Monetary rules in an open economy with distortionary subsidies and inefficient shocks: A DSGE approach for Bolivia

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14 July 2020

MONETARY RULES IN AN OPEN ECONOMY WITH DISTORTIONARY SUBSIDIES AND INEFFICIENT SHOCKS: A DSGE APPROACH FOR BOLIVIA

by

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2020

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Abstract

Monetary rules in an open economy with distortionary subsidies and inefficient shocks: A DSGE approach for Bolivia

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Through an estimated and calibrated DSGE model with imperfect competition and nominal rigidities, this work aims to assess the dynamic effects of exogenous perturbations in a small open economy to provide a prescription of a simple monetary policy rule associated with the minimal welfare losses in the case of Bolivia.

Following Gali and Monacelli (2005) and De Paoli (2009), I display the baseline model in a canonical representation. Yet, unlike them, I consider the presence of efficient and inefficient perturbations, namely government spending, productivity, foreign demand, and cost-push shocks, to analyze its effects in terms of observable variables but also on the relevant output gap. Moreover, considering the significance of raw materials as a proportion of the Bolivian exports, I extend the model by taking into account a distortionary subsidy on consumption financed by the positive profits of the commodity sector,

Further, in the style of Gali and Monacelli (2005), I compare the welfare implications under two scenarios: A monetary rule focus on maintain a nominal exchange rate peg (fixed) regime[1] and a Taylor rule. The main results reveal that the latter outperforms the former when the full set of shocks occurs simultaneously, showing the importance of inflation targeting.

Yet, by focusing only on inefficient exogenous perturbations, and taking into account a pegged regime and a simple Taylor rule based on consumer and producer price inflation, the ranking of monetary policy aligns in the first place an exchange rate peg. This scenario shows the potential success of alternative simple monetary rules under these circumstances[2].

[1]I analyze the dynamic implications of a monetary rule focus on maintain a fixed exchange rate regime, given that according to the IMF report on exchange arrangements and exchange restrictions, the Central Bank of Bolivia focus on a "monetary policy aggregates" system, keeping a "de facto" exchange rate anchor of the Bolivian currency to the US dollar.

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Chapter 1

Introduction

The systematic application of DSGE models is not necessarily common practice in emerging small open economies, where the understanding of the dynamic effects of exogenous perturbations is imperative for an appropriate monetary policy recommendation. Specifically, the focal point of this analysis is Bolivia, given that it provides a realistic scenario to examine the repercussions of efficient and inefficient shocks in a small open economy.

Since 2011, this country maintained a fixed exchange rate regime pegged to the US dollar. In the last twenty years, its exports, mainly explained by commodities, represented 24 percent of GDP, showing a procyclical behavior and a high degree of growth volatility of the order of 7.5 percent[1]. The government is the owner of commodity products, whose profits fund distortionary subsidies on consumption. Further, government spending, financed by taxes, played an important role, as it has been growing at an average rate of 4.2 percent over the last two decades. On the other hand, Bolivian agriculture, which represents almost 15 percent of GDP, is particularly vulnerable to real shocks, as the issue of natural disasters such as floods and climatic atmospheric changes in the Amazon and Altiplano, affects unmistakably the level of prices and generates lower levels of output[2]. Thence, the presence of inefficient perturbations (cost-push shocks) and distortionary subsidies in this economy is undeniable.

In the context of the basic New Keynesian model, inflation and relevant output gap stability are simultaneously achieved only when cost-push shocks are not present. Otherwise, the monetary authority faces a dilemma and targeting inflation conveys an inefficient level of economic activity. Hence, to close the relevant output gap, a flexible inflation targeting would be preferable, however, that complicates the welfare-maximizing strategy, even in the context of an optimal monetary policy. Thus, according to Shuterland (2004) [43] in view of such difficulties, the most practical strategy would be the application of non-optimal but simple monetary policy rules and follow the one that maximizes the gains for society.

Thence, this work portrays a model to understand the dynamic effects and welfare implications of efficient and inefficient shocks in the context of a small open economy under the presence of distortionary subsidies. After understanding its implications, I evaluate alternative simple monetary policy rules, in order to select the one associated with the minimal welfare losses.

Unlike in the literature before, the baseline model of this analysis provides a canonical representation based not only on productivity and foreign demand exogenous perturbations as in Gali and Monacelli (2005) but also on government spending and cost-push shocks. Thanks to this representation, it is feasible to analyze its welfare effects by focussing not

[1] Author computations based on Bolivian national accounts. Check table 2.1 for further details.

[2] Social conflicts, such as strikes and roadblocks also affects positively the level of prices, since the transportation of products faces difficulties.

only on observable macroeconomic variables but also on the output gap and the theoretical efficient level of output. Further, the extended model centers on the everlasting paradigm of emerging commodity-dependent countries. To represent this issue, I introduce a time-varying distortionary subsidy on consumption financed by the profits of the commodity sector, to illustrate a channel of transmission towards the representative agent. Thence introducing this additional perturbation, it is possible to complete the picture of the analysis of a typical developing country such as Bolivia.

In the case of the baseline model, I compare a Taylor rule with four different combinations of weights on the output gap and inflation, and an exchange rate peg. The results show that in terms of the minimal welfare losses and the joint presence of efficient and inefficient shocks, a pegged regime is situated always in the last place, in line with Gali and Monacelli (2005) [27]. Yet, when I concentrate only on inefficient exogenous perturbations, the ranking of monetary policy rules aligns in the first place an exchange rate peg. This scenario sheds light on the Central Bank‘s dilemma arising as a result of cost-push shocks, showing the typical ineffectiveness of inflation targeting under this case, and the potential success of alternative simple monetary rules under these circumstances.

As mentioned before, the analysis of the Bolivian economy would not be complete without taking into account the commodity sector as it introduces additional distortions in this economy. In this context, by comparing a Taylor rule and an exchange rate peg regime I find that in the joint presence of efficient and inefficient exogenous perturbations, the preferred policy rule is the former over the latter, as in the baseline case. The same result holds when commodity price shocks are evaluated individually.

The model provides many other insights relevant to the analysis. In particular, each exogenous perturbation elicits a different set of dynamics. First, government spending based on the purchase of domestic goods leads to an upsurge of the efficient level of output, producing downward pressures on private consumption given the negative wealth effect. Second, cost-push shocks lead to a deeper slowdown in production under a Taylor rule given the emerging dilemma between inflation and output gap stabilization. Third, under both regimes, a technological shock triggers an expansion of the efficient output as a result of the lift in the capacity of production. Fourth, external demand for home goods generates a contemporaneous expansion on the efficient output, along with considerable deviations of the relevant gap under a peg, and upward pressures on domestic consumption given the switching effect. And fifth, commodity price shocks lead to downward pressures on the real exchange rate and terms of trade, generating a hike in consumption for foreign goods.

To perform the analysis, I did a Bayesian estimation of those parameters for which there is only a prior belief about their value. For that purpose, I used quarterly data from Bolivia along the periods between 2009 to 2019.

Besides, I analyze an additional simple monetary policy rule that reflects the commitment to maintain a managed exchange rate but also targets inflation as in Monacelli (2004) [39]. The welfare evaluation under this policy shows that it outperforms a pure pegged regime but underperforms a Taylor rule.

Concerning the theoretical construction of the model, the main structure follows the seminar paper of Gali and Monacelli (2005) [27], by taking into account imperfect competition, staggered prices à la Calvo (1983) [13] and the dynamics of interaction between the small open economy and the rest of the world. Further, unlike their representation of the world as a continuum of economies, I consider the analysis of De Paoli (2009) [21] in which the limit of the size of the domestic country tends to zero, such that it does not influence the performance of the rest of the world, but it is attained by external demand fluctuations. Yet, as stated above, I extend their models by incorporating a canonical representation including together efficient and inefficient shocks.

The typical issue of assessing the dynamic impacts of government spending in an open economy context is explored by Lubik and Schorfheide (2006) [33]. In their style, I incorporate public purchases of domestic goods. Moreover, the importance of considering cost-push shocks in this framework is highlighted by Adolfson et al (2007) [1]. They develop a DSGE model including several frictions and describing the dynamics of multiple exogenous perturbations in a small open economy. Unlike Lubik et al (2006) and Adolfson et al (2007), I also focus on the welfare implications of government spending and cost-push shocks.

To incorporate a commodity sector in the model, I take into account the analyses of Medina (2005) [36] and Ferrero (2017) [24]. Although, unlike both of them, I consider that the profits generated in the commodity sector finance a distortionary subsidy on consumption, as it is the case in Bolivia. The first author includes oil in the consumption basket, and the second one represents the production function of the commodity sector by displaying decreasing returns to scale.

Bolivia and the DSGE literature: One of the most important analysis through the lens of a DSGE model for this country, is the paper of Cerezo (2010) [15]. The author presents the effects of productivity and external demand shocks and finds that a monetary policy targeting only inflation is less successful than a Taylor-type rule that takes into account output and inflation simultaneously.

Valdivia et al (2017) [47] perform a Bayesian estimation to evaluate the dynamic effects of public investment and cost-push shocks. However, their analysis is based on the context of a closed economy model.

Concerning previous efforts in understanding the effects of the commodity sector in Bolivia, Valdivia (2016) [46] assumes that government spending is financed by the profits of the natural gas sales. Finally, the more recent contribution is the one of Zeballos et al (2018) [46] that estimates a Markov-Switching DSGE model in the context of a small open economy, considering monetary policy, preference, productivity, and external demand shocks. They take into account a "monetary aggregate targeting" regime but do not represent the "de facto" exchange rate anchor to the US dollar holding in this nation.

The description of the recent strand of DSGE literature for this country, among others[3], provides the base for the calibration of the model presented here. Therefore, I focus on the Bayesian estimation of those parameters for which there is only a prior belief about their value. Further, I contribute to the Bolivian literature, by analyzing the welfare implications and the dynamic effects of efficient and inefficient shocks in an open economy context by comparing alternative monetary policies and taking into account a fixed exchange rate regime.

This work is organized as follows: Chapter 2 presents a brief description of the context of analysis. Chapter 3 sets up the theoretical model and its canonical representation considering the effect of four kinds of exogenous perturbations in a small open economy: external demand, government spending, productivity, and cost-push shocks. In the first section of chapter 4, I display the simulation of the model based on both, estimated and calibrated parameters. The second section of chapter 4 exhibits the welfare analysis and the comparison and evaluation of alternative simple monetary rules. Chapter 5 presents a concise extension of the model for the analysis of commodity price shocks. Finally, chapter 6 concludes. The appendixes show the mathematical derivations and the relevant graphs linked to this work.

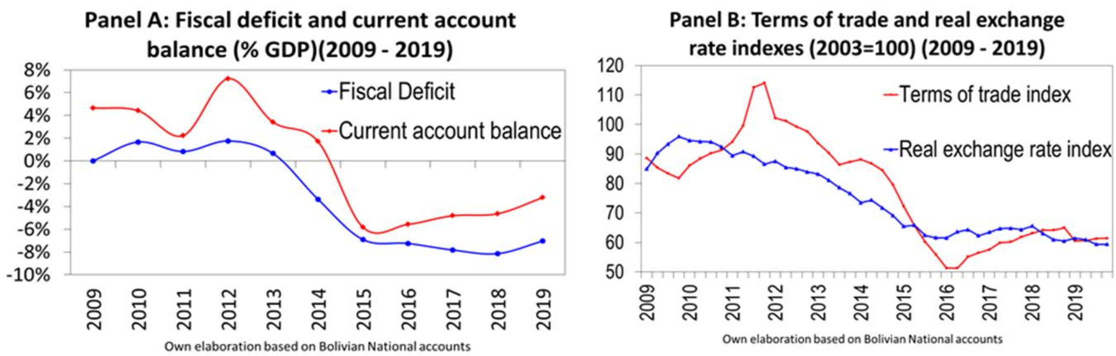
[3]See also Garron (2016) [22] Machicado (2012) [34] and Salas (2018) [42]

Chapter 2

Context of analysis

According to the International Monetary Fund [10], Bolivia registered annual real GDP growth of 4.8 percent on average in the last ten years, and built up sizable foreign reserves. However, since the commodity price drop in 2014, authorities have focused on the expansion of public spending in order to strengthen further economic growth. This approach led to large fiscal and external current account deficits and a sharp increase in public debt.

Figure 2.1: Twin deficits and competitiveness



Panel (a) of figure 2.1 shows the behavior of the twin deficits over the last ten years. The fiscal deficit reached its most critical performance in 2018 at a level of 8 percent of GDP. There was also a persistent negative current account balance of -3 percent on average since 2015. As the theory suggest, these facts implies that the country is a net debtor to the rest of the world, which means capital inflows triggering an appreciation of the real exchange rate. This is known as the transfer problem, as there are upward pressures on the demand for home tradable and nontradable goods, leading to higher relative domestic prices and finally a terms of trade worsening [17].

In this regard, Panel (b) of figure 2.1 shows the terms of trade improvement and the real exchange rate appreciation. This makes Bolivian exports less competitive in comparison to their foreign counterparts. Although, according to Gali and Monacelli (2005) [27], in the case of a pegged regime there is an excess smoothing of the terms of trade, which suggests that the real appreciation of the domestic currency could have been even deeper[1].

As it is shown in panel (a) of figure two, despite the negative performance of the current account, the Bolivian economy grew up on average 1.2 percent more than their foreign counterparts in the last ten years. Concerning annual inflation, the country registered on average, a rate of 4.2 percent in the same period, substantially lower than other economies

[1] In this regard check IMF/Combes et al (2011) [17]

such as Venezuela (annual inflation above 1000 percent), but greater than Chile and Peru (average annual inflation below 3 percent).

Figure 2.2: GDP growth in Bolivia and south America, CPI inflation and composition of foreign exports

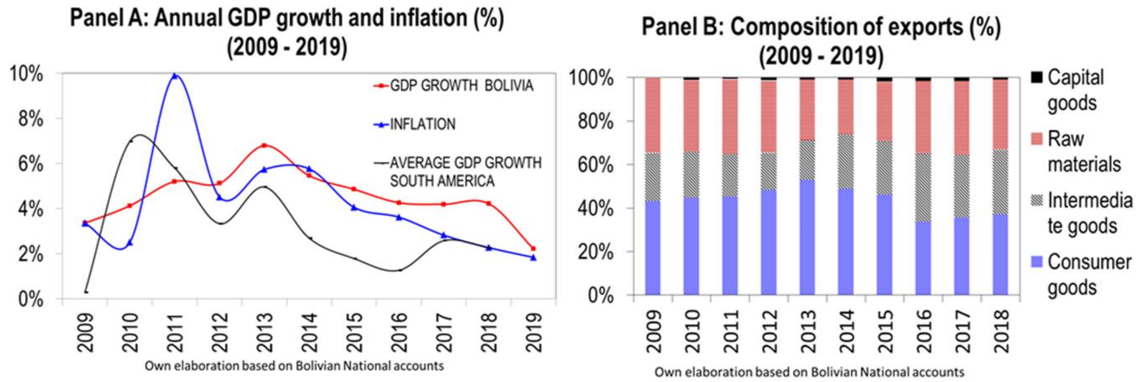


Table 1.1 summarizes the key business cycle moments for Bolivia since 1990. I report mean, standard deviation, persistence, and contemporaneous correlation across variables. As it is shown, a number of properties of this economy are in line with the traditionally observed properties in business cycles: consumption and government spending, are procyclical variables, and investment is the most volatile one.

Table 2.1: **Business cycles moments 1990-2019, In terms of Growth rates**

	Y	C	I	G	X	M	IND	COM	CP
Mean	4.09	3.81	6.86	4.18	4.66	5.32	4.04	3.56	4.09
S.D	1.43	1.2	12.42	1.91	7.62	8.59	1.85	2.96	15.42
Persistence	0.351	0.351	0.115	0.619	0.055	0.178	0.168	0.133	0.083
Corr with Y	1	***0.72***	***0.61***	***0.46***	0.23	***0.51***	***0.51***	***0.59***	0.06
Corr with C	0.72	1	***0.64***	***0.59***	-0.23	***0.38***	0.17	0.19	-0.14
Corr with I	0.61	0.64	1	0.27	-0.08	***0.76***	-0.05	0.13	0.04
Corr with G	0.46	0.59	0.27	1	-0.10	0.14	0.22	0.10	-0.23
Corr with X	0.23	-0.23	-0.08	-0.10	1	***0.45***	0.12	***0.39***	***0.53***
Corr with M	0.51	0.38	0.76	0.14	0.45	1	-0.23	0.27	***0.32***
Corr with IND	0.51	0.17	-0.05	0.22	0.12	-0.23	1	0.28	0.02
Corr with COM	0.59	0.19	0.13	0.10	0.39	0.27	0.28	1	***0.47***
Corr with CP	0.06	-0.14	0.04	-0.23	0.53	0.32	0.02	0.47	1

Note: Y=Output, C=Consumption, I=Investment, G=Government Spending, X=Exports, M= Imports, IND=Industrial sector, COM=Commodity sector, CP=Global commodity price Index IMF (2003=100). Own elaboration based on Bolivian national accounts and IMF. Persistence is the coefficient from an estimated AR(1) process. Annual frequency, real and per capita terms.

Interestingly, government spending presents the highest level of annual persistence along the last twenty years, and its average growth surpasses the one of the output, highlighting its importance on the overall economic performance.

Also, there is a high correlation between the growth rate of imports and output, suggesting the relevance in the consumption of foreign goods in this country.

Finally, on the supply side, I take into account only two sectors of the Bolivian economy: the commodity and the industrial one. The former represents around 60 percent of the total tradable output, is highly procyclical, and is explained mainly by agriculture, natural gas, and metals. This fact suggests two ideas: First, the importance of the commodity sector, given that according to panel B of figure 2.2, exports of raw materials represent on average 32 percent of the total. And second, the relevance of agriculture that is subject to potential real shocks, given the frequency of natural disasters related to floods, storms, and dramatic atmospheric changes in the Altiplano and Amazon[2].

Not surprisingly, there is a positive correlation between the commodity sector and exports. There is also a strong relationship between exports and the world commodity price index published by the International Monetary Fund.

[2]Also, it is important to consider that in Bolivia, social conflicts, strikes and roadblocks also exert upward pressures in the level of prices of agricultural products.

Chapter 3

The model

Following in large part Gali & Monacelli (2005) [27], the main features of this model consist of imperfect competition and staggered price setting à la Calvo (1983) [13][1]. Unlike their representation of the world as a continuum of economies, I consider the analysis of De Paoli (2009) [21] in which the limit of the size of the domestic country tends to zero, such that it does not influence at all the performance of the rest of the world, but is attained by external demand fluctuations. There are two types of exogenous perturbations: demand shocks emerging from the external and public consumption of domestic goods, and supply shocks arising from productivity and cost-push shocks. In this way, I consider efficient and inefficient shocks and represent a canonical representation of the model.

There is a continuum of identical infinite-lived households of unit mass populating the world. The segment [0,n) lives in the home country H, and the remaining (n,1] belongs to the rest of the world or the foreign country F. Each household owns a competitive-monopolistic firm producing a differentiated good, and each one of them employs only labor for production, assuming constant returns to scale. In the same way, agents produce a continuum of goods indexed on the intervals [0, n) and [n, 1], respectively.

Home and foreign goods are tradable, and the law of one price holds for all of them, such that the only reason for deviations from purchasing power arises as a result of home bias in consumption. Further, the international financial market is complete.

3.1 Preferences

The representative household in country H maximizes her inter-temporal separable utility function defined over consumption C and hours worked N, which is continuously differentiable in both arguments, increasing and concave in C, and decreasing and convex in N:

$$U_t = E_0 \sum_{t=0}^{\infty} \beta^t U(C_t, N_t) \tag{3.1}$$

Therefore, the agent obtains utility from a consumption bundle and contributes to the production of goods, attaining disutilty from labor. $\beta \in [0, 1]$ is the inter-temporal discount factor of preferences and E_0 is the expectation conditional on the information at time 0. Consumption and labor are respectively represented by iso-elastic functional forms:

$$U(C, N) = \frac{C^{1-\rho} - 1}{1-\rho} - \frac{N^{1+\eta}}{1+\eta}$$

[1]See also: De Paoli (2008) [20], Bertholt (2012) [9], Benigno(2009) [7] and Corsetti et al (2005) [18]

where the parameter ρ is the coefficient of relative risk aversion, and η is the inverse of the elasticity of labor supply. Each monopolistic competitive firm produces a differentiated consumption good h, using labor as the only input and employing the production function $y_t(h) = A_t N_t(h)$, where A_t is the aggregate technology.

Consumption C is a composite index of home C_H and foreign C_F goods, defined by the following constant elasticity of substitution function:

$$C = \left(v^{\frac{1}{\theta}} C_H^{\frac{\theta-1}{\theta}} + (1-v)^{\frac{1}{\theta}} C_F^{\frac{\theta-1}{\theta}} \right)^{\frac{\theta}{\theta-1}} \tag{3.2}$$

$\theta > 0$ denotes the intra-temporal elasticity of substitution between home and foreign-produced goods. As in De Paoli (2009), the parameter determining home consumers' preferences for foreign goods, $(1-v)$, is a function of the relative size of the foreign economy $(1-n)$ and the domestic degree of openness "λ", namely:

$$(1-v) = (1-n)\lambda \tag{3.3}$$

hence, home bias is inversely related to the degree of openness of the domestic economy. Similar preferences yield for the rest of the world, where C_H^* and C_F^* denote the foreign consumption of home and foreign products respectively:

$$C_s^* = \left(v^{*\frac{1}{\theta}} C_H^{*\frac{\theta-1}{\theta}} + (1-v^*)^{\frac{1}{\theta}} C_F^{*\frac{\theta-1}{\theta}} \right)^{\frac{\theta}{\theta-1}} \tag{3.4}$$

where the foreign consumers' preferences for home goods v^* depend on the relative size of the home economy n and its degree of openness λ:

$$v^* = n\lambda \tag{3.5}$$

C_H and C_H^* are indexes of home and foreign consumption of differentiated goods produced in-country H, represented by the following constant elasticity of substitution function:

$$C_H = \left(\left(\frac{1}{n} \right)^{\frac{1}{\sigma_t}} \int_0^n c(h)^{\frac{\sigma_t-1}{\sigma_t}} dh \right)^{\frac{\sigma_t}{\sigma_t-1}} \tag{3.6}$$

$$C_H^* = \left(\left(\frac{1}{n} \right)^{\frac{1}{\sigma_t}} \int_0^n c^*(h)^{\frac{\sigma_t-1}{\sigma_t}} dh \right)^{\frac{\sigma_t}{\sigma-1}} \tag{3.7}$$

σ_t denotes a time varying elasticity of substitution across the differentiated goods h within a country. In the same way, the consumption indexes of differentiated goods elaborated in country F are:

$$C_F = \left(\left(\frac{1}{1-n} \right)^{\frac{1}{\sigma_t}} \int_{(1-n)}^1 c(f)^{\frac{\sigma_t-1}{\sigma_t}} df \right)^{\frac{\sigma_t}{\sigma_t-1}} \tag{3.8}$$

$$C_F^* = \left(\left(\frac{1}{1-n} \right)^{\frac{1}{\sigma_t}} \int_{(1-n)}^1 c^*(f)^{\frac{\sigma_t-1}{\sigma_t}} df \right)^{\frac{\sigma_t}{\sigma_t-1}} \tag{3.9}$$

Consumption-based price indexes represent a composition of prices of domestic and foreign goods since the representative household consumes both of them:

$$P = (vP_H^{1-\theta} + (1-v)P_F^{1-\theta})^{\frac{1}{1-\theta}} \tag{3.10}$$

$$P^* = (v^* P_H^{*1-\theta} + (1-v^*) P_F^{*1-\theta})^{\frac{1}{1-\theta}} \tag{3.11}$$

The price sub-indexes for domestic goods consumed in countries H and F are respectively:

$$P_H = \left(\left(\frac{1}{n} \right) \int_0^n p(h)^{1-\sigma_t} dh \right)^{\frac{1}{1-\sigma_t}} \tag{3.12}$$

$$P_H^* = \left(\left(\frac{1}{n} \right) \int_0^n p^*(h)^{1-\sigma_t} dh \right)^{\frac{1}{1-\sigma_t}} \tag{3.13}$$

where $p^*(h)$ is the price of a generic differentiated good h produced in country H and consumed in country F. In the same fashion, the price sub-indexes of foreign- produced goods are the following:

$$P_F = \left(\left(\frac{1}{1-n} \right) \int_{(1-n)}^1 p(f)^{1-\sigma_t} df \right)^{\frac{1}{1-\sigma_t}} \tag{3.14}$$

$$P_F^* = \left(\left(\frac{1}{1-n} \right) \int_{(1-n)}^1 p^*(f)^{1-\sigma_t} df \right)^{\frac{1}{1-\sigma_t}} \tag{3.15}$$

Not less important, P and P^* are expressed in units of domestic and foreign currency respectively. The real exchange rate $\tilde{Q}$ is the following:

$$P \neq SP^* \quad \rightarrow \quad \tilde{Q} = \frac{SP^*}{P} \tag{3.16}$$

where S is the Nominal exchange rate. On the other hand, the law of one price holds for identical goods produced in country H but traded at separate markets H and F:

$$p(h) = Sp^*(h) \quad and \quad p(f) = Sp^*(f) \tag{3.17}$$

3.1.1 Intratemporal consumption choice

The optimal domestic allocation of home and foreign goods is the following[2]:

$$C_H = v \left(\frac{P_H}{P} \right)^{-\theta} C \tag{3.18}$$

$$C_F = (1-v) \left(\frac{P_F}{P} \right)^{-\theta} C \tag{3.19}$$

where the demand function for a home-produced good depends on its relative price, the elasticity of intratemporal substitution, and is weighted by the home bias. Similarly, the optimal allocation of foreign agents in terms of domestic prices is:

$$C_H^* = v^* \left(\frac{P_H}{P\tilde{Q}} \right)^{-\theta} C^* \tag{3.20}$$

$$C_F^* = (1-v^*) \left(\frac{P_F}{P\tilde{Q}} \right)^{-\theta} C^* \tag{3.21}$$

[2]Derivations available in appendix A.1

Agents of the domestic country maximize their consumption of differentiated goods, thus the demands functions for generic goods h and f are respectively:

$$c(h) = \frac{1}{n}\left(\frac{p(h)}{P_H}\right)^{-\sigma_t} C_H \tag{3.22}$$

$$c(f) = \frac{1}{1-n}\left(\frac{p(f)}{P_F}\right)^{-\sigma_t} C_F \tag{3.23}$$

Analogously, the foreign demand for a generic good h and f in terms of domestic prices is the following:

$$c^*(h) = \frac{1}{n}\left(\frac{p(h)}{P_H}\right)^{-\sigma_t} C_H^* \tag{3.24}$$

$$c^*(f) = \frac{1}{1-n}\left(\frac{p(f)}{P_F}\right)^{-\sigma_t} C_F^* \tag{3.25}$$

According to De Paoli (2009) [21], a country-specific level of domestic (foreign) public expenditure is allocated only among the home (foreign) goods, describing the following demands[3]:

$$g(h) = \frac{1}{n}\left(\frac{p(h)}{P_H}\right)^{-\sigma_t} G \tag{3.26}$$

$$g^*(f) = \frac{1}{1-n}\left(\frac{p(f)}{P_F}\right)^{-\sigma_t} G^* \tag{3.27}$$

In this way, the government budget constraint in the home and foreign country are respectively:

$$-\tau W_t N_t = P_{H,t}(G_t - Tr_t) \tag{3.28}$$

where proportional subsidies on labor τ, and exogenous fluctuations in government spending G_t are financed by lump-sum transfers Tr_t given in the form of domestic goods.

