Worker Household Debt, Functional Income Distribution and Growth: a neo-Kaleckian Perspective

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12 August 2020

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Abstract

In a stock-flow consistent neo-Kaleckian macro-model, along with worker households' debt dynamics, in the long-run, we incorporate distributional dynamics, and demonstrate the possibility of multiple equilibria. Dynamic stability of the economy is also examined. Both debt-led and debt-burdened demand and growth regimes are possible in short-run as well as in the long-run. We find that mergers, acquisitions and hostile takeovers play a crucial role for (de)stabilizing the economy. In some instances, the speed of the adjustment parameter of the distributional dynamics becomes crucial for stabilizing the economy. Otherwise, the economy may lose its stability and gives birth to limit cycles.

Keywords: Capital Accumulation, Income Distribution, Worker Household Debt, Kaleckian Model, Limit Cycle, Stock-flow Consistency,

JEL classification: C62, E12, E25, G34, O41.

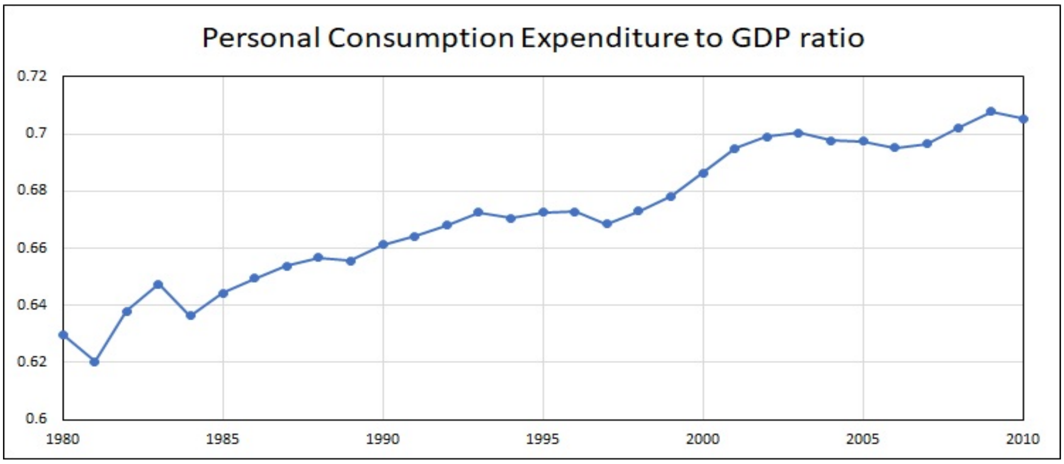
*This paper is part of ongoing work on the author's doctoral thesis to be submitted to Jawaharlal Nehru University, New Delhi, India. The author is indebted to Subrata Guha, Gogol Mitra Thakur, Mausumi Das and Soumya Datta for their valuable comments. However, the author is solely responsible for the remaining errors.

1 *Introduction*

Since the 1980s, there has been a massive deterioration in the functional income distribution from workers' point of view in the US economy. Over the last four decades, the real wage rate increased at a lower rate than labour productivity (Setterfield; 2013, pp. 163). Labour income share declined from 67.26% in 1961-73 to 63.66% in 2001-08 (Hein; 2014, pp. 14). However, the share of overall consumption demand in GDP has not declined over the same period (see Figure 1.1). This has happened due to workers' borrowing for consumption (see Figure 1.2). We also observe an unprecedented amount of mergers, acquisitions and hostile takeovers in the 1980s and 1990s in the US economy (Rohit; 2013, pp. xxiv; see Figure 1.3 as well)[1]. Rising indebtedness of worker households on the one hand and a declining income share of workers on the other hand are two of the most important phenomena in the US economy for the last several decades. While most of the neo-Kaleckian literature focuses on the macro-dynamics of consumer debt (or debt of workers), the interaction between the debt dynamics and distributional dynamics is missing in this kind of literature. In this paper, in a stock-flow consistent neo-Kaleckian macro-model, we incorporate the long-run distributional dynamics of the economy along with the debt dynamics of worker households, and demonstrate the possibility of multiple equilibria. We examine the dynamic stability of the economy. We find that both debt-led and debt-burdened growth (and demand) regimes are possible in the short-run as well as in the long-run. There can arise a unique stable equilibrium in the debt-led demand and growth regime. The debt-burdened demand and growth regime can be categorized as (*i*) weak debt-burdened (*ii*) moderate debt-burdened and (*iii*) strong debt-burdened demand and growth regime. In case of a weak debt-burdened demand (and growth) regime, there is a possibility of multiple equilibria where one of them is stable (the other one is a saddle point unstable). However, in case of a moderate or a strong debt-burdened demand regime, along with stable or unstable equilibrium, limit cycles are also possible. In our model, the speed of adjustment parameter related to the distributional dynamics that represents the ability of firms to adjust their actual share of profit to the desired one explains the limit cycles of the economy. We also investigate how mergers, acquisitions and hostile takeovers influence the debt-capital ratio, profit share and rate of capital accumulation in the long run.

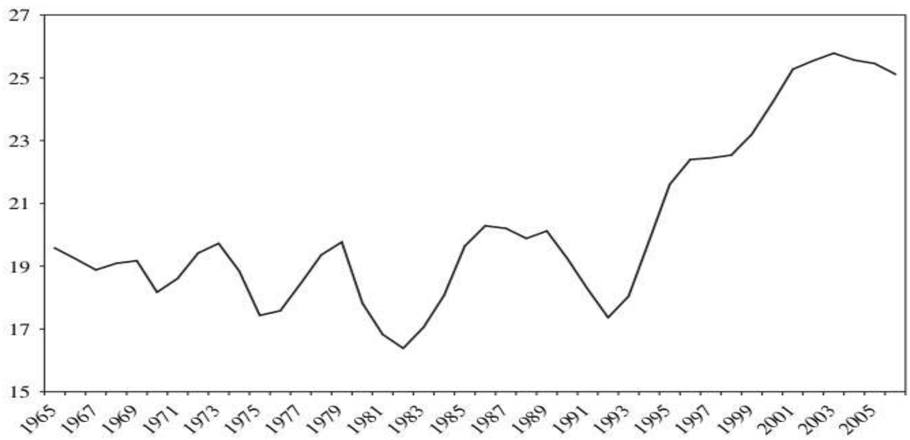
Starting from 15% in the 1980s, the financial sector's profit share (out of all profits) tripled in 2007 with a peak of 45% in 2002 (Tomaskovic-Devey and Lin, 2011; Lin and

[1] Mergers in the 1980s were different in nature from those of 1990s. The most important distinction comes from the fact that there was a massive utilization of stock as a method of payment during the 1990s. As quoted by Andrade *et al.* (2001, pp. 105), "[A]bout 70 per cent of all deals in the 1990s involved stock compensation, with 58 per cent entirely stock financed. These numbers are approximately 50 per cent more than in the 1980s."



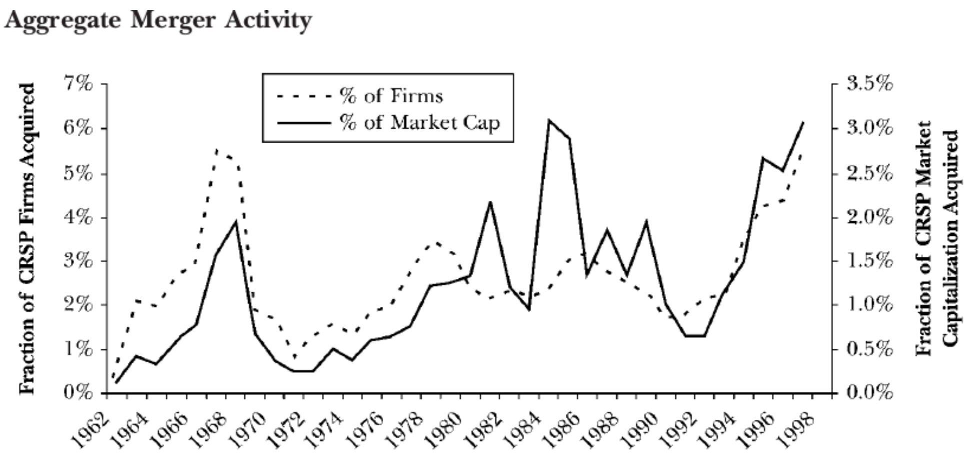
Source: Economic Report of the President, February 2012, table B-1; author's calculations.

Figure 1.1: Personal consumption expenditure to GDP ratio (1980-2010)



Source: Barba and Pivetti; 2009, pp. 115

Figure 1.2: Consumer credit outstanding as a percentage of disposable personal income for the US, 1965-2006.



Source: Andrade *et al.* 2001, pp. 105

Figure 1.3: Aggregate Merger Activity Since 1060s for the US.

Tomaskovic-Devey, 2013). In case of non-financial sector firms, the ratio of financial income to realized profits more than doubled from 15% to 32% with a peak of 42% in 2001 (Lin and Tomaskovic-Devey, 2013). Both the share of capital in national income and the compensation of top corporate executives increased significantly. Needless to say, income inequality increased tremendously. The shift of power towards rentiers and away from workers (because of financialization), as Van Arnum and Naples (2013) point out, is one of the primary reason for rising income inequality. The relationship between financialization and inequality, therefore, plays a crucial role in the analysis of Crotty (2003), Palley (2012) and Stockhammer (2015).

According to Orhangazi (2008), non-financial corporations, by increasing their financial investments (relative to real investments), derive increasing part of their income from financial sources. Through the allocation of funds, away from real investment into financial investment, financial investments crowd out real investment. On the other hand, shareholders, with their increased power, pressurize managers to adopt higher financial payout ratios and short-term planning horizons, which lead to a reduction in levels of investment. Stockhammer (2004) finds strong support from the empirical results for the USA and France regarding the hypothesis that financialization affects capital accumulation negatively.

However, the above literature focuses only on investment demand and neglects to capture the effect of financialization on consumption demand. A more general analysis can be found in Onaran *et al.* (2011). Onaran *et al.* (2011) point out how the reduction in investment demand, due to the change in income distribution in favour of capital (because of financialization), has been compensated by a rise in consumption demand through the wealth effect which has come from the redistribution of income because of financialization. As worker incomes decrease dramatically, to keep up their consumption patterns, workers borrow from rentiers. The impact of worker borrowing on the economy is captured by Kim *et al.* (2014), Setterfield *et al.* (2016), Dutt (2006) and Hein (2012a) as well[2].

Emphasizing the relative income hypothesis (of Duesenberry (1949)) and debt finance, to understand the household consumption behaviour, Kim *et al.* (2014) construct a Keynesian model of aggregate consumption. In this model, there are two types of households (working and rentier households) that consist of three types of income recipients (production and non-supervisory workers, supervisory workers, and capitalists). In this model, under the 'conventional' case of debt-servicing (where workers service their debt through an initial deduction from income and then consume a fraction of the rest), we find that

[2] Cynamon and Fazzari (2015), Kapeller and Schütz (2015), Kim (2012), and Setterfield and Kim (2016), are among others who contribute in this area.

current borrowing by workers enhances the aggregate level of consumption while a rise in indebtedness leads to a reduction in the same. Under the 'unconventional' case of debt-servicing (where workers, considering savings as a luxury that must be foregone first, consume a part of their current income and then service their debts), one can nonetheless find that current borrowing of workers enhances the aggregate level of consumption while the aggregate level of consumption is invariant with respect to a rise in indebtedness. This result can be sustained unless a critical point is arrived where debt-servicing obligations exceed current income less consumption expenditures. At this critical point, however, the burden of accumulated debt may exert a sudden negative influence on aggregate consumption spending. Nevertheless, it is worth remembering that this analysis is based on a static model where firms do not engage in investment, and as a consequence, there is no growth in the economy.

In a neo-Kaleckian stock-flow consistent macro-model, Setterfield *et al.* (2016) show that the way in which debtor households (a section of workers) service their debt plays a crucial role in the overall macroeconomic performance both in the short run as well as in the long run. In this model, worker borrowing depends on the difference between the target level of consumption to which working households aspire and the actual level of consumption by workers out of their wage income. This target level of consumption to which working households aspire depends on how much rentier households consume. So, worker households try to emulate the consumption pattern of the rentier class. Under the 'conventional' case of debt-servicing (where debtors service their debt through an initial deduction from income and then consume a fraction of the rest), a rise in the interest rate or the debt to capital ratio leads to an ambiguous impact on the equilibrium rate of capital accumulation in the short run. The reason is two-fold. First, *ceteris paribus*, a redistribution of income from workers to rentiers (who have a lower marginal propensity to consume) immediately reduces total consumption spending. Second, as redistribution of income to the rentiers as a result of debt servicing increases rentier income and their consumption demand, it in turn also increases the target level of consumption to which working households aspire. Thus the gap between the workers' targeted level of consumption and actual level of consumption rises. Consequently, there is an increment in workers' consumption spending as well. In the long run, under the 'conventional' case of debt-servicing, one can get two equilibrium debt-capital ratios where the lower value of the debt-capital ratio corresponds to a stable steady state and the higher value to an unstable one. On the other hand, under the 'unconventional' case of debt-servicing (where debtors first consume a part of their current income and then service their debts), one can again get two equilibrium debt-capital ratios. However, here the higher value of the debt-capital ratio is a stable steady state, whereas the lower value is unstable.

Dutt (2006), in a Steindlian framework, argues that as long as workers are able to borrow, their consumption can increase beyond their wage income at least in the short run. An increase in borrowing by workers, through a rise in consumption demand, improves the capacity utilization and accumulation rate in the short run. However, worker borrowing has a limit. The desired lending of capitalists (or rentiers) to workers, or the desired debt of workers from the perspective of the capitalists (or rentiers) is determined by worker incomes net of interest payments. In the long run, the impact of a rise in borrowing on growth is ambiguous, since an increase in borrowing also increases the debt burden on the workers. As the debt–capital ratio rises, net income of the workers declines which in turn can potentially depress their capacity to borrow further. The increased interest payment, which is essentially a redistribution of income away from the workers to the capitalists (or rentiers), can cause and exacerbate under-consumption in the long run. At the same time, if investment demand is not sufficiently high to compensate this under-consumption, the capacity utilization and accumulation rate will deteriorate in the long run.

Unlike Dutt (2006), in Hein (2012a, 2012b), borrowing by workers is independent of their net income and is determined completely by how much rentiers want to lend. This departure from Dutt (2006) allows for potential instability in the model in the long run. Using a Kaleckian distribution and growth model, which consists of workers, rentiers and firms, Hein (2012a) explains the effect of workers' debt on a finance-dominated capitalist economy in the short and long run. Consumption of workers depends on their income (i.e. on wages since no assets are being held by the workers), fresh borrowing and interest payment on debt. Fresh borrowing (or new loans) is independent of workers' income and entirely depends on how much rentiers provide as loans to the workers. Hein assumes a fixed proportion of rentiers' savings goes to the workers as loans. Two types of assets, he assumes, exist in the economy - deposits (or loans to the workers) and equities. Firms have only one source of funds for investment - equities. The entire profit is completely distributed as dividend payments to rentiers and hence the dividend rate and the profit rate are same in his model. Rentiers have two sources of income: dividends, and interest income (which they earn on issuing loans to the workers). Rentiers consume a fraction of their income and save the rest for purchasing new assets. In Hein's model, in the short run, an increase in the provision of loans to the workers has a positive effect on the equilibrium degree of capacity utilization and the growth rate. But the effect of an increase in the interest rate or the debt-capital ratio is contractionary on the capacity utilization rate and the rate of accumulation. This is due to the fact that workers have a higher propensity to consume than the rentier class, and so, as the interest rate or the debt-capital ratio rises, there is a redistribution of income from workers to the rentiers. Paradox of cost also prevails in the short run i.e. higher real wages lead to higher profit rates[3]. Hein endogenizes the debt-capital ratio in the long run and obtains two equilibrium values of the debt-capital ratio where the lower one is stable. The effect of higher lending by rentiers to workers in the long run can be debt-led or debt-burdened depending on the interest rate on debt and the profit rate. As long as the debt-capital ratio lies below the unstable upper steady state value, the workers' debt-capital ratio will approach towards the stable steady state value. When the debt-capital ratio exceeds this unstable upper steady state value, the system loses its stability. A higher animal spirits, lower interest rates and a higher profit share each have a positive effect on the upper equilibrium value of workers' debt-capital ratio and thus improves the upper limit of stability.

The basic structure of our model in this paper is the same as in Hein (2012a). However, compared to Hein (2012a), our work has a few distinct features.

First, none among Dutt (2006), Hein (2012a), Kim *et al.* (2014) and Setterfield *et al.* (2016) in their analyses account for the effect of wealth on rentiers' consumption demand. In the USA, in the last few decades (especially in the 1980s, 1990s, and early 2000s) a massive increment has occurred in rentiers' wealth, and it has had a significant impact on their consumption demand. In our analysis, rentiers' consumption not only depends on their income but also on their assets.

Second, in the era of financialization, 'shareholder value orientation' leads to a shift in the preferences of managers (here firms) from retaining profit and reinvesting it for capital accumulation to downsizing the labour force and distributing the profit to shareholders (Lazonick and O'Sullivan; 2000). As a consequence, the dividend-payout ratio to rentiers has increased in the last few decades (see Hein (2012b), Rohit (2013)). The novelty of our model compared to Hein (2012a) is that we explicitly capture the impact of a rise in the dividend-payout ratio on the short run equilibrium values of aggregate demand and on the equilibrium rate of capital accumulation. In Hein (2012a), one cannot analyse this impact due to the assumption that profit is completely distributed to rentiers.

Third, most of the literature (and all of the literature discussed above) treat share of profit as a parameter and analyse the impact of few financialization parameters (for e.g. interest rate, dividend–payout ratio etc.) on the share of profit. However, we attempt to analyse explicitly the distributional dynamics of the economy in the long run i.e. how the share of profit evolves through time in the long run. We then show, how the interaction between two subsystems can cause instability in the whole system.

Fourth, there was a massive volume of mergers, acquisitions and hostile takeovers in the 1980s and 1990s in the US economy (Rohit, 2013; Hein, 2012b). Although these mergers and acquisitions have a positive effect on the mark-up and hence on the share of profit

[3] See Rowthorn (1981, pp. 18) for more on Paradox of cost.

in the short run, we are interested to know whether this positive relationship prevails in the long run as well. We show that in a particular scenario, a higher degree of mergers, acquisitions etc. can cause instability for an otherwise stable economy.

The outline of the rest of the paper is as follows. Section 2 sets up the model, talks about the short run analysis and the short run comparative statics. Section 3 discusses the long run where we endogenize the debt-capital ratio and the income distribution of the economy. Section 4 explains different possible cases which may arise due to the interaction between the debt and the distributional dynamics. This is followed by the discussion of some comparative statics in section 5. Section 6 discusses a special case where no consumption demand is generated out of wealth (of rentiers). Section 7 offers some concluding remarks.

2 *The Model*

We assume a simple one-sector, closed economy, neo-Kaleckian growth model in which the economy consists of workers, rentiers and firms. Neither government intervention nor technical progress is there. Income is distributed between wages and profits as

$$Y = W + R \tag{2.1}$$

where, Y is nominal income, W is nominal wage income and R is nominal profit income. We assume that excess supply of labour and under-utilization of capacity prevail in the economy. For simplicity we refrain from depreciation of capital stock, overhead labour, raw materials and intermediate products. We assume that two types of households are there in the economy- workers and rentiers. Workers do not hold any kind of assets and consume their entire wage income and whatever they borrow (net of interest payment) from the banks (effectively they borrow from the rentiers). So,

$$C_W = W + \dot{D} - iD = [(1-\pi)u + \frac{\dot{D}}{K} - id]K \tag{2.2}$$

where, C_W is consumption of workers, K is the existing capital stock, $u = \frac{Y}{K}$ is the degree of capacity utilization[4], D is total debt of workers to the rentiers, d is debt-capital ratio, $\dot{D}$ is the change in debt (amount borrowed from rentiers in that period), $\pi = \frac{R}{Y}$ is share of profit, and $r = \frac{R}{K}$ is profit rate. So, $r = \pi$u. Rentiers hold two types of assets (i) deposit with the banks and (ii) equities that are issued by firms. Equities are

[4]As long as potential output-capital ratio is fixed, actual output-capital ratio can be used as a proxy for degree of capacity utilization.

considered to be a more risky asset compared to bank deposits. Banks play a passive role of allotting those deposits to the workers as credit[5]. Rentiers earn their income from two sources, interest income on the funds they lend to the workers and from a fraction of profit $(1 - s_f)$ given to them as dividend by the firms. Rentiers spend a fraction of their income and a fraction of their assets for consumption purposes (see Modigliani (1986), Boyer (2000), Maki and Palumbo (2001), and Godley and Lavoie (2007, pp. 66), for example). Therefore, consumption of rentiers (C_R) can be represented as

$$C_R = c_r\{(1 - s_f)R + iD\} + c_q(P_e E + D) \tag{2.3}$$

where c_r is the consumption propensity of rentiers out of income and c_q is the consumption propensity out of wealth[6], i is the interest rate on both deposits and loans, s_f is the retention rate of firms. Equation (2.3) can be rewritten as

$$C_R/K = c_r\{(1 - s_f)\pi u + id\} + (1 + \lambda)c_q d \tag{2.4}$$

where $(P_e E + D)/K = (1 + \lambda)d$, and $\lambda = \frac{P_e E}{D}$ is the equity (or value of equity) to deposit (or debt) ratio. Total savings of the renters is used for purchasing new assets i.e.

$$S_R = P_e \dot{E} + \dot{D} \tag{2.5}$$

Rentiers' savings is the difference between total income and consumption. Consequently,

$$\frac{S_R}{K} = [(1 - c_r)\{(1 - s_f)\pi u + id\} - (1 + \lambda)c_q d] \tag{2.6}$$

Let's assume a fixed fraction (δ) of rentiers' saving is being spent on purchasing new equities by them i.e.

$$\frac{P_e \dot{E}}{K} = \delta \frac{S_R}{K} = \delta(1 - c_r)\{(1 - s_f)\pi u + id\} - \delta(1 + \lambda)c_q d \tag{2.7}$$

Hence, the rest fraction $(1 - \delta)$ of the savings is deposited in banks (which effectively goes to the workers as lending[7]) i.e.

$$\frac{\dot{D}}{K} = (1 - \delta)\frac{S_R}{K} = (1 - \delta)(1 - c_r)\{(1 - s_f)\pi u + id\} - (1 - \delta)(1 + \lambda)c_q d \tag{2.8}$$

[5] So effectively the rentiers lend money to the workers.

