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Abstract

This paper develops a general equilibrium model of international trade in homogenous intermediate inputs. In the model, trade between countries is driven exclusively by uncertainty in the delivery of inputs. Because their managers are risk-averse, final good firms contract with multiple suppliers located in different countries in an attempt to decrease the variability of their profits. The analysis shows that risk diversification provides an incentive for international trade over and above such reasons as comparative advantages (emphasized in classical models of international trade) and economies of scale (emphasized in new trade models), and highlights a new channel – a reduction in uncertainty – through which trade liberalization increases welfare.

Keywords: Intermediate inputs, international trade, sourcing, trade liberalization, uncertainty.

JEL Classification Numbers: F1.

1 Introduction

Understanding the determinants of bilateral trade patterns is one of the most important questions in international economics. Although countries' incomes and the trade barriers between them occupy a central place in explaining international trade flows since Tinbergen (1962), recent studies point to the importance of a new margin: supply-chain risk management. For example, Wolak and Kolstad (1991) develop an empirical methodology to estimate the role of risk aversion in explaining Japanese steam-coal imports. According to their estimates, Japan is willing to pay 29 to 50 percent above the current market price for a supply of coal having no price uncertainty.[1] This rationalizes the fact that, in the mid-80s, Japan imported more than twice the amount of steam-coal from Australia than it did from South Africa although both countries had approximately the same mean price over the period. More recently, Gervais (2016, 2018) reports statistically significant negative associations between exporter-level input prices variance (a measure of uncertainty) and import demand, and provides evidence that input demand is spread more evenly across suppliers when there is more uncertainty in the upstream industry. As a final example, Heise et al. (2017) show, both theoretically and empirically, that changes in trade policy uncertainty affect the firm's optimal procurement strategy. Overall, these studies suggest there may be a link between uncertainty and sourcing decisions.

A number of recent papers extend new trade theories to study the effects of uncertainty on the export decisions of firms, the production location decisions of multinational enterprises, and the effect of trade on income volatility.[2] However, the role of supply-chain risk management as *impetus* to international trade has, so far, received much less attention. This is surprising for at least two reasons. First, the growth of world trade in recent decades is mainly due to the increase in intermediate goods trade. According to available estimates, inputs account for as much as two-thirds of international trade flows (e.g. Feenstra (1998), Hummels et al. (2001), Johnson and Noguera (2012), and Timmer et al. (2014)). Second, supply-chain risk management is known to be an important determinant of firm performance (e.g., Tang (2006), and Tang and Musa (2011)). Together, these two observations suggest that supply-chain risk management potentially plays an important role in explaining international trade patterns.

In this context, the current paper's main objectives are to develop a general equilibrium model of international sourcing that features uncertainty, but intentionally omits the standard

[1] Building on the work of Wolak and Kolstad (1991), Appelbaum and Kohli (1997) estimate the impact of uncertainty on oil and non-oil import demand functions for the United States, Appelbaum and Kohli (1998) estimate the effects of import-price uncertainty on factor income in Switzerland, and Muhammad (2012) estimates carnation demand in the United Kingdom.

[2] See for example, Ramondo et al. (2013), Caselli et al. (2015); Fillat and Garetto (2015), Fillat et al. (2015), Esposito (2019), and De Sousa et al. (2020).

motives for trade (such as technological differences, relative factor endowment differences, increasing returns to scale, and product differentiation), and to use the model to study the general equilibrium effects of trade liberalization on welfare in the presence of uncertainty.

For the theoretical model, I combine elements from modern portfolio theory and international trade theory to develop a general equilibrium model of firm decisions which rationalizes multi-sourcing strategies. In the model, the benefits of multi-sourcing are the same as those of portfolio diversification in theoretical finance. When firms spread their input demand across a larger number of geographically diverse suppliers their profits become less variable, much like diversifying investment across a larger number of assets with imperfectly correlated returns reduces the variance of a portfolio's return (e.g., Markowitz (1952) and Sharpe (1964)). In equilibrium, firms select a portfolio of suppliers and a distribution of input demand across these suppliers that optimally trades off expected profits with variability. The analysis of the model shows that risk diversification provides an incentive for trade over and above such reasons as comparative advantage and economies of scale, and highlights a new channel – a reduction in uncertainty – through which trade can increase welfare.

The key assumption of the model – that the behavior of firms is consistent with risk aversion – can be motivated by the existence and size of insurance markets for businesses. As a first example, consider the business insurance industry. In the United States (U.S.), commercial lines (which include the many kinds of insurance products designed specifically for businesses) account for about half of U.S. property/casualty insurance industry premium or about $287 billion. A second example is the export credit and investment insurance industry.[3] The Berne Union, the leading global association that industry, reports that its members provided $2.5 trillion of payment risk protection to banks, exporters, and investors in 2018 - equivalent to about 13% of world cross border trade for goods and services. As a final example, consider the foreign exchange markets. When a business contract is entered into with the agreement that payment will be settled at a future date, the exchange rates that exist on the date the contract is entered into and the date that the contract is settled may be different. In order to manage currency exchange rate risks, firms trade derivatives on the foreign exchange market, one of the largest and deepest of all markets.

In the theoretical model, I define uncertainty as unexpected variation in output delivery originating from supplier-level productivity shocks. This captures in a simple way the fundamental impact of a broad range of potential events associated with supply-chain risk (e.g., unexpected increases in input costs, delayed shipments, or lower than expected input

[3]This type of insurance is available to exporters to hedge against the potential risks in shipment and non-payment by importers. It can also protect exporters against the risk of non-payment by a foreign buyer and cover commercial risks (e.g., insolvency of the buyer, bankruptcy, or protracted defaults/slow payment) and certain political risks (e.g., war, terrorism, riots, and revolution) that could result in non-payment. It also covers currency inconvertibility, expropriation, and changes in import or export regulations.

quality). The key mechanism at play is straightforward. Suppliers' productivity shocks can be decomposed into idiosyncratic and country-specific components. Together, these components govern the dispersion of (realized) production costs across suppliers within each country as well as the correlation between suppliers' production costs both within and across countries. In a closed economy, a multi-sourcing strategy reduces only the impact of the idiosyncratic components of productivity shocks. In contrast, firms in an open economy can simultaneously diversify away the idiosyncratic and the country-specific components of risk by purchasing inputs from domestic and foreign suppliers. Because trade provides access to more efficient diversification opportunities, a smaller share of resources is devoted to risk diversification activities (i.e., supplier-level fixed costs in the model) in an open economy and, as a result, equilibrium output per worker, a natural measure of welfare, increases following trade liberalization.

In the model, the imperfect correlation between country-level shocks provides an incentive for international trade in homogeneous inputs. Anecdotal evidence is consistent with this type of behavior. For example, gas from Russia has stopped flowing through Ukrainian pipelines three times since 2006. These events prompted the European Union to develop a strategy for energy security, which aims to limit dependence on any one source (The Economist (2019)). As another example, following the 2011 earthquake, automakers faced a shortage of a purl-luster pigment, called Xirallic, which makes paint sparkle. As a result, automakers such as Ford and Chrysler were force to suspend sales of vehicles in certain colors. The shortage happened because Merck KGaA, a German chemical company, produced the entire world's production in a single plant in Fukushima Prefecture (Dawson (2011)). Following this adverse shock, Merck began a second Xirallic production line in Germany (Tajitsu (2016)). More recently, the COVID-19 global pandemic revealed the importance of resilient supply-chains. As much as 94 percent of the Fortune 1000 companies reported supply-chain disruptions due to the coronavirus outbreak (Sherman (2020)). One of the reasons for this is the dependence of large firms on components from China (The Economist (2020)). In particular, the crisis brought to light the impact of the increasing dependence of American pharmaceutical firms on imported active pharmaceutical ingredients (e.g., Woodcock (2019)). This lead to the introduction of the "Strengthening America's Supply Chain and National Security Act," which aims to encourage geographically diverse sourcing strategies.

The extant literature provides several estimates of the impact of reduced uncertainty on trade flows derived from quantitative models of trade. While the specific mechanisms through which uncertainty affects trade flows are not the same as in the current paper, these estimates nevertheless suggest that the novel supply-chain risk management channel could contribute significantly in explaining trade flows and, as a result, generate important additional gains from trade. First, Handley and Limão (2017) develop a general equilibrium model of international trade under monopolistic competition that features trade policy uncertainty. They use their model to quantify the impact of China's accession to the WTO in 2001 on its export boom to the U.S. They find the reduced policy uncertainty in the period 2000-2005 led to an average export increase of 28 log points and increased U.S. consumers' real income by the equivalent of a 13 percentage point permanent tariff decrease. Second, Heise et al. (2017) present quantitative simulations of their procurement choice model that exploits the U.S. extension of Permanent Normal Trade Relations to China in October 2000. They find that the change in procurement patterns induced by the policy change increases U.S. imports from China by about 20%. They also show that the re-optimization of procurement practices associated with the change in trade policy increases welfare by 0.2% via a decline in final goods prices. Third, Esposito (2019) extends a standard heterogeneous-firm model of international trade to include risk-aversion and foreign demand uncertainty (i.e., firms must make export decisions before foreign demand is realized). He estimates the model in Portuguese data and finds that the risk-diversification motive explains about 15% of the observed trade patterns and increases welfare gains from trade by 16% relative to a model with risk neutrality. Importantly, these three papers control for other motives for trade and still find that uncertainty plays a significant role in explaining trade flows.