- **Aggregating the intratemporal demand**

Applying the definition of home bias and taking into account the limit when the size of the domestic economy tends to zero ($n \to 0$), the total demand for a generic good produced in the country H and F are respectively[4]:

$$y(h) = \left(\frac{p(h)}{P_H}\right)^{-\sigma_t}\left(\left(\frac{P_H}{P}\right)^{-\theta}\left((1-\lambda)C + \lambda C^*\left(\frac{1}{\tilde{Q}}\right)^{-\theta}\right) + G\right) \tag{3.29}$$

$$y(f) = \left(\frac{p^*(f)}{P_F^*}\right)^{-\sigma_t}\left(\left(\frac{P_F^*}{P^*}\right)^{-\theta} C^* + G\right) \tag{3.30}$$

Hence, external changes in consumption and real exchange rate alter the aggregate demand of the small open economy. But, domestic consumption does not influence international demand, as the size of the home country tends to zero.

[3] Derivations available in appendix A.2
[4] Derivations available in appendix A.3

Finally, aggregating and loglinearizing around the zero inflation steady state[5], the aggregate demands of the home and foreign country are:

$$y_t = (1-\lambda)c_t + \lambda c_t^* + \gamma q_t + g_t \tag{3.31}$$

$$y_t^* = c_t^* + g_t^* \tag{3.32}$$

where the public demand for differentiated home goods follows an autoregressive process with white noise: $\epsilon_g \sim \ iid \ \ N(0, \sigma_g)$:

$$g_t = \rho_g g_{t-1} + \epsilon_{g,t} \tag{3.33}$$

To allow potential external shocks in the small open economy, the domestic demand for foreign goods follows an autoregressive process with white noise: $\epsilon_{c^*} \sim \ iid \ \ N(0, \sigma_{c^*})$:

$$c_t^* = \rho_{c^*} c_{t-1}^* + \epsilon_{c^*,t} \tag{3.34}$$

3.1.2 Intertemporal consumption choice

The intertemporal budget constraint of the household is the following:

$$(1+i_{t-1})B_{t-1} + (p_{s,t} + d_t)Z_{t-1} + (1+\tau)W_t N_t - P_t Tr_t \geq B_t + p_{z,t}Z_t + P_t C_t$$

where B_{t-1} is the stock of nominal bonds denominated in domestic currency, which pays a nominal interest rate. Z_{t-1} are the shares of an asset which has price $p_{z,t}$ and pays dividend d_t. Moreover, $P_t Tr_t$ denotes lump-sum taxes made in the form of domestic goods and $(1+\tau)$ is a subsidy on labor. $W_t N_t$ is labor income, where W_t is the home nominal wage and N_t is the amount of time spending in working. Also, the representative agent consumes both domestic and foreign goods, such that $P_t C_t = P_{H,t}C_{H,t} + P_{F,t}C_{F,t}$. Setting the Lagrangian and deriving the first-order conditions[6], the Euler equation is the following:

$$\beta(1+i_t)E_t\left(\left(\frac{C_{t+1}}{C_t}\right)^{-\rho}\frac{P_t}{P_{t+1}}\right) = 1 \tag{3.35}$$

Here, the stochastic discount factor $Q_{t,t+1}$, that governs the rate at which the consumer is willing to intertemporaly substitute consumption is defined by:

$$Q_{t,t+1} = \beta E_t\left(\left(\frac{C_{t+1}}{C_t}\right)^{-\rho}\frac{P_t}{P_{t+1}}\right) \tag{3.36}$$

The labor optimality condition is:

$$\frac{N_t^\eta}{C_t^{-\rho}} = \frac{W_t(1+\tau)}{P_t} \tag{3.37}$$

Finally, the loglinearized equations are given by:

$$c_t = E_{t+1}(c_{t+1}) - \frac{1}{\rho}(i_t - E_t\pi_{t+1} - b) \tag{3.38}$$

$$\eta n_t + \rho c_t - v = w_t - p_t \tag{3.39}$$

Where $b = -ln(\beta)$, $v = ln(1+\tau)$ and $\pi_{t+1} = p_{t+1} - p_t$ is the home consumer price index inflation.

[5]Derivations in appendix A.3 and A.3

[6]Derivations available in appendix A.4

3.2 Firms

There is a unit mass of competitive monopolistic firms, such that those on the interval $[0, n)$ are established in the Home country, while those on the interval $(n, 1]$ are located in the Foreign economy. Firms use only a homogeneous type of labor for production, whose market is competitive and there is no investment. Each domestic firm produces a differentiated good h, and the production function, as stated before, is the following:

$$y_t(h) = N_t(h)A_t \tag{3.40}$$

Labor N(h) describes constant returns to scale and A_t is productivity. Hence, at the aggregate level, the loglinearized production function (around the zero inflation steady state, and up to a first order approximation) is described by:

$$y_t = a_t + n_t \tag{3.41}$$

All domestic firms operate with the same technology, that faces an AR(1) process with Gaussian shocks $\epsilon_{a,t} \sim \ iid \ \ N(0, \sigma_a)$:

$$a_t = \rho_a a_{t-1} + \epsilon_{a,t}$$

3.2.1 Cost minimization

Intermediate home producers face a common nominal wage. By minimizing the total variable cost, the nominal marginal cost of producing one additional unit of the generic good h (which is common across firms) is[7]:

$$MC^n = \frac{W_t}{A_t} = \frac{W_t}{MPL} \tag{3.42}$$

where MPL is a decreasing function of the marginal cost and denotes the marginal productivity of labor. Finally, the loglinearized equation of the real marginal cost in terms of domestic prices is the following:

$$mc_t^r = w_t - p_{H,t} - a_t \tag{3.43}$$

3.2.2 Price setting mechanism

In line with Gali and Monacelli (2005), the representative monopolistic competitive firm knows the form of their demand functions derived previously, and follows a partial adjustment rule à la Calvo (1983) for the price setting. In each period, a fraction $\alpha \ \epsilon \ [0, 1]$ of randomly chosen producers do not change the nominal price of the goods they produce[8], but the remaining proportion of firms $(1 - \alpha)$ selects prices optimally, independently of the time elapsed since its last price modification.

A firm that does not re-optimize its prices will choose $\tilde{p}_t(h)$ that maximizes the present value of its future stream of profits discounted by the stochastic discount factor $Q_{t,t+k}$. Therefore, the representative firm deals with the following problem:

$$\begin{aligned} \underset{\tilde{p}_t(h)}{\text{maximize}} \quad & \sum_{t=0}^{\infty} \alpha^k E_t(Q_{t,t+k} \left(\left(y_{t+k}(h)(\tilde{p}_t(h) - MC^n_{t+k}) \right) \right) \\ \text{subject to} \quad & \tilde{y}_{t+k}(h) = \left(\frac{\tilde{p}_t(h)}{P_{H,t+k}} \right)^{-\sigma_t} Y_{t+k} \end{aligned}$$

[7]Derivations are available in appendix A.5

[8]There are potential adjustment costs associated with resetting prices, such as the menu costs.

where MC^n_{t+k} corresponds to the future stream of nominal marginal cost, Y_{t+k} is the sequence of demand constraints in period $t+k$. Appendix A.6 shows the derivations that finally end up in the New Keynessian Philips curve for the home country:

$$\pi_{H,t} = \beta\pi_{H,t+1} + \zeta\tilde{mc}^r_t \tag{3.44}$$

with $\zeta = \frac{(1-\alpha\beta)(1-\alpha)}{\alpha}$. Further, the definition of the marginal cost when the only distortions of the economy are monopolistic competition and sticky prices (nominal rigidities) is defined as: $\hat{mc}^r_t = mc^r_t - (-\mu)$, where $-\mu$ is equal to the constant markup of firms. This distortion leads to prices higher than those in perfect competition, and therefore an inefficient level of output (if no subsidy is in place)[9]. However, a subsidy that perfectly offset the markup triggers an increase in production, leading to an efficient level of output, which coincides with its natural level (i.e. no cost-push shocks are present)[10].

This model considers a time-varying markup given by $\mu^n_t = \frac{\sigma_t}{\sigma_t - 1}$. In this way, the representative firm takes into account the following marginal cost:

$$\tilde{mc}^r_t = mc^r_t + \mu^n_t \tag{3.45}$$

Thus, the New Keynesian Phillips curve is written as:

$$\pi_{H,t} = \beta\pi_{H,t+1} + \zeta(mc^r_t + \mu^n_t) = \beta\pi_{H,t+1} + \zeta(mc^r_t + \mu + \mu^n_t \ - \mu)$$

$$\pi_{H,t} = \beta\pi_{H,t+1} + \zeta(\hat{mc_t}^r) + \zeta(\mu^n_t - \mu)$$

$$\pi_{H,t} = \beta\pi_{H,t+1} + \zeta(\hat{mc_t}^r) + \zeta(\mu_t) \tag{3.46}$$

Finally, the term $\mu_t = \mu^n_t - \mu$ essentially captures the deviations of the efficient output from the natural output[11], and follows an autoregressive AR(1) process with Gaussian shocks $\epsilon_{\mu,t} \sim \ iid \ \ N(0, \sigma_\mu)$:

$$\mu_t = \rho_\mu \mu_{t-1} + \epsilon_{\mu,t} \tag{3.47}$$

3.3 Interaction between the small open economy and the rest of the world

3.3.1 Terms of trade and inflation

The home country's terms of trade[12] is the relative price of imported goods in terms of the domestic goods. An improvement or a decrease in this relationship represents a more competitive domestic country with respect to its counterpart:

$$TOT_t = \frac{P_{F,t}}{P_{H,t}} \quad \rightarrow \quad tot_t = p_{F,t} - p_{H,t} \tag{3.48}$$

After some straightforward computations[13], and loglinearizing around the zero inflation steady state, the following relationship holds:

$$\pi_t = \pi_{H,t} + (1-v)\Delta tot_t \tag{3.49}$$

[9] Efficient level of output = Output under flexible prices and constant markup offset by a subsidy.

[10] Natural level of output = output under flexible prices, but do not rule out the presence of time varying markups

[11] μ=markup that can be offset by a constant subsidy; μ^n_t=Time varying markup

[12] This section is mainly based on Gali and Monacelli (2005) [27], unless otherwise specified.

[13] Derivations are available in Appendix A.7

Thus, the gap between consumer and domestic inflation is given by terms of trade weighted by the share of preferences of foreign goods. For instance, a terms of trade appreciation, enlarge the gap between consumer and domestic inflation, as agents would prefer relatively cheaper foreign goods.

3.3.2 Terms of trade and exchange rate

- **Terms of trade and nominal exchange rate**

As stated in section 3.1, the law of one price holds only for generic goods produced in a specific country, namely:

$$p_t(h) = S_t p_t^*(h) \;\; and \;\; p_t(f) = S_t p_t^*(f)$$

Plugging the above expression in the definition of foreign prices 3.14, holds:

$$P_{F,t} = S_t P_{F,t}^* \tag{3.50}$$

Recalling the definition of the Foreign price index in equation 3.11, the weight of home goods in the foreign preferences ($v^* = n\lambda$) and the fact that the size of the home country tends to zero, the following expression yields:

$$P^* = (v^* P_H^{*1-\theta} + (1-v^*) P_F^{*1-\theta})^{\frac{1}{1-\theta}} = P_F^* \tag{3.51}$$

thus, the small open economy do not influence international prices. Plugging 3.51 in equation 3.50, and loglinearizing:

$$p_{F,t} = s_t + p_t^* \tag{3.52}$$

Finally, combining 3.52 with the definition of terms of trade 3.48, one can observe the positive relationship between a nominal depreciation and a terms of trade worsening[14]:

$$tot_t = s_t - p_{H,t} + p_t^*$$

In terms of first differences:

$$\Delta tot_t = \Delta s_t - \pi_{H,t} + \pi_t^* \tag{3.53}$$

A nominal depreciation means that home products are relatively cheaper than foreign ones. That determines that per each unit of domestic products, the home country can purchase a lower proportion of foreign goods.

- **Terms of trade and real exchange rate**

The real exchange is defined as follows:

$$Q_t = \frac{S_t P_t^*}{P_t} \;\; \rightarrow \;\; q_t = s_t + p_t^* - p_t \tag{3.54}$$

where the real and nominal exchange rate shows a positive correlation. Also, an increase in the world price triggers a depreciation of the real exchange rate since the domestic goods would be comparatively cheaper than its counterparts in the rest of the world. After some computations shown in appendix A.8, the following relationships hold[15]:

$$q_t = v tot_t \tag{3.55}$$

$$q_t = (1-\lambda) tot_t \tag{3.56}$$

Where it is clear the positive co-movement between terms of trade and real exchange rate.

[14] A terms of trade worsening means an increase in the terms of trade.

[15] Following Gali and Monacelli (2005) [27]

3.3.3 Uncovered interest Parity

Under complete markets households can invest in home and foreign bonds, thus the budget constraint can be written as:

$$(1+\tau)W_tN_t - P_tTr_t + \Pi_t + (1+i_{t-1})B_{t-1} + (\mathbf{1}+\mathbf{i^*_{t-1}})\mathbf{S_{t-1}B^*_{t-1}} + (p_{z,t}+d_t)Z_{t-1}$$
$$= B_t + p_{z,t}Z_t + P_tC_t + \mathbf{S_tB^*_t}$$

After obtain the optimality conditions and loglinearizing[16], the following relationship holds:

$$i_t - i_t^* = E_t(\Delta s_{t+1}) \tag{3.57}$$

When home nominal interest rates are higher than its foreign counterpart, there is an appreciation of the nominal exchange rate at time t, since there are capital inflows as returns on domestic investments are higher. In the opposite case, when $i_t{<}i_t^*$, there are capital outflows that triggers a nominal depreciation at t.

Although the uncovered interest parity condition is not an additional independent equilibrium condition, it is meaningful to have an insight into the reasons behind potential variations in nominal exchange rate fluctuations.

3.3.4 International Risk Sharing

Under the assumption of complete markets[17], a first-order condition analogous to the Euler equation must also hold for the representative household in any other country. After some computations[18] and the assumption of symmetric initial conditions, which means zero net foreign asset holding, the following relationship arises:

$$c_t = c_t^* + \frac{1}{\rho}q_t \tag{3.58}$$

Using equation 3.55, which corresponds to the relationship between terms of trade and real exchange rate, the international risk sharing can be written as:

$$c_t = c_t^* + \frac{v}{\rho}tot_t \tag{3.59}$$

Therefore, under perfect asset diversification, there is a perfect consumption correlation across countries. However, home bias allows for a gap between the home and foreign households' consumption, even if the international financial market structure is complete. When the size of the economy tends to zero[19], the influence of the terms of trade depends on the degree of openness of the economy. The less open the economy, the lower the consumption correlation between the home and foreign country, and the higher the effect of terms of trade on domestic consumption.

Finally, another representation of the risk sharing derived in equation 3.59 when the size of the home economy tends to zero $n \to 0$, and using the definition of home bias ($(1-v) = (1-n)\lambda$) yields as:

$$c_t = c_t^* + \frac{(1-\lambda)}{\rho}tot_t \tag{3.60}$$

[16] Derivations are available in appendix A.9

[17] Following De Paoli (2009) [21] and Gali and Monacelli (2005) [27]

[18] Derivations are available in appendix A.10

[19] Recall that $(1-v) = (1-n)\lambda$)

3.4 Equilibrium - Demand side

Following De Paoli (2009) [21], the aggregate demand, loglinearized around the symmetric, zero inflation steady state, is the following:

$$y_t = (1-\lambda)c_t + \lambda c_t^* + \gamma q + g_t \tag{3.61}$$

where $\gamma = \frac{\theta\lambda(2-\lambda)}{(1-\lambda)}$ using equation 3.55, the aggregate demand can be written as:

$$y_t = (1-\lambda)c_t + \lambda c_t^* + v\gamma tot_t + g_t \tag{3.62}$$

Hence, the home aggregate output relies on the home (private and public) and foreign aggregate consumption. The impact of deterioration or amelioration of the terms of trade depends on the degree of openness of the small open economy as the home bias is defined as $(1-v) = (1-n)\lambda$. In addition, an improvement in tot_t would trigger a decrease in the demand for domestic goods since foreign ones are cheaper.

The total demand for foreign-produced goods is the following:

$$y_t^* = c_t^* + g_t^* \tag{3.63}$$

which depends only on the international private and public consumption of foreign goods. The domestic demand for foreign goods is negligible as the size of the home economy tends to zero $(n \to 0)$.

Using the risk sharing conditions, and after some computations[20] yields:

$$y_t = y_t^* - g_t^* + \frac{tot_t}{\rho_v} + g_t \tag{3.64}$$

$$where \quad \rho_v = \frac{\rho}{(1-\lambda)\gamma\rho + (1-\lambda)^2} \quad and \quad \gamma = \frac{\theta\lambda(2-\lambda)}{(1-\lambda)}$$

Thus, the impact of the terms of trade on the aggregate domestic demand depends strictly on the parameters, namely: the elasticity of intertemporal substitution in consumption ρ, the degree of openness λ, the home bias v and the elasticity of intratemporal substitution θ.

3.4.1 The Euler equation in terms of output

The Euler equation in terms of output is written as:[21]:

$$y_t - g_t = y_{t+1} - g_{t+1} - \frac{1}{\rho_v}(i_t - \pi_{t+1,H} - \gamma) + \omega_\rho(c_{t+1}^* - c_t^*) \tag{3.65}$$

where openness increases the sensitivity of aggregate demand for home goods to the interest rate given the parameter ρ_v[22]. Variations in the interest rates generate an expenditure switching effect on foreign goods. That is because a rise in the domestic nominal interest rate triggers a fall in domestic consumption greater than in the closed economy case since agents can switch consumption towards foreign goods.

[20]Derivations available in appendix A.11

[21]Derivations in appendix A.16

[22]Recall that $\rho_v = \frac{\rho}{(1-\lambda)\gamma\rho + (1-\lambda)^2}$ and $\gamma = \frac{\theta\lambda(2-\lambda)}{(1-\lambda)}$

Further, according to the uncovered interest parity condition, an increase in domestic nominal interest rates produces an appreciation of the nominal exchange rate (capital inflows). That triggers an improvement of the terms of trade, since foreign goods are relatively cheaper, lowering domestic output. The effect of foreign consumption depends on the combination of different parameters, such as the degree of openness, intertemporal and intratemporal elasticity of substitution contained in ω_ρ[23].

3.4.2 The dynamic IS equation

To derive the dynamic version of the IS equation is essential to consider the definition of the real interest rate r_t according to the Fisher rule:

$$r_t = i_t - E_t(\pi_{t+1}) \tag{3.66}$$

After some computations and expressing output in terms of deviations from its efficient level ($x_t = y_t - y_t^e$) [24] the dynamic IS equation for the Small open economy is the following:

$$x_t = x_{t+1} - \frac{1}{\rho_v}(i_t - E_t(\pi_{H,t+1}) - r_t^e) \tag{3.67}$$

The main difference of this equation with its closed economy counterpart, is that the degree of openness impacts the sensitivity of the efficient output gap to variations on the interest rate. This causes an expenditure switching effect, as stated previously. Also, it relates deviations of the real interest rate ($r_t \approx i_t - E_t(\pi_{H,t+1})$) from its efficient level r^e.

Hence, when the real interest rate is below the efficient one, there are positive deviations of output from its efficient level. Further, expected domestic inflation triggers incentives to produce more, and increments in the nominal interest rates exert downward pressures on the current aggregate demand, by intertemporal substitution of consumption.

3.5 Equilibrium - Supply side

3.5.1 The marginal cost

Taking into account the labor optimality condition and the international risk sharing, the real marginal cost is the following[25]:

$$mc_t = -v + \rho c_t^* + tot_t + \eta y_t - a_t(\eta + 1) \tag{3.68}$$

Equation 3.68 is important to understand the sources of inflationary pressures. First, a rise in output push up the home aggregate labor demand increasing the home real wage and real marginal cost. The size of this effect is captured by the parameter η, which is the inverse of the elasticity of labor supply.

The effect of foreign consumption in the marginal cost is positive. That is because a hike on domestic production leads to an increase in the demand for labor, rising marginal costs, and thus generating inflation coming from those firms allowed to reset its prices.

[23] $\omega_\rho = (\frac{\omega\rho_v - \lambda\rho}{\rho_v(1-\lambda)})$

[24] Derivations in appendix A.16

[25] Derivations in appendix A.12

In addition, a deterioration of the terms of trade makes domestic goods comparatively cheaper, indirectly triggering an increase in its demand, and thus marginal costs. Not less important, the marginal cost is inversely proportional to first, productivity as employers demand fewer workers to achieve the same output, and second, the labor subsidy as it promotes an expansion on labor, lowering the cost per worker effectively paid by the firm.

3.5.2 The natural output

By definition, the natural level of output prevails in the absence of nominal rigidities (i.e. flexible prices) but considers the existence of potential markup shocks. Thus, the marginal cost is equivalent to μ_t^n and 3.68 is written as:

$$-\mu_t^n = -v + y_t^n(\eta + \rho_v) + c_t^*(\rho - \rho_v) - g_t\rho_v - a_t(\eta + 1) \tag{3.69}$$

Isolating the natural output, the following expression yields:

$$y_t^n = \Gamma_o + \Gamma_\mu \mu_t^n + \Gamma_* c_t^* + \Gamma_g g_t + \Gamma_a a_t \tag{3.70}$$

$$\Gamma_o = \frac{v}{\eta + \rho_v} \quad ; \quad \Gamma_\mu = -\frac{1}{\eta + \rho_v} \quad ; \quad \Gamma_* = -\frac{\rho - \rho_v}{\eta + \rho_v} \quad ; \quad \Gamma_g = \frac{\rho_v}{\eta + \rho_v} \quad ; \quad \Gamma_a = \frac{\eta + 1}{\eta + \rho_v}$$

Government spending affects positively the natural level of output, due to a negative wealth effect. That is because it absorbs resources and makes the agent feel poorer by the present discounted value of taxes used to finance this spending. So, the agent consumes less and works more, which in turn implies a rise in the natural level of output (higher labor supply and lower marginal cost).

The effect of foreign demand is ambiguous since depends on the degree of openness and the elasticities of intertemporal and intratemporal substitution between home and foreign goods given by the parameter ρ_v. The natural output depends negatively on the markup[26], as it potentially generates an increase in inflation at the same time than reducing output. Also, the labor subsidy generates upward pressures in the natural output, as there is an increment in production. Finally, a technological improvement leads to a greater capacity of production.

3.5.3 The Efficient output

The efficient output y_t^e prevails in the absence of markup shocks and nominal rigidities. Consequently, equation 3.68 is written in the following way:

$$-\mu = -v + y_t^e(\eta + \rho_v) - g_t\rho_v + c_t^*(\rho - \rho_v) - a_t(\eta + 1) \tag{3.71}$$

Isolating y_t^e, the efficient output is:

$$y_t^e = \Gamma_o + \Gamma_* c_t^* + \Gamma_g g_t + \Gamma_a a_t \tag{3.72}$$

$$\Gamma_o = \frac{-\mu + v}{\eta + \rho_v} \quad ; \quad \Gamma_* = -\frac{\rho - \rho_v}{\eta + \rho_v} \quad ; \quad \Gamma_g = \frac{\rho_v}{\eta + \rho_v} \quad ; \quad \Gamma_a = \frac{\eta + 1}{\eta + \rho_v}$$

The only difference between the efficient and the natural output is that the former now depends on the constant markup whose impact on y_t^e is still negative, as it pushes output below its capacity. Although in this case, the markup would be completely offset by the subsidy, and no cost-push shocks are present.

[26]The markup fluctuates over time as a result of the time-varying subsidy

3.5.4 The New-Keynessian Phillips curve

The relevant output gap is defined as a function of deviations from the efficient product:

$$x_t = y_t - y_t^e \tag{3.73}$$

Thus, the real marginal cost is written as a function of x_t[27], and the New Keynessian Philips curve is the following:

$$\pi_{H,t} = \beta\pi_{H,t+1} + \kappa_v x_t + \kappa_v \mu_t \tag{3.74}$$

$$\kappa_v = \zeta(\eta + \rho_v) \;\; ; \;\; \zeta = \frac{(1-\alpha\beta)(1-\alpha)}{\alpha} \;\; ; \;\; \rho_v = \frac{\rho}{(1-\lambda)\gamma\rho + (1-\lambda)^2} \;\; ; \;\; \gamma = \frac{\theta\lambda(2-\lambda)}{(1-\lambda)}$$

. Where μ_t captures the difference between the efficient and the natural output. First, the open economy New Keynessian Phillips curve nests the special case of a closed economy case when $\lambda = 0$, (i.e. zero degree of openness).

Second, the Phillips curve of the small open economy now depends on the relevant output gap. According to this framework, targeting a zero level of domestic inflation does not close automatically the gap between the actual output and the efficient one, given the potential cost-push shocks. Therefore, there is no divine coincidence in this economy.

Also, the greater the degree of substitutability between home and foreign goods, the less responsive is inflation to changes in the output gap. That is because expansion in domestic aggregate demand triggers a nominal appreciation of the exchange rate, which makes foreign goods relatively cheaper. Thus, if the elasticity of substitution between home and foreign goods is high, the real appreciation would break a further expansion of the demand and thus, of domestic prices.

3.5.5 The efficient interest rate

$$r_t^e = \gamma + \omega_\rho \Delta c_{t+1}^* + \rho_v \Gamma_g \Delta g_{t+1} - a_t \rho_v \Gamma_a (1-\rho_a) \tag{3.75}$$

Intuitively, the efficient interest rate defines the level of the real rate required to keep aggregate demand equal to its efficient level. This model portrays an economy in which nominal rigidities and inefficient shocks are present, thus the theoretical efficient interest rate abstracts from cost-push shocks.Derivations are available in appendix A.17

[27]Derivations in appendix A.15

Chapter 4

Simulation and welfare analysis

An application of the model developed in section 3 is the simulation of the dynamic effects of external demand, government spending, productivity, and cost-push shocks. This chapter explores in detail its main repercussions not only on observable variables such as output, consumption, inflation, and terms of trade but also on the efficient level of output, and therefore the relevant output gap for the Bolivian economy. This application in the style of a rigorous theoretical approach as in De Paoli (2009) and Galí and Monacelli (2005) is a novelty in the analysis of this country.

4.1 Estimation and parametrization

To simulate the model, I calibrate those parameters supported by the literature based on previous DSGE models for Bolivia and Latin-American countries. On the other hand, I estimate those parameters for which there is only a prior belief about their value.