[6] $c_r \in (0, 1)$, $c_q \in (0, 1)$ and we assume $c_r > c_q$.

[7] Following Hein (2012a) we assume this fraction $(1 - \delta)$ (or δ) depends on several factors like workers' willingness to borrow, rentiers' willingness to supply credit to workers, workers' creditworthiness as perceived by rentiers, regulations of the credit markets and the standards for creditworthiness set by various institutions (e.g. banks) and so on so forth. We assume δ to be an exogenous variable which may shift over time.

Table 2.1: Balance sheet matrix

	Workers' households	Rentiers' households	Firms	$\sum$
Loans	$-D$	$+D$		0
Equities		$+P_eE$	$-P_eE$	0
Capital			K	K
$\sum$	$-D$	$D+P_eE$	$K-P_eE$	K

Table 2.2: Transaction flow matrix

	Workers' households	Rentiers' households	Firms current	Firms capital	$\sum$
Consumption	$-C_W$	$-C_R$	C_W+C_R		0
Investment			I	$-I$	0
Wages	W		$-W$		0
Retained profits			$-s_fR$	s_fR	0
Distributed profits (dividends)		$(1-s_f)R$	$-(1-s_f)R$		0
(Value of) Change in equities		$-P_e\dot{E}$		$P_e\dot{E}$	0
Interest on loans	$-iD$	iD			0
Change in loans	$\dot{D}$	$-\dot{D}$			0
$\sum$	0	0	0	0	0

Following Kaleckian literature we assume a fraction of profit, $(1-s_f)$, is given to the rentiers as dividends. Following Hein (2012a, 2012b) we assume investment function (I) is determined by the expected sales and hence by the rate of capacity utilization (as it is used as a proxy for expected rate of sales) and by the animal spirits of the firms (α) so that we obtain the basic Kaleckian function for investment in the next equation as

$$I=[\alpha+\beta u]K \tag{2.9}$$

The basic structure of the model is summarized by the balance sheet matrix in Table 2.1 and the transaction flow matrix in Table 2.2.

In the short run equilibrium, $Y=C_W+C_R+I$ must hold which implies,

$$u^*=\frac{\alpha+\delta\{(1+\lambda)c_q-(1-c_r)i\}d}{\{s_f+(1-s_f)(1-c_r)\delta\}\pi-\beta}=\frac{\alpha+\delta Ad}{\psi\pi-\beta} \tag{2.10}$$

Where $\psi=\{s_f+(1-s_f)(1-c_r)\delta\}>0$, and $A=\{(1+\lambda)c_q-(1-c_r)i\}\gtreqless 0$. Keynesian stability condition requires responsiveness of investment demand due to a unit change in aggregate demand to be less than that of the savings for the same unit change in aggregate demand, i.e. $\beta<\psi\pi$. Let's assume the Keynesian stability condition is satisfied. For a meaningful degree of capacity utilization, the numerator of the equation (2.10) must be positive i.e. $[\alpha+\delta Ad]>0$. We assume that this condition is also satisfied.

The equilibrium growth rate g^* is equal to $\alpha + \beta u^*$ which in turn implies

$$g^* = \frac{\alpha\pi\{s_f + (1-s_f)(1-c_r)\delta\} + \beta\delta Ad}{\psi\pi - \beta} \tag{2.11}$$

The equilibrium rate of profit is

$$r^* = \pi u^* \tag{2.12}$$

Differentiating u^*, g^* and r^* w.r.t. π we get,

$$\frac{\partial u^*}{\partial \pi} = -\frac{[\alpha + \delta Ad]\,\psi}{[\psi\pi - \beta]^2} < 0,\ \frac{\partial g^*}{\partial \pi} = -\frac{\beta\,[\alpha + \delta Ad]\,\psi}{[\psi\pi - \beta]^2} < 0,\ \frac{\partial r^*}{\partial \pi} = -\frac{\beta\,[\alpha + \delta Ad]}{[\psi\pi - \beta]^2} < 0 \tag{2.13}$$

As profit share rises, the equilibrium degree of capacity utilization, the profit rate and the equilibrium rate of capital accumulation, all decrease i.e. the paradox of costs occurs in our analysis. So the economy is always in a wage-led demand and wage-led growth regime.

The short run comparative statics are summarized as follows:

$$\frac{\partial u^*}{\partial \delta} = -\frac{[(1-s_f)(1-c_r)\pi u^* - Ad]}{\{s_f + (1-s_f)(1-c_r)\delta\}\pi - \beta} \gtreqless 0,\ \frac{\partial u^*}{\partial i} = \frac{-\delta(1-c_r)d}{\psi\pi - \beta} < 0,$$

$$\frac{\partial u^*}{\partial d} = \frac{\delta A}{\psi\pi - \beta} \gtreqless 0,\ \frac{\partial u^*}{\partial \lambda} = \frac{\delta c_q d}{\psi\pi - \beta} > 0,\ \frac{\partial u^*}{\partial c_q} = \frac{\delta(1+\lambda)d}{\psi\pi - \beta} > 0,$$

$$\frac{\partial u^*}{\partial c_r} = \frac{(1-s_f)\delta\pi u^* + id\delta}{\psi\pi - \beta} > 0,\ \frac{\partial u^*}{\partial s_f} = -\frac{\{\alpha + \delta Ad\}\,\{1 - (1-c_r)\delta\}\,\pi}{[\psi\pi - \beta]^2} < 0$$

For a given debt-capital ratio, an increase in the proportion of rentiers' savings lent to workers in Hein (2012) is expansionary in the short run i.e. $\frac{\partial u^*}{\partial \delta} < 0$. However, in our model that may not necessarily be the case and it depends on the sign and the absolute value of A. The possibility that $\frac{\partial u^*}{\partial \delta}$ is positive arises because we have considered the possibility of rentiers' consumption out of wealth. If rentiers' propensity to consume out of wealth and the equity to debt ratio are sufficiently high, these can more than compensate the loss in consumption demand of workers due to a reduction in the proportion of rentiers' savings lent to workers and we get $\frac{\partial u^*}{\partial \delta} > 0$. The same is true for the equilibrium rate of capital accumulation as well.

A rise in the rate of interest lowers the equilibrium degree of capacity utilization and accumulation rate. This is happening due to the fact that income is redistributed from workers (who have higher propensity to consume than rentiers) to rentiers through interest payments. However, for the debt-capital ratio, the result is ambiguous. For each unit rise in debt-capital ratio, from equations (2.2) & (2.8) it is clear that workers' consumption demand (in form of the fraction of capital stock) falls by $[(1-\delta)A + i]$ unit while from equation (2.4) we get that rentiers' consumption demand (in form of the fraction of capital stock) rises by $[(1+\lambda)c_q + c_r i]$ unit. Thus overall change in consumption demand in the economy equals to δA. As long as $A > 0$, a rise in d rises the equilibrium degree of capacity utilization. Otherwise a rise in d has a contractionary effect on the aggregate demand. Thus whether a debt-led or a debt-burdened demand regime prevails in the economy depends on the sign of A. Same is applicable for a debt-led or a debt-burdened growth regime. Hence $A > 0$ ensures that the economy is in a debt-led demand and a debt-led growth regime. On the other hand $A < 0$ ensures that the economy is in a debt-burdened demand and a debt-burdened growth regime.

For a given debt-capital ratio (d), if the equity-debt ratio (λ) rises by one unit, rentiers' consumption demand (per unit of capital) rises by $c_q d$ units (see equation (2.3)) . However, higher the consumption demand (in form of the fraction of capital stock) of the rentiers, lower is their savings and so lower is the addition to asset in terms of deposits which go as loans to the workers. As a result, from equations (2.2) & (2.8), workers' consumption demand (per unit of capital stock) falls by $(1-\delta)c_q d$ unit. Thus the rise in consumption demand (per unit of capital) of rentiers' is mitigated to some extent by the fall in workers' consumption demand. Nonetheless, the overall effect of a rise in λ on aggregate demand is positive. A rise in λ raises the equilibrium rate of capital accumulation and profit rate as well.

So, *ceteris paribus*, higher the dividend payout ratio (or lower the retention rate s_f), higher is the equilibrium degree of capacity utilization and accumulation rate. The reason is two fold. First, a rise in dividend-payout ratio raises the rentiers' consumption demand through a rise in their income. Second, the rest of the increase in income is saved by rentiers. As we know, a fraction of rentiers assets is in the form of bank deposit. So, when rentiers' saving increases, workers borrowing also rises, and as a result workers consumption demand rises. So, a higher dividend-payout ratio means higher consumption demand for the entire economy since firms save the entire undistributed profit. Note that in our model, investment demand is not constrained by any kind of savings including the internal source of funds. In fact in our model, as it represents the demand-constrained economy, savings are not necessarily invested.

The above short run comparative static results are encapsulated in Table 2.3.

3 *Long Run*

In this section, we analyse the distributional dynamics of the economy and the dynamics of the debt-capital ratio. We assume that the short run equilibrium values are always

Table 2.3: Impact of changes in various parameters on u^*, g^* and r^*

	u^*	g^*	r^*
π	-	-	-
δ	+/-	+/-	+/-
i	-	-	-
d	+/-	+/-	+/-
λ	+	+	+
c_q	+	+	+
c_r	+	+	+
s_f	-	-	-

attained in the long run. The long run equilibrium is defined as where the share of profit and the debt-capital ratio remain constant over time.

3.1 *Dynamics of the debt-capital ratio*

As $d = \frac{D}{K}$, change in debt-capital ratio with respect to time can be represented as

$$\dot{d} = \frac{\dot{D}}{K} - dg \tag{3.1}$$

Inserting the values of $\frac{\dot{D}}{K}$, g^*, and u^* from equations (2.8), (2.11) and (2.10) yields,

$$\dot{d} = \frac{-\beta\delta A d^2 + [A(1-\delta)(\beta - s_f\pi) - \psi\pi\alpha]\, d + (1-\delta)(1-s_f)(1-c_r)\alpha\pi}{\psi\pi - \beta} \tag{3.2}$$

In the long run equilibrium $\dot{d} = 0$. Therefore, from equation (3.2) we get,

$$A\beta\,\{\delta d - (1-\delta)\}\, d + \{Ed - F\}\,\pi = 0 \tag{3.3}$$

$$\implies \pi\Big|_{\dot{d}=0} = \left\{\frac{-\beta A d\{\delta d - (1-\delta)\}}{Ed - F}\right\} \qquad (\text{provided } d \neq \frac{F}{E}) \tag{3.4}$$

where $E = \{A(1-\delta)s_f + \psi\alpha\} \gtreqless 0$ and $F = (1-\delta)(1-s_f)(1-c_r)\alpha > 0$. Vertical intercept of the $\dot{d} = 0$ isocline is $\pi\Big|_{\dot{d}=0}^{d=0} = 0$. There are two non-negative values of d for which $\pi\Big|_{\dot{d}=0} = 0$: $d = 0$ and $d = \frac{1-\delta}{\delta}$. We can write equation (3.2) as,

$$\dot{d} = \frac{-hd^2 + ld + m}{\psi\pi - \beta} \tag{3.5}$$

where $h = \beta\delta A$, $m = (1-\delta)(1-s_f)(1-c_r)\alpha\pi > 0$ and $l = [A(1-\delta)(\beta - s_f\pi) - \psi\pi\alpha]$. If the economy is in a debt-burdened growth regime (i.e. if $A < 0$) then $h = \beta\delta A < 0$. Then for achieving a steady state d we need l to be negative here. $l = [A(1-\delta)(\beta - s_f\pi) - \psi\pi\alpha] =$

$[A(1-\delta)\beta - E\pi]$. If $E > 0$ then l is unambiguously negative. But if $E < 0$ then $l < 0$ provided $\beta > \frac{E\pi}{A(1-\delta)}$. Differentiating equation (3.5) w.r.t. d we get,

$$J_{11} = \frac{\partial \dot{d}}{\partial d} = \frac{-2hd + l}{\psi\pi - \beta} \gtreqless 0 \text{ depending on} d \gtreqless \hat{d} = \frac{l}{2h} \tag{3.6}$$

Differentiating equation (3.2) w.r.t. π and rearranging it we get,

$$J_{12} = \frac{\partial \dot{d}}{\partial \pi} = \frac{-(Ed - F)}{\psi\pi - \beta} \tag{3.7}$$

First we assume $a < 0$ i.e $E\delta < 0$, which implies $E < 0$. So $(Ed - F)$ is unambiguously negative. Hence $J_{12} = \frac{\partial \dot{d}}{\partial \pi} > 0$. Now, let us consider $a > 0$ i.e $E\delta > 0$, which implies $E > 0$. If $d > \frac{F}{E}$ then $(Ed - F) > 0$, and therefore $J_{12} = \frac{\partial \dot{d}}{\partial \pi} < 0$. On the other hand, if $d < \frac{F}{E}$, $(Ed - F)$ becomes negative. Hence, $J_{12} = \frac{\partial \dot{d}}{\partial \pi} > 0$. Slope of the $\dot{d} = 0$ isocline is $\frac{d\pi}{dd}\Big|_{\dot{d}=0} = -\frac{J_{11}}{J_{12}} = \frac{-2hd+l}{(Ed-F)}$. If $(Ed - F) < 0$, the slope is positive for all $d < \frac{l}{2h}$, and negative for all $d > \frac{l}{2h}$. See Figure 3.3a & 3.3b for the diagram of the $\dot{d} = 0$ isocline. On the other hand if $(Ed - F) > 0$, the slope is negative for all $d < \frac{l}{2h}$ and positive for all $d > \frac{l}{2h}$. See Figure 3.3a for the diagram of the $\dot{d} = 0$ isocline. Remember that $-2hd + l = 0$ implies $\pi = \frac{A(1-\delta)\beta}{E} - \frac{2\beta\delta A}{E} d$. When $a < 0$ we get a negatively sloped straight line for $-2hd + l = 0$, and when $a > 0$, we get a positively sloped straight line for $-2hd + l = 0$.

In what follows we explain equations (3.6), and (3.7) respectively. J_{11} shows the effect of an increase in the debt-capital ratio on a change in the debt-capital ratio itself. When the economy is in a debt-led growth regime, for $d \in (\frac{F}{E}, \frac{1-\delta}{\delta})$, a rise in d has a negative effect on the change in the debt-capital ratio itself i.e. $J_{11} < 0$. The reason is as follows. A rise in d increases the the capacity utilization rate by $\frac{A\delta}{\psi\pi-\beta}$ unit. As a result, the debt level (normalized by the capital stock) changes by $\frac{A(1-\delta)(\beta - s_f\pi)}{\psi\pi-\beta}$ unit (as from equation (2.8) we get $\frac{\partial\left(\frac{\dot{D}}{K}\right)}{\partial d} = \frac{A(1-\delta)(\beta-s_f\pi)}{\psi\pi-\beta}$). For a rise in d, second term of the right hand side of equation (3.1) i.e. dg rises by $\frac{2A\delta\beta d+\alpha\psi\pi}{\psi\pi-\beta}$ unit (as $\frac{\partial(dg)}{\partial d} = g + d\frac{\partial g}{\partial d} = \frac{2A\delta\beta d+\alpha\psi\pi}{\psi\pi-\beta}$). As we know $\dot{d} = \frac{\dot{D}}{K} - dg$, for a rise in d, $\dot{d}$ changes by $\frac{-A\beta\{2\delta d-(1-\delta)\}-\{A(1-\delta)s_f+\alpha\psi\}\pi}{\psi\pi-\beta}$ unit. Here $\{A(1-\delta)s_f + \alpha\psi\}\pi = E\pi > 0$ and dominates $A\beta\{2\delta d - (1-\delta)\}$ (as here the value of d is so high that $A\beta\{2\delta d - (1-\delta)\}$ becomes small in size making the difference between $2\delta d$ and $(1-\delta)$ is very small) and hence we get $J_{11} = \frac{\partial d^*}{\partial d} < 0$ i.e. the self-feedback effect of the debt-capital ratio is negative in $\frac{F}{E} < d \leq \frac{1-\delta}{\delta}$ region .

However, when the economy is in a debt-burdened growth regime, a rise in d has an ambiguous effect on the change in the debt-capital ratio and it depends on the level of debt-capital ratio. The reason is as follows. A rise in d increases the capacity utilization rate by $\frac{A\delta}{\psi\pi-\beta}$ unit. As a result, the debt level (normalized by the capital stock) changes by $\frac{A(1-\delta)(\beta-s_f\pi)}{\psi\pi-\beta}$ unit. On the other hand, as $\dot{d} = \frac{\dot{D}}{K} - dg^*$, in the debt-burdened growth regime a rise in d reduces g^* and so the net effect on (dg^*) is ambiguous. So the final result of a rise in d on $\dot{d}$ is ambiguous here and depends on the combination of d and π.[8] If $d < \frac{l}{2h}$, a rise in d negatively affects the change in the debt-capital ratio i.e. $J_{11} < 0$. On the other hand a higher level of d (i.e. if $d > \frac{l}{2h}$) has a positive effect on the change in the debt-capital ratio and so $J_{11} > 0$.

J_{12} shows the effect of an increase in profit share on change in debt-capital ratio. Let us first concentrate on the first term of the right hand side of equation (3.1) i.e. on $\frac{\dot{D}}{K}$. As the economy is in a wage-led demand regime, a rise in profit share reduces u^*. This fall in u^* overcompensates the rise in π. So, there is a deterioration in rentiers' level of income. As a result, rentiers' consumption as well as savings both fall. As rentiers save a fraction $(1-\delta)$ of their savings in banks as deposits (which effectively goes to the workers as lending), the debt level (normalized by the capital stock) decreases. The economy is in a wage-led growth regime as well. Hence, for a given d, a rise in π leads to a deterioration in the investment rate. Hence the second term of the right hand side of equation (3.1) i.e. dg^* falls. Consequently, the final result of a rise in π on $\dot{d}$ is ambiguous and depends on the level of d.

There is another way of deriving the slope of the $\dot{d} = 0$ isocline. This will be useful for drawing the diagram of the $\dot{d} = 0$ isocline. In what follows we explain the other method now. Differentiating equation (3.4) w.r.t. d and rearranging it we get the slope as,

$$\left.\frac{d\pi}{dd}\right|_{\dot{d}=0} = \frac{-A\beta\left[ad^2 - bd + c\right]}{\left[Ed - F\right]^2} \tag{3.8}$$

where $a = E\delta \gtreqless 0$, $c = F(1-\delta) > 0$, $b = 2\delta F > 0$ and $\eta = [A(1-\delta)s_f + s_f\alpha] \gtreqless 0$.[9] Depending on the sign of A two different cases are possible: *case 1* and *case 2* respectively.

Case 1: In *case 1* we assume $A > 0$. Therefore, $\eta > 0$, $E > 0$, and $a > 0$. As $A > 0$, we conclude that in *case 1*, the economy is in a debt-led demand and growth[10] regime. *Case 1* is represented in Figure 3.2a.[11]

When $0 < d < \frac{F}{E} < \frac{1-\delta}{\delta}$, $A\beta\{\delta d - (1-\delta)\}$ and $(Ed - F)$ both are negative. So here the profit share is negative (see equation (3.4). From $d = 0$, as d rises, first term of the

[8] As l is a function of π.

[9] One can show that $\left.\frac{d\pi}{dd}\right|_{\dot{d}=0} = \frac{-A\beta\left[ad^2-bd+c\right]}{[Ed-F]^2} = \frac{-2hd+l}{(Ed-F)}$. Consider $(-2hd+l)$. $(-2hd+l) = [-2\beta\delta Ad + A\beta(1-\delta) - E\pi]$. Now putting the equilibrium value of π from equation (3.4) and rearranging we get $[-2\beta\delta Ad + A\beta(1-\delta) - E\pi] = \frac{(-2\beta\delta Ad + A\beta(1-\delta))(Ed-F) + A\beta\delta Ed^2 - (1-\delta)A\beta Ed}{(Ed-F)^2} = \frac{-A\beta\{\delta Ed^2 - 2\delta Fd + (1-\delta)F\}}{(Ed-F)^2}$ $= \frac{-A\beta\{ad^2-bd+c\}}{(Ed-F)^2}$. So $\frac{-2hd+l}{(Ed-F)} = \frac{-A\beta\left[ad^2-bd+c\right]}{[Ed-F]^2}$.

[10] As $\frac{\partial g^*}{\partial d} = \beta\frac{\partial u^*}{\partial d} > 0$ here.

