Correcting the Error in Gamma Discounting

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Abstract: In "Gamma Discounting" Martin L. Weitzman concludes that certainty equivalent discount rates should decline significantly over time. He draws this conclusion from fitting a Gamma distribution to the responses of 2,160 economists asked to give a discount rate estimate and calculating effective discount rates from it. This paper shows that Weitzman's model is based on an erroneous definition of expected present value. Correcting the definition changes Weitzman's conclusions, explains the Pazner and Razin discrepancy and solves the Weitzman-Gollier puzzle under risk neutrality. The assertions of this paper are corroborated by calculations based on data found in Weitzman (2001).

Keywords: Weitzman-Gollier puzzle; declining discount rates; discounting.

JEL Codes: D61, H43

Martin L. Weitzman's articles "Gamma Discounting" (2001), which followed up on his "Why the far-distant future should be discounted at its lowest possible rate?" (1998) are arguably the two most important antecedents of the adoption by several countries of his recommendation that discount rates should decline over time (DDR). In setting official policy on this matter for the UK, The Green Book gives the following arguments: "Where the appraisal of a proposal depends materially upon the discounting of effects in the very long term, the received view is that a lower discount rate for the longer term (beyond 30 years) should be used. The main rationale for declining long-term discount rates results from uncertainty about the future. This uncertainty can be shown to cause declining discount rates over time." (HMT 2003) A footnote to the last sentence cites Weitzman (1998) and Weitzman (2001). The OECD (2018) Cost-Benefit Analysis manual cites Weitzman's work and states "it can be shown that the certainty-equivalent decreases with time."

In its abstract Weitzman (2001) presents the following summary: "A numerical example is constructed from the results of a survey based on the opinions of 2,160 economists. The main finding is that even if every individual believes in a constant discount rate, the wide spread of opinion on what it should be makes the effective social discount rate decline significantly over time." This assertion has two components: (1) that the "effective discount rate" is a declining function of time and (2) that it is so because of the wide spread of the opinions received in the survey.

One great advantage of Weitzman (2001) is that it provides the results of the survey, which makes it possible to use its own numerical example to check the validity of its claims. This paper challenges both of Weitzman's assertions. Section 1 will address the claim that effective discount rates should be declining; Section 2 will process the numerical example, corroborate the conclusions of Section 1, and address the claim that the first assertion is the consequence of the responses to the survey; Section 3 will present conclusions.

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1. What is the correct certainty equivalent discount rate?

Before addressing matters of substance, it is important to define what "effective social discount rate" means. In Weitzman (2001) the term effective is used to describe the function defined by expression (1) which clearly aims to compute an expected present value. Weitzman derives a "certainty equivalent discount rate" in Weitzman (1998) from an expression analogous to (1). For this reason, this paper uses the term certainty equivalent discount rate, or more briefly, certainty equivalent rate (CER) in the same sense that Weitzman (2001) uses the term "effective discount rate."

As for the "social" qualifier, we interpret this the same way as the OECD (2018) Cost-Benefit Analysis manual does: "As long as projects and policies are being evaluated from society's point of view, *s* is a social discount rate" in the belief that this is the sense given by Weitzman to this term.

As neither Weitzman (1998) nor Weitzman (2001) mentions utility functions, the expected present or future values calculated are interpreted as risk neutral, and we assume that the interest rates mentioned pertain to some capital market.

Weitzman (2001) *defines* "the expected value today of an extra expected dollar at time *t*" as follows:

$$A(t) \stackrel{\text{def}}{=} \int_0^{\infty} e^{-xt}\ f(x)\, d(x) \tag{1}$$

where *f(x)* is a probability density function giving the probability of the interest rate being *x*.

For this discussion, we convert the above expression to the expression (2). The only difference is that instead of having a continuous probability density function of interest rates *x*, we assume that interest rates can be any $\{r_i\}$ with probability $\{p_i\}$:

$$A(t) \stackrel{\text{def}}{=} \sum p_i e^{-r_i t} \tag{2}$$

These are alternative forms of the same definition of present value, on which all of Weitzman's conclusions are based. We will show that this definition is wrong.