4.1.1 Empirical methodology

The model is estimated using a Bayesian approach. According to Mancini (2008) [35], "Bayesian estimation is a bridge between calibration and maximum likelihood", as the former focus on the specification of priors, and the latter is based on confronting the model with the data. Priors are described by a density function of the form:

$$p(\mathcal{V}_A|A)$$

where A symbolizes a specific model, $\mathcal{V}_A$ represents its parameters, and $p(\cdot)$ stands for a probability density function. On the other hand, the likelihood function is given by:

$$\mathcal{L}(\mathcal{V}_A|Y_T, A) = p(Y_T|\mathcal{V}_A, A)$$

where Y_T are observations until period T. Together, the prior density, the likelihood function and the Bayes theorem, gives the posterior density as:

$$p(\mathcal{V}_A|Y_T, A) = \frac{p(Y_T|\mathcal{V}_A, A)p(\mathcal{V}_A|A)}{p(Y_T|A)}$$

Finally, the posterior Kernel assumes that Y_T conditional on the model A (marginal density[1]) is always constant, and is given by[2]:

$$p(\mathcal{V}_A|Y_T, A) \propto \mathcal{K}(\mathcal{V}_A|Y_T, A)$$

Therefore, following Mancini (2008) [35], this is the fundamental equation to rebuild the posterior moments of the parameters of interest. I use the software dynare to find its posterior mean.[3]. In this case, the vector of parameters to be estimated is:

$$\mathcal{V} = [\rho_{c^*}, \sigma_{c^*}, \rho_a, \sigma_a, \rho_u, \sigma_u]$$

And the vector of observable variables is:

$$Y_T = [y_t, c_t, q_t, \pi_t, tot_t]$$

4.1.2 Data and parametrization

Table 4.1: **Parametrization**

Description	Par.	Value	Source
Subjective discount factor	β	0.98	Zeballos (2018) [49]
Intertemporal elasticity of substitution	ρ	1	Zeballos (2018)* [49]
Inverse of labor supply elasticity	η	0.5	Cerezo (2011)** [15]
Degree of openness	λ	0.32	Zeballos (2018)*** [49]
Intratemporal elasticity of substitution	θ	0.8	Amado (2014)† [3]
Calvo parameter	α	0.39	Cerezo (2011)† † [15]
Persistence of government spending	ρ_g	0.12	Salas (2018)† † † [42]
Shock of government spending	σ_g	0.0056	Salas (2018) [42]

* Consistent with Medina (2007) [37] in the case of Chile.
**In line with the estimation of Duncan for emerging countries (2004) [23]
***Consistent with Garcia-Cicco (2017) [28] focused on Latin-American countries.
† Estimation for Peru, and consistent with the Brazilian case (Meneses) (2016) [38]
†† In line with Valdivia (2008) [48]
† † † Computations in line with the Bayessian estimation of Lubik (2005) [33]

To estimate the model I take into account Bolivian quarterly data from 2009 Q1 to 2019 Q4. As stated in the previous section, the observable variables are real GDP (y_t), real domestic consumption (c_t), real exchange rate (q_t), consumer price inflation (π_t), and terms of trade (tot_t). The sources of data are the Central Bank of Bolivia (BCB) [14] and the National

[1]"Univariate" probability distribution

[2]The symbol $\propto$ is a binary mathematical operator indicating that the left value is proportional to the right value.

[3]This computational package estimates the likelihood function applying 1)the Kalman Filter: According to Hamilton (1994) [30], this is an algorithm for sequentially updating a system based on the information available. In simple words, the Kalman filter tells how to update the knowledge of a latent variable (not directly observable but inferred from observed variables) when new information is available. and then 2)simulates the posterior distribution using a Monte Carlo method (Technique that use random sampling to simulate a draw from some distribution, such as the Metropolis-Hasting)

Institute of Statistics (INE) [40]. To proceed with the analysis, first, I deseasonalize the original series related to real GDP and consumption using the method ARIMA X-13 from the US Census Bureau. Appendix B shows the raw data and the corresponding deseasonalized series. Finally, to deal with non-stationary processes, I compute first differences of the log-variables.

According to the literature based on previous DSGE analysis for Bolivia and Latin American countries, the parameters stated in table 4.1 hold for the simulation of this model.

The subjective discount factor set at 0.98 (quarterly) follows Zeballos (2018), and implies an annual interest rate of around 4 percent. The intertemporal elasticity of consumption ρ is 1, in line with DSGE analysis for Latin American countries and emerging economies[4]. The inverse of the labor supply elasticity η is set at 0.5 following Cerezo (2010) [15]. This implies an elasticity of labor supply of 2, related to the high degree of informality typical in Latin America and thus, with the important job turnover[5].

The degree of openness λ, or the share of foreign consumption goods is calibrated at 0.32 following Zeballos (2018) [49], who computes the proportion of imported consumption goods between 2000-2017. Also, according to Amado (2014) [3], the intratemporal elasticity of substitution among home and foreign goods θ is set at 0.8 for the Peruvian economy, consistent with the Brazilian case analyzed by Meneses (2018) [38].

The parameter corresponding to the degree of price rigidity α is calibrated at 0.39, implying that firms keep their prices fixed, on average, 1.6 quarters[6]. Finally, the parameter regarding the autoregressive process of government spending and the standard deviation of the stochastic component is based on the estimation of Salas (2018) [42].

4.1.3 Priors and Posteriors distributions

The last three columns of Table 4.2 reports the mean and 90 percent confidence intervals of the Bayesian estimation. I select priors based on evidence from previous analysis in the case of Bolivia and Latin-American countries. Appendix C shows the plots of prior and posterior distributions as well as the convergence diagnostic.

Table 4.2: Bayesian Estimation of Parameters

		Prior Distribution			Posterior Distribution		
Description	Par	Density	Domain	Mean	Mean	90 percentinterval	
Cost-push shock persist.	ρ_u	Beta	[0, 1)	0.5	0.3218	0.1870	0.4487
S.D. of cost-push shock	σ_u	Inv.Gam	$[0,\infty)$	0.0013	0.0074	0.0057	0.0091
Ext. demand persist.	ρ_{c^*}	Beta	[0, 1)	0.7	0.7941	0.6666	0.9235
S.D. ext. demand shock	σ_{c^*}	Inv.Gam	$[0,\infty)$	0.01	0.0286	0.0233	0.0337
Productivity persist.	ρ_a	Beta	[0, 1)	0.4	0.2659	0.1392	0.3913
S.D. prod. shock	σ_a	Inv.Gam	$[0,\infty)$	0.0071	0.0140	0.0112	0.0166

The autoregressive parameters of the stochastic processes ρ_u, ρ_{c^*} and ρ_a have beta distributions because their values should lie in the [0, 1) interval[7].

In particular, the mean of the prior about the persistence of cost-push shocks is set at 0.5. For the case of Bolivia, there is only a single analysis related to these types of perturbations

[4]Medina (2007) [37], Meneses (2016) [38], Lahcen (2014) [31], Aguiar et al (2007) [2] among others,

[5]Céspedes (2012) [16]

[6]Recall that the average duration of the price is $\frac{1}{1-\alpha}$

[7]This guarantees a stationary process that does not "explode".

(Valdivia (2017) [47]) in a closed economy context. Finally, according to the Bayesian process performed in this section, I estimate an autoregressive coefficient of 0.3218 and a stochastic process with a standard deviation of 0.0074.

Regarding the external demand shocks, I take into account the analysis of Cerezo (2011) [15] who computed an autoregressive coefficient of 0.7[8]. Thence, I set a prior belief of 0.7 and obtain a posterior mean of 0.79. In the same way, analyses for Latin American countries suggest values between 0.01 and 0.02 for the standard deviation of external demand shocks. I set a value of 0.01 and estimated a posterior mean of 0.0286.

Finally, I estimate productivity shocks given the large discordance in the Bolivian literature regarding this parameter. Concerning the persistence of the autoregressive process, Salas (2019) [42] and Cerezo (2011) [15] estimate values around 0.3, while Zeballos (2018) [49] fixes this parameter around 0.8. My Bayesian estimation suggests a coefficient of persistence of 0.27 and a standard deviation of 0.014.

4.1.4 Monetary policy

Until now there was missing an essential part of the analysis: The specification of the monetary policy rule. In this context, I **compare** the effects of exogenous perturbations under two different types of simple monetary policy rules: First, a typical Taylor rule[9], and second a fixed exchange rate regime. I analyze the latter rule given that according to the IMF report on "Exchange Arrangements and Restrictions" 2019 [5], the Central Bank of Bolivia focus on a "monetary policy aggregates" system, keeping a "de facto" exchange rate anchor of the Bolivian currency to the US dollar.

For the Taylor rule, the weights assigned to inflation ψ_π and output gap ψ_x are set at 1.5 and 0.5 respectively. The parameter corresponding to the interest rate smoothing ρ_r is 0.7:

$$i_t = \rho_r i_{t-1} + \psi_\pi \pi_t + \psi_x x$$

On the other hand, in order to take into account the fixed exchange rate currently in place in Bolivia, the only condition is a constant nominal exchange rate over time:

$$\Delta s_t = 0$$

Thus, I **compare** both policies separately in the style of Gali and Monacelli(2005) [27].

4.2 Welfare Analysis

A natural criteria to compare alternative monetary policy regimes in a microfunded model, is the sum of the utilities of the consumers[10]:

$$W = E_t \sum_{t=0}^{\infty} \beta^t w_t$$

[8]The analysis of Cerezo was executed in 2011, thus it is relevant to compute this parameter with updated information

[9]A typical Taylor rule generally considers a Consumer Price Inflation - CPI - targeting, as it is easily communicated to the public. However, a recent strand of literature points out to the welfare improvements of a Producer Price Inflation - PPI - targeting. Appendix D shows the impulse response functions under the later policy and its welfare implications

[10]Benigno (2008) [7]

where $w_t = \left(\frac{C^{1-\rho}-1}{1-\rho} - \frac{N^{1+\eta}}{1+\eta}\right)$. Also, consumption and labor are respectively represented by iso-elastic functional forms.

Appendix E shows that a second order approximation of W around the steady state, assuming intertemporal elasticity of substitution equal to the unity (i.e. $\rho = 1$), yields the following welfare criterion:

$$W = -\frac{\mathcal{C}(1+\eta)}{2} var(\hat{n}_t) \tag{4.1}$$

where the term $\mathcal{C} = \frac{U_n N}{U_c C}$ keeps constant over time. Hence, using equation 4.1 it is possible to assess each monetary policy rule and select the one that minimizes variations in labor from the steady-state. But why is that the case? As this is a micro-founded model, the notion that individuals get satisfaction from consuming goods and leisure is summarized by the utility function, which in the end measures the agent's level of felicity or happiness (Borjas (2008)) [11]. In this context, the representative household maximizes consumption, since utility is an increasing and concave[11] in C and minimize labor as the function is decreasing and convex in N. Hence, welfare losses (or "individual unhappiness") are explained by variations in labor from the steady-state, showing the key role of this variable in the analysis.

Also, following Gali and Monacelli (2005), and assuming intratemporal and intertemporal elasticity of substitution equal to the unity, i.e. $\rho = \theta = 1$, the following relationship is derived[12] from a second-order Taylor approximation:

$$W = -(1-\lambda) \sum_{t=0}^{\infty} \beta^t \left(\frac{1}{2} (\sigma \zeta^{-1} \pi_{Ht}^2 + (1+\eta) \hat{x}_t^2) \right) \tag{4.2}$$

where $\zeta^{-1} = \frac{\alpha}{(1-\alpha)(1-\beta\alpha)}$. However, this representation ties the analysis to a very restrictive case given the assumption on the parameters. Therefore the following evaluation is based on the application of equation 4.1, but also shows the variations on inflation and output gap associated to each simple alternative monetary policy.

According to Clarida et al (1999) [12], the foremost goal of the monetary authority is to minimize squared deviations of the output gap and inflation from their targets to maximize *welfare*. Those fluctuations arise as a result of efficient and inefficient exogenous perturbations:

When shocks are *inefficient*, the natural level of output does not coincide with the efficient one, as cost-push shocks are present. In the baseline model, the leading component of those frictions is the time-varying markup of firms. The New Keynesian Phillips curve, depicted by equation 3.74, reveals a trade-off between output and inflation stabilization, as both of them do not concur simultaneously. Consequently, it is clear that even if the Central Bank achieves completely its target on inflation, it will convey an inefficient level of economic activity.

On the other hand, *efficient* exogenous perturbations (such as productivity, foreign demand, and government spending shocks) do not create a dilemma between output and inflation stabilization. This scenario indicates that the so-called divine coincidence prevails. Thus, when the subsidy on labor is in place, the natural level of output matches the efficient one since no cost-push shocks are present.

But why is it so important to attain output and inflation stabilization to minimize *welfare losses*? In accordance with Gali (2015) [26], the former should fluctuate in line with its efficient level[13], given that real shocks (such as exogenous perturbations in the aggregate

[11] In this particular setting I assume intertemporal elasticity of substitution equal to the unity $\rho = 1$ i.e. log-utility which is a function also increasing and concave in C

[12] Derivations in appendix E.2

[13] The natural output is equivalent to the efficient one if cost-push shocks are not present and the subsidy on labor perfectly offsets the markup of firms

productivity) represent a source of changes on its level. Besides, deviations of output from its target generates inflationary (deflationary) pressures as the marginal costs of firms rise (decrease) due to the expansion (shrinkage) of aggregate demand, leading output above (below) its capacity[14]. Thence, when output is at its efficient level, there is no inflation, as those firms allowed to reset prices every period select the same price as their counterparts facing price stickiness[15]. Further, price stability is desirable for many reasons including the maintenance of the purchasing power, the anchoring of expectations, and the Central Bank‘s transparency in communicating explicitly an inflation targeting.

The next part exhibits the implications of supply and demand shocks on the relevant output gap defined as the difference between the actual and the efficient output. I subsequently show the welfare implications associated with the second-order approximation of the felicity function and the effects of exogenous perturbations under alternative monetary policy rules. Therefore, as explained previously, it is going to be clear that deviations from the relevant output gap and excessive fluctuations in inflation drive also welfare losses.

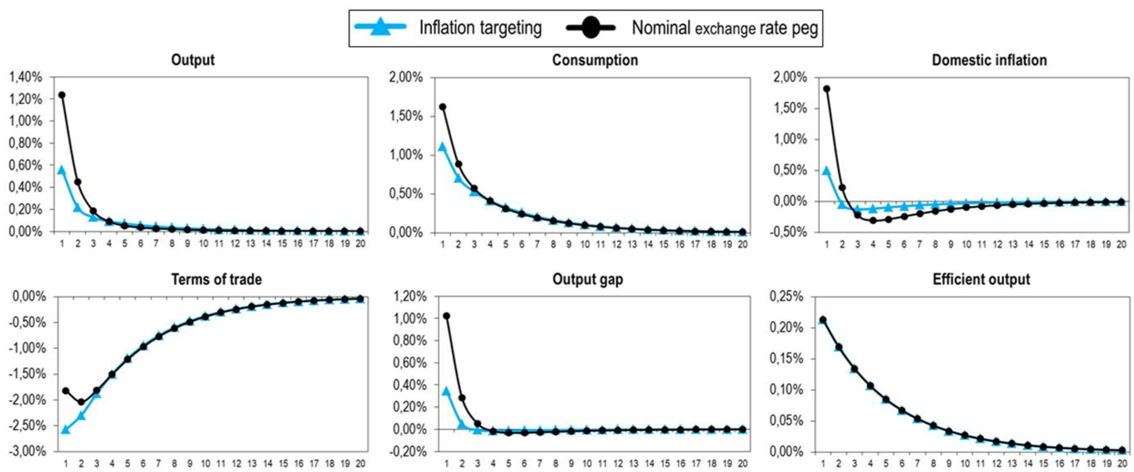
Note that, the simulation and welfare analysis are performed using the software Dynare. In particular, variances corresponds to the unconditional theoretical moments of variables in the estimated model.

4.3 Simulation

This section displays the effects of efficient and inefficient exogenous perturbations on six variables: output, consumption, domestic inflation, efficient output, and the relevant output gap. Appendix D shows the impulse response functions to the full set of variables. Also, I compare the repercussions under two different scenarios: A Taylor rule and an exchange rate peg regime.

4.3.1 Impulse responses to external demand shocks under alternative policy rules

Figure 4.1: Impulse responses to external demand shocks



[14]The output capacity is the natural level of output

[15]That is because of the intrinsic characteristic of price flexibility when the output is at its efficient level.

First of all, an inflation-targeting rule maintains greater stabilization of prices and output gap than an exchange rate peg regime. More precisely, under inflation targeting, a shock of 2.9 percent in external demand leads to a rise in domestic inflation of only 0.5 percent compared to a potential increase of approximately 2 percent under an exchange rate peg. In the same fashion, the output gap is almost two times higher in the context of the latter.

According to the equilibrium conditions, an expansion of foreign consumption of domestically produced goods generates an increase of output and hence domestic inflation coming from those firms allowed to reset their prices. Thus, there is an improvement in the terms of trade, triggering the expenditure switching effect, as home consumption grows since agents can purchase more foreign goods per each unit of domestic ones. This intuition accompanies the idea of perfect consumption correlation illustrated by the risk-sharing condition.

Not less important, [16] an expansion in foreign demand encourages domestic labor demand and thus wages, pushing an enlargement in the marginal cost and inflation. The amplification of production leads to a rise in the natural output which coincides with the efficient one in the absence of cost-push shocks. Finally, given an increase in the efficient output less than proportional than the actual hike in the level of output, there is an expansionary impact in the efficient interest rate. However, the output gap remains positive, as the real interest rate is below its efficient level. As a result of the greater deviations of the output gap and inflation under a nominal exchange rate peg, there are negative welfare implications of this policy. According to the literature [17], floating exchange rate regimes are in theory the most effective mean to absorb external shocks. When domestic prices are sticky, a shock in foreign demand leads to an appreciation of the nominal exchange rate. That in turns increases the relative prices of home goods precisely the moment when the demand for them has risen, and therefore partially offsets the effect of higher deviations of output and weakens inflation.

The same intuition holds in the context of a negative foreign demand shock, which leads to a nominal exchange rate depreciation, making domestic goods more competitive (automatic stabilizer). This is not the case under a fixed exchange rate regime, where a shock in the foreign demand is translated in an expansion of inflation, limited by the price stickiness.

4.3.2 Impulse responses to government spending shocks under alternative policy rules

The first element to note here is the effect of government spending on output. In this case public spending is explained by the purchase of home differentiated goods, thus the positive transmission of government consumption is focus on the demand side of this economy, affecting unmistakably the output gap through the dynamics of the IS curve.

More precisely, in the context of a Taylor rule, the peak fiscal multiplier is 0.75 and 0.875 under the competing monetary policy (given an upsurge of government spending of 5.6 percent). But, concerning the cumulative fiscal multiplier, the difference between a peg regime and a Taylor rule only arises as a result of the contemporaneous effect on output.

On the other hand, there is a sudden decrease in private consumption [18] due to a negative wealth effect. That is because government spending absorbs resources and makes the agent feel poorer by the present discount value of taxes that are used to finance this spending. Thence,

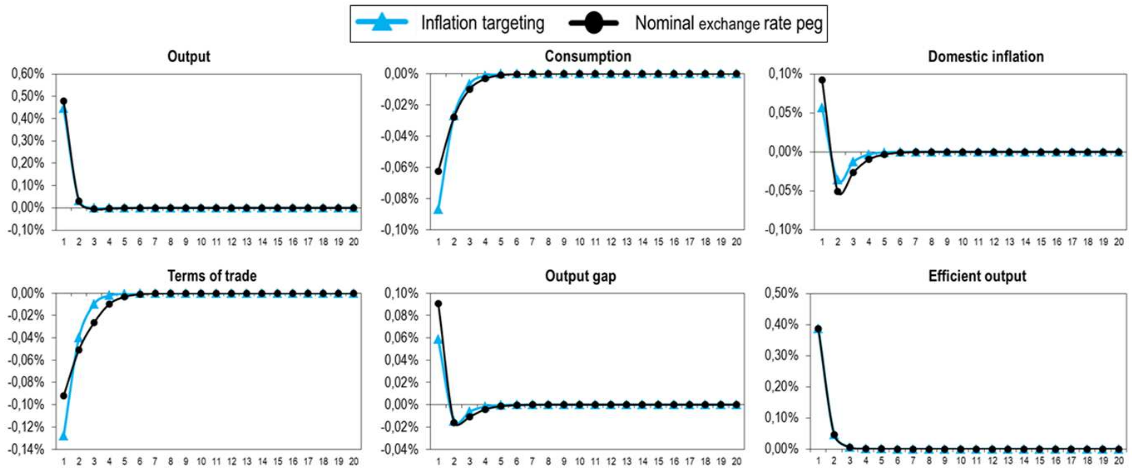
[16] Recall equation 3.68: $\mathrm{mc}_t = \rho c_t^* + tot_t + \eta y_t - a_t(\eta + 1)$

[17] (Broda (2001) [12])

[18] This result is contrary to the data and empirical analysis (Ambler (2017)) [4]

the representative household consumes less and works more for a given wage, which at the same time implies an increase in output[19].

Figure 4.2: Impulse responses to government spending shocks



Interestingly, in the context of a nominal exchange rate peg regime, the effect on output is slightly higher than under an inflation targeting. This result coincides with the literature (Corsetti (2013)) [19] and the textbook Mundell-Fleming model, which suggests that fiscal policy is more effective under a fixed regime.

Finally, considering a nominal exchange rate peg, the terms of trade cutback is 4 percent lower than in the inflation targeting case, given the typical smoothing effect in the context of a fixed regime. Although, the augment of inflation is almost double under this scenario, given the increase in domestic prices as a result of the demand expansion.

In general, a positive shock of government spending represents a transient expansion of output, that last for less than three quarters. In contrast, the effect on consumption endures for at least five quarters.

Even though there are apparent benefits in terms of a greater hike in output in the context of an exchange rate peg, the volatility of inflation and the output gap implies negative welfare effects of this policy.

4.3.3 Impulse responses to productivity shocks under alternative policy rules

First of all, a productivity shock is tied to a reduction in the domestic interest rate, expanding consumption and output[20]. Therefore, the uncovered interest parity condition implies a nominal depreciation, leading to a terms of trade worsening which boost consumption and output given the greater competitiveness of the domestic economy.

Under both monetary rules, there is a fall in domestic inflation, as the marginal cost of production falls as a result of the positive shock in productivity. In the context of a pegged regime, the impossibility of lowering the nominal exchange rate leads to a limited response in the terms of trade, which undermine competitiveness.

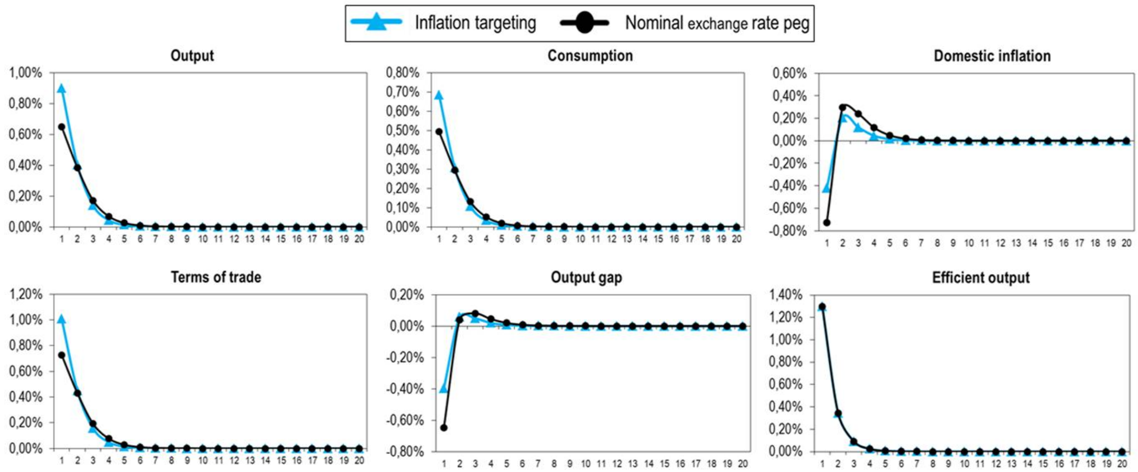
In addition, upward pressures on productivity has multiple effects on the labor market: firms rise labor demand to produce more, raising wages and allowing the recovery of inflation.

[19]Baxter and King (1993) [6]

[20]Gali and Monacelli (2005) [27]

Also, technological improvements lead to a greater capacity of production, triggering an expansion of the efficient output. Although there is an increase in output, it remains below its efficient level for at least two quarters after the shock, which illustrates the time that firms need to adapt production to the new technology.

Figure 4.3: Impulse responses to productivity shocks



Finally, variations in the efficient output are independent of the monetary policy regime in place. This scenario illustrates the idea of the classical dichotomy.

From the welfare perspective, inflation targeting leads to a greater stabilization of the output gap and inflation, as it is clear on the graphs.

4.3.4 Impulse responses to cost-push shocks under alternative policy rules

First of all, the upsurge of domestic inflation is independent of the monetary policy regime in place. However, the reduction in output is 0.5 percent higher under an inflation targeting. The reason behind is that cost-push shocks create a dilemma for the central bank, as it is inefficient to target output gap and inflation simultaneously[21]. On the one hand, price stabilization is desirable, but on the other, it generates negative variations in the output gap since the economy produces below its capacity[22]. Therefore, the presence of cost-push shocks involves that monetary policy should allow for some flexibility in prices to reach the stabilization of the output gap. At the same time, upward pressures in prices imply a cutback in the terms of trade given that domestic goods are relatively more expensive than their foreign counterparts.

In terms of welfare, the analysis is not clear, as inflation targeting leads to greater deviations of the output gap, and a pegged regime generates a considerable variability of prices. The next section is going to explore this issue in detail.

In the context of an open economy and a flexible exchange rate regime, a cost-push shock is the leading component of an increase in the interest rate. That supports a transitory downturn in consumption and output. Given the constancy of the world nominal interest rates, the uncovered interest parity implies a nominal appreciation and therefore a further contraction in demand and output through a decrease in exports, as they are less competitive. On the

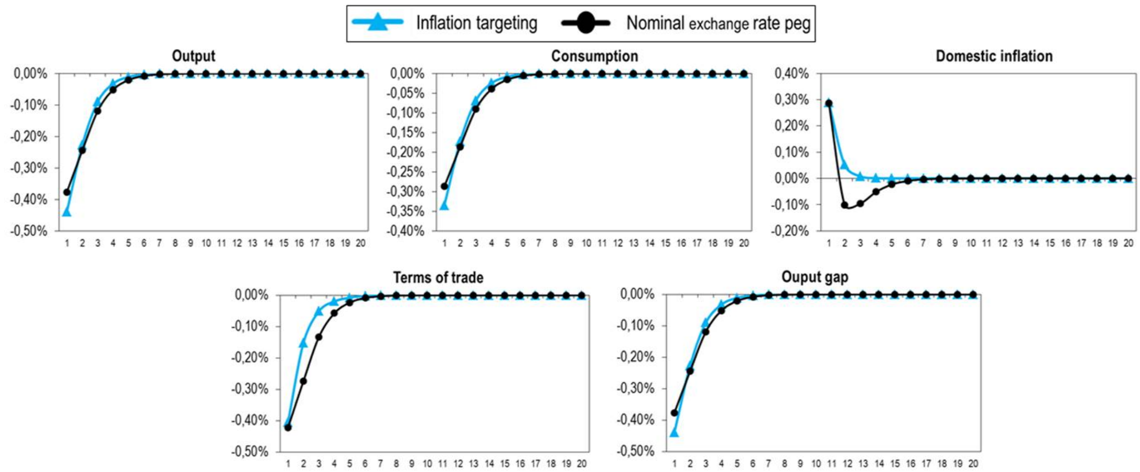
[21] Shuterland (2004) [43]

[22] Recall the Phillips curve (equation A.63): $\pi_{H,t} = \beta\pi_{H,t+1} + \kappa_v x_t + \kappa_v \mu_t$

other hand, under an exchange rate peg regime, this is no longer the case, and the contraction in output, and thus consumption is mitigated.

Therefore, cost-push shocks are related to upward pressures on inflation and weakness of output. For instance, an increase in the price of inputs leads to lower production since a proportion of firms maintains prices fixed.

Figure 4.4: Impulse responses to cost-push shocks



4.3.5 Variance Decomposition of shocks

Table 4.3: **Variance Decomposition of shocks**- in percent

	Foreign demand		Productivity		Gov spending		Cost-push shock	
	Taylor	Peg	Taylor	Peg	Taylor	Peg	Taylor	Peg
σ y	21.65	62.88	53.68	21.26	10.84	8.10	13.83	7.76
σ tot	93.18	94.25	5.36	4.17	0.08	0.06	1.38	1.52
σ c	77.09	89.62	18.01	7.53	0.26	0.10	4.64	2.75
$\sigma\ \pi_h$	40.73	80.22	31.67	14.67	0.62	0.25	26.99	4.86
σ x	22.6	63.27	30.12	24.01	0.67	0.48	46.6	12.25
$\sigma\ y^e$	5.86	5.88	86.81	86.81	7.31	7.31	0.00	0.00

-Computations are performed in Dynare, and corresponds to the unconditional variance of shocks from the estimated model. The variance decomposition is the contribution of each shock explaining fluctuations in endogenous variables.

