the one of the DF test for the corresponding no-break case.

The new test is used over a subset of data of the recently updated and improved Maddison database. Since one of the purposes of this paper is to reassess the results obtained with other variants of Fourier-type tests and, particularly, by King and Ramlogan-Dobson (2014) with LM tests, I have selected the same set of 25 countries as in this paper. These are, besides the reference economy, the US, Australia, Austria, Belgium, Canada, Denmark, Finland, France, Germany, Greece, Hungary, Ireland, Israel, Italy, Japan, Netherlands, New Zealand, Norway, Poland, Portugal, South Korea, Spain, Sweden, Switzerland, and the UK. The series that is selected to represent output *per capita* is called *CGDPpc* in Bolt et al. (2018), and it is considered the most reliable measure for assessing the degree of income convergence because it is based on multiple benchmark comparisons of prices and incomes across countries.

Broadly speaking, three distinct generations or waves of empirical results can be associated with the assessment of the hypothesis (see, in particular, Durlauf et al., 2005, and Islam, 2003). The first wave resorted to cross section data and provided an optimistic view for the cases of advanced economies. A second wave focused

on the hypothesis' time series implications and gathered much less favourable evidence, even when focusing only on the same type of (advanced) economies. More recently, a third wave, using both a panel data approach and a time series/unit root testing one but allowing for breaks in the series restored at least some of the hypothesis' initial aura, and may have even went much further than the first generation including in the converging group some countries not usually viewed as advanced. In this paper I find that such an optimistic view is not well sustained empirically. That is, even allowing for multiple breaks in the series of relative incomes and making an extra effort to maximize the power of the tests, i. e., to reject non-convergence, the favourable evidence is relatively weak, thereby tending to agree with the (not very distant past) results of the second generation.

The remainder of this paper contains the following material. In the next section the arguments for level stationarity rather than trend stationarity as the alternative hypothesis in unit root tests for income convergence are briefly reviewed. Section 3 reviews the current versions of Fourier-type Dickey-Fuller tests for unit roots. In section 4, after adapting and criticizing those versions I propose the new test statistic. Its size and power performance in finite samples is analysed in the following section. Section 5 contains also an examination of a possible testing sequence, the union of rejections of the new test with the standard DF test. Section 6 presents the empirical results and the final section contains a comparison with recent empirical evidence and a further detailed discussion of the results.

2 Unit root tests: LSP rather than TSP

Let me represent the logarithms of *per capita* output for countries i and j with $y_{i,t}$ and $y_{j,t}$, respectively, the latter associated with the technological leader. As (hopefully convincingly) argued in Lopes (2016), the most adequate definition of income convergence, provided in Bernard and Durlauf (1996), requires that the income discrepancy $y_{i,t} - y_{j,t}$ is a level stationary process (LSP) rather than a trend stationary (TSP) one.

Strictly speaking, as the requirement is that the long-run (MSE) optimal fore-

casts for the logs of both countries should not diverge, i. e.,

$$\lim_{k\to\infty} \mathrm{E}(y_{i,t+k} - y_{j,t+k} |\, \mathcal{F}_t) = 0, \tag{1}$$

with $\mathcal{F}_t$ denoting the set all information available at time t, stationarity around zero could be imposed. However, as the two economies may differ in important structural characteristics — rates of population growth, for instance —, a non-zero mean for the output gap is admissible.

In Pesaran (2007) and in some further literature, the zero mean condition is considered as overly stringent. In Pesaran's simple growth model it requires that several deep structural parameters must be identical for both economies; for instance, the savings rate and the steady-state growth rate of employment must be the same. Therefore, it is very frequently disregarded. When a (constant) non-zero mean is allowed for the output gap, due to different saving rates or population growth rates, for example, and only both trends, deterministic and stochastic, are ruled out, there is "long-run convergence" (Oxley and Greasley, 1995), or "deterministic convergence" (Li and Papell, 1999) or "asymptotically relative convergence" (Hobijn and Franses, 2000).

Transposed to the (DF) unit root testing framework, this condition implies that while a constant term is admissible in the regression equation, a (linear) trend term is not. Its inclusion would allow detecting the existence of *catching-up*, which is a weaker notion of convergence, but it is ruled out by equation (1). Rather clearly, the presence of a trend in the income discrepancy would mean that its long-run forecasts would diverge instead of converging.

Moreover, as the power of unit root tests decreases as deterministic regressors are added to the test equation, the simple omission of the usual trend term in convergence tests, alone, is liable to improve their performance. Finally, as also shown in Lopes (2016), such omission guarantees that convergence tests are consistent when the income discrepancy is a TSP, i. e., when the divergence is dominated by the presence of a (linear) deterministic trend (and deviations around that trend are stationary). Formally, it assures that

$$\lim_{T\to\infty} \Pr\left[\text{rejecting convergence} |\, (y_{i,t} - y_{j,t}) \sim TSP\right] = 1.$$

3 Fourier-type Dickey-Fuller tests: an overview

Although there are now several unit root tests incorporating the flexible Fourier approximation, due to its simplicity the most promising version appears to be the Dickey-Fuller one, proposed in Enders and Lee (2012a) and improved in Omay (2015) [2].

To simplify the notation, let me represent the income discrepancy or gap simply with y_t $(= y_{i,t} - y_{j,t})$. The most general version of these tests departs from the basic equation

$$y_t = d(t) + \rho y_{t-1} + \gamma t + \epsilon_t,$$

where $d(t)$ denotes a general deterministic function of the time index, $t = 1, 2, \ldots, T$, and ϵ_t is assumed as $iid(0, \sigma^2)$. As usual, interest lies in testing the null hypothesis of a unit root ($\rho = 1$). But the function $d(t)$ is indeed very general because it encompasses any type of non-linearity and particularly one or several breaks of any kind (particularly when they are gradual or smooth[3]). This is precisely the strongest and most appealing feature of these tests: the nature, the number and the date(s) of the break(s) can be left unspecified; it is not necessary to know them *a priori* and it is not even necessary to know whether they really exist. Moreover, in case they do exist, it is also not even necessary to estimate them, avoiding any negative contamination arising from the estimation error. However, sometimes it is acknowledged that sudden and sharp breaks may require the traditional modeling with dummy variables because the approximation works better with gradual changes than with sharp ones.

This flexibility and robustness to breaks is achieved approximating the function $d(t)$ with a Fourier series expansion

$$d(t) = \alpha_0 + \sum_{k=1}^{n} \alpha_k \sin\left(\frac{2\pi kt}{T}\right) + \sum_{k=1}^{n} \beta_k \cos\left(\frac{2\pi kt}{T}\right),\ n \leq \frac{T}{2},$$

[2]The first version of these Fourier-type tests was proposed by Enders and Lee (2004) in the framework of Lagrange Multiplier tests but it lacks interest for the case that we study here because it applies only to trending time series. A KPSS-type, stationary test, was proposed in Becker, Enders and Lee (2006), and a DF-GLS-type, unit root test, is analysed in Rodrigues and Taylor (2012).

[3]When the breaks are abrupt the approximation that will be mentioned below can be poor. However, there is almost no research about this topic.

where n denotes the number of approximating frequencies, k is used to index the frequencies and T represents the sample size.

In practice, using many frequencies will likely provoke an over-fitting problem and can lead to a substantial power loss. Therefore, at most two frequencies should be considered but the most frequent recommendation is to use only one. Accordingly, the most general test regression becomes

$$\Delta y_t = c_1 + c_2 t + \phi y_{t-1} + \alpha_1 \sin\left(\frac{2\pi k t}{T}\right) + \beta_1 \cos\left(\frac{2\pi k t}{T}\right) + u_t, \tag{2}$$

where $\phi = \rho - 1$, k now denotes the single selected frequency and u_t represents a zero mean stationary error term[4].

The asymptotic distribution of the unit root test statistics is invariant to the values of α_1 and β_1 but depends on the value of k. Since breaks push the spectral density function of the series towards zero, the usual recommendation is that the value of k should be low. Sometimes the value $k = 1$ is mentioned as particularly adequate, as a reasonable approximation to many cases but, in practice, a selection/estimation problem now emerges. The initial break specification problem — which shape? When? How many? — transforms into one of the selection of the particular frequency.

The most frequent solution consists of choosing k from a small set of (small) integer values. Also, since the usual DF tests emerges as a special case when there is no non-linear deterministic component, a joint procedure to estimate k and to decide which test to employ is recommended in Enders and Lee (2012a):

1. estimate k using a grid search procedure over all integer values in the interval $[1, k_{MAX}]$. Usually $k_{MAX} = 5$ and

$$\widehat{k} = \arg\min_k SSR(k),$$

   SSR denoting the sum of squared residuals of equation (2). As is usual in DF test regressions, this equation may require augmentation with lags of Δy_t to whiten the residuals.

[4] In this particular case with a single frequency, α_1 and β_1 simply represent the amplitude and the displacement of the sinusoidal component.