[11] Here, $\frac{F}{E} = \frac{(1-\delta)(1-s_f)(1-c_r)\alpha}{\{A(1-\delta)s_f + \psi\alpha\}} = \frac{(1-\delta)(1-s_f)(1-c_r)\alpha}{\underbrace{[A(1-\delta)s_f + s_f\alpha]}_{+} + (1-s_f)(1-c_r)\alpha\delta} < \frac{(1-\delta)(1-s_f)(1-c_r)\alpha}{(1-s_f)(1-c_r)\alpha\delta} = \frac{1-\delta}{\delta}$.

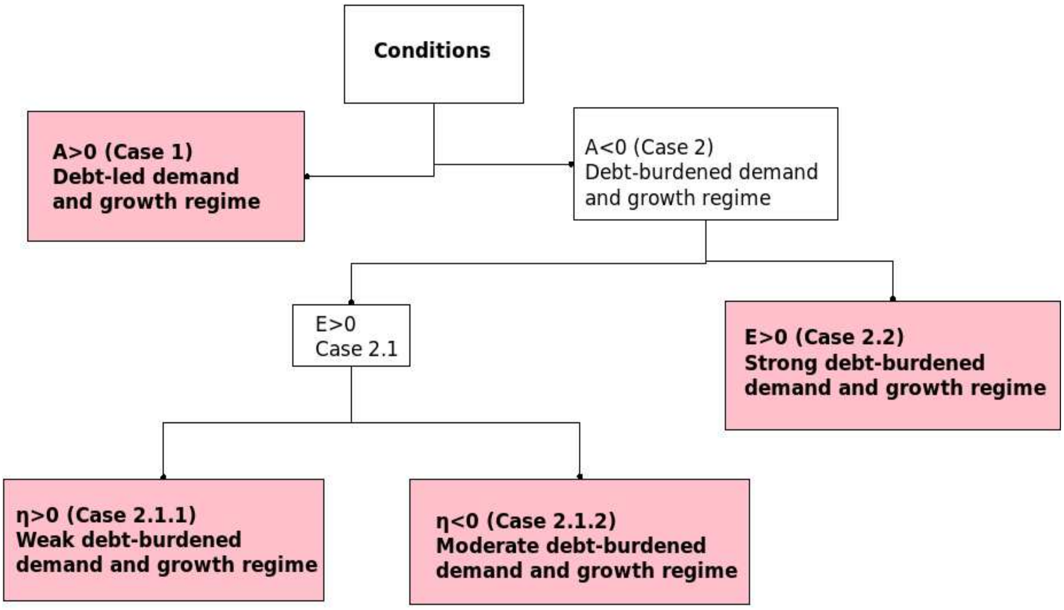


Figure 3.1: Flowchart of various cases related to $\dot{d} = 0$ isocline

left hand side of equation (3.3) falls (i.e. the magnitude of $A\beta\{\delta d - (1-\delta)\}d$ rises). This is happening due to the fact that as d is very small in size, the linear term $(1-\delta)d$ dominates the quadratic term δd^2. Consequently, for the debt market to be in equilibrium (i.e. for $\dot{d} = 0$ to achieve), profit share must fall. Note that as d approaches toward $\frac{F}{E}$, $(Ed - F)$ becomes smaller and smaller. Therefore for a small rise in d, a larger decrease in π is required. That is why we get a vertical asymptote at $d = \frac{F}{E}$.

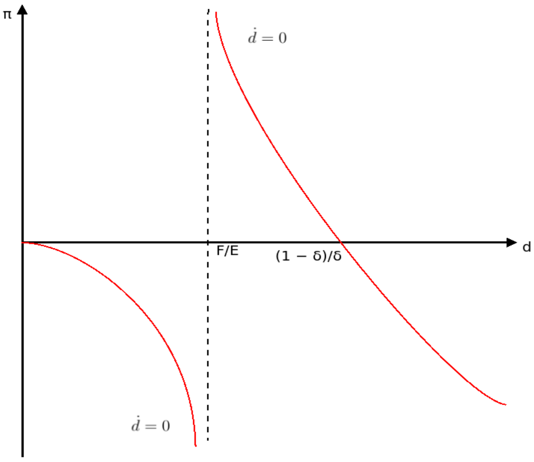
Now consider $\frac{F}{E} < d < \frac{1-\delta}{\delta}$ region. Here $A\beta\{\delta d - (1-\delta)\} < 0$ but $(Ed - F) > 0$. As a result, equation (3.4) suggests π to be positive. From $\frac{F}{E}$, as d rises towards $\frac{1-\delta}{\delta}$, the quadratic term δd^2 dominates the linear term $(1-\delta)d$, and so the magnitude of $A\beta\{\delta d - (1-\delta)\}d$ now falls. For the debt market to be in equilibrium, as $(Ed - F) > 0$ here, profit share must fall. Note that near $\frac{F}{E}$, as $(Ed - F)$ becomes very small, for a small fall in d, a larger increase in π is required for achieving equilibrium in the debt market.

$\forall d \in (0, \frac{F}{E})$ we get $\pi\Big|_{\dot{d}=0} = \left\{\frac{\overbrace{-\beta Ad}^{+}\overbrace{\{\delta d-(1-\delta)\}}^{-}}{\underbrace{(Ed-F)}_{-}}\right\} < 0$; $\forall d \in (\frac{F}{E}, \frac{1-\delta}{\delta})$ we get $\pi\Big|_{\dot{d}=0} = \left\{\frac{\overbrace{-\beta Ad}^{+}\overbrace{\{\delta d-(1-\delta)\}}^{-}}{\underbrace{(Ed-F)}_{+}}\right\} > 0$ and $\forall d \in (\frac{1-\delta}{\delta}, \infty)$ we get $\pi\Big|_{\dot{d}=0} = \left\{\frac{\overbrace{-\beta Ad}^{+}\overbrace{\{\delta d-(1-\delta)\}}^{+}}{\underbrace{(Ed-F)}_{+}}\right\} < 0$. Note that the $\dot{d} = 0$ isocline has vertical asymptote at $d = \frac{F}{E}$.

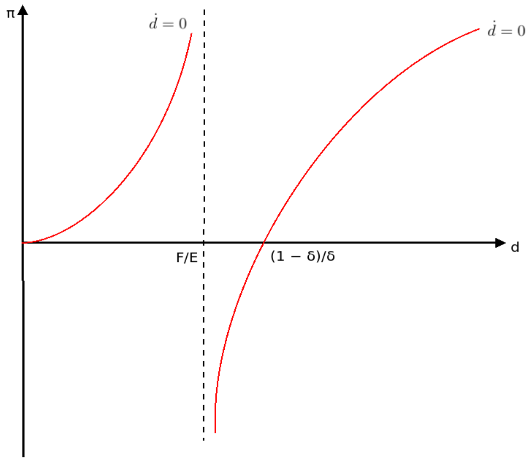
When $d = 0$, from equation (3.8) we get $\frac{d\pi}{dd}\Big|_{\dot{d}=0}^{d=0} = \frac{-A\beta c}{F^2} < 0$ and as d approaches to infinity we get $\lim_{d\to\infty}\left\{\frac{d\pi}{dd}\Big|_{\dot{d}=0}\right\} = -\frac{A\beta\delta}{E} < 0$. Now differentiating equation (3.8) we get, $\frac{d^2\pi}{dd^2}\Big|_{\dot{d}=0} = \frac{2AF\beta}{(Ed-F)^3}(1-\delta)\{(1-\delta)As_f + s_f\alpha\}$. Therefore, $\forall d < \frac{F}{E}$, we get $\frac{d^2\pi}{dd^2}\Big|_{\dot{d}=0} < 0$ and $\forall d > \frac{F}{E}$ we get $\frac{d^2\pi}{dd^2}\Big|_{\dot{d}=0} > 0$.

Table 3.1: Illustrative values of parameters and variables in different *cases*

Case	c_r	c_q	s_f	α	β	δ	λ	i	π	d
Case 1	0.4	0.03	0.33	0.02	0.125	0.5	0.8	0.01	0.34	0.5
Case 2.1.1	0.3	0.01	0.33	0.02	0.125	0.5	0.5	0.03	0.34	0.5
Case 2.1.2	0.4	0.01	0.33	0.01	0.125	0.3	0.3	0.05	0.34	0.5
Case 2.2	0.4	0.007	0.33	0.01	0.125	0.3	0.2	0.05	0.34	0.5



(a) $\dot{d}=0$ isocline: *case 1*



(b) $\dot{d}=0$ isocline: *case 2.1.1*

Figure 3.2

Beyond $\frac{1-\delta}{\delta}$, however, a positive value of d is related to a negative share of profit. This is happening because here $A\beta\{\delta d-(1-\delta)\}$ and $(Ed-F)$ both are positive.

Let us take a numerical example. Table 3.1 represents an illustrative set of possible values of parameters and variables[12]. From those values we get $\psi=0.531$, $\psi\pi=0.18054$, $A=0.048>0$, $\eta=0.01452>0$, $E=0.016932>0$ and $u^*=0.57616$. Keynesian stability condition $(\psi\pi>\beta)$ is also satisfied here. Therefore, *case 1* is very much possible.

Case 2: Here $A<0$ i.e. the economy is in a debt-burdened demand and growth regime. Two sub-cases are possible here. *Case 2.1*, where $E>0$, and *case 2.2,* where $E<0$. However, *case 2.1* can be split into two more sub-cases (i) *case 2.1.1* where $\eta>0$ and (ii) *case 2.1.2* where $\eta<0$. Let's analyse all of them one by one.

Case 2.1.1: Here $A<0$, $E>0$, $a>0$, and $\eta>0$. So, here even though $A<0$, it is not strong enough to make E negative. Therefore, the economy is in a weak debt-burdened demand and growth regime in *case 2.1.1.* Figure 3.2b illustrates it diagrammatically.[13]

[12] We take $\pi=0.34$ from Stockhammer (2006), $\beta=0.125$ from Skott and Ryoo (2008) and $s_f=0.33$ from Ryoo (2016) and c_r in between 0.3 and 0.4 from Godley and Lavoie (2007). Onaran *et al.* (2011) finds c_q to be 0.007 whereas $c_q=0.02$ is the conventional value for the USA (Girouard and Blondal, 2001).

[13] Here, $\frac{F}{E}=\frac{(1-\delta)(1-s_f)(1-c_r)\alpha}{\{A(1-\delta)s_f+\psi\alpha\}}=\frac{(1-\delta)(1-s_f)(1-c_r)\alpha}{\underbrace{[A(1-\delta)s_f+s_f\alpha]}_{+}+(1-s_f)(1-c_r)\alpha\delta}<\frac{(1-\delta)(1-s_f)(1-c_r)\alpha}{(1-s_f)(1-c_r)\alpha\delta}=\frac{1-\delta}{\delta}.$

Let us discuss the diagram intuitively. As $A<0$, as long as $d<\frac{1-\delta}{\delta}$, $A\beta\{\delta d-(1-\delta)\}>0$ and as long as $d<\frac{F}{E}$, $(Ed-F)<0$. When d is very small, for a rise in debt-capital ratio (d), first term of the left hand side of equation (3.3) rises by $A\beta\{2\delta d-(1-\delta)\}$ unit,[14] and the second term of L.H.S. of equation (3.3) rises by $E\pi$ unit. So, a rise in d leads to a rise in the L.H.S. of equation (3.3) by $A\beta\{2\delta d-(1-\delta)\}+E\pi$ unit.[15] As $(Ed-F)<0$, for the debt market to be in equilibrium, from equation (3.3), π must rise. As a result, we get a positively sloped $\dot{d}=0$ isocline for $d\in(0,\frac{F}{E})$. However, as d approaches towards $\frac{F}{E}$, $(Ed-F)$ approaches to zero. Hence, for a small rise in d, for the debt market to be in equilibrium, now a very large increment in π is required. That is why $\dot{d}=0$ isocline has a vertical asymptote at $d=\frac{F}{E}$.

When $\frac{F}{E}<d<\frac{1-\delta}{\delta}$, $A\beta\{\delta d-(1-\delta)\}$ and $(Ed-F)$ both are positive. Therefore, π is negative in this region (see equation (3.4)). If d rises, the gap between δd and $(1-\delta)$ shrinks and it shrinks so much that a fall in the magnitude of $A\beta\{\delta d-(1-\delta)\}$ overcompensate the rise in d. Hence the first term of equation (3.3) falls. For the debt market to be in equilibrium, the second term must rise and it is possible if the profit share rises. Thus for $d\in(\frac{F}{E},\frac{1-\delta}{\delta})$, we get a positively sloped $\dot{d}=0$ isocline. However, for $d>\frac{1-\delta}{\delta}$, $A\beta\{\delta d-(1-\delta)\}<0$ and $(Ed-F)>0$. Consequently, d and π both are positive here. As debt-capital ratio rises beyond $\frac{1-\delta}{\delta}$, first term of equation (3.3) falls (the magnitude $A\beta\{\delta d-(1-\delta)\}d$ rises). As a result, for the debt market to be in equilibrium, share of profit must rise. Thus for $d>\frac{1-\delta}{\delta}$, we get a positively sloped $\dot{d}=0$ isocline.

The numerical example in Table 3.1 suggests $\psi=0.5645$, $\psi\pi=0.19193$, $A=-0.006<0$, $\eta=0.00561>0$, $E=0.0103>0$, and $u^{*}=0.276408188$. Keynesian stability condition $(\psi\pi>\beta)$ is also satisfied here.

From equation (3.4) we get that at $d=0$ and at $d=(\frac{1-\delta}{\delta})$, $\pi\Big|_{\dot{d}=0}=0$. $\forall d\in(0,\frac{F}{E})$ we get $\pi\Big|_{\dot{d}=0}=\left\{\dfrac{-\beta A\overbrace{d}^{-}\overbrace{\{\delta d-(1-\delta)\}}^{-}}{\underbrace{(Ed-F)}_{-}}\right\}>0$; $\forall d\in(\frac{F}{E},\frac{1-\delta}{\delta})$ we get $\pi\Big|_{\dot{d}=0}=\left\{\dfrac{-\beta A\overbrace{d}^{-}\overbrace{\{\delta d-(1-\delta)\}}^{-}}{\underbrace{(Ed-F)}_{+}}\right\}<0$ and $\forall d\in(\frac{1-\delta}{\delta},\infty)$ we get $\pi\Big|_{\dot{d}=0}=\left\{\dfrac{-\beta A\overbrace{d}^{-}\overbrace{\{\delta d-(1-\delta)\}}^{+}}{\underbrace{(Ed-F)}_{+}}\right\}>0$. Note that the $\dot{d}=0$ isocline has vertical asymptote at $d=\frac{F}{E}$.

When $d=0$, from equation (3.8) we get $\frac{d\pi}{dd}\Big|_{\dot{d}=0}^{d=0}=\frac{-A\beta c}{F^{2}}>0$ and as d approaches to infinity we get $\lim_{d\to\infty}\left\{\frac{d\pi}{dd}\Big|_{\dot{d}=0}\right\}=-\frac{A\beta\delta}{E}>0$. Now from equation (3.8) we get,

$$\frac{d^{2}\pi}{dd^{2}}\Big|_{\dot{d}=0}=\frac{2AF\beta}{(Ed-F)^{3}}(1-\delta)\left\{(1-\delta)As_{f}+s_{f}\alpha\right\} \tag{3.9}$$

So, $\forall d<\frac{F}{E}$, we get $\frac{d^{2}\pi}{dd^{2}}\Big|_{\dot{d}=0}>0$ and $\forall d>\frac{F}{E}$ we get $\frac{d^{2}\pi}{dd^{2}}\Big|_{\dot{d}=0}<0$.

[14] This is because for small values of d the linear term $(1-\delta)d$ dominates the quadratic term δd^{2}.

[15] $\frac{\partial}{\partial d}\left[A\beta\{\delta d-(1-\delta)\}d+(Ed-F)\pi\right]=A\beta\{2\delta d-(1-\delta)\}+E\pi$

Case 2.1.2: Here $A < 0$, $E > 0$, $\eta < 0$, and $a > 0$. Although A is negative and it makes η negative, it (A) is not strong enough to make E negative. Therefore, here the economy is in a moderate debt-burdened demand and growth regime. *Case 2.1.2* is represented in Figure 3.3a.[16]

Let us discuss Figure 3.3a intuitively. As $A < 0$, as long as $d < \frac{1-\delta}{\delta}$, $A\beta\{\delta d - (1-\delta)\} > 0$. Starting from zero as debt-capital ratio (d) rises, initially the linear term $(1-\delta)d$ dominates the quadratic term δd^2 and leads to a rise in the first term $A\beta\{\delta d - (1-\delta)\}d$ of the left hand side (L.H.S.) of equation (3.3) by $A\beta\{2\delta d - (1-\delta)\}$ unit. At the same time, second term $(Ed - F)\pi$ of the the left hand side of equation (3.3) rises by $E\pi$ unit. Thus, for a rise in d, left hand side of equation (3.3) rises by $A\beta\{2\delta d - (1-\delta)\} + E\pi$ unit. To achieve equilibrium in the debt market, as $(Ed - F) < 0$, profit share must rise. That is why initially we get a positively sloped $\dot{d} = 0$ isocline. As d rises further, the quadratic term δd^2 dominates the linear term $(1-\delta)d$. As a consequence, $A\beta\{2\delta d - (1-\delta)\}d$ becomes negative and dominates $E\pi$[17]. So, for a rise in d, L.H.S. of equation (3.3) falls. As $(Ed - F) < 0$, profit share must fall to satisfy $\dot{d} = 0$. Hence, for a relatively large debt-capital ratio (i.e. when $d \in (d'', \frac{1-\delta}{\delta}]$), a negatively sloped $\dot{d} = 0$ isocline is obtained. Thus, for $d \in [0, \frac{1-\delta}{\delta}]$, we get an inverted "U" shaped $\dot{d} = 0$ isocline.

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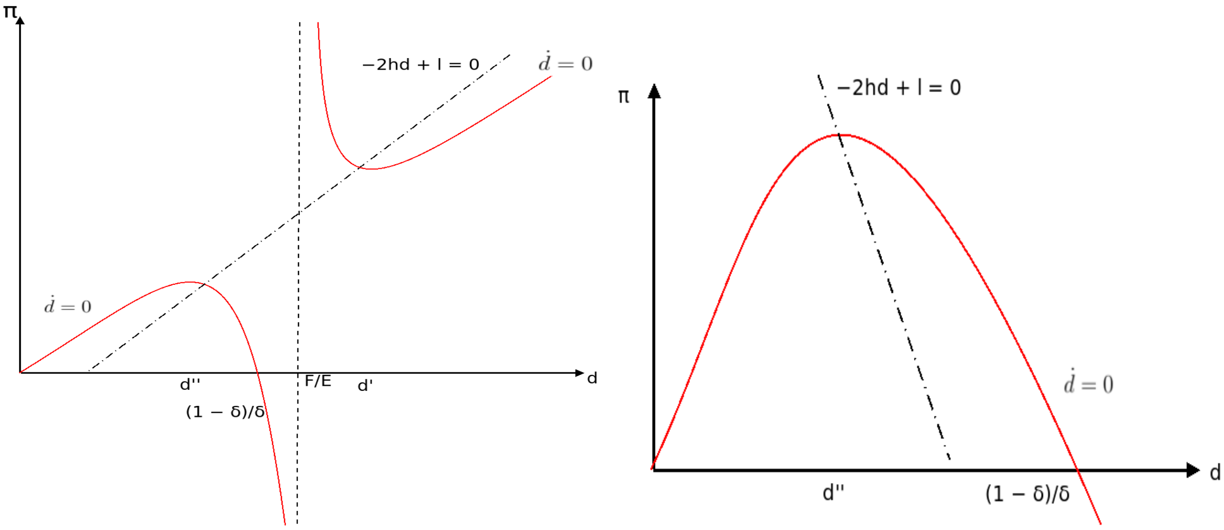
[16]Here, $\frac{F}{E} = \frac{(1-\delta)(1-s_f)(1-c_r)\alpha}{\{A(1-\delta)s_f + \psi\alpha\}} = \frac{(1-\delta)(1-s_f)(1-c_r)\alpha}{\underbrace{[A(1-\delta)s_f + s_f\alpha]}_{-} + (1-s_f)(1-c_r)\alpha\delta} > \frac{(1-\delta)(1-s_f)(1-c_r)\alpha}{(1-s_f)(1-c_r)\alpha\delta} = \frac{1-\delta}{\delta}$. From equation (3.4) we get that at $d = 0$ and at $d = (\frac{1-\delta}{\delta})$, $\pi\Big|_{\dot{d}=0} = 0$. $\forall d \in (0, \frac{1-\delta}{\delta})$ we get $\pi\Big|_{\dot{d}=0} = \left\{\frac{-\overbrace{\beta A}^{-} d \overbrace{\{\delta d - (1-\delta)\}}^{-}}{\underbrace{(Ed - F)}_{-}}\right\} > 0$; $\forall d \in (\frac{1-\delta}{\delta}, \frac{F}{E})$ we get $\pi\Big|_{\dot{d}=0} = \left\{\frac{-\overbrace{\beta A}^{-} d \overbrace{\{\delta d - (1-\delta)\}}^{+}}{\underbrace{(Ed - F)}_{-}}\right\} < 0$ and $\forall d \in (\frac{F}{E}, \infty)$ we get $\pi\Big|_{\dot{d}=0} = \left\{\frac{-\overbrace{\beta A}^{-} d \overbrace{\{\delta d - (1-\delta)\}}^{+}}{\underbrace{(Ed - F)}_{+}}\right\} > 0$. Note that the $\dot{d} = 0$ isocline has vertical asymptote at $d = \frac{F}{E}$. From equation (3.8), $\left[ad^2 - bd + c\right] = 0$ implies $d' = \frac{b+\sqrt{b^2-4ac}}{2a}$ and $d'' = \frac{b-\sqrt{b^2-4ac}}{2a}$. Consider the discriminant $\Delta = b^2 - 4ac$. For the discriminant to be positive we need $\Delta = b^2 - 4ac > 0$. After some calculation, we get that $\Delta = b^2 - 4ac > 0$ is equivalent to $[A(1-\delta)s_f + s_f\alpha] = \eta < 0$. So, the discriminant ($\Delta$) is positive if and only if $\eta < 0$. This is satisfied here. Rearrangement of equation (3.8) yields $\frac{d\pi}{dd}\Big|_{\dot{d}=0} = \frac{-A\beta a(d-d')(d-d'')}{[Ed-F]^2}$. Here $d' = \frac{\overbrace{b + \sqrt{b^2 - 4ac}}^{+}}{\underbrace{2a}_{+}} > 0$. d'' is also positive as $b > \Delta > 0$. Thus we get $0 < d'' < d'$. Here $\frac{d\pi}{dd}\Big|_{\dot{d}=0} = \frac{\overbrace{-A\beta a}^{+}(d-d')(d-d'')}{[Ed-F]^2}$ and so $\forall d \in [0, d'')$, $\frac{d\pi}{dd}\Big|_{\dot{d}=0} > 0$; $\forall d \in (d'', d')$, $\frac{d\pi}{dd}\Big|_{\dot{d}=0} < 0$ and finally $\forall d \in (d', \infty)$, $\frac{d\pi}{dd}\Big|_{\dot{d}=0} > 0$. Again $d' = \frac{\overbrace{b + \sqrt{b^2 - 4ac}}^{+}}{\underbrace{2a}_{+}} = \frac{b}{2a} + \frac{\sqrt{b^2-4ac}}{2a} = \frac{(1-\delta)(1-s_f)(1-c_r)\alpha}{\{A(1-\delta)s_f + \psi\alpha\}} + \frac{\sqrt{b^2-4ac}}{2a} = \frac{F}{E} + \frac{\sqrt{b^2-4ac}}{2a} > \frac{F}{E} > \frac{1-\delta}{\delta} > 0$.