Table 4.3 shows the contribution of each shocks to the variation in output, terms of trade, consumption, domestic inflation, output gap and the efficient output.

First, foreign demand explains a large proportion of the output variability in the context of a pegged regime. The reason is related to the upsurge in inflation (coming from those firms allowed to reset their prices) which enables an enlargement of production as a result of the greater demand. Moreover, the terms of trade smoothness characteristic of this monetary policy rule, limits a further appreciation of the real exchange rate and enhances production, as the currency is overvalued.

On the other hand, considering a Taylor rule, a flexible exchange rate (automatic stabilizer), limits variations in the output gap as a result of foreign demand perturbations. Thus, alterations of output are mainly explained by productivity shocks as it is common on the literature[23]

[23] For instance the analysis of Tenreyro et al (2017) [44]

Also note that under any competing monetary policy, the contribution of foreign demand shocks in the variability of the terms of trade is greater than 90 percent. This shows the influence of the external demand on domestic prices and therefore competitiveness of the small open economy. Consequently, foreign demand is also an important source of volatility in consumption, given the expenditure switching effect.

Finally, the contribution of government spending to fluctuations in the variables analyzed here is negligible compared to the rest of the shocks. The reason behind is given by the low standard deviation of the stochastic process corresponding to the exogenous process.

4.4 Evaluation of monetary policies

Table 4.4: **Welfare Losses** - Evaluation of simple monetary policy rules

	Taylor Rule				PEG
ϕ_π	1.5	1.5	5	1.5	-
ϕ_y	0.125	0	0	1	-
σy	1.366391	1.370662	1.458503	1.350135	1.682293
σx	0.806274	0.831424	0.857563	0.665638	1.339094
$\sigma \pi_H$	0.892987	0.911011	0.914880	0.868607	2.163713
$\sigma \Delta s$	2.162405	2.119744	2.023694	2.436389	0
σtot	4.784296	4.773591	4.754520	4.844872	4.256270
Welfare loss	0.009313	0.009708	0.010094	0.007292	0.021014

-The analysis considers productivity shocks, external demand shocks, government spending shocks, and cost-push shocks at the same time
-Assumption: $\rho = 1$
-ϕ_π and ϕ_y are parameters corresponding to the Taylor rule
-σy, σx, $\sigma \pi$ $\sigma \Delta s$ and σtot represent variations in output, output gap, inflation, nominal exchange rate and terms of trade.

This section shows the welfare implications of two alternative monetary policy rules. First, a Taylor rule that considers four different combinations of weights on inflation and output gap, going from an inflation targeting to a balanced target on output and inflation. And second, a nominal exchange rate peg, which is currently in place in Bolivia. According to table 4.4, in the presence of simultaneous exogenous perturbations, (namely, productivity, foreign demand, government spending, and cost-push shocks) the monetary policy that minimizes welfare losses correspond to a Taylor rule focus on targeting inflation and output gap. The least preferred policy is an exchange rate peg regime[24].

On the other hand, variations in the output gap and inflation are greater in the context of a peg. Also, in line with Gali (2015) [26], an excess smoothness in the terms of trade arises under a pegged regime, which is a consequence of the inability of prices (sticky) to compensate for the constancy of the nominal exchange rate [39].

[24]Note that, the simulation and welfare analysis are performed using the software Dynare. In particular, variances corresponds to the theoretical moments of variables under the estimated and calibrated model.

In case shocks are only efficient, the most preferred monetary policy would be the one focus only on inflation targeting[25]. However, given the presence of cost-push shocks on this analysis, the most preferred monetary policy targets inflation and the output gap at the same time, as the divine coincidence no longer holds.

4.4.1 Simple monetary policy rules

This section shows the welfare implications of two alternative simple monetary policy rules in terms of observable variables. First, a Taylor rule that considers a producer and consumer inflation targeting. And second, a nominal exchange rate peg, which is currently in place in Bolivia[26]:

$$i_t = \rho_r i_{t-1} + \psi_\pi \pi_t$$

$$i_t = \rho_r i_{t-1} + \psi_\pi \pi_{Ht}$$

$$\Delta s_t = 0$$

The main results reveal the welfare benefits of inflation targeting relative to an exchange rate peg.

According to table 4.5, in the presence of simultaneous exogenous perturbations, (namely, productivity, foreign demand, government spending, and cost-push shocks) the monetary policy that minimizes welfare losses is a Taylor rule targeting domestic prices. The least preferred policy is an exchange rate peg regime[27].

On the other hand, variations on key variables such as output and domestic inflation are greater in the context of a peg regime. Also, in line with Gali (2015) [26], an excess smoothness in the terms of trade arises under this policy, given the inability of prices (sticky) to compensate for the constancy of the nominal exchange rate [39].

As mentioned before, a typical Taylor rule generally considers a Consumer Price Inflation index. Emerging countries such as Peru and Brazil focus on this objective. However, as shown here and in line with De Paoli (2009) [21], a producer price inflation targeting is the one with the best performance. These results also coincide with the welfare analysis of Parrado (2004) [41], focus on the case of Chile. According to this author, the response of interest rates to changes on inflation is greater in the context of a producer price inflation targeting. Hence, given the uncovered interest parity condition, the fluctuations in the nominal exchange rate performs the role of an automatic stabilizer. That explains the relatively low variability of output, the high volatility of the real exchange rate, and thus the considerable variation of the terms of trade.

Also is remarkable the fact that lower welfare losses are related to lower variability of domestic inflation instead of CPI inflation. This result is in line with De Paoli (2009) [20] who suggests that domestic price targeting conveys to the lowest welfare costs when multiple types of perturbations ocurrs simultaneously.

When government spending shocks are present, a Taylor rule is desired given the upward pressures on domestic inflation as a result of the expansion in the aggregate demand. The same intuition holds in the case of external demand shocks. In the presence of productivity perturbations, a Taylor rule is better than a pegged regime, as a result of the potential

[25] Indeed, as it is shown in the appendix, in the presence of efficient shocks, welfare losses are minimized when the monetary authority focus only on inflation targeting.

[26] Following the same calibration as in the previous section

[27] Note that, the simulation and welfare analysis are performed using the software Dynare. In particular, variances correspond to the unconditional variance under the estimated model.

Table 4.5: **Welfare Losses** - Evaluation of simple monetary policy rules according to the type of shock

	Taylor Rule		PEG
	CPI	PPI	PEG
ϕ_π	1.5	1.5	-
σy	1.37066201	1.36079207	1.682293
σx	0.83142429	0.70942044	1.339094
$\sigma \pi_H$	0.91101129	0.46151344	2.163713
$\sigma \Delta s$	2.11974488	3.27175809	0
σtot	4.77359149	5.06121415	4.256270
Productivity	0.00231938	0.00129827	0.004301
Ext. demand	0.00343266	0.00098918	0.013346
Gov.spending	0.00154771	0.00143604	0.001719
Cost-push	0.00240842	0.00301173	0.001646
Welfare loss	0.00970817	0.00673521	0.021014

-Assumption: $\rho = 1$
-ϕ_π is a parameter corresponding to the Taylor rule

deflationary effects and the efforts of the central bank to cut down interest rates to reactivate the economy.

Finally, considering cost-push shocks, an exchange rate peg is preferred rather than a pure inflation targeting based either on producer or consumer prices. According to Berger (2006) [8], in this case, policies that focus on target output and inflation simultaneously, outperform those concentrated only in maintain price stability since the divine coincidence no longer hold under these circumstances.

But why do cost-push shocks perform better under a peg regime? When there is a markup shock, there is a sudden reduction in output, as firms that are not allowed to reset their prices immediately reduce production. Along with the sudden increase in prices, follows an improvement of the terms of trade. This situation makes domestic products relatively more expensive, lowering its competitiveness and generating downward pressures on the demand for exports. Since the nominal exchange rate is fixed, it remains underappreciated which stimulates the aggregate demand and thus production. That finally end up tightening the output gap and improving welfare.

4.4.2 An alternative monetary policy rule

In this section, I characterize an alternative simple monetary rule that reflects the commitment of the central bank to maintain a managed nominal exchange rate and target inflation. Following Monacelli (2004) [39], I analyze the performance of this rule. As mentioned before, more than 120 countries in the world maintain either a hard or a soft pegged regime, and Bolivia is not the exception.

The monetary policy rules would be the following (Monacelli (2004) [39]):

$$i_t = \psi_\pi \pi_t + \phi_s \Delta s_t \tag{4.3}$$

$$i_t = \psi_\pi \pi_{H,t} + \phi_s \Delta s_t \tag{4.4}$$

where $\phi_s = \frac{w_s}{(1-w_s)}$, and in particular:

- $w_s = 0 \rightarrow$ is equivalent to a floating exchange rate regime
- $w_s \in (0, 1] \rightarrow$ represents a managed/fixed exchange rate regime

I will calibrate the value of w_s at 0.76[28].

Table 4.6: **Welfare losses**-Evaluation of Monetary policy rule à la Monacelli

	Hybrid rule		Pure inflation targeting		Peg
	CPI	PPI	CPI	PPI	-
ω_s	0.76	0.76	-	-	-
ϕ_π	1.5	1.5	1.5	1.5	
Welfare loss	0.01708941	0.01552005	0.00970817	0.00673521	0.021014

-ϕ_π is a parameter corresponding to the Taylor rule

-Assumption: $\varphi = 1$

Table 4.7 shows the welfare implications of the monetary policy rules analyzed in this section. However, they are all inferior compared to a Taylor rule focus only on inflation targeting of domestic prices.

There is a fundamental reason explaining why a pure inflation targeting is better than the hybrid rule considered here. A potential depreciation of the nominal exchange rate (ceteris paribus), trigger upward pressures on the interest rate. That channels a reduction in consumption by intertemporal substitution. However, a nominal depreciation might be also due to a fall in domestic prices, for instance as a result of a backward in the domestic demand. Thus, the former could misguide a rise in interest rates when in fact, the appropriate policy is a reduction of the monetary instrument to avoid a further deflation.

Besides, a hybrid rule outperforms a pegged regime in terms of welfare. In this way, those countries that currently focus only on maintaining a pure fixed exchange rate regime, such as Bolivia, can reduce welfare losses by implementing a hybrid rule. The latter policy has multiple advantages: First, it allows for clear communication of the Central Bank, improving the effectiveness of the monetary policy. Second, and most important, it takes into account the "fear to float" typical of those countries that currently target only an exchange rate peg.

According to the impossible trilemma, the Central Bank can commit to an independent monetary policy, i.e. set interest rates, and manage the exchange rate peg if and only if there are interventions in the capital markets to prevent investors from taking advantage of arbitrage opportunities. Thus, if the monetary authorities implement a hybrid simple rule, they should take into account the relevance of capital controls for the success of its policy.

[28]Monacelli (2004) [39] computes this value to reproduce a proportional change in the volatility of the nominal exchange rate in line with the data observed during the decade of the 60s when industrialized countries maintained a fixed /managed exchange rate regime.

Chapter 5

Commodity price shocks in a small open economy

The analysis of the Bolivian economy would not be complete without taking into account the commodity sector, whose revenues finance a distortionary subsidy on consumption.

According to the United Nations Conference of Trade and Development [45], Bolivia is considered a commodity-dependent country, whose trade of raw materials accounts for 32 percent of exports[1] and only 1.4 percent of imports[2].

5.1 The model

The main features of the model extension are: The home country is the only producer of commodities; the representative agent consumes a bundle of domestic (commodities and non-commodities goods) and foreign goods; domestic households are the owners of firms and contribute with labor to the production of home goods. The production function of commodities displays decreasing returns to scale, and requires only labor as input. This specification implies that profits coming from the aforementioned sector are higher than zero. In contrast, the production function of non-commodity goods employs labor with constant returns to scale.

In the case of Bolivia, the government is the owner of most of the natural resources[3], including gas, hydrocarbons, metals, and production of raw agricultural commodities[4]. Also, the government makes direct transfers to citizens, through various mechanisms such as periodic distribution of conditional transfers to the aging population, students and pregnant women, and also subsidizes the price of imported fuels and diesel. Under these particular characteristics, I will assume that profits from the commodity sector (which are higher than zero given the decreasing returns to scale) are channeled through a distortionary subsidy on consumption.

Further, the main features regarding the notation or concepts (such as the home bias in consumption) are identical to the baseline model of chapter three, any additional modification is clearly stated.

[1] World Trade Organization [45]

[2] Thus, it is perfectly reasonable to assume in the model that the only producer of commodities is the home country

[3] See Lasa (2016) [32]

[4] There are several State-Owned enterprises in Bolivia dedicated to the production of natural gas, lithium and basic alimentary products. See IMF Bolivia Article IV (2018) [10]Gysel (2016) [29]

5.1.1 Households - Intratemporal choice

As in the baseline model, the composite consumption C and price index P are respectively given by:

$$C = \left(v^{\frac{1}{\theta}} C_H^{\frac{\theta-1}{\theta}} + (1-v)^{\frac{1}{\theta}} C_F^{\frac{\theta-1}{\theta}}\right)^{\frac{\theta}{\theta-1}} \tag{5.1}$$

$$P = (v P_H^{1-\theta} + (1-v) P_F^{1-\theta})^{\frac{1}{1-\theta}} \tag{5.2}$$

where C_H and C_F are the indices of consumption of home and foreign goods, $\theta > 0$ is the elasticity of intratemporal substitution between both of them and v is the home bias in consumption. The demands for home and foreign goods are the following:

$$C_H = v\left(\frac{P_H}{P}\right)^{-\theta} C \quad ; \quad C_F = (1-v)\left(\frac{P_F}{P}\right)^{-\theta} C \tag{5.3}$$

where the home bias v is a function of the degree of openness λ and the size of the economy n, such that $(1-v) = (1-n)\lambda$. As in the baseline model, the size of the domestic economy tends to zero ($n \to 0$).

In this model, the index of consumption of home goods C_H is now given by:

$$C_H = ((1-\omega)^{\frac{1}{\iota}} C_N^{\frac{\iota-1}{\iota}} + \omega^{\frac{1}{\iota}} C_O^{\frac{\iota-1}{\iota}})^{\frac{\iota}{\iota-1}} \tag{5.4}$$

where C_N and C_O are the indexes of consumption of non-commodity and commodity goods, $\iota > 0$ denotes the intratemporal elasticity of substitution between both of them and ω is the share of commodities in the consumption bundle. Equation 5.4 assumes that country H is the only producer of commodities[5]. The consumption based price index is the following:

$$P_H = ((1-\omega) P_N^{1-\iota} + \omega P_O^{1-\iota})^{\frac{1}{1-\iota}} \tag{5.5}$$

Thus, the optimal allocation of commodities and non-commodity goods in the home economy is:

$$C_N = (1-\omega)\left(\frac{P_N}{P_H}\right)^{-\iota} C_H \quad ; \quad C_O = \omega\left(\frac{P_O}{P_H}\right)^{-\iota} C_H \tag{5.6}$$

As in the baseline model, agents of the domestic country maximize their consumption of differentiated goods. Thus the domestic demand functions of a generic good z produced in country H (non-commodity and commodity) and country F are:

$$c_N^H(z) = \frac{1}{n}\left(\frac{p(z)}{P_N}\right)^{-\sigma_t} C_N \;\; ; \;\; c_O^H(z) = \frac{1}{n}\left(\frac{p(z)}{P_O}\right)^{-\sigma_t} C_O \;\; ; \;\; c_O^F(z) = \frac{1}{1-n}\left(\frac{p(z)}{P_F}\right)^{-\sigma_t} C$$

where σ_t is the elasticity of substitution among varieties. Also, a country-specific domestic public expenditure is allocated among home commodity and non-commodity goods according to the following demands:

$$g(z) = \frac{1}{n}\left(\frac{p(z)}{P_N}\right)^{-\sigma_t} G_N \;\; ; \;\; g(z) = \frac{1}{n}\left(\frac{p(z)}{P_O}\right)^{-\sigma_t} G_O$$

[5]This assumption is reasonable, given that in the case of Bolivia, commodities represent 32% of exports, while only 1.4% of imports [45]

The government budget constraint in the home country is

$$P_t^N G_t^N + P_t^O G_t^O = P_{H,t} Tr_t$$

where fluctuations in government spending G_t^N and G_t^O are exogenous and financed by lump-sum taxes Tr_t.

Further, the representative agent's budget constraint now takes into account the profits from the sale of commodity and non-commodity goods, such that $\Pi_t = \Pi_t^O + \Pi_t^N$:

$$(1+i_{t-1})B_{t-1} + (p_{s,t} + d_t)Z_{t-1} + W_t N_t + \mathbf{\Pi_t^N} + \mathbf{\Pi_t^O} - P_t Tr_t \geq B_t + p_{z,t} Z_t + P_t C_t$$

where total consumption is a composite of foreign and home goods: $P_T C_T = P_F C_F + P_N C_N + P_O C_O$. On the other hand, total labor in the domestic economy is described by the following function:

$$N = (\phi^{\frac{1}{u}} (N^N)^{\frac{u-1}{u}} + (1-\phi)^{\frac{1}{u}} (N^O)^{\frac{u-1}{u}})^{\frac{u}{u-1}} \tag{5.7}$$

where N^N and N^O are the indexes of non-commodity and commodity labor, $u > 0$ is the intratemporal elasticity of substitution between both of them, and ϕ is the share of non-commodity labor of total labor. The wage index is written as:

$$W = (\phi (W^N)^{u-1} + (1-\phi)(W^O)^{u-1})^{\frac{1}{1-u}}$$

And the optimal labor supply in the commodity and non-commodity sector in the home country is:

$$N^N = \phi \left(\frac{W^N}{W}\right)^{-u} N \quad ; \quad N^O = (1-\phi) \left(\frac{W^O}{W}\right)^{-u} N \tag{5.8}$$

5.1.2 Firms

- **Non-Commodity firms**

The representative non-commodity firm use only labor as an input. Its production function displays constant returns to scale:

$$Y_t^N(z) = A_t N_t^N(z)$$

where z is a generic differentiated good. After minimizing the cost subject to the production function, the loglinearized real marginal cost in terms of domestic prices p_H is the following:

$$mc_t^N = w_t^N - p_H - a_t$$

As in the baseline model, productivity shocks follows an auto-regressive process $\epsilon_t \sim \ iid \ \ N(0, \sigma_a)$:

$$a_t = \rho_a a_{t-1} + \epsilon_{a,t} \tag{5.9}$$

And, as described in section 3.2, the Calvo price setting mechanism yields the Phillips curve for the non-commodity producing firms. Here, I also assume a time varying markup:

$$\pi_{H,t}^N = \beta \pi_{H,t+1}^N + \zeta \hat{mc}_t^r + \zeta \mu_t \tag{5.10}$$

with $\zeta = \frac{(1-\alpha\beta)(1-\alpha)}{\alpha}$ and μ_t capturing deviations of the efficient output from the natural output, such that $\epsilon_{\mu,t} \sim \ iid \ \ N(0, \sigma_\mu)$:

$$\mu_t = \rho_\mu \mu_{t-1} + \epsilon_{\mu,t}$$

- **Commodity firms**

Domestic commodity firms use only labor, and minimize their costs subject to the production function:

$$Y_t^O(z) = A_t^O N_{O,t}^k(z)$$

where the parameter $k < 1$ implies decreasing returns to scale. Thus, under this specification, profits from the commodity sales are[6]:

$$\Pi_t^O = (1-k)P_{O,t}(z)Y_{O,t}(z) \tag{5.11}$$

Following the same steps as in section 3.2, the loglinearized real marginal cost in terms of domestic prices p_H is the following:

$$mc_t^o = kw_t^O - p_H - a_{o,t}$$

where $a_{o,t}$ is aggregate productivity in the commodity sector and follows an auto-regressive process AR(1) with Gaussian shocks $\epsilon_{ao,t} \sim \ iid \ \ N(0, \sigma_{ao})$:

$$a_{ot} = \rho_{ao} a_{o,t-1} + \epsilon_{ao,t} \tag{5.12}$$

Commodity firms in the small open economy take prices as given. Those follow a stochastic process $\epsilon_{o,t} \sim \ iid \ \ N(0, \sigma_o)$:

$$p_{o,t} = \rho_o p_{o,t-1} + \epsilon_{o,t} \tag{5.13}$$

5.1.3 Intertemporal choice

The production function of the representative commodity firm displays decreasing returns to scale. Its profits finance a distortionary subsidy on consumption τ_t, such that the following equality holds[7]:

$$\Pi_t^O = (1-k)P_{O,t}(z)Y_{O,t}(z) = \tau_t P_t C_t \tag{5.14}$$

Hence, the representative agent's budget constraint now is written as:

$$(1+i_{t-1})B_{t-1} + (p_{s,t} + d_t)Z_{t-1} + W_t N_t + \mathbf{\Pi_t} - P_t Tr_t \geq B_t + p_{z,t} Z_t + (1-\tau_t)P_t C_t$$

Also, with the subsidy in place, the intertemporal optimization problem yields as:

$$\beta(1+i_t)E_t\left(\left(\frac{C_{t+1}}{C_t}\right)^{-\rho} \frac{P_t(1-\tau_t)}{P_{t+1}(1-\tau_{t+1})}\right) = 1 \tag{5.15}$$

$$\frac{N_t^\eta}{C_t^{-\rho}} = \frac{W_t}{P_t(1-\tau_t)} \tag{5.16}$$

The loglinearized equations are given by:

$$c_t = E_{t+1}(c_{t+1}) - \frac{1}{\rho}(i_t - E_t\pi_{t+1} - \gamma + v_{t+1} - v_t) \tag{5.17}$$

$$\eta n_t + \rho c_t = w_t - p_t + v_t \tag{5.18}$$

Where $v_t = -ln(1-\tau_t)$, $\gamma = -ln(\beta)$, $i_t = ln(1+i_t)$ and $\pi_{t+1} = p_{t+1} - p_t$ is the home consumer price index inflation.

[6] Derivations in appendix F

[7] Alternatively, it can be the case that the profits from the oil sector are collected by the government and finance a distortionary subsidy on consumption

5.1.4 Interaction between the small open economy and the rest of the world

- **Terms of trade and inflation**

As the representative consumer in the domestic country consumes home and foreign goods, the loglinearized price indexes imply:

$$\pi_t = (1-\lambda)\pi_{H,t} + \lambda\pi_{F,t} \tag{5.19}$$

$$\pi_{H,t} = (1-\omega)\pi_{N,t} + \omega\pi_{O,t} \tag{5.20}$$

Plugging 5.19 in 5.20, the overall CPI inflation is:

$$\pi_t = \omega\pi_{Ot} + (1-\omega)\pi_{N,t} + \lambda\Delta tot_t \tag{5.21}$$

where $\Delta tot_t = \pi_t^F - \pi_t^H$. Therefore, a shock in commodity prices triggers an increase in the domestic inflation. There is also an improvement in the terms of trade, which is associated with an overall appreciation of the real exchange rate.

- **Terms of trade and the exchange rate**

As in the baseline model, the relationship between the nominal exchange rate s_t and the terms of trade is the following:

$$tot_t = s_t - p_t^H + p_t^* \tag{5.22}$$

Thus, a nominal depreciation means that home products are relatively cheaper than their foreign counterparts. That in turn determines that per each unit of domestic goods, the home country can purchase a lower proportion of foreign ones. Rewriting equation 5.22 in terms of variations: $\Delta tot_t = \Delta s_t - \pi_t^H + \pi_t^*$. Finally, the relationship between the exchange rate and the terms of trade is identical to the baseline model, as the definition of terms of trade remains unchanged:

$$q_t = (1-\lambda)tot_t \tag{5.23}$$

where λ is the degree of openness. Under the assumption that country H is the only producer of commodities, an exogenous increase on its prices triggers a real exchange rate appreciation. That is because Home prices are a composite of commodity and non-comomodity goods.

- **International risk sharing**

Since the international risk sharing is derived from the home and foreign Euler equations, as in the baseline model based on Gali and Monacelli (2005) [27], the loglinearized risk sharing condition is written as:

$$c_t = c_t^* + \frac{(1-\lambda)}{\rho}tot_t + \frac{1}{\rho}v_t \tag{5.24}$$

where ρ is the coefficient of risk aversion, λ is the degree of openness and v_t is the subsidy on consumption financed by the profits from the commodity sector.

5.1.5 Equilibrium

For each generic good z produced in the domestic non-commodity sector, the equilibrium implies that total production equalizes home and foreign consumption. Therefore, the following condition must be satisfied:

$$y^N(h) = n(c_N^H(z) + g(z)) + (1-n)({c_N^*}^H(z))$$

replacing subsequently the equations derived in section 5.1.1:

$$y(h) = n\frac{1}{n}\left(\frac{p(h)}{P_N}\right)^{-\sigma_t}(C_N + G^N) + (1-n)\frac{1}{n}\left(\frac{p(h)}{P_N}\right)^{-\sigma_t} C_N^*$$

$$y(h) = \left(\frac{p(h)}{P_N}\right)^{-\sigma_t}\left((1-\omega)\left(\frac{P_N}{P_H}\right)^{-\iota} C_H + G^N\right) + (1-n)\frac{1}{n}\left(\frac{p(h)}{P_N}\right)^{-\sigma_t}(1-\omega^*)\left(\frac{P_N}{QP_H}\right)^{-\iota} C_H^*$$

$$\begin{aligned} y(h) = {} & \left(\frac{p(h)}{P_N}\right)^{-\sigma_t}\left((1-\omega)\left(\frac{P_N}{P_H}\right)^{-\iota} v\left(\frac{P_H}{P}\right)^{-\theta} C + G^N\right) \\ & + (1-n)\frac{1}{n}\left(\frac{p(h)}{P_N}\right)^{-\sigma_t}(1-\omega^*)\left(\frac{P_N}{QP_H}\right)^{-\iota} v^*\left(\frac{P_H}{QP}\right)^{-\theta} C \end{aligned}$$

Recalling the definition of home bias $(1-v) = (1-n)\lambda$; $v^* = n\lambda$, simplifying and finally taking the limit when $n \to 0$, holds:

$$\begin{aligned} y^N(h) = {} & \left(\frac{p(h)}{P_N}\right)^{-\sigma_t}\left(\left(\frac{P_N}{P_H}\right)^{-\iota}\left(\frac{P_H}{P}\right)^{-\theta}((1-\omega)(1-\lambda)C) + G^N\right) \\ & + \left(\frac{p(h)}{P_N}\right)^{-\sigma_t}\left(\frac{P_N}{QP_H}\right)^{-\iota}\left(\frac{P_H}{QP}\right)^{-\theta}((1-\omega^*)\lambda C^*) \end{aligned}$$

Assuming symmetric preferences, i.e. $\omega = \omega^*$, yields the market clearing condition in the home non-commodity sector:

$$y^N(h) = \left(\frac{p(h)}{P_N}\right)^{-\sigma_t}\left(\left(\frac{P_N}{P_H}\right)^{-\iota}\left(\frac{P_H}{P}\right)^{-\theta}(1-\omega)\left((1-\lambda)C + Q^{\theta+\iota}\lambda C^*\right) + G^N\right)$$

Similarly, for each generic good h produced in the commodity sector, the equilibrium implies:

$$y^O(h) = \left(\frac{p(h)}{P_O}\right)^{-\sigma_t}\left(\left(\frac{P_O}{P_H}\right)^{-\iota}\left(\frac{P_H}{P}\right)^{-\theta}(1-\omega)\left((1-\lambda)C + Q^{\theta+\iota}\lambda C^*\right) + G^O\right)$$

5.1.6 Monetary Policy

For this particular experiment, I will consider a nominal exchange rate peg that simply implies $\Delta s_t = 0$

5.2 Simulation

The model is estimated using a Bayessian approach for those parameters for which there is only a prior belief about their value:

$$\mathcal{V} = [k, \iota, u, \omega, \rho_o, \sigma_o, \rho_{ao}, \sigma_{ao}]$$

where the parameter k represents the share of labor in the commodity production and displays decreasing returns to scale. ι is the elasticity of substitution between commodity and non-commodity goods, u is the elasticity of substitution between commodity and non-commodity labor and ω is the share of commodities in the consumption bundle. ρ_o, σ_o, ρ_{ao}, σ_{ao} are the coefficients corresponding to the auoregressive process with Gaussian shocks of the commodity prices and aggregate technology in that sector.

