The Uniqueness of Utility and Production Functions of African Music: Implications for African Non-Music Industries

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Abstract: I struggle with a *simple* question, but one that may have *liberating* implications. The question is: How has African music been a nearly imperialist force on the global scale whereas African non-music goods and services, besides raw materials and slaves, have had *limiting* effects? The answer is not in the sophistication of inputs; African traditional musical instruments are basic, requiring low capital investment. The difference is in the music consumption (utility) and production (profit/surplus) functions. The utility function is endogenous, deriving its value from local social demand that always exists irrespective of the product price of the music itself. The production function is characterized at least in part by (im)pulses that may or may not share the same properties with ordinary functions generally used to describe production activities. The unique functions guarantee that African music will always be globally influential. African non-music industries may want to imitate African music industries as their attempts to copy functions that have led to success in countries outside Africa have put them at a huge disadvantage.

Keywords: Unique utility function, unique production function, music consumption, music production/making, constrained social welfare, endogenous consumption and production, Africa

JEL Code: Z11, O55, A13, D69

The Uniqueness of Utility and Production Functions of African Music: Implications for African Non-Music Industries

1. Introduction

Currently the Africa region ranks lowest on nearly all economic performance indicators (IMF, 2020; UNDP, 2020). Even if we accept Mishan's (1974) charge that some of the observed research outcomes on Africa are often "translations (sic) [of political] prejudices into respectable-looking numerals" (pp. 93-4), a nagging question remains about how the cradle of human life has come to fall behind others. Lately I have become convinced that we are asking the wrong questions, and consequently looking at things in wrong ways. African music has had one of the most 'imperialistic' influences on other musics, and by inference other cultures, and hence the well-being of many worldwide (Azam, 1993). Jazz, rock-n-roll (at least the beginnings of it), reggae, and now varieties of HipHop are just obvious examples of an African musical genesis. The question becomes: *What gives value to African music but not to African non-music goods and services?* Are there lessons to be learned which African non-music industries can imitate? In this note I argue that the answer is not in the sophistication of inputs; African traditional musical instruments are simple, and require low capital investment. The difference is in the music utility and production functions. The utility function is endogenous, deriving its value from local social demand that always exists irrespective of the price of music itself. The production function is characterized at least in part by (im)pulses, which may or may not share the same properties with well-defined and easily differentiable functions so often used to describe production activities. The uniqueness of these functions guarantees that African music will always be globally influential.

In this note I think through this question in a simple and clear way. In Section 2 I describe the uniqueness of the utility function deriving from consuming African music, without specifying its precise mathematical form. Section 3 characterizes the complexity of producing African music. In this case I assume a simple Romer endogenous model of which the Cobb-Douglas functional form is one special case. The simplicity of musical instruments suggests low investment in capital inputs, and little, if any, foreign investment is needed. In Section 4 I speculate about the implied social welfare following Amavilah's (2016) characterization of integral social welfare, a concept due to Pope Francis, given endogenous constraints (Amavilah, 2018; Fiocca, 1994). The argument enables me to urge non-music African industries to take a look at the music industry in Section 5,which states concluding implications for policy and future research.

2. A socially-constrained endogenous utility function

Received theory says that the objective of the consumer is maximum satisfaction (utility). The consumer derives utility (U_i) from the use/consumption of goods and services (q_i). Consumption possibilities – the "field of choice" in Mundall's (1968) phrasing – depends on the consumer's purchasing power or budget constraint (B_i), given exogeneous prices of goods and services (P_i). The preceding is similar to solving the following Lagrangian:

$$V_i = \max U_i(q_i(x_i)) + \lambda[B_i - P_i q_i(x_i)], \quad (1)$$

where x_i are productive resources available to the ith music producer. However, this theory is inadequate because music in Africa is a *social good/service* (q_s); it is *created for the social value* of activities like work, childbirth, marriage, death, hunting, and so on. – implying the social value of music is larger than the sum of the individual values. In other words, the local demand for African music is a derived social demand – which leaves little distinction between the product and factor markets. This further means $q_i(q_s(x_i))$ and (1) becomes

$$W_i = \max U_i\big(q_i(q_s(x_i))\big) + \lambda[B_i - P_i q_i(q_s(x_i))] \neq V_i. \quad (2)$$