[17]As $-2hd + l = -2\beta\delta Ad + (1-\delta)A\beta - E\pi = -[A\beta\{2\delta d - (1-\delta)\} + E\pi] > 0$ here. This is possible only when $A\beta\{2\delta d - (1-\delta)\}$ is negative and dominates $E\pi$.



(a) $\dot{d} = 0$ isocline: *case 2.1.2* (b) $\dot{d} = 0$ isocline: *case 2.2*

Figure 3.3

If d rises beyond $\frac{1-\delta}{\delta}$, $A\beta\{\delta d - (1-\delta)\}$ becomes negative. Then, as debt-capital ratio rises, the first term of equation (3.3) falls. For $\dot{d} = 0$ to achieve, not only π must be negative now, if d rises, π must fall. As d approaches towards $\frac{F}{E}$, $(Ed - F)$ approaches to zero. Hence, for a small fall in d, for the debt market to be in equilibrium, now a very large rise in π is required. That is why for all $d \in (\frac{1-\delta}{\delta}, \frac{F}{E})$, the $\dot{d} = 0$ isocline in negatively sloped and has a vertical asymptote at $d = \frac{F}{E}$.

For $d > \frac{F}{E}$, $A\beta\{\delta d - (1-\delta)\} < 0$ and $(Ed - F) > 0$. Hence, as debt-capital ratio rises beyond $\frac{F}{E}$, for every unit rise in d first term of the L.H.S. of equation (3.3) falls by $A\beta\{2\delta d - (1-\delta)\}d$ unit[18] whereas the second term rises by $E\pi$ unit. Initially, for a rise in d, left hand side of equation (3.3) rises (because $E\pi$ outweighs $A\beta\{2\delta d - (1-\delta)\}d$).[19] For the debt market to be in equilibrium, as $(Ed - F) > 0$, share of profit must fall. Thus we get a negatively sloped $\dot{d} = 0$ isocline. However, as d rises further, $A\beta\{2\delta d - (1-\delta)\}d$ overcompensates $E\pi$, and so the L.H.S. of equation (3.3) falls. For the debt market to be in equilibrium now the share of profit must rise. Hence we get a positively sloped $\dot{d} = 0$ isocline now.

From the numerical example in Table 3.1 we obtain $\psi = 0.4506$, $\psi\pi = 0.153204$, $A = -0.017 < 0$, $\eta = -0.000627 < 0$, $E = 0.000579 > 0$, and $u^* = 0.26414693$. Keynesian stability condition $(\psi\pi > \beta)$ is fulfilled too.

Case 2.2: Here $A < 0$, and $E < 0$. $E < 0$ ensures that $\eta =< 0$, and $a < 0$. So, here $A < 0$, and it is strong enough to make E negative. Consequently, here the economy is

[18]Note that $A < 0$ and $A\beta\{\delta d - (1-\delta)\} < 0$ implies $A\beta\{2\delta d - (1-\delta)\}d < 0$.

[19]As $-2hd + l = -2\beta\delta A d + (1-\delta)A\beta - E\pi = -[A\beta\{2\delta d - (1-\delta)\} + E\pi] < 0$ here. As $A\beta\{2\delta d - (1-\delta)\}$ is negative, this inequality holds if $E\pi$ dominates.

in a strong debt-burdened demand and growth regime. *Case 2.2* is represented in Figure 3.3b.[20]

$(Ed - F)$ is always negative in case 2.2. As long as the debt-capital ratio is sufficiently small (i.e. when $d < \frac{1-\delta}{\delta}$), $A\beta\{\delta d - (1-\delta)\}$ is positive. Starting from zero, for every unit rise in d, first term of the L.H.S. of equation (3.3) rises by $A\beta\{2\delta d - (1-\delta)\}$ unit (as for the smaller values of d the linear term $(1-\delta)d$ dominates the quadratic term δd^2) whereas the second term falls by $E\pi$ unit. Initially, for a rise in d, left hand side of equation (3.3) rises (because $A\beta\{2\delta d - (1-\delta)\}$ dominates $E\pi$).[21] As $(Ed - F) < 0$, for the debt market to be in equilibrium, profit share must rise. Eventually, as d rises further, the gap between $2\delta d$ and $(1-\delta)$ shrinks. As a consequence, $E\pi$ outweighs $A\beta\{2\delta d - (1-\delta)\}$, and therefore, the L.H.S. of equation (3.3) falls. For the debt market to be in equilibrium now the share of profit must fall too. Hence, we get an inverted "U" shaped $\dot{d} = 0$ isocline in the $0 < d \leqslant \frac{1-\delta}{\delta}$ region.

From the numerical example in Table 3.1 we get $\psi = 0.4506$, $\psi\pi = 0.153204$, $A = -0.0216 < 0$, $\eta = -0.0016896 < 0$, $E = -0.00051 < 0$, $u^* = 0.239682315$. Keynesian stability condition ($\psi\pi > \beta$) and $\beta = 0.125 > \frac{E\pi}{A(1-\delta)} = 0.011468254$ are also satisfied here.

3.2 *Distributional Dynamics*

Let us now focus on the distributional dynamics of the economy. Let us assume that in the long run the share of profit changes according to the difference between the desired share of profit of the firms and the actual share of profit. We assume that the desired

[20] Note that here $\frac{F}{E} < 0$. From equation (3.4) we get that at $d = 0$ and at $d = (\frac{1-\delta}{\delta})$, $\pi\Big|_{\dot{d}=0} = 0$. $\forall d \in (0, \frac{1-\delta}{\delta})$ we get $\pi\Big|_{\dot{d}=0} = \left\{\frac{\overbrace{-\beta A d}^{-}\{\overbrace{\delta d - (1-\delta)}^{-}\}}{\underbrace{(Ed-F)}_{-}}\right\} > 0$; $\forall d \in (\frac{1-\delta}{\delta}, \infty)$, $\pi\Big|_{\dot{d}=0} = \left\{\frac{\overbrace{-\beta A d}^{-}\{\overbrace{\delta d - (1-\delta)}^{+}\}}{\underbrace{(Ed-F)}_{-}}\right\} < 0$. Similar to *case 2.1.2,* as $\eta < 0$, here also we obtain $\Delta > 0$ Re-arranging of equation (3.8) we obtain $\frac{d\pi}{dd}\Big|_{\dot{d}=0} = \frac{-A\beta a(d-d')(d-d'')}{[Ed-F]^2}$. Here $d' = \frac{\overbrace{b + \sqrt{b^2-4ac}}^{+}}{\underbrace{2a}_{-}} < 0$. Since $a < 0$, $\left(b - \sqrt{b^2-4ac}\right) < 0$ and $d'' = \frac{\left(b-\sqrt{b^2-4ac}\right)}{2a} > 0$. Thus we get $d' < 0 < d''$. Here $\frac{d\pi}{dd}\Big|_{\dot{d}=0} = \frac{\overbrace{-A\beta a}^{-}(d-d')(d-d'')}{[Ed-F]^2}$ and so $\forall d \in [0, d'')$, $\frac{d\pi}{dd}\Big|_{\dot{d}=0} > 0$; and $\forall d \in (d'', \infty)$, $\frac{d\pi}{dd}\Big|_{\dot{d}=0} < 0$.

[21] Here $-2hd + l = -2\beta\delta A d + (1-\delta)A\beta - E\pi = -[A\beta\{2\delta d - (1-\delta)\} + E\pi] < 0$. As $E\pi < 0$, this is possible only when $A\beta\{2\delta d - (1-\delta)\}$ is positive and dominates $E\pi$.

share of profit of firms depends positively on the economic condition which is captured by the degree of capacity utilization. Dutt (1992, pp.583) argues "firms attempt to raise markups to take advantage of buoyant markets and reduce markups when sales are low".[22] Similarly, in line with Rowthorn (1977, pp. 119) we can argue that when there is ample amount of unutilized capacity, firms cannot aspire for higher profit share in the fear that other firms may invade their markets. However, higher the capacity utilization rate, firms become stronger and can use their market power asking for higher profit share. So much so, higher the aggregate demand (the degree of capacity utilization), more the pressure put by the rentiers on firms for higher dividends. As a result, firms desire higher share of profit to keep rentiers (shareholders) happy.

Mergers, acquisitions and hostile takeovers also influence the desired share of profit of firms. Higher the tendency to mergers, acquisitions and hostile takeovers, higher is the desired share of profit of firms. More mergers and acquisitions lead to higher market concentrations of firms (as there are few firms after the mergers and acquisitions), and therefore fimrs/capitalists become stronger in the product market. So, they can aspire for obtaining higher profit share.

Finally, a rise in π is expected to reduce $\dot{\pi}$. This is happening because, *ceteris paribus*, higher profit share will provide incentive to outsiders (outside firms) to enter in the market and therefore the monopoly power will be decreased (see for example Dutt; 2012).

Therefore the profit share changes according to the following equation as

$$\dot{\pi} = \rho[\pi^d - \pi] = \rho[\gamma_0 + \gamma_1 u^* - \pi]; \quad \rho > 0 \tag{3.10}$$

where $\pi^d (= \gamma_0 + \gamma_1 u^*)$ is the share of profit desired by the firms (managers). γ_0 and γ_1 are positive parameters. Here the influence of mergers, acquisitions and hostile takeovers is represented by the parameter γ_0. γ_1 represents the responsiveness of the desired share of profit of firms to a unit change in the short-run equilibrium degree of capacity utilization. For the model to be economically meaningful we assume $\pi^d \in (0, 1)$. [23] However the change is not instantaneous and depends on the parameter ρ, where $\rho > 0$. We assume that the value of ρ depends on the ability of the firms to adjust its actual share of profit to the

[22] However, Dutt (1992) points out that a negative relationship between firms desired/target profit share and capacity utilization rate is also possible.

[23] One obvious question that may arise is that why we have not incorporated explicitly the workers' bargaining dynamics in equation (3.10). In the era of financialization, there has been a shift in management strategies "from retain and reinvest to downsize and distribute" (Lazonick and O'Sullivan, 2000). Hein (2012b) points out that there has been a sectoral shift in the economic structure from public and non-financial corporate sectors (who have strong trade unions) to the financial sectors with weaker trade unions. At the same time there has been immense deregulation of labour markets. All of the above play a crucial role for weakening the bargaining power of workers. This is why we feel that exclusion of the workers' role in determining the change in share of profit (or π dynamics) will not change the qualitative results significantly.

desired one. This ability depends on the institutional features such as labour law, trade union representation,[24] the degree to which downsizing, outsourcing or plant relocation etc. are practiced, and so on (Setterfield; 2007, 2009). A more stringent labour law and a higher bargaining power of workers vis-à-vis firms leads to the deterioration of ρ.

Differentiating equation (3.10) partially w.r.t. d and π respectively, and rearranging we get,

$$J_{21} = \frac{\partial \dot{\pi}}{\partial d} = \rho \left(\frac{\gamma_1 \delta A}{\psi\pi - \beta} \right) \gtreqless 0 \tag{3.11}$$

$$J_{22} = \frac{\partial \dot{\pi}}{\partial \pi} = \rho \left[\frac{-2\psi\pi + (\psi\gamma_0 + \beta)}{\psi\pi - \beta} \right] < 0 \tag{3.12}$$

So, the slope of the $\dot{\pi} = 0$ isocline is

$$\left.\frac{d\pi}{dd}\right|_{\dot{\pi}=0} = -\frac{J_{21}}{J_{22}} = \frac{\gamma_1 A\delta}{2\psi\pi - (\psi\gamma_0 + \beta)} \tag{3.13}$$

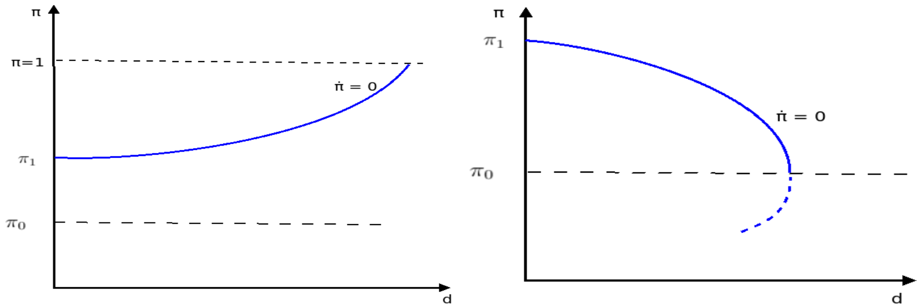
Note, however, that as in equilibrium $\pi = \gamma_0 + \gamma_1 u^* > \gamma_0$ (for all $u^* > 0$), and as we assume that the Keynesian stability condition $\psi\pi > \beta$ holds, $2\psi\pi - (\psi\gamma_0 + \beta) = \psi\overbrace{(\pi - \gamma_0)}^{+} + \overbrace{(\psi\pi - \beta)}^{+} > 0$. Hence, the slope depends on the sign of A. We shall consider the meaningful case only i.e. we shall consider $\pi > \pi_0 = \frac{\gamma_0\psi+\beta}{2\psi}$. When $A > 0$, the slope of the $\dot{\pi} = 0$ isocline is positive, and for $A < 0$, the slope is negative.

Let us explain equations (3.11), and (3.12) respectively. J_{21} shows the effect of a rise in debt-capital ratio on a change in profit share. When the economy is in a debt-led demand regime, a rise in d raises the aggregate demand which in turn raises the desired profit share of firms. Hence, when debt-capital ratio rises, for a given π, the change in profit share becomes positive i.e. $J_{21} = \frac{\partial \dot{\pi}}{\partial d} > 0$ holds. On the other hand if the economy is in a debt-burdened demand regime, the reverse occurs.

J_{22} shows the effect of an increase in profit share on the change in profit share itself. As the economy is in a wage-led demand regime, a rise in π causes a deterioration in the aggregate demand which in turn leads to a fall in firms' desired profit share. Due to a rise in π, the change in the profit share falls by $\rho\gamma_1 \frac{\partial u^*}{\partial \pi} = \frac{\rho\gamma_1\psi(\alpha+\delta Ad)}{(\psi\pi-\beta)^2}$ unit. On the other hand, the profit share will erode its own change at a speed of ρ, holding π^d constant. Thus, the net effect is unambiguously negative and therefore we get $J_{22} < 0$.

For vertical intercept, we insert $d = 0$ in equation (3.10) and obtain $\pi\Big|_{\dot{\pi}=0}^{d=0} = \gamma_0 + \frac{\gamma_1\alpha}{(\psi\pi-\beta)}$. Therefore, for $d = 0$, we get two values of π, $\pi_1 = \frac{(\psi\gamma_0+\beta)+\sqrt{(\psi\gamma_0+\beta)^2-4\psi(\gamma_0\beta-\gamma_1\alpha)}}{2\psi}$ and $\pi_2 = \frac{(\psi\gamma_0+\beta)-\sqrt{(\psi\gamma_0+\beta)^2-4\psi(\gamma_0\beta-\gamma_1\alpha)}}{2\psi}$. The discriminant is $(\psi\gamma_0 + \beta)^2 - 4\psi(\gamma_0\beta - \gamma_1\alpha) =$

[24] These affect the bargaining powers of workers vis-à-vis firms



(a) *Case 1* $(A > 0)$ (b) *Case 2* $(A < 0)$

Figure 3.4: $\dot{\pi} = 0$ isocline

$(\psi\gamma_0 - \beta)^2 + 4\psi\gamma_1\alpha$ which is always positive. When $(\gamma_0\beta - \gamma_1\alpha) < 0$ holds, we get $(\psi\gamma_0 + \beta)^2 - 4\psi(\gamma_0\beta - \gamma_1\alpha) > (\psi\gamma_0 + \beta)^2$, and therefore $\pi_1 > 0 > \pi_2$. When the reverse of that occurs i.e. when $(\gamma_0\beta - \gamma_1\alpha) > 0$, $(\psi\gamma_0 + \beta)^2 - 4\psi(\gamma_0\beta - \gamma_1\alpha) < (\psi\gamma_0 + \beta)^2$ holds. As a result, we get $\pi_1 > \pi_2 > 0$. For economically viable results, for the rest of the paper we assume $\pi_1 < 1$.

Depending on the sign of A two cases are possible: *case 1* and *case 2* respectively.

Case 1: A debt-led demand and growth regime prevails in *case 1* (as $A > 0$ here). As a result, the slope of the $\dot{\pi} = 0$ isocline $\frac{d\pi}{dd}\Big|_{\dot{\pi}=0} = \frac{\gamma_1 A\delta}{2\psi\pi - (\psi\gamma_0 + \beta)} > 0$ for all $\pi > \frac{\gamma_0\psi + \beta}{2\psi}$ and $\frac{d\pi}{dd}\Big|_{\dot{\pi}=0} = \frac{\gamma_1 A\delta}{2\psi\pi - (\psi\gamma_0 + \beta)} < 0$ for all $\pi < \frac{\gamma_0\psi + \beta}{2\psi}$. But for a meaningful analysis we have to restrict ourselves to $\pi > \frac{\gamma_0\psi + \beta}{2\psi} = \pi_0$ since $\{2\psi\pi - (\psi\gamma_0 + \beta)\} > 0$ (see Figure 3.4a).

Case 2: In *case 2* one obtains a debt-burdened demand and growth regime i.e. $A < 0$ holds here. The slope of the $\dot{\pi} = 0$ isocline $\frac{d\pi}{dd}\Big|_{\dot{\pi}=0} = \frac{\gamma_1 A\delta}{2\psi\pi - (\psi\gamma_0 + \beta)} < 0$ for all $\pi > \frac{\gamma_0\psi + \beta}{2\psi}$ and $\frac{d\pi}{dd}\Big|_{\dot{\pi}=0} = \frac{\gamma_1 A\delta}{2\psi\pi - (\psi\gamma_0 + \beta)} > 0$ for all $\pi < \frac{\gamma_0\psi + \beta}{2\psi}$. But for a meaningful analysis we have to restrict ourselves to $\pi > \frac{\gamma_0\psi + \beta}{2\psi} = \pi_0$ as $\{2\psi\pi - (\psi\gamma_0 + \beta)\} > 0$ (see Figure 3.4b).

4 *Possible Cases*

This section explains different possible cases which may arise due to the interaction between the debt and the distributional dynamics. We get four different cases. These are (i) debt-led demand and growth regime (ii) weak debt-burdened demand and growth regime, (iii) moderate debt-burdened demand and growth regime, and (iv) strong debt-burdened demand and growth regime. Before we proceed, note that the sign of J_{22} is always negative. However, the sign of J_{11}, J_{12}, and J_{21} are ambiguous.