The vast majority of the recent literature on uncertainty in international trade focuses on foreign demand uncertainty. A notable exception is Heise et al. (2017) which studies the impact of policy uncertainty on the firms' selection of procurement systems. The key distinction with my work is that they focus on the optimal type of contract between a firm and its supplier, whereas I focus on the multi-sourcing behavior of firms, a key feature of trade data.[4] Among other results, they find that firms tend to enter long-term, more costly contracts when there is a high level of uncertainty, but prefer cheaper "just-in-time" practices when there is little uncertainty. This implies that a decrease in uncertainty reduces transaction costs and, as a result, increases welfare. A similar channel is active in my model. A reduction in uncertainty increases the firm's optimal input-demand per supplier, thereby increasing output per firm and decreasing the number of firms in equilibrium. These adjustments imply that a decrease in uncertainty reduces the amount of resources devoted to fixed costs and, as a result, increases welfare.

[4]First, Antras et al. (AER, 2017) use firm-level information on trade transactions and report that the average firm buys the same product from 3 countries. One has to keep in mind that this statistic is obtained from a sample that includes all products, not only homogeneous ones, such that it may overstates the number of sources for homogeneous products. At the same time, because of data limitation a source is defined as an origin country. Because it is possible that a firm buys from more than one supplier in a given country, this statistic may understate the number of suppliers per firm. Second, using U.S. Census microdata, Chung (2017) reports that U.S. firms buy most of their inputs from multiple sources (defined as countries of origin). More precisely, she finds that multi-sourced goods account for about 80 percent of import value.

While research on the impact of uncertainty on sourcing decisions is burgeoning in the field of economics, there are important theoretical literatures on supply-chain-risk mitigation in the fields of logistic management and operational research (see Snyder et al. (2012) for a review). Typically, these papers use numerical methods to solve partial equilibrium models of a single firm choosing the optimal allocation of demand across a known set of suppliers. My approach contrasts with this literature in two important aspects. First, my model provides analytical expressions for both the optimal distribution of input demand across suppliers and for the optimal set of suppliers. Therefore, instead of assuming multi-sourcing, the model explicitly describes the conditions that give rise to multi-sourcing. Furthermore, because I obtain tractable analytical expressions, I can study the impact of changes in uncertainty on the intensive and the extensive margins of sourcing using comparative statics. Second, I embed my sourcing decision framework into a general equilibrium model of international trade. The theoretical model shows how trade barriers interact with uncertainty to determine the optimal sourcing strategy. Among other results, the model predicts that, conditional on the level of uncertainty, a move from autarky to free trade will increase the optimal number of suppliers per firm (as long as the correlation between suppliers shocks is lower across-countries than within-country).

The rest of the paper proceeds as follows. In the next section, I set out an analytical model of international sourcing decisions under uncertainty. In section 3, I use the model to evaluate the effects of trade liberalization on the optimal sourcing strategy and welfare. Finally, in section 4, I present some concluding comments.

2 Theory

In this section, I develop a general equilibrium model of international sourcing under input-delivery uncertainty. Classical models of international trade show that trade between countries arises because of comparative advantages associated with differences in technologies or relative endowments, whereas new trade models show that increasing returns to scale and taste for variety are enough to generate trade. I eliminate both of these motives for trade by assuming that goods are homogeneous and that (expected) production costs are the same in every country. Together, these assumptions focus the analysis entirely on the role of uncertainty.

2.1 Production

Consider a world composed of $j = 1, ..., J$ identical countries. Each country produces two types of homogeneous output, final goods and intermediate inputs. The representative final good firm faces two technology constraints. First, it must pay a fixed set-up cost, equal to F units of labor, before production can begin. The presence of a firm-level fixed cost provides an incentive to expand production and (potentially) contract with multiple suppliers. Second, the firm must purchase intermediate inputs from suppliers. Essentially, I assume away the make-or-buy decision to focus the analysis on sourcing decisions. As in Antràs (2003), intermediate inputs can be transformed at no further cost into final goods and, without loss of generality, physical units are chosen such that

$$q = M, \tag{1}$$

where q denotes the quantity of final goods produced and M is the amount of inputs purchased.

Intermediate inputs are produced under increasing returns to scale using only labor. The supplier-level fixed cost, denoted f, is specific to each downstream firm. It reflects the resources devoted to preparing the workplace to produce inputs that meet the specifications of the downstream firm. As a result, there are no economies of scope and each supplier produces materials for a single final good firm.[5] After the fixed cost is paid, labor can be transformed at a constant rate into intermediate inputs

$$m = zl. \tag{2}$$

While the fixed cost (f) is deterministic and common to all suppliers, the productivity parameter (z) is stochastic and varies across suppliers. As equation (2) makes clear, suppliers that receive high productivity shocks (i.e., high z) need fewer workers to produce a given quantity of inputs compared to suppliers that receive low productivity shocks.

Suppliers learn their productivity, a draw from a common distribution, only after they begin production and the fixed cost is sunk. For simplicity, I assume that the mean and the variance of productivity shocks are the same in every country and denote them, respectively, by

$$\mathbb{E}(z_k) = \mu, \text{ and } \operatorname{var}(z_k) = \sigma^2, \forall\, k \in \mathbb{S}, \tag{3}$$

where $\mathbb{S}$ is the set of suppliers in the world. In Appendix A, at the end of the paper, I develop a richer model where foreign suppliers are associated with higher variance. The main results of the analysis are not affected by this generalization.[6]

[5] As in Antràs (2003) and Antràs and Chor (2013), there is an unbounded mass of *ex ante* identical (potential) suppliers. A random subset of these suppliers are matched to final good firms. Once they are matched, suppliers transact with only one final good firm.

[6] The analysis of the extended model shows that the optimal distribution of sourcing between domestic and foreign suppliers depends on the variance of productivity shocks in each country. As expected, firms find it optimal to source a larger fraction of their inputs from low variance countries.

Uncertainty in productivity reflects aggregate, country-level shocks (e.g., natural disaster, acts of war and terrorism, or epidemic) as well as idiosyncratic, supplier-level shocks (e.g., problems with machines or production defects). The correlation between the productivity shocks of any two suppliers k and h therefore depends on the location of each supplier as follows

$$\text{corr}(z_k, z_h) = \begin{cases} \rho & \text{if } k, h \in \mathbb{S}_j, \\ \delta \leq \rho & \text{if } k \in \mathbb{S}_j \text{ and } h \in \mathbb{S} \backslash \mathbb{S}_j, \end{cases} \tag{4}$$

where $\mathbb{S}_j \subset \mathbb{S}$ denotes the set of suppliers located in country j.

To make the model interesting, I assume contracts between suppliers and final good firms must be signed before the realization of uncertainty. To focus the analysis on the role of uncertainty, I assume that contract terms are enforceable by third parties and specify the distribution of conglomerate profits between final good firms and suppliers. As in Antràs and Chor (2013), suppliers can either engage in intermediate inputs production or in an alternative activity that provides zero profits. In that case, risk averse suppliers are indifferent between the outside option and producing materials for final good firms if, and only if, the contract terms adjust so that they break even in every state of the world. As a result, profit sharing contracts are structured such that final good firms' managers bear all the risk.

2.2 Consumers

In each country, there is a mass of consumers, L. Consumers are each endowed with one unit of labor and have no taste for leisure. As a result, they supply their unit of labor to the market at the prevailing wage, w. Because final goods are homogeneous and preferences are unique up to a monotonic transformation, any increasing function of consumption is a candidate to characterize the preferences of the representative consumer. Therefore, I do not impose any specific form on the utility function of consumers. Because there is no savings in the model, aggregate expenditure is equal to aggregate income in equilibrium. This implies that the aggregate demand for final goods in each country is given by $D = wL/p$, where p is the unit price of final goods.

2.3 Managers

The preferences of the final good firms' managers are represented over the firm's profits, π, by a continuously differentiable concave utility function $U(\pi)$. Managers of final good firms are risk-averse and profits are unknown when they make decisions. As a result, they maximize the expected utility of profits, $\mathbb{E}[U(\pi)]$. A difficulty with using the expected utility of profits as the objective function of the optimization problem is that it requires the full specification of the utility function. Instead, I follow the same method as Eeckhoudt et al. (2005) to obtain a tractable approximation to the managers' objective function.