The certainty equivalent discount rate that can be derived from (2) is the following[1]:

$$r^w(t) = -\left({}^1\!/_t\right)\ ln\left(\textstyle\sum_i p_i e^{-r_i t}\right) \tag{3}$$

Pazner and Razin (1975) had already observed that CERs derived from the expected value of discount factors, such as (2), are lower than those that can be derived from the expected value of compound factors. We define the expected future value (*EFV*) of \$1, using the same terminology as before, as follows:

$$EFV(t) \stackrel{\text{def}}{=} \sum p_i e^{r_i t} \tag{4}$$

The certainty equivalent discount rate that can be derived from (4) is the following:

[1] Superscript *w* identifies Weitzman's method, while the asterisk superscript will be used to denote the correct method.

$$r^*(t) = \left(1/_t\right)\ ln(\textstyle\sum_i p_i e^{r_i t}) \tag{5}$$

Gollier (2004) pointed out, thereby launching the Weitzman-Gollier puzzle, that while $r^w(t)$ is a declining function of time, $r^*(t)$ is an increasing one.

An unstated assumption of Weitzman's models explains this behavior: interest rates are perfectly autocorrelated. This is evident in (2), as for any given scenario *i* the interest rate will be the same r_i regardless of the value of *t*. This creates the following mechanism. For high values of r_i, distant future values will become very high, far exceeding those resulting from lower r_is. The disproportionally high contribution to *EFV*s by higher r_is will cause *CER*s to tend to the highest possible interest rate. The same happens in reverse with Weitzman's proposed discounting method. In that case it is the low r_is that generate the largest present values and thus pull CERs downwards. As these effects get stronger as *t* increases, $r^w(t)$ will decline towards the lowest possible r_i, whereas $r^*(t)$ will increase towards the highest possible r_i.

Regarding $r^w(t)$ and $r^*(t)$ Pazner and Razin (1975) concludes that "as the two criteria discussed here are equally likely, on *a priori* grounds, to be used as guides to investment decision making, and as their use may provide different rankings of investment prospects, the question arises as to what is the correct way to approach the problem in general." The confusion resurfaced after Gollier's (2004) observation. The discrepancy was universally regarded as troublesome. Ben Groom, Cameron Hepburn, Phoebe Koundouri and David Pearce (2005) characterized the puzzle as follows:

> "So, confusingly, whereas in the absence of uncertainty the two decision criteria are equivalent, once uncertainty regarding the discount rate is introduced the appropriate discount rate for use in CBA depends upon whether we choose ENPV or ENFV as our decision criterion. In the former case, discount rates are declining and in the latter they are rising through time. It is not immediately clear which of these criteria is correct."

Because $r^w(t)$ is derived from a certain future value (FV) and a stochastic EPV, and $r^*(t)$ is derived from a certain present value (PV) and a stochastic EFV, Gollier (2004) stated that "Taking the expected net future value is equivalent to assuming that all risks will be borne by the future generation. […] Using the expected net present value implicitly means that it is the current generation who bears the risk." This is a strange remark, given the assumption of risk-neutrality implicit in the fundamental papers of the puzzle. But Gollier (2016) went further: "the risk-neutrality assumption underlying the two discounting rules is technically incompatible with an uncertain interest (or discount) rate […] Thus, in order to reconcile the basic ingredient of the gamma discounting approach (*i.e.*, uncertain interest rates with economic theory), a model with a risk-averse representative agent must be considered."

Indeed, most of the literature trying to reconcile the two approaches appeals to the notion of risk-aversion, and this is the basis on which Gollier and Weitzman (2010) claimed to have solved the puzzle[2].

But this is not how this paper will proceed. Whenever a common set of premises yields mathematically conflicting results, either there must be calculation errors, or the premises must be inconsistent. We can discard calculation errors; consequently, the puzzle can only

[2] A claim disputed in Szekeres, Sz. (2017) Checking Gollier and Weitzman's solution of the "Weitzman–Gollier puzzle".

be due to conflicting premises. Therefore, we must examine the two definitions: the definition EFV and the definition of EPV.

The EFV of \$1 is defined by expression (4), repeated for convenience:

$$EFV(t) \stackrel{\text{def}}{=} \sum p_i e^{r_i t} \tag{6}$$

This expression describes a stochastic process in which the initial \$1 is compounded alternatively by all possible r_is and each of the i results obtained is weighted by the corresponding probability p_i. As this is simply the application of the definition of expected value $E[f(x)] = \Sigma f(x_i)p_i$ to $f(x) = exp(tx)$, we will assume that the definition of $EFV(t)$ is correct.

