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Abstract

We consider two differentiated products mixed markets, comprised of a state holding corporation (SHC) and private firms, which decide strategic corporate social responsibility (CSR) and merger between the multiple plants. In the model of managerial delegation, we show that the level of unilateral CSR by a single plant under merger is higher than that under non-merger, but merged (non-merged) private firms can generate higher social welfare when products are low (high) substitutes. We also show that the level of bilateral CSR by both plants is lower than that of unilateral CSR under non-merger competition, and partial privatization is always optimal regardless of merger decisions but its degree under merger is lower than that under non-merger. Finally, we discuss policy implications of the firm's strategic alliance in the case that merger is not voluntarily supportive.

Keywords: Strategic corporate social responsibility; Merger; Mixed market; State holding corporation;

1. Introduction

Over the last decades, privatization policy has been a prevailing trend in the world. The welfare consequence of privatization programs is a fundamental question and thus, it is a controversial subject among policy-makers and public watchers.[1] Yet, several significant industries such as banking, finance, energy, electricity, manufacture and transportation sectors remain under the government's control and management.[2]

Since the government runs multiple enterprises with various sector of production, the state holding corporation (SHC) is considered as an alternative way of retaining assets in addition to privatizing to potential buyers. That is, state-owned enterprises that would not be privatized can be formed as a single entity and

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[1] For comprehensive research on privatization, see Armstrong et al. (1994) and Vickers and Yarrow (1988, 1991). For recent cases of privatization programs, see Nepal and Foster (2015) and Winston and Yan (2011).

[2] According to an OECD report by Kowalski et al. (2013), of the 2000 largest public companies in the world, more than 10%are either SOEs or have significant government ownership; these government-associated companies' sales are equivalent to approximately 6% of the global GDP.

administered under a new management established as a SHC.[3] For example, in China, Korea and Japan, state owned enterprise in postal service entered logistic sectors, that in railways industry is supplying travel services, and that in banking industry is providing insurance services. All those industries are mixed markets where the state owned enterprise competes with private enterprises.[4]

In reality, many private enterprises also comprise private holding corporations (PHCs) that produces various types of goods at various production plants.[5] As a result, corporations are indeed multiproduct firms. Typical examples include consumer markets for automobiles, PCs, and air travels, as well as business markets for processed materials, computer servers, and various types of industrial machinery.[6]

These facts stress the importance of investigating the market performace with PHCs that produce several differentiated goods in the presence of SHC. Further, in the process of various privatization programs it is urgent for antitrust agency to investigate the welfare effect of merger behaviors of PHCs in mixed markets. A few recent works including Bárcena-Ruiz and Garzón (2017), Dong et al. (2018), and Kawasaki and Matsuzaki (2020) have considered mixed markets where the SHC competes with private firms or PHC. However, previous works assumed that an industry is consist of pure private firms that have homogeneous objectives to maximize their profits only.

There is another trend in organization and ownership structure in globalized market environment. Corporate social responsibility (CSR) waves are explosively expanding today and the firm's voluntary engagement of CSR activities has become a global business practice.[7] Among many other classifications of CSR, as an instrument

[3]The OECD (2005) reports that the EU governments are the largest shareholders in many partially privatized firms in EU countries. State control is also signicant in Asian countries such as Japan, China, Indonesia, Korea, Malaysia, Singapore and Thailand. In these countries, publicly-owned firms are usually organised as SHCs and produce various types of products at various production plants. See also The Reporter (17 November 2018), "Government contemplates merger of public enterprises under a holding company."

[4]De Fraja and Delbono (1989) and Matsumura (1998) firstly examined a mixed market and justified privatizing public firms when the number of private firms is sufficient or they are efficient. Since they open a new and interesting field of mixed oligopolies, controversial debates among researchers are on going until now. For discussion on optimal privatization policy, see Lee and Hwang (2003), Matsumura and Kanda (2005), Fujiwara (2007), Lee et al. (2018), and Xu and Lee (2019) among others.

[5]Multiproduct firms are omnipresent in modern economies as pointed out in Kumar (1992) and Eckel and Neary (2010). Recent studies on multiproduct competition include Bárcena-Ruiz and Garzón (2017); Bárcena-Ruiz et al. (2020), Armstrong and Vickers (2018) and Johnson and Rhodes (2019).

[6]The factors that influence the decision to merge by private firms and the effects of that decision on social welfare have been heavily analyzed by the literature on mergers. For recent works, Farrell and Shapiro (2010), Nocke and Whinston (2010, 2013) and Jaffe and Weyl (2013) have influenced antitrust practice.

[7]The global phenomenon that firms concern with CSR has been also confirmed by various surveys, such as KPMG (2013, 2015) and UN Global Compact-Accenture CEO Study (2010, 2013). According to the PWC Global CEO survey (2016), for example, 64% of the CEOs see CSR as a core part of their business and 59% of them believe social values are important to attract top employees. Furthermore, the importance of CSR is expected to rise within the next 5 years, and 87% of the companies become aware of the strategic dimension of societal outcome measurement.

for profit creation, there has been also great amount of studies seek to explain or understand firms' strategic profit-maximizing use of CSR.[8] These studies help to explain the role of CSR in competitive markets dominated by a few private profit-maximizing firms.

Recent theoretical study on incentives of CSR concerned by private firms when they compete with state-owned firms has just caught attention of research scholars. For examples, Ouattara (2017), Itano (2017) and Kim et al. (2019) examined optimal privatization policies with the CSR activities of a private firm in mixed markets. These works assumed that the level of CSR is exogenously given. To our knowledge, studies on how private firms decide on the strategic level of CSR competing with welfare maximizing firms are limited. One exception is the paper by Dong and Wang (2019), who consider a mixed oligopoly where a private firm in a managerial delegation contract competes with a SHC with differentiated products and show that the level of CSR depends on the degree of privatization of the SHC and the degree of substitution between products. However, it does not consider strategic merger decision of the firms as a PHC.

The purpose of this paper is to fill this gap in the literature on the subject of merger and CSR in mixed markets where many heterogenous firms coexist. Our approach is close to Bárcena-Ruiz and Garzón (2003), but they did not consider the CSR activities of the merged firms. We will extend and study how private firms decide on the level of CSR competiting with a welfare-maximizing SHC and how the strategic CSR is related with merger decision among the private firms. We analyze the decision of private firms whether to merge when SHC exists, and how to use the strategic CSR for its own profit-maximizing purpose.

In particular, we consider two product differentiated mixed markets comprised of a SHC and a private sector. The SHC owns two production plants, each of which produces a differentiated good. The private sector also comprises two private plants, each of which produces a differentiated good. One or two of private firms might adopt CSR and also consider whether to merge with rival private firm to form a PHC or not. We then examine the private firm's decisions of strategic CSR and merger in the process of output competition in mixed markets with SHC. Our goal is to investigate how the merger decision of the private firm is relate to the strategic incentives of CSR.

Our main findings are as follows: First, the strategic level of unilateral CSR by a single plant under merger is higher than that under non-merger even though private firms compete with either a fully nationalized or semi-public SHC. That is, the CSR-firm is more aggressive in production under merger in mixed markets. This is because merger can internalize the inter-products substitution effects between the two competing markets, which brings about more reduction of the SHC's output especially when the substitutability between the two products is low.

[8]Many researchers have considered the strategic approach in consumer-friendly activities since Goering (2007), Kopel and Brand (2012), Kopel (2015), and Lambertini and Tampieri (2015). Further, recent works on the environmental concerns have been also gaining academic attention. See Liu et al. (2015), Lee and Park (2019), and Hirose et al. (2017, 2020).

In presence of a fully nationalized SHC, we then show that merged (non-merged) competition with a unilateral CSR can generate the highest social welfare unless products are (not) close substitutes. We also show that the strategic CSR between the two private firms is strategic substitutes and thus the level of bilateral CSR by both private plants is lower than that of unilateral CSR under non-merged competition. However, bilateral CSR encourages more competition in the market and thus generates lower market prices and higher consumer surplus than those of unilateral CSR under non-merged competition. As a result, merged competition with a bilateral CSR might yield the lowest social welfare regardless of products differentiation.

In presence of a semi-public SHC, we also show that partial privatization is always optimal, regardless of merger, but its degree under merger is lower than that under non-merger. We then confirm that our findings under a fully nationalized SHC still hold under a semi-public SHC.

Finally, we show that merger decisions depend on whether or not side payments between firms are allowed in the case of unilateral CSR,[9] whereas a merger is always voluntarily achievable and joint profits are increasing in the case of bilateral CSR. We then show that a merger is achievable and always profitable for both private firms if side payments between insiders within the merged firm are permitted. If such side payments are not permitted and thus a merger is not achievable, we further consider a strategic alliance and reveal that strategic alliance is not only profitable for the private firms but socially beneficial, which is Pareto-improving result.

The organization of this article is as follows: In section 2, we provide a basic model of a unilateral CSR. In section 3, we analyze non-merged case and merged case, respectively, and compare the equilibrium outcomes. We discuss some policy implications in section 4, which includes the competition effect of bilateral CSR, the optimal degree of privatization policy, and the welfare effect of strategic alliance. Finally, section 5 concludes the paper.