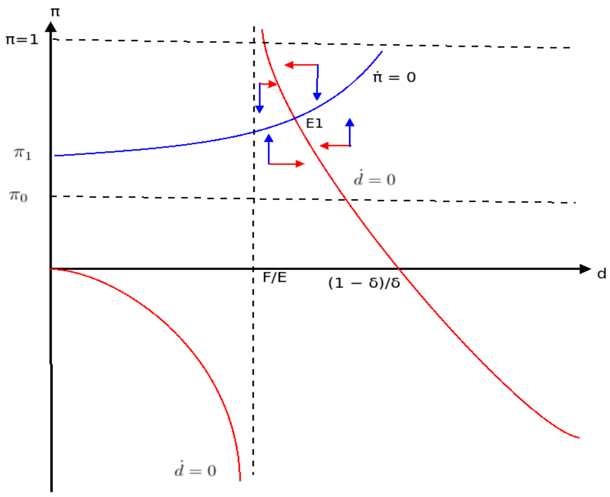


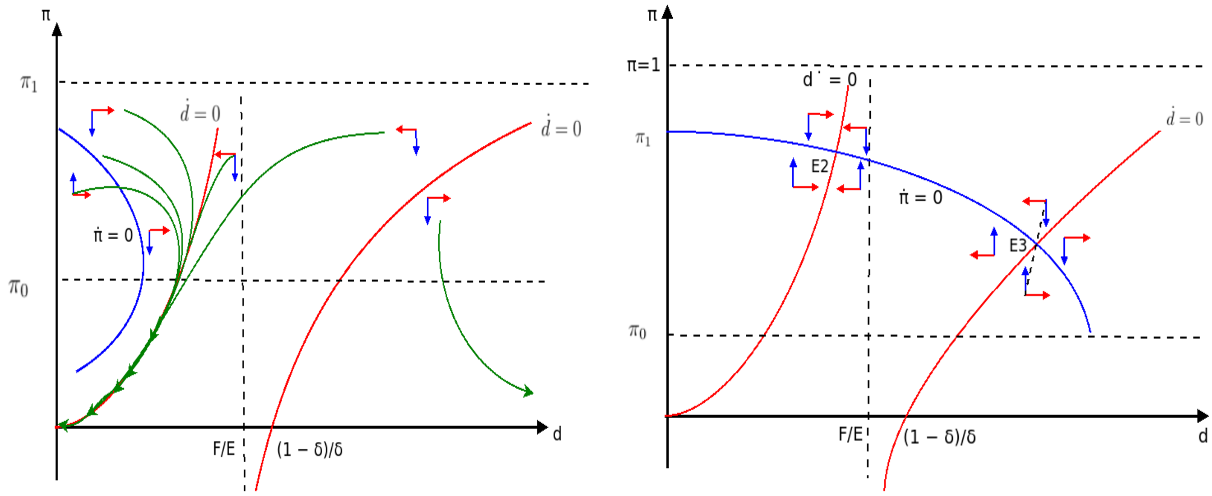
Figure 4.1: Multiple cases of the long run equilibria: *Case 1*

4.1 *Case 1: debt-led demand and growth regime*

Here the economy is in a debt-led demand and growth regime (i.e. $A > 0$). $A > 0$ implies $E > 0$, $a > 0$ and $\eta = [A(1-\delta)s_f + s_f\alpha] > 0$. $A > 0$ and equation (3.11) imply that $J_{21} > 0$. From Figure 4.1 it is clear that the $\dot{d} = 0$ isocline is negatively sloped. For $\pi \in (0,1)$, therefore, $\left.\frac{d\pi}{dd}\right|_{\dot{d}=0} = -\frac{J_{11}}{J_{12}} < 0$. At the steady state E_1, $d > \frac{F}{E}$. Equation (3.7) therefore implies $J_{12} < 0$. Consequently, J_{11} must be negative at E_1. Here, as the $\dot{\pi} = 0$ isocline is positively sloped and $\dot{d} = 0$ isocline is negatively sloped, the sufficient condition for an equilibrium to exist is that the $\dot{\pi} = 0$ isocline must cross the $d = \frac{F}{E}$ line within the relevant range of π.[25] At point E_1, the determinant of the Jacobian matrix $\mathrm{Det(J)} = (\overbrace{J_{11}}^{-}\overbrace{J_{22}}^{-} - \overbrace{J_{12}}^{-}\overbrace{J_{21}}^{+}) > 0$, and the trace of the matrix $\mathrm{tr(J)} = J_{11} + J_{22} < 0$. As a result, point E_1 emerges as a stable steady state. It is represented in Figure 4.1. As the arrows show, the economy will converge to E_1 either monotonically or by spiraling around E_1.

Let us explain the stability of the steady state E_1 intuitively. Consider the debt-capital ratio deviates from its steady state value due to the occurrence of some exogenous shock. Suppose that the debt-capital ratio is higher than its steady state value, for instance. First, in the debt-led demand and growth regime, if d is higher than the steady state value d^*, it must fall due to $J_{11} < 0$. This is the direct effect. Second, as the debt-capital ratio is higher than its steady state value, the profit share increases due to $J_{21} > 0$, which leads to a fall in the debt-capital ratio due to $J_{12} < 0$. This is the indirect effect. In the debt-led demand and growth regime, both the direct and indirect effects are stable. As a result, in this case, if the debt-capital ratio rises from the steady state value, it again comes back to the steady state. Consequently, the steady state is stable.

[25] Note that the relevant range of π lies between π_0 and 1.



(a) No equilibrium: *Case 2.1.1*

(b) *Case 2.1.1*

Figure 4.2

4.2 *Case 2.1.1: weak debt-burdened demand and growth regime*

Here the economy is in a debt-burdened demand and growth regime, i.e. $A < 0$ holds here. Nevertheless, A is too weak to make E and η negative ($E > 0$ and $\eta > 0$ here). This is why in *case 2.1.1,* the economy is in a weak debt-burdened demand and growth regime. $A < 0$ and equation (3.11) imply that $J_{21} < 0$. It is noticeable that $d = \frac{F}{E}$ line divides the diagram (i.e Figure 4.2a, or 4.2b) into two segments. The necessary and sufficient condition for no meaningful equilibrium to exist is that the $\dot{\pi} = 0$ isocline changes its slope (from negative to positive) to the left of the upward sloped portion of the $\dot{d} = 0$ isocline. In that case, d will either converge to zero or increase without bound (see Figure 4.2a). Thus if the $\dot{\pi} = 0$ isocline changes its slope (from negative to positive) to the right of the $\dot{d} = 0$ isocline in the left segment, this ensures the existence of the equilibrium point E_2. On the other hand if the $\dot{\pi} = 0$ isocline changes its slope (from negative to positive) to the right of the $\dot{d} = 0$ isocline in the right segment, this ensures the existence of multiple equilibria: E_2 and E_3.

Consider point E_2 : At the steady state E_2, $d < \frac{F}{E}$, and equation (3.7) together imply that $J_{12} > 0$. As the $\dot{d} = 0$ isocline is positively sloped (see Figure 4.2b), $J_{11} < 0$ must hold at E_2. Here, the determinant Det(J) $= (\overbrace{J_{11}}^{-}\overbrace{J_{22}}^{-} - \overbrace{J_{12}}^{+}\overbrace{J_{21}}^{-}) > 0$, and the trace is negative (tr(J) $= J_{11} + J_{22} < 0$). Therefore, E_2 is a stable steady state. Let us explain it intuitively. Suppose, because of some exogenous shock, the debt-capital ratio deviates and is higher than its initial steady state value. First, near E_2, as $d > d^*$, it must fall due to $J_{11} < 0$. This is the direct stable adjustment process. On the other hand, as the economy is in a debt-burdened demand regime, a higher value of d decreases u^* which in turn reduces π^d. As a result, the profit share decreases due to $J_{21} < 0$. This in turn decreases the debt-capital ratio due to $J_{12} > 0$. This is the indirect effect which is also stable. As the direct and the indirect effects both are stable, in this scenario, if the debt-capital ratio rises from the steady state value, it again comes back to the steady state. Hence, the steady state is stable.

Consider point E_3 : At the steady state E_3, $d > \frac{F}{E}$. Therefore, equation (3.7) implies $J_{12} < 0$. As illustrated in Figure 4.2b, the $\dot{d} = 0$ isocline is positively sloped around E_3. Therefore, J_{11} must be positive here. Hence, the determinant $\mathrm{Det(J)} = (\overbrace{J_{11}}^{+}\overbrace{J_{22}}^{-} - \overbrace{J_{12}}^{-}\overbrace{J_{21}}^{-}) < 0$, which results E_3 a saddle point unstable. Let us explain the stability of the steady state E_3 intuitively. Suppose the profit share is higher than its steady state value. Here two opposing effects exist. First, as the profit share is higher than its steady state value, it must fall due to $J_{22} < 0$. This is the direct stable effect. Second, the rise in profit share leads to a fall in the debt-capital ratio due to $J_{12} < 0$. As $J_{21} < 0$, this fall in debt-capital ratio leads to a rise in the profit share. This second effect is an indirect unstable effect. As debt-capital ratio is very high near E_3 ($\because d > \frac{1-\delta}{\delta} > \frac{F}{E}$), for a rise in π, the debt-capital ratio falls significantly (see equation (3.7)). As a consequence, the profit share increases by a large amount via equation (3.11). Hence, the indirect unstable effect dominates the direct stable effect, and makes the steady state unstable. There is only one stable arm that reaches to the equilibrium point E_3. Hence point E_3 emerges as a saddle point.

4.3 *Case 2.1.2: moderate debt-burdened demand and growth regime*

A negative value of A and η but a positive value of E (which in turn implies $a > 0$) is associated with *case 2.1.2.* These imply the economy to be in a moderate debt-burdened demand and growth regime. For simplicity, we assume away the possibility that $\dot{\pi} = 0$ and $\dot{d} = 0$ isoclines coincide with each other. Thus whenever the equilibrium exists, it is because either these isoclines intersect with each other or they are tangent to each other for a single point. We consider six different sub-cases here. These are *cases 2.1.2.a, 2.1.2.b, 2.1.2.c, 2.1.2.d, 2.1.2.e, and 2.1.2.f* respectively (see Figure 4.3a, 4.3b, 4.3c, 4.3d, 4.3e, and 4.3f for respective diagrams). Similar to *Case 2.1.2,* here also the $d = \frac{F}{E}$ line divides the diagram into two segments. As long as the steady state exists, for the left segment of the diagram (where $d < \frac{F}{E}$), the necessary and sufficient condition for a unique equilibrium to exist is that within the relevant range of π, the $\dot{\pi} = 0$ isocline must not intersect the $\dot{d} = 0$ isocline from below. *Cases 2.1.2.a, 2.1.2.b,* and *2.1.2.d* satisfy this condition. On the other hand, we get multiple equilibria in *case 2.1.2.c.* For the right segment of the diagram (where $d > \frac{F}{E}$), necessary and sufficient condition for multiple equilibria to exist is that within the relevant range of π, the $\dot{\pi} = 0$ isocline must intersect the $\dot{d} = 0$ isocline. Thus whenever the equilibrium exists, as long as these two isoclines are not tangent to each other, there must be multiple equilibria.

In cases *2.1.2.a, 2.1.2.b, 2.1.2.c,* and *2.1.2.d,* we get all the equilibria where $d < \frac{F}{E}$. Consequently, equation (3.7) ensures $J_{12} > 0$. The opposite happens in *cases 2.1.2e,* and *2.1.2f.* For a relatively higher debt-capital ratio ($\forall d > \frac{l}{2h}$), the sign of J_{11} is positive, whereas $d < \frac{l}{2h}$ makes J_{11} negative. Finally, $A < 0$ implies (from equation (3.11)) $J_{21} < 0$. Analysis of *cases 2.1.2.a, 2.1.2.b, 2.1.2.c,* and *2.1.2.d* resemble with the analysis of *cases 2.2.a, 2.2.b, 2.2.c,* and *2.2.e* respectively. Therefore, let us concentrate on *cases 2.1.2.e* and *2.1.2.f* only.

4.3.1 *Case 2.1.2.e*

As depicted in Figure 4.3e, two equilibria- E_7, and E_8 are possible here. As at E_7, $d < \frac{l}{2h}$, J_{11} becomes negative. Slope of the $\dot{d} = 0$ isocline is less than the slope of the $\dot{\pi} = 0$ isocline at E_7 i.e.

$$0 > \frac{d\pi}{dd}\Big|_{\dot{\pi}=0} = -\frac{J_{21}}{J_{22}} > \frac{d\pi}{dd}\Big|_{\dot{d}=0} = -\frac{J_{11}}{J_{12}}$$

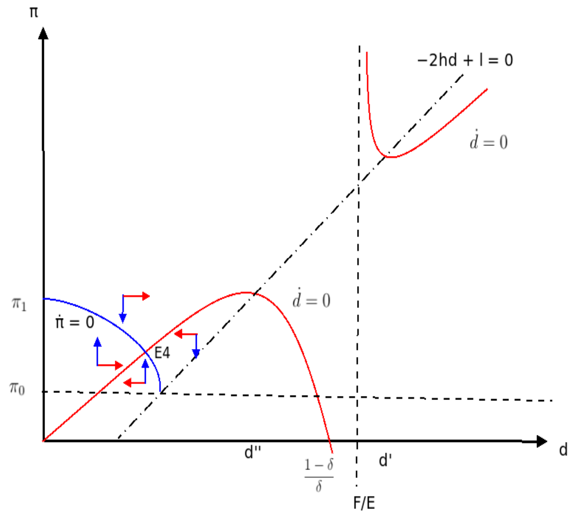
$$\Rightarrow (J_{11}J_{22} - J_{12}J_{21}) > 0 \qquad (\because J_{12} < 0 \text{ and } J_{22} < 0)$$

Hence, the determinant is positive, and the trace is negative ($\text{tr(J)} = J_{11} + J_{22} < 0$). As a result, point E_7 emerges as a stable steady state.[26]

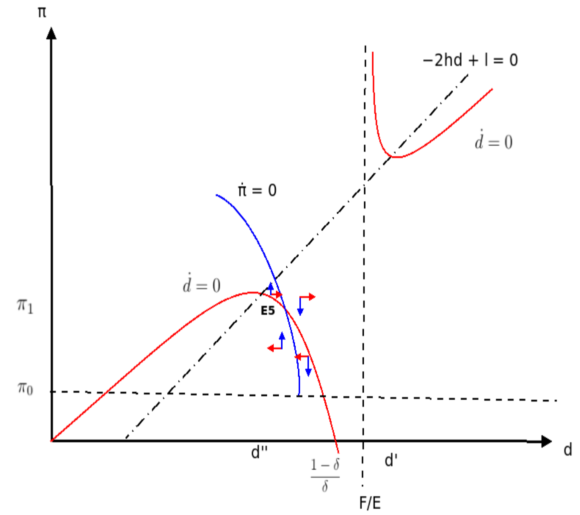
Share of profit, suppose due to some reason, deviates from the steady state and is now higher than its steady state value. Two opposite effects are in work here. As the profit share is higher than its steady state value, it must fall due to equation (3.12). This is the direct stable effect. Second, the rise in profit share leads to a fall in the debt-capital ratio due to $J_{12} < 0$. As $J_{21} < 0$, this fall in debt-capital ratio leads to a rise in the profit share. This second effect is an indirect unstable effect. However, as the debt-capital ratio is very close to $\frac{F}{E}$, $(Ed - F)$ is very small in size. Therefore the negative effect of J_{12} is very weak (see equation (3.7)). As a result, the rise in profit share leads to a negligible amount of fall in the debt-capital ratio, which in turn through equation (3.11) leads to a negligible amount of rise in the profit share. Therefore, the direct stable effect dominates the indirect unstable effect and results the steady state to be stable.

As $d > \frac{l}{2h}$ at E_8, $J_{11} > 0$. Here, $\text{Det(J)} = (\overbrace{J_{11}}^{+}\overbrace{J_{22}}^{-} - \overbrace{J_{12}}^{-}\overbrace{J_{21}}^{-}) < 0$. Consequently, point E_8 emerges as a saddle point. The intuitive analysis here is the same as in steady

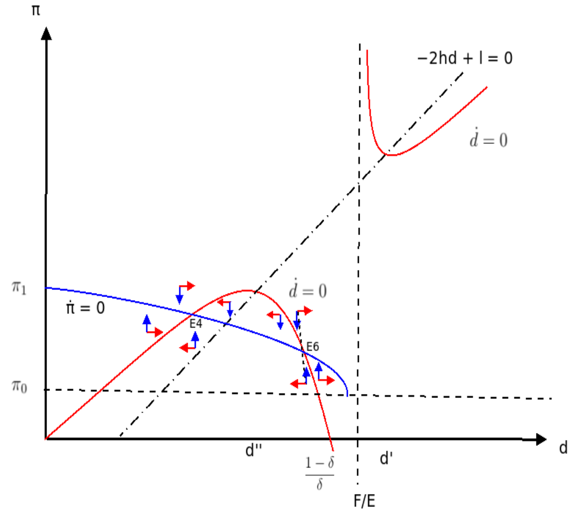
[26]Here as $J_{12} < 0$ and $J_{21} < 0$ so, $tr(J)^2 - 4Det(J) = (J_{11} - J_{22})^2 + 4J_{12}J_{21} > 0$ and hence the steady state is a stable node.



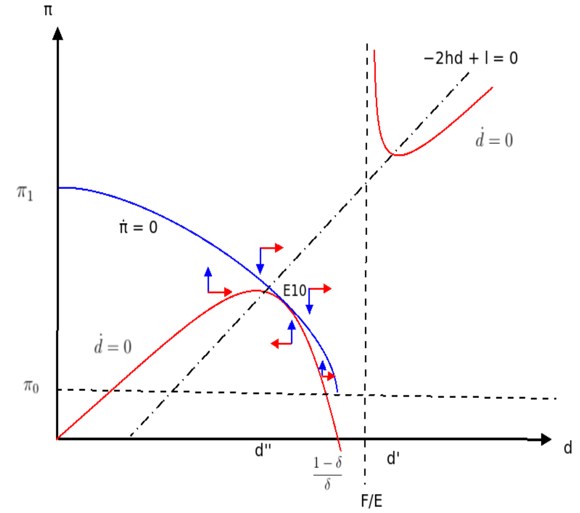
(a) Multiple cases of the long run equilibria: *Case 2.1.2.a*



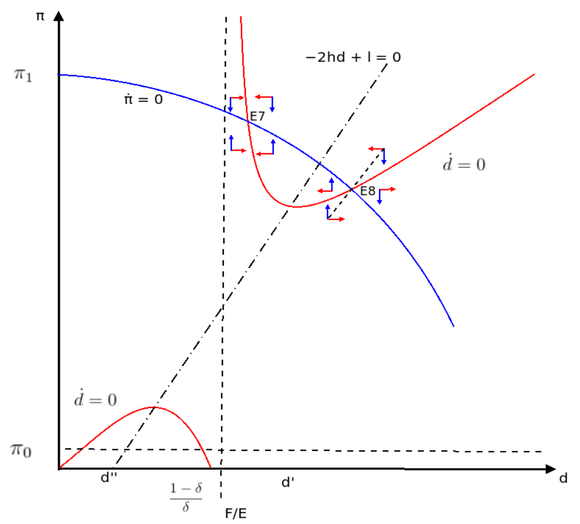
(b) Multiple cases of the long run equilibria: *Case 2.1.2.b*



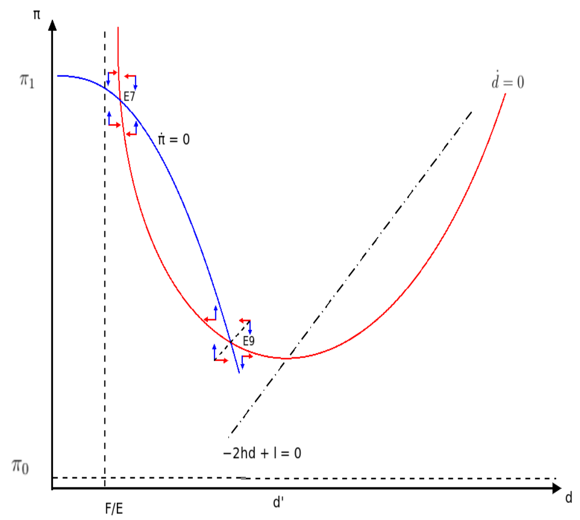
(c) Multiple cases of the long run equilibria: *Case 2.1.2.c*



(d) Multiple cases of the long run equilibria: *Case 2.1.2.d*



(e) Multiple cases of the long run equilibria: *Case 2.1.2.e*



(f) Multiple cases of the long run equilibria: *Case 2.1.2.f*

Figure 4.3

state E_7. Only difference is that as the debt-capital ratio is sufficiently high near E_8 (and $d > \frac{F}{E}$), $(Ed - F)$ is significantly large in size. Therefore, the debt-capital ratio falls significantly because of the strong negative effect of J_{12}. As a consequence, the profit share increases by a large amount via equation (3.11). Hence, the indirect unstable effect dominates the direct stable effect and makes the steady state unstable. There is only one stable arm that reaches to the equilibrium point E_8. Hence point E_8 emerges as a saddle point.

4.3.2 *Case 2.1.2.f*

As Figure 4.3f shows, two equilibria E_7 and E_9 may emerge here. We already have discussed about the steady state E_7 in *case 2.1.2.e.* On the other hand, at E_9, slope of the $\dot{d} = 0$ isocline is greater than the slope of the $\dot{\pi} = 0$ isocline i.e.

$$0 > \frac{d\pi}{dd}\bigg|_{\dot{d}=0} - \frac{J_{11}}{J_{12}} > \frac{d\pi}{dd}\bigg|_{\dot{\pi}=0} = -\frac{J_{21}}{J_{22}}$$

$$\Rightarrow (J_{11}J_{22} - J_{12}J_{21}) < 0 \qquad (\because J_{12} < 0 \text{ and } J_{22} < 0)$$

So the determinant of the Jacobian matrix $Det(J) = (J_{11}J_{22} - J_{12}J_{21}) < 0$. As a result, point E_9 emerges as a saddle point.