Then it must be $A(t)$, the definition of expected present value used both in Pazner and Razin (1975) and in Weitzman (1998, 2001), that is wrong. Odd, given that it also involves probability weighting an expression that is correct in deterministic cases, which is why this definition has been seldom questioned. But as it is incompatible, none the less, with the definition of EFV, it is best to go back to first principles.

The correct expected present value (*EPV*) of a future \$*1* can be derived directly from the textbook definition of present value. It is the amount that will compound to the *EFV* of \$1 at the going (stochastic) market rate *r*. If $EFV(t) = 1$, as in (2), then

$$EPV(t) \sum p_i e^{r_i t} \stackrel{\text{def}}{=} 1 \tag{7}$$

It is a fact of mathematics that the correct EPV derived from (7) is the following:

$$EPV(t) = \frac{1}{\sum p_i e^{r_i t}} \tag{8}$$

The correct *EPV(t)* is different from Weitzman's *A(t),* as defined in (2):

$$EPV(t) \stackrel{\text{def}}{=} \frac{1}{\sum p_i e^{r_i t}} \neq \sum p_i e^{-r_i t} \stackrel{\text{def}}{=} \mathrm{A(t)} \tag{9}$$

It might be difficult to accept that the expected value of the scenario specific discount factors does *not* equal the certainty equivalent discount factor. To help see it, it might be useful to closely examine the simplest possible practical example. Let the stochastic r_i be one of $\{r_1, r_2\}$ with probabilities $\{p_1, p_2\}$. Expression (9) then becomes:

$$EPV(t) \stackrel{\text{def}}{=} \frac{1}{p_1 e^{r_1 t} + p_2 e^{r_2 t}} \neq \frac{p_1}{e^{r_1 t}} + \frac{p_2}{e^{r_2 t}} \stackrel{\text{def}}{=} A(t) \tag{10}$$

As exponentiation is not distributive over addition, the following reformulation highlights the difference:

$$(p_1 e^{r_1 t} + p_2 e^{r_2 t})^{-1} \neq p_1 (e^{r_1 t})^{-1} + p_2 (e^{r_2 t})^{-1} \tag{11}$$

A numerical example will help illustrate why Weitzman CERs are lower than those obtained from correct discounting, and support an argument made in Section II. Following with the two states of the world example, let's assume that $p_1 = p_2 = 0.5$, and taking numbers from

Weitzman (2001), let us assume that r_1 = one percent, the lowest survey response that was higher than zero, and r_2 = 27 percent , the highest response received, and let t = 300.

The marginal or instantaneous discount rate corresponding to *A(t)* for any time *t* can be calculated using equation (8) in Weitzman (2001), which takes the following form with our two-states of the world example:

$$R(t) = -\frac{\dot{A}(t)}{A(t)} = -\frac{-p_1 r_1 e^{-r_1 t} - p_2 r_2 e^{-r_2 t}}{p_1 e^{-r_1 t} + p_2 e^{-r_2 t}} \tag{12}$$

Using the assumed values of the numerical example we get:

$$R^w(300) = -\frac{-0.5\frac{0.01}{e^3} - 0.5\frac{0.27}{e^{81}}}{\frac{0.5}{e^3} + \frac{0.5}{e^{81}}} = 0.01 \tag{13}$$

where the first exponent of *e* is the product of *t* and r_1 (300 0.01 = 3) and the second is the product of *t* and r_2 (300 0.27 = 81). Notice that r_2 in the numerator is weighted by $1/e^{81}$, which means that the higher interest rate hardly contributes to the result.

Proceeding the same way for *EPV(t)*, we get the following after differentiating and simplifying:

$$R^*(300) = \frac{p_1 r_1 e^{r_1 t} + p_2 r_2 e^{r_2 t}}{p_1 e^{r_1 t} + p_2 e^{r_2 t}} = \frac{0.005e^3 + 0.135e^{81}}{0.5e^3 + 0.5e^{81}} = 0.27 \tag{14}$$

where the factors in the numerator are the probability weighted interest rates assumed. Notice that in this case it is the low interest rate that has hardly any weight, and therefore the result is equal to the higher rate.