Let $\mathcal{P} \geq 0$ denote the risk premium, defined as the amount of money that makes an agent indifferent between the risky return and the expected return, so that

$$\mathbb{E}\left[U(\pi)\right] = U\left(\mathbb{E}(\pi) - \mathcal{P}\right). \tag{5}$$

Taking the expectation of a second order Taylor expansion of any concave function $U(\pi)$, evaluated at $\mathbb{E}(\pi)$, yields

$$\mathbb{E}\left[U(\pi)\right] \approx U\left(\mathbb{E}(\pi)\right) + (1/2)\, U''\left(\mathbb{E}(\pi)\right) \cdot \mathrm{var}(\pi). \tag{6}$$

This equation shows that expected utility of profits depends not only on the expected level of profits but also on the variance of profits. A first order Taylor series expansion of the utility of the certainty equivalent at $\mathcal{P} = 0$ yields

$$U\left(\mathbb{E}(\pi) - \mathcal{P}\right) \approx U\left(\mathbb{E}(\pi)\right) - \mathcal{P} \cdot U'\left[\mathbb{E}(\pi)\right]. \tag{7}$$

Substituting with equations (6) and (7) into equation (5) yields a solution for the risk premium

$$\mathcal{P} \approx (\beta/2)\, \mathrm{var}(\pi), \tag{8}$$

where $\beta \equiv -U''\left(\mathbb{E}(\pi)\right)/U'\left(\mathbb{E}(\pi)\right)$ is the Arrow-Pratt measure of absolute risk aversion.[7]

Using the definition of the risk premium in equation (8) to substitute for $\mathcal{P}$ in equation (5) yields

$$\mathbb{E}\left[U(\pi)\right] \approx U\left(\mathbb{E}(\pi) - (\beta/2)\, \mathrm{var}(\pi)\right). \tag{9}$$

Because the utility function is increasing, it follows that the managers' objective function can be approximated by

$$\mathbb{E}\left[U(\pi)\right] \approx \mathbb{E}(\pi) - (\beta/2)\, \mathrm{var}(\pi). \tag{10}$$

This approximation to the general objective function is widely used in classical portfolio selection models to represent the preferences of investors (e.g., Sharpe (1964)). In the special cases where the utility function is quadratic or the productivity shocks follow a multivariate normal distribution, the expression on the right-hand side of equation (10) is exact (e.g., Samuelson (1970) or Sargent (1979)). In general, the approximation is valid only in the

[7] Using a first and a second order Taylor expansion gives a lot of tractability to the model and allows me to derive analytical solutions in a general equilibrium context. While it is straightforward to add higher terms to the expansion, analytical solutions quickly become intractable.

neighborhood of $\mathbb{E}(\pi)$, or when the skewness, kurtosis, and other higher moments of the shocks distribution tend to zero.

2.4 Firm behavior

Because the manager bears all the risk of the conglomerate, he takes the production costs of the final good firm as well as those of all of the firm's suppliers into account when making decisions. Under the maintained assumptions, the total employment associated with each firm is given by

$$\Gamma = F + \sum_{k=1}^{S} (f + l_k), \tag{11}$$

where l_k is employment at the firm's k^{th} supplier, and S is the total number of suppliers per firm.

Together equations (1), (2), and (11) imply that firm profits depend on the price of final goods and the distribution of labor across suppliers as follows

$$\pi = p \sum_{k=1}^{S} z_k l_k - w \left[F + \sum_{k=1}^{S} (f + l_k) \right]. \tag{12}$$

The first term on the right-hand side is revenue, which depends on the output price and the quantity produced. The second term is costs, which comprise firm-level fixed costs, supplier-level fixed costs, and production workers' payroll. Equation (12) makes clear that profits are stochastic because of z_k. While the distribution of employment is known *ex ante* (because suppliers are contractually obligated to commit to a certain amount of labor to the production of materials), the amount of materials delivered is learned *ex post* (once production has begun and productivity is revealed).

The final goods industry is perfectly competitive, so that managers treat output prices as given when making decisions.[8] Profit maximization involves two interrelated choices. Managers choose the set of suppliers with which to contract and the expected output (i.e., the level of employment) at each of the selected suppliers. Together, the fact that suppliers' characteristics are independent of their location and the absence of trade costs imply that, in equilibrium, employment is the same at every supplier and suppliers are evenly distributed across countries. As a result, expected profits can be expressed as a function of the number

[8] The analysis shows that risk aversion allows increasing returns to scale to be reconciled with perfect competition. This is, in a way, reminiscent of the contestable markets literature where increasing returns to scale and perfect competition are made consistent by the presence of an outside threat (e.g., Baumol et al. (1982)).

of suppliers per country (n) and the size of each supplier (l) as follows

$$\mathbb{E}(\pi) = pJnl\mu - w(F + Jnf + Jnl). \tag{13}$$

The first term on the right-hand side is expected total revenue, which is given by the product of input price (p), total number of suppliers ($S = Jn$), and expected output per supplier ($l\mu$). The second term is the deterministic total production costs. Similarly, the variance of profits can be expressed as follows

$$\text{var}(\pi) = p^2 \,\text{var}\left(\sum_{k=1}^{S} z_k l_k\right) = \{1 - \rho + [(J-1)\delta + \rho]\, n\}\, p^2\sigma^2 l^2 Jn. \tag{14}$$

The term in curly brackets accounts for the separate contributions of the variance of productivity shocks, the covariance between the shocks of suppliers located in the same country, and the covariance between the shocks of suppliers located in different countries.

Substituting with equations (13) and (14) into the objective function (10), the manager's problem can be expressed in terms of choosing the number of suppliers per country and the number of workers employed at each supplier as follows

$$\max_{n,\, l} \mathbb{E}\left[U(\pi)\right] = pJnl\mu - w(F+Jnf+Jnl) - (\beta/2)\,\{1 - \rho + [(J-1)\delta + \rho]\, n\}\, p^2\sigma^2 Jnl^2. \tag{15}$$

This optimization problem differs from the classical portfolio choice with identical assets in one important aspect: the number of assets (i.e., the number of suppliers under contract) is endogenous. In the portfolio literature, investors typically do not face transaction costs, so they always purchase shares of every available asset and simply choose the optimal weights of each asset in the portfolio. Instead, in the current model, final good firms face supplier-level fixed production costs, such that the number of suppliers to contract with, in addition to the relative importance of each supplier, are endogenous.

The key role of uncertainty in the model is apparent from equation (15). First, suppose that managers are risk neutral. In that case, as illustrated in Figure 1, increasing returns in production imply the (expected) average cost of a unit of material is monotonically decreasing in firm size and converges to the expected cost of a unit of material, w/μ. The equilibrium of the model is not well defined in that case. This happens because there is only one firm in the economy, such that the price taking assumption is no longer reasonable. However, because managers are risk averse in the model, changes in production also have an impact on the expected utility of profits through changes in the variance of profits. Raising employment at each supplier increases the firm's exposure to idiosyncratic shocks, while increasing the number of suppliers raises fixed costs. As a result, the average "disutility" of production (the sum of labor costs and uncertainty) is U-shaped as illustrated in Figure 1. Because of uncertainty, perfectly competitive firms have a finite size. In order to maximize profits, they operate at the efficient scale, where average costs (inclusive of uncertainty) are at their minimum.[9]

The two first-order conditions for the problem defined in (15) are

$$\frac{\partial \mathbb{E}\left[U(\pi)\right]}{\partial l} = 0 \quad \Leftrightarrow \quad p\mu - w = \beta p^2 \sigma^2 l \left\{1 - \rho + \left[(J-1)\delta + \rho\right] n\right\}, \tag{16}$$

$$\frac{\partial \mathbb{E}\left[U(\pi)\right]}{\partial n} = 0 \quad \Leftrightarrow \quad (p\mu - w)l - wf = (\beta/2)\, p^2 \sigma^2 l^2 \left\{1 - \rho + \left[(J-1)\delta + \rho\right] n\right\}. \tag{17}$$

Equation (16) states that, conditional on the number of suppliers, the marginal profits associated with an increase in employment at each supplier must be equal to the corresponding marginal increase in uncertainty associated with the larger exposure to idiosyncratic productivity shocks. Equation (17) states that, conditional on employment per supplier (l), the marginal revenue from contracting with an additional supplier must be just equal to the marginal increase in risk associated with adding an extra supplier (and increasing expected output by μl).[10] Together, conditions (16) and (17) imply that firms operate at the efficient scale depicted in Figure 1.

By combining the two first-order conditions (16) and (17), we obtain an analytical expression for the optimal number of production workers per supplier

$$l = \left[\frac{2wf}{\beta p^2 (1-\rho)\sigma^2}\right]^{\frac{1}{2}}. \tag{18}$$

To make progress, I impose two restrictions. First, and without loss of generality, I use the wage rate as numraire such that $w = 1$. Second, instead of solving for the coefficient of absolute risk aversion β (which is a function of profits π), I follow Wolak and Kolstad (1991) and choose a specific functional form. Specifically, I assume that $\beta p^2 = \eta$. Under those conditions, optimal employment becomes

$$l^* = \left[\frac{2f}{\eta (1-\rho)\sigma^2}\right]^{\frac{1}{2}}. \tag{19}$$

[9] In the new trade theory beginning with Krugman (1979) firms face only labor costs and produce differentiated products. Because consumers have taste for variety, firms face a downward sloping demand curve which limits their size. Here, firms produce homogenous goods and face a perfectly elastic demand curve. Instead, firm size is limited by the risk-aversion of managers.

[10] For some parameter values, the optimal number of suppliers may be a fraction, in which case the optimal number of suppliers would be equal to the nearest feasible integer point. For the remainder of the paper, I restrict the analysis to cases where the first order conditions (17) and (16) are reasonable approximations.

This specification of β has two attractive features. First, it makes the distribution of employment across suppliers invariant to prices which simplifies the characterization of the equilibrium. Second, as shown in Appendix B at the end of the paper, the optimal distribution of employment across suppliers under quantity uncertainty is equal, up to a normalization, to the optimal distribution of input demand across suppliers under cost uncertainty. Importantly, in Appendix C at the end of the paper, I outline the case of constant β and show that the main characteristics of the equilibrium are not affected by this assumption.