5.2.1 Data and Parametrization

In this case the vector of observable variables is:

$$Y_T = [y_n, y_o, c_t, q_t, \pi_t]$$

As in the baseline model the sources of the quarterly data are the Central Bank of Bolivia and the National Institute of Statistics. In particular, the commodity output y_o corresponds to agricultural and livestock products, hunting, fishing, oil, natural gas non-metallic minerals, and metallic minerals. The remaining economic activities corresponds to the non-commodity output y_n. Concerning the data, the deseasonalization and detrending are identical to the baseline model. The following table describes the parameters calibrated:

Table 5.1: **Parametrization**

Description	Par	Value	Source
Subjective discount factor	β	0.98	Zeballos (2018)
Intertemporal elasticity of substitution	ρ	1	Zeballos (2018)
Inverse of labor supply elasticity	η	0.5	Cerezo (2011)
Degree of openness	λ	0.32	Zeballos (2018)
Intratemporal elasticity of substitution	θ	0.8	Amado (2014)
Calvo parameter	α	0.39	Cerezo (2011)
Share of non-oil labor	ϕ	0.8	National Accounts
Government spending persistence	ρ_g	0.12	Salas (2018)
S.D. government spending	σ_g	0.0056	Salas (2018)
Productivity persistence non-commodity sector	ρ_a	0.2659	Baseline estimation
S.D. prod. shock non-commodity sector	σ_a	0.0140	Baseline estimation
Cost-push shock persistence	ρ_u	0.3218	Baseline estimation
S.D. of cost-push shock	σ_u	0.0074	Baseline estimation
Ext. demand persist.	ρ_{c^*}	0.7941	Baseline estimation
S.D. ext. demand shock	σ_{c^*}	0.0286	Baseline estimation
Government spending persistence	ρ_g	0.12	Salas (2018)
S.D. government spending	σ_g	0.0056	Salas (2018)

The only additional calibrated parameter with respect to the baseline model is the share of non-commodity labor ϕ, which is directly computed from the national accounts considering the average along 2009 and 2019.

5.2.2 Priors and Posteriors distributions

Table 5.2 reports the main results of the Bayesian estimation. Regarding the selection of priors, the share of labor in the commodity production k is set at 0.3. This specification is based on the analysis of Ferrero et al (2017) [24] who estimates this parameter in particular for the oil sector. The elasticity of substitution between commodity and non-commodity goods takes into account the research of Medina et al (2007) [37] that considers the elasticity of substitution between oil and non-oil goods in the case of Chile.

There is no prior reference about the elasticity of substitution between commodity and non-commodity labor. Thereby, to set a prior belief, I consider that the share of labor in the production of raw materials has been maintained close to 20% along the last decade with minimal variations over time. The prior belief about the share of commodities in consumption is set at 0.2 based upon the evaluation of the US Department of Agriculture [25]. The prior related to the standard deviation of the commodity price shock is based on the analysis of Tenreyro (2019) [44] who performs an empirical analysis for the case of Argentina.

Finally, the shape of the prior distributions are: Normal for elasticities, Beta for shares and persistence parameters and Inverse Gamma for standard deviations of shocks.

Table 5.2: Bayesian Estimation of Parameters

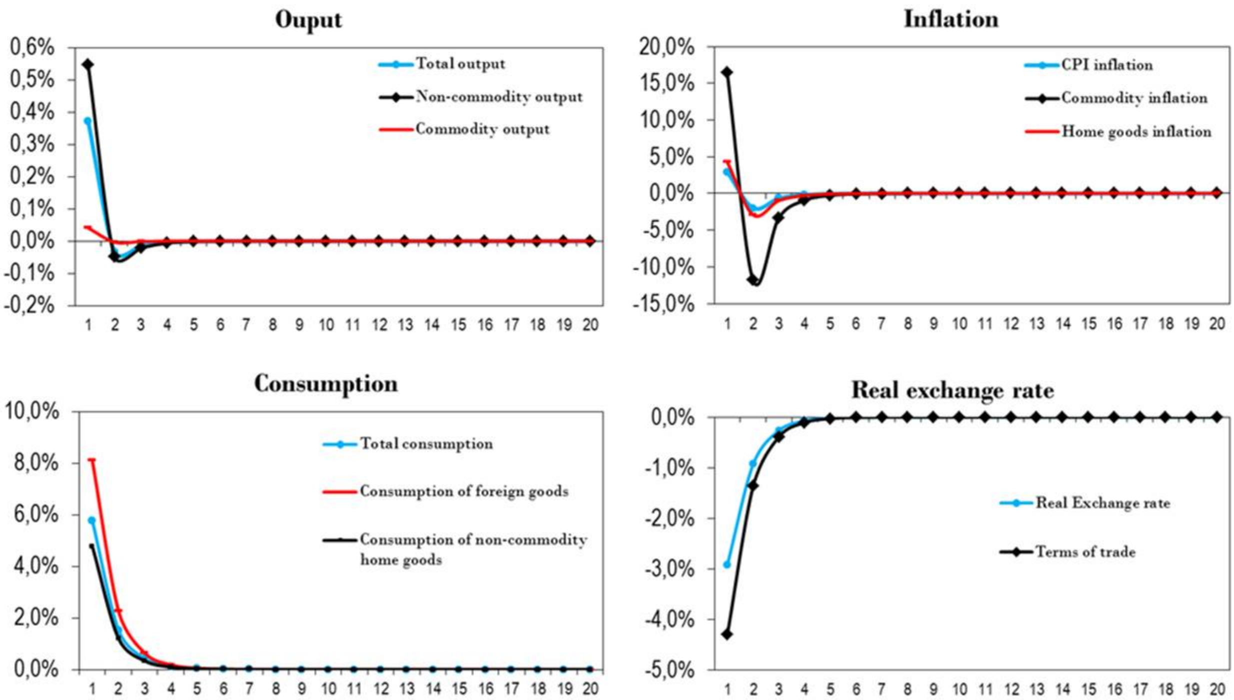
		Prior Dist.		Posterior Dist.		
Description	Par	Density	Mean	Mean	90%interval	
Labor share commodity prod.	k	Beta	0.3	0.2979	0.1329	0.4562
Elasticity subst. bet. commodity and non-commodity goods	ι	Normal	0.3	0.0173	0.0078	0.0265
Elasticity subst. bet. commodity and non-commodity labor	u	Normal	0.3	0.0131	0.0045	0.0209
Commodity share in consumption	ω	Beta	0.2	0.3466	0.1382	0.5485
Commodity price persistence	ρ_o	Beta	0.5	0.2817	0.1581	0.4034
Persistence in commodity sector productivity	ρ_{ao}	Beta	0.5	0.3511	0.1964	0.4980
SD commodity price shock	σ_o	Inv.Gam	0.15	0.1641	0.1322	0.1949
SD commodity sector productivity shock	σ_{ao}	Inv.Gam	0.2	0.1978	0.1585	0.2356

5.2.3 Results and Welfare analysis

A commodity price shock affects the economy in two different ways. On the one hand, it unmistakably increases the prices of home goods. And on the other, it affects total consumption, as it finances a distortionary subsidy on this variable. Therefore, as a result of the first channel of transmission, there are upward pressures on domestic inflation, and thus a terms of trade shock. That improves the competitiveness of the home country as foreign goods are relatively cheaper than domestic ones. Second, a rise in commodity prices leads to higher profits in the commodity sector which leads to a rise in its production. However the increment in production is limited, given the decreasing returns to scale

On the other hand, a shock on the subsidy in consumption leads also to multiple effects. First, there is an expansion in output (the effect is similar to a rise in the demand for total goods). Consequently there are inflationary pressures coming from those firms able to reset their prices. Second, the relative improvement in competitiveness generates upward pressures on the consumption of foreign goods. Although, demand for domestic products is not far behind given its relative share on the consumption bundle.

Figure 5.1: Impulse responses to commodity price shocks



Welfare analysis: Here, I follow exactly the same process of evaluation as in section 4.4, i.e. I compute the second order approximation of the utility function. Besides an exchange rate peg, I consider a Taylor rule of the form: $i_t = \rho i_{t-1} + (1-\rho) * (\psi_\pi \pi + \psi_y y)$, where calibration of the parameters follows the Chilean analysis of Medina et al(2007) [37]: $\rho = 0.7$, $\psi_\pi = 1.5$ and $\psi_y = 0.5$. The welfare evaluation shows that the most preferred monetary policy is a Taylor rule rather than a PEG in the case of every shock analyzed separately and all of them evaluated jointly, although, the welfare differences are tiny little, as it is common in the business cycles literature. Appendix F shows this results in detail.

Chapter 6

Conclusions

The purpose of this work was twofold. First, assess the dynamic effects of efficient and inefficient exogenous perturbations in a small open economy, by comparing their repercussions under alternative monetary policies. And second, provide a prescription of a simple monetary policy rule associated with the minimal welfare losses in the case of Bolivia.

To reach that goal, I built a model for a small open economy with imperfect competition and staggered prices. I took into account the presence of external demand, government spending, productivity, and cost-push shocks to finally derive a canonical representation of the system. To simulate it, I calibrated and estimated the parameters based on quarterly data from Bolivia for the periods between 2009 to 2019. Then, I extended the model by taking into account a distortionary subsidy on consumption financed by the profits of the commodity sector.

Concerning the first aim of this work, two utmost results were presented. First, under a Taylor rule there are lower deviations of the relevant output gap and inflation. And second, there is a terms of trade smoothness under a pegged exchange rate regime, in line with Gali and Monacelli (2005) [27].

In particular, each exogenous perturbation elicits a different set of dynamics. First, a government spending shock leads to a contemporaneous peak fiscal multiplier of 0.875 under a pegged regime, and 0.75 under a Taylor rule. However, there are downward pressures on private consumption given the negative wealth effect.

Second, cost-push shocks trigger a deeper slowdown in output under a Taylor rule given the emerging dilemma between inflation and output gap stabilization. Also, there are downward pressures in the real exchange rate, which are lower in the context of a pegged regime, given the terms of trade smoothness characteristic of this policy. Third, a technological shock expands the efficient output, as there is a rise in the capacity of production in the economy.

And fourth, an increase of 2.8 percent in the external demand generates a contemporaneous rise in output, which is almost 1 percent higher under a pegged regime. There are also greater deviations of the relevant output gap and inflation under that policy.

Thus, an exchange rate peg triggers in general greater expansions in output in comparison to a Taylor rule. However, it exerts upward pressures on inflation and deviations of output from its efficient level, which generates negative welfare implications.

Concerning the prescription of a simple monetary policy rule associated with the minimum welfare losses for Bolivia, the results suggest that when the full set of shocks occurs simultaneously, a Taylor rule is the preferred one. Yet, by focusing only on inefficient exogenous perturbations, the ranking of monetary policy aligns in the first place, an exchange rate peg. This scenario shows the typical ineffectiveness of inflation targeting under this case and the potential success of alternative simple monetary rules.

Regarding the extension of the model, I considered the dynamic effects of a shock in the commodity prices. In this context, profits coming from firms producing commodities (whose production function displays decreasing returns to scale) finance a distortionary subsidy on consumption. This exogenous perturbation show an expansion in consumption for foreign goods, given downward pressures on the real exchange rate.

The natural refinement of this work would be the extension of the model considering incomplete markets, the analysis of financial frictions, and an informal sector. Also, it would be useful to contrast the behavior of the actual data with the results provided by the model, and the computation of the output gap through sophisticated techniques such as the Kalman Filter. Finally, it is possible to model a number of different scenarios given that DSGE models are the laboratory of experimentation in macroeconomics.

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Appendix A

Derivations

A.1 Demand of home and foreign goods

The home country maximizes total consumption 3.2 by choosing both home and foreign goods:

$$\begin{aligned} \underset{C_H, C_F}{\text{maximize}} \quad & C(C_H, C_F) = (v^{\frac{1}{\theta}} C_H^{\frac{\theta-1}{\theta}} + (1-v)^{\frac{1}{\theta}} C_F^{\frac{\theta-1}{\theta}})^{\frac{\theta}{\theta-1}} \\ \text{subject to} \quad & PC \geq P_H C_H + P_F C_F \end{aligned}$$

Where the nominal consumption of home and foreign goods exhaust the nominal value of total consumption. Therefore, setting the Lagrange and deriving the first- order conditions, yields:

$$\mathcal{L}(C_H, C_F, \lambda) = (v^{\frac{1}{\theta}} C_H^{\frac{\theta-1}{\theta}} + (1-v)^{\frac{1}{\theta}} C_F^{\frac{\theta-1}{\theta}})^{\frac{\theta}{\theta-1}} + \lambda(PC - P_H C_H - P_F C_F)$$

$$\frac{\partial \mathcal{L}}{\partial C_H} = C^{\frac{1}{\theta}} v^{\frac{1}{\theta}} C_H^{-\frac{1}{\theta}} - \lambda P_H = 0 \quad \rightarrow \quad C_H = (\lambda P_H)^{-\theta} v C$$

$$\frac{\partial \mathcal{L}}{\partial C_F} = C^{\frac{1}{\theta}} (1-v)^{\frac{1}{\theta}} C_F^{-\frac{1}{\theta}} - \lambda P_F = 0 \quad \rightarrow \quad C_F = (\lambda P_F)^{-\theta} (1-v) C$$

$$\Rightarrow C_H = C_F \left(\frac{P_F}{P_H}\right)^{\theta} \frac{v}{1-v} \tag{A.1}$$

Plugging A.1 in the objective function and rearranging terms using the home price index 3.10, the optimal allocation for home and foreign goods is:

$$C = (v^{\frac{1}{\theta}} (C_F \left(\frac{P_F}{P_H}\right)^{\theta} \frac{v}{1-v})^{\frac{\theta-1}{\theta}} + (1-v)^{\frac{1}{\theta}} C_F^{\frac{\theta-1}{\theta}})^{\frac{\theta}{\theta-1}}$$

$$C^{\frac{\theta-1}{\theta}} = v(1-v)^{\frac{1-\theta}{\theta}} C_F^{\frac{\theta-1}{\theta}} P_H^{1-\theta} P_F^{\theta-1} + (1-v)^{\frac{1}{\theta}} C_F^{\frac{\theta-1}{\theta}} \mathbf{P_F^{\theta-1} P_F^{1-\theta}}$$

$$C^{\frac{\theta-1}{\theta}} = C_F^{\frac{\theta-1}{\theta}} P_F^{\theta-1} (1-v)^{\frac{1-\theta}{\theta}} \mathbf{P^{1-\theta}}$$

$$C_H = v \left(\frac{P_H}{P}\right)^{-\theta} C \tag{A.2}$$

$$C_F = (1-v) \left(\frac{P_F}{P}\right)^{-\theta} C \tag{A.3}$$

Similarly, foreign agents choose the demand of home and foreign goods which can be expressed in terms of domestic prices by applying the law of one price 3.17 and the definition of the real exchange rate 3.16:

$$C_H^* = v^* \left(\frac{P_H^*}{P^*}\right)^{-\theta} C^* \quad \rightarrow \quad C_H^* = v^* \left(\frac{P_H}{PQ}\right)^{-\theta} C^* \tag{A.4}$$

$$C_F^* = (1 - v^*) \left(\frac{P_F^*}{P^*}\right)^{-\theta} C^* \quad \rightarrow \quad C_F^* = (1 - v^*) \left(\frac{P_F}{PQ}\right)^{-\theta} C^* \tag{A.5}$$

A.2 Demand among the differentiated home and foreign goods

At this stage, agents of the domestic country maximize their consumption of the different varieties of domestically produced goods, thus:

$$\begin{aligned} \underset{c(z)}{\text{maximize}} \quad & C_H(c(h)) = \left(\left(\frac{1}{n}\right)^{\frac{1}{\sigma}} \int_0^n c(h)^{\frac{\sigma-1}{\sigma}} dh\right)^{\frac{\sigma}{\sigma-1}} \\ \text{subject to} \quad & P_H C_H \geq \int_0^n p(h)c(h)dh \end{aligned}$$

In the same way, the Lagrange and the first-order condition with respect to domestic consumption of the differentiated home good is:

$$\frac{\sigma}{\sigma-1}\left(\left(\frac{1}{n}\right)^{\frac{1}{\sigma}} \int_0^n c(h)^{\frac{\sigma-1}{\sigma}} dh\right)^{\frac{1}{\sigma-1}} \frac{\sigma-1}{\sigma}\left(\frac{1}{n}\right)^{\frac{1}{\sigma}} (c(h))^{-\frac{1}{\sigma}} - \lambda_t p(h) = 0$$

$$\left(\frac{1}{n}\right)^{\frac{1}{\sigma}} C_H^{\frac{1}{\sigma}} (c(h))^{-\frac{1}{\sigma}} = \lambda_t p(h)$$

$$c(h) = \frac{1}{n}(\lambda_t p(h))^{-\sigma} C_H \tag{A.6}$$

Plugging A.6 in the budget constraint and using the definition of the price index of domestic goods 3.12, the representation of the Lagrangian operator is λ_t:

$$\int_0^n p(h)(\frac{1}{n}(\lambda_t p(h))^{-\sigma} C_H)dh = P_H C_H$$

$$\lambda^{-\sigma} P^{1-\sigma} = P_H \quad \rightarrow \quad \lambda_t = \frac{1}{P_H} \tag{A.7}$$

Thus, the domestic demand for home and foreign differentiated goods is the following:

$$c(h) = \frac{1}{n}\left(\frac{p(h)}{P_H}\right)^{-\sigma} C_H \tag{A.8}$$

$$c(f) = \frac{1}{1-n}\left(\frac{p(f)}{P_F}\right)^{-\sigma} C_F \tag{A.9}$$

An analogous process stands for the foreign demand for goods produced in country H and F, which can be expressed in terms of domestic prices by applying the law of one price 3.17

$$c^*(h) = \frac{1}{n}\left(\frac{p^*(h)}{P_H^*}\right)^{-\sigma} C_H^* \;\rightarrow\; c^*(h) = \frac{1}{n}\left(\frac{p(h)}{P_H}\right)^{-\sigma} C_H^* \tag{A.10}$$

$$c^*(f) = \frac{1}{1-n}\left(\frac{p^*(f)}{P_F^*}\right)^{-\sigma} C_F^* \;\rightarrow\; c^*(f) = \frac{1}{1-n}\left(\frac{p(f)}{P_F}\right)^{-\sigma} C_F^* \tag{A.11}$$

According to De Paoli (2009), the government of the home country has preferences for differentiated domestic goods, therefore its maximization problem yields as:

$$\begin{aligned} \underset{g(z)}{\text{maximize}} \quad & G(g(h)) = \left(\left(\frac{1}{n}\right)^{\frac{1}{\sigma}} \int_0^n g(h)^{\frac{\sigma-1}{\sigma}} dh\right)^{\frac{\sigma}{\sigma-1}} \\ \text{subject to} \quad & P_H G \geq \int_0^n p(h) g(h) dh \end{aligned} \tag{A.12}$$

Thus, the public domestic demand is the following:

$$g(h) = \frac{1}{n}\left(\frac{p(h)}{P_H}\right)^{-\sigma} G \tag{A.13}$$

Similarly, the foreign government has preferences only for differentiated foreign goods:

$$g^*(f) = \frac{1}{1-n}\left(\frac{p(f)}{P_F}\right)^{-\sigma} G^* \tag{A.14}$$

A.3 Total demand for differentiated home and foreign goods

The total demand for a generic good h produced in country H is given by the aggregation of its domestic and foreign demands:

$$y(h) = n(c(h) + g(h)) + (1-n)c^*(h) \tag{A.15}$$

Replacing the corresponding demands derived previously:

$$y(h) = \frac{n}{n}\left(\frac{p(h)}{P_H}\right)^{-\sigma} (C_H + G) + \frac{(1-n)}{n}\left(\frac{p(h)}{P_H}\right)^{-\sigma} C_H^*$$

$$y(h) = \left(\frac{p(h)}{P_H}\right)^{-\sigma} (C_H + \frac{(1-n)}{n} C_H^* + G)$$

$$y(h) = \left(\frac{p(h)}{P_H}\right)^{-\sigma} \left(\left(\left(\frac{P_H}{P}\right)^{-\theta} vC + \frac{(1-n)}{n}\left(\frac{P_H}{PQ}\right)^{-\theta} v^* C^*\right) + G\right)$$

$$y(h) = \left(\frac{p(h)}{P_H}\right)^{-\sigma} \left(\left(\frac{P_H}{P}\right)^{-\theta} \left(vC + \frac{(1-n)}{n} v^* C^* \left(\frac{1}{Q}\right)^{-\theta}\right) + G\right)$$

Applying the definition of home bias described earlier (v, v^*), and then taking into account the limit of the home economy size to zero $(n \to 0)$ in order to represent the small open economy and the rest of the world, the total demand for the differentiated goods is the following:

$$y(h) = \left(\frac{p(h)}{P_H}\right)^{-\sigma} \left(\left(\frac{P_H}{P}\right)^{-\theta} \left((1-\lambda+n\lambda)C + \frac{(1+n)}{n} n\lambda C^* \left(\frac{1}{Q}\right)^{-\theta}\right) + G\right)$$

$$y(h) = \left(\frac{p(h)}{P_H}\right)^{-\sigma} \left(\left(\frac{P_H}{P}\right)^{-\theta} \left((1-\lambda)C + \lambda C^* \left(\frac{1}{Q}\right)^{-\theta}\right) + G\right) \tag{A.16}$$

Therefore:

$$Y_t = \left(\left(\frac{P_H}{P}\right)^{-\theta} \left((1-\lambda)C + \lambda C^* \left(\frac{1}{Q}\right)^{-\theta}\right) + G\right) \tag{A.17}$$

Following De Paoli(2009), the log linearied equation A.17 around the symmetric steady state is[1]:

$$y_t = -\theta(p_h) + (1-\lambda)c_t + \lambda c_t^* + \theta\lambda q_t + g_t \tag{A.18}$$

where $p_h = P_h/P$ and variables in lowercase denotes deviations from the steady state and capital letters variables at the steady state. Section A.3 shows the derivations of the aggregate demand only in terms of c_t, c_t^*, q_t and g_t.

Similarly, the total consumption for differentiated foreign goods is:

$$y(f) = nc(f) + (1-n)(c^*(f) + g^*(f)) \tag{A.19}$$

$$y(f) = \frac{n}{1-n}\left(\frac{p(f)}{P_F}\right)^{-\sigma} C_F + \frac{1-n}{1-n}\left(\frac{p(f)}{P_F}\right)^{-\sigma} (C_F^* + G^*)$$

$$y(f) = \left(\frac{p(f)}{P_F}\right)^{-\sigma} \left(\left(\frac{P_F}{P}\right)^{-\theta} \left(\frac{n}{1-n}(1-v)C + (1-v^*)\left(\frac{1}{Q}\right)^{-\theta} C^*\right) + G^*\right)$$

$$y(f) = \left(\frac{p(f)}{P_F}\right)^{-\sigma} \left(\left(\frac{P_F}{P}\right)^{-\theta} \left(\frac{n}{1-n}(1-n)\lambda C + (1-n\lambda)\left(\frac{1}{Q}\right)^{-\theta} C^*\right) + G^*\right)$$

$$y(f) = \left(\frac{p(f)}{P_F}\right)^{-\sigma} \left(\left(\frac{P_F}{PQ}\right)^{-\theta} C^* + G\right)$$

$$y(f) = \left(\frac{p^*(f)}{P_F^*}\right)^{-\sigma} \left(\left(\frac{P_F^*}{P^*}\right)^{-\theta} C^* + G\right) \tag{A.20}$$

Now, I am going to show an alternative representation of equation A.18, such that it is written only in terms of c_t, c_t^*, q_t and g_t.