The Weitzman discounting CER that corresponds to the assumptions of the example can be calculated using expression (3) and is the following:

$$r^w(300) = -\left(1/_{300}\right)\, ln\left(\frac{0.5}{e^3} + \frac{0.5}{e^{81}}\right) = 0.012 \tag{15}$$

The CER calculated by the correct discounting method can be derived from (8) and is the same *r*(t)* that is defined by (5).

$$r^*(300) = \left(1/_{300}\right)\, ln\left(0.5e^3 + 0.5e^{81}\right) = 0.268 \tag{16}$$

The fact that $R^w(300) < r^w(t)$ shows that Weitzman CERs decline as a function of time, while that $R^*(300) > r^*(t)$ indicates that correct CERs are a growing function of time.

In this very same context, Pazner and Razin (1975) already observed that by Jensen's inequality $r^w(t) < r^*(t)$, which the above numerical example confirms. Given that both rates have been defined for the same *EFV(t)* = \$1, their relative magnitude implies the following, since a lower rate implies a higher present value:

$$\sum p_i e^{-r_i t} > \frac{1}{\sum p_i e^{r_i t}} \tag{17}$$

While the cause of this inequality is clear, what explains the magnitude of the difference between the two sides of the inequality? Let us define random variable *X* to be *exp*($r_i t$) and random variable *Y* to be *1/exp*($r_i t$). The expected values of random values *X* and *Y* are related as follows:

$$E[XY] = \; E[X]E[Y] + cov(X, Y) \tag{18}$$

Given that *E[XY] = 1* because *Y* is the reciprocal of *X,* we can rearrange (18) as follows:

$$E[Y] = \; \frac{1 - cov(X,Y)}{E[X]} \tag{19}$$

which means that

$$\sum p_i e^{-r_i t} \; = \; \frac{1 - cov(\, e^{r_i t}, \; e^{-r_i t})}{\sum p_i e^{r_i t}} \tag{20}$$

This relationship will be corroborated in Section II with data from Weitzman (2001). The difference between Weitzman's *A(t)* and the correct *EPV(t)* is not a puzzle, but the predictable difference between an incorrect and the correct definition of EPV.

If the right-hand side of (17) is the correct *EPV(t),* why would anyone choose the left-hand side, *A(t),* to define it? Possibly because it would make no difference if the covariance in (20) were equal to zero. In other words, if there were no uncertainty, it would be true that:

$$EPV(t) = \; e^{-xt} \tag{21}$$

To convert (21) into a stochastic expression by making *x* a random variable to be probability weighted is risky when *x* is the argument of a nonlinear function. It turns out that to be a mistake in this case. This is the seductive trap into which Weitzman and many others fell. Expression (1) is a fallacy because the expected value of the inverses is not equal to the inverse of the expected value.

That *A(t)* is not the correct certainty equivalent discount factor is proven by the fact that *A(t)* does not compound to the assumed *EFV(t)* of 1, thus violating the definition of present value:

$$\sum p_i e^{-r_i t} \; \sum p_i e^{r_i t} \neq 1 \tag{22}$$

Both the Pazner and Razin discrepancy and the Weitzman-Gollier puzzle disappear when the textbook definition of expected present value is used. Notice that while the EFV definition describes the process of compounding, the correct EPV definition merely states "find a value *EPV(t)* such that it compounds to value *EFV(t)*," without defining a computational procedure of its own. There can be no conflict between the two definitions, as the same computational procedure is used for both compounding and discounting, in which taking a reciprocal is the only additional step. *EPV(t)* and *EFV(t)* only differ in proportion to a positive constant: the expected compound factor. For this reason, the NPV and NFV investment rules cannot conflict. When correctly calculated, they will only differ by a positive factor, which ensures their equivalency.

EPV(t) could also be found by numerical methods, to any desired degree of precision, iterating until the compounded value of *EPV(t)* equals *EFV(t)*. Correctly calculated *EPV(t)* and *EFV(t)* pairs will always be congruent and will always be related to each other by the following expression:

$$EPV(t) = \frac{EFV(t)}{\sum p_i e^{r_i t}} \tag{23}$$

No value other than the above *EPV(t)* is the present value of *EFV(t).* Weitzman's *A(t),* which uses a different computational procedure, is therefore not the present value of *EFV(t).* For a conceptual interpretation of what Weitzman's *A(t)* actually computes, see Szekeres (2017).