The conditions necessary for optimizing (1) and (2) are no longer the same. Whereas $V' = U_i' - \lambda P_i = 0$, by the Chain Rule

$$W_i' = \left(\frac{dU_i}{dq_i}\right)\left(\frac{dq_i}{dq_s}\right) - \lambda P_i \begin{cases} = 0 \Rightarrow \frac{dq_i}{dq_s} = 1 \\ \neq 0 \Rightarrow \frac{dq_i}{dq_s} \neq 1 \end{cases}. \quad (3)$$

Eq. (3) suggests that even if we were to assume a similarly fixed budget, $\frac{dU_i}{d\lambda} = B_i - \lambda P_i = 0$, and exogenous P_i, $V' = W_i' \Rightarrow q_i = q_s$. Again, (3) looks like an indirect value function, but the interpretation is very different because the variables and parameters are endogenous rather than exogenous. Received theory is deficient, because for $\frac{dq_i}{dq_s} = 1, (dU_i dq_j)/(dq_i\, dU_j) = \frac{p_i}{p_j}$ (the familiar marginal rate of substitution) and $q_i^*(p_i, B_i)$ is the ith individual's Marshallian demand. This is no longer the outcome for $\frac{dq_i}{dq_s} \neq 1, because\ q_i$ is produced only if it leads to $U_i \leq U_s$, suggesting that for given $B_i, P_i,$ and λ, there is always a social demand for African music. Moreover, since music itself can induce enjoyment, reflection, sadness, or all simultaneously, (3) is *endogenous*, *socially constrained*, and *always exists, but not necessarily substitutable*.

3. Producing complex music with simple instruments (inputs)

Clearly $q_i \neq q_s$. How informative is the endogenous growth theory model, for instance, for the production of African music? Using Romer (1990), we could say

$$q_i(q_s(x_i)) = A^{\delta} q_i = A^{\delta} f(x_i), \quad (4)$$

where A is an endogenous Romer technology, and $f(x)$ is the classical Solow aggregate production function of inputs x, and can take a Cobb-Douglas form, such that $q_i(q_s) = A^{\delta+1}x^{\alpha}, \alpha < 1$, ignoring the subscript i. In that case (2) becomes

$$W_i = \max U_i\big(q_i(q_s(x))\big) + \lambda[B_i - P_i A^{\delta+1}x^{\alpha}] \quad (5)$$

The simplicity of African traditional musical instruments suggests that the power of (4) is not in the sophistication of x, but rather in the uniqueness of f – the production rule itself. Musicologists tell us that f is really complex – it is polyphony and collective, timed, recurrent, cross-rhythmic, interlocking, and pulsating. According to the Encyclopedia Britannica, "the Ghanaian musicologist J.H. Kwabena Nketia (1974) pointed out the function of this African form of hocket technique in achieving overall effects of continuity, [and] for building up interlocking, and

sometimes complex structures, out of relatively simple elements" ((https://www.britannica.com/art/African-music/Musical-structure, [] added). Prima facie, the elements of African music are not entirely different from those of Western European music. However, the structure is very different in its polycentricity, and in making music a collective enterprise (https://www.britannica.com/art/African-music/Musical-structure). *African music is never made; it is always being made* – a natural barrier to piracy and property rights violations. In fact, African music and dance are inseparable; the dancer responds to the music and the musician "improvises" in response to the dancer, sometimes acknowledging and often challenging the dancer's ability. Hence, the degree of this complexity distinguishes *f* in (4) from the conventional *f* imposed on economic activities. I illustrate and explain this idea further in Figure 1. Note that if $q_i(q_s(x))$ evolves according to conventional $f(x)$, the received production function theory obtains. However, if an (im)pulse occurs in the production process, say from x_1 to x_2, then f is no longer consistent with received theory, and we can describe the "jump" with a Heaviside function just as an illustration, so that

$$q_i(q_s(x)) = A^{\delta+1} f(x) H(x^*) = A^{\delta+1} x^{\alpha} H(x^*)\,,\ x^* = [H(x - x_1) - H(x - x_2)] \tag{6}$$

Eq. (6) is complex, indeed! It encompasses three different functions (A which is an endogenous Romer technology of which the exogenous Solow residual is an element, $f(x)$ which is a Solow aggregate production function, so that $Af(x)$ is endogenous, and $H(x^*)$ which is a Heaviside function (convolution). A cumulative convolution is a Fourier transform, and is consistent with the production of African music, because it allows for the introduction of time and frequency, filtering, and interactive sampling. The complexity is that it is only empirically solved by the music maker. If so, then (5) can be restated as

$$W_i = \max U_i\big(q_i(q_s(x))\big) + \lambda[B_i - P_i A^{\delta+1} x^{\alpha} H(x^*)] \tag{7}$$