2. Perform a pre-test for non-linearity testing $H_0 : \alpha_1 = \beta_1 = 0$ *vs.* $H_1 : \alpha_1 \neq 0 \vee \beta_1 \neq 0$ using the usual F-statistic. Small sample conservative critical values, i. e., that are valid when the unit root null is imposed, are available in Enders and Lee (2012a)[5].

3. Decide which test to use on the basis of the previous test. In case the previous null hypothesis is rejected, tables of small sample critical values for the "with trend" and "no-trend" cases are also available in Enders and Lee (2012a) for $k \in \{1, 2, 3, 4, 5\}$. Otherwise, the usual ("linear") DF test should be performed.

Although the possibility of fractional frequencies was initially entertained (see, e.g., Enders and Lee (2004)), it was subsequently abandoned until it was recently recuperated by Omay (2015). Actually, a fractional frequency can provide a better fit to the data, i.e., a better approximation to the non-linear deterministic component, and hence it may improve substantially the power of the tests. Indeed, the simulation study by Nordström (2018) indicates that the tests allowing only integer frequencies can be completely powerless (i.e., have zero power) when the actual frequency in the DGP is fractional. This corroborates the idea that an incorrectly specified break can be as harmful to the properties of unit root tests as simply neglecting its presence.

On the other hand, fractional frequencies are considered to be better than integer ones to capture breaks occurring near the extremes of the sample. Therefore, hereafter I will consider the case where fractional frequencies are allowed. Unfortunately, Omay (2015) imposed $k_{MAX} = 2$ and tabulated the small sample distributions only for $k \in \{1.1, 1.2, 1.3, \ldots, 1.9\}$.

4 Adapting and criticizing. A simple proposal

The first task is to adapt the previous tests to the income convergence problem. The first and most obvious modification consists of dropping the linear trend term from equation (2). But it is also advisable to examine the estimated deterministic

[5]Since it is a function of $\widehat{k}$ and since minimizing the sum of squared residuals is equivalent to maximizing this F-statistic, it is also sometimes denoted with $\max F(\widehat{k})$.

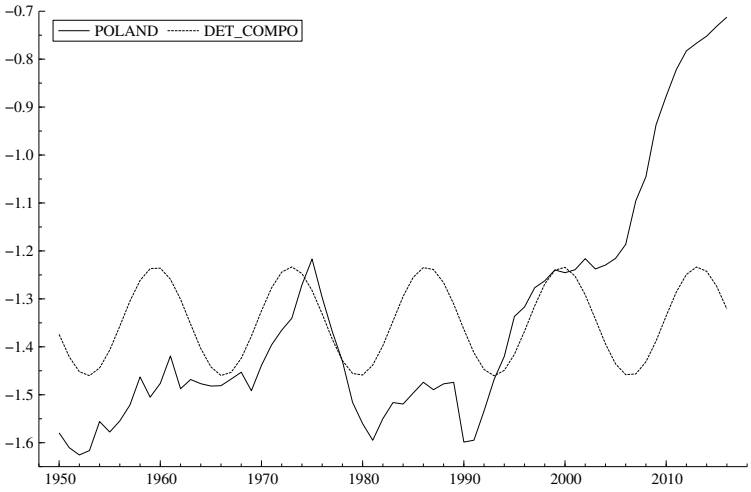

Figure 1: The output gap and the estimated deterministic component for Poland ($\widehat{k} = 5$)

component to assess whether it is reasonable, i.e., whether it makes sense from an economic point of view.

Actually, the graphical representation of $\widehat{d(t)}$ shows that simply excluding the linear trend is not sufficient. In some cases the estimated frequency is 5, which appears as clearly excessive, producing a deterministic component seemingly overfitting the actual series, hardly justifiable, meaningless and possibly distorting inference.

This is the case, for instance, for Poland, which is represented in figure 1. Using the estimation method of the previous section (with $k_{MAX} = 5$) produces precisely $\widehat{k} = 5$. Therefore, in figure 1, besides the gap for Poland the graph exhibits the estimated function

$$\widehat{c_1} + \widehat{\alpha_1} \sin\left(\frac{2\pi 5t}{T}\right) + \widehat{\beta_1} \cos\left(\frac{2\pi 5t}{T}\right),$$

which is represented with `det_comp`. One immediately wonders what sort of economic mechanism could have generated such a regular cyclical process to approximate the gap from the technology leader.

The main problem is that so many changes in level do not appear to be plausible

to approximate the gap. The estimated function contains 10 turning points, i.e., 10 breaks in level (that can even be seen as changes in the sign of a trend), which is clearly excessive for a series with only 67 observations. In case the deviations around this function behave as a stationary process, then the gap series could be called a "snake stationary process", a process that is devoid of any economic meaning. Moreover, such a process is not only unexplainable but it is also slippery to forecast, the in-sample fitting transmitting a misleading level of confidence that is completely erroneous; and indeed we can even observe already in the figure the last observations behaving rather differently, escaping completely from the snake like pattern and rapidly approaching a value much closer to zero.

Moreover, the case of Poland is not the only one in our data. Three more cases — Finland, Israel and Sweden — could be presented as illustrations of the same problem, totaling 4 out of the 24 countries, i.e., 16.6(6)% of the cases, and hence far from a negligible fraction. Such a large estimate of k entails removing from the data information that appears to be unrelated with the long-run or zero frequency but that, in practice, may be relevant to determine correctly their long-run properties; in particular, the effect of shocks that are only temporary, whose persistence is low, appears to be incorrectly removed.

As previously mentioned, it is well known that structural breaks distort the spectral density function of time series at low frequencies. This justifies the usual recommendation that $k = 1$ or $k = 2$ (for the integer case) should be sufficient to handle the large majority of breaks. Since, on the other hand, one cannot find any argument supporting the use of $k_{MAX} = 5$, this upper limit appears as somewhat arbitrary, overly cautious, wasting power in irrelevant cases and, as previously illustrated, sometimes producing incredible attractor lines. As Enders and Lee (2012, p. 576) emphasize, "*there is little point in claiming that a series reverts to an arbitrarily evolving mean*". To this, one might add that there is also little point in claiming that a series *does not* revert to an arbitrarily evolving mean. Therefore, still based on some size concerned orientation, the first modification I propose is to restrict the upper limit of k to 3, i.e., to consider the set of possible values for k as $K = \{0, 0.1, 0.2, 0.3, \ldots, 3\}$. Notice also that, besides including fractional values between 0 and 1, the lower limit is set equal to zero – that is, in practice no trigonometric terms —, so that the standard linear case is also clearly included.

The main origin of the proposed test, however, relies on the feebleness of available critical values: besides referring to rather different sets for k, these assume that the selected frequency is known a priori, i. e., that it is given exogenously to the data, and that it is also previously known that there is a non-linear, preferably smooth component. In other words, both the data-dependent nature of the selection process for k and the pre-test procedure for the test are completely neglected in published critical values.

More precisely, as the distribution of the test statistic depends on the frequency parameter k, published critical values are obtained as if it is previously known: its value is fixed in advance in the test regressions of the replications employed to produce the critical values and these regressions are run for each value belonging to the assumed set (which we denote with K). Therefore, although similar to the usual criticism made to Perron's (1989) initial work, this one here is much stronger: in complete contrast with the assumption made to tabulate critical values, k needs to be estimated from the data, making the data-exogeneity assumption clearly inadequate.

Moreover, available critical values do not accommodate the pre-test sequence, that is, they implicitly assume that the presence of a break (or of any type of non-linear component) is certain, thereby *a priori* excluding an option that may be followed in practice: the implementation of a linear (standard) Dickey-Fuller test. Since both the usual DF critical values and those of the Fourier-type variety neglect this testing sequence, it is hard to believe that they produce tests that are free from size distortion problems. In this regard, notice that my proposal for the inclusion of the value 0 (zero) in the admissible set K ($0 \in K$) permits, from the outset, the case where there is no-break (or, more generally, no non-linear component).

To circumvent the previous problems (and besides the previous enlargement of K) my proposal is to consider instead the minimum of the Fourier-type DF test statistics over the set of admissible values for k, i.e.,

$$\tau_{min}^{FDF} = \min_{k \in K} t_{\phi},$$

where t_{ϕ} denotes the t-ratio for ϕ in equation (2) without the linear trend term, as explained, and $K = \{0, 0.1, 0.2, \ldots, 3\}$. Besides relatively simple to compute, this test statistic neither assumes that k is known *a priori* nor requires its previous

Table 1. Critical values for the τ_{min}^{FDF} test

T	1%	5%	10%
	no constant case, $\tau_{nc,min}^{FDF}$		
50	−4.57	−3.93	−3.61
100	−4.43	−3.87	−3.58
200	−4.40	−3.83	−3.55
1000	−4.35	−3.82	−3.55
	no trend case, $\tau_{c,min}^{FDF}$		
50	−5.22	−4.51	−4.19
100	−4.98	−4.40	−4.11
200	−4.90	−4.35	−4.07
1000	−4.84	−4.30	−4.03
	with trend case, $\tau_{ct,min}^{FDF}$		
50	−5.70	−4.99	−4.65
100	−5.40	−4.82	−4.53
200	−5.30	−4.76	−4.48
1000	−5.22	−4.69	−4.43

estimation; instead, in the simplest case, this estimation is simultaneous with the computation of the test statistic. Moreover, in line with the flexibility of the Fourier approach, it also dispenses with the pre-test F statistic. However, as is usual, the test regression may need to be augmented with lags of the dependent variable. Concerning this issue, I assume that this augmentation occurs only after obtaining the minimizer, i. e., after the estimation of k.