In the light of the above, it can be concluded that expression (1), the premise of Weitzman (2001), is wrong. Correct results can only be derived from the following expression:

$$A(t) \stackrel{\text{def}}{=} \frac{1}{\int_0^{\infty} e^{xt} f(x)\, d(x)} \tag{24}$$

Further, only CERs derived from (24) would be congruent with the assumed future values of $1 for all time horizons. Using *R(t),* the marginal discount rate, as Weitzman proposed, would produce incongruent (lower) results.

These findings will be empirically corroborated in the next Section using the survey responses reported on in Weitzman (2001). The error introduced by using marginal discount rates instead of CERs will be shown in Appendix A.

2. Results of the correct and incorrect discounting methods

Weitzman's (2001) reports on the results of a survey of 2,160 economists who were asked to name the real interest rate to be used in appraising climate change mitigation projects. The responses ranged between –3 percent and 27 percent. Weitzman fitted a Gamma distribution to the responses received, as in Figure 1, and proceeded to compute marginal discount rates. He used the Weitzman discounting method implicit in incorrect expression (1).

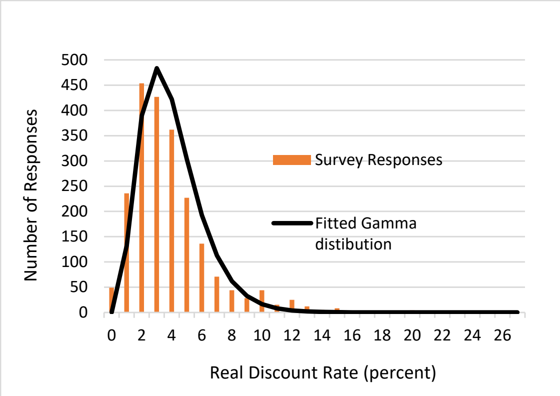
We repeated the calculations using both the Weitzman and the correct discounting methods. For transparency and ease of replicability, we used the discrete probability distribution derived from the raw survey responses, without the intervening step of fitting a Gamma distribution, which is not essential to test the basic claim of Weitzman (2001), namely that CERs are a declining function of time[3]. We discarded the three negative interest rate responses[4]. The calculations, computing expressions (3) and (5) with the survey data, are straightforward and easy to follow in the accompanying Excel[5] workbook, so no details are included here.

[3] See Appendix A for a comparison between the results obtained using the frequency distribution of responses directly with those obtained using Weitzman's fitted Gamma distribution.

[4] Weitzman probably did the same, for two reasons: (1) negative interest rates are explicitly ruled out in Weitzman (1998) and (2) the Gamma distribution is not defined for negative values. Leaving the negative observations in would result in negative CERs for large enough values of *t* when using Weitzman discounting.

[5] https://doi.org/10.3886/E119568V1

FIGURE 1. ACTUAL AND FITTED FREQUENCY DISTRIBUTIONS



As will be shown below, the actual content of the probability distribution used is of no consequence. In Footnote 5, page 264, Weitzman (2001) states "To the extent that some panel members may believe in declining discount rates, the basic conclusions of this paper will only be strengthened. In this spirit, the main conclusion might be stated as follows: Even if everyone believes in a constant discount rate, the effective discount rate declines strongly over time." This is true. *Any* probability distribution will do so, regardless of its skewness value, provided Weitzman discounting is used. The expression "Gamma discounting" is a misnomer therefore, for the fitted Gamma distribution only describes the distribution of survey results, not the method of discounting, which is Weitzman's.