2. The Model

We consider two differentiated products markets with a state-holding corporation (SHC), firm A, which has two plants and produces goods in market 1 and 2, denoted by $A1$ and $A2$, respectively, and two uniplant private firms in each market, denoted by $B1$, and $B2$, respectively. When firm $B1$ and firm $B2$ merge (or when one of them acquires the other), we assume that they can set up a multiproduct corporation after the merger, which has two divisions/plants, each producing one goods.

On the demand side, there is a continuum of consumers of the same type. The representative consumer has a utility function $U(q_1, q_2)$, which is quadratic, strictly concave and symmetric in q_1 and q_2: $U(q_1, q_2) =$

[9] Although cross-subsidization is pretty widespread within multi-product firms, agreeing on side payments may often be difficult. Peck and Temple (2002) emphasize that technological change may have important effects on many different aspects of the firms' operation, including production costs, demand, and product lines, thus making it difficult for the merging parties to agree on suitable transfers between them.

$(q_1+q_2)-((q_1^2+2\gamma q_1q_2+q_2^2))/2$, where $\gamma \in [0,1)$ measures the degree of product differentiation and q_{ki} is the output produced by firm or plant ki, $k=A,B$; $i=1,2$. Then, the consumer maximizes $U(q_1,q_2)-p_1q_1-p_2q_2$, where p_i is the price of good i, $q_1=q_{A1}+q_{B1}$ and $q_2=q_{A2}+q_{B2}$ are the quantity of good 1 and 2 respectively. The inverse demand functions are linear and given by:

$$p_i = 1-(q_{Ai}+q_{Bi})-\gamma(q_{Aj}+q_{Bj}),\ \ i\neq j,\ \ i,j=1,2; \tag{1}$$

Note that the products are regarded as highly substitutes if γ approaches 1 while independent if $\gamma=0$.

On the supply side, we assume that firms have identical technologies represented by the following quadratic cost function: $C(q_{ki})=\frac{q_{ki}^2}{2}$, $k=A,B$; $i=1,2$. Thus, the profit function of a plant or firm ki is:

$$\pi_{ki} = p_iq_{ki}-\frac{q_{ki}^2}{2} \tag{2}$$

The social welfare is defined as the sum of consumer surplus and producer surplus in both markets:

$$W = CS+PS = CS+\pi_A+\pi_{B1}+\pi_{B2}; \tag{3}$$

where $CS=((q_{A1}+q_{B1})^2+2\gamma(q_{A1}+q_{B1})(q_{A2}+q_{B2})+(q_{A2}+q_{B2})^2)/2$ and $\pi_A=\pi_{A1}+\pi_{A2}$.

We assume that a SHC is fully owned by a government and maximizes the social welfare.[10] Regarding the objective of the private firms, firm $B1$ and firm $B2$, we assume that both are profit-maximizing firms with the asymmetry between the private firms, as in Fanti and Buccella (2017) and Kim et al. (2019). In specific, we regard firm $B1$ as a managerial firm where production decision is delegated to a manager. That is, the owner of firm $B1$ specifies an incentive contract with the manager in order to maximize its profit. Then, the manager is assumed to maximize the following objective function, i.e., the profit of firm $B1$ plus a fraction (θ) of consumer surplus (CS) in production:[11]

$$V = \pi_{B1}+\theta_1CS \tag{4}$$

where parameter $\theta_i \in [0,1]$ measures the degree of concern on consumer surplus when the firm adopts CSR activities, i.e., the level of CSR. That is, the private firm $B1$ is a CSR firm. However, we assume that the private firm $B2$ is a pure profit-maximizing firm.[12]

The private firms, $B1$ and $B2$, can decide to merge and set up a multiproduct firm with two divisions, 1 and 2. If they merge, the profit of the owner and the objective function of the manager in a multiplant

[10] We will also consider a semi-public SHC and examine optimal degree of privatization in section 5 and Appendix A.

[11] Note that we consider consumer surplus as the proxy of CSR, which has been widely utilized in related literature. For more discussion in recent works, see Lambertini and Tampieri (2015), Bian and Li (2016), Garcia et al. (2019), Dong and Wang (2019), and Leal et al. (2018, 2019, 2020a) among others.

[12] In section 5, we will consider a symmetric case that both firms are managerial firms where production decision is delegated to a manager while the level of CSR is determined by the owner. We then examine the homogeneity of the objectives between the firms in the competitive decisions of CSR.

CSR-corporation (CC) are, respectively:[13]

$$\pi_B = \pi_{B1} + \pi_{B2} \tag{5}$$

$$V_B = \pi_B + \theta CS \tag{6}$$

Our goal is to investigate the merger incentives of the CSR firm when it competes with a SHC, and the relations between strategic CSR and merger decisions.

The game runs as follows[14]: In stage 1, both CSR and private firms decide whether to merge and set up a multiproduct CSR firm. In stage 2, the owner of a CSR firm determines its degree of CSR. Finally, both firms compete with outputs in the last stage. The subgame perfect equilibrium will be derived by backward induction.

3. Analysis

Given that private firms will decide whether to merge or not, we have two cases. In the first case, private firms do not merge, in which a SHC competes with two independent private firms, which will be denoted by the superscript $\star$. In the second case, firms merge and set up a multiproduct CSR firm, in which a SHC competes with a CSR-corporation, which will be denoted by the superscript †. We will analyze the two cases, respectively.

3.1. Non-merged competition

In the last stage, we solve equilibrium output decisions of the game under non-merger. When there is a SHC competing with two independent private firms. The SHC chooses the output level of its two plants, q_{A1} and q_{A2} that maximise eqn. (3). The manager of firm $B1$ chooses the output q_{B1} that maximises (4) while firm $B2$ chooses output q_{B2} to maximise its profit given by eqn. (2).

From the first-order conditions, the respond functions are derived as

$$\begin{aligned} q_{Ai} &= \frac{1 - \gamma q_{Aj} - q_{Bi} - \gamma q_{Bj}}{2}, \;\; i \neq j, \;\; i, j = 1, 2 \\ q_{B1} &= \frac{1 - (1 - \theta_1) q_{A1} - (\gamma - \gamma\theta_1) q_{A2} - (\gamma - \gamma\theta_1) q_{B2}}{3 - \theta_1} \\ q_{B2} &= \frac{1}{3}\left(1 - \gamma q_{A1} - q_{A2} - \gamma q_{B1}\right) \end{aligned} \tag{7}$$

[13]Following Garcia et al. (2020), we assume that a CSR-corporation (CC) maximizes the joint profits in a managerial delegation contract in which the owners of the CC delegate the managers the output production in order to maximize the joint profits plus a fraction (θ) of consumer surplus (CS). Note also that if the two merged firms have different objective functions before the merger, then it is reasonable to assume that the merged firm maximizes the weighted average of the payoff of the two firms. For related analysis, see Matsumura (1998) and Lee and Hwang (2003) examined partial ownership and Bárcena-Ruiz and Garzón (2003) examined the merger between the welfare-maximizing public firm and the profit-maximizing private firm, respectively.

[14]Note that the decision of whether to merge or not precedes the CSR decision, as the former is typically a longer-term issue.

From (7), it is easy to see that output of firm A ($q_{Ai},\ i=1,2$) decreases with that of private firm Bi, and the output produced by one plant of firm A decreases with that of the other, i.e. outputs are strategic substitutes. In addition, the output of private firm Bi decreases with that of firm A ($q_{Ai},\ i=1,2$). Solving them, we obtain the following Lemma.

Lemma 1. *When a SHC competes with two uniplant private firms, the outputs are*

$$q_{A1}^{\star}=\frac{10-6\gamma-\left(5-3\gamma^{2}\right)\theta_{1}}{25-9\gamma^{2}-\left(5-3\gamma^{2}\right)\theta_{1}};\qquad q_{A2}^{\star}=\frac{2(5-3\gamma-\theta_{1})}{25-9\gamma^{2}-\left(5-3\gamma^{2}\right)\theta_{1}}$$
$$q_{B1}^{\star}=\frac{5-3\gamma+\left(5+3\gamma-3\gamma^{2}\right)\theta_{1}}{25-9\gamma^{2}-\left(5-3\gamma^{2}\right)\theta_{1}};\qquad q_{B2}^{\star}=\frac{5-3\gamma-\theta_{1}}{25-9\gamma^{2}-\left(5-3\gamma^{2}\right)\theta_{1}}$$

By Lemma 1 it can be checked that $\frac{\partial q_{Ai}^{\star}}{\partial\theta_{1}}<0$, $\frac{\partial q_{B1}^{\star}}{\partial\theta_{1}}>0$ and $\frac{\partial q_{B2}^{\star}}{\partial\theta_{1}}<0$. Therefore, $\frac{\partial q_{1}^{\star}}{\partial\theta_{1}}>0$ and $\frac{\partial q_{2}^{\star}}{\partial\theta_{1}}<0$ where $q_{i}=q_{Ai}+q_{Bi}$. Then, there exists an inter-products substitution effect between the firms in market i and an inter-markets substitution effect between the markets A and B. Both effects go to the same direction where the CSR-firm's output always increases with CSR effort while both SHC and private firms' outputs always decrease with CSR effort. Further, from last 2 inequalities, we have that the market 1's output always increases with CSR effort while market 2's output always decreases with CSR effort.