[1] The steady state conditions considers:

- $P_t^H/P_{t-1}^H = P_t^F/P_{t-1}^F = 1$
- The price index is normalized, such that $\bar{P}_H = \bar{P}_F$
- The steady state versions of the demand equation in the domestic and foreign country are: $\bar{Y} = (1-\lambda)\bar{C} + \lambda\bar{C}^* + G$ and $\bar{Y}^* = \bar{C}^* + \bar{G}^*$

Dividing the domestic price index by $P_{H,t}$, using the definition of terms of trade and the fact that $n \to 0$ such that $(1-v) = \lambda$:

$$\frac{P_t}{P_{H,t}} = \frac{((1-\lambda)P_H^{1-\theta} + \lambda P_F^{1-\theta})^{\frac{1}{1-\theta}}}{P_{H,t}}$$

$$\frac{P_t}{P_{H,t}} = \frac{((1-\lambda)P_H^{1-\theta} + \lambda P_F^{1-\theta})^{\frac{1}{1-\theta}}}{P_{H,t}^{\frac{1-\theta}{1-\theta}}} = \left((1-\lambda) + \lambda\left(\frac{P_{F,t}}{P_{H,t}}\right)^{1-\theta}\right)^{\frac{1}{1-\theta}}$$

$$\frac{P_t}{P_{H,t}} = \left((1-\lambda) + \lambda TOT_t^{1-\theta}\right)^{\frac{1}{1-\theta}}$$

$$\frac{P_{H,t}}{P_t} = \left(\lambda(TOT_t^{1-\theta} - 1) + 1\right)^{\frac{1}{\theta-1}} \tag{A.21}$$

Following De Paoli, the normalization of the price index leads to $P_F = P_H$ at the steady state. Hence, loglinearizing equation A.21 yields:

$$\frac{p_{H,t}}{p_t} = -\lambda tot_t \tag{A.22}$$

Plugging equation in A.18 and using the relationship between terms of trade and real exchange rate described in equation ($q_t = (1-\lambda)tot_t$), the following relationship holds:

$$y_t = (1-\lambda)c_t + \lambda c_t^* + \gamma q_t + g_t \tag{A.23}$$

where $\gamma = \frac{\theta\lambda(2-\lambda)}{(1-\lambda)}$

Similarly, the aggregate foreign output when all goods are in equilibrium is:

$$y_t^* = c_t^* + g_t^* \tag{A.24}$$

A.4 Utility maximization

The representative household maximizes his utility and optimally shares risk with the rest of the world by trading securities:

$$\begin{aligned}
\text{maximize} \quad & U(C,N) = E_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{C_t^{1-\rho}}{1-\rho} - \frac{N_t^{1+\eta}}{1+\eta}\right) \\
\text{subject to} \quad & W_t N_t - P_t Tr_t + (1+i_{t-1})B_{t-1} + (p_{z,t} + d_t)Z_{t-1} \\
\geq \quad & B_t + p_{z,t}Z_t + P_t C_t
\end{aligned}$$

The intertemporal problem can be written as:

$$\mathcal{L} = E_t \sum_{t=0}^{\infty} \beta^t \left(\frac{C_t^{1-\rho}}{1-\rho} - \frac{N_t^{1+\eta}}{1+\eta}\right) + \lambda_t(W_t N_t - P_t Tr_t + \Pi_t + (1+i_{t-1})B_{t-1} + (p_{z,t} + d_t)Z_{t-1}$$

$$-B_t - p_{z,t}Z_t - P_t C_t)$$

$$\frac{\partial \mathcal{L}}{\partial C_t} = \beta^t C_t^{-\rho} - \lambda_t P_t = 0$$

$$\frac{\partial \mathcal{L}}{\partial N_t} = \beta^t N_t^\eta - \lambda_t W_t = 0$$
$$\frac{\partial \mathcal{L}}{\partial B_t} = \lambda_{t+1}(1+i_t) - \lambda_t = 0$$

The Euler Equation:

$$\beta(1+i_t)E_t\left(\left(\frac{C_{t+1}}{C_t}\right)^{-\rho}\frac{P_t}{P_{t+1}}\right) = 1 \tag{A.25}$$

Here, the stochastic discount factor $Q_{t,t+1}$, that governs the rate at which the consumer is willing to intertemporaly substitute consumption is defined by:

$$Q_{t,t+1} = \beta E_t\left(\left(\frac{C_{t+1}}{C_t}\right)^{-\rho}\frac{P_t}{P_{t+1}}\right) \tag{A.26}$$

The labor optimality condition is:

$$\frac{N_t^\eta}{C_t^{-\rho}} = \frac{W_t}{P_t} \tag{A.27}$$

A.5 Cost minimization

The representative domestic firm mimimizes costs in the following way:

$$\begin{aligned} \underset{n_t(h)}{\text{minimize}} \quad & W_t n_t(z) \\ \text{subject to} \quad & A_t n_(z) = y_t(z) \end{aligned}$$

$$\mathcal{L} = W_t n_t(z) + \lambda_t(z)(y_t(z) - A_t n(z))$$

The first order condition with respect to labor stands as:

$$\frac{\partial \mathcal{L}}{\partial n_t(z)} = w_t - \lambda_t(z)A_t = 0$$

where the multiplier on the constraint has the interpretation of marginal cost which is common across all firms and can be written as:

$$\lambda_t = \frac{W_t}{A_t} \tag{A.28}$$

A.6 Calvo Price Setting mechanism

$$\begin{aligned} \underset{\tilde{p}(h)}{\text{maximize}} \quad & \sum_{k=0}^{\infty} \alpha^k E_t(Q_{t,t+k}\left((y_{t,t+k}(h)(\tilde{p}_t(h) - \mathbf{MC^n}_{t,t+k}))\right) \\ \text{subject to} \quad & y_{t,t+k}(h) = \left(\frac{\tilde{p}_t(h)}{P_{H,t+k}}\right)^{-\sigma}\left(\left(\frac{P_{H,t+k}}{P_{t+k}}\right)^{-\theta}\left((1-\lambda)C_{t+k} + \lambda C^*_{t+k}\left(\frac{1}{Q_{t+k}}\right)^{-\theta}\right) + G_{t+k}\right) \\ \Rightarrow \quad & \tilde{y}_{t+k}(h) = \left(\frac{\tilde{p}_t(h)}{P_{H,t+k}}\right)^{-\sigma} Y_{t+k} \end{aligned}$$

Note that the future stream of nominal marginal costs MC_{t+k} corresponds to the previous derivation in 3.43. Substituting the demand constraint in the objective function and reorganizing terms:

$$\rightarrow \sum_{k=0}^{\infty} \alpha^k E_t \left(Q_{t,t+k} \left(\left(\left(\frac{\tilde{p}_t(h)}{P_{H,t+k}} \right)^{-\sigma} Y_{t+k} \right) \left(\tilde{p}_t(h) - MC^n_{t+k} \right) \right) \right)$$

$$\rightarrow \sum_{k=0}^{\infty} \alpha^k E_t \left(Q_{t,t+k} \left(\left(\frac{1}{P_{H,t+k}} \right)^{-\sigma} Y_{t+k} (\tilde{p}_t(h))^{(1-\sigma)} - (\tilde{p}_t(h))^{(-\sigma)} \left(\frac{1}{P_{H,t+k}} \right)^{-\sigma} Y_{t,t} MC^n_{t+k} \right) \right)$$

Deriving the first order conditions with respect to the optimal price $\tilde{p}_t(h)$:

$$\sum_{k=0}^{\infty} \alpha^k E_t \left(Q_{t,t+k} \left((1-\sigma) \left(\frac{\tilde{p}_t(h)}{P_{H,t+k}} \right)^{-\sigma} Y_{t+k} + \sigma (\tilde{p}_t(h))^{(-1)} \left(\frac{\tilde{p}_t(h)}{P_{H,t+k}} \right)^{-\sigma} Y_{t+k} MC^n_{t+k} \right) \right) = 0$$

$$\sum_{k=0}^{\infty} \alpha^k E_t \left(Q_{t,t+k} \left((1-\sigma) y_{t+k}(h) + \sigma (\tilde{p}_t(h))^{(-1)} y_{t+k}(h) MC^n_{t+k} \right) \right) = 0$$

Multiply both sides of the equation by $-\frac{(\tilde{p}_t(h))}{(\sigma-1)}$ and factorizing:

$$\sum_{k=0}^{\infty} \alpha^k E_t \left(Q_{t,t+k} y_{t+k}(h) \left(\tilde{p}_t(h) - \frac{\sigma}{(\sigma-1)} \frac{MC^n_{t+k}}{} \right) \right) = 0$$

In the style of De Paoli (2009), I take into account markup shocks providing from a time varying elasticity of substitution between goods, which generates inefficiency given the markup fluctuations over time:

$$\mu_t = \frac{\sigma_t}{(1-\sigma_t)} \tag{A.29}$$

that follows an autoregressive process (AR(1)) with white noise: $\epsilon_{\mu,t} \sim \ iid \ \ N(0, \sigma_\mu)$:

$$\mu_t = \rho_\mu \mu_{t-1} + \epsilon_{\mu,t} \tag{A.30}$$

Replacing the stochastic discount factor: $Q_{t,t+k} = \beta^k E_t \left(\frac{C_{t+k}}{C_t} \right)^{-\sigma} \frac{P_t}{P_{t+k}}$ and simplifying some common terms:

$$\sum_{k=0}^{\infty} (\alpha\beta)^k E_t \left(\frac{(C_{t+k})^{-\sigma}}{P_{t+k}} y_{t+k}(h) \left(\tilde{p}_t(h) - \mu_t MC^n_{t+k} \right) \right) = 0$$

$$\sum_{k=0}^{\infty} (\alpha\beta)^k E_t \left((C_{t+k})^{-\sigma} \frac{\mathbf{p_{t-1}(h)}}{P_{t+k}(h)} y_{t+k}(h) \left(\frac{\tilde{p}_t(h)}{\mathbf{p_{t-1}(h)}} - \frac{P_{t+k}(h)}{P_{t+k}(h)} \frac{\mu_t MC^n_{t+k}}{\mathbf{p_{t-1}(h)}} \right) \right) = 0$$

According to Gali and Monacelli(2005), the following definitions of inflation and real marginal cost holds:

$$\pi_{H,t-1,t+k} = \frac{P_{t+k}}{P_{t-1}} \ \text{ and } \ MC_{t+k} = \frac{MC^n_{t+k}}{P_{H,t+k}}$$

$$\Rightarrow \sum_{k=0}^{\infty} (\alpha\beta)^k E_t \left((C_{t+k})^{-\sigma} \frac{\mathbf{p_{t-1}(h)}}{P_{t+k}(h)} y_{t+k}(h) \left(\frac{\tilde{p}_t(h)}{\mathbf{p_{t-1}(h)}} - \mu_t \pi_{H,t-1,t+k} MC_{t+k} \right) \right) = 0$$

Log-linearizing the infinite sum [2] around the zero-inflation steady state and taking into account the following properties:

$$mc_t = -log\frac{\sigma_t}{(\sigma_t - 1)} = -\mu_t \;\; ; \;\; MC_{t+k} - mc_{t+k} = \tilde{mc}_{t+k}$$

$$\Rightarrow \tilde{p}_t(h) - p_{t-1}(h) = (1-\alpha\beta)\sum_{k=0}^{\infty}(\alpha\beta)^k(\tilde{mc}_{t+k} + \pi_{H,t}) \tag{A.31}$$

The previous equation obeys a first order stochastic difference equation,[3] thus:

$$\tilde{p}_t(h) - p_{t-1}(h) = \alpha\beta(\tilde{p}_{t+1}(h) - p_t(h)) + (1-\alpha\beta)(\tilde{mc}_t + \pi_{H,t}) \tag{A.32}$$

Under the Calvo price setting specification, the price index evolves according to the following law of motion:

$$(P_{H,t})^{1-\sigma} = \alpha P_{H,t-1}^{1-\sigma} + (1-\alpha)(\tilde{p}_t(h))^{(1-\sigma)} \tag{A.33}$$

Its corresponding log-linearized version is given by:

$$p_{H,t} = \alpha p_{H,t-1} + (1-\alpha)\tilde{p}_{H,t}$$

$$p_{H,t} - p_{H,t-1} = (1-\alpha)(\tilde{p}_{H,t} - p_{H,t-1})$$

$$p_{H,t} - p_{H,t-1} = (1-\alpha)(\tilde{p}_{H,t} - p_{H,t-1}) \;\; \rightarrow \;\; \pi_{H,t} = (1-\alpha)(\tilde{p}_{H,t} - p_{H,t-1}) \tag{A.34}$$

Finally plugging A.34 in A.32 the New Keynessian Philips curve for the home country is:

$$\pi_{H,t} = \beta\pi_{H,t+1} + \zeta\tilde{mc}_t \tag{A.35}$$

where $\zeta = \frac{(1-\alpha\beta)(1-\alpha)}{\alpha}$.

A.7 Terms of trade and inflation

The log-linearized version of the terms of trade is the following:

$$TOT_t = \frac{P_{F,t}}{P_{H,t}} \;\; \Rightarrow \;\; tot_t = p_{F,t} - p_{H,t} \tag{A.36}$$

The log-linearized version of the domestic price index described in equation 3.10 can be expressed as:

$$p_t = vp_{t,h} + (1-v)p_{t,f} \tag{A.37}$$

Combining equations A.36 and A.37, the level of prices of the small open economy is positively correlated to the domestic prices adjusted by the terms of trade and the home bias:

$$p_t = p_{t,F} - v(tot_t)$$

$$p_t = p_{t,F} - \mathbf{p_{t,h}} - v(tot_t) + \mathbf{p_{t,h}} \rightarrow p_t = tot_t - v(tot_t) + p_{t,h}$$

$$p_t = p_{t,h} + (1-v)tot_t \tag{A.38}$$

[2] $\sum_{k=0}^{\infty} x^k = \frac{1}{1-x}$

[3] $y_t = a\sum_{k=0}^{\infty} b^k E_t x_{t+k} \Rightarrow y_t = ax_t + bE_t y_{t+1}$ $\;$ (y, x random variables *and* a, b constants)

By definition, the log-linearized version of inflation is the following:

$$\pi_t = p_t - p_{t-1} \;\; and \;\; \pi_{t,H} = p_{t,H} - p_{t-1,H}$$

plugging it in equation A.38 and using A.36, the relationship between home inflation and terms of trade changes:

$$\pi_t + p_{t-1} = \pi_{t,H} + p_{t-1,H} + (1-v)(p_{t,F} - p_{t,H})$$
$$\pi_t = \pi_{t,H} + p_{t-1,H} + (1-v)p_{t,F} - (1-v)p_{t,H} - \mathbf{p_{t-1}}$$
$$\pi_t = \pi_{t,H} + p_{t-1,H} + (1-v)p_{t,F} - (1-v)p_{t,H} - \mathbf{v p_{t-1,H}} - \mathbf{(1-v) p_{t-1,F}}$$
$$\pi_t = \pi_{t,H} + (1-v)p_{t-1,H} - (1-v)p_{t-1,F} + (1-v)p_{t,F} - (1-v)p_{t,H}$$
$$\pi_t = \pi_{t,H} - (1-v)(p_{t-1,F} - p_{t-1,H}) + (1-v)(p_{t,F} - p_{t,H})$$
$$\pi_t = \pi_{t,H} + (1-v)\Delta tot_t \tag{A.39}$$

A.8 Terms of trade and real exchange rate

Combining equations 3.54 and 3.53 holds:

$$q_t = s_t + p_t^* - p_t$$
$$q_t = tot_t + p_{H,t} - p_t \tag{A.40}$$

Finally, using equations A.40 and A.38, it is obvious the positive relationship between the real exchange rate and the terms of trade adjusted by the home bias:

$$q_t = tot_t + p_{t,H} - p_t \;\; \Rightarrow \;\; q_t = tot_t - (1-v)tot_t$$
$$q_t = v tot_t \tag{A.41}$$

A.9 Uncovered interest Parity

Under complete markets, households can invest in home and foreign bonds. So, the budget constraint can be written as:

$$W_t N_t - P_t Tr_t + \Pi_t + (1+i_{t-1})B_{t-1} + \mathbf{(1+i_{t-1}^*) S_{t-1} B_{t-1}^*} + (p_{z,t} + d_t)Z_{t-1}$$
$$= B_t + p_{z,t} Z_t + P_t C_t + \mathbf{S_t B_t^*}$$

The first order conditions with respect to home and foreign goods result in the following expressions:

$$1 = \beta(1+i_t)\left(\frac{C_{t+1}}{C_t}\right)^{-\rho} \frac{P_t}{P_{t+1}}$$
$$1 = \beta(1+i_t^*)\left(\frac{C_{t+1}}{C_t}\right)^{-\rho} \frac{P_t}{P_{t+1}} \frac{S_{t+1}}{S_t}$$

Taking the ratio between both equations:

$$1 = \frac{(1+i_t)S_t}{(1+i_t^*)S_{t+1}}$$

Simplifying and loglinearizing:

$$i_t - i_t^* = E_t(\Delta s_{t+1}) \tag{A.42}$$

A.10 Risk Sharing conditions

According to Gali and Monacelli (2005) considering complete markets, the Euler Equation corresponding to the intertemporal optimality condition in country H, yields for the representative agent in country F. Thus, expressing foreign consumption in terms of the domestic currency, and assuming that the nominal interest rate paid on the bond B_t is the same across countries (Berthold, 2012):

$$\beta R_t E_t\left(\left(\frac{C^*_{t+1}}{C^*_t}\right)^{-\rho}\frac{P^*_t S_t}{P^*_{t+1}S_{t+1}}\right)=1$$

Equalizing the expression above with its domestic counterpart and recalling the definition of the real exchange rate 3.16:

$$E_t\left(\left(\frac{C_{t+1}}{C_t}\right)^{-\rho}\frac{P_t}{P_{t+1}}\right)=E_t\left(\left(\frac{C^*_{t+1}}{C^*_t}\right)^{-\rho}\frac{P^*_t S_t}{P^*_{t+1}S_{t+1}}\right)=1$$

$$E_t\left(\left(\frac{C_{t+1}}{C_t}\right)^{-\rho}\right)=E_t\left(\left(\frac{C^*_{t+1}}{C^*_t}\right)^{-\rho}\frac{Q_t}{Q_{t+1}}\right)=1$$

After isolating C_t, the domestic consumption depends mainly on foreign consumption and the real exchange rate:

$$C_t=E_t\left(\frac{C_{t+1}}{C^*_{t+1}}Q_{t+1}^{\frac{1}{\rho}}\right)C^*_t Q_t^{\frac{1}{\rho}}$$

According to Gali and Monacelli (2005), it is possible to assume symmetric initial conditions, which implies zero net foreign asset holdings and thus same levels of future consumption in county H and F. Hence, the expression above can be written as:

$$C_t=C^*_t Q_t^{\frac{1}{\rho}}$$

And its log-linearized version:

$$c_t=c^*_t+\frac{1}{\rho}q_t \tag{A.43}$$

Combining equation A.43 with the alternative definition of the real exchange rate as a function of the terms of trade derived in A.41:

$$c_t=c^*_t+\frac{v}{\rho}tot_t \tag{A.44}$$

A.11 Equilibrium - Demand side

As derived previously, the total demand for home goods comes from the domestic (private and public) and foreign consumption:

$$y_t=(1-\lambda)c_t+\lambda c^*_t+\gamma q+g_t \tag{A.45}$$

where $\gamma = \frac{\theta\lambda(2-\lambda)}{(1-\lambda)}$. Plugging equation A.41 in A.45 one obtains the demand for home goods in terms of the terms of trade. Thus, an improvement in tot_t produces a decrease in the demand for domestic goods since foreign ones are cheaper:

$$y_t = (1-\lambda)c_t + \lambda c_t^* + v\gamma tot_t + g_t \tag{A.46}$$

The total demand for foreign-produced goods corresponds to the log-linearized version of the already derived equation A.24:

$$y_t^* = c_t^* + g_t^* \tag{A.47}$$

Inserting equation A.44, which corresponds to the definition of home consumption as a function of the terms of trade, into equation A.46 yields:

$$y_t = (1-\lambda)(c_t^* + \frac{vtot_t}{\rho}) + \lambda c_t^* + v\gamma tot_t + g_t$$

$$y_t = c_t^* + (1-\lambda)\frac{vtot_t}{\rho} + v\gamma tot_t + g_t$$

$$y_t = c_t^* + tot_t(\frac{v\gamma\rho + (1-\lambda)v}{\rho}) + g_t$$

$$y_t = c_t^* + \frac{tot_t}{\rho_v} + g_t \quad where \quad \rho_v = \frac{\rho}{v\gamma\rho + (1-\lambda)v}$$

Implementing the world demand clearing condition, the equation above is written as:

$$y_t = y_t^* - g_t^* + \frac{tot_t}{\rho_v} + g_t \tag{A.48}$$

Subsequently, isolating c_t from the aggregate demand of domestic products A.45 and plugging it in the log-linearized version of the home Euler equation 3.38 yields:

$$y_t - \lambda c_t^* - v\gamma tot_t - g_t = y_{t+1} - \lambda c_{t+1}^* - v\gamma tot_{t+1} - g_{t+1} - \frac{(1-\lambda)(i_t - \pi_{t+1} - \gamma)}{\rho}$$

$$y_t = y_{t+1} - \frac{(1-\lambda)(i_t - \pi_{t+1} - \gamma)}{\rho} - \lambda(c_{t+1}^* - c_t^*) - v\gamma(tot_{t+1} - tot_t) - \Delta g_{t+1}$$

$$y_t = y_{t+1} - \frac{(1-\lambda)(i_t - \pi_{t+1} - \gamma)}{\rho} - \lambda\Delta y_{t+1}^* + \lambda\Delta g_{t+1}^* - v\gamma\Delta tot_{t+1} - \Delta g_{t+1} \tag{A.49}$$

A.12 Supply side - The marginal cost

Recalling the log-linearized version of the production function up to a first order approximation: $n_t = y_t - a_t$, the domestic Philips curve A.35 and the definition of marginal cost defined previously:

$$\tilde{mc}_t^r = mc_t + \mu_t$$

. The log-linearized marginal cost 3.43 can be written in the following way:

$$mc_t = w_t - p_{t,H} - a_t \tag{A.50}$$

$$mc_t = (w_t - p_t) + (p_t - p_{t,H}) - a_t$$

Plugging the relationship between marginal utility and real wage derived in 3.39 and the definition of domestic prices as a function of the terms of trade A.38, the equation above holds as:

$$mc_t = (-v + \eta n_t + \rho c_t) + (1-v)tot_t - a_t$$

$$mc_t = -v + \rho c_t + \eta n_t + (1-v)tot_t - a_t \tag{A.51}$$

Taking into account the world market equilibrium A.47, the log-linearized production function and the relationship between domestic consumption and terms of trade A.44:

$$mc_t = -v + \rho(c_t^* + \frac{vtot_t}{\rho}) + \eta n_t + (1-v)tot_t - a_t$$

$$mc_t = -v + \rho(y_t^* - g_t^*) + vtot_t + \eta(y_t - a_t) + (1-v)tot_t - a_t$$

$$mc_t = -v + \rho(y_t^* - g_t^*) + tot_t + \eta y_t - a_t(\eta+1) \tag{A.52}$$

Finally, isolating tot_t from equation A.48: $tot_t = (y_t - g_t)\rho_v - (y_t^* - g_t^*)\rho_v$ and plugging it in the equation derived above, the marginal cost as a function of domestic and foreign variables is the following:

$$mc_t = -v + \rho(y_t^* - g_t^*) + tot_t + \eta y_t - a_t(\eta+1)$$

$$mc_t = -v + \rho(y_t^* - g_t^*) + (y_t - g_t)\rho_v - (y_t^* - g_t^*)\rho_v + \eta y_t - a_t(\eta+1)$$

$$mc_t = -v + c_t^*(\rho - \rho_v) + y_t(\eta + \rho_v) - g_t\rho_v - a_t(\eta+1) \tag{A.53}$$

A.13 The natural output

A.53 is written as:

$$-\mu_t^n = -v + y_t^n(\eta + \rho_v) + c_t^*(\rho - \rho_v) - g_t\rho_v - a_t(\eta+1) \tag{A.54}$$

Isolating the natural output, the following expression holds:

$$y_t^n = \Gamma_o + \Gamma_\mu \mu_t^n + \Gamma_* c_t^* + \Gamma_g g_t + \Gamma_a a_t \tag{A.55}$$

$$\Gamma_o = \frac{v}{\eta+\rho_v} \quad ; \quad \Gamma_\mu = -\frac{1}{\eta+\rho_v} \quad ; \quad \Gamma_* = -\frac{\rho-\rho_v}{\eta+\rho_v} \quad ; \quad \Gamma_g = \frac{\rho_v}{\eta+\rho_v} \quad ; \quad \Gamma_a = \frac{\eta+1}{\eta+\rho_v}$$

A.14 The Efficient output

The efficient output y_t^e prevails in the absence of real and nominal rigidities. Consequently, equation A.53 is written in the following way:

$$-\mu = -v + y_t^e(\eta + \rho_v) - g_t\rho_v + c_t^*(\rho - \rho_v) - a_t(\eta+1) \tag{A.56}$$

Isolating y_t^e, the efficient output is:

$$y_t^e = \Gamma_o + \Gamma_* c_t^* + \Gamma_g g_t + \Gamma_a a_t \tag{A.57}$$

$$\Gamma_o = \frac{-\mu+v}{\eta+\rho_v} \quad ; \quad \Gamma_* = -\frac{\rho-\rho_v}{\eta+\rho_v} \quad ; \quad \Gamma_g = \frac{\rho_v}{\eta+\rho_v} \quad ; \quad \Gamma_a = \frac{\eta+1}{\eta+\rho_v}$$

A.15 The New Keynesian Phillips curve

An alternative version of the Phillips curve derived in the previous section as a function of the marginal cost and the markup relies on the definition of the output gap $\tilde{y}_t$:

$$\tilde{y}_t = y_t - y_t^n \tag{A.58}$$

Subtracting equation A.54 from A.53, the marginal cost $\tilde{mc}_t$ as a function of the output gap with respect to the natural output is the following:

$$\begin{gathered} mc_t + \mu_t^n = (\rho - \rho_v)(y_t^* - g_t^*) + y_t(\eta + \rho_v) - g_t\rho_v - a_t(\eta + 1) \\ -yt^n(\eta + \rho_v) - (\rho - \rho_v)(y_t^* - g_t^*) + g_t\rho_v + a_t(\eta + 1) \\ \tilde{mc}_t = \tilde{y}_t(\eta + \rho_v) \end{gathered} \tag{A.59}$$

However, in order to obtain an expression for the New Keynesian curve defined previously, one should obtain the marginal cost as a function of the efficient output gap: $\hat{mc} = mc_t - mc_t^e = mc_t + \mu$. In this case, the relevant output gap is defined as the difference between the actual output and the efficient one:

$$x_t = y_t - y_t^e \tag{A.60}$$

Therefore, subtracting equation A.56 from A.53, and taking into account equation A.60:

$$\begin{gathered} \hat{mc}_t^r = (\eta + \rho_v)(y_t - y_t^e) \\ \hat{mc}_t^r = (\eta + \rho_v)(x_t) \end{gathered} \tag{A.61}$$

Now, just remain the specification of the markup $\mu_t = \mu_t^n - \mu$, which can be obtained as the difference between equation A.54 and A.56:

$$\mu_t = \mu_t^n - \mu = (\eta + \rho_v)(y_t^e - y_t^n) \tag{A.62}$$

Lastly, inserting equation A.61 and A.62 into the domestic Phillips curve already derived A.35:

$$\begin{gathered} \pi_{H,t} = \beta\pi_{H,t+1} + \zeta(\hat{mc_t}^r) + \zeta(\mu_t) \\ \pi_{H,t} = \beta\pi_{H,t+1} + \zeta(\eta + \rho_v)x_t + \zeta(\eta + \rho_v)\mu_t \\ \pi_{H,t} = \beta\pi_{H,t+1} + \kappa_v x_t + \kappa_v\mu_t \end{gathered} \tag{A.63}$$

where $\zeta = \frac{(1-\alpha\beta)(1-\alpha)}{\alpha}$ *and* $\kappa_v = \zeta(\eta + \rho_v)$.