FIGURE 2. WEITZMAN AND CORRECT DISCOUNT FACTORS

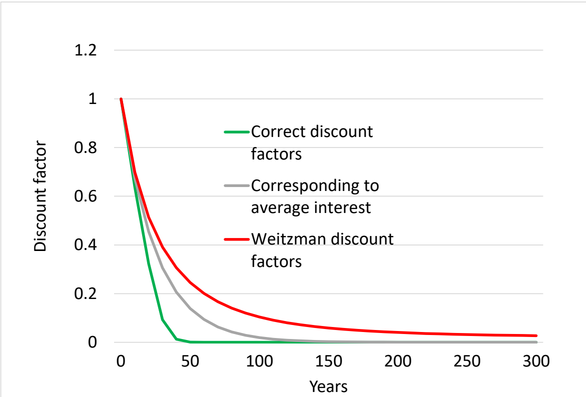
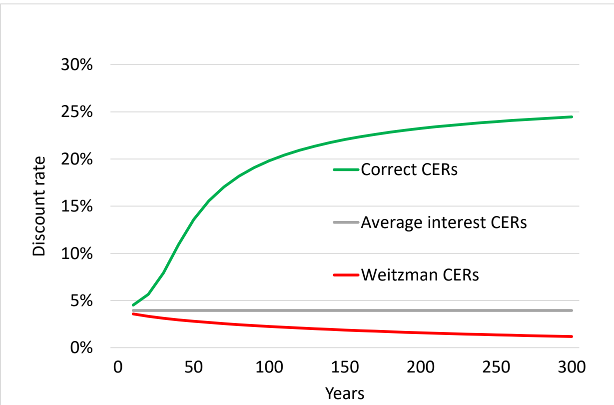


FIGURE 3. WEITZMAN AND CORRECT CERS



The differences between the correct and incorrect calculation results are huge, and the correct results invalidate the conclusion of Weitzman (2001) that "society should be using effective discount rates that decline from a mean value of, say, around 4 percent per annum for the immediate future down to around zero for the far-distant future." Weitzman's year 300 CER is 1.2 percent, but the correct value is 24.5 percent. Figure 2 shows how the Weitzman and correct discount factors change as a function of time, and Figure 3 depicts the corresponding CERs. In both figures the values corresponding to the deterministic average interest rate of 3.96 percent are shown.

Weitzman's assertion that it is "the very wide spread of professional opinion on discount rates" that causes CERs to be a negative function of time is untrue, as only the most extreme values matter. If we invert the survey results by attributing the response frequency of the highest interest rate to the lowest, and so on in sequence, in effect flipping the distribution horizontally, Weitzman's CERs still decline monotonically and tend towards the lowest possible value.

Figure 4 shows the inverted frequency distribution of responses and Figure 5 the corresponding CERs. Inverting the frequency distribution increases the mean value of the observations from 3.96 percent to 20.4 percent. Consequently, the correctly calculated CERs

FIGURE 4. INVERTED FREQUENCY DISTRIBUTION

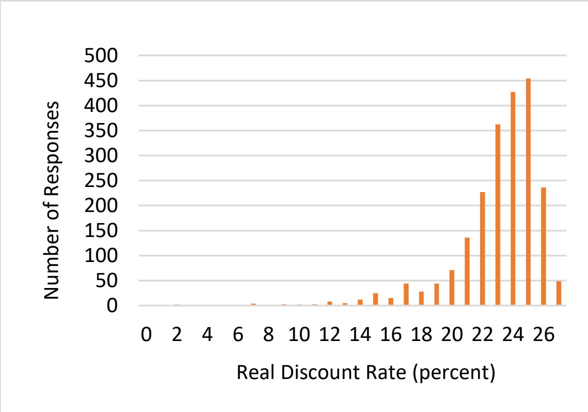
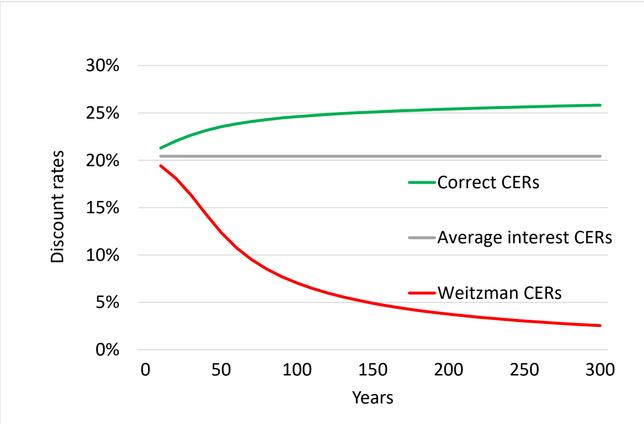


FIGURE 5. INVERTED PROBABILITY WEITZMAN AND CORRECT CERS



increase from the mean, reflecting the yield-boosting effect of the perfect correlation assumption, while Weitzman's CERs decline towards the lowest possible interest rate, which is still zero percent. This low value now has a lower probability than before, so it will take longer for Weitzman CERs to converge to it, but they will get there eventually, regardless of how all respondents other than those whose answer was zero percent opined.