Table 1 contains some simulated critical values for this test. In all simulations the data generating mechanism is a random walk process, $y_t = y_{t-1} + \epsilon_t$, $\epsilon_t \sim iid\mathcal{N}(0,1)$ and the percentiles were obtained from simulations with $50,000$ replications. Sample sizes with $T = 50, 100, 200$ and 1000 observations were considered, this last case serving to approximate the asymptotic distribution. In spite of the focus in the case where, beyond the trigonometric terms, the test regression contains only the intercept term, the critical values for the other two usual cases are also presented. The three statistics are denoted with $\tau_{nc,min}^{FDF}$, $\tau_{c,min}^{FDF}$ and $\tau_{ct,min}^{FDF}$ for the no constant, no trend, and with trend cases, respectively.

5 Finite sample performance

To assess the finite sample performance of the proposed test two benchmarks were used: the standard DF test and the FDF test, a logical and natural competitor, considering only integer frequencies, albeit from 1 to 5, as in Enders and Lee (2004, 2012a, 2012b) and Rodrigues and Taylor (2012); i. e., for the FDF test, $K = \{1, 2, 3, 4, 5\}$. To circumvent the dependence of this test on the frequency parameter k, its estimate was neglected in the tabulation of the test statistic. That is, a Gaussian driftless random walk performed the role of the DGP and the critical values were collected regardless of the estimate $\widehat{k}$; actually, in each regression, these estimates were not even retained. These critical values are presented in table A.1 of the Appendix.

The DGP of the Monte Carlo experiments is

$$y_t = \rho y_{t-1} + \alpha_1 \sin\left(\frac{2\pi kt}{T}\right) + \beta_1 \cos\left(\frac{2\pi kt}{T}\right) + \varepsilon_t,$$

with $\varepsilon_t \sim iid\mathcal{N}(0,1)$, $k = 0.4, 0.8, 1.2, 1.6$ and 2 and each experiment consisted of $10,000$ Monte Carlo replications. Sample sizes with $T = 50, 100$ and 200 observations were considered. In all the cases, the reported rejection frequencies are based in 5% nominal critical values. While for the proposed test and for the FDF test these critical values were derived from the specific sample sizes, the usual and popular (-2.86) asymptotic critical value was used for the DF test (including a constant as the only deterministic term).

The experimental design follows those of Enders and Lee (2004, 2012a, 2012b) and Su and Nguyen (2013). Besides the no-break, linear case, with $(\alpha_1, \beta_1) = (0, 0)$, the pairs $(0, 3), (3, 0), (0, 5)$ and $(3, 5)$ were used to generate data.

5.1 Size

Size estimates (obtained with $\rho = 1$) for the three tests are presented in table 2. [6]. For the simple, exclusively linear case of the unit root null hypothesis, the size performance of all the tests is very good, with empirical size barely deviating from

[6]Notice that this table contains also the results for a different test procedure, the "UR" test, that will be presented only later.

the nominal 5%.

However, when a non-linear component is added to the unit root, in general a situation of under-rejection emerges. This is most frequent and severe for the standard DF test but it also occurs very often for the two Fourier-type tests. Overall, the proposed test is the least severely affected by this problem.

The most serious problem for the integrated plus break case is, however, one of spurious rejections of the unit root null, the series appearing to be level stationary. This occurs mostly when k is very low (0.4 and 0.8) but also when $k = 1.2$. While the problem is extremely severe in some cases, with sizes estimates rapidly attaining 100% of rejections when T is only 50, it vanishes completely for $k > 1.2$. This is the well known "converse Perron phenomenon", firstly reported by Leybourne, Mills and Newbold (1998, LMN) for standard DF tests: a break in an I(1) series confounds the unit root test and it is considered as I(0).

What appears to be new or previously unreported (as far as I know) concerning this "phenomenon" is that:

a) it can also affect the size performance of tests designed to be robust to breaks (in terms of power), i.e., the Fourier-type tests;

b) it affects also standard DF tests when breaks are smooth (recall that the case reported in LMN is for abrupt breaks);

c) it is even more severe for the two Fourier-type tests than for the standard DF test, both in terms of the possible cases and of the magnitude of the size distortion.

This last evidence is somewhat unexpected and it is challenging: robustness to breaks appears to be hard to obtain in terms of size properties. As will become clear below, the flexibility of the Fourier approach is very useful in terms of power but methods to detect breaks and to handle them still cannot be dismissed when size properties are concerned.

Table 2. Size estimates for 5% nominal tests (in %)

			$T = 50$				$T = 100$				$T = 200$			
k	α_1	β_1	DF	FDF	$\tau_{c,min}^{FDF}$	UR	DF	FDF	$\tau_{c,min}^{FDF}$	UR	DF	FDF	$\tau_{c,min}^{FDF}$	UR
	0	0	5.56	4.74	4.85	6.40	5.85	4.84	4.97	5.25	5.45	4.64	5.09	5.19
0.4	0	3	98.18	99.61	99.72	99.68	99.94	100.0	100.0	100.0	99.97	100.0	100.0	100.0
	3	0	0.01	0.00	2.14	1.71	0.00	0.00	1.75	0.93	0.01	0.00	1.55	0.67
	0	5	99.98	100.0	100.0	100.0	99.97	100.0	100.0	100.0	99.98	100.0	100.0	100.0
	3	5	99.97	100.0	100.0	100.0	99.95	100.0	100.0	100.0	99.97	100.0	100.0	100.0
0.8	0	3	0.01	0.00	1.32	1.01	0.00	0.00	1.29	0.74	0.01	0.00	1.41	0.57
	3	0	3.24	50.35	97.98	97.26	25.16	92.81	100.0	100.0	86.37	99.98	100.0	100.0
	0	5	0.01	0.00	1.24	0.96	0.00	0.00	1.29	0.74	0.01	0.00	1.41	0.57
	3	5	0.01	0.00	1.24	0.96	0.00	0.00	1.29	0.74	0.01	0.00	1.41	0.57
1.2	0	3	0.01	12.29	22.61	18.27	0.10	42.89	93.60	86.29	0.11	92.78	100.0	100.0
	3	0	0.02	0.09	3.97	3.35	0.01	0.00	3.29	2.24	0.05	0.00	2.18	1.36
	0	5	0.11	51.74	72.25	64.71	0.12	97.24	100.0	100.0	0.13	100.0	100.0	100.0
	3	5	0.00	0.00	0.63	0.47	0.00	0.01	0.72	0.43	0.00	0.02	0.73	0.31
1.6	0	3	0.01	0.00	3.47	2.91	0.08	0.00	3.86	2.69	0.00	0.00	4.32	2.80
	3	0	0.00	0.01	1.07	0.81	0.00	0.00	0.91	0.55	0.02	0.01	0.62	0.31
	0	5	0.00	0.00	2.41	1.98	0.01	0.00	2.99	1.88	0.00	0.00	4.18	2.62
	3	5	0.00	0.00	0.46	0.40	0.00	0.00	0.63	0.32	0.00	0.00	0.60	0.30
2.0	0	3	0.00	1.27	0.77	0.53	0.00	1.17	0.55	0.26	0.00	1.16	0.60	0.25
	3	0	0.02	1.27	0.74	0.52	0.00	1.17	0.66	0.37	0.00	1.16	0.60	0.43
	0	5	0.00	1.27	0.49	0.38	0.00	1.17	0.37	0.15	0.00	1.16	0.43	0.17
	3	5	0.00	1.27	0.32	0.22	0.00	1.17	0.23	0.09	0.01	1.16	0.34	0.14

Note: the DGP is $y_t = y_{t-1} + \alpha_1 \sin\left(\frac{2\pi kt}{T}\right) + \beta_1 \cos\left(\frac{2\pi kt}{T}\right) + \varepsilon_t (\rho = 1)$, with $\varepsilon_t \sim iid\mathcal{N}(0, 1)$.

5.2 Power

Table 3, containing the finite sample power results for $\rho = 0.9$, clearly justifies the employment of the proposed test and a preference over the FDF test. Indeed, with a few exceptions, the proposed test generally dominates the two competitors in terms of estimated power performance, sometimes even smashing them, particularly when $T = 200$ but also when $T = 100$.