To solve for the optimal number of suppliers and the equilibrium price of final goods, I need to impose additional restrictions on the problem. I assume that there is an unbounded pool of identical prospective entrants into the final good industry and that entry into the industry is unrestricted. Under those conditions, firms will continue to enter in the industry until the expected utility of profits is equal to zero. From (15) and the assumption that $\beta p^2 = \eta$, this free-entry condition requires that

$$pJnl\mu - (F + Jnf + Jnl) = (\eta/2)\left\{1 - \rho + \left[(J-1)\delta + \rho\right]n\right\}\sigma^2 Jnl^2. \tag{20}$$

Equation (20) states that, in the free-entry equilibrium, expected profits should compensate exactly for the risk borne by managers. Combining the first order condition (17) and the free-entry condition (20), and substituting with the equilibrium number of production workers per supplier (19) provide an analytical expression for the optimal number of suppliers per country for each final good firm

$$n^* = \left\{\left[\frac{1-\rho}{(J-1)\delta + \rho}\right]\frac{F}{Jf}\right\}^{\frac{1}{2}}. \tag{21}$$

Next, substituting with the equilibrium number of suppliers and the equilibrium number of production workers per supplier into the free-entry condition (20) yields the free-entry equilibrium price

$$p^* = \frac{1}{\mu}\left\langle 1 + \left(2\eta\sigma^2\right)^{\frac{1}{2}}\left\{(1-\rho)^{\frac{1}{2}} f^{\frac{1}{2}} + \left[\frac{(J-1)\delta + \rho}{J}\right]^{\frac{1}{2}} F^{\frac{1}{2}}\right\}\right\rangle. \tag{22}$$

In equilibrium, final good firms charge a markup over the (expected) marginal costs of producing a unit of materials, $1/\mu$. As expected, this markup is a function of the fixed production costs (f and F), but it also depends on the degree of risk aversion of managers (η), the characteristics of the distribution of productivity shocks (σ^2, ρ, and δ), and the number of countries in the world (J). This completes the characterization of the optimal sourcing strategy.

2.5 General equilibrium

In this section, I complete the description of the general equilibrium of the model by solving for the equilibrium number of final goods markets using the labor-market-clearing condition.

Replacing with the optimal employment per supplier (19) and the number of suppliers (21) into the definition of firm-level employment (11) provides an expression for the equilibrium total number of workers associated with each final good firm

$$\Gamma^* = F + \left[\frac{(1-\rho)JfF}{(J-1)\delta+\rho}\right]^{\frac{1}{2}} + \left\{\frac{2JF}{[(J-1)\delta+\rho]\eta\sigma^2}\right\}^{\frac{1}{2}}. \tag{23}$$

The first term captures the labor associated with the firm's fixed production costs, the second accounts for the labor used to pay suppliers' fixed production costs, while the third term represents labor used in the production of materials. By definition, the aggregate demand for labor in equilibrium is given by the product of the optimal labor demand per firm and the equilibrium mass of firms in the economy, which I denote N^*. Because consumers have no taste for leisure, the supply of labor is always equal to the mass of consumers in the economy, L. The labor-market-clearing condition, $N\Gamma = L$, can be used to obtain an expression for the equilibrium mass of firms in a country

$$N^* = \frac{L}{\Gamma^*}, \tag{24}$$

where Γ^* is defined in equation (23). This result shows that an increase in the size of the labor force increases the equilibrium mass of final good firms.

The total number of suppliers per firm in equilibrium is

$$S^* = Jn^*, \tag{25}$$

where n^* is defined in equation (21). Because each firm purchases materials from a disjoint set of suppliers, the equilibrium number of suppliers in the economy, $\mathbb{S}_j^*$, is given by the product of the equilibrium mass of final good firms and the optimal number of suppliers per firm

$$\mathbb{S}_j^* = N^*n^*, \tag{26}$$

where N^* is defined in equation (24). This completes the characterization of the general equilibrium of the model. Before I move to the analysis of the impact of trade liberalization, I briefly discuss the time series properties of the equilibrium, the impact of changes in uncertainty, and the empirical validity of the main mechanism of the model.

2.6 Aggregate fluctuations

As seen in equations (19), (21), (22), and (24), the equilibrium values for the number of suppliers per firms, the number of workers per supplier, the output price, and the mass of final good firms depend only on the moments of the shocks distribution. Because the moments of the distribution are time invariant, all of these variables remain constant over time. Conversely, aggregate output, aggregate expenditure, and aggregate income depend on the current realizations of the productivity shocks and, as a result, fluctuate over time. To see this, first define average productivity in country j at time t as

$$\overline{\mu}_{jt} = \frac{1}{N^*}\int_{i=1}^{N^*} \mu_{jt}(i)di, \quad \text{where} \quad \mu_{jt}(i) = \frac{1}{S^*}\sum_{k=1}^{S^*} z_k(i) \tag{27}$$

is the average productivity of firm-i's suppliers in period t.[11] Using this result, it is possible to express aggregate output in the final goods sector as the product of the mass of firms and average output per firm as follows

$$Q_{jt} = N^* J n^* l^* \overline{\mu}_{jt}. \tag{28}$$

When an economy is in a good state, the average productivity of suppliers is higher then the expected productivity (i.e., $\overline{\mu}_{jt} > \mu$) and, as a result, aggregate output is higher then expected (i.e., $Q_{jt} > N^* J n^* l^* \mu$). Conversely, when an economy is in a bad state, the average productivity and the aggregate output are lower than expected.

Final goods market clearing implies that aggregate expenditure in each country (E_j) is equal to aggregate income. There are two sources of income, payroll and return on equity, such that $E_{jt} = L + N^*\overline{\pi}_{jt}$, where $\overline{\pi}_{jt}$ is average realized profits per firm in country j at time t.[12] From equation (12), realized profits for firm i are given by the difference between revenue and labor costs such that $\pi(\mu_{jt}(i)) = p^* J n^* l^* \mu_{jt}(i) - \Gamma^*$. It follows that average realized profits in country j is

$$\overline{\pi}_{jt} \equiv \pi(\overline{\mu}_{jt}) = \frac{1}{N^*}\int_{i=1}^{N^*} \pi(\mu_{jt}(i))di = p^* J n^* l^* \overline{\mu}_{jt} - \Gamma^*. \tag{29}$$

[11] Because all firms have the same size, a simple average across firms is required. The same is true for within-firm average supplier productivity.

[12] For simplicity, I assumed that consumers own only shares of domestic firms. However, because realized average productivity in any given period can be different in each country, there is room for international risk sharing in the model. International trade in homogenous final good across different states of nature could provide a form of insurance to consumers.

This ensures that total income is equal to total expenditure in every period

$$\begin{aligned} E_{jt} &= L + N^*(p^* J n^* l^* \overline{\mu}_{jt} - \Gamma^*) = L + N^* p^* J n^* l^* \overline{\mu}_{jt} - N^* \Gamma^* \\ &= N^* p^* J n^* l^* \overline{\mu}_{jt} = p^* Q_{jt}. \end{aligned} \tag{30}$$

This result makes clear that workers are always able to consume exactly what they produce. When workers are more (less) productive than average, production and consumption are higher (lower) than average.

2.7 Changes in productivity shocks distribution

This section describes the main impact of changes in distribution of suppliers' productivity shocks on the optimal sourcing decisions. The impact of these changes on the size and the number of suppliers are summarized in the following proposition:

PROPOSITION 1. In the free trade equilibrium,

(a) an increase in uncertainty (σ^2) decreases the size of each supplier, but has no impact on the number of suppliers per firm.

(b) an increase in within-country correlation (ρ) increases the size of suppliers and decreases the number of suppliers per firm.

(c) an increase in across-country correlation (δ) has no impact on the size of suppliers, but decreases the number of suppliers per firms.

Proof. See Appendix D.1 □

Overall, these results are intuitive. An increase in uncertainty (σ) or a decrease in the correlation of suppliers shocks (ρ or δ) increases the managers' incentive to diversify. As shown in proposition 1, these changes can lead to a decrease in the equilibrium size of suppliers or an increase in the number of suppliers per firm depending on the nature of the shock. As seen in part (a) of the proposition, a change in uncertainty has no impact on the number of suppliers per firm. This happens because a change in uncertainty has two opposite effects on the optimal number of suppliers. On the one hand, a reduction in uncertainty decreases the optimal number of suppliers conditional on firm size. On the other hand, a reduction in uncertainty increases the optimal size of each firm which increases the number of suppliers per firm. In equilibrium, these two effects offset each other exactly, such that the increase in input demand operates at the intensive supplier margin only.

2.8 Discussion

The theoretical model developed in this section predicts that, in the presence of risk-aversion, firms purchase the same homogeneous input from multiple suppliers. The fact that multi-sourcing is prevalent (see footnote 4) implies that the model captures a salient feature of the data that has received little attention so far, especially in general equilibrium models of international trade. While uncertainty may not be the only reason why firms multi-source their inputs, it appears to be a credible one.[13] Gervais (2018) reports that "multi-sourcing is more prevalent for products characterized by high levels of uncertainty," where uncertainty is defined as unexpected price shocks. Similarly, using U.S. import transaction data, Chung (2017) finds that "the estimated coefficients on country business operating risk and industry risk suggest that multi-sourcing is more likely when the dominant supplier is more risky." Overall, these empirical results suggest that uncertainty is a plausible motive for multi-sourcing.

With that in mind, in the next section, I explore how trade liberalization in the presence of supply-chain risk affects the optimal sourcing decisions of firms, the general equilibrium structure of the industry, and welfare.

3 Trade liberalization

In this section, I use the theoretical model to evaluate the impact of trade liberalization on equilibrium outcomes. In the current framework, trade liberalization can be thought of either as a move from autarky to free trade ($J = 1 \rightarrow J \geq 2$), or as an increase in the number of countries in the world ($J \rightarrow J + y$, with $y \geq 1$). These two interpretations are related to discrete changes in the number of countries. For simplicity, however, I present the results of comparative static exercises which are qualitatively equivalent. I presents results, in turn, for the impact of trade liberalization on the optimal sourcing strategy, on industry characteristics, and, finally, on welfare. In each case, I discuss how the predictions of the theoretical model relate to the available empirical evidence.