The numerical example of Section I bears out this conclusion. In the equiprobable two-scenario example $r^w(300)$ was 1.2 percent, the same as the CER calculated from the frequency distribution of survey responses.

It is easy to check in the accompanying Excel workbook that Weitzman discounting is inconsistent with the definition of present value. For example, discounting a safe \$1 due in year 100 has a Weitzman expected "present value" of \$0.103895747 (The correct value is 2.47E-09). Compounding that amount with the same probability distribution of interest rates yields an expected future amount of \$42,108,494. This kind of inconsistency never happens with correctly calculated expected discount factors because they are the inverses of the corresponding expected compound factors. Consequently, all correctly calculated *EPV(t)* compound to the *EFV(t)* of \$1.

We can corroborate the validity of expression (20) of Section 1 by using the computed values for year 100. The requisite expected compound factor and covariance are calculated in the accompanying Excel workbook.

$$\sum p_i e^{-r_i t} = \frac{1 - cov(e^{r_i t},\ e^{-r_i t})}{\sum p_i e^{r_i t}} = \frac{1 -(-42108493)}{405295645.4} = 0.103895747 \qquad (25)$$

This result, already cited above, confirms that the magnitude of the error in Weitzman discounting can be readily computed and is not a puzzle.

The results obtained from replicating Weitzman's calculations with both his discounting method and the correct one allows us to draw the following conclusions:

1. Weitzman's discounting method is incorrect, because it does not correctly calculate the present values of future sums. This is demonstrated by the fact that his calculated results do not compound back to the amount originally discounted.

2. Weitzman's recommendation that "society should be using effective discount rates that decline" with time is not a correct conclusion from his model and the survey data he employed. The opposite is true based on his assumptions: CERs should be increasing.
3. Weitzman's assertion that his conclusions derive from the "the wide spread of opinion" in the survey is untrue. Ultimately only the opinion of those giving the lowest response matters.

3. Conclusions

In the preceding two sections we have shown that Weitzman (2001) is wrong in its two principal claims: (1) based on his data and assumptions, CERs should not be declining, but increasing, and (2) his claimed result was not a reflection of the diversity of opinions of the 2,160 economist surveyed, but is an intrinsic property of his discounting method.

The fundamental flaw in Weitzman (2001) is that it is premised on an incorrect definition of expected present value. Once this is recognized and remedied, the correct results emerge. Both the Pazner and Razin discrepancy and the Weitzman-Gollier puzzle disappear when the textbook definition of expected present value is used. No consideration of risk aversion is germane to the original puzzle.

Weitzman's model is certainly not characteristic of any real capital market from which the opportunity cost of capital could be derived for the appraisal of real-world projects, so the correct results shown above are not a basis for advocating *increasing* discount rates. Further, arguments other than Weitzman's have been advanced for DDRs, and those are not addressed here. But the problem with Weitzman's DDR proposal is that it suffers from a double burden: it assumes an unrealistic degree of autocorrelation of interest rates and uses an incorrect calculation method. His recommendations should not be followed.

Weitzman's results caused well founded unease for some: how could a market characterized by the yield-boosting effect of perfectly autocorrelated interest rates yield declining CERs? But his results might have been welcomed by others, those eager to find a solution to the problem that many dubbed the "tyranny of the present over the future associated with constant rate discounting" (Groom et al 2005). Expression (1) was taken to be a definition; it had not yet been recognized to be a fallacy[6]. And Gollier and Weitzman (2010) claimed to have solved the puzzle leaving the DDR recommendation intact.

"The successful deployment and dissemination of DDRs suggests that, for better or worse, academic economists can enslave practical men with economic ideas." (Groom and Cameron, 2017) Regrettably, it was for worse in this case. There are always twists and turns in science, however, and it is better to correct errors than to let them remain undetected.