4.4 *Case 2.2: strong debt-burdened demand and growth regime*

Here, $A < 0$, $E < 0$[27] (which in turn implies $a < 0$) and $\eta < 0$. Thus here the economy is in a strong debt-burdened demand and growth regime. For simplicity, we assume away the possibility that $\dot{\pi} = 0$ and $\dot{d} = 0$ isoclines coincide with each other. Thus whenever the equilibrium exists, it is because either these isoclines intersect with each other or they are tangent to each other for a single point. Five different sub-cases are considered here. These are *cases 2.2.a, 2.2.b, 2.2.c, 2.2.d* and *2.2.e* respectively. These are illustrated in Figure 4.4. As long as the steady state exists, the necessary and sufficient condition for a unique equilibrium to exist is that within the relevant range of π, the $\dot{\pi} = 0$ isocline must not intersect the $\dot{d} = 0$ isocline from below. *Cases 2.2.a, 2.2.b* and *2.2.e* satisfy this very condition. On the other hand, the necessary and sufficient condition for multiple equilibria to exist is that the $\dot{\pi} = 0$ isocline must intersect the $\dot{d} = 0$ isocline from below in the negatively sloped section of the latter curve. *Case 2.2.c and 2.2.d* satisfy this condition. As $E < 0$, for all positive values of d we get $(Ed - F) < 0$. Equation (3.7),

[27] So here the absolute value of A is strong enough to make E negative.

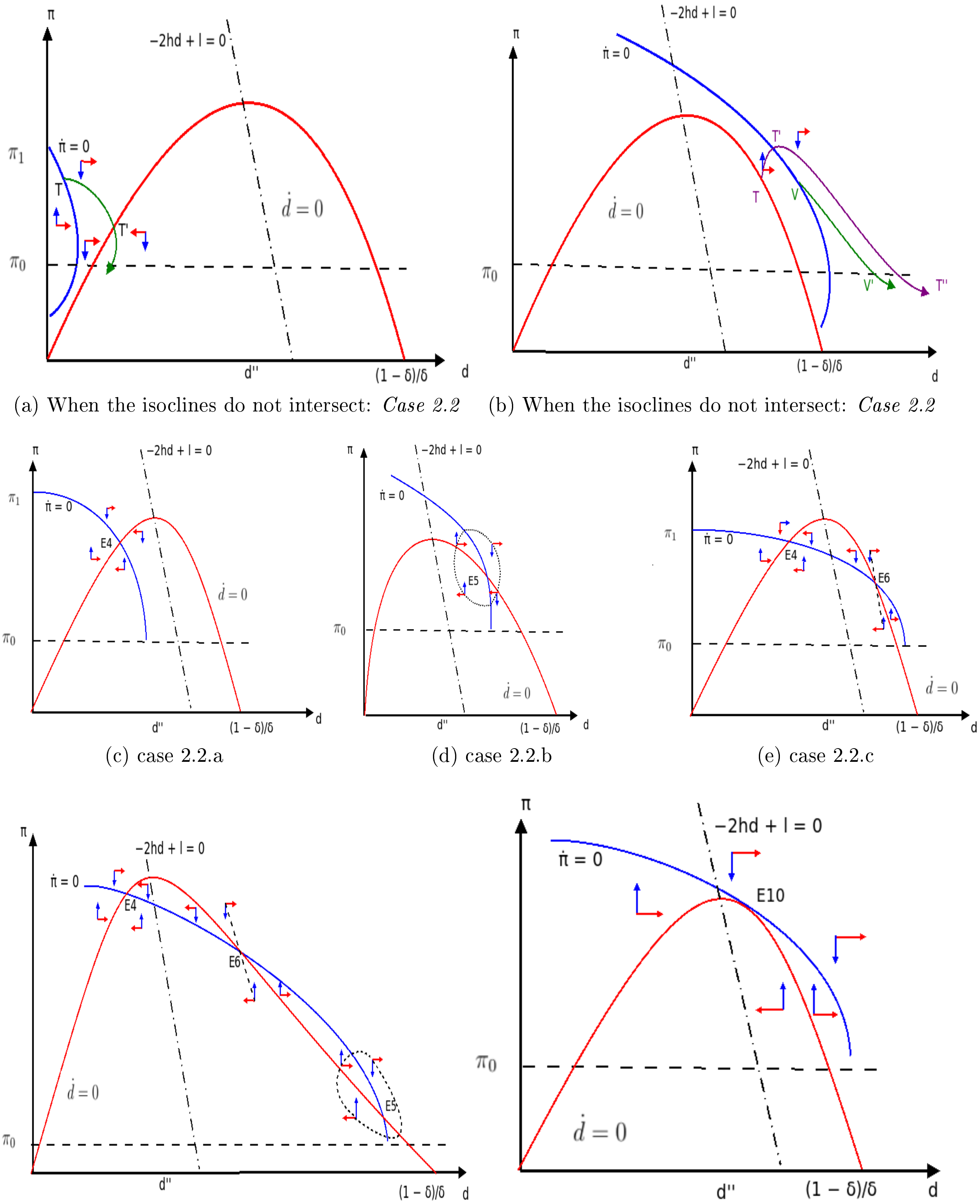
therefore, suggests $J_{12} > 0$. For a relatively high debt-capital ratio ($\forall d > \frac{l}{2h}$), $J_{11} > 0$, while $d < \frac{l}{2h}$ suggests $J_{11} < 0$. $A < 0$ implies (from equation (3.11)) $J_{21} < 0$.

Figure 4.4a and 4.4b depicts two possible scenarios where no equilibrium exists. Consider Figure 4.4a first. Here, suppose the economy starts from point T. At point T, although $\dot{\pi} = 0$ is satisfied, the debt-capital ratio is lower than the required one to make the debt market in equilibrium. As a result (as here$J_{11} < 0$) d must rise. However, as d rises, it puts pressure on profit share through $\frac{\partial \dot{\pi}}{\partial d}$, and thereby, decreases the profit share (as $\frac{\partial \dot{\pi}}{\partial d} = J_{21} < 0$ here). Because of fall in π and rise in d, point T' is achieved. However, at T' although debt market is in equilibrim, as d is higher than need to achieve $\dot{\pi} = 0$, π fall further (through equation (3.11), which is negative in the debt-burdened demand regime). Because of a fall in π, the debt market is no more in equilibrium. Rather, now d is more than needed to achieve $\dot{d} = 0$ and therefore d falls due to $J_{11} < 0$. These fall in π and d ultimately pushes the economy below the economically meaningful range of profit share (i.e. π is below π_0 now). On the other hand for Figure 4.4b, suppose the economy starts from point V. At point V, although $\dot{\pi} = 0$ is satisfied, level of debt-capital ratio is higher than required for achieving the debt market equilibrium. As a result (as here$J_{11} > 0$) d rises further . A rise in d inserts negative pressure on profit share through equation (3.11) and causes a decline in the profit share. This fall in π and the rise in d ultimately push the economy below the economically meaningful range of profit share (i.e. π is below π_0 now). Similarly, if the economy starts from point T in Figure 4.4b, it ultimately reaches below the the economically meaningful range of profit share.

4.4.1 *Case 2.2.a*

Here only one unique equilibrium (E_4) is possible which is shown in Figure 4.4c. At E_4, as $d < \frac{l}{2h}$, $J_{11} < 0$. So, the determinant of the Jacobian matrix Det(J) $= (\overbrace{J_{11}}^{-}\overbrace{J_{22}}^{-} - \overbrace{J_{12}}^{+}\overbrace{J_{21}}^{-}) > 0$, and the trace tr(J) $= J_{11} + J_{22} < 0$. As a result, point E_4 emerges as a stable steady state. As the arrows around E_4 show, the economy will converge to E_4 either monotonically or by spiraling around E_4.

Let us explain the stability of the steady state E_4 intuitively. Suppose the debt-capital ratio is above its steady state value. First, as $d > d^*$, d must fall due to the negative self-feedback effect ($\because J_{11} < 0$). On the other hand, as the debt-capital ratio is higher than its steady state value, the profit share decreases (due to $J_{21} < 0$), which in turn causes a decline in the the debt-capital ratio ($\because J_{12} > 0$). This is the indirect effect. Here, both the direct and indirect effects are stable. As a result, in this case, if the debt-capital ratio rises from the steady state value, it again comes back to the initial steady state.



(a) When the isoclines do not intersect: *Case 2.2*

(b) When the isoclines do not intersect: *Case 2.2*

(c) case 2.2.a

(d) case 2.2.b

(e) case 2.2.c

(f) *Case* 2.2.d

(g) *Case 2.2.e*

Figure 4.4: *Case 2.2*

4.4.2 *Case 2.2.b*

As shown in Figure 4.4d, a unique equilibrium E_5 is possible here. As $d > \frac{l}{2h}$, equation (3.6) suggests J_{11} to be positive. At point E_5, slope of the $\dot{d} = 0$ isocline is greater than the slope of the $\dot{\pi} = 0$ isocline i.e.

$$0 > \left.\frac{d\pi}{dd}\right|_{\dot{d}=0} = -\frac{J_{11}}{J_{12}} > \left.\frac{d\pi}{dd}\right|_{\dot{\pi}=0} = -\frac{J_{21}}{J_{22}}$$

$$\Rightarrow (J_{11}J_{22} - J_{12}J_{21}) > 0 \quad (\because J_{12} > 0 \text{ and } J_{22} < 0)$$

Hence, the determinant is positive. Trace of the matrix $\text{tr}(\text{J}) = J_{11} + J_{22} = \frac{\overbrace{\rho[-2\psi\pi + (\psi\gamma_0 + \beta)]}^{-}}{(\psi\pi - \beta)} + \frac{\overbrace{(l - 2hd)}^{+}}{(\psi\pi - \beta)} \gtreqless 0$. As a result, point E_5 can be either stable or unstable equilibrium depending on whether the speed of adjustment parameter $\rho > \hat{\rho} = \frac{l-2hd}{[2\psi\pi - (\psi\gamma_0 + \beta)]}$ or $\rho < \hat{\rho}$ respectively. If $\rho = \hat{\rho}$, limit cycles occur due to Hopf-bifurcation. More discussion regarding Hopf-bifurcation is provided in section 4.5.

Intuition behind the stability at point E_5 is as follows. First, when the debt-capital ratio rises above its steady state value, as $J_{11} > 0$, the positive self-feedback effect leads to a further rise in the debt-capital ratio. This is the direct unstable effect. Second, a rise in d through equation (3.11) leads to a fall in the share of profit ($\because J_{21} < 0$) which in turn through equation (3.7) causes a fall in the debt-capital ratio ($\because J_{12} > 0$). This is the indirect stable effect. When the speed of adjustment parameter of the profit share (ρ) is sufficiently high, the dynamics of the system could become stable because the negative indirect-feedback mechanism of the debt-capital ratio becomes strong and dominates the unstable self-feedback effect. On the other hand, for a lower value of ρ (i.e. when $\rho < \hat{\rho}$), the steady state E_5 becomes unstable.

4.4.3 *Case 2.2.c*

Two equilibria, E_4 and E_6, are possible here (see Figure 4.4e). Analysis of E_4 is the same as in *case 2.2.a*. At E_6, as $d > \frac{l}{2h}$,J_{11} is positive. At E_6, slope of the $\dot{d} = 0$ isocline is steeper than the slope of the $\dot{\pi} = 0$ isocline i.e.

$$0 > \left.\frac{d\pi}{dd}\right|_{\dot{\pi}=0} = -\frac{J_{21}}{J_{22}} > \left.\frac{d\pi}{dd}\right|_{\dot{d}=0} = -\frac{J_{11}}{J_{12}}$$

$$\Rightarrow (J_{11}J_{22} - J_{12}J_{21}) < 0 \quad (\because J_{12} > 0 \text{ and } J_{22} < 0)$$

Hence, E_6 is a saddle point.

Table 4.1: *Summary of stability of the steady states*

Case	steady state	Sign of the elements of Jacobian Matrix	Nature of the steady state
1	E_1	$J_{11} < 0,\ J_{12} < 0,\ J_{21} > 0,\ J_{22} < 0$	stable
2.1.1	E_2	$J_{11} < 0,\ J_{12} > 0,\ J_{21} < 0,\ J_{22} < 0$	stable
	E_3	$J_{11} > 0,\ J_{12} < 0,\ J_{21} < 0,\ J_{22} < 0$	saddle point unstable
2.1.2.a	same as in case *2.2.a*		
2.1.2.b	same as in case *2.2.b*		
2.1.2.c	same as in case *2.2.c*		
2.1.2.d	same as in case *2.2.e*		
2.1.2.e	E_7	$J_{11} < 0,\ J_{12} < 0,\ J_{21} < 0,\ J_{22} < 0$	stable
	E_8	$J_{11} > 0,\ J_{12} < 0,\ J_{21} < 0,\ J_{22} < 0$	saddle point unstable
2.1.2.f	E_7	$J_{11} < 0,\ J_{12} < 0,\ J_{21} < 0,\ J_{22} < 0$	stable
	E_9	$J_{11} < 0,\ J_{12} < 0,\ J_{21} < 0,\ J_{22} < 0$	saddle point unstable
2.2.a	E_4	$J_{11} < 0,\ J_{12} > 0,\ J_{21} < 0,\ J_{22} < 0$	stable
2.2.b	E_5	$J_{11} > 0,\ J_{12} > 0,\ J_{21} < 0,\ J_{22} < 0$	stable/unstable/limit cycle
2.2c	E_4	$J_{11} < 0,\ J_{12} > 0,\ J_{21} < 0,\ J_{22} < 0$	stable
	E_6	$J_{11} > 0,\ J_{12} > 0,\ J_{21} < 0,\ J_{22} < 0$	saddle point unstable
2.2.d	E_4	$J_{11} < 0,\ J_{12} > 0,\ J_{21} < 0,\ J_{22} < 0$	stable
	E_6	$J_{11} > 0,\ J_{12} > 0,\ J_{21} < 0,\ J_{22} < 0$	saddle point unstable
	E_5	$J_{11} > 0,\ J_{12} > 0,\ J_{21} < 0,\ J_{22} < 0$	stable/unstable/limit cycle
2.2.e	E_{10}	$J_{11} > 0,\ J_{12} > 0,\ J_{21} < 0,\ J_{22} < 0$	saddle point unstable

4.4.4 *Case 2.2.d*

Three equilibria, E_4, E_5 and E_6 are possible here (see Figure 4.4f).

4.4.5 *Case 2.2.e*

As illustrated in Figure 4.4g, both the isoclines are tangent to each other at E_{10}, and makes E_{10} a saddle point unstable.

From the above analysis of different cases, the next point is worth remembering.

Remark 1. *Suppose the economy is either in a moderate or in a strong debt-burdened demand and growth regime. As long as a unique steady state exists, $d < \frac{l}{2h}$ is sufficient for ensuring stability. For $d < \frac{l}{2h}$, if we observe multiple equilibria, one of them must be stable (for example see case 2.1.2.f, Figure 4.3f). However, $d < \frac{l}{2h}$ is not a necessary condition for achieving stability (for example see case 2.2b, Figure 4.4d).*

4.5 *Hopf Bifurcation*

Consider the steady state E_5 of the *case 2.2.b*.

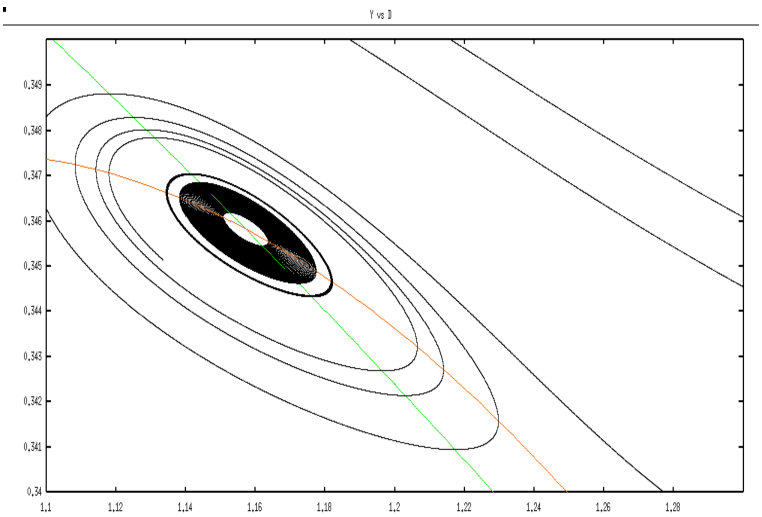
Proposition 1. *For an appropriate value of the speed of adjustment parameter, ρ, the characteristic equation to (3.2) & (3.10) evaluated at the steady state E_5 of the case 2.2.b has purely imaginary roots, and for the same dynamical system, $\rho = \hat{\rho} = \frac{l-2hd}{2\psi\pi-(\psi\gamma_0+\beta)}$ provides a point of Hopf bifurcation.*

Proof. See Appendix A.1. □

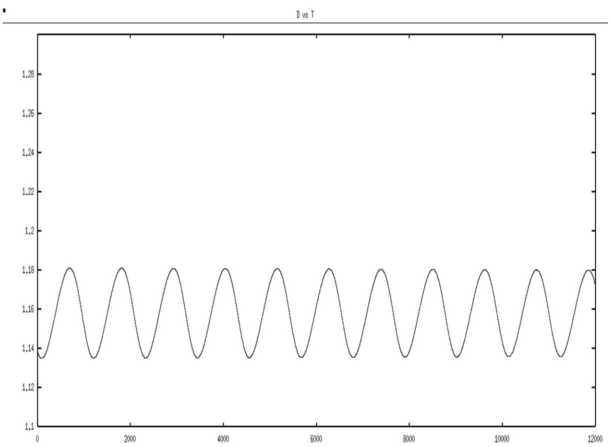
Note that the objective of this very numerical study is not to calibrate a real economy. Rather, the prime purpose is to confirm whether the model produces the limit cycle and to observe its basic properties. Therefore, the values introduced are set for these purposes to obtain economically meaningful outcomes. Using XPPAUT software, we find that the Hopf bifurcation is sub-critical in nature i.e. an unstable limit cycle exists. We draw the solution path from $t = 0$ to $t = 12000$, and we find that the solution path is not a perfect closed orbit in the sense that for an initial condition close to the long-run equilibrium the solution path converges to the equilibrium whereas for the initial condition further away from the long-run equilibrium, the solution path diverges from the equilibrium. As a result, we confirm that in this numerical example, the sub-critical Hopf bifurcation occurs and the periodic solution is unstable. Note that the purpose of this numerical study is not to calibrate a real economy. Rather, the primary objective is to confirm whether the model produces the limit cycle and to observe its basic properties. Therefore, for the simulation, we set $\alpha = 0.01$, $\beta = 0.132$, $\delta = 0.298$, $\lambda = 0.2$, $c_q = 0.007$, $s_f = 0.33$, $i = 0.05$, $c_r = 0.4$, $\gamma_0 = 0.27$, $\gamma_1 = 0.7$, $\hat{\rho} = 0.0024523$. We get the equilibrium values of d and π for the steady state E_5 of *case 2.2.b* as $d^* = 1.1572$ and $\pi^* = 0.34583$.

Note that for $\rho > \hat{\rho}$, the trace become negative and hence we have a stable equilibrium. However when $\rho < \hat{\rho}$, the equilibrium is unstable. When ρ falls to $\hat{\rho}$, the system with a stable steady state loses its stability and gives birth to a limit cycle.

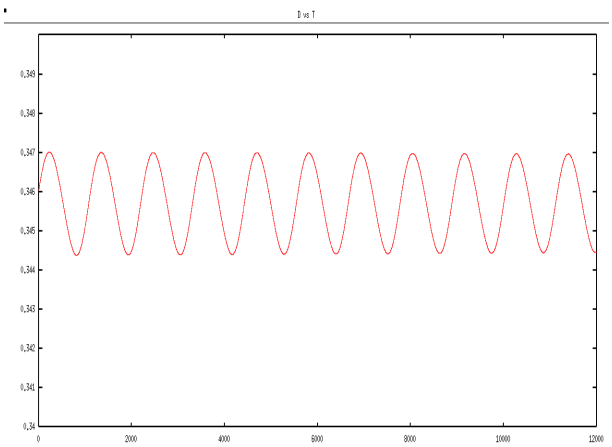
A more flexible labour law and a weaker bargaining power of workers vis-à-vis firms may result a higher level of ρ and make the equilibrium stable. Hence, when there is a strong debt-burdened demand and growth regime, if the economy is at the steady state E_4, a flexible labour law is desirable for ensuring stability in the economy.



(a) *Unstable limit cycles due to sub-critical Hopf bifurcation*



(b) *Transitional dynamics of debt-capital ratio*



(c) *Transitional dynamics of profit share*

Figure 4.5: *Transitional dynamics*

5 *Comparative Statics*

In our model, γ_0 represents the positive influence of mergers, acquisitions and hostile takeovers on the desired profit share of firms (managers). Therefore, the comparative static of a change in γ_0 deserves some attention. For comparative statics analysis, we assume the economy is in a stable steady state.[28] The total differentiation of equations (3.2) and (3.10) shows the effects of parametric change of γ_0 in the economy which imply

$$\begin{bmatrix} J_{11} & J_{12} \\ J_{21} & J_{22} \end{bmatrix} \begin{bmatrix} dd \\ d\pi \end{bmatrix} = \begin{bmatrix} 0 \\ -\rho \end{bmatrix} d\gamma_0 \tag{5.1}$$

[28]Depending on the size of the parameter ρ, steady state E_5 can be either stable or unstable or limit cycles can emerge. However, for comparative statics analysis we assume E_5 to be a stable steady state.