[13] The literature suggests other mechanisms to explain multi-sourcing. First, when suppliers are capacity constraints, downstream firms may not be able to increase their input demand in response to positive demand shock. Instead of carrying inventories, firms may decide to source from multiple suppliers (e.g., Tomlin (2006) and Farrokhi (2020)). Second, when the final good can be imitated by suppliers, multisourcing can serve as a deterrent to entry into the downstream market by lowering the potential profits (e.g., Mukherjee and Tsai (2013)). Third, multisourcing can be the equilibrium outcome when there is a relationship-specific investment that is non-contractible and subject to a usual hold-up problem (Andrabi et al. (2006)).

3.1 Sourcing

The impact of trade liberalization on the optimal sourcing strategy is summarized in the following proposition.

> PROPOSITION 2. Trade liberalization (i.e., a marginal increase in the number of trading partners):
>
> (a) has no impact on the size of suppliers;
>
> (b) increases the total number of suppliers per firm;
>
> (c) decreases the number of suppliers per firm in each country.
>
> *Proof.* See Appendix D.3 □

To get a sense of the channels through which trade liberalization affects sourcing decisions, it is useful to look at the variance of profits which, from equation (14), can be expressed as follows

$$\operatorname{var}(\pi) = \left[\frac{1-\rho}{Jn} + \left(\frac{J-1}{J}\right)\delta + \frac{\rho}{J}\right] p^2 L_p^2 \sigma^2, \tag{31}$$

where $L_p = Jnl$ denotes production employment per firm. Equation (31) shows that, conditional on L_p, an increase in the number of countries in the world decreases the variance of profits. This reduction in uncertainty induces firms to employ more workers. Because changes in the number of countries have no impact on the firm's exposure to any particular supplier's productivity shock, the optimal employment per supplier is invariant to the number of countries. This implies that firms' employment grows exclusively because of adjustments at the extensive suppliers' margin.

From equation (31), it follows that, as the number of countries grows large, the variance of profits converges to the contribution of the covariance of productivity shocks for suppliers located in different countries

$$\lim_{J\to\infty} \operatorname{var}(\pi) = \delta p^2 L_p^2 \sigma^2. \tag{32}$$

Intuitively, when the number of countries in the world tends to infinity firms can completely diversify away the idiosyncratic components of uncertainty and are left facing only the undiversifiable worldwide uncertainty. Using equation (31), it is also possible to show that the level of diversification when the number of countries approaches infinity will always be greater than what can be achieved in a closed economy even if the number of suppliers tends to infinity (i.e, $n \to \infty$)

$$\begin{aligned} \lim_{n\to\infty} \text{var}(\pi) &= \left[\frac{(J-1)\delta+\rho}{J}\right] p^2 L_p^2 \sigma^2 \\ &> \left[\frac{(J-1)\delta+\delta}{J}\right] p^2 L_p^2 \sigma^2 = \delta p^2 L_p^2 \sigma^2 = \lim_{J\to\infty} \text{var}(\pi), \end{aligned} \tag{33}$$

where the inequality comes from the assumption that the correlation between suppliers' shocks is stronger within-countries than across-countries (i.e., $\rho \geq \delta$ in equation (4)). This last results highlights the efficiency of across-country diversification relative to across-supplier (within-country) diversification. Overall, these results explain why, when the number of countries increases, firms increase their total number of suppliers and relocate some of their input demand to their new trading partners.

The international trade literature currently provides only a limited number of empirical results with which to confront the predictions of the theoretical model. However, what is available is broadly consistent with the theory. Part (b) of proposition 1 states that trade liberalization increases the number of suppliers per firm. This prediction is in line with the results of Gervais (2018) who reports that the number of suppliers is higher on average for products characterized by low trade costs. Part (c) of proposition 1 states that trade liberalization decreases the number of suppliers per firm in each country. Again, this is consistent with the empirical results of Gervais (2018) who reports the distribution of input demand across suppliers is more geographically dispersed in input markets characterized with low trade costs. These results provide empirical support to the main mechanism of the model. When trade costs are lower, firms take advantage of the available diversification opportunities by increasing the spatial distribution of their input demand.

In contrast with parts (b) and (c), part (a) of proposition 1, which states that trade liberalization has no impact on the size of suppliers, is at odds with the evidence. The empirical results of Wolak and Kolstad (1991) and Gervais (2016) suggest that the variance of the prices, in addition to the mean of prices, have an impact on the optimal size of suppliers. Both studies interpret the second moment of the price distribution has a measure of uncertainty. Instead, Chung (2017) uses three different "direct" measures of risk to disentangle the two channels: country-industry risk, product "downstreamness" risk, and contracting risk. Using U.S. Census microdata, she finds that firm-level input demand is more dispersed when the dominant supplier is associated with higher measures of risk. Together, these three studies suggest that an increase in a supplier's uncertainty reduces the downstream firm's optimal demand for its input. Fortunately, it is straightforward to reconcile the theory with the empirical results. First, Appendix A at the end of the paper develops a straightforward extension of the model which allows the degree of uncertainty to vary across domestic and foreign suppliers. Among other results, the analysis shows that the optimal input-demand per foreign suppliers is lower than the optimal demand per domestic supplier as long as foreign suppliers are associated with higher uncertainty. Second, Appendix C at the end of the paper develops a version of the model with constant coefficient of absolute risk aversion. In that case, trade liberalization increases the optimal size of supplier.

3.2 Industry characteristics

The impacts of trade liberalization on the equilibrium characteristics of the industry is summarized in the following proposition.

PROPOSITION 3. Trade liberalization (i.e., a marginal increase in the number of trading partners):

(a) decreases output price;

(b) increases employment per firm;

(c) decreases the mass of firms.

Proof. See Appendix D.4 □

The intuition for these results is as follows. As seen in equation (31), the variance of profits is a decreasing function of the number of countries in the world. Because a reduction in the variance of profits reduces the break-even markup, firms can charge lower prices when the number of countries increases. Put differently, trade liberalization lowers the cost of diversification, thereby decreasing prices. For similar reasons, a reduction in the variance of profits increases the optimal employment per firm. Finally, because the supply of labor is fixed, the increase in employment per firm reduces the mass of firms in equilibrium.

The results on the impact of trade on industry characteristics presented in proposition 2 are different from those obtained using "standard" new trade models with CES preferences *à la* Krugman (1980). In that context, trade liberalization has no impact on output price, the size of firms, or the number of firms. The only firm-level adjustment in that model is the reallocation of sales across domestic and foreign markets. However, when using the more flexible Krugman (1979) framework, which allows for variable demand elasticity, one finds that trade liberalization lowers output price, increases the size of firms and decreases the number of firms, which are exactly the same predictions as those reported in proposition 2 of the current paper. The underlying cause of these adjustments are of course quite different. Krugman (1979) emphasizes increasing returns to scale and taste for variety, whereas my model relies on supply-chain uncertainty and risk aversion. Nevertheless, this similarity implies that the predictions of the current model in terms of industry adjustments to trade shocks are just as consistent with the available empirical evidence as are those that emerge from the Krugman (1979) framework. Further empirical works would be required to disentangle the separate contributions of each channel in explaining these adjustments to trade liberalization.

3.3 Welfare

Aggregate welfare in any given period depends on the welfare of both the managers and the consumers. For the comparative static exercises presented in this section, I am interested in comparing long-run, or expected, welfare across equilibria. First note that, by definition of the free-entry equilibrium, expected managers' welfare is always equal to zero (i.e., $\mathbb{E}[U(\pi)] = 0$). Therefore, managers' welfare is not included in the long-run measure of welfare. Second, because consumers' utility depends only on the consumption of final goods, "long-run" equilibrium welfare is proportional to purchasing power such that

$$\mathbb{W}^* \equiv \mathbb{E}\left(\frac{E_{jt}/L}{p^*}\right) = \frac{\mu}{1 + \left(\frac{\eta\sigma^2}{2}\right)^{\frac{1}{2}}\left\{(1-\rho)^{\frac{1}{2}} f^{\frac{1}{2}} + \left[\frac{(J-1)\delta+\rho}{J}\right]^{\frac{1}{2}} F^{\frac{1}{2}}\right\}}. \tag{34}$$

The first equality defines long-run aggregate welfare as expected consumption per worker. The second equality uses the definitions for the equilibrium number of firms, suppliers per firms, workers per suppliers, and number of suppliers to obtain an expression for welfare that depends only on parameters. Equation (34) shows that output per worker is less than expected output per production worker, μ, because some workers are employed for fixed costs. The impact of fixed costs, and other parameters of the model, on output per worker (i.e., welfare) can be clearly seen in the denominator of the equation.

We can use equation (34) to derive the following proposition.

PROPOSITION 4. Trade liberalization (i.e., a marginal increase in the number of trading partners) increases welfare.