A.16 The Dynamic IS equation

In order to derive the dynamic version of the IS equation, the definition of the real interest rate rr_t according to the Fisher rule is:

$$r_t = i_t - E_t(\pi_{t+1}) \tag{A.64}$$

Secondly, in order to derive an expression of the output gap as a function of the natural interest rate, I follow five additional steps:

- Recalling the definition of inflation as a function of terms of trade A.39, an alternative representation of equation A.49 is:

$$y_t = y_{t+1} - \frac{(1-\lambda)(i_t - \pi_{t+1,H} - (1-v)\Delta tot_{t+1} - \gamma)}{\rho} - \lambda\Delta y^*_{t+1} + \lambda\Delta g^*_{t+1} - v\gamma\Delta tot_{t+1} - \Delta g_{t+1}$$

$$y_t = y_{t+1} - \frac{(1-\lambda)(i_t - \pi_{t+1,H} - \gamma)}{\rho} - \lambda\Delta y^*_{t+1} + \lambda\Delta g^*_{t+1} - \Delta g_{t+1} + \Delta tot_{t+1}(\frac{(1-\lambda)(1-v)}{\rho} - v\gamma)$$

$$y_t = y_{t+1} - \frac{(1-\lambda)(i_t - \pi_{t+1,H} - \gamma)}{\rho} - \lambda\Delta y^*_{t+1} + \lambda\Delta g^*_{t+1} - \Delta g_{t+1} + \Delta\frac{\Delta tot_{t+1}}{\rho}((1-\lambda)(1-v) - v\gamma\rho)$$

$$y_t = y_{t+1} - \frac{(1-\lambda)(i_t - \pi_{t+1,H} - \gamma)}{\rho} - \lambda\Delta y^*_{t+1} + \lambda\Delta g^*_{t+1} - \Delta g_{t+1} - \frac{\Delta tot_{t+1}}{\rho}\omega \tag{A.65}$$

Where $\omega = v\gamma\rho - (1-\lambda)(1-v)$

- Now, isolating the terms of trade tot_t from equation A.48 ($tot_t = \rho_v(y_t - y^*_t + g^*_t - g_t)$) and plugging it in the just derived equation A.65, the domestic demand as a function of foreign output, government spending, interest rate and expected domestic inflation is written as:

$$\begin{aligned}
\rightarrow y_t = y_{t+1} - \frac{(1-\lambda)}{\rho}(i_t - \pi_{t+1,H} - \gamma) - \lambda\Delta y^*_{t+1} + \lambda\Delta g^*_{t+1} - \Delta g_{t+1} \\
-\frac{\omega\rho_v}{\rho}((y_{t+1} - y^*_{t+1} + g^*_{t+1} - g_{t+1}) - (y_t - y^*_t + g^*_t - g_t)) \\
\rightarrow y_t(1 - \frac{\omega\rho_v}{\rho}) - g_t(1 - \frac{\omega\rho_v}{\rho}) = (y_{t+1} - g_{t+1})(1 - \frac{\omega\rho_v}{\rho}) \\
-\frac{(1-\lambda)}{\rho}(i_t - \pi_{t+1,H} - \gamma) + (\Delta y^*_{t+1} - \Delta g^*_{t+1})(\frac{\omega\rho_v}{\rho} - \lambda)
\end{aligned}$$

The term $\rho(1 - \frac{\omega\rho_v}{\rho})$ can be written as $\rho_v(1-\lambda)$, thus, the previous equation holds as:

$$y_t - g_t = y_{t+1} - g_{t+1} - \frac{1}{\rho_v}(i_t - \pi_{t+1,H} - \gamma) + (\Delta y^*_{t+1} - \Delta g^*_{t+1})(\frac{\omega\rho_v - \lambda\rho}{\rho_v(1-\lambda)}) \tag{A.66}$$

$$\rightarrow y_t - g_t = y_{t+1} - g_{t+1} - \frac{1}{\rho_v}(i_t - \pi_{t+1,H} - \gamma) + \omega_\rho\Delta c^*_{t+1}$$

where $\omega_\rho = \frac{\omega\rho_v - \lambda\rho}{\rho_v(1-\lambda)}$

- Using the expression of the real interest rate A.64 in A.66, the domestic demand can be written as:

$$y_t - g_t = y_{t+1} - g_{t+1} - \frac{(r_t - \gamma)}{\rho_v} + (\Delta y^*_{t+1} - \Delta g^*_{t+1})(\frac{\omega\rho_v - \lambda\rho}{\rho_v(1-\lambda)})$$

- Knowing that the efficient output is a function of the efficient real interest rate:

$$y^e_t - g_t = y^e_{t+1} - g_{t+1} - \frac{(r^n_t - \gamma)}{\rho_v} + (\Delta y^*_{t+1} - \Delta g^*_{t+1})(\frac{\omega\rho_v - \lambda\rho}{\rho_v(1-\lambda)}) \tag{A.67}$$

- Finally, the dynamic IS equation emerges by subtracting equation A.67 from A.66

$$\rightarrow y_t - g_t - y_t^e + g_t = y_{t+1} - g_{t+1} - \frac{1}{\rho_v}(i_t - \pi_{t+1,H} - \gamma) + (\Delta y_{t+1}^* - \Delta g_{t+1}^*)(\frac{\omega\rho_v - \lambda\rho}{\rho_v(1-\lambda)})$$

$$-y_{t+1}^e + g_{t+1} + \frac{(r_t^e - \gamma)}{\rho_v} - (\Delta y_{t+1}^* - \Delta g_{t+1}^*)(\frac{\omega\rho_v - \lambda\rho}{\rho_v(1-\lambda)})$$

$$\rightarrow x_t = x_{t+1} - \frac{1}{\rho_v}(i_t - r_t^e - \pi_{t+1,H}) \tag{A.68}$$

A.17 The efficient interest rate

Equation A.66, A.57 and A.68 can be written respectively as:

$$\Delta y_{t+1} - \Delta g_{t+1} = \frac{1}{\rho_v}(i_t - \pi_{t+1,H} - \gamma) - \omega_\rho \Delta c_{t+1}^* \tag{A.69}$$

$$\Delta y_{t+1}^e = \Gamma_g \Delta g_{t+1} + \Gamma_* \Delta c_{t+1}^* + \Gamma_a \Delta a_{t+1} \tag{A.70}$$

$$\Delta x_{t+1} = \frac{1}{\rho_v}(i_t - r_t^e - \pi_{t+1,H}) \tag{A.71}$$

Isolating the efficient real interest rate from equation A.71 and using the definition of the relevant output gap $x_t = y_t - y_t^e$:

$$r_t^e = (i_t - \pi_{t+1,H}) - \rho_v(\Delta y_{t+1} - \Delta y_{t+1}^e)$$

Consequently, plugging A.69 and A.70 in the equation above, the efficient interest rate is the following:

$$\rightarrow r_t^e = (i_t - \pi_{t+1,H}) - \rho_v((\Delta g_{t+1} + \frac{1}{\rho_v}(i_t - \pi_{t+1,H} - \gamma) - \omega_\rho \Delta c_{t+1}^*$$

$$-(\Gamma_*(\Delta c_{t+1}^*) + \Gamma_g \Delta g_{t+1} + \Gamma_a \Delta a_{t+1}))$$

$$\rightarrow r_t^e = \gamma - \rho_v \Delta g_{t+1} + \rho_v \Delta c_{t+1}^*(\omega_\rho + \Gamma_*) + \rho_v \Gamma_g \Delta g_{t+1} + \rho_v \Gamma_a(\rho_a a_t - a_t)$$

Simplifying:

$$r_t^e = \gamma + \rho_v \Delta c_{t+1}^*(\omega_\rho + \Gamma_*) + \rho_v \Gamma_g \Delta g_{t+1} + \rho_v \Gamma_a(\rho_a a_t - a_t) \tag{A.72}$$

Appendix B

Deseasonalized Data

Source: Institute of National Statistics - Bolivia and author's computation.

Figure B.1: Original and seasonal adjustment ARIMA X-13: Gross Domestic Product - Constant USD - Quarterly data

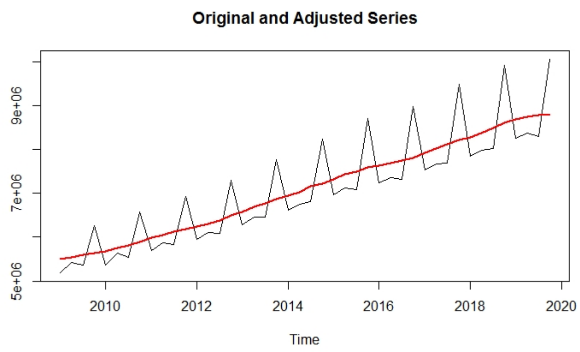


Figure B.2: Original and seasonal adjustment ARIMA X-13: Private Consumtpion - Constant USD - Quarterly data

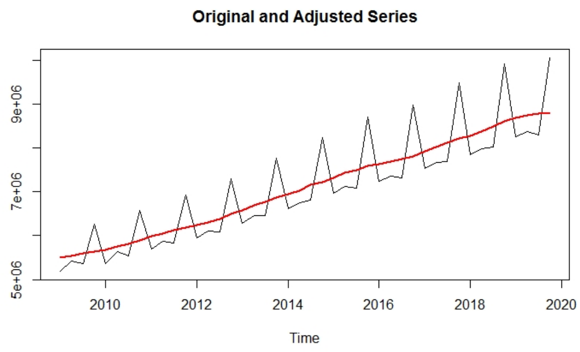


Figure B.3: Original and seasonal adjustment ARIMA X-13: Governement Spending - Constant USD - Quarterly data

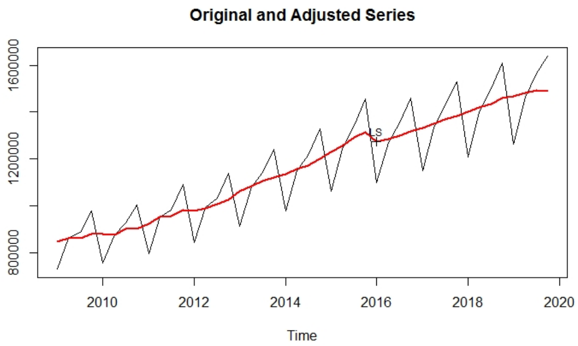


Figure B.4: Original and seasonal adjustment ARIMA X-13: Exports - Constant USD - Quarterly data

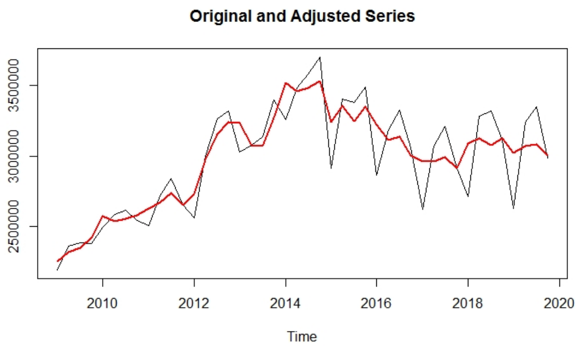
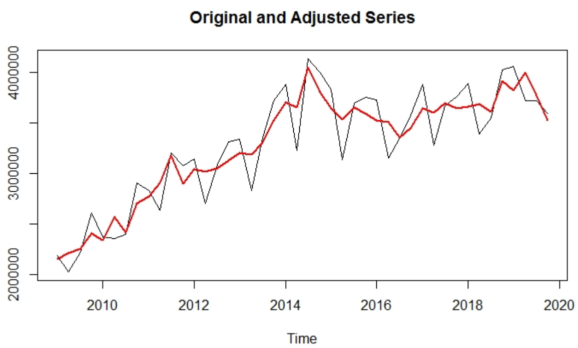


Figure B.5: Original and seasonal adjustment ARIMA X-13: Imports - Constant USD - Quarterly data



Appendix C

Bayessian estimation - Baseline model

Figure C.1: Prior and Posterior distributions

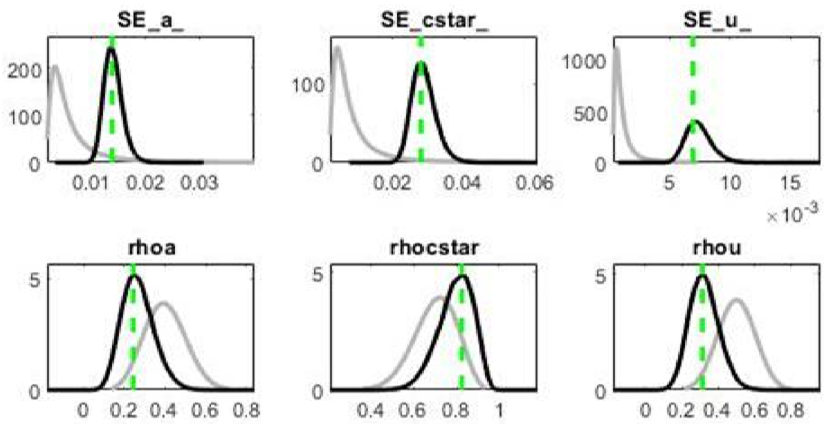
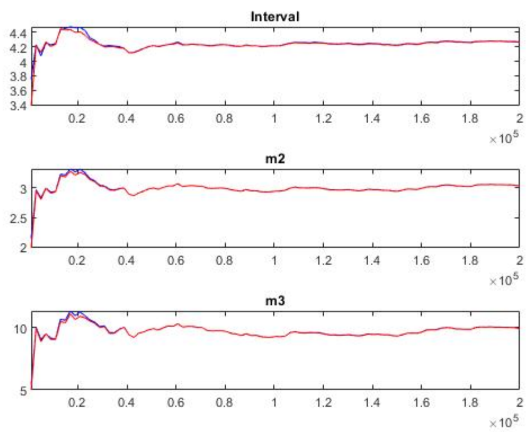


Figure C.2: Multivariate convergence diagnostic



Appendix D

Baseline model - Impulse response functions

D.1 Welfare under PPI,CPI and PEG

The least preferred policy is marked in red color.

	Taylor Rule (PPI)				PEG
ϕ_π	1.5	1.5	5	1.5	-
ϕ_y	0.125	0	0	1	-
Productivity	0.0123007	0.0129827	0.0045509	0.0089328	0.0430188
Ext.demand	0.0098652	0.0098918	0.0094063	0.0097231	0.1334600
Gov.spending	0.0142477	0.0143604	0.0127876	0.0136479	0.0171980
Cost-push	0.0281007	0.0301173	0.0384760	0.0183349	0.0164681
Welfare loss	0.00645142	0.00673521	0.00652207	0.00506387	0.021014

Assumption: $\rho = 1$
-ϕ_π and ϕ_y are parameters corresponding to the Taylor rule

	Taylor Rule (CPI)				PEG
ϕ_π	1.5	1.5	5	1.5	-
ϕ_y	0.125	0	0	1	-
	Welfare losses under each type of shock				
Productivity	0.002204	0.002319	0.001786	0.001619	0.004301
Ext. demand	0.003308	0.003432	0.004120	0.002664	0.013346
Gov.spending	0.001533	0.001547	0.001477	0.001457	0.001719
Cost-push	0.002267	0.002408	0.002709	0.001550	0.001646
Welfare loss	0.009313	0.009708	0.010094	0.007292	0.021014

Assumption: $\rho = 1$
-ϕ_π and ϕ_y are parameters corresponding to the Taylor rule

D.2 Inflation targeting (CPI and PPI) and peg

Figure D.1: Inflation targeting (CPI) - Productivity shock

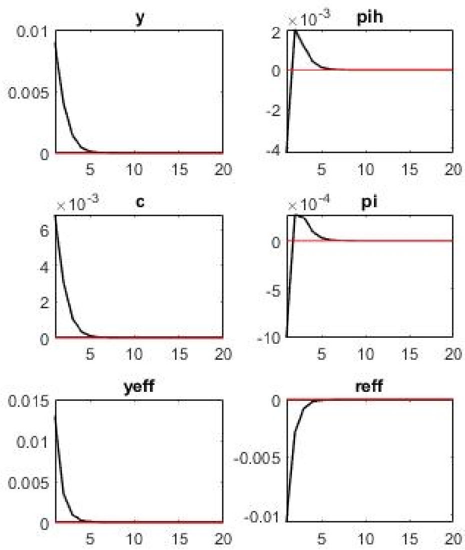


Figure D.2: Inflation targeting (CPI)- Government Spending Shock

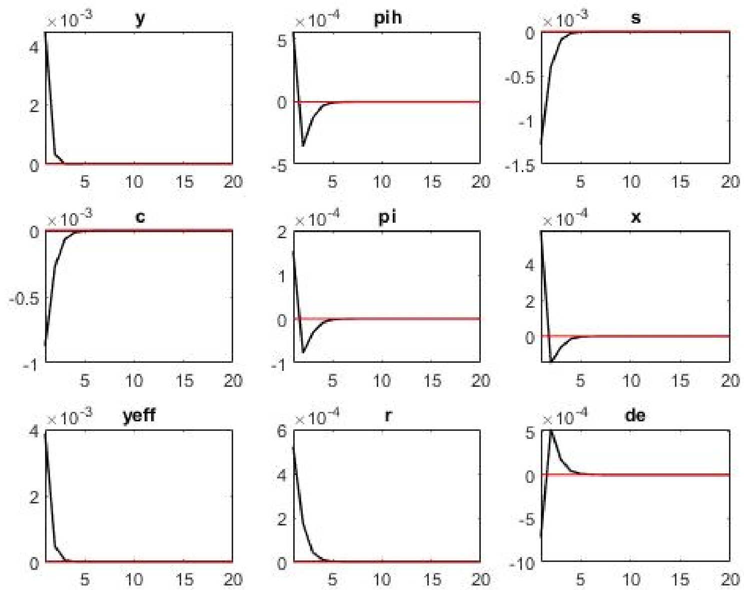


Figure D.3: Inflation targeting (CPI)- Foreign Demand Shock

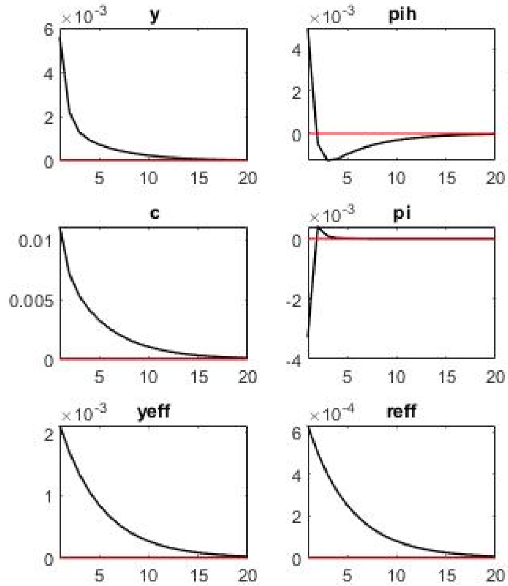


Figure D.4: Inflation targeting (CPI) - Markup Shock

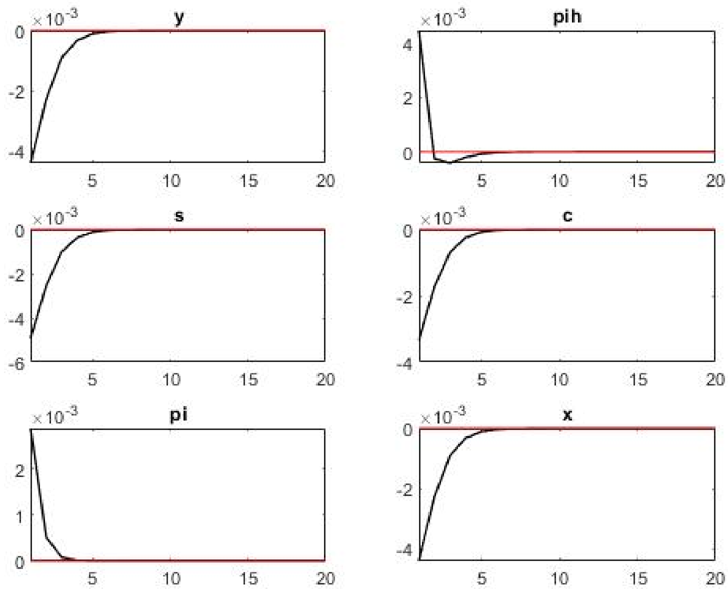


Figure D.5: Nominal exchange rate peg - Productivity shock

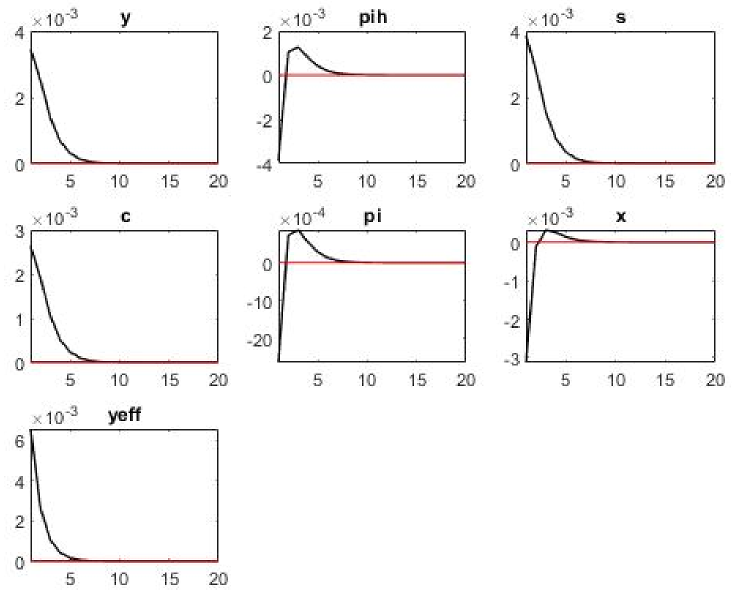


Figure D.6: Nominal exchange rate peg - Government Spending Shock

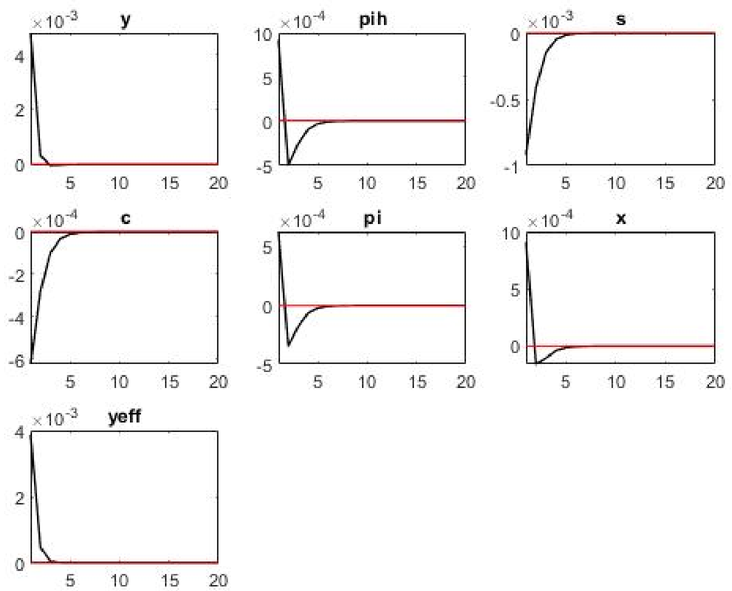


Figure D.7: Nominal exchange rate peg - Foreign Demand Shock

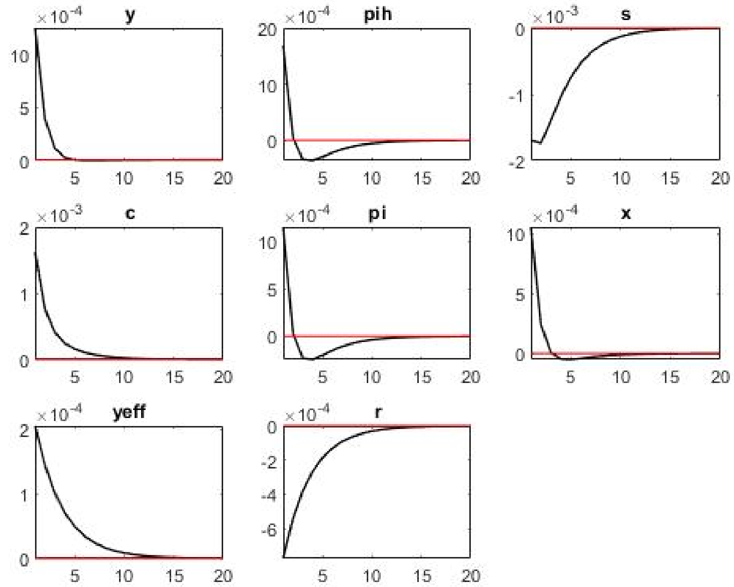


Figure D.8: Nominal exchange rate peg - Markup Shock

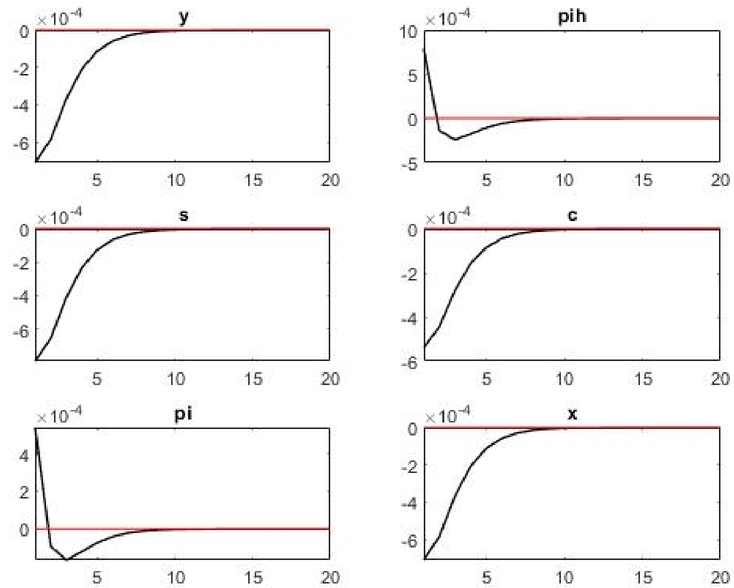


Figure D.9: Taylor (PPI) - Markup Shock

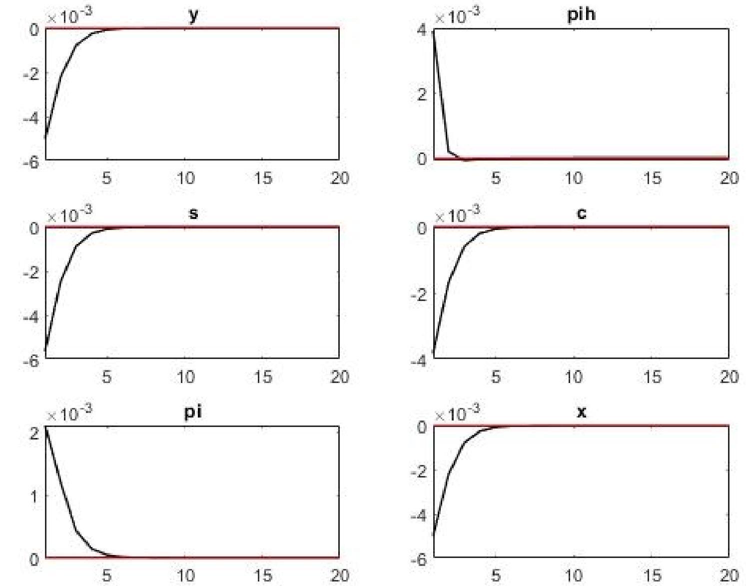


Figure D.10: Taylor (PPI) - Government Spending Shock

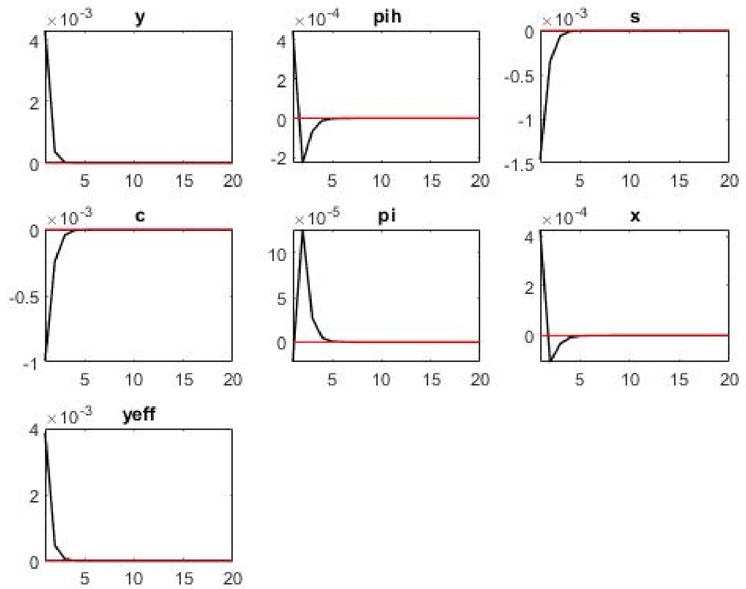


Figure D.11: Taylor (PPI) - Productivity Shock

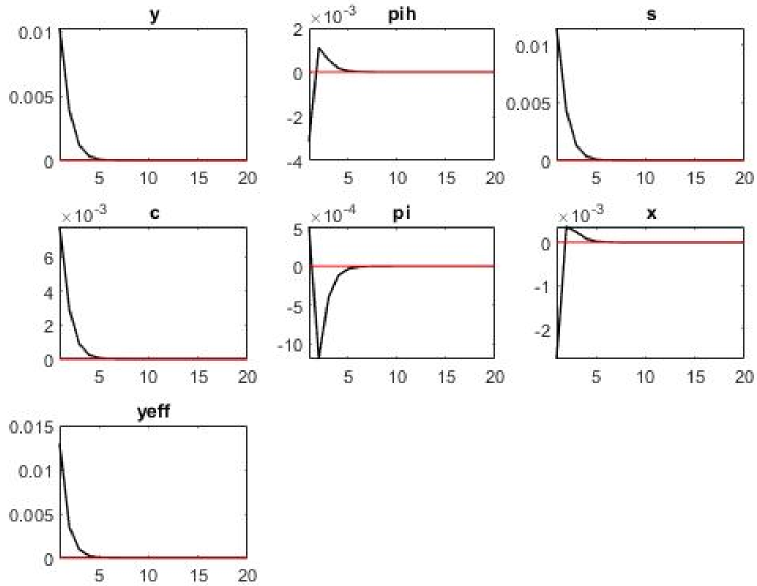
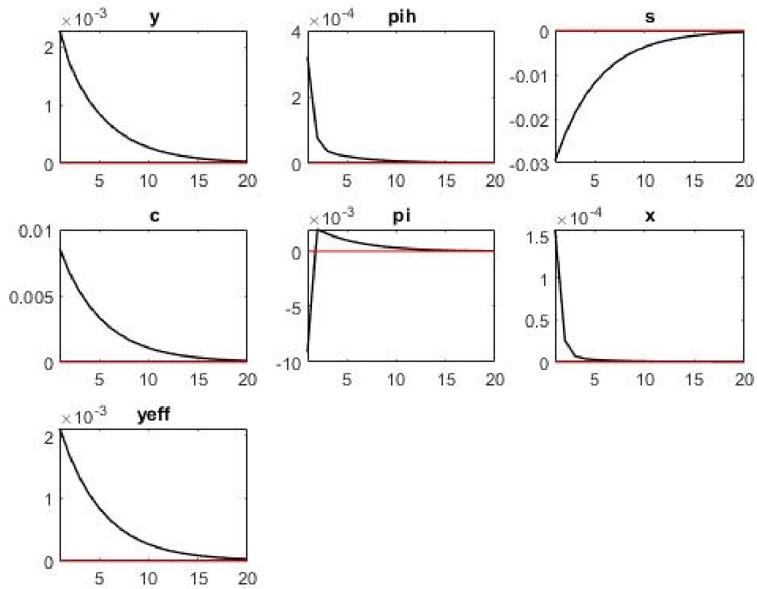