One of the exceptions is the standard, purely linear (non-break) unit root case, where the standard DF test unsurprisingly dominates and the new test is the worst of the three. This means that the inclusion of the value zero in the set of admissible parameter values for k is insufficient to warrant a reasonable performance when there is no non-linear component. This also means that a union of rejections strategy between the new test and the DF test may be beneficial, at least in that case.

The other exceptions are some of the cases when $T = 50$ only, where the power of the proposed $(min-)$test is low, and in few cases even lower than the power of the FDF test. Anyway, on one hand, provided that a non-linear component is present in the data, the new test is always more powerful than the DF test, sometimes much more so. On the other hand, when T grows from 50 to 100 the growth of power of the proposed test is usually much faster than those of the two rival tests. Therefore, although not uniformly, the dominance of the new test is very clear.

Moreover, notice that in many break cases the estimated power of the new test is even larger than that of the DF test for the same sample size with no break case. Thus, the presence of breaks is often a very powerful boost to the power of the *min*-test. Furthermore, the results also suggest that the proposed test is always consistent, but the same cannot be said in some cases for the FDF test (for instance, when $k = 0.4$ and $(\alpha_1, \beta_1) = (3, 5)$) and, much less surprisingly, in most break cases for the DF test.

Table 3. Power estimates for 5% nominal tests (in %)

			$T = 50$				$T = 100$				$T = 200$			
k	α_1	β_1	DF	FDF	$\tau_{c,min}^{FDF}$	UR	DF	FDF	$\tau_{c,min}^{FDF}$	UR	DF	FDF	$\tau_{c,min}^{FDF}$	UR
	0	0	13.34	10.98	7.94	12.50	32.94	21.95	13.72	25.33	86.52	64.38	43.93	76.94
0.4	0	3	0.06	0.00	11.17	9.12	0.06	0.00	63.50	51.16	0.07	0.00	99.81	99.28
	3	0	5.14	0.32	5.97	6.52	4.77	0.29	10.55	7.57	0.58	0.36	31.13	18.96
	0	5	0.06	0.00	33.12	28.99	0.07	0.00	99.57	98.88	0.07	0.00	100.0	100.0
	3	5	0.08	0.03	35.21	30.71	0.07	0.00	98.78	97.31	0.08	0.00	100.0	100.0
0.8	0	3	0.09	0.02	8.23	6.66	0.09	0.09	69.46	58.51	0.09	0.52	99.95	99.83
	3	0	0.03	0.00	11.13	9.19	0.02	0.00	13.08	8.03	0.02	0.00	29.89	17.23
	0	5	0.09	0.00	22.99	19.53	0.09	0.00	99.92	99.59	0.09	0.00	100.0	100.0
	3	5	0.01	0.00	2.97	2.36	0.08	0.00	81.18	73.08	0.09	0.00	100.0	100.0
1.2	0	3	0.08	84.28	57.99	52.00	0.05	99.11	98.45	96.19	0.05	94.59	100.0	100.0
	3	0	0.01	0.00	15.90	13.03	0.01	0.00	21.75	14.41	0.01	0.00	40.85	26.25
	0	5	0.12	99.99	99.33	98.70	0.06	100.0	100.0	100.0	0.05	100.0	100.0	100.0
	3	5	0.11	88.01	91.93	87.84	0.12	100.0	100.0	100.0	0.13	100.0	100.0	100.0
1.6	0	3	0.02	0.00	11.41	9.44	0.03	0.00	75.06	63.00	0.04	0.00	99.97	99.99
	3	0	0.02	0.00	4.84	3.90	0.02	0.00	22.61	15.16	0.01	0.00	49.26	33.20
	0	5	0.02	0.00	48.29	42.70	0.03	0.00	99.95	99.87	0.04	0.00	100.0	100.0
	3	5	0.00	0.00	1.18	0.92	0.01	0.00	25.62	15.72	0.03	0.00	100.0	100.0
2.0	0	3	0.00	3.84	3.59	2.86	0.00	38.91	36.87	26.69	0.04	99.91	99.52	98.72
	3	0	0.02	6.19	3.74	3.02	0.02	43.09	21.69	14.47	0.01	83.70	57.10	41.13
	0	5	0.00	5.97	8.53	7.06	0.02	93.84	89.95	83.85	0.04	100.0	100.0	100.0
	3	5	0.00	3.31	0.86	0.64	0.01	16.89	6.02	3.56	0.03	99.99	99.87	99.66

Note: the DGP is $y_t = 0.9y_{t-1} + \alpha_1 \sin\left(\frac{2\pi kt}{T}\right) + \beta_1 \cos\left(\frac{2\pi kt}{T}\right) + \varepsilon_t(\rho = 0.9)$, with $\varepsilon_t \sim iid\mathcal{N}(0, 1)$.

5.3 The "union of rejections" (UR) testing sequence

As mentioned previously, in spite of the general power dominance of the new test, its weakness when breaks are absent may possibly be overcome through a union of rejections decision rule, combining the new test with a (previous) standard DF test[7]. This is indeed a very simple testing strategy, much simpler than the testing sequence proposed by Enders and Lee, and may reduce the cost associated with the robustness to breaks (or other non-linear components) of the proposed test. As noted previously, this cost is represented by the significant power loss when there are no breaks.

The raw version of this strategy consists simply of the decision rule "reject the unit root null if either DF or τ_{min} rejects" and it is frequently adopted by many practitioners, albeit with tests different from the τ_{min}. In such a crude form, it is quite obvious that it can easily attain a high power performance, but at the expense of some size distortion, possibly far exceeding the nominal level. Moreover, this is also a feature that it shares with the Enders and Lee testing sequence. A preferable size-adjusted variant is, however, easily implemented following Harvey et al. (2009): reject the unit root null hypothesis if either

$$\{DF < \gamma^{\lambda}\, cv^{\lambda}_{DF}\} \text{ or } \{\tau_{min} < \gamma^{\lambda}\, cv^{\lambda}_{\tau_{min}}\}$$

where λ is the desired significance level, γ^{λ} is a common scaling constant and cv^{λ}_{DF} and $cv^{\lambda}_{\tau_{min}}$ are the corresponding $100\lambda\%$ asymptotic critical values of the DF and τ_{min} tests, respectively. Therefore, the constant γ^{λ} ensures that the asymptotic size of the rule is γ, and it can be approximated through Monte Carlo simulation using a grid search procedure. In table 4 this constant is presented for the usual significance level $\gamma = 0.05$ for the three usual cases. It is this corrected version that is considered in the remainder of this paper.

Both size and power simulation results for this sequence are also available in tables 2 and 3, respectively. In terms of size, the UR strategy improves somewhat significantly the performance of the new test in only 3 of the most severe over-rejection cases, but it leave us very far from eliminating them completely. On

[7]The combination with a DF-GLS test was also considered but it was excluded *a priori* due to the problem of the initial condition, which is often present when testing for convergence with samples starting in 1950.

Table 4. 5% asymptotic critical values and size-
-correcting parameter (ascp) for the UR testing strategy

	$cv_{DF}^{0.05}$	$cv_{\tau_{min}}^{0.05}$	5% ascp
no const.	−1.95	−3.82	1.094
no trend	−2.86	−4.30	1.072
with trend	−3.41	−4.69	1.054

Notes: the 5% asymptotic DF critical values are taken from Fuller (1996), table 10.A.2 (p. 642) and the 5% asymptotic critical values of the τ_{min} statistic are from table 1 of this paper with $T = 1000$. The size-correcting parameters are obtained with $10,000$ replications.

the other hand, in many other cases, it inherits the conservative character of both component tests, which is not necessarily a desirable feature.

Concerning power, when breaks are absent the UR sequence really improves considerable the performance of the *min*-test, increasing the estimated rejection frequencies in more than 50%. But in many other cases the power of the UR combined test is significantly lower than the proposed test. For instance, when $T = 50$ this occurs with $k = 0.4$, $(\alpha_1, \beta_1) = (0, 5)$ and $(3, 5)$, when $k = 1.2$ with the pair $(0, 3)$ and with $k = 1.6$ with $(0, 5)$, and when $T = 100$ and $T = 200$ with many other cases.

All in all, the main benefit of the UR sequence lies in disciplining the informal procedure consisting of the sequential application of the two tests, searching for a rejection. As a formal procedure, it allows us controlling the overall size. However, its estimated size benefits are scarce and unclear and the power gains when breaks are absent are outweighed by significant power losses in many break cases. Hence, for the empirical case under analysis, the straightforward application of the *min*-test appears to be more adequate than the UR strategy: with such a long span of data and with time series depending on the economies of two countries it is likely that they include at least one break.

6 Empirical results

The empirical analysis will follow sequentially a "classical" perspective, based on standard unit root test statistics, the Enders and Lee testing strategy and the new test proposed in this paper, as well as the UR strategy.