Proof. See Appendix D.5 □

Because there are no comparative advantages and no product-differentiation in the model, this result highlights a new source of gains from trade: risk diversification.[14] In the current

[14] I note that, while economies of scale do not drive trade flows, they are necessary to generate gains from trade. Without them, equilibrium prices and, as a result, welfare would remain unchanged. In a sense, returns to scale play a role similar to product differentiation in new trade models with CES preferences and representative firms (e.g., Krugman (1980)). While economies of scales generate an incentive for trade in those models, taste for varieties are necessary for welfare gains because firm-level prices and output do not change following trade liberalization.

model, welfare goes up because trade liberalization decreases the cost of diversification thereby increasing output per worker. The increase in productivity is the result of two opposing effects. On the one hand, firms source from a greater number of suppliers which increases total fixed costs associated with the production of each final good firms. On the other hand, each firm demands a greater quantity of inputs from each of its suppliers and produces more output. Overall, the increase in output outweighs the increase in fixed production costs, such that average (expected) production costs per firm goes down following trade liberalization.

The prediction, reported in proposition 3, that trade liberalization increases welfare is obviously not novel. We know from both classical and new trade theories that there are potential welfare gains from trade. The source of gains from trade is quite different here, however. The important point to be gained from this analysis is that uncertainty is shown to give rise to both trade and gains from trade even when there are no comparative advantages (i.e., international differences in tastes, technology, or factor endowments) and no product differentiation. As mentioned in the introduction, recent studies provide evidence that the uncertainty channel could generate significant gains from trade.

The theoretical model predicts that trade liberalization decreases the share of labor devoted to fixed costs. This implies that, on average, more output can be produced in equilibrium. Therefore, the reallocation of input demand and final good production brought about by trade liberalization leads to increased efficiency:

> Corrolary 1. Trade liberalization (i.e., a marginal increase in the number of trading partners) increases (expected) firm-level and aggregate labor productivity.

This prediction is consistent with the empirical results of Topalova and Khandelwal (2011). They take advantage of India's trade reform to estimate the impact of trade liberalization on firm-level productivity. Their results suggest that reductions in both input and final goods tariffs appear to have increased firm-level productivity. Interestingly, according to their estimates the reduction in input tariffs, which led to an increase in the number and volume of imported inputs from abroad, boosted firm-level productivity more then the reduction in output tariffs. Because the study does not explore the exact channels through which improved access to input increases firm-level productivity, it is not clear what fraction of the increase in productivity, if any, is due to a reduction in sourcing costs, as emphasized by the current model. Nevertheless, the theoretical predictions are consistent with their empirical findings.

4 Conclusion

The analysis in this paper shows that management of supply-chain risk provides an independent motive for trade and highlights a new channel through which trade can increase welfare. To focus the analysis on the role of uncertainty and make matters as simple as possible, these results were derived under strict assumptions. A natural next step is to explore the extent to which these results generalize. In particular, it would be interesting to develop a general equilibrium model of international trade that would include not only uncertainty in the delivery of inputs, but also other motives for trade such as product differentiation and cross-country productivity differences. This framework could be used to evaluate the relative quantitative importance of the different mechanisms in explaining trade flows and generating gains from trade.

The extant literature provides indications on how the sourcing-decision model developed in the current paper could be imbedded into workhorse models of trade such as Eaton and Kortum (2002) or Melitz (2003) (e.g.,Handley and Limão (2017), Heise et al. (2017), and Esposito (2019)). Esposito (2019) is a particularly interesting example because the mechanism he explores is quite similar to the current paper's. In both models, risk-averse entrepreneurs exploit the imperfect correlation of shocks across countries to lower the variance of their profits.[15] The key distinction with my work is that the uncertainty in his model comes from the demand-side instead of the supply-side. Because Esposito (2019) develops an extension of the Melitz model which features across-country differences in costs and foreign demand uncertainty, his model includes both the classical and new trade motives for trade, as well as uncertainty. Among other findings, Esposito (2019) reports that the risk diversification channel explains about 15% of the observed trade patterns in Portuguese data, and increases welfare gains from trade by 16% relative to models with risk neutrality. These results suggest that a quantitative version of the sourcing-decision model developed in this paper could also predict significant additional gains from trade.

[15] In his framework, firms must chose the profit-maximizing distribution of their sales across destinations before the realization of demand shocks. In my model, firms chose the optimal distribution of their input demand across origins before the realization of supply shocks.

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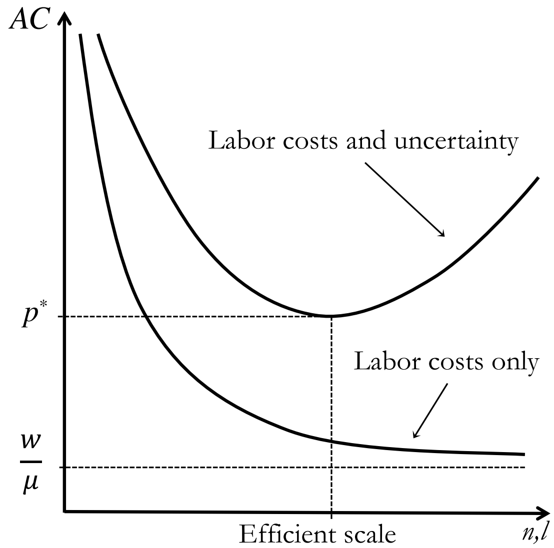

Figure 1: Average production costs

A Heterogeneous variance model

Domestic suppliers may be associated with lower risk compared to foreign suppliers. This happens because it is harder to enforce contracts and monitor suppliers across countries, and there is a greater potential for delivery delays associate with international trade. Therefore, in this section, I extent the benchmark set-up used in the main text to allow for differences in the variance of productivity shocks across countries. To simplify the analysis, I develop the case of two countries. It would be straightforward to generalize to an arbitrary number of identical countries, $J \geq 2$.

As in the main text, the mean of the distribution of productivity shocks is the same in every country

$$\mathbb{E}(z_k) = \mu, \quad \forall\, k \in \mathbb{S}, \tag{A.1}$$

and the correlation between the productivity shocks of any two suppliers k and h depends on the location of each supplier as follows

$$\mathrm{corr}(z_k, z_h) = \begin{cases} \rho & \text{if } k, h \in \mathbb{S}_j, \\ \delta \leq \rho & \text{if } k \in \mathbb{S}_j \text{ and } h \in \mathbb{S} \backslash \mathbb{S}_j, \end{cases} \tag{A.2}$$

where $\mathbb{S}_j \subset \mathbb{S}$ denotes the set of suppliers located in country j. However, the variance of productivity shock now depends on the location of each supplier as follows

$$\mathrm{var}(z_k) = \begin{cases} \sigma_D^2 & \text{if } k, h \in \mathbb{S}_j, \\ \sigma_X^2 = \gamma^2 \sigma_D^2 & \text{if } k \in \mathbb{S}_j \text{ and } h \in \mathbb{S} \backslash \mathbb{S}_j. \end{cases} \tag{A.3}$$

Here, the underlying variance of productivity shock is the same in every country, but international trade magnifies the variance of productivity shocks.

When the variance of productivity shocks varies across countries, the optimal number and size of suppliers will also vary across sourcing country. This implies that there are now four unknowns to solve for: the optimal number of domestic and foreign suppliers (n_D and n_X, respectively), and the optimal demand per domestic and foreign suppliers (l_D and l_X, respectively). The representative firm's problem is given by

$$\begin{aligned} \max_{n_D, l_D, n_X, l_X} \mathbb{E}\left[U(\pi)\right] = p(n_D l_D + n_X l_X)\mu - w\left[F + (n_D + n_X)f + n_D l_D + n_X l_X\right] \\ - \frac{\beta}{2}\Big\{ \left[(1-\rho)n_D + \rho n_D^2\right] l_D^2 \sigma_D^2 \\ + \left[(1-\rho)n_X + \rho n_X^2\right] l_X^2 \sigma_X^2 + \delta \sigma_D \sigma_X n_D n_X l_D l_X \Big\} p^2. \end{aligned} \tag{A.4}$$

The four first order conditions for this problem are given by

$$\frac{\partial \mathbb{E}\left[U(\pi)\right]}{\partial l_D} = p\mu - w - \left(\frac{\beta}{2}\right) p^2\sigma_D^2 \left[(1-\rho+\rho n_D)2l_D + \delta\gamma l_X n_X\right] = 0, \tag{A.5}$$

$$\frac{\partial \mathbb{E}\left[U(\pi)\right]}{\partial n_D} = (p\mu - w)l_D - wf - \left(\frac{\beta}{2}\right) p^2\sigma_D^2 l_D \left[(1-\rho+2\rho n_D)l_D + \delta\gamma l_X n_X\right] = 0, \tag{A.6}$$

$$\frac{\partial \mathbb{E}\left[U(\pi)\right]}{\partial l_X} = p\mu - w - \left(\frac{\beta}{2}\right) p^2\sigma_X^2 \left[(1-\rho+\rho n_X)2l_X + \left(\frac{\delta}{\gamma}\right) l_D n_D\right] = 0, \tag{A.7}$$

$$\frac{\partial \mathbb{E}\left[U(\pi)\right]}{\partial n_X} = (p\mu - w)l_X - wf - \left(\frac{\beta}{2}\right) p^2\sigma_X^2 l_X \left[(1-\rho+2\rho n_X)l_X + \left(\frac{\delta}{\gamma}\right) l_D n_D\right] = 0. \tag{A.8}$$

We can use these first-order conditions to obtain analytical solutions for the optimal demand per supplier (l_D and l_X, respectively), and obtain a relationship between the optimal number of domestic and foreign suppliers.