We now proceed to the second stage of the game in which the owner of CSR firm determines its degree of CSR. That is, the owner of firm $B1$ chooses θ to maximize its profits given by (2). Thus, we obtain the following result:

Lemma 2. *When a SHC competes with two uniplant private firms:*

$$\theta^{\star}=\frac{25-15\gamma}{125+45\gamma-120\gamma^{2}-18\gamma^{3}+27\gamma^{4}};$$
$$q_{A1}^{\star}=\frac{3(5-3\gamma)\left(5-2\gamma^{2}\right)}{(10-3\gamma^{2})(20-9\gamma^{2})};\qquad q_{A2}^{\star}=\frac{2\left(40-25\gamma-15\gamma^{2}+9\gamma^{3}\right)}{(10-3\gamma^{2})(20-9\gamma^{2})};$$
$$q_{B1}^{\star}=\frac{5-3\gamma}{20-9\gamma^{2}};\qquad q_{B2}^{\star}=\frac{40-25\gamma-15\gamma^{2}+9\gamma^{3}}{(10-3\gamma^{2})(20-9\gamma^{2})};$$
$$\pi_{B1}^{\star}=\frac{(5-3\gamma)^{2}}{2\left(200-150\gamma^{2}+27\gamma^{4}\right)};\qquad \pi_{B2}^{\star}=\frac{3\left(40-25\gamma-15\gamma^{2}+9\gamma^{3}\right)^{2}}{2\left(200-150\gamma^{2}+27\gamma^{4}\right)^{2}}$$

By Lemma 2 it can be shown that $\theta^{\star}>0$ and thus the positive level of CSR can be utilized by the CSR firm for the profit-maximizing purpose. Also, it can be checked that the relationship between θ and γ is non-monotone, i.e., $\frac{\partial\theta^{\star}}{\partial\gamma}\gtrless 0$ if $\gamma\gtrless\bar{\gamma}$ and $\frac{\partial^{2}\theta^{\star}}{\partial\gamma^{2}}>0$, where $\bar{\gamma}$ satisfies that $\frac{\partial\theta^{\star}}{\partial\gamma}=0$ and $\bar{\gamma}\approx 0.636$. That is, when the differentiation between products is sufficiently high, the optimal CSR level decreases with the degree of substitution. However, when the degree of substitution is increased past a certain level, the optimal CSR begins increasing with the degree of substitution. Therefore, the degree of product differentiation plays a key role in determining the strategic level of CSR under non-merged competition.

3.2. Merged competition

In the last stage, we solve equilibrium output decisions of the game under merger. When the two private firms merge, they set up a multiproduct CC that chooses the values of q_{B1} and q_{B2} that maximize (6). Then, according the first-order conditions, the respond functions for the SHC are the same as the previous case, while the respond functions for CC are derived as:

$$q_{Bi} = \frac{1-(1-\theta)q_{Ai}-(\gamma-\gamma\theta)q_{Aj}-\gamma(2-\theta)q_{Bj}}{3-\theta}, \;\; i \neq j, \;\; i,j = 1,2$$

Solving them, we obtain the following Lemma.

Lemma 3. *When a SHC competes with a CC, the outputs are*

$$q_{Ai}^{\dagger} = \frac{2+\gamma-\theta(1+\gamma)}{5+5\gamma+\gamma^2-\theta(1+\gamma)}, \qquad q_{Bi}^{\dagger} = \frac{1+\theta(1+\gamma)}{5+5\gamma+\gamma^2-\theta(1+\gamma)} \quad i=1,2$$

By Lemma 3 it can be checked that $\frac{\partial q_{Ai}^{\dagger}}{\partial \theta} < 0$, $\frac{\partial q_{Bi}^{\dagger}}{\partial \theta} > 0$ and $\frac{\partial q_{i}^{\dagger}}{\partial \theta} > 0$. Then, merger generates the internalization effect between the differentiated products a multiproduct CC internalizes how the output of one of its plants affect that of its other plant, which encourages to increase the total outputs of its plants. That is, from last inequality, unlike the non-merged case, since merger can generate more aggressive production, the market outputs always increase with CSR effort. However, an intra-products substitution effect in the market still exists where the CC's output always increases with CSR effort while SHC's outputs always decrease with CSR effort.

We now proceed to the second stage of the game in which a CC determines its degree of CSR. That is, the owner(s) of CC chooses θ to maximize the joint-profits given by (5). Thus, we get:

Lemma 4. *When a SHC competes with a CC:*

$$\theta^{\dagger} = \frac{1+\gamma}{5+3\gamma};$$
$$q_{Ai}^{\dagger} = \frac{3+2\gamma}{(2+\gamma)(4+3\gamma)}; \qquad q_{Bi}^{\dagger} = \frac{1}{4+3\gamma}$$
$$\pi_{B}^{\dagger} = \frac{1}{(2+\gamma)(4+3\gamma)}$$

By Lemma 4 it can be checked that $\frac{\partial \theta^{\dagger}}{\partial \gamma} > 0$ and $\frac{\partial^2 \theta^{\dagger}}{\partial \gamma^2} < 0$. It means that the degree of product differentiation also plays a key role in determining the strategic level of CSR under merged competition. However, unlike non-merged case, the CSR monotonically increases with the degree of substitution, and this effect is weakened as the degree of substitution increases.

3.3. Comparisons

We will compare the levels of strategic CSR in the two cases:

Proposition 1. *The strategic level of CSR under merger is higher than that under non-merger.*

Proof. From Lemma 2 and Lemma 4, we have that for any $\gamma \in [0,1)$

$$\theta^{\dagger} - \theta^{\star} = \frac{\gamma\left(170 - 30\gamma - 138\gamma^2 + 9\gamma^3 + 27\gamma^4\right)}{(5+3\gamma)\left(125 + 45\gamma - 120\gamma^2 - 18\gamma^3 + 27\gamma^4\right)} \geq 0$$

□

Proposition 1 states that merger will encourage the level of CSR, but its effect depends non-monotonically on the degree of product differentiation. In addition, the difference between the levels of CSR increases (decreases) as the substitutability increases when the substitutability is low (high), i.e., $\frac{\partial(\theta^{\dagger}-\theta^{\star})}{\partial\gamma} \gtreqless 0$ if $\gamma \lesseqgtr 0.77$. Thus, the CSR-firm can be more aggressive in production under merger in the presence of a SHC since it can internalize its own inter-products substitution effects between the two markets, which brings about more reduction of the SHC's output especially when the substitutability is not high.[15]

Regarding the equilibrium outputs and prices in the two cases, we have the followings:

Lemma 5. *Comparing the equilibrium outputs and prices in the two cases, we have:*

(i) $q_{A1}^{\star} + q_{A2}^{\star} \lesseqgtr q_{A1}^{\dagger} + q_{A2}^{\dagger}$ *if* $\gamma \gtreqless \gamma_1$ *where* γ_1 *satisfies that* $q_{A1}^{\star} + q_{A2}^{\star} = q_{A1}^{\dagger} + q_{A2}^{\dagger}$ *and* $\gamma_1 \approx 0.790$.

(ii) $q_{B1}^{\star} \geq q_{B1}^{\dagger}$, $q_{B2}^{\star} < q_{B2}^{\dagger}$ *while* $q_{B1}^{\star} + q_{B2}^{\star} \lesseqgtr q_{B1}^{\dagger} + q_{B2}^{\dagger}$ *if* $\gamma \lesseqgtr \gamma_1$.

(iii) $q_1^{\star} \geq q_1^{\dagger}$ *while* $q_2^{\star} < q_2^{\dagger}$ *where* $q_i = q_{Ai} + q_{Bi}$, $i = 1, 2$.

(iv) $p_1^{\star} \leq p_1^{\dagger}$ *while* $p_2^{\star} \lesseqgtr p_2^{\dagger}$ *if* $\gamma \gtreqless 0.913$

Lemma 5 shows that (i) a SHC increases (decreases) its outputs from rivals' merger when products are relatively high (low) substitutes. (ii) Regarding CSR firm's output after the merger, due to the internalization effect under merged case, firm $B1$'s output decreases while firm $B2$'s output increases, but the joint production of the private firms increases (decreases) when products are relatively low (high) substitutes. These two findings imply that there is output substitution effect between the SHC and the merged CC while cost saving effect arises unless products are relatively high substitutes.[16] Also, (iii) total outputs in market 1 (2) can decrease (increase) after the merger, but (iv) price in market 1 before merger is lower than that after merger while price in market

[15] As shown in Lee and Xu (2018) and Leal et al. (2020b), in an endogenous timing game in a mixed market where the public firm competes with a private firm and both firms decide the timing of movement, it is well-known that in the equilibrium the private firm produces the production more aggressively as a Stackelberg leader while the public firm follows, which can also increase the social welfare because the output substitution effect between the firms can also accompany with cost-saving effect of the public firm. Regarding these effects, see the descriptions in Lemma 5.

[16] Note that SHC cares for the whole amount of CS whereas CC cares for a partial amount of CS. It brings about higher output and higher production cost to the SHC. Therefore, if output substitution arises between the SHC and CC, then cost-saving effect also arises when the SHC decreases its output.

2 after merger can be lower than that before merger when products are sufficiently differentiated. That is, when products are not close substitutes, consumers are also better off.