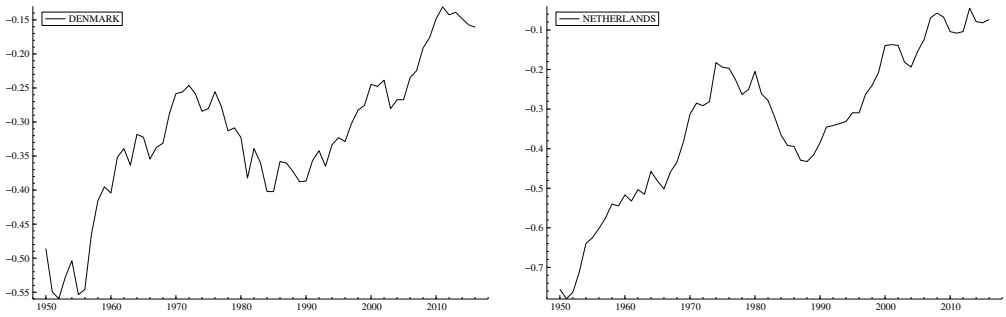

Figure 2: The cases of Denmark and the Netherlands

6.1 Standard tests

Our standard tests are the well known and are the simple but adequate τ_c^{DF} and the inadequate τ_{ct}^{DF} test statistics, and the τ_c^{DF-GLS} statistic of Elliot et al. (1996).

The evidence for non-divergence for Austria, France, Germany, Italy and Japan produced by the τ_c^{DF} which was previously found in Lopes (2016) is confirmed, in some cases with a change in the significance level. But compared with those results, now the same type of evidence for non-divergence for Denmark and the Netherlands has evaporated. While for Denmark there appears indeed to have occurred a widening of the gap with the US in the last years of the sample, the result for the Netherlands is much more surprising because one cannot find a similar graphical evidence.

What appears to be disconcerting is the favourable outcome for Greece, in spite of the rather visible reversal of the converging process that has occurred in the last 8 years of the sample. This result appears to be due mainly to some stability of the income gap in a very substantial and final fraction of the sample, that is, albeit at a much lower level than the US, it appears that Greece has attained its steady state. A similar phenomenon appears to be responsible for the result for Israel: the graphical analysis suggests that Israel has attained its steady state in the seventies and since then the income gap has been rather stable, with a rather stationary visual shape. In other words, although neither Greece nor Israel had attained the same level of *per capita* output as the US, the difference has remained limited and rather stable in the last 40-45 years of the sample.

Finally notice that the DF-GLS test allows rejecting the unit root null only for

Table 5 – Standard unit root test statistics

	τ_c^{DF}(nlag)	τ_{ct}^{DF}	τ_c^{DF-GLS}
Australia	−1.32 (3)	−1.37 (3)	−1.04(3)
Austria	−3.53 (0) **	−2.91 (0)	0.09(2)
Belgium	−1.92 (5)	−1.74 (0)	0.07(0)
Canada	−2.42 (0)	−2.80 (0)	−2.07(1)**
Denmark	−1.89 (3)	−2.49 (4)	−0.52(3)
Finland	−2.07 (4)	−1.86 (4)	0.04(1)
France	−2.87 (0) **	−2.03 (0)	−0.27(1)
Germany	−4.04 (1) ***	−3.11 (6)	−0.02(3)
Greece	−3.38 (0) **	−1.01 (0)	−0.28(5)
Hungary	−2.22 (6)	−3.33(6)*	0.09(6)
Ireland	0.03 (6)	−1.96 (6)	0.43(6)
Israel	−3.89 (2) ***	−2.62 (2)	0.02(0)
Italy	−4.51 (5) ***	−2.47 (5)	−0.24(6)
Japan	−3.26 (1) **	−1.34 (6)	−0.19(2)
Netherlands	−1.56 (6)	−2.85 (6)	−0.22(6)
New Zealand	−1.27 (3)	−0.58 (3)	−0.64(3)
Norway	−0.85 (5)	−2.66 (5)	−0.34(5)
Poland	1.08 (5)	−0.16 (5)	1.24(5)
Portugal	−1.49 (1)	−2.61 (3)	0.27(3)
South Korea	−0.66 (6)	−2.17 (6)	0.40(6)
Spain	−1.32 (0)	−2.54 (2)	0.50(0)
Sweden	−1.47 (0)	−2.23 (0)	0.05(0)
Switzerland	−2.21 (1)	−2.70 (1)	0.77(0)
U.K.	−1.22 (0)	−2.64 (3)	−0.69(0)

Notes: 1) "***", "**", and "*" represent rejections of the (unit root) null hypothesis at the 1%, 5% and 10%, respectively; 2) the general-to-specific t-sig (GTS) procedure was employed to select the lag truncation parameter, with asymptotic 10% level tests and initiating the testing sequence with (k_{MAX} =) 6 lagged terms (the AIC method was also employed, producing identical or very similar results).

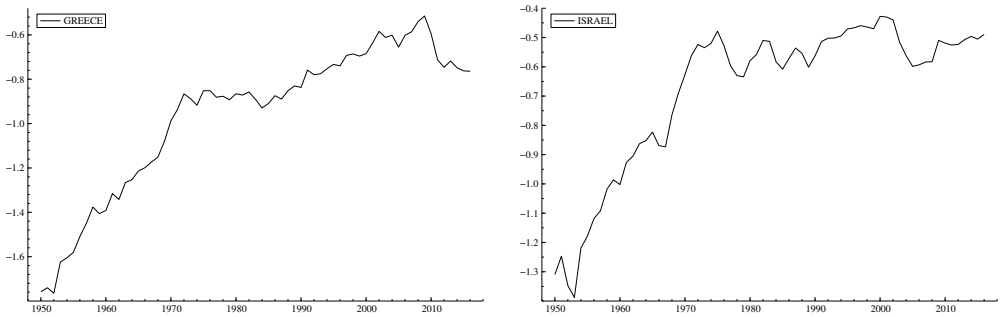

Figure 3: The cases of Greece and Israel

Canada, an outcome seemingly contradicting the superior power properties of this test over the simpler DF-OLS version. I believe that this evidence must be devalued because it is known that these tests are very sensitive to the initial conditions and these are large for most countries because in 1950 their economies were still greatly affected by the Second World War. Therefore, this clearly appears to be a case where the recommendation by Muller and Elliot (2003) should be adopted, i.e., the simpler and usually less powerful DF-OLS test must supersede its DF-GLS relative, except for the case of Canada. Moreover, notice that the only rejection with this test concerns precisely a country whose territory was not involved in WW2, i.e., one whose initial condition is closer to the remaining sample points.

On the other hand, this evidence further illustrates the improved power performance of the level stationarity analysis compared to the trend stationarity one: the previous evidence for the rejection of the unit root null disappears for all the previous seven countries when the τ_{ct}^{DF} is used and somewhat surprisingly it appears only for Hungary. Recalling the discussion of section 2, this only means that the Hungarian income gap can be considered as trend stationary, not that Hungary has converged; instead, it appears to be in a catching-up process, the gap steadily decreasing as time passes, following a process dominated by a linear deterministic trend, with deviations or fluctuations around that trend that behave in a stationary fashion.

6.2 Enders and Lee testing strategy results

In table 6 I present the results of the Enders and Lee (2012a) procedure. The conservative $\max F(\widehat{k})$ allows rejecting the linear null hypothesis at the 5% level

for 3 countries only: Australia, New Zealand and Poland. Of these, the τ_c^{FDF} test statistic is able to reject the unit root null at 5% level only for Australia. For the remaining 21 countries the sequence produces exactly the same results as in the previous subsection because it neglects its pre-testing character.

These results appear unsatisfactory and unreliable:

a) given the number of countries and the length and nature of the sample period, the small number of rejections resulting from the $\max F(\widehat{k})$ test seems to testimony that its power is low, affecting also the overall power properties of the procedure;

b) the rejections of the second test in the sequence may reflect a serious problem of size distortion, affecting both the DF and the τ^{FDF} tests.

Concerning this, recall that both statistics result from a sequential procedure whose nature is ignored in the derivation of their null distributions; moreover, recall also that available critical values for τ_c^{FDF} incorrectly assume the exogeneity of $\widehat{k}$.

6.3 The FDF and τ_{min}^{FDF} test results

In table 7 I present the results for the FDF_c and $\tau_{c,min}^{FDF}$ and $\tau_{nc,min}^{FDF}$ test statistics. Recall that the first test is not the one by Enders and Lee (2012a) and that it is not strictly valid because it neglects the dependence of the distribution on the estimated value of k. It is a useful benchmark against which the properties of the new test could be assessed but its use cannot be straightforwardly recommended.

Nonetheless, the results of the first two columns are disappointing: the small sample properties of the FDF and the $\tau_{c,min}^{FDF}$ statistics and, in particular, the power properties of this one promised a number of rejections of the unit root null superior to the one produced by the DF statistic but rather the opposite occurs. Actually, the new test is able to reject the unit root at 5% or lower only for Israel, Italy and South Korea. At the 10% level one gets rejections for Australia and France as well, i.e., in total only 5 countries, less than the corresponding 7 rejections obtained with the much simpler and non-robust, prone to power deficiencies due to breaks τ_c^{DF}. Although the statistic is (negative and) large for all these cases, now one does not get a rejection neither for Austria nor for Germany, Greece and Japan.