By combining equations (A.5) and (A.6), we obtain the optimal demand per domestic supplier

$$l_D^* = \left[\frac{2wf}{\beta p^2(1-\rho)\sigma_D^2}\right]^{\frac{1}{2}}. \tag{A.9}$$

This result shows that the optimal employment per domestic supplier is the same as in the benchmark model. Similarly, by combining equations (A.7) and (A.8), we obtain the optimal demand per foreign supplier

$$l_X^* = \left[\frac{2wf}{\beta p^2(1-\rho)\sigma_X^2}\right]^{\frac{1}{2}} = \frac{l_D^*}{\gamma}. \tag{A.10}$$

This last result implies that the optimal size of domestic suppliers is greater than the optimal size of foreign suppliers $l_D^* > l_X^*$ as long as the variance of foreign suppliers is greater than the variance of domestic suppliers (i.e., $\gamma > 1$).

Next, I show that the optimal number of domestic suppliers is greater then the optimal number of foreign suppliers (i.e., $n_D^* > n_X^*$). To do this, I use the equilibrium conditions of the model to derive two mappings between n_D and n_X. The first relationship between the optimal number of domestic and foreign suppliers is obtained from the free entry condition. From equation (A.4), the free entry condition is given by

$$p(n_D l_D + n_X l_X)\mu - w\left[F + (n_D + n_X)f + n_D l_D + n_X l_X\right] = \tag{A.11}$$
$$\frac{\beta}{2}\Big\{ \left[(1-\rho)n_D + \rho n_D^2\right] l_D^2\sigma_D^2 + \left[(1-\rho)n_X + \rho n_X^2\right] l_X^2\sigma_X^2 + \delta\sigma_D\sigma_X n_D n_X l_D l_X \Big\} p^2.$$

Substituting with first order conditions (A.5) and (A.7) into the free entry condition (A.11), we obtain a first mapping between the optimal number of domestic and foreign suppliers

$$g(n_D, n_X) \equiv \rho n_D^2 + \rho n_X^2 + \delta n_D n_X - (1-\rho)\left(\frac{F}{f}\right) = 0. \tag{A.12}$$

It is straightforward to solve for the two intercept as follows

$$n_D(n_X = 0) = n_X(n_D = 0) = \left[\left(\frac{1-\rho}{\rho}\right)\frac{F}{f}\right]^{\frac{1}{2}}, \tag{A.13}$$

which is exactly the optimal number of suppliers predicted by the benchmark model in the case of one country. Furthermore, the derivative of the optimal number of foreign suppliers with respect to the optimal number of domestic suppliers along the mapping $g(\cdot)$ is given by

$$\left.\frac{\partial n_X}{\partial n_D}\right|_{g(n_D,n_X)} = -\frac{2\rho n_X + \delta n_D}{2\rho n_D + \delta n_X} < 0. \tag{A.14}$$

This last result shows that there is a negative association between the number of domestic and foreign suppliers along the free entry condition.

The second relationship between the optimal number of domestic and foreign suppliers is obtained from the first order conditions (A.5) and (A.7). Combining these conditions and using the fact that $l_X = \gamma^{-1} l_D$ and $\sigma_X^2 = \gamma^2 \sigma_D^2$, we obtain

$$h(n_D, n_X) \equiv (2\rho - \gamma\delta)n_D - (2\gamma\rho - \delta)n_X - 2(\gamma - 1)(1-\rho) = 0. \tag{A.15}$$

The two intercepts for this mapping are given by

$$n_D(n_X = 0) = \frac{2(\gamma-1)(1-\rho)}{2\rho - \gamma\delta}, \tag{A.16}$$

$$n_X(n_D = 0) = -\frac{2(\gamma-1)(1-\rho)}{2\gamma\rho - \delta}, \tag{A.17}$$

and the derivative of the optimal number of foreign suppliers with respect to the optimal number of domestic suppliers is

$$\left.\frac{\partial n_X}{\partial n_D}\right|_{h(n_D,n_X)} = -\frac{2\rho - \gamma\delta}{2\gamma\rho - \delta} \in (0,1). \tag{A.18}$$

Using the two assumptions $\rho > \delta$ and $\gamma > 1$, we can show that $2\rho - \gamma\delta > \rho(2-\gamma) > \rho > 0$ and that $(2\rho - \gamma\delta) - (2\gamma\rho - \delta) > 0$. Together, these results imply that the derivative in equation (A.18) is positive and less than unity. The two mappings between the number of domestic and foreign suppliers described in equations (A.12) and (A.15) imply that

$$0 < n_X^* < n_D^* < \left[\left(\frac{1-\rho}{\rho} \right) \frac{F}{f} \right]^{\frac{1}{2}}. \tag{A.19}$$

The results presented in this section show that the optimal distribution of the input demand and suppliers depends on the variance of productivity shocks of each supplying countries. Under the assumption that foreign suppliers are more risky, the number of suppliers and the optimal demand per suppliers are smaller for foreign countries. When the ratio of variance become large relative to other parameters of the model, it is possible that firms buy only from domestic suppliers. More precisely, if $g(n_D, 0) < h(n_D, 0)$, which is equivalent to

$$\frac{\tau(\gamma - 1)}{2\tau - \gamma} < \left[\left(\frac{\rho}{1-\rho} \right) \frac{F}{f} \right]^{\frac{1}{2}}, \tag{A.20}$$

where τ is defined as $\tau = \rho/\delta \geq 1$. The right-hand side of the equation is alway positive, so greater than 0 and the left-hand-side of the equation converges to 0 as the correlation between productivity shock across-countries goes to 0 (i.e., $\tau \to 0$). This implies that, if the incentive to diversify is strong enough, firms will continue to source from foreign suppliers even if they are more risky compared to domestic suppliers. In other words, for any γ there is a τ such that $n_X^* > 0$.

B Costs uncertainty model

In this section, I evaluate the impact of the nature of uncertainty on the optimal sourcing strategy. In the model presented in the main text, firms choose employment at each supplier and uncertainty arises from unexpected changes in input quantity delivered. Suppose that instead of contracting on the expected amount of inputs to be delivered (i.e., the number of workers each suppliers should employ) final good firms contracted on the actual amount of materials to be delivered.[16] This would move the uncertainty from input quantities to input costs. In this case, firm profits are given by

$$\pi = pq - w\left(F + nf + \sum_{k=1}^{n} c_k m_k\right), \tag{B.1}$$

where n denotes the number of suppliers from which the firm buys inputs. Profits are still stochastic, but the source of the uncertainty shifted from revenue to the production costs. This happens because the number of units delivered by each supplier, m_k, is now fixed but the cost of these units, c_k, depends on worker's productivity which in unknown *ex ante*.

Managers maximize the expected utility of profits by choosing how much output to produce conditional on the market price. This involves two interrelated decisions. First, the manager chooses the set of suppliers to contract with. Second, he chooses the allocation of input demand across the selected suppliers. Because suppliers are identical *ex ante* (i.e., at the moment contracts are signed), the optimal share of demand is constant across suppliers and given by $1/n$. It follows that output is given by the product of the number of suppliers and the quantity of inputs demanded from each supplier, $q = nm$.

Similar to the model developed in the main text, I assume that the expected costs, the variance of costs, and the correlation between the production costs are common across suppliers and, respectively, given by

$$\mathbb{E}(c_k) = \mu, \ \text{var}(c_k) = \sigma^2, \ \text{and} \ \text{corr}(c_k, c_h) = \rho \in (0,1), \ \forall\, k, h \in \mathbb{S}, \ k \neq h. \tag{B.2}$$

For simplicity, I present the case where $\delta = \rho$; extending to $\delta \leq \rho$ is trivial but provides little additional insight here. Using these assumptions and equation (B.1), it follows that

[16] In terms of general equilibrium analysis, the benchmark is more tractable because there is no uncertainty in aggregate employment (contractual obligations ensure that suppliers hire a given number of workers). Conversely, when suppliers are constrained to deliver a given quantity, productivity shocks lead to unexpected changes in employment. In that case, the labor-market clearing condition does not necessarily hold *ex post*. There may be excess-demand for labor or excess-supply of labor.

the variance of profits can be expressed as

$$\text{var}(\pi) = \text{var}\left(\sum_{s=1}^{n} c_s m_s\right) = \left(\frac{1-\rho}{n} + \rho\right) q^2\sigma^2. \tag{B.3}$$

Substituting with this result into the expected utility function (10), the firm's problem can be expressed in terms of choosing the number of suppliers and the quantity of inputs purchased from each supplier as follows

$$\max_{n,m} \mathbb{E}\left[U(\pi)\right] = pnm - w(F + nf + n\mu m) - (\beta/2)\left(1 - \rho + n\rho\right)\sigma^2 m^2 n. \tag{B.4}$$

The two first-order conditions for the firm's problem defined in (B.4) are

$$\frac{\partial \mathbb{E}\left[U(\pi)\right]}{\partial n} = 0 \quad \Leftrightarrow \quad (p - w\mu)\, m - wf = (\beta/2)\left(1 - \rho + 2n\rho\right)\sigma^2 m^2, \tag{B.5}$$

$$\frac{\partial \mathbb{E}\left[U(\pi)\right]}{\partial m} = 0 \quad \Leftrightarrow \quad p - w\mu = \beta\left(1 - \rho + n\rho\right)\sigma^2 m. \tag{B.6}$$

Equation (B.5) states that, conditional on demand per supplier (m), the marginal revenue from contracting with an additional supplier must be just equal to the marginal increase in risk associated with adding an extra supplier (and increasing output by m). Equation (B.6) states that the marginal revenue from increasing the quantity of inputs demanded from each supplier must be equal to the corresponding marginal increase in risk. Together, conditions (B.5) and (B.6) imply that firms operate at the efficient scale.