These findings are different with the single product case. As shown in Lambertini and Tampieri (2015), Leal et al. (2018) and Garcia et al. (2019), a higher level of CSR implies aggressive output production in a single product market. However, Lemma 5 shows that a higher level of CSR in a multi-product market does not always implies more output production. Only when products are sufficiently differentiated, a higher level of CSR encourages not only the output production of CC but total industry outputs.

We shall now analyze whether private firms want to merge and set up a CC. We will consider two cases of merger decision. The first case is a voluntary decision on merger in which both firms will accept the merger if the profit that they will obtain in the multiproduct firm, $\frac{\pi_B^\dagger}{2}$, is greater than the profit obtained when competing as 2 uniplant private firms, $\pi_{Bi}^\star$ $i = 1, 2$. It implies that the merger decision does not allow for side payment in the process of merger.[17] The second case allows for side payment between the firms and thus both firms will merge if total joint profits, $\pi_B^\dagger$, is greater than the sum of profits obtained when competing as 2 uniplant private firms, $\pi_{B1}^\star + \pi_{B2}^\star$.

Then, we have the followings:

Lemma 6. *Comparing profits in the two cases, we have:*

(i) $\pi_{B1}^\star > \frac{\pi_B^\dagger}{2}$ *while* $\pi_{B2}^\star < \frac{\pi_B^\dagger}{2}$ *for any* $\gamma \in [0, 1)$.

(ii) $\pi_{B1}^\star + \pi_{B2}^\star < \pi_B^\dagger$ *for any* $\gamma \in [0, 1)$.

Lemma 6 states that (i) CSR firm $B1$ prefers to remain as a uniplant firm while firm $B2$ prefers to merge. Therefore, without side-payments they will not voluntarily merge. It also states that (ii) both firms will better off after merger when side payments are allowed. Therefore, our result depends on whether or not side payments between firms are allowed.

Proposition 2. *When side payments between insiders within the merged firm are permitted, a merger is achievable and always profitable for both firms. When such side payments are not permitted, however, a merger is not voluntarily achievable even though its joint profits are increasing.*

Finally, we will compare the consumer surplus and welfare in the two cases:

Proposition 3. *Comparing the consumer surplus and welfare in the two cases, we have:*

[17] Note that although cross-subsidization is pretty widespread within multi-product firms, agreeing on side payments may often be difficult. According to Peck and Temple (2002), technological change may have important effects on many different aspects of the firms' operation, including production costs, demand, and product lines, thus making it difficult for the merging parties to agree on suitable transfers between them. The assumption of no side payments between insiders within the merged firm is in line with the concept of pairwise stability in network formation. For more discussion, see Kesavayuth et al. (2018).

(i) $CS^{\star} \lessgtr CS^{\dagger}$ *if* $\gamma \lessgtr \gamma_2$ *where* γ_2 *satisfies that* $CS^{\star} = CS^{\dagger}$ *and* $\gamma_2 \approx 0.784$

(ii) $W^{\star} \lessgtr W^{\dagger}$ *if* $\gamma \lessgtr \gamma_3$ *where* γ_3 *satisfies that* $W^{\star} = W^{\dagger}$ *and* $\gamma_3 \approx 0.972$

Proof. (i) The difference of consumer surplus between under non-merged and under merged competition is as follows: $CS^{\dagger} - CS^{\star} = \frac{78400+28000\gamma-200500\gamma^2-94020\gamma^3+156855\gamma^4+80550\gamma^5-48762\gamma^6-26082\gamma^7+5346\gamma^8+2916\gamma^9}{2(2+\gamma)^2(4+3\gamma)^2(10-3\gamma^2)^2(20-9\gamma^2)^2}$ and this expression is equal to zero when $\gamma = \gamma_2$. (ii) The difference of welfare between under non-merged and under merged competition is as follows: $W^{\dagger} - W^{\star} = \frac{10400-3200\gamma-20850\gamma^2+4255\gamma^3+14850\gamma^4-1800\gamma^5-4482\gamma^6+243\gamma^7+486\gamma^8}{(2+\gamma)(4+3\gamma)^2(10-3\gamma^2)^2(20-9\gamma^2)^2}$ and this expression is equal to zero when $\gamma = \gamma_3$.

□

Proposition 3 states that the differences of consumer surplus and welfare under merger decision depends crucially on the degree of product differentiation. That is, merged competition can generate higher consumer surplus and welfare when products are not close substitutes where total industry outputs under merger increases. Meanwhile, non-merged competition enhances higher consumer surplus and welfare when products are very similar where total industry outputs under merger decreases. In the case that the degree of product differentiation is intermediate, i.e., $0.784 < \gamma < 0.972$, output decreasing effect from merger will be outweighed by the cost-saving effect arised from the intra-products substitution effect between the SHC and CC.

From the viewpoint of society, therefore, merger decision by the antitrust agency in mixed markets should consider not only the degree of product differentiation but the strategic level of CSR. This result suggests detailed information on the products and market coverage is required if a government intends to encourage private firm be more consumer-friendly in mixed markets where merger decision by the private firms is important.

4. Discussions

In this section, we extend the analysis and examine policy implications of the findings. We first consider a symmetric case of bilateral CSR where both private firms adopt CSR competitively in a same managerial delegation framework. We also examine the optimal degree of privatization of SHC. Finally, we discuss on the welfare effect of strategic alliance when the side payments are not allowed between the private firms in the process of merger decision.

4.1. Bilateral CSR

We consider the symmetric case that firm *B2* is also a CSR firm, whose manager maximizes $\pi_{B2} + \theta_2 CS$ and its owner maximizes π_{B2}, as analyzed in Bian and Li (2016) and Fanti and Buccella (2017). The game runs as before, except stage 2 in which both CSR firms determine the levels of CSR simultaneously.[18] The subgame

[18]Note that the results under merger are the same as in Lemma 4 in which an SHC competes with a CC, since the merger takes place between two CSR firms.

perfect equilibrium will be derived by backward induction.

In the last stage, we solve equilibrium output decisions of the game under non-merger. The SHC chooses the output level of its two plants, q_{A1} and q_{A2} that maximise eqn. (3). The firm Bi´s manager chooses the output q_{Bi} that maximises $\pi_{Bi} + \theta_i CS$; $i = 1, 2$, respectively.

Solving the first-order conditions, we obtain the following Lemma where the superscript $\star\star$ denotes the equilibrium of the bilateral CSR under non-merged competition.

Lemma 7. *When a SHC competes with two uniplant CSR firms, the outputs are*

$$q_{A1}^{\star\star} = \frac{2(5-3\gamma-\theta_2)-\theta_1\left(5-3\gamma^2-\left(1-\gamma^2\right)\theta_2\right)}{25-9\gamma^2-(5-3\gamma^2)\theta_2-\theta_1(5-3\gamma^2-(1-\gamma^2)\theta_2)}; \qquad q_{A2}^{\star\star} = \frac{10-6\gamma-\left(5-3\gamma^2\right)\theta_2-\theta_1\left(2-\left(1-\gamma^2\right)\theta_2\right)}{25-9\gamma^2-(5-3\gamma^2)\theta_2-\theta_1(5-3\gamma^2-(1-\gamma^2)\theta_2)}$$

$$q_{B1}^{\star\star} = \frac{5-3\gamma-\theta_2+\theta_1\left(5+3\gamma-3\gamma^2-\left(1-\gamma^2\right)\theta_2\right)}{25-9\gamma^2-(5-3\gamma^2)\theta_2-\theta_1(5-3\gamma^2-(1-\gamma^2)\theta_2)}; \qquad q_{B2}^{\star\star} = \frac{5-3\gamma+\left(5+3\gamma-3\gamma^2\right)\theta_2-\theta_1\left(1+\left(1-\gamma^2\right)\theta_2\right)}{25-9\gamma^2-(5-3\gamma^2)\theta_2-\theta_1(5-3\gamma^2-(1-\gamma^2)\theta_2)}$$

We now proceed to the second stage of the game in which both CSR firms´ owners determine their CSR degree simultaneously. That is, the owner of firm Bi chooses θ_i to maximize its profits π_{Bi}. In the process of examining the first-order conditions, we can note that the reaction functions are downward sloping; i.e. $\frac{\partial \theta_i}{\partial \theta_j} < 0, i \neq j = 1, 2$, which implies that the strategic levels of CSR by the both firms are strategic substitutes.[19] Further, taking into account that firms are symmetric, then the equilibrium levels of CSR from the reaction curves are symmetric. Then, setting $\theta_1 = \theta_2 = \theta^{\star\star}$ at equilibrium, we obtain the following result:

Lemma 8. *When a SHC competes with two uniplant CSR firms:*

$$\theta^{\star\star} = \frac{26 + 25\gamma - 9\gamma^2 - 9\gamma^3 - \psi}{2(1+\gamma)(5-3\gamma^2)};$$

$$q_{Ai}^{\star\star} = \frac{-6 - 25\gamma - 3\gamma^2 + 9\gamma^3 + \psi}{24 + 5\gamma - 21\gamma^2 - 9\gamma^3 + \psi}; \qquad q_{Bi}^{\star\star} = \frac{36 + 25\gamma - 15\gamma^2 - 9\gamma^3 - \psi}{24 + 5\gamma - 21\gamma^2 - 9\gamma^3 + \psi}$$

$$\pi_{Bi}^{\star\star} = \frac{3\left(16 + 25\gamma - 3\gamma^2 - 9\gamma^3 - \psi\right)\left(-36 - 25\gamma + 15\gamma^2 + 9\gamma^3 + \psi\right)}{2\left(24 + 5\gamma - 21\gamma^2 - 9\gamma^3 + \psi\right)^2} \quad i = 1, 2.$$

where $\psi := \sqrt{576 + 1200\gamma + 217\gamma^2 - 858\gamma^3 - 369\gamma^4 + 162\gamma^5 + 81\gamma^6} > 0$ for any $\gamma \in [0, 1)$. As in the case where an SHC competes with two uniplant private firms, we can show that $\theta^{\star\star} > 0$ and the relationship between θ and γ is non-monotone, i.e., $\frac{\partial \theta^{\star\star}}{\partial \gamma} \gtreqless 0$ if $\gamma \gtreqless \hat{\gamma}$ and $\frac{\partial^2 \theta^{\star\star}}{\partial \gamma^2} > 0$, where $\hat{\gamma}$ satisfies that $\frac{\partial \theta^{\star\star}}{\partial \gamma} = 0$ and $\hat{\gamma} \approx 0.694$.