Table 5.1: *Summary of comparative statics results for a change in* γ_0

Case	steady state	Sign of elements of Jacobian Matrix	Effect on d^*	Effect on π^*	Effect on g^*
1	E_1	$J_{11} < 0,\ J_{12} < 0,\ J_{21} > 0,\ J_{22} < 0$	negative	positive	negative
2.1.1	E_2	$J_{11} < 0,\ J_{12} > 0,\ J_{21} < 0,\ J_{22} < 0$	positive	positive	negative
2.1.2	E_7	$J_{11} < 0,\ J_{12} < 0,\ J_{21} < 0,\ J_{22} < 0$	negative	positive	ambiguous
2.2	E_4	$J_{11} < 0,\ J_{12} > 0,\ J_{21} < 0,\ J_{22} < 0$	positive	positive	negative
	E_5	$J_{11} > 0,\ J_{12} > 0,\ J_{21} < 0,\ J_{22} < 0$	positive	negative	ambiguous
	E_{10}	$J_{11} = 0,\ J_{12} > 0,\ J_{21} < 0,\ J_{22} < 0$	positive	negative	ambiguous

Therefore we get, $\frac{dd^*}{d\gamma_0} = \frac{\rho J_{12}}{(J_{11}J_{22} - J_{12}J_{21})}$ and $\frac{d\pi^*}{d\gamma_0} = \frac{-\rho J_{11}}{(J_{11}J_{22} - J_{12}J_{21})}$. Table 5.1 summarizes the results of the comparative statics.

Case 1: At E_1, as γ_0 increases, d^* decreases and π^* increases. As illustrated in Figure 5.1a, a rise in γ_0 shifts the $\dot{\pi} = 0$ isocline upward.[29] Consequently, we get a new equilibrium point at E_1'. Intuitively, a rise in γ_0, *ceteris paribus*, raises the desired profit share of firms and thereby pushes the $\dot{\pi} = 0$ isocline upwards. For a given π, at the old steady state E_1, the debt-capital ratio is higher than required for $\dot{\pi} = 0$ to be satisfied. This higher level of d puts upward pressure on profit share through equation (3.11) (as $J_{21} > 0$ here). As a result, profit share starts rising. As soon as profit share rises, debt market deviates from its equilibrium position. Given the level of d, profit share is now higher than required for $\dot{d} = 0$ to be satisfied. As $J_{12} = \frac{\partial \dot{d}}{\partial \pi} < 0$, debt-capital ratio must fall. Combination of higher profit share and lower debt-capital ratio ultimately ensure to achieve the new equilibrium point E_2 either monotonically or spiraling around E_2.

At point E_1, the impact of a rise in γ_0 on the long run equilibrium rate of capital accumulation is unambiguously negative, and can be shown mathematically as, $\frac{dg^*}{d\gamma_0} = \left\{ \overbrace{\frac{\partial g^*}{\partial d}}^{+} \overbrace{\frac{\partial d^*}{\partial \gamma_0}}^{-} + \overbrace{\frac{\partial g^*}{\partial \pi}}^{-} \overbrace{\frac{\partial \pi^*}{\partial \gamma_0}}^{+} \right\} < 0$. At point E_1, as γ_0 rises, π^* increases which in turn reduces the equilibrium rate of capital accumulation. On the other hand, a rise in γ_0 decreases d^*. As the economy is in a debt-led growth regime, a fall in d^* decreases g^*. Therefore, for a rise in γ_0, g^* decreases. Summary of the above analysis yields the following proposition.

[29] Slope of the $\dot{\pi} = 0$ isocline is $\left.\frac{d\pi}{dd}\right|_{\dot{\pi}=0} = \frac{\gamma_1 A \delta}{2\psi\pi - (\psi\gamma_0 + \beta)}$. Partially differentiating it with respect to γ_0 we get, $\frac{\partial}{\partial \gamma_0}\left(\left.\frac{d\pi}{dd}\right|_{\dot{\pi}=0}\right) = \frac{\gamma_1 A \delta \psi}{\{2\psi\pi - (\psi\gamma_0 + \beta)\}^2} > 0$. Thus as γ_0 rises, $\dot{\pi} = 0$ isocline becomes steeper. The vertical intercept of the $\dot{\pi} = 0$ isocline is $\pi_1 = \frac{(\psi\gamma_0 + \beta) + \sqrt{(\psi\gamma_0 + \beta)^2 - 4\psi(\gamma_0\beta - \gamma_1\alpha)}}{2\psi}$. So partially differentiating it with respect to γ_0 we get, $\frac{\partial \pi_1}{\partial \gamma_0} = \frac{\psi + \left\{(\psi\gamma_0 + \beta)^2 - 4\psi(\gamma_0\beta - \gamma_1\alpha)\right\}^{-\frac{1}{2}}\psi(\psi\gamma_0 - \beta)}{2\psi} > 0$. Note that if $(\psi\gamma_0 - \beta) > 0$, $\frac{\partial \pi_1}{\partial \gamma_0}$ is unambiguously positive. But if $(\psi\gamma_0 - \beta) < 0$, then $\frac{\partial \pi_1}{\partial \gamma_0} > 0 \iff 1 + \left\{(\psi\gamma_0 + \beta)^2 - 4\psi(\gamma_0\beta - \gamma_1\alpha)\right\}^{-\frac{1}{2}}(\psi\gamma_0 - \beta) > 0$. Further calculation implies $\frac{\partial \pi_1}{\partial \gamma_0} > 0$ if and only if $(\psi\gamma_0 - \beta)^2 < \left\{(\psi\gamma_0 - \beta)^2 + 4\psi\gamma_1\alpha\right\}$ holds. This indeed is satisfied. Thus irrespective of the sign of $(\psi\gamma_0 - \beta)$, $\frac{\partial \pi_1}{\partial \gamma_0}$ is unambiguously positive.

Proposition 2. *Suppose the economy is in the stable steady state E_1. Then a rise in γ_0, the degree of mergers, acquisitions and hostile takeovers causes both d^* and g^* to fall, even though it leads to a rise in π^*.*

Case 2.1.1: Here, as γ_0 increases, both d^* and π^* rises. As illustrated in Figure 5.1b, for a rise in γ_0, the $\dot{\pi} = 0$ isocline shifts upward and becomes steeper, and intersects the $\dot{d} = 0$ isocline at E_2' and E_3'.[30]

The economy in our model is always in a wage-led growth regime. Hence, $\frac{\partial g^*}{\partial \pi} < 0$. Moreover, $\frac{\partial g^*}{\partial d}$ is also negative here (as $A < 0$). At point E_2, the impact of a rise in γ_0 on the long run equilibrium rate of capital accumulation is unambiguously negative and can be shown as $\frac{dg^*}{d\gamma_0} = \left\{ \overbrace{\frac{\partial g^*}{\partial d}}^{-} \overbrace{\frac{\partial d^*}{\partial \gamma_0}}^{+} + \overbrace{\frac{\partial g^*}{\partial \pi}}^{-} \overbrace{\frac{\partial \pi^*}{\partial \gamma_0}}^{+} \right\} < 0$. At point E_2, as γ_0 increases, π^* rises. This in turn reduces the equilibrium rate of capital accumulation. On the other hand a rise in γ_0 raises d^* which in turn reduces g^*. Hence, the final result of a rise in γ_0 on g^* is unambiguously negative. Summary of the above analysis yields the following proposition.

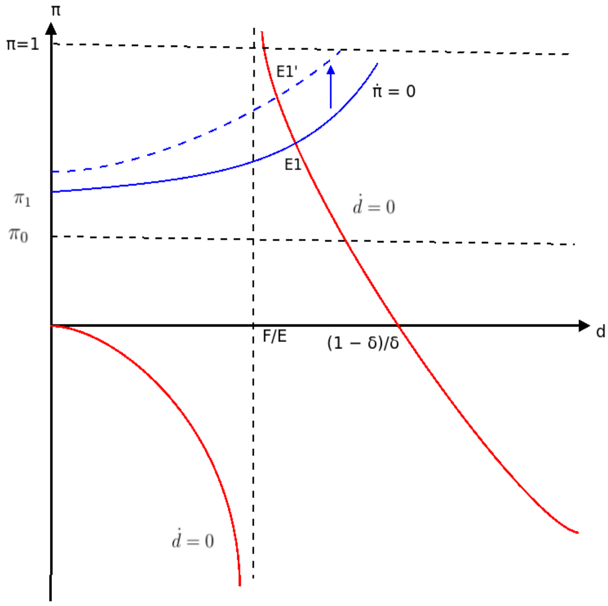
Proposition 3. *Suppose the economy is in the stable steady state E_2. Then a rise in γ_0 increases π^*, but both d^* and g^* fall.*

Case 2.1.2: Analysis of *cases 2.1.2.a, 2.1.2.b, 2.1.2.c* and *2.1.2.d* are the same as in *case 2.2.a, 2.2.b, 2.2.c* and *2.2.e* respectively. Therefore, we focus only on *cases 2.1.2.e,* and *2.a.2.f.* At E_7, as γ_0 increases, as shown in Figure 5.1c, d^* falls and π^* rises. At point E_7, the impact of a rise in γ_0 on the long run equilibrium rate of capital accumulation is ambiguous and is shown as $\frac{dg^*}{d\gamma_0} = \left\{ \overbrace{\frac{\partial g^*}{\partial d}}^{-} \overbrace{\frac{\partial d^*}{\partial \gamma_0}}^{-} + \overbrace{\frac{\partial g^*}{\partial \pi}}^{-} \overbrace{\frac{\partial \pi^*}{\partial \gamma_0}}^{+} \right\} \gtreqless 0$. Summing up the above analysis we get the following proposition.

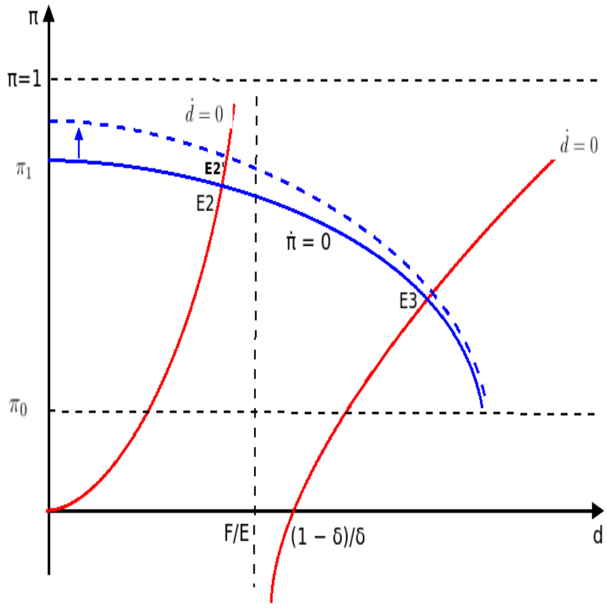
Proposition 4. *Suppose the economy is in the stable steady state E_7. Then a rise in γ_0 increases π^* and decreases d^*. However, the effect of γ_0 on g^* is ambiguous.*

Case 2.2: A rise in γ_0 enhances both d^* and π^* in E_4. For a diagrammatic illustration, see Figure 5.2a. A rise in γ_0 decreases the slope of the $\dot{\pi} = 0$ isocline (as $\frac{\partial}{\partial \gamma_0}\left(\frac{d\pi}{dd}\Big|_{\dot{\pi}=0}\right) = \frac{\gamma_1 A \delta \psi}{\{2\psi\pi - (\psi\gamma_0 + \beta)\}^2} < 0$) whereas it causes the rise in the vertical intercept

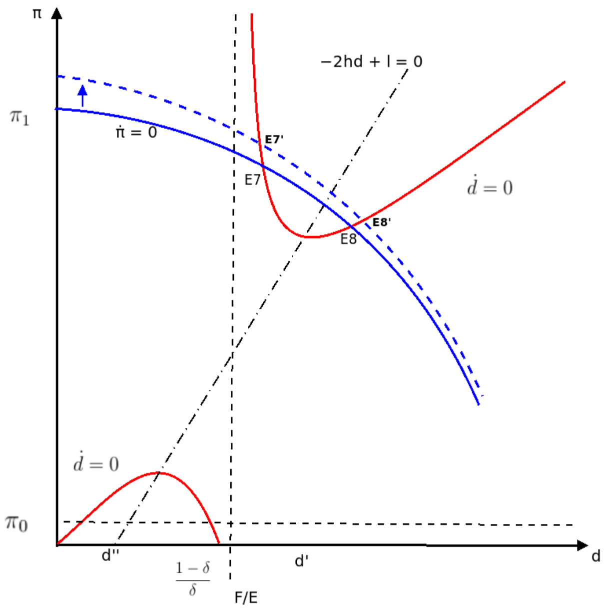
[30] Partial differentiation of the slope of the $\dot{\pi} = 0$ isocline with respect to γ_0 yields $\frac{\partial}{\partial \gamma_0}\left(\frac{d\pi}{dd}\Big|_{\dot{\pi}=0}\right) = \frac{\gamma_1 A \delta \psi}{\{2\psi\pi - (\psi\gamma_0 + \beta)\}^2} < 0$. Consequently, a rise in γ_0 makes the slope steeper. Partially differentiating the vertical intercept of the $\dot{\pi} = 0$ isocline with respect to γ_0 we get $\frac{\partial \pi_1}{\partial \gamma_0} = \frac{1 + \left\{(\psi\gamma_0 + \beta)^2 - 4\psi(\gamma_0\beta - \gamma_1\alpha)\right\}^{-\frac{1}{2}}(\psi\gamma_0 - \beta)}{2} > 0$. Thus when γ_0 rises, the vertical intercept rises.



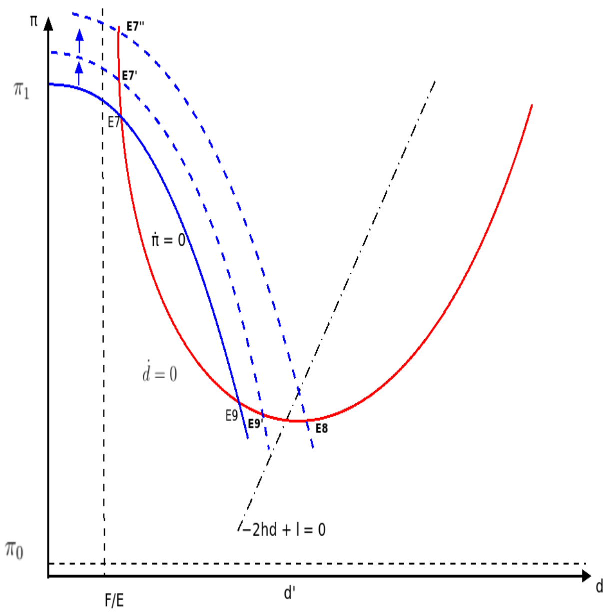
(a) *case 1*



(b) *case 2.1.1*



(c) Effect of a change in γ_0



(d) Effect of a change in γ_0

Figure 5.1

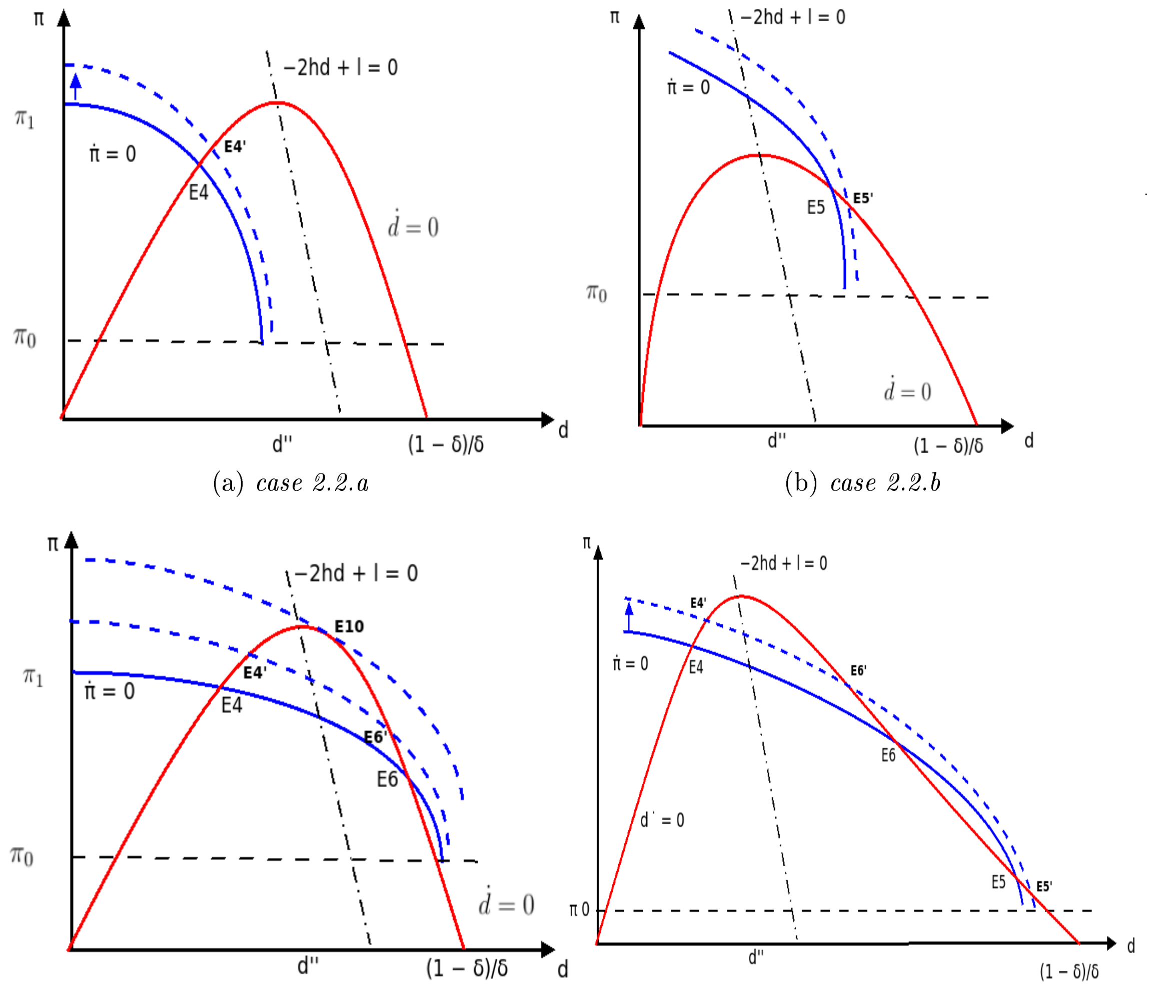
$(\because \frac{\partial \pi_1}{\partial \gamma_0} = \frac{1+\left\{(\psi\gamma_0+\beta)^2-4\psi(\gamma_0\beta-\gamma_1\alpha)\right\}^{-\frac{1}{2}}(\psi\gamma_0-\beta)}{2} > 0)$. Hence the $\dot{\pi} = 0$ isocline shifts upward and becomes steeper, and intersects the $\dot{d} = 0$ isocline at E_4'.

The economy in our model, in *case 2.2.a*, is in a wage-led and debt-burdened growth regime. Hence $\frac{\partial g^*}{\partial \pi}$ and $\frac{\partial g^*}{\partial d}$ are both negative. At point E_4, the impact of a rise in γ_0 on the long run equilibrium rate of capital accumulation is unambiguously negative, and can be illustrated as $\frac{dg^*}{d\gamma_0} = \left\{ \overbrace{\frac{\partial g^*}{\partial d}}^{-} \overbrace{\frac{\partial d^*}{\partial \gamma_0}}^{+} + \overbrace{\frac{\partial g^*}{\partial \pi}}^{-} \overbrace{\frac{\partial \pi^*}{\partial \gamma_0}}^{+} \right\} < 0$. A higher degree of mergers, acquisitions and so on always increases the share of profit in the short run, and as a result, the equilibrium rate of capital accumulation unambiguously decreases in the short run (as there is wage-led growth regime in the economy). In *case 2.2.a*, a higher degree of mergers, acquisitions and hostile takeovers also decreases the equilibrium rate of capital accumulation in the long run. But the reason is different. At point E_4, as γ_0 increases, the equilibrium value of π^* increases, which in turn reduces the equilibrium rate of capital accumulation. On the other hand, a rise in γ_0 also increases d^*, which in turn reduces g^*. Hence, the final result of a rise in γ_0 on g^* is unambiguously negative. We sum up this analysis with the following proposition.

Proposition 5. *Suppose the economy is in the stable steady state E_4. Then a rise in γ_0 increases both π^* and d^*, but g^* falls.*

At E_5, as γ_0 increases, the equilibrium value of d^* increases and π^* decreases (see Figure 5.2b). One of the most important objective for firms, as they go for mergers, acquisitions and hostile takeovers, is to increase the profit share. Although this objective is fulfilled in the short run, we get opposite result in the long run in *case 2.2.b*. The reason is as follows. As γ_0 increases, *ceteris paribus*, the desired share of profit of firms rises and it pushes the $\dot{\pi} = 0$ isocline upwards. For a given π, at the old steady state E_5, the debt-capital ratio is lower than required for $\dot{\pi} = 0$ to be satisfied. This lower level of d puts pressure on profit share through equation (3.11) (as $J_{21} < 0$ here). As a result, profit share starts rising initially. However, as the economy is in a wage-led demand regime, a rise in profit share reduces u^*. This fall in u^* overcompensates the rise in π. So, there is a deterioration in rentiers' level of income[31]. As a result, rentiers' consumption and savings both fall. Rentiers save a fraction $(1-\delta)$ of its savings in banks as deposits which goes to the workers as lending and therefore workers' debt level (normalized by the capital stock) decreases. On the other hand, as the economy is in a wage-led growth regime, for a given d, a rise in π leads to a deterioration in the investment rate (i.e. dg^* falls). Here the

[31] For every unit rise in profit share, *ceteris paribus,* rentiers' income level falls by $(1-s_f)\{u^*+\pi\frac{\partial u^*}{\partial \pi}\} = \frac{-(1-s_f)(\alpha+\delta A d)\beta}{(\psi\pi-\beta)^2}$ unit.