Combining the two first-order conditions (B.5) and (B.6) provides an analytical expression for the optimal demand per supplier

$$m^* = \left[\frac{2wf}{\beta(1-\rho)\sigma^2}\right]^{\frac{1}{2}}. \tag{B.7}$$

Substituting $\beta = \eta$ and using the wage rate as numeraire (such that $w = 1$) yield the optimal employment per supplier under quantity uncertainty defined in equation (19).

C Constant risk aversion parameter

In this appendix, I explore the properties of the model when the managers' marginal rate of substitution between expected profits and risk is constant. First, I note that the first order conditions in equations (16) and (17) remains the same, as does the free-entry condition (20). Furthermore, because the expression for the optimal number of suppliers per firm is derived from equations (17) and (20), it is not effected by the choice of specification for β.

The price equation (22) is still valid, but under constant β it provides an implicit definition of output price (because $\eta = \beta p^2$). It is straightforward to solve for the equilibrium price as a function of the parameters of the model as follows

$$p_\beta^* = \frac{1}{\mu - (2\beta\sigma^2)^{\frac{1}{2}} \left\{ (1-\rho)^{\frac{1}{2}} f^{\frac{1}{2}} + \left[\frac{(J-1)\delta+\rho}{J}\right]^{\frac{1}{2}} F^{\frac{1}{2}} \right\}}. \tag{C.1}$$

Prices will be positive if and only if productivity is large enough relative to the costs and risk parameter

$$\mu > \left(2\beta\sigma^2\right)^{\frac{1}{2}} \left\{ (1-\rho)^{\frac{1}{2}} f^{\frac{1}{2}} + \left[\frac{(J-1)\delta+\rho}{J}\right]^{\frac{1}{2}} F^{\frac{1}{2}} \right\}. \tag{C.2}$$

While the expression for equilibrium price in (C.1) is different then in the benchmark model, its qualitative properties are the same. The equilibrium price is increasing in fixed costs (F and f) and risk related parameters (β and σ^2), but decreasing in the number of countries in the world.

Substituting with equilibrium price defined in equation (C.1) into (18) yields an analytical expression for the optimal number of workers per supplier which depends only on parameters of the model

$$l_\beta^* = \frac{1}{p_\beta^*} \left[\frac{2f}{\beta(1-\rho)\sigma^2}\right]^{\frac{1}{2}}, \tag{C.3}$$

where p_β^* is defined in equation (C.1). In contrast with the benchmark result, the optimal size of each supplier under constant β depends on the firm-level fixed production costs (F), the correlation between domestic and foreign shocks (δ), and the number of countries in the world (J).

As before, the total number of workers associated with each final-goods firm is given by

$$\Gamma^* = F + \left[\frac{(1-\rho)JfF}{(J-1)\delta+\rho}\right]^{\frac{1}{2}} + \left\{\frac{2JF}{[(J-1)\delta+\rho]\beta\sigma^2}\right\}^{\frac{1}{2}} \frac{1}{p_\beta^*}. \tag{C.4}$$

It follows from the labor-market-clearing condition that the equilibrium mass of firms is given by

$$N_\beta^* = \frac{L}{\Gamma_\beta^*}, \tag{C.5}$$

and the equilibrium number of suppliers in the economy

$$S^* \equiv N_\beta^* n^* = \left(\frac{L}{\Gamma_\beta^*}\right)\left[\left(\frac{1-\rho}{\rho}\right)\frac{F}{f}\right]^{\frac{1}{2}}. \tag{C.6}$$

The key distinction between these results and the benchmark is the definition of output price. This completes the characterization of the equilibrium for the case of constant marginal rate of substitution between expected profits and risk.

So far, the analysis revealed that the equilibrium under constant β is quite similar to the benchmark. The main difference being the dependence of the optimal size of suppliers on additional parameters of the model. I now contrast the predicted impacts of changes in uncertainty and trade liberalization on the optimal sourcing strategy and on welfare. Using the same approach as in appendix D below, it is straightforward to show that the sign of the comparative statics presented in propositions 1-5 of sections 3 and 4 of the main text are the same, except for part (a) of proposition 3. From (C.3), it follows that

$$\frac{\partial l_\beta^*}{\partial J} = \frac{\partial l_\beta^*}{\partial p_\beta^*}\frac{\partial p_\beta^*}{\partial J} = -\frac{l_\beta^*}{p_\beta^*}\frac{\partial p_\beta^*}{\partial J} \geq 0, \tag{C.7}$$

where the inequality follows from part (a) of proposition 4. This result shows that in the constant β case, an increase in the number of countries (J) increases the optimal size of each supplier.

D Proofs

D.1 Proposition 1

Part (a): From equations (19) and (21), it follows that

$$\frac{\partial n^*}{\partial \sigma^2} = 0 \quad \text{and} \quad \frac{\partial l^*}{\partial \sigma^2} = -\frac{1}{2}\left(\frac{l^*}{\sigma^2}\right) < 0. \tag{D.1}$$

These results show that the number of suppliers per firm is invariant to the variance of productivity shocks, whereas the size of each supplier is decreasing in the variance of productivity shocks. □

Part (b): From equations (23) and (24), it follows that

$$\frac{\partial \Gamma^*}{\partial \sigma^2} = -\frac{1}{2}\left(\frac{\Gamma^*}{\sigma^2}\right) < 0 \quad \text{and} \quad \frac{\partial N^*}{\partial \sigma^2} = -L\left(\frac{1}{\Gamma^*}\right)^2 \frac{\partial \Gamma^*}{\partial \sigma^2} > 0. \tag{D.2}$$

These results show that total employment per firm is decreasing in the variance of productivity shocks, whereas the equilibrium mass of firms is decreasing in the variance of productivity shocks. □

Part (c): From the definition of equilibrium price given in equation (22), it follows that

$$\frac{\partial p^*}{\partial \sigma^2} = \frac{1}{\mu}\left(\frac{1}{2\sigma^2}\right)^{\frac{1}{2}} \left[(1-\rho)^{\frac{1}{2}} f^{\frac{1}{2}} + \rho^{\frac{1}{2}} F^{\frac{1}{2}}\right] > 0. \tag{D.3}$$

These results show that equilibrium price is increasing in the variance of productivity shocks. □

D.2 Proposition 2

From the definition of equilibrium expected welfare given in equation (34), it follows that

$$\frac{\partial W^*}{\partial \sigma^2} = -\frac{W^2}{\mu}\left(\frac{1}{2\sigma^2}\right)^{\frac{1}{2}} \left[(1-\rho)^{\frac{1}{2}} f^{\frac{1}{2}} + \rho^{\frac{1}{2}} F^{\frac{1}{2}}\right] < 0. \tag{D.4}$$

These results show that equilibrium expected aggregate welfare is decreasing in the variance of productivity shocks. □

D.3 Proposition 3

From equations (19) and (21) it follows that

$$
\begin{aligned}
\text{part (a):} \quad & \frac{\partial l^*}{\partial J} = 0, \\
\text{part (b):} \quad & \frac{\partial J n^*}{\partial J} = n^* + J\frac{\partial n^*}{\partial J} = \frac{(2J-1)\delta + \rho}{2[(J-1)\delta + \rho]} > 0, \\
\text{part (c):} \quad & \frac{\partial n^*}{\partial J} = -\frac{(\rho - \delta)n^*}{2[(J-1)\delta] + \rho} \leq 0.
\end{aligned}
$$

where the last (weak) inequality follows from $\delta \leq \rho$ as defined in equation (4). □

D.4 Proposition 4

From equations (22), (23), and (24), it follows that

$$
\begin{aligned}
\text{part (a):} \quad & \frac{\partial p^*}{\partial J} = -\frac{\rho - \delta}{\mu}\left(\frac{\eta\sigma^2 F}{2}\right)^{\frac{1}{2}}\left[\frac{J}{(J-1)\delta + \rho}\right]^{\frac{1}{2}} \leq 0, \\
\text{part (b):} \quad & \frac{\partial \Gamma^*}{\partial J} = \frac{\rho - \delta}{2}\left[\frac{(J-1)\delta + \rho}{J}\right]^{\frac{1}{2}}\left\{[(1-\rho)fF]^{\frac{1}{2}} + \left(\frac{2F}{\eta\sigma^2}\right)^{\frac{1}{2}}\right\} \geq 0, \\
\text{part (c):} \quad & \frac{\partial N^*}{\partial J} = -\left(\frac{N^*}{\Gamma^*}\right)\frac{\partial \Gamma^*}{\partial J} < 0.
\end{aligned}
$$

where the (weak) inequalities follow from $\delta \leq \rho$ as defined in equation (4). □

D.5 Proposition 5

Using the definition of output price in equation (22) to substitute for the denominator of equation (34), it follows that welfare can be expressed as

$$
\mathbb{W}^* = 2\left(\frac{\mu}{1 + \mu p^*}\right).
$$

Using this result, it is straightforward to show that

$$
\frac{\partial \mathbb{W}^*}{\partial J} = \frac{\partial \mathbb{W}^*}{\partial p^*}\frac{\partial p^*}{\partial J} = -2\left(\frac{\mu}{1 + \mu p^*}\right)^2 \frac{\partial p^*}{\partial J} \geq 0,
$$

where the inequality follows from part (a) of proposition 4. □