Proposition 4. *Bilateral CSR reduces the level of CSR compared to unilateral CSR, that is, $\theta^{\star} \geq \theta^{\star\star}$.*

Proof. From Lemma 2 and Lemma 8, we have that for any $\gamma \in [0, 1)$

$$\theta^{\star} - \theta^{\star\star} = \frac{-1917\gamma^5+81\gamma^6+243\gamma^7-27\gamma^4(31-\psi)-125(24-\psi)+60\gamma^2(47-2\psi)+6\gamma^3(823-3\psi)-5\gamma(839-9\psi)}{2(1+\gamma)(5-3\gamma^2)(125+45\gamma-120\gamma^2-18\gamma^3+27\gamma^4)} \geq 0$$

□

[19] As well-known in the managerial delegation literature, strategic choices of bonus incentives are strategic substitutes and thus there is another version of prisoner's dilemma. Regarding CSR competition, see Fanti and Buccella (2017, 2018).

Proposition 4 comes from the fact that the levels of CSR are strategic substitutes. That is, when the CSR-firm adopts CSR, it chooses higher level of CSR when the rival firm does not adopt CSR but it reduces its level of CSR when the rival firm also adopts CSR. Thus, competitive adoption of CSR discourage the level of CSR in order to reduce over-adoption of CSR. It also implies that merger can encourage higher level of CSR since it can also eliminate the competitive effect of CSR.

Corollary 1. *The strategic level of CSR under merger is higher than the bilateral CSR under non-merger.*

Proof. From Proposition 1 and 4, it easy to show that $\theta^{\dagger} \geq \theta^{\star} \geq \theta^{\star\star}$ for any $\gamma \in [0, 1)$. □

Regarding the equilibrium outputs and prices between unilateral and bilateral CSR, we have the followings:

Lemma 9. *Comparing the equilibrium outputs and prices between unilateral and bilateral CSR, we have that for any* $\gamma \in [0, 1)$

(i) $q^{\star}_{A1} + q^{\star}_{A2} > q^{\star\star}_{A1} + q^{\star\star}_{A2}$

(ii) $q^{\star}_{B1} \geq q^{\star\star}_{B1}$, $q^{\star}_{B2} < q^{\star\star}_{B2}$ *while* $q^{\star}_{B1} + q^{\star}_{B2} < q^{\star\star}_{B1} + q^{\star\star}_{B2}$

(iii) $q^{\star}_{1} \geq q^{\star\star}_{1}$ *while* $q^{\star}_{2} < q^{\star\star}_{2}$ *where* $q_i = q_{Ai} + q_{Bi}$, $i = 1, 2$.

(iv) $p^{\star}_{1} \geq p^{\star\star}_{1}$ *and* $p^{\star}_{2} > p^{\star\star}_{2}$.

(v) $CS^{\star} < CS^{\star\star}$ *and* $W^{\star} < W^{\star\star}$

Lemma 9 represents the welfare consequence of output substitution effect between the three firms. First, (i) a SHC decreases its outputs under bilateral CSR. Second, (ii) regarding each firm's output, due to the competitive effect of CSR, firm $B1$ decreases its level of CSR and thus decreases its output while firm $B2$ increases its level of CSR and thus increases its output. Since a SHC decreases its output, the total outputs of the two CSR-firms also increase. Third, (iii) total outputs in market 1 (2) decreases (increases) under bilateral CSR. However, (iv) market prices in both markets decrease under bilateral CSR. As a result, the competitive adoption of CSR under bilateral CSR is always better than unilateral CSR in terms of consumer surplus and welfare.[20]

Finally, we consider the decision of whether bilateral CSR-firms want to merge and set up a CC, and measure the welfare effect of merger decision. Then, we will compare the profits, consumer surplus and welfare between under non-merger and merger in the case of bilateral CSR:

Proposition 5. *Under bilateral CSR, we have that for any* $\gamma \in [0, 1)$,

(i) *Both uni-plant CSR firms prefer to merge and they will voluntarily do so. That is,* $\pi^{\star\star}_{Bi} \leq \frac{\pi^{\dagger}_{B}}{2}$

[20] Note that single plant firm $B2$ obtains higher profit when it adopts CSR. We can show that $\pi^{\star}_{B1} \geq \pi^{\star\star}_{B1}$ meanwhile $\pi^{\star}_{B2} < \pi^{\star\star}_{B2}$. That is, CSR-firm B1 reduces its output, which results in the reduction of profit. However, CSR-firm B2 increases its output even though market prices decrease. Thus, the firm B2 can increase its profit since the output effect outweighs the price effect.

(ii) *Both consumer surplus and social welfare under merger decrease compared to those under non-merged competition. That is,* $CS^{\star\star} \geq CS^{\dagger}$*, and* $W^{\star\star} \geq W^{\dagger}$

Proof. First, it is easy to compare the profits between the merged case and the non-merged competition with bilateral CSR and show the results. Second, the consumer surplus and social welfare under non-merged competition are the following, respectively:

$$CS^{\star\star} = \frac{36(1+\gamma)\left(5-3\gamma^2\right)^2}{(24+5\gamma-21\gamma^2-9\gamma^3+\psi)^2};$$
$$W^{\star\star} = \frac{4\left(-486+459\gamma^4-135\gamma^5-81\gamma^6+36\psi-25\gamma(51-\psi)+\gamma^3(858-9\psi)-\gamma^2(421+15\psi)\right)}{(24+5\gamma-21\gamma^2-9\gamma^3+\psi)^2}$$

Therefore, we can show that $CS^{\star\star} - CS^{\dagger} \geq 0$ for any $\gamma \in [0,1)$. Also, $W^{\star\star} - W^{\dagger} \geq 0$ for any $\gamma \in [0,1)$.

□

Proposition 5 states that in the case of bilateral CSR, the consumer surplus and welfare under merger always decrease irrespective of the degree of product differentiation. That is, merger decision by the private firms is anti-competitive if a government can encourage all the private firms in the market to be more consumer-friendly in mixed markets. Therefore, merger decision by the antitrust agency in mixed markets should take care of each firm's strategic level of CSR into policy consideration.

4.2. Optimal privatization

We consider the case that a semi-public SHC exists instead of a fully nationalized SHC, and investigate the optimal degree of privatization policy.[21] Then, the objective of the semi-public SHC, firm A, is given as the weighted average of social welfare and its own profits, i.e., private investor owns s percent of shares in plant(s) of the state corporation:

$$\Omega = (1-s)W + s\pi_A \tag{8}$$

Following similar procedures, we obtain the equilibrium outcomes in Appendix A. In that case, in stage 0, the government determines 's' to maximize welfare under non-merged competition and merged competition, respectively.

First, from the the detailed analysis and outcomes in the Appendix A1, the welfare under non-merged competition in mixed markets is as follows:

$$W^{\star} = \frac{25925 - 16750\gamma - 28000\gamma^2 + 18060\gamma^3 + 9855\gamma^4 - 6345\gamma^5 - 1134\gamma^6 + 729\gamma^7 + \eta_8}{\left(10 - 3\gamma^2 + s\left(11 - 5\gamma^2\right) + s^2\left(3 - 3\gamma^2\right)\right)^2 H_2^2} \tag{9}$$

Let $s^{\star}$ satisfies $\frac{\partial W^{\star}}{\partial s} = 0$. Then, under non-merged competition, we have the equilibrium outcomes in Appendix A1. Figure 1 illustrates the following findings: (i) the optimal degree of privatization is decreasing in γ, (ii)

[21] In the recent literature, a few works consider partial privatization in a multiproduct mixed markets. See Dong et al. (2018), Dong and Wang (2019) and Bárcena-Ruiz et al. (2020).

the strategic level of CSR is non-monotone in γ and (iii) the welfare is decreasing in γ. Hence, both the optimal degree of privatization, which is determined by the government, and the strategic level of CSR, which is determined by the CSR firm, move the same direction when the degree of product differentiation is low. Then, these two decision variables have the relations of strategic complements, i.e., both levels are decreasing as the degree of product differentiation increases. However, both levels move to different direction when the degree of product differentiation is high, i.e., the level of CSR is increasing as the degree of product differentiation increases.