While (7) is a more realistic representation of the production of African music than (1), assessing its necessary and sufficient conditions for (7) has become a little complicated, to say the least. One can use the Chain Rule to differentiate the first RHS term of (7). It is not clear whether or not a Product or Chain Rule would be appropriate or even enough for the last term because of the Heaviside (im)pulse. For example,

$$\frac{dW_i}{dx} = q_s'(x)\ q_i'(q_s(x)U'\left(q_i\big(q_s(x)\big)\right))) - \lambda[P_i\ A^{\delta+1} H(x^*)\ \ \alpha x^{\alpha-1}] = 0 \tag{8.1}$$

$$\frac{dW_i}{dx^*} = -\lambda\big[P_i\ A^{\delta+1}\delta(x^*) f(x)\big] = \begin{cases} 0, x^* < 0 \\ \frac{1}{2}, x^* = 0 \\ 1, x^* > 0 \end{cases} \tag{8.2}$$

$$\frac{dW_i}{dq_i} = q_s'(x)\ q_i'(q_s(x)U\left(q_i\big(q_s(x)\big)\right))) - \lambda[P_i\ A^{\delta+1} H(x^*)\ \ \alpha x^{\alpha-1}] = 0 \tag{8.3}$$

$$\frac{dW_i}{dq_s} = q_s'(x) U'(q_i(q_s) - \lambda[P_i\ \theta_s] \begin{cases} = 0, \theta > 0, x^* \le x_1, q_i = q_s \\ < 0, \theta > 0, x^* = 0, q_i > q_s \\ > 0, \theta > 0, x^* < x_2, q_i < q_s \end{cases} \tag{8.4}$$

where social production adjusted for the $H(x^*)$ (im)pulse is $\theta_s = \alpha A^{\delta+1} x^{\alpha-1} H(x^*)$, $\alpha x^{\alpha-1} = f'(x)$, $\delta(x^*) = H'(x^*)$ is the derivative of $H(x^*)$ is the so-called Dirac delta function. The cumulative distribution of the Dirac delta function is a Kronecker delta function and both functions can be approximated as a logistic function, i.e.,

$$\int_{-\infty}^{x^*} \delta(x^*) dx^* = \sum_{k=-\infty}^{n} \delta(x^*) = \sum_{k=-\infty}^{n} H'(x^*) = \frac{1}{1+e^{-2kx^*}}.$$

Eq. (8) is even more complex than (6)! Wheareas (8.1) is standard, (8.2), (8.3), and (8.4) require additional assumptions to have standard interpretation. For example (8.4) alone requires $\theta_s > 0$, which is possible only if $U(q_i) \leq U(q_s)$. Even if that happens, it remains unclear whether the (im)pulse $H(x^*)$ will lead to a jump forward, backward, downward, and or upward in $q_i(q_s)$. Figure 1 assumes a sustained rectangular (im)pulse – complex stuff – well, given my math level. However, while (7) and (8) are intractable theoretically, empirically African music producers solve these equations routinely even without us understanding them or knowing how to explain them – a modified Polanyi paradox that says people know more than they tell and they can do more than they tell (McAfee and Brynjolfsson, 2017, pp. 3-5).

In Figure 1 again: at Point B, x_1 inputs produce output $q_i = q_s$. The (im)pulse $H(x^*)$ increases the efficiency of x_1, leading to $q_i < q_s$ (Point D), but that level of q_i would have been produced with x_2 inputs, which is a compensation equivalent of producing at Point E. Since Point E is not possible (no resources), and Point D is not permissible ($q_i > q_s$), the production function is really ABDE $= H(x^*)$.