Figure D.12: Taylor (PPI) - Foreign Demand Shock



Appendix E

Welfare analysis

E.1 Welfare function in terms of consumption and labor

The main assumption for the derivation of a tractable welfare function in terms of consumption and labor is that the elasticities of intertemporal and intratemporal substitution are equal to one. (i.e. $\theta = \rho = 1$). According to the definition of the second-order Taylor expansion, the utility function is written in the following way:

$$U_t - U = U_c C \frac{C_t - C}{C} + U_n N \frac{N_t - N}{N} + \frac{1}{2} U_{cc} C^2 (\frac{C_t - C}{C})^2 + \frac{1}{2} U_{nn} N^2 (\frac{N_t - N}{N})^2 \tag{E.1}$$

Using the fact that: $\frac{X_t - X}{X} = x_t + \frac{1}{2} x_t^2$, the equation above holds as:

$$U_t - U = U_c C(\hat{c}_t + \frac{1}{2}\hat{c}_t^2) + U_n N(\hat{n}_t + \frac{1}{2}\hat{n}_t^2) + \frac{1}{2} U_{cc} C^2 \hat{c}_t^2 + \frac{1}{2} U_{nn} N^2 \hat{n}_t^2 + (||o||^3)$$

where $(||o||^3)$ considers terms higher than second order. Distributing:

$$\frac{U_t - U}{U_c C} = \hat{c}_t + \frac{1}{2}\hat{c}_t^2 \left(1 + \frac{U_{cc} C}{U_c}\right) + \frac{U_n N}{U_c C}\left(\hat{n}_t + \frac{1}{2}\hat{n}_t^2 \left(1 + \frac{U_{nn} N}{U_n}\right)\right) \tag{E.2}$$

According to the functional form of utility, and the assumption of the intertemporal elasticity of substitution equal to the unity:

$$\frac{U_{cc} C}{U_c} = -1 \quad and \quad \frac{U_{nn} N}{U_n} = \eta \tag{E.3}$$

Plugging E.3 in E.2 and simplifying:

$$\frac{U_t - U}{U_c C} = \hat{c}_t + \frac{U_n N}{U_c C}\left(\hat{n}_t + \frac{1}{2}\hat{n}_t^2 (1 + \eta)\right) \tag{E.4}$$

Thus, writing the previous equation as an infinite sum, the welfare function in terms of consumption and labor is:

$$W_t = \sum_{t=0}^{\infty} \beta^t \left(c_t - (1 - \lambda)\hat{n}_t - \frac{(1 - \lambda)(1 + \eta)}{2}\hat{n}_t^2\right) \tag{E.5}$$

E.2 The Central Bank loss function

The main assumption for the derivation of the central bank loss function in terms of output gap and inflation is that the elasticities of intertemporal and intratemporal substitution are equal to one (i.e. $\theta = \rho =1$) which implies that the analysis is analogous to the closed economy case.

According to the definition of the second-order Taylor expansion, the utility function is written in the following way:

$$U_t - U = U_c C \frac{C_t - C}{C} + U_n N \frac{N_t - N}{N} + \frac{1}{2} U_{cc} C^2 (\frac{C_t - C}{C})^2 + \frac{1}{2} U_{nn} N^2 (\frac{N_t - N}{N})^2$$

Using the fact that: $\frac{X_t - X}{X} = x_t + \frac{1}{2} x_t^2$, the equation above holds as:

$$U_t - U = U_c C(\hat{c}_t + \frac{1}{2}\hat{c}_t^2) + U_n N(\hat{n}_t + \frac{1}{2}\hat{n}_t^2) + \frac{1}{2} U_{cc} C^2 \hat{c}_t^2 + \frac{1}{2} U_{nn} N^2 \hat{n}_t^2 + (||o||^3)$$

where $(||o||^3)$ considers terms higher than second order. Distributing terms:

$$\frac{U_t - U}{U_c C} = \hat{c}_t + \frac{1}{2}\hat{c}_t^2 \left(1 + \frac{U_{cc} C}{U_c}\right) + \frac{U_n N}{U_c C}\left(\hat{n}_t + \frac{1}{2}\hat{n}_t^2 \left(1 + \frac{U_{nn} N}{U_n}\right)\right) \tag{E.6}$$

Taking into account the relationship between domestic and foreign output given by equation A.48 and isolating the terms of trade:

$$tot_t = \rho_v (y_t - g_t) - \rho_v c_t^* \tag{E.7}$$

Plugging E.7 in the the risk sharing condition given by A.44:

$$c_t = c_t^* + \frac{(1-\lambda)\rho_v}{\rho} y_t + \frac{(1-\lambda)}{\rho} \rho_v (g_t^* - y_t^* - g_t) \tag{E.8}$$

Also, note that according to the functional form of utility and the assumption of intertemporal elasticity of substitution equal to the unity:

$$\frac{U_{cc} C}{U_c} = -1 \quad and \quad \frac{U_{nn} N}{U_n} = \eta \tag{E.9}$$

Plugging E.8 into E.6 and taking into account equation E.9:

$$\frac{Ut - U}{U_c C} = \frac{(1-\lambda)\rho_v}{\rho} y_t + \frac{U_n N}{U_c C} (\hat{n}_t + \frac{(1+\eta)}{2} n_t^2) + t.i.p. \tag{E.10}$$

where $t.i.p$ accounts for terms independent of policy. Further, from the utility function, the aggregate labor is written as:

$$\int_0^n N(z) dz = \int_0^n \frac{y(z)}{A_t}$$

Plugging the aggregate demand given by equation A.16:

$$\int_0^n \left(\frac{p(z)}{P_H}\right)^{-\sigma} \frac{Y_{t,H}}{A_t} d(z)$$

Where $Y_{t,H} = \left(\left(\frac{P_H}{P}\right)^{-\theta}\left((1-\lambda)C + \lambda C^*\left(\frac{1}{Q}\right)^{-\theta}\right) + G\right)$. Organizing terms, the following log-linear representation yields:

$$n_t = y_{t,H} - a_t + ln(\int_0^n \left(\frac{p(z)}{P_H}\right)^{-\sigma} d(z))$$

$$n_t = y_{t,H} - a_t + d_t \tag{E.11}$$

Following Gali (2015), the term d_t is written as:

$$d_t = \frac{\sigma}{2} var_i(p_{t,H}) \tag{E.12}$$

Finally, plugging equation E.11 and E.12 in E.10:

$$\frac{Ut-U}{U_cC} = \frac{(1-\lambda)\rho_v}{\rho}y_t + \frac{U_nN}{U_cC}(\hat{y}_t + \frac{\sigma}{2}var_i(p_{t,H}) + \frac{(1+\eta)}{2}(\hat{y}_t - \hat{a}_t)^2) + t.i.p. + (||o||^3) \tag{E.13}$$

As shown previously, the maximization problem of the social planner yields:$-\frac{U_n}{U_c} = (1-\lambda)\frac{C}{N}$ (Derivation of this relationship in section E.2.1) (Equation E.27), therefore equation E.13 simplifies as follows:

$$\frac{Ut-U}{U_cC} = (1-\lambda)\rho_v y_t - (1-\lambda)(\hat{y}_t + \frac{\sigma}{2}var_i(p_{t,H}) + \frac{(1+\eta)}{2}(\hat{y_t}^2 - 2\hat{y}_t\hat{a}_t + a_t^2)) + t.i.p. + (||o||^3)$$

Reorganizing terms and factorizing:

$$\frac{Ut-U}{U_cC} = (1-\lambda)\rho_v y_t - \frac{(1-\lambda)}{2}(\hat{y}_t + \sigma var_i(p_{t,H}) + (1+\eta)(\hat{y_t}^2 - 2\hat{y}_t\hat{a}_t)) + t.i.p. + (||o||^3)$$

Recalling the definition of the efficient output gap: $\hat{x}_t = \hat{y}_t - \hat{y}_t^e$:

$$\frac{Ut-U}{U_cC} = (1-\lambda)(\rho_v - 1)\hat{x}_t + -\frac{(1-\lambda)}{2}(\sigma var_i(p_{t,H}) + (1+\eta)(\hat{y_t}^2 - 2\hat{y}_t\hat{a}_t)) + t.i.p. + (||o||^3)$$

Using the expression of the efficient output as a function of productivity, simplifying and factorizing:

$$\frac{Ut-U}{U_cC} = (1-\lambda)(\rho_v - 1)\hat{x}_t - \frac{(1-\lambda)}{2}(\sigma var_i(p_{t,H}) + (1+\eta)(\hat{y_t}^2 - 2\hat{y}_t\hat{y}_t^e + \hat{y}_t^{e2} - \hat{y}_t^{e2})) + t.i.p. + (||o||^3)$$

$$\frac{Ut-U}{U_cC} = (1-\lambda)(\rho_v - 1)\hat{x}_t - \frac{(1-\lambda)}{2}(\sigma var_i(p_{t,H}) + (1+\eta)((\hat{y}_t - \hat{y}_t^e)^2 - \hat{y}_t^{e2})) + t.i.p. + (||o||^3)$$

$$\frac{Ut-U}{U_cC} = (1-\lambda)(\rho_v - 1)\hat{x}_t - \frac{(1-\lambda)}{2}(\sigma var_i(p_{t,H}) + (1+\eta)\hat{x}_t^2) + t.i.p. + (||o||^3) \tag{E.14}$$

Following Woodford (2003), the next equality holds:

$$\sum_{t=0}^{\infty}\beta^t var_i(p_t, H) = \frac{\alpha}{(1-\alpha)(1-\beta\alpha)}\sum_{t=0}^{\infty}\beta^t\pi_t^2 = \zeta^{-1}\sum_{t=0}^{\infty}\beta^t\pi_t^2$$

Where the coefficients corresponding to ζ match exactly to those in the Phillips curve's equation derived previously. Hence, writing equation E.14 as an infinite sum and factorizing:

$$\sum_{t=0}^{\infty}\beta^t\frac{Ut-U}{U_cC} = \sum_{t=0}^{\infty}\beta^t\left((1-\lambda)(\rho_v - 1)\hat{x}_t - \frac{(1-\lambda)}{2}(\sigma\zeta^{-1}\pi_t^2 + (1+\eta)\hat{x}_t^2) + t.i.p. + (||o||^3)\right)$$

$$\sum_{t=0}^{\infty}\beta^t\frac{Ut-U}{U_cC} = -(1-\lambda)\sum_{t=0}^{\infty}\beta^t\left((1-\rho_v)\hat{x}_t+\frac{1}{2}(\sigma\zeta^{-1}\pi_t^2+(1+\eta)\hat{x}_t^2)+t.i.p.+(||o||^3)\right)$$

Finally, when the elasticities of intertemporal and intratemporal substitution are equal to one, the term $\rho_v = 1$, and the welfare loss function is written as:

$$W = -(1-\lambda)\sum_{t=0}^{\infty}\beta^t\left(\frac{1}{2}(\sigma\zeta^{-1}\pi_t^2+(1+\eta)\hat{x}_t^2)\right) \tag{E.15}$$

E.2.1 The social planner problem

The optimization problem of the social planner seeks to maximize the utility subject to the production function and the aggregate demand:

$$\underset{C_t,N_t}{\text{maximize}} \quad E_t(\sum_0^{\infty}U(C_t,N_t)) \quad \text{subject to} \quad Y_t = A_tN_t\ ; \quad Y_t = \left(\frac{P_H}{P}\right)^{-1}((1-\lambda)C_t+\lambda C_t^*Q)$$

It is useful to transform the above equations, such that the maximization problem is focused only in N_t. To do that, I follow some additional steps, taking into account that $\theta = \rho = 1$:

- The risk sharing condition (equation 3.60) is rewritten as:

$$Tot_t = \left(\frac{C_t}{C_t^*}\right)^{\frac{1}{1-\lambda}} \tag{E.16}$$

- In particular, when $\theta = 1$ and the size of the economy tends to zero, the price index (equation 3.10) holds as:

$$P_t = P_{H,t}^{1-\lambda}P_{F,t}^{\lambda} \tag{E.17}$$

which divided by $P_{H,t}$ and $P_{F,t}$ gives respectively:

$$\rightarrow \frac{P_t}{P_{H,t}} = \left(\frac{P_{F,t}}{P_{H,t}}\right)^{\lambda} \rightarrow \frac{P_t}{P_{H,t}} = Tot_t^{\lambda} \tag{E.18}$$

$$\rightarrow \frac{P_t}{P_{F,t}} = \left(\frac{1}{Tot_t}\right)^{1-\lambda} \rightarrow \frac{P_{F,t}}{P_t} = Tot_t^{1-\lambda} \tag{E.19}$$

- Using equations 3.50 ($P_{F,t} = S_tP_{F,t}^*$) and 3.51 ($P^* = P_F^*$), the real exchange rate is written as:

$$Q_t = \frac{S_tP_t^*}{P_t} = \frac{S_tP_{F,t}^*}{P_t} = \frac{S_tP_{F,t}}{S_tP_t}$$

$$Q_t = \frac{P_{F,t}}{P_t} \tag{E.20}$$

Equalizing equation E.20 and E.19, the following equality holds:

$$Q_t = Tot_t^{1-\lambda} \tag{E.21}$$

- Plugging equations E.18 and E.21 in the demand function stated above and distributing terms:

$$Y_t = \left(\frac{P_H}{P}\right)^{-1} ((1-\lambda)C_t + \lambda C_t^* Q)$$

$$Y_t = Tot_t^{\lambda}((1-\lambda)C_t + \lambda C_t^* Tot_t^{1-\lambda})$$

$$Y_t = (1-\lambda)Tot_t^{\lambda} C_t + \lambda Tot_t C_t^* \tag{E.22}$$

- Plugging equation E.16 in E.22 and simplifying:

$$C_t = Y_t^{1-\lambda} C_t^{*\lambda} \tag{E.23}$$

- Finally, inserting the production function in equation E.23:

$$C_t = (A_t N_t)^{1-\lambda} C_t^{*\lambda} \tag{E.24}$$

- Using equation E.24, the period optimization problem of the social planner is:

$$\underset{C_t, N_t}{\text{maximize}} \quad E_t(\sum_0^{\infty} U(C_t, N_t)) \quad \rightarrow E_t(\sum_0^{\infty} U((A_t N_t)^{1-\lambda} C_t^{*\lambda}, N_t))$$

$$F.O.C \;\; wrt \;\; N_t \;\rightarrow\; U_{t,C}((1-\lambda)A_t N_t^{-\lambda} C_t^{*\lambda} + U_{t,N} = 0 \tag{E.25}$$

Note that equation E.24 can be written as $A_t N_t^{-\lambda} C_t^{*\lambda} = \frac{C_t}{N_t}$: , thus, equation E.25 is:

$$U_{t,C}(1-\lambda)\frac{C_t}{N_t} + U_{t,N} = 0$$

$$\rightarrow -\frac{U_{t,N}}{U_{t,C}} = (1-\lambda)\frac{C_t}{N_t} \tag{E.26}$$

- At the steady state, equation E.26 holds as:

$$\rightarrow -\frac{U_N}{U_C} = (1-\lambda)\frac{C}{N} \tag{E.27}$$

Appendix F

Application of the Baseline model

F.1 Profits of the oil sector

The profit maximization problem is written as:

$$\Pi_t^O = \frac{1}{n}\int_0^n p(z)y^O(z) - N_t^O W_t^O$$

Knowing that: $y^O(z) = \left(\frac{p(z)}{P_t^O}\right)^{-\sigma} Y_t^O$ and $P_t^O = \left(\frac{1}{n}\int_0^n (p_t(z))^{1-\sigma} dz\right)^{\frac{1}{1-\sigma}}$, the following equality holds:

$$\frac{1}{n}\int_0^n p(z)y^O(z) = \frac{1}{n}\int_0^n p(z)\left(\frac{p(z)}{P_t^O}\right)^{-\sigma} Y_t^O = \frac{1}{n}\int_0^n (p(z))^{1-\sigma}\frac{Y_t^O}{(P_t^O)^{-\sigma}} = (P_t^O)^{1-\sigma}\frac{Y_t^O}{(P_t^O)^{-\sigma}}$$

$$\rightarrow \frac{1}{n}\int_0^n p(z)y^O(z) = P_t^O Y_t^O$$

Thus, the profit maximization problem is:

$$\Pi_t^O = P_t^O Y_t^O - N_t^O W_t^O \tag{F.1}$$

Using the production function, the equation above can be written as:

$$\Pi_t^O = P_t^O A_t^O (N_t^O)^k - N_t^O W_t^O$$

First order conditions with respect to labor:

$$P_t^O A_t^O k (N_t^O)^{k-1} = W_t^O$$

$$\rightarrow W_t = \frac{P_t^O A_t^O k}{(N_t^O)^{1-k}} \tag{F.2}$$

Replacing equation F.2 in F.1, yields:

$$\Pi_t^O = P_t^O Y_t^O - P_t^O A_t^O (N_t^O)^k k$$

$$\Pi_t^O = P_t^O Y_t^O (1-k) \tag{F.3}$$

F.2 Welfare losses

Table F.1: **Welfare Losses** - Evaluation of simple monetary policy rules in the presence of commodity price shocks, productivity shocks (commodity and non-commodity sectors) and foreign demand shock

	PEG	Taylor Rule (CPI)
Non-commodity sector productivity shock	0.000697575723672130834575496	0.000697575723672126070015167
Commodity sector productivity shock	0.0451024984134719962060311 45	0.045102498413471752260542336
Commodity price shock	0.034668429708940014884101938	0.034668429708939655742132302
Foreign demand shock	0.001355384925110290918578082	0.001355384925110269530996164
Cost-push shock	0.000155595208925536038328141	0.000155595208925535614811667
Government spending shock	0.000385203521161498050274163	0.000385203521161496144450031
Welfare loss	0.083765410254551232412979778	0.083765410254551151097816841

-Assumption: $\rho = 1$

-The Taylor rule considered in this case is $i_t = \rho_r * i_{t-1} + (1 - \rho_r) * (\psi_y * y + \psi_p i * pi)$, where $\rho_r = 0.7$, $\psi_y = 0.5$ and $\psi_\pi = 1.5$ (Similar to the baseline model)

Appendix G

Bayessian estimation - Commodity shocks

Figure G.1: Prior and Posterior distributions

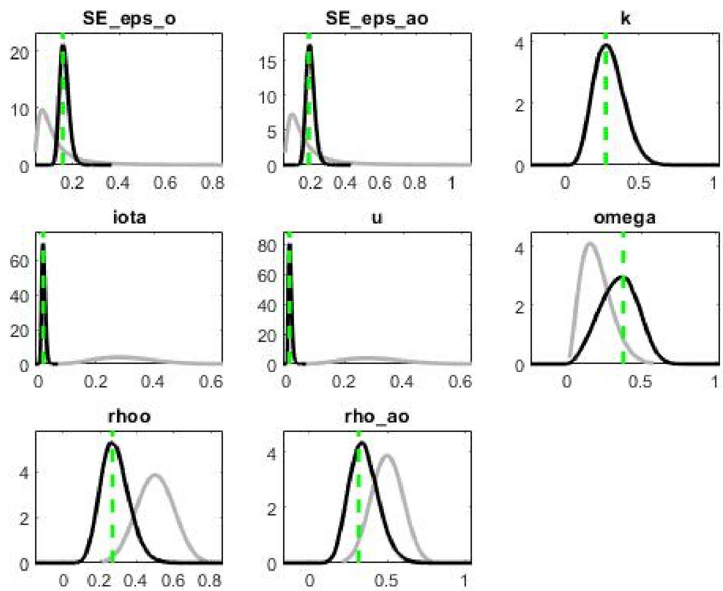
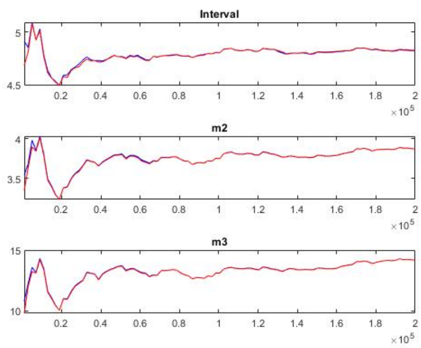


Figure G.2: Multivariate convergence diagnostic



Appendix H

Commodity sector - Impulse response functions

Figure H.1: Commodity price shock (1)

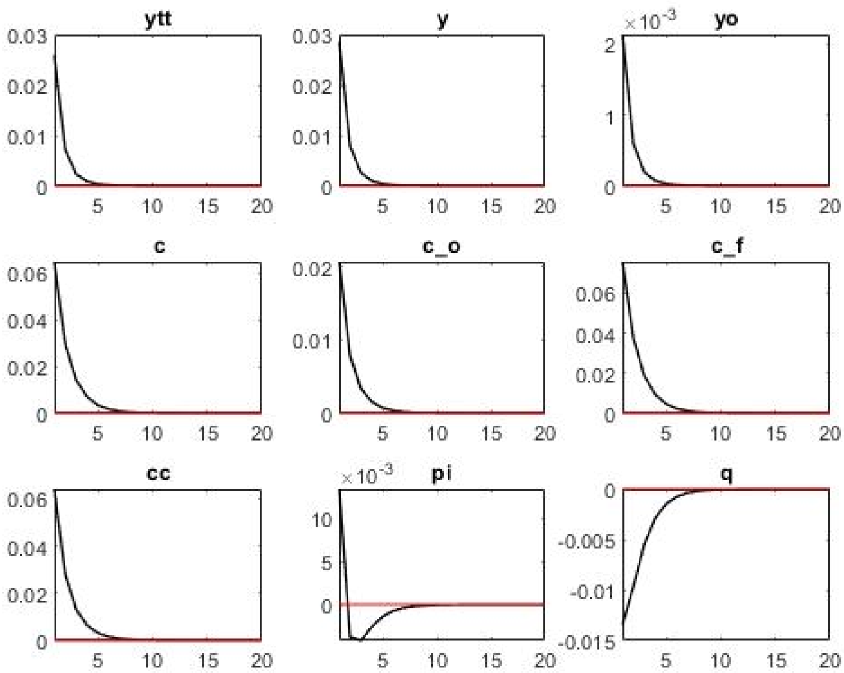


Figure H.2: Commodity price shock (2)

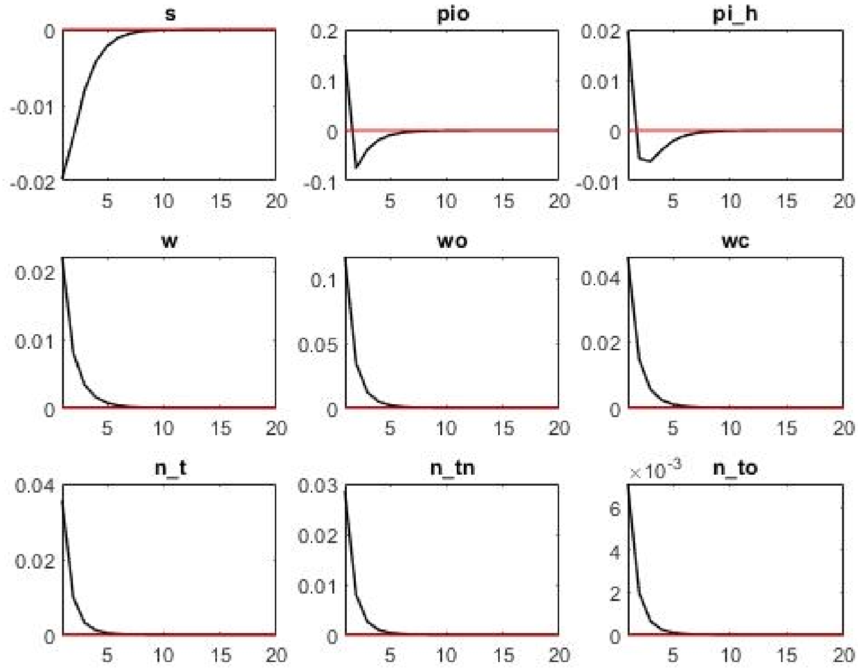


Figure H.3: Productivity shock in the commodity sector (1)

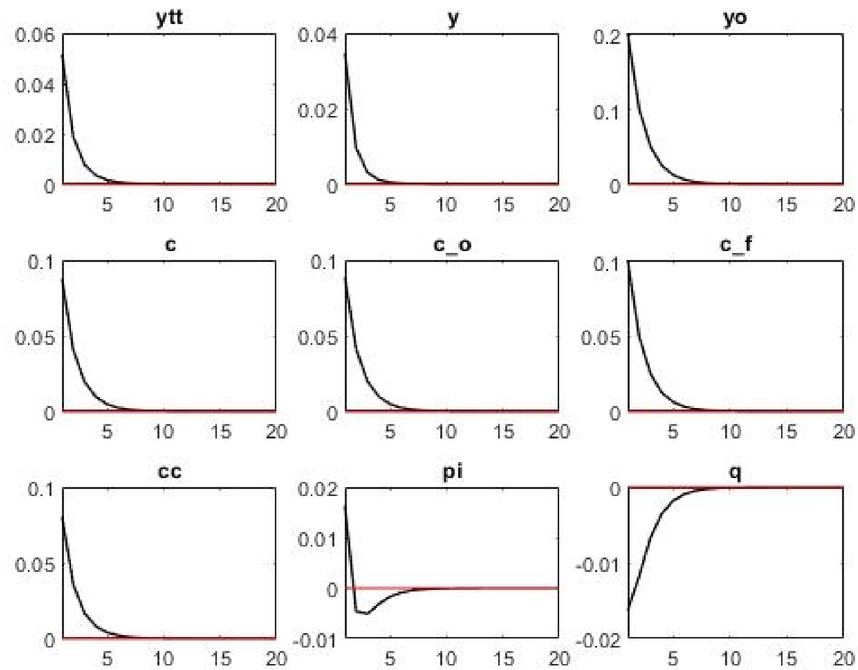


Figure H.4: Productivity shock in the commodity sector (2)