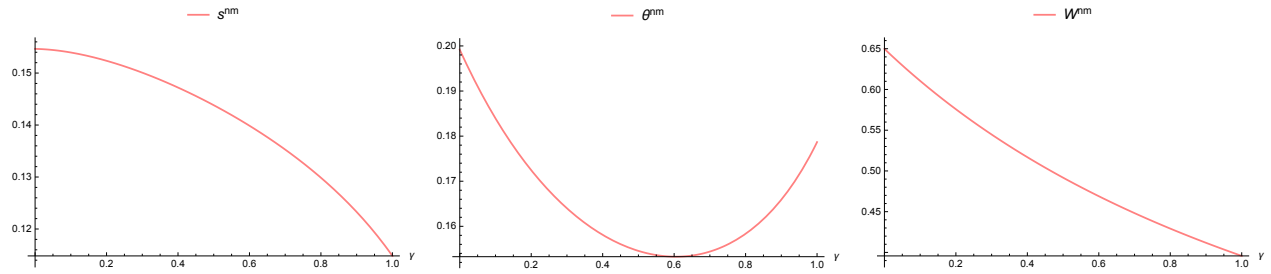

Figure 1

Next, from the the detailed analysis and outcomes in the Appendix A2, the welfare under merged competition in mixed markets is as follows:

$$W^{\dagger} = \frac{42 + 77\gamma + 46\gamma^2 + 9\gamma^3 + \nu_1}{((2 + \gamma + s(1 + \gamma))N_1)^2} \tag{10}$$

Let $s^{\dagger}$ satisfies $\frac{\partial W^{\dagger}}{\partial s} = 0$. Then, under merged competition, we have the equilibrium outcomes in the Appendix A2. Figure 2 illustrates the following findings: (i) the optimal degree of privatization is decreasing in γ, (ii) the strategic level of CSR is increasing in γ and (iii) the welfare is decreasing in γ. Then, both the optimal degree of privatization, which is determined by the government, and the strategic level of CSR, which is determined by the CSR firm, move to the reverse direction irrespective of the degree of product differentiation. Hence, these two decision variables have the relations of strategic substitutes irrespective of the degree of product differentiation.

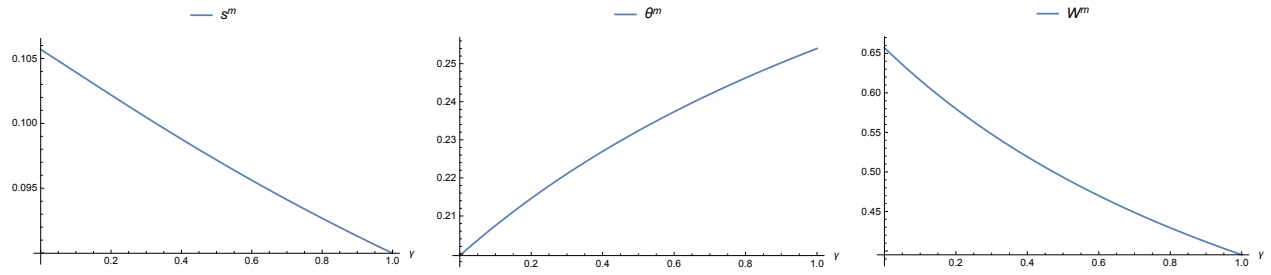

Figure 2

Then, we have the following comparisons.[22]

Proposition 6. *Partial privatization is always optimal, regardless of merger, but its degree under merger is lower than without merger. Furthermore, strategic CSR is higher under merger.*

Proposition 6 reminds the statement in Proposition 1 that merger will encourage the level of CSR. Thus, due to the strategic substitutes relationship, higher level of CSR under merger will bring about lower degree of optimal degree of privatization.

Proposition 7. $W^{nm} \lesseqgtr W^m$ *if* $\gamma \lesseqgtr \tilde{\gamma}$ *where* $\tilde{\gamma}$ *satisfies that* $W^{nm} = W^m$ *and* $\tilde{\gamma} \approx 0.8271$.

Proposition 7 reminds the statement in Proposition 2 that merged competition can generate higher social welfare when products are not close substitutes while non-merged competition enhances higher social welfare when products are very similar. Therefore, it confirms that our findings under a fully nationalized SHC still hold.

4.3. Strategic alliance

In the case that side payments are not allowed,[23] we consider a variant of a voluntary collusive agreement such as cooperative contract on strategic alliance,[24] which provide similar effect of merger under imperfect contract between the firms.

In specific, we consider the case that both firms agree on the profit constraint that strategic alliance guarantees the competition profit of CSR firm by providing some advantages to CSR firm. Note that private firm $B2$ is willing to merge but CSR firm $B1$ does not want even though total joint profits can be increasing. One possible contract is that both firms agree that CSR firm can choose θ to maximize its profit, not joint profit, while the CSR firm can get its original profit under competition environment at least. Then, we can define the following maximization problem of the firms under strategic alliance[25]

$$L = V_B + \mu_(\pi^{\star\star}_{B1} - \pi_{B1}) \quad \text{where} \quad \pi^{\star\star}_{B1} = \frac{3\left(5 - 3\gamma + \left(5 + 3\gamma - 3\gamma^2\right)\theta\right)\left(5 - 3\gamma - \left(5 + \gamma - 3\gamma^2\right)\theta\right)}{2\left(25 - 9\gamma^2 - \left(5 - 3\gamma^2\right)\theta\right)^2} \tag{11}$$

where $\pi^{\star\star}_{B1}$ is the profit of CSR firm under non-merged competition. Under strategic alliance, the game runs as follows: In stage 1, CSR firm $B1$ determines θ to maximize its profit. In last stage, firms compete with outputs, that is, $B1$ and $B2$ cooperate in production to maximize (11).

[22] The proofs are provided in Appendix B.

[23] Note that one of key factors in our analysis is whether side payments between insiders within the merged firm are permitted or not. That is, from lemma 6, we show that when side payments are not allowed, the merger will not be achieved.

[24] A few examples include code sharing in airline alliance or co-sharing the transportation routes in logistic industry.

[25] Note that the optimal solution of program L in (11) violates the profit constraint of CSR firm, $\pi^{\star\star}_{B1} \leq \pi_{B1}$. Thus, we can show that the constraint holds as an equality in the equilibrium analysis, i.e., $\pi^{\star\star}_{B1} = \pi_{B1}$.

As the developed model generates relatively complicated mathematical expressions, we will now use a numerical analysis in order to show equilibrium outcomes. The results of the calculations for various levels of parameter γ are given in the following table.

	θ^{sa}	π_{B1}^{sa}	π_{B2}^{sa}	W^{sa}	$\theta^{\star}$	$\pi_{B1}^{\star}$	$\pi_{B2}^{\star}$	$W^{\star}$
γ= 0	0.2	0.0625	0.0625	0.65625	0.2	0.0625	0.0625	0.65625
γ=0.1	0.183186	0.0556416	0.0553161	0.615226	0.183186	0.0556416	0.0553161	0.615226
γ=0.2	0.170412	0.0498858	0.0493005	0.579457	0.170412	0.0498858	0.0473399	0.574012
γ=0.3	0.160869	0.0450142	0.0442505	0.548062	0.160869	0.0450142	0.0424015	0.543008
γ=0.4	0.154047	0.0408623	0.0399463	0.520309	0.154047	0.0408623	0.0381435	0.515209
γ=0.5	0.149653	0.0373049	0.0361973	0.495557	0.149653	0.0373049	0.03443	0.490146
γ=0.6	0.147586	0.0342477	0.0328738	0.473292	0.147586	0.0342477	0.0311506	0.46744
γ=0.7	0.147946	0.0316207	0.0298912	0.45314	0.147946	0.0316207	0.0282103	0.446777
γ=0.8	0.151087	0.0293762	0.0271762	0.434842	0.151087	0.0293762	0.0255208	0.4279
γ=0.9	0.157766	0.0274906	0.0246429	0.418215	0.157766	0.0274906	0.0229882	0.410593

Table 1: Comparisons between strategic alliance and independent competition

Superscript *sa* stands for 'strategic alliance'. Now comparing the strategic CSR and welfare under strategic alliance with that under non-merged competition in the Table 1, we get the following findings:

Proposition 8. *Under strategic alliance, we get the followings:*

(i) *The strategic level of CSR under strategic-alliance is tantamount to that under non-merged competition, and so does the profit of CSR firm:* $\theta^{sa} = \theta^{\star}$ *and* $\pi_{B1}^{sa} = \pi_{B1}^{\star}$.

(ii) *The profit of private firm* $B2$ *increases under strategic alliance:* $\pi_{B2}^{sa} \geq \pi_{B2}^{\star}$.

(iii) *The social welfare under strategic alliance is higher than that under non-merged competition:* $W^{sa} \geq W^{\star}$.

Therefore, strategic alliance is not only profitable for the private firms but socially beneficial, which is a Pareto-improving result.