Figure 1 Hypothetical production function of African music

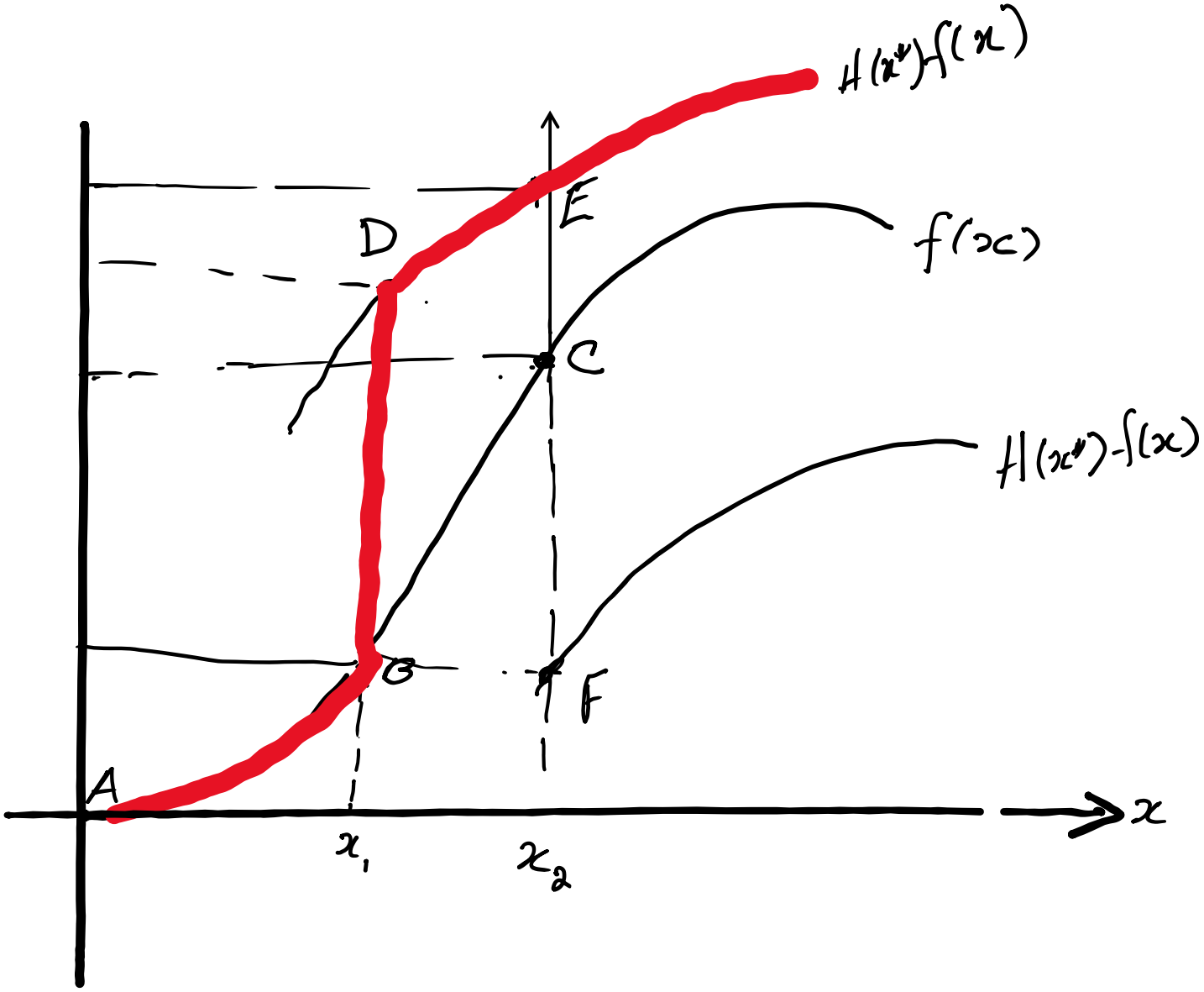

Table 1. African musical instruments by type

Technical name	Instrument
Idiophones	Slit drums, Rattles, Shakers, Clappers, Bells, Gongs, Kalimba/Karimba, Sanza, or Sansu, Jew's Harp
Xylophones	Marimba, Balo/Balafon, Baan, Amadinda, Akadinda
Chordophones	Harps, Bows, Lyres, Fiddles, Lutes, Zithers
Aerophones	Flutes, Whistles, Reed Pipes, Trumpets, Horns
Membranophones	Breasted Drum, Two-headed Drum, Shangaan Drum, Tonga Drum, Yangere slit Drum, Buganda royal fetish Drum, Royal Lunda Drum
Percussion	Voice, Yodeling, Whispering, Whistling, Body and Tube stamping, Hand Capping, Foot Stomping

Source: https://www.britannica.com/art/African-music/Musical-instruments.

Table 2. Prevalence of drum indicative of it universality in African music and dance

Region	Local name for drum
North Africa (Mali)	Bara, Tama, Krin
West Africa	Djembe, Dundun, Dunno, Gome, Sabar, Bougarabou, Bara, Gungon, Atumpan, Fontomfro, Bamileke, Sakara, Gbedu, Bata, Gudugudu, Chamba, Udu
Central Africa	Kili, Mondo, Bata
East and Southern Africa	Udu, Ngoma

Source: https://www.contemporary-african-art.com/african-drums.html.