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Appendix A

In this Appendix we compare the results obtained in our calculations based on the frequency distribution of the survey responses to those that can be calculated using the formulas developed in Weitzman (2001) with the parametrization of the Gamma distribution that it contains.

Weitzman (2001) shows that the solution of the integral in expression (1) of the main text, after *f(x)* has been replaced by the Gamma distribution with parameters α and β, is the following:

$$A(t) = \left(\frac{\beta}{\beta+t}\right)^{\alpha} \tag{26}$$

The values of α and β relate to the mean (μ) and standard deviation (σ) of the observed frequency distribution of responses in the following way:

$$\alpha = \frac{\mu^2}{\sigma^2} \tag{27}$$

$$\beta = \frac{\mu}{\sigma^2} \tag{28}$$

The expected value *A(t)* can be computed by reference the statistical descriptors of the survey responses as follows:

$$A(t) = \frac{1}{(1+t\sigma^2/\mu)^{\mu^2/\sigma^2}} \tag{29}$$

As *A(t)* is the "present value" calculated by Weitzman's method, the corresponding CER can be computed from the following:

$$e^{-tr_t^w} = \frac{1}{(1+t\sigma^2/\mu)^{\mu^2/\sigma^2}} \tag{30}$$

From which:

$$r^w(t) = -\frac{1}{t}ln\left(\frac{1}{(1+t\sigma^2/\mu)^{\mu^2/\sigma^2}}\right) \tag{31}$$

Taking μ = 3.96 percent and σ = 2.94 percent we calculated the $r^w(t)$ that are comparable to the ones we calculated from the frequency distribution of responses.

This is not how Weitzman calculated his proposed discount rates, however, even though this is the method that gives the exact CER corresponding to (1). Instead, he calculated the marginal discount rate using the following (equation 22 in Weitzman 2001), which will yield lower values than $r^w(t)$, as Table 1 shows.

$$R^w(t) = \frac{\mu}{(1+t\sigma^2/\mu)} \tag{32}$$

Even with correct discounting, the marginal discount rate ought not be used for discounting, as it would not ensure the congruence between *EPV(t)* and *EFV(t).* For comparability we computed $R^w(t)$ retaining the same μ = 3.96 percent and σ = 2.94 percent values, even though Weitzman rounded both up to 4 and 3 percent, respectively.

Table 1 compares the variously calculated CERs and marginal discount rates for selected years. The results are presented in the following sequence: $r^w(t)$ from the frequency distribution based calculation, $r^w(t)$ from equation (31), $R(t)$ from equation (32) and, for reference, $r^*(t)$ from the frequency distribution based calculation.

TABLE 1 – COMPARISON OF DISCOUNT RATES (IN PERCENT)

	Calculation based on incorrect EPV definition			Correct Def.
	$r^w(t)$	$r^w(t)$	$R^w(t)$	$r^*(t)$
	Freq.	Gamma	Gamma	Freq.
Years	Dist.	Eq. 31	Eq. 32	Dist.
10	3.6	3.6	3.3	4.5
20	3.3	3.3	2.8	5.7
30	3.1	3.0	2.4	7.9
40	3.0	2.8	2.1	10.9
50	2.8	2.7	1.9	13.6
60	2.7	2.5	1.7	15.6
70	2.6	2.4	1.6	17.0
80	2.5	2.3	1.4	18.2
90	2.4	2.2	1.3	19.1
100	2.3	2.1	1.2	19.8
150	1.9	1.8	0.9	22.1
200	1.6	1.5	0.7	23.2
250	1.4	1.4	0.6	24.0
300	1.2	1.2	0.5	24.5

By comparing the first two columns we can see that the rates are nearly the same, which is not surprising given the good fit of the Gamma distribution to the survey responses. However, it is not the shape of the Gamma distribution that is responsible for this agreement. The observed $r^w(300) = 1.2$ was also obtained in the two states of the world numerical example of Section 1. It is only the values of the lowest observations that matter in Weitzman discounting.

Weitzman's recommendation to use $R^w(t)$ instead of $r^w(t)$ is wrong, as it understates the applicable discount rate. For year 300 it is less than half of the corresponding $r^w(t)$, which is also wrong, of course. The correct result for year 300, derived from Weitzman's assumptions and the survey results, is 24.5 percent.