5. Concluding Remarks

We considered two differentiated products mixed markets comprised of a SHC and a private sector, and examined the private firm's decisions of strategic CSR and merger between the two plants. We showed that the strategic level of unilateral CSR by a single plant under merger is higher than that under non-merger even though private firms compete with either a fully nationalized or semi-public SHC. We then showed that in

presence of a fully nationalized SHC, merged (non-merged) competition with a unilateral CSR can generate the highest social welfare unless products are (not) close substitutes. We also showed that the level of bilateral CSR by both private plants is lower than that of unilateral CSR under non-merger competition. However, bilateral CSR encourages more competition in the market and thus generates lower market prices and higher consumer surplus than those of bilateral CSR. As a result, merged competition with a bilateral CSR might yield the lowest social welfare regardless of products differentiation.

in presence of a semi-public SHC we showed that partial privatization is always optimal, regardless of merger, but its degree under merger is lower than that under non-merger. Further, we confirmed that our findings under a fully nationalized SHC still hold.

Finally, We found that a merger is achievable and always profitable for both private firms if side payments between insiders within the merged firm are permitted. Nevertheless if such side payments are not permitted, then a merger is not voluntarily achievable even though its joint profits are increasing. Thus, our result depends on whether or not side payments between firms are allowed. We then considered a strategic alliance in the case that side payments are not allowed, and showed that strategic alliance is not only profitable for the private firms but socially beneficial.

It should be noticed that we have considered that all the firms are owned by domestic shareholders. However, due to globalization and open economy in the real world, a mixed oligopoly includes foreign private firms, which are operated in the country under foreign ownership. Then, not only the objective function of the SHC but also the strategic decisions on merger and CSR will be affected by the distribution of ownership shares in foreign firms. This issue is left for further research.

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Appendix A. The analysis of optimal privatization

A1. Non-merged competition

When a semi-public SHC competes with 2 uniplant private firms, semi-public state corp. maximizes Ω while private firm $B1$ maximizes (4) and firm $B2$ maximizes (2), respectively. Then, the outputs are as follows:

$$q_{A1}^{\star} = \frac{2\left(5-3\gamma+s\left(3-4\gamma+\gamma^2\right)\right)+\left(-5+3\gamma^2+s\left(-3+2\gamma+\gamma^2\right)\right)\theta}{H}$$
$$q_{A2}^{\star} = \frac{2\left(5-3\gamma+s\left(3-4\gamma+\gamma^2\right)\right)+\left(-2-s\left(2-3\gamma+\gamma^3\right)\right)\theta}{H}$$
$$q_{B1}^{\star} = \frac{5-3\gamma-2s\left(-4+\gamma+\gamma^2\right)+s^2\left(3-\gamma-3\gamma^2+\gamma^3\right)+\left(5+3\gamma-3\gamma^2+s\left(3+2\gamma-3\gamma^2\right)+s^2\left(\gamma-\gamma^3\right)\right)\theta}{H}$$
$$q_{B2}^{\star} = \frac{5-3\gamma-2s\left(-4+\gamma+\gamma^2\right)+s^2\left(3-\gamma-3\gamma^2+\gamma^3\right)+\left(-1+s^2\left(-1+\gamma^2\right)+s\left(-2-\gamma+\gamma^3\right)\right)\theta}{H}$$

where $H = 25-9\gamma^2+s\left(30-14\gamma^2\right)+s^2\left(9-10\gamma^2+\gamma^4\right)+\left(-5+3\gamma^2+4s\left(-2+\gamma^2\right)-s^2\left(3-4\gamma^2+\gamma^4\right)\right)\theta > 0$.

Then, we have the equilibrium outcomes:

$$\theta^{\star} = \frac{25-15\gamma+\eta_1}{H_1}; \qquad q_{A1}^{\star} = \frac{75-45\gamma-30\gamma^2+18\gamma^3+\eta_2}{(10-3\gamma^2+s(11-5\gamma^2)+s^2(3-3\gamma^2))H_2}; \qquad q_{A2}^{\star} = \frac{80-50\gamma-30\gamma^2+18\gamma^3+\eta_3}{(10-3\gamma^2+s(11-5\gamma^2)+s^2(3-3\gamma^2))H_2}$$
$$q_{B1}^{\star} = \frac{5-3\gamma+\eta_4}{H_2}; \qquad q_{B2}^{\star} = \frac{40-25\gamma-15\gamma^2+9\gamma^3+\eta_5}{(10-3\gamma^2+s(11-5\gamma^2)+s^2(3-3\gamma^2))H_2}; \qquad \pi_{B1}^{\star} = \frac{(5-3\gamma)^2+\eta_6}{2(10-3\gamma^2+s(11-5\gamma^2)+s^2(3-3\gamma^2))H_2};$$
$$\pi_{B2}^{\star} = \frac{3\left(40-25\gamma-15\gamma^2+9\gamma^3+\eta_7\right)^2}{2(10-3\gamma^2+s(11-5\gamma^2)+s^2(3-3\gamma^2))^2H_2^2}; \qquad W^{\star} = \frac{25925-16750\gamma-28000\gamma^2+18060\gamma^3+9855\gamma^4-6345\gamma^5-1134\gamma^6+729\gamma^7+\eta_8}{(10-3\gamma^2+s(11-5\gamma^2)+s^2(3-3\gamma^2))^2H_2^2}$$

where

$$H_1 = 125 + 45\gamma - 120\gamma^2 - 18\gamma^3 + 27\gamma^4 - s^4\left(-1+\gamma^2\right)^2\left(-9-6\gamma+3\gamma^2+\gamma^3\right) + s\big(275+87\gamma-287\gamma^2-39\gamma^3+72\gamma^4\big)$$
$$+ s^3\left(75+31\gamma-134\gamma^2-42\gamma^3+67\gamma^4+11\gamma^5-8\gamma^6\right) + s^2\big(220+71\gamma-294\gamma^2-60\gamma^3+104\gamma^4+9\gamma^5-6\gamma^6\big) > 0;$$

$$H_2 = 20 - 9\gamma^2 + s\left(27-13\gamma^2\right) + s^2\left(9-11\gamma^2+2\gamma^4\right) > 0;$$

$$\eta_1 = s\left(55-19\gamma-15\gamma^2+3\gamma^3\right) + s^2\left(39-11\gamma-24\gamma^2+4\gamma^3-3\gamma^4+3\gamma^5\right) + s^3\left(9-3\gamma-4\gamma^2+2\gamma^3-7\gamma^4+\gamma^5+2\gamma^6\right)$$
$$- s^4(-3+\gamma)\gamma^2\left(-1+\gamma^2\right)^2;$$

$$\eta_2 = s\left(140-122\gamma-43\gamma^2+51\gamma^3-6\gamma^4\right) + s^2\left(87-97\gamma-40\gamma^2+58\gamma^3-5\gamma^4-3\gamma^5\right) - 3s^3(-1+\gamma)^2\big(-6-4\gamma$$
$$+ 3\gamma^2+\gamma^3\big);$$

$$\eta_3 = s\left(148-125\gamma-49\gamma^2+52\gamma^3-6\gamma^4\right) + s^2\left(90-97\gamma-43\gamma^2+57\gamma^3-5\gamma^4-2\gamma^5\right) + s^3(-1+\gamma)^2\big(18+12\gamma-8\gamma^2$$
$$- \gamma^3+\gamma^4\big);$$

$$\eta_4 = -2s\left(-4+\gamma+\gamma^2\right) + s^2\left(3-\gamma-3\gamma^2+\gamma^3\right);$$

$$\eta_5 = s\left(114-50\gamma-62\gamma^2+20\gamma^3+6\gamma^4\right) + s^2\left(119-41\gamma-107\gamma^2+33\gamma^3+20\gamma^4-4\gamma^5\right) + s^3\big(54-17\gamma-77\gamma^2$$
$$+ 22\gamma^3+24\gamma^4-5\gamma^5-\gamma^6\big) - s^4\left(-1+\gamma^2\right)^2\left(-9+3\gamma+\gamma^2\right);$$

$$\eta_6 = s\left(80-68\gamma-8\gamma^2+12\gamma^3\right) - 2s^2\left(-47+30\gamma+26\gamma^2-18\gamma^3+\gamma^4\right) - 4s^3\left(-12+7\gamma+14\gamma^2-8\gamma^3-2\gamma^4+\gamma^5\right)$$
$$+ s^4\left(3-\gamma-3\gamma^2+\gamma^3\right)^2;$$

$$\eta_7 = s\left(114-50\gamma-62\gamma^2+20\gamma^3+6\gamma^4\right) + s^2\left(119-41\gamma-107\gamma^2+33\gamma^3+20\gamma^4-4\gamma^5\right) + s^3\big(54-17\gamma-77\gamma^2+22\gamma^3$$
$$+ 24\gamma^4-5\gamma^5-\gamma^6\big) - s^4\left(-1+\gamma^2\right)^2\left(-9+3\gamma+\gamma^2\right);$$

$$\eta_8 = s\left(127985-81700\gamma-150445\gamma^2+95656\gamma^3+58093\gamma^4-36774\gamma^5-7371\gamma^6+4644\gamma^7\right) + s^2\big(272233-175825\gamma$$
$$- 367002\gamma^2+237007\gamma^3+163540\gamma^4-105910\gamma^5-25074\gamma^6+16470\gamma^7+405\gamma^8-324\gamma^9\big) + s^3\big(325655-214926\gamma$$
$$- 520104\gamma^2+343214\gamma^3+275128\gamma^4-182109\gamma^5-52886\gamma^6+35446\gamma^7+2143\gamma^8-1545\gamma^9\big) + s^4\big(239329-160830\gamma$$
$$- 460594\gamma^2+308492\gamma^3+296764\gamma^4-198427\gamma^5-73309\gamma^6+49158\gamma^7+5301\gamma^8-3637\gamma^9-43\gamma^{10}+36\gamma^{11}\big)$$
$$+ s^5\big(110433-74187\gamma-257981\gamma^2+172094\gamma^3+206652\gamma^4-136600\gamma^5-66382\gamma^6+43368\gamma^7+7463\gamma^8-4793\gamma^9$$
$$- 185\gamma^{10}+118\gamma^{11}\big) - s^6\left(-1+\gamma^2\right)^2\big(-31149+20112\gamma+26263\gamma^2-16379\gamma^3-5714\gamma^4+3367\gamma^5+309\gamma^6$$
$$- 170\gamma^7+\gamma^8\big) + s^7\left(-1+\gamma^2\right)^3\left(-4887+2799\gamma+2283\gamma^2-1187\gamma^3-239\gamma^4+110\gamma^5+\gamma^6\right) + s^8\left(-1+\gamma^2\right)^4(324$$
$$- 135\gamma-81\gamma^2+30\gamma^3+2\gamma^4);$$