4. A socially-constrained endogenous social welfare

Instead of formalizing a social welfare from (8), here emphasis is solely on the assumption that a socially-constrained social welfare function ($2SWF \neq SWF$) exists and is unique, which is precisely what explains how African music continues its global influence where other African non-music goods and services have failed. These insights I gleaned from Amavilah (2016) and Amavilah (2018).

Drawing on Pope Francis's economics of well-being Amavilah (2016) has argued that, because $q_i < q_s \Rightarrow U_i(U_s), U_i < U_s$, the time-discounted present value of social welfare implied by (7) is:

$$2SWF = \max \{\int_0^\infty U_i(q_i(U_s(q_s(x)))) + \lambda\left[P_i A^{\delta+1} x^\alpha H(x^*) - C\left(q_i(q_s(x))\right)\right]e^{-\rho t}\}\, dt, \quad (9)$$

where ρ is the time discount factor. The Hamiltonian for (9) is:

$$H = \{U_i(q_i(U_s(q_s(x)))) + \lambda[P_i A^{\delta+1} x^\alpha H(x^*) - C(q_i(q_s(x)))]\} = U^* + \lambda \Pi^*, \quad (10)$$

where $U^* = \{U_i(q_i(U_s(q_s(x))))\}$ is the discounted present value of utility from African music, and $\Pi^* = [P_i A^{\delta+1} x^\alpha H(x^*) - C(q_i(q_s(x)))]$ is discounted net social surplus. I wish the reader good luck in working out the first and higher order conditions for (10). Agenor and Montiel (1996) set $\{U_i(q_i(U_s(q_s(x))))\}\mathrm{e}^{-\rho t} = \ln C_t e^{-\rho t}$ and extend analysis to what they call the "temporariness hypothesis" (pp.345-352) that allows for government involvement. Whatever they are, the solutions to (10) will show that $MC_i = C'(q_i)C(q_s)$ and $MC_s = C(q_i)C'(q_s)$, implying MC_i and MC_s are inseparable, and $MC_i = MC_s$ only if $C(q_i) = C(q_s))$ or $q_i = q_s$, which is impossible because society always prefers $MC_i > MC_s$.

The meaning of (10) is consistent with Pope Francis's argument for an integral in which consumption and production and associated time-discounted social satisfaction and surplus depend on the common good. This uniqueness guarantees local demand and supply of African music, implying "consumption and production functions are a system of endogenous (dynamical) simultaneous equations in which the utility of the integral person is a function of social utility" (Amavilah, 2016, p.). The liberating forces of endogenous consumption and endogenous production technology is responsible for the continued influence of African music even as the production inputs are not sophisticated and require little to low investment.

5. Concluding implications for non-music industries

African non-music industries have adopted consumption and production functions that have "overstressed the importance of exogenous constraints in the economic performance of African countries at the expense of endogenous constraints, although the latter are longer-lasting and more self-propagating than the former" (Amavilah, 2018, p.). The adoption has unleashed limiting forces that weaken endogenous social demand and comparative advantages. Local factors and forces have become constraints making investment necessary and the sources of funds foreign. That art (music) can explain science (economics) is no longer a new idea given what we know about the influence of African art on Pablo Picasso and the Cubist revolution and how the latter affected modern physics (Shlain, 2007). I find a key lesson for the African non-music industry in relation to this in Simon Newcomb's (2012[1885], p. 76.) statement that

> In nearly every branch of business a person possessed of the proper talents gradually acquires a sort of knowledge by which he instinctively avoids mistakes, forms correct judgments of what should be done under various circumstances, and thus acquires a wealth-producing power which inexperienced persons would not possess. If all the producers of the country should lose the special skill and faculties which they have acquired by experience, a severe blow would be struck at the production of wealth.[1]

The statement applies to African non-music industries, and they may benefit by looking to the African music industry, where cumulative experience, even without elegant theory, has created the "knowledge requisite" essential to enhancing other "requisites of production" (Newcomb, 2012, Book III, pp. 70-126).

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[11] I have a special fond for this work, because a relative of the author, the late Dr. Richard T. Newcomb, was my Ph.D. dissertation director and he talked to me a lot about this book.

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