Table 6 – Enders and Lee test statistics (k integer)

	$\max F(\widehat{k})$	$\widehat{k}$	τ_c^{FDF}(nlag)	τ_c^{DF}
Australia	7.65**	1	−4.07 (2) **	—
Austria	5.68	4	—	−3.53 (0) **
Belgium	2.81	2	—	−1.92 (5)
Canada	5.43	2	—	−2.42 (0)
Denmark	2.31	2	—	−1.89 (3)
Finland	1.69	5	—	−2.07 (4)
France	6.12	2	—	−2.87 (0) **
Germany	5.28	4	—	−4.04 (1) ***
Greece	2.85	2	—	−3.38 (0) **
Hungary	1.09	4	—	−2.22 (6)
Ireland	3.21	1	—	0.03 (6)
Israel	2.84	5	—	−3.89 (2) ***
Italy	2.88	4	—	−4.51 (5) ***
Japan	2.26	3	—	−3.26 (1) **
Netherlands	3.89	2	—	−1.56 (6)
New Zealand	8.02**	1	−3.14 (3)**	—
Norway	1.79	2	—	−0.85 (5)
Poland	8.07**	5	1.73 (0)	—
Portugal	3.47	3	—	−1.49 (1)
South Korea	5.86	4	—	−0.66 (6)
Spain	2.37	4	—	−1.32 (0)
Sweden	2.43	5	—	−1.47 (0)
Switzerland	1.79	2	—	−2.21 (1)
U.K.	6.28	3	—	−1.22 (0)

Notes: 1) a "**" in the $\max F(\widehat{k})$ statistic represents a rejection at the 5% level using the critical value for $T = 100$ in Enders and Lee (2012b, EL) (7.58); for 10% it is 6.35; 2) again the general-to-specific t-sig (GTS) procedure was employed to select the lag truncation parameter, with asymptotic 10% level tests and initiating the testing sequence with (k_{MAX} =) 6 lagged terms; 3) the "**" in the τ_c^{FDF} test statistic denotes a rejection at the 5% level (the 5% critical value for $T = 100$ and $k = 1$ from EL is −3.81).

Table 7 – FDF and τ_{min}^{FDF} test statistics

	$FDF_c(\widehat{k})$ [nlag]	$\tau_{c,min}^{FDF}(\widehat{k})$ [nlag]	$\tau_{nc,min}^{FDF}(\widehat{k})$ [nlag]
Australia	$-4.07(1)[2]^{**}$	$-4.43(1.4)[0]^{*}$	$-3.02(0.3)[0]$
Austria	$-3.13(4)[0]$	$-3.56(1.2)[0]$	$-5.75(2.9)[0]^{***}$
Belgium	$-1.90(2)[5]$	$-2.51(2.7)[5]$	$-2.64(2.5)[5]$
Canada	$-3.72(2)[0]^{*}$	$-3.72(2.0)[0]$	$-2.65(0.1)[0]$
Denmark	$-1.10(2)[2]$	$-2.52(0.1)[4]$	$-3.28(0.8)[2]$
Finland	$-2.07(5)[4]$	$-3.44(2.2)[5]$	$-2.87(2.3)[4]$
France	$-4.45(2)[5]^{**}$	$-4.38(2.0)[5]^{*}$	$-3.33(0.7)[5]$
Germany	$-0.18(4)[6]$	$-3.98(0.0)[1]^{+++}$	$-7.24(0.6)[0]^{***}$
Greece	$-3.08(2)[0]$	$-3.84(2.4)[0]$	$-4.30(2.4)[0]^{**}$
Hungary	$-2.04(4)[6]$	$-2.64(1.9)[0]$	$-3.90(0.7)[0]^{*}$
Ireland	$-1.82(1)[6]$	$-2.53(0.7)[6]$	$-2.93(1.0)[6]$
Israel	$-3.81(5)[2]^{*}$	$-4.72(2.8)[5]^{**+}$	$-3.66(2.7)[2]^{**}$
Italy	$-4.80(4)[5]^{***}$	$-4.64(2.5)[4]^{**}$	$-3.79(2.8)[1]^{*}$
Japan	$-3.84(3)[1]^{*}$	$-3.80(2.7)[6]$	$-3.99(2.6)[1]^{**}$
Netherlands	$-2.12(2)[0]$	$-0.85(2.2)[3]$	$-1.78(2.4)[6]$
New Zealand	$-3.14(1)[3]$	$-2.71(1.0)[5]$	$-3.76(0.3)[3]^{*}$
Norway	$-0.92(2)[0]$	$-2.77(0.4)[5]$	$-2.47(0.8)[5]$
Poland	$1.73(5)[0]$	$-1.98(0.1)[3]$	$-3.62(0.7)[6]^{*}$
Portugal	$-2.10(3)[4]$	$-3.02(0.1)[5]$	$-3.29(2.6)[4]$
South Korea	$-0.52(4)[5]$	$-5.65(0.5)[0]^{***}$	$-2.60(1.1)[6]$
Spain	$-1.01(4)[0]$	$-2.40(0.8)[0]$	$-3.10(2.4)[0]$
Sweden	$-1.78(5)[2]$	$-2.13(0.1)[1]$	$-2.24(0.1)[0]$
Switzerland	$-0.92(2)[0]$	$-2.60(3.0)[3]$	$-3.26(3.0)[1]$
U.K.	$-0.73(3)[2]$	$-3.76(0.6)[6]$	$-2.90(0.1)[3]$

Notes: 1) "***", "**" and "*" denote rejections at the 1%, 5% and 10% levels, respectively, using the critical values presented in this paper for $T = 50$; 2) again the general-to-specific t-sig (GTS) procedure was employed to select the lag truncation parameter in every case, with asymptotic 10% level tests and initiating the testing sequence with ($k_{MAX} =$) 6 lagged terms; 3) "+++" and "**+" denote rejections at 1% but with the GTS t-sig method employing 5% level tests.

One possible explanation for this apparent contradiction lies in the likely interference or noise produced by the lag length selection method, which has simply not played any role in the simulation study. To this one may object, on one hand, that all the rival methods must be equally affected. On the other hand, trying to remedy the problem one may think of reversing the order of the procedures for the estimation (minimization) of k and for the lag augmentation but this does not seem reasonable. Anyway, to further investigate this issue, the calculation of the test statistic was redone with a less size concerned t$-sig$ method, more power oriented, using 5% level tests for the simplification process. This has produced shorter lag lengths for four countries but only two different decisions: the test statistic for Germany changes to $-5.17(\widehat{k} = 0)$, allowing a rejection at 1%, and the one for Italy changes to -5.12 ($\widehat{k} = 0$ also), changing only the rejection level from 5% to 1%. For the remaining cases there is no relevant change. Anyway, notice also that the estimate originally obtained for Germany for k (zero, with one lag) already implied the rejection of the unit root null via the DF test statistic obtained previously, i. e., in a reversed UR testing sequence.

Inspired by the same evidence, a rather different argument could sustain that the results do not support the existence of breaks in the series: after all, this arguing goes, since the test which is robust to breaks produces less, not more evidence for stationarity, breaks must be largely absent from the data. However, this argument is rather feeble: although the proposed test is indeed more powerful — sometimes much more powerful — than the DF test under the stationary alternative, in many cases its power is still very low, as a closer inspection of table 4 confirms. For instance, when T is only 50 (here $T = 67$), $\rho = 0.9$, $k = 2$ and $(\alpha_1, \beta_1) = (3, 5)$, the false unit root null is expected to be rejected in only 0.86% of the cases. This is an extreme case but other cases exist, mostly when $(\alpha_1, \beta_1) = (3, 5)$, where the estimated power of the min-test is rather low, and particularly below the fixed nominal size.

Overall, these results inspired a further search for a more powerful test. While in statistical terms this search lead to a small step away, in terms of the concrete problem the adopted solution appears to be counter-intuitive because the requirement for non-divergence becomes more demanding, not less. In fact, additional power can be obtained adopting the strict interpretation of equation (1), i.e., drop-

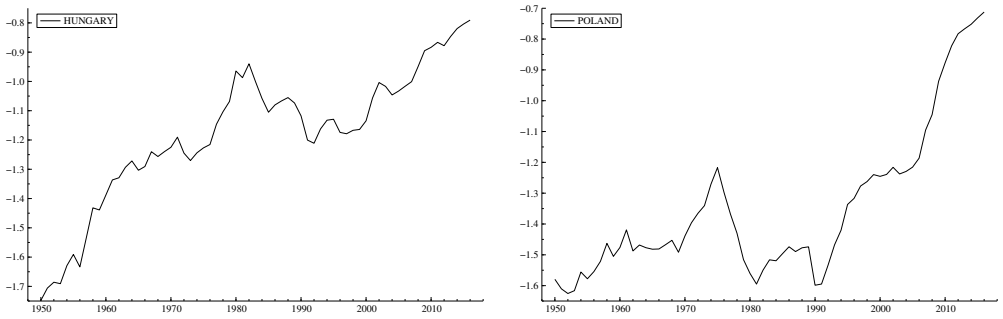

Figure 4: The cases of Hungary and Poland

ping the constant term from the test regression. Requiring stationarity around a zero mean provides, simultaneously, a more stringent condition for convergence and, insofar as a deterministic regressor that becomes irrelevant is omitted, a more powerful unit root test.