Now in stage 0, the government determines 's' to maximize welfare. The parameter s is complicated to find explicitly. However, it can be shown that for any $\gamma \in [0,1)$:

$$\frac{\partial W^\star}{\partial s}\Big|_{s\to 0} = \frac{O_1}{\left(200-150\gamma^2+27\gamma^4\right)^3} > 0 \text{ and } \frac{\partial W^\star}{\partial s}\Big|_{s\to 1} = -\frac{O_2}{\left(24-11\gamma^2\right)^3\left(56-33\gamma^2+2\gamma^4\right)^3} < 0.$$

where

$O_1 = 190500 + 75000\gamma - 588250\gamma^2 - 47600\gamma^3 + 667645\gamma^4 - 67800\gamma^5 - 353745\gamma^6 + 76212\gamma^7 + 88641\gamma^8 - 25758\gamma^9 - 8505\gamma^{10} + 2916\gamma^{11} > 0;$

$O_2 = 78303232 - 34750464\gamma - 255362048\gamma^2 + 136585728\gamma^3 + 329929664\gamma^4 - 209027520\gamma^5 - 208809168\gamma^6 + 159722904\gamma^7 + 62808018\gamma^8 - 64476531\gamma^9 - 5299357\gamma^{10} + 13138448\gamma^{11} - 1210117\gamma^{12} - 1135177\gamma^{13} + 210585\gamma^{14} + 30142\gamma^{15} - 6753\gamma^{16} - 440\gamma^{17} + 110\gamma^{18} > 0.$

These facts imply the existence of $s^\star \in (0,1)$ such that $\frac{\partial W^\star}{\partial s}\big|_{s=s^\star} = 0$. We can also show that $\frac{\partial^2 W^\star}{\partial s^2}\big|_{s=s^\star} < 0$. Therefore, under non-merged competition, we have the equilibrium outcomes, $s^{nm} = s^\star$; $\theta^{nm} = \theta^\star(s^{nm})$; $W^{nm} = W^\star(s^{nm})$, respectively, which are shown in Figure 1.

A2. Merged competition

When the two private firms merge, they set up a CC that chooses q_{B1} and q_{B2} that maximize (6) while semi-public SHC chooses q_{A1} and q_{A2} that maximize Ω. Then, the outputs are as follows:

$$q^\dagger_{Ai} = \frac{2+\gamma+(-1-\gamma)\theta}{N}, \qquad q^\dagger_{Bi} = \frac{1+s+s\gamma+(1+\gamma)\theta}{N} \quad i = 1,2$$

where $N = 5 + 5\gamma + \gamma^2 + s\left(3 + 5\gamma + 2\gamma^2\right) - (1+\gamma)(1+s+s\gamma)\theta > 0$.

Then, we have the equilibrium outcomes:

$$\theta^\dagger = \frac{1+\gamma+s(1+\gamma)^2}{5+3\gamma+s\left(5+7\gamma+2\gamma^2\right)+s^2(1+\gamma)^2}; \quad q^{\dagger M}_{Ai} = \frac{3+2\gamma+s\left(2+3\gamma+\gamma^2\right)}{(2+\gamma+s(1+\gamma))N_1}; \quad q^{\dagger M}_{Bi} = \frac{1+s(1+\gamma)}{N_1}$$

$$\pi^{\dagger M}_B = \frac{(1+s(1+\gamma))^2}{(2+\gamma+s(1+\gamma))N_1}; \qquad W^\dagger = \frac{42+77\gamma+46\gamma^2+9\gamma^3+\nu_1}{((2+\gamma+s(1+\gamma))N_1)^2}$$

where

$N_1 = 4 + 3\gamma + s\left(3 + 5\gamma + 2\gamma^2\right) > 0$

$\nu_1 = 2s\left(53 + 137\gamma + 125\gamma^2 + 47\gamma^3 + 6\gamma^4\right) + s^2(1+\gamma)^2\left(94 + 125\gamma + 46\gamma^2 + 4\gamma^3\right) + 2s^3(1+\gamma)^3\left(17 + 18\gamma + 4\gamma^2\right) + s^4(1+\gamma)^4(4+3\gamma);$

Now in stage 0, the government determines 's' to maximize welfare. The parameter s is complicated to find explicitly. However, it can be shown that for any $\gamma \in [0,1)$:

$$\frac{\partial W^\dagger}{\partial s}\big|_{s\to 0} = \frac{2(1+\gamma)^3}{(2+\gamma)(4+3\gamma)^3} > 0, \text{ and } \frac{\partial W^\dagger}{\partial s}\big|_{s\to 1} = -\frac{2(1+\gamma)^2\left(154+485\gamma+616\gamma^2+402\gamma^3+142\gamma^4+26\gamma^5+2\gamma^6\right)}{(3+2\gamma)^3(7+8\gamma+2\gamma^2)^3} < 0.$$

The fact that $\frac{\partial W^\dagger}{\partial s}\big|_{s\to 0} > 0$ and $\frac{\partial W^\dagger}{\partial s}\big|_{s\to 1} < 0$ implies the existence of $s^\dagger \in (0,1)$ such that $\frac{\partial W^\dagger}{\partial s}\big|_{s=s^\dagger} = 0$. Moreover, $\frac{\partial^2 W^\dagger}{\partial s^2}\big|_{s=s^\dagger} < 0$.

Therefore, under merged competition, we have the equilibrium outcomes: $s^m = s^\dagger$; $\theta^m = \theta^\dagger(s^m)$; $W^m = W^\dagger(s^m)$ which are shown in Figure 2.

Appendix B. Proofs of Propositions 4 and 5

From appendix $A1$ and $A2$, we have that for any $\gamma \in [0,1)$ and $s \in [0,1]$

$$\theta^{\star} - \theta^{\dagger} = -\frac{\gamma(1+s+s\gamma)O}{(5+3\gamma+s\,(5+7\gamma+2\gamma^2)+s^2(1+\gamma)^2)\,H_1} \leq 0$$

where $O = 170 - 30\gamma - 138\gamma^2 + 9\gamma^3 + 27\gamma^4 + s\left(317 - 58\gamma - 311\gamma^2 + 24\gamma^3 + 72\gamma^4\right) - 2s^2\big(- 112 + 32\gamma + 158\gamma^2 - 25\gamma^3$

$- 55\gamma^4 + 3\gamma^5 + 3\gamma^6\big) + s^3\left(72 - 49\gamma - 155\gamma^2 + 54\gamma^3 + 86\gamma^4 - 13\gamma^5 - 11\gamma^6\right) + s^4(1+\gamma)^2\big(9 - 38\gamma + 33\gamma^2 + 5\gamma^3 -$

$10\gamma^4 + \gamma^5\big)$

$+ s^5(-3+\gamma)(-1+\gamma)^2\gamma(1+\gamma)^3 > 0.$

Then, using figures 1 and 2, we have the following relations:

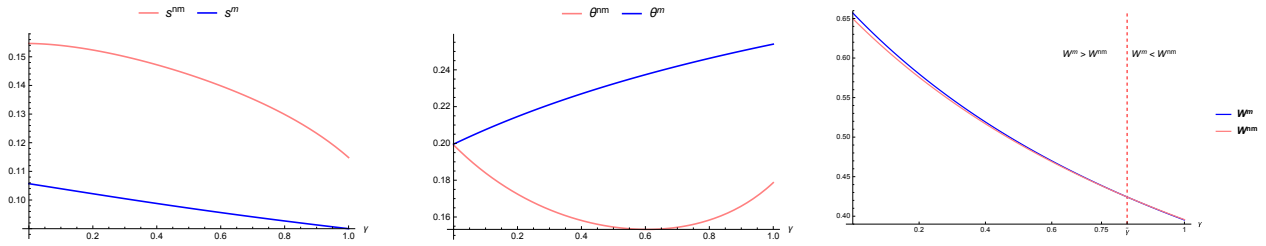

Figure B.3