The results for the *min* version of this test are also presented in table 7, in its last column. Although, as expected, the number of rejections of divergence increases, three distinct cases are worth considering:

a) for Germany, Israel and Italy the rejection may be viewed as simply confirming identical previous results;

b) for Austria, Greece, Japan and New Zealand the novelty of the rejection is far from surprising because previous results were already close to it. Therefore, these cases seem to serve as good illustrations of the gain in power.

c) However, the cases of Hungary and Poland appear as dubious, not only because the rejections necessitate a size of 10% but mostly because previous tests were rather unfavourable to the hypothesis and the graphical analysis (see figure 4) does not lend any support to such a decision. In both cases, a catching-up process initiated around 1990 is clearly visible but it seems far from attaining stability, even at a (much) lower level than the leader (as in the cases of Greece and Israel).

A closer inspection of this last case does not allow drawing any firm conclusion. The simulation results of table 2 show that indeed the spurious rejection problems

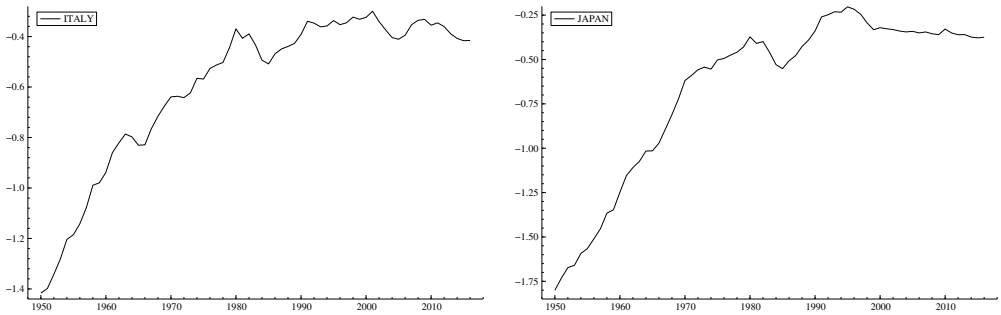

Figure 5: The cases of Italy and Japan

tends to concentrate around the values of k which lie close to the (common) estimate for Hungary and Poland ($k = 0.4$ and 0.8 and $\widehat{k} = 0.7$, respectively). But, on the other hand, the estimates for α_1 and β_1 are far from all the cases of the simulation study: again in both cases they are small and symmetric. Moreover, a unit root test allowing for a smooth break under the null, the one by Lanne et al. (2003), produces strong supporting evidence for the unit root, contradicting the result of the $\tau^{FDF}_{nc,min}$ test, thereby confirming the suspicion that this test is producing spurious rejections. While acknowledging that the ground is not solid, I believe that this conclusion is the most plausible, and consider, at least provisionally, that both Hungary and Poland are still divergent cases.

7 Comparison and final discussion

Summing up the evidence produced by the proposed *min* test, only 10 decisions for non-divergence were obtained: 2 at the 10% level (Australia, France and New Zealand), 5 at the 5% level (Greece, Israel, Italy, Japan and South Korea), and only 2 at the 1% level (Austria and Germany). Moreover, recall that in some cases — most notably those of Greece and Israel, but also, for instance, Italy and Japan at a closer level to the leader — an outcome for non-divergence means that the relative income difference to the reference economy has remained limited and rather stable for some time, not that the level of *per capita* income of this latter country has been attained.

These results are presented in table 8, together with a summary of some previous

evidence reported in the literature[8]. The main inference that this comparison delivers is that Fourier-type unit root tests are not always as favourable to the hypothesis as previous studies implied. Both in Christopoulos and Leon-Ledesma (2011, CL-D) and in King and Ramlogan-Dobson (2014, KR-D) the evidence for non-divergence was overwhelming, with only the exception of one country (Japan) in the former study. That is, although not starkly contrasting with it, the evidence gathered here is much less benevolent to the hypothesis.

However, in both cases a strict comparison is not feasible: in CL-D the sample size is much larger but it is sectionally restricted to 13 countries that are usually considered as developed or high-income. The set of countries considered here coincides with the one of KR-D, their sample ending in 2008 but, most importantly, their Fourier-type tests allow a linear trend term in the deterministic component[9], which I consider inadmissible in my approach.

Nonetheless, the results of this paper clearly contrast with those previously produced with the Fourier approach to unit root tests, and more generally they contradict a recent wave of optimism concerning the convergence hypothesis, represented, for instance, by Desli and Gkoulgkoutsika (2019, DG): "*most of the studies concerned with developed countries find evidence of convergence*". Note, however, that mostly due to the procedures employed, these studies that DG refer are not comparable with this one. A parallel optimistic perspective has also recently appeared concerning low and medium income countries but it is soundly refuted in Johnson and Papageorgiou (2018).

Qualitatively, the evidence gathered here is much closer to the one in Chong, Hinich, Liew and Lim (2008, CHLL), where the framework is not formally one of allowing for breaks but non-linearities are allowed, both in the deviations around the trend and in this latter component as well (in dissonance with the approach adopted here). The general picture painted by this evidence is, therefore, much less favourable to the hypothesis than the one that usually transpires from recent studies and, particularly, from those that resort to the Fourier approach. A partial explanation for this lies in the diversity of the conditions embedded in the testing

[8]Surprisingly, the number of empirical studies that are comparable to this one is very small. This is because very often the methods that are employed are rather disparate.

[9]The Lagrange-multiplier framework used in KR-D requires the presence of this component and hence it is not adaptable to the framework adopted here.

Table 8 — Comparison of test results

	CHLL(2008)	CL-D(2011)	KR-D(2014)	L(2016)	this paper
database	PWT	Maddison	Maddison	Maddison	Maddison
sample	1950-2000	1900-2008	1950-2008	1950-2008	1950-2016
Australia	nd1	nd1	nd5		nd10
Austria	nd5	nd5	nd5	nd5	nd1
Belgium		nd1	nd5		
Canada		nd5	nd5	nd5	
Denmark	—	nd1	nd5	nd5	
Finland		nd5	nd5		
France		nd5	nd5	nd10	nd10
Germany	—	nd10	nd5	nd1	nd1
Greece	—	—	nd5	—	nd5
Hungary	—	—	nd5	—	
Ireland	—	—	nd5	—	
Israel	—	—	nd5	—	nd5
Italy	—	nd5	nd5	nd5	nd5
Japan			nd5	nd1	nd5
Netherlands	nd1	nd1	nd5	nd1	
New Zealand	—	—	nd5	—	nd10
Norway		nd1	nd5		
Poland	—	—	nd10	—	
Portugal	—	—	nd5	—	
South Korea	—	—	nd5	—	nd5
Spain	—	—	nd5	—	
Sweden		nd5	nd5		
Switzerland		—	nd5		
U.K.	nd1	nd5	nd5		

Notes: 1) "PWT" represents the Penn World Tables. 2) a blank entry represents a non rejection of the UR hypothesis, i.e., a result for divergence. 3) "ndx" represents a rejection of the UR hypothesis at the x% level, i.e., a result for non divergence. 4) CHLL (2008) represents Chong, Hinich, Liew and Lim (2008), who use a KSS (Kapetanios, Shin and Snell (2003)) unit root test only in those cases where, in a first stage, a linearity test finds evidence for non-linearity in the series; this excludes Denmark, Germany and Italy from further consideration. 5) CL-D denotes Christopoulos and Leon-Ledesma (2011), where the unit root test is a two-step FDF test where the non-linear deterministic component is removed in the first step with a Fourier expansion with no trend (as in this paper), and a unit root is tested in the residuals against both a linear and a logistic (stationary) smooth transition autoregressive (LSTAR) alternative using the $(inf - t)$ test of Park and Shintani (2016). 6) KR-D (2014) represents King and Ramlogan-Dobson (2014), where 2 different Fourier type tests are used over a LM unit root test variant but allowing a linear trend term in the deterministic component. In one of the tests the deviations from the trend are allowed to follow an exponential STAR (ESTAR). In both these papers the joint nature of the procedure is not considered in the evaluation of the p-value. 7) L(2016) denotes Lopes (2016).

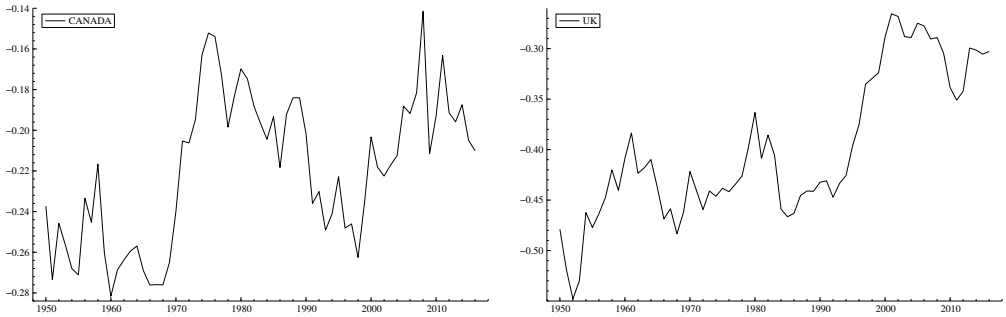

Figure 6: The cases of Canada and the U.K.

procedures. For instance, in Ceylan and Abiyev (2016), the optimistic view concerning the countries of the European Union is likely to derive (at least) partially from the adopted benchmark: not the technological leader but the country-average (see also Islam, 2003, on this subject)[10].

The present evidence is also qualitatively very close to my previous one (Lopes, 2016), based on a previous version of the Maddison database, ending in 2008 and therefore with a significantly smaller sample size, but making no allowance for the possibility of breaks in the level of the series. The shifting composition of the evidence therefore seems to result mainly from two conflicting forces:

a) both the augmentation of the sample size and the allowance for breaks tend to produce tests with improved power properties, lending support to convergence;

b) however, the developments that have followed the burst of the global financial crisis appear having derailed some economies from their steady path, at least in relative terms, making them move away from the convergence trajectory.

This seems to be the case particularly for Canada and Denmark, now negatively labelled as non-converging, in stark contrast with my previous results (see figure 2 for the case of Denmark and figure 6 below for Canada). Moreover, the graphical analysis indicates that several other economies appear having been particularly disturbed by the events initiated in 2007–2008: this is the case clearly for Australia,

[10]For an opposite outcome concerning the EU but using rather different methods see Franks et al. (2018).

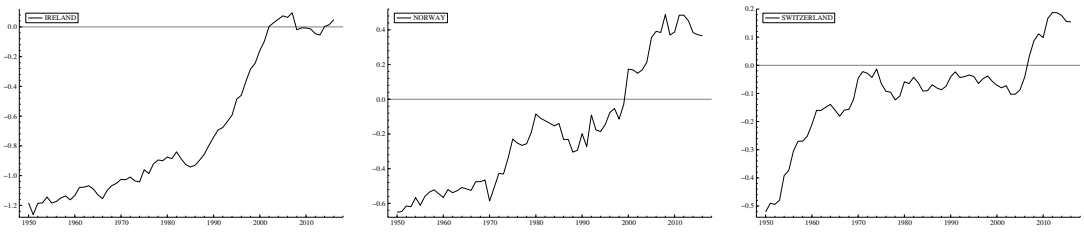

Figure 7: The cases of Ireland, Norway and Switzerland

Finland, and Spain, and not so significantly for Belgium and Sweden; the case of the U.K. is similar but the relative decay began sooner, closer to the beginning of the century (see figure 6). The inauguration of the financial crisis therefore seems to mark the beginning of a new phase of transitional dynamics for many countries and it is yet unclear when and how (and whether) it will stabilize.

On the other hand, I reiterate that the view of convergence that I adopt here is distinct from the usual catching-up approach, which allows and even rests on the presence of a trend component in the logged output gap. If that was the case, the economies of Ireland and, most notoriously, those of Norway and Switzerland had not only converged with the leader but have even overtook it, becoming the new leaders. In the case of these three economies, the interpretation of the outcome of unit root tests is that a stable path has not yet been achieved; they appear to be still in a process of transitory dynamics as well, approaching their steady state from a level above that of the (current) leader.

A final remark concerns the economic significance of the statistical evidence. Both Durlauf, Johnson and Temple (2005, DJT) and Johnson and Papageorgiou (2018, JP) question the relevance of unit root tests allowing for breaks to assess income convergence. In particular, JP argue that the "*interpretation of breaks is unclear*". However, on one hand, as DJT emphasize, the time series approach to convergence is "*largely statistical in nature*" (p. 589). Now, the failure of standard unit root tests to reject divergence when it is false because the gap series displays some sort of discontinuity is a failure of these statistical methods, which need to be patched to produce a less fragile result. On the other hand, JP themselves provide the key to the interpretation problem when they make the question "*do the breaks represent large exogenous shocks?*" Indeed, at least returning to Perron's (1989)

original argumentation, time series breaks may be viewed as a "*device*" to remove large shocks from the mean (trend) function without investing a lot of effort with its adequate modelling. It then follows that, in fact, these results are not free from the testing model. They are conditional upon it, as is usually the case.

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8 Appendix

Table A. Critical values for the FDF test

T	1%	5%	10%
	no constant case, FDF_{nc}		
50	−3.38	−2.48	−1.99
100	−3.40	−2.60	−2.11
200	−3.41	−2.65	−2.17
1000	−3.48	−2.76	−2.29
	no trend case, FDF_c		
50	−4.65	−3.97	−3.62
100	−4.52	−3.91	−3.60
200	−4.46	−3.88	−3.57
1000	−4.41	−3.85	−3.55
	with trend case, FDF_{ct}		
50	−5.32	−4.65	−4.31
100	−5.12	−4.53	−4.24
200	−5.04	−4.47	−4.20
1000	−4.96	−4.43	−4.16

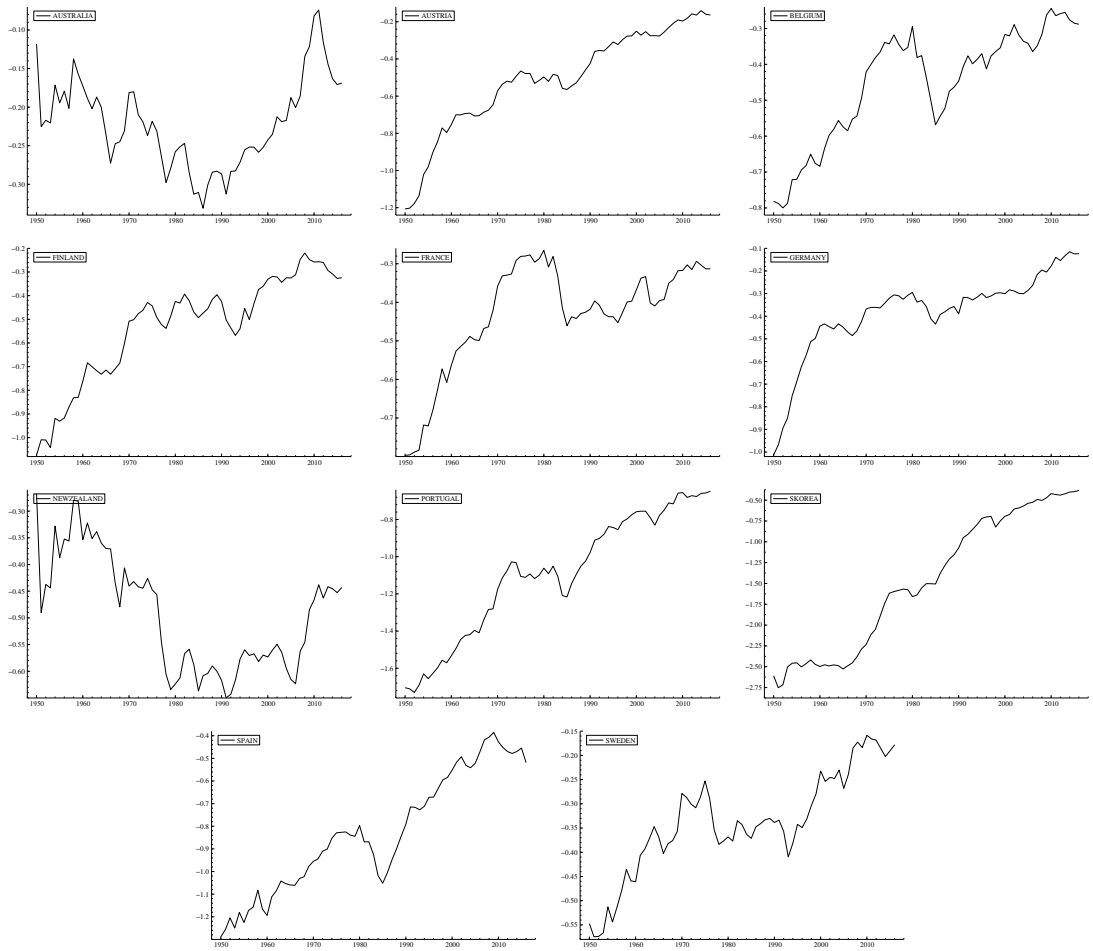

Figure 8: The remaining countries: Australia, Austria, Belgium, Finland, France, Germany, New Zealand, Portugal, South Korea, Spain and Sweden.