(a) *case 2.2.a* (b) *case 2.2.b*

(c) *case 2.2.c* (d) *case 2.2.d*

Figure 5.2: Effect of a change in γ_0

latter dominates the former and consequently, the debt-capital ratio increases. Further, as the economy is in a strong debt-burdened demand and growth regime, magnitude of A is very high here. Near E_5, π is also small which makes $(\psi\pi - \beta)$ a small positive number. Due to a high magnitude of A and a small positive value of $(\psi\pi - \beta)$, as debt-capital ratio rises, given the profit share, the desired profit share (π^d) falls by a significantly large amount. Therefore, to achieve $\dot{\pi} = 0$, π falls by a large amount. This fall in π more than compensates the initial rise in π. Hence, because of a rise in γ_0, there is finally a fall in the share of profit.

At point E_5, the impact of a rise in γ_0 on the long run equilibrium rate of capital accumulation is ambiguous (as $\frac{dg^*}{d\gamma_0} = \left\{ \overbrace{\frac{\partial g^*}{\partial d}}^{-} \overbrace{\frac{\partial d^*}{\partial \gamma_0}}^{+} + \overbrace{\frac{\partial g^*}{\partial \pi}}^{-} \overbrace{\frac{\partial \pi^*}{\partial \gamma_0}}^{-} \right\} \gtreqless 0$). At point E_5, as γ_0 increases, the equilibrium value of π^* decreases which in turn enhances the equilibrium rate of capital accumulation. On the other hand a rise in γ_0 increases d^* which in turn reduces g^*.

Hence, the final result of a rise in γ_0 on g^* is ambiguous, and depends on whether its impact on g^* through a reduction in π^* dominates the other one or not. Proposition 6 summarizes the results.

Proposition 6. *Suppose the economy is in the stable steady state E_5. Then a rise in γ_0 increases d^* and decreases π^*. However, the effect of γ_0 on g^* is ambiguous.*

Note that the the analysis in *case 2.2.c* is same as in *case 2.2.a.* However, if γ_0 increases significantly, the two equilibria E_4 and E_6 can coincide. As a result, a saddle point unstable equilibrium (E_{10}) emerges (see Figure 5.2c). Thus, for a massive rise in mergers, acquisition and hostile takeovers, instability arises in the economy which was initially stable (at E_4). Proposition 7 summarizes the results.

Proposition 7. *Suppose the economy is in the stable steady state E_4 in case 2.2.c. Then a rise in γ_0 increases d^* and decreases π^*. However, the effect of γ_0 on g^* is ambiguous. Moreover, for a sufficiently large rise in γ_0, instability arises in the economy.*

From the above analysis of different cases, few points are worth remembering.

Remark 2. *A rise in mergers, acquisitions and hostile takeovers need not necessarily increase the share of profit in the long run. In certain cases, on the contrary, a rise in γ_0 may decrease the profit share (e.g. case 2.2.b).*

In the next section we consider a special case which is very similar to Hein (2012a).[32] Here, we first investigate the long run dynamics, and then consider how γ_0 parameter influences the equilibrium values of debt-capital ratio and the share of profit.

6 *A Special Case*

In this section, we assume that firms distribute their entire profit to the rentiers (i.e. $s_f = 0$) and rentiers have zero consumption propensity out of assets (i.e. $c_q = 0$ which in turn implies $A = -(1 - c_r)i < 0$), and hence, rentiers' consumption demand comes out of their income only. So effectively we are going back to Hein's (2012a) model. Inserting $s_f = 0$, and $c_q = 0$ in equation (3.2) we get,

$$\dot{d} = \frac{(1 - c_r)\{\delta d - (1 - \delta)\}\{\beta i d - \alpha \pi\}}{\psi \pi - \beta} \tag{6.1}$$

[32] Only major difference is that rather than a parameter, here the share of profit is treated as a long run variable.

In equilibrium, $\dot{d} = 0$, which implies either $d\big|_{\dot{d}=0} = \frac{1-\delta}{\delta}$ or $d\big|_{\dot{d}=0} = \frac{\alpha\pi}{\beta i}$.[33] When $d\big|_{\dot{d}=0} = \frac{\alpha\pi}{\beta i}$, a straight line passing through the origin with slope $\frac{\beta i}{\alpha}$ represents the $\dot{d} = 0$ isocline. On the other hand, when $d\big|_{\dot{d}=0} = \frac{1-\delta}{\delta}$, the $\dot{d} = 0$ isocline is represented by the straight line parallel to the ordinate (π-axis). However, if we assume $d = \frac{\alpha\pi}{\beta i}$, from equation (2.11), the equilibrium rate of capital accumulation (g^*) become zero. Therefore, to achieve a positive value of g^*, let us assume $d < \frac{\alpha\pi}{\beta i}$. Thus $\dot{d} = 0$ implies $d\big|_{\dot{d}=0} = \frac{1-\delta}{\delta}$. Note that the only economically meaningful region lies above the $d\big|_{\dot{d}=0} = \frac{\alpha\pi}{\beta i}$ line where $\pi \in (\pi_0, 1)$. The only economically meaningful segment of the $\dot{d} = 0$ isocline lies on the $d\big|_{\dot{d}=0} = \frac{1-\delta}{\delta}$ line where $\pi > \frac{\beta i d}{\alpha}$.[34] The red solid line in Figure 6.1b represents this.

Partial differentiation of equation (6.1) w.r.t. d yields,

$$J_{11} = \frac{\partial \dot{d}}{\partial d} = \frac{(1-c_r)\delta\{\beta i d - \alpha\pi\}}{\psi\pi - \beta} \quad (\because \{\delta d - (1-\delta)\} = 0) \tag{6.2}$$

As we only focus on $d < \frac{\alpha\pi}{\beta i}$ we get, $J_{11} < 0$. Partial differentiation of equation (6.1) w.r.t. π yields,

$$J_{12} = \frac{\partial \dot{d}}{\partial \pi} = \frac{(1-c_r)\left[\alpha\beta(\delta d - 1 + \delta)) - \beta i d(\delta d - 1 + \delta)\psi\right]}{(\psi\pi - \beta)^2} = 0 \quad (\because \{\delta d - (1-\delta)\} = 0) \tag{6.3}$$

Let's focus on the distributional dynamics now. From equation (3.10) we get

$$\dot{\pi} = \rho\left[\frac{-\psi\pi^2 + (\gamma_0\psi + \beta)\pi - (\beta\gamma_0 - \gamma_1\alpha) - \gamma_1\psi i d}{\psi\pi - \beta}\right] \tag{6.4}$$

Partial differentiation of equation (6.4) w.r.t. d and π yields

$$J_{21} = \frac{\partial \dot{\pi}}{\partial d} = \rho\left(\frac{-\gamma_1\psi i}{\psi\pi - \beta}\right) < 0 \tag{6.5}$$

$$J_{22} = \frac{\partial \dot{\pi}}{\partial \pi} = \rho\left[\frac{-2\psi\pi + (\psi\gamma_0 + \beta)}{\psi\pi - \beta}\right] \tag{6.6}$$

So, the slope of the $\dot{\pi} = 0$ isocline $\frac{d\pi}{dd}\Big|_{\dot{\pi}=0} = -\frac{J_{21}}{J_{22}} = -\frac{\gamma_1\psi i}{2\psi\pi - (\psi\gamma_0 + \beta)} < 0$ for $\pi > \pi_0 = \frac{\gamma_0\psi + \beta}{2\psi}$, and $\frac{d\pi}{dd}\Big|_{\dot{\pi}=0} > 0$ for $\pi < \pi_0$.[35]

[33] Note that now, $A = -(1-c_r)i < 0$; $\psi = (1-c_r)\delta > 0$; $E = \psi\alpha = (1-c_r)\delta\alpha > 0$; $F = (1-\delta)(1-c_r)\alpha > 0$; $h = \beta A\delta = -(1-c_r)i\beta\delta < 0$; $l = -(1-c_r)[(1-\delta)\beta i + \delta\alpha\pi] < 0$; $m = (1-\delta)(1-c_r)\alpha\pi > 0$; $u^* = \frac{\alpha - \psi i d}{\psi\pi - \beta}$; and $g^* = \frac{\psi\alpha\pi - \psi\beta i d}{\psi\pi - \beta}$.

[34] Inserting $d\big|_{\dot{d}=0} = \frac{1-\delta}{\delta}$ into this inequality we get $\pi > \frac{(1-\delta)\beta i}{\alpha\delta}$.

[35] For vertical intercept, we insert $d = 0$ in equation (6.4) and obtain $\pi\Big|_{\dot{\pi}=0}^{d=0} = \gamma_0 + \frac{\gamma_1\alpha}{(\psi\pi - \beta)}$. Therefore, for $d = 0$, we get two values of π, $\pi_1 = \frac{(\psi\gamma_0 + \beta) + \sqrt{(\psi\gamma_0 + \beta)^2 - 4\psi(\gamma_0\beta - \gamma_1\alpha)}}{2\psi}$ and $\pi_2 = \frac{(\psi\gamma_0 + \beta) - \sqrt{(\psi\gamma_0 + \beta)^2 - 4\psi(\gamma_0\beta - \gamma_1\alpha)}}{2\psi}$. The discriminant is $(\psi\gamma_0 + \beta)^2 - 4\psi(\gamma_0\beta - \gamma_1\alpha) = (\psi\gamma_0 - \beta)^2 + 4\psi\gamma_1\alpha$

Necessary and sufficient condition for existence of the equilibrium is that the $\dot{\pi} = 0$ isocline changes its slope (from negative to positive) to the right of the $\dot{d} = 0$ isocline. Suppose it is occurring here.[36] As a result we get a unique equilibrium X where Det(J)$=(J_{11}J_{22} - J_{12}J_{21}) > 0$ and tr(J) $= J_{11} + J_{22} < 0$. So, point X is a stable steady state (see Figure 6.1b).

Suppose due to some reason the profit share rises above its steady state level. As the profit share is higher than its steady state value, it must fall due to equation (6.6). This is the direct stable effect. However, as the profit share has no effect on debt-capital ratio (because $J_{12} = 0$), π cannot have any indirect effect on $\dot{\pi}$. Thus, because of this direct stable effect, profit share again must fall to its initial steady state level. Similarly, if due to some reason debt-capital ratio rises above its steady state, due to equation (6.2) it must fall. This is the direct stable effect. Second, the rise in debt-capital ratio leads to a fall in the profit share due to $J_{21} < 0$ (see equation (6.5)). However, profit share itself has no effect on the debt-capital ratio (as $J_{12} = 0$). Hence, there is no indirect effect. Thus, as there is only a direct stable effect, debt-capital ratio again must fall to its initial steady state level.

Now let us focus on how mergers, acquisitions etc. influence the equilibrium levels of debt-capital ratio and the share of profit. Total differentiation of equations (6.1) and (6.4) shows the effects of parametric change of γ_0 in the economy which imply

$$\begin{bmatrix} J_{11} & J_{12} \\ J_{21} & J_{22} \end{bmatrix} \begin{bmatrix} dd \\ d\pi \end{bmatrix} = \begin{bmatrix} 0 \\ -\rho \end{bmatrix} d\gamma_0 \tag{6.7}$$

Therefore we get, $\frac{dd^*}{d\gamma_0} = \frac{\rho J_{12}}{(J_{11}J_{22} - J_{12}J_{21})} = 0$ and $\frac{d\pi^*}{d\gamma_0} = \frac{-\rho J_{11}}{(J_{11}J_{22} - J_{12}J_{21})} > 0$.[37] Figure 6.1c explains it diagrammatically.[38]

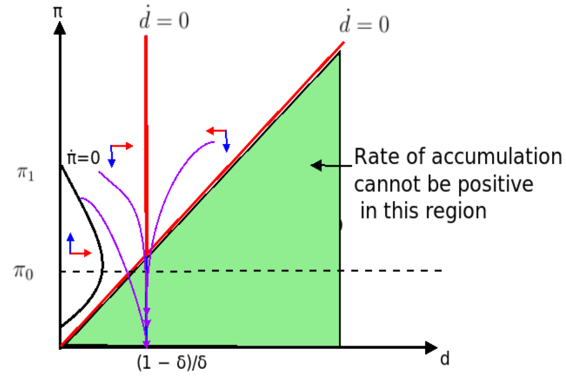
Note that as γ_0 increases, π^* rises. However the equilibrium value of d^* remains unchanged. The reason is as follows. As γ_0 rises, *ceteris paribus*, the desired profit share of firms increases and thereby pushes the $\dot{\pi} = 0$ isocline upwards. For a given d, at the

which is always positive. When $(\gamma_0\beta - \gamma_1\alpha) < 0$ holds, we get $(\psi\gamma_0 + \beta)^2 - 4\psi(\gamma_0\beta - \gamma_1\alpha) > (\psi\gamma_0 + \beta)^2$, and therefore $\pi_1 > 0 > \pi_2$. When the reverse of that occurs i.e. when $(\gamma_0\beta - \gamma_1\alpha) > 0$, $(\psi\gamma_0 + \beta)^2 - 4\psi(\gamma_0\beta - \gamma_1\alpha) < (\psi\gamma_0 + \beta)^2$ holds. As a result, we get $\pi_1 > \pi_2 > 0$.

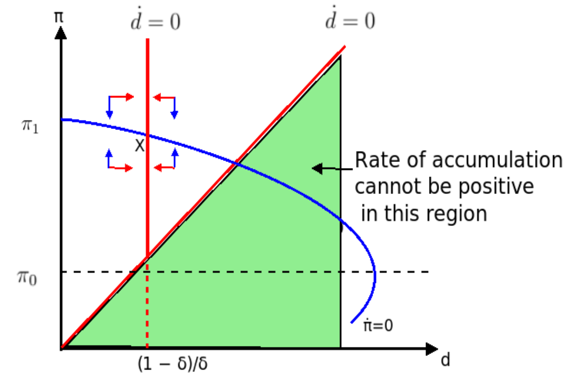
[36] Otherwise there will not be any equilibrium and the economy will converge to the point where $d = \frac{1-\delta}{\delta}$ and $\pi = 0$. It is illustrated in Figure 6.1a.

[37] As $J_{11} < 0$, and $J_{12} = 0$.

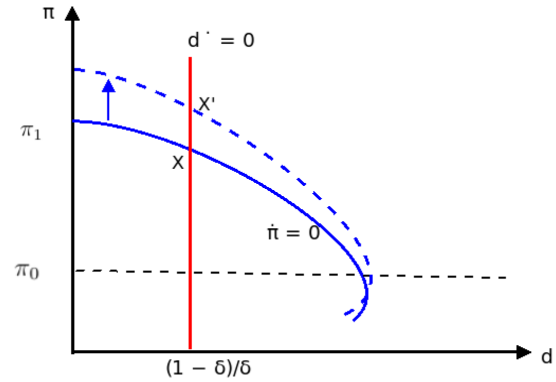
[38] Slope of the $\dot{\pi} = 0$ isocline is $\left.\frac{d\pi}{dd}\right|_{\dot{\pi}=0} = \frac{\gamma_1\psi i}{-[2\psi\pi - (\psi\gamma_0 + \beta)]}$. So partially differentiating it with respect to γ_0 we get, $\frac{\partial}{\partial\gamma_0}\left(\left.\frac{d\pi}{dd}\right|_{\dot{\pi}=0}\right) = -\frac{\gamma_1\psi^2 i}{\{2\psi\pi - (\psi\gamma_0 + \beta)\}^2} < 0$. Therefore, the slope decreases further for a rise in in γ_0. The vertical intercept of the $\dot{\pi} = 0$ curve is $\pi_1 = \frac{(\psi\gamma_0+\beta)+\sqrt{(\psi\gamma_0+\beta)^2 - 4\psi(\gamma_0\beta - \gamma_1\alpha)}}{2\psi}$. Therefore, $\frac{\partial\pi_1}{\partial\gamma_0} = \frac{1 + \left\{(\psi\gamma_0+\beta)^2 - 4\psi(\gamma_0\beta - \gamma_1\alpha)\right\}^{-\frac{1}{2}}(\psi\gamma_0 - \beta)}{2} > 0$. Hence, when γ_0 rises, the vertical intercept of the $\dot{\pi} = 0$ isocline rises (See footnote 29 for why $\frac{\partial\pi_1}{\partial\gamma_0} > 0$).



(a) When the isoclines do not intersect



(b) Long run stable equilibrium



(c) Effect of a change in γ_0

Figure 6.1: Diagram for a special case

old steady state X, the profit share is lower than required for $\dot{\pi} = 0$ to be satisfied. This lower level of π puts upward pressure on profit share through equation (6.6) (as $J_{22} < 0$). As a result, profit share starts rising. However, irrespective of the level of profit share, here the debt market is always in equilibrium. Consequently, there is no change in d. The profit share, however, must rise till the gap between the desired profit share of firms and the actual level of π vanishes. Thus eventually the new equilibrium point X' is achieved where π^* rises but d^* remains unchanged. Note that here the movement from X to X' happens along the $\dot{d} = 0$ isocline.

The economy in our model is always in a wage-led growth regime, and hence $\frac{\partial g^*}{\partial \pi} < 0$. Moreover, $\frac{\partial g^*}{\partial d} = \frac{-\psi\beta i}{\psi\pi - \beta}$ is also negative here. At point X, the impact of a rise in γ_0 on the long run equilibrium rate of capital accumulation is unambiguously negative as $\frac{dg^*}{d\gamma_0} = \left\{ \overbrace{\frac{\partial g^*}{\partial d}}^{-} \overbrace{\frac{\partial d^*}{\partial \gamma_0}}^{0} + \overbrace{\frac{\partial g^*}{\partial \pi}}^{-} \overbrace{\frac{\partial \pi^*}{\partial \gamma_0}}^{+} \right\} = \frac{\partial g^*}{\partial \pi} \frac{\partial \pi}{\partial \gamma_0} < 0$. Summary of the above analysis yields the following proposition.

Proposition 8. *Suppose the economy is in the stable steady state X. Then, a rise in γ_0 causes π^* to rise and g^* to fall. However d^* remains unaffected.*

7 *Conclusion*

In this paper, we dealt with a neo-Kaleckian growth model in which in the long run, the share of profit and the debt-capital ratio of workers evolve endogenously. We examined the short-run stability condition and analysed some comparative statics. We reached the conclusion that the economy can be either in a debt-led or in a debt-burdened demand and growth regime. However, the economy is always in a wage-led demand and growth regime. Unlike Hein (2012a), we found that an increase in rentiers' loans to the workers is not necessarily expansionary for the aggregate demand and the growth rate of the economy. Instead, if rentiers' consumption out of wealth is sufficiently high, it can overcompensate the loss in consumption demand of workers due to a reduction in the proportion of rentiers' lending to workers. We also see that for a given debt-capital ratio (d), if the equity-debt ratio (λ) rises, both the equilibrium rate of capacity utilization and capital accumulation rise unambiguously. We find that higher the dividend payout ratio (or lower the retention rate of firms), higher is the equilibrium degree of capacity utilization and accumulation rate.

The main departure of our analysis from the earlier literature, however, lies in the long run, where along with the debt-capital ratio, we endogenized the share of profit as well. In the long-run, we found richer dynamics than Hein (2012a). Because of the incorporation of the distributional dynamics, we found that the interaction between debt-capital ratio and distributional dynamics can lead to instability in the economy. The economy can be either in a debt-led or in a debt-burdened demand and growth regime. It is found that there is always a unique stable equilibrium in the debt-led demand and growth regime. However, in the debt-burdened demand and growth regime, several cases can occur. If the economy is in a weak debt-burdened demand and growth regime, the steady-state equilibrium is either stable or saddle point unstable. However, in a moderate or in a strong debt-burdened demand and growth regime, along with stable or unstable equilibrium, a limit cycle is also possible.

We found that when the economy is either in a moderate or in a strong debt-burdened demand and growth regime, the level of debt-capital ratio plays an important role for achieving stability in the economy. As long as a unique steady state exists, a lower value of debt-capital ratio ($d < \frac{l}{2h}$) ensures the steady state to be stable. However, in case of multiple equilibria, a lower value of debt-capital ratio ($d < \frac{l}{2h}$) ensures one of those equilibria to be stable. This stability occurs mainly because of the fact that as long as the debt-capital ratio is lower than $\frac{l}{2h}$, it has a direct stable feedback effect on itself. On the other hand, the indirect feedback on d is either stable, or unstable and weak. Whenever debt-capital ratio rises beyond $\frac{l}{2h}$, either the economy is unstable (saddle point) or its stability depends on the value of ρ, the speed of adjustment parameter of the distributional dynamics (for example in *case 2.2b*, point E_5). Here, the debt-capital ratio has an unstable self-feedback effect. Further, the indirect stable effect is either relatively weak (irrespective of the size of ρ and consequently, it causes saddle point instability) or the indirect stable effect depends on the size of ρ. In the latter case, a higher value of ρ (when ρ exceeds $\hat{\rho}$) ensures stability in the economy. Further, (in *case 2.1.2.b* and *case 2.2.b*), when ρ falls to $\hat{\rho}$ the system with a stable steady state loses its stability and gives birth to limit cycles.

We observed that in the short run, a rise in the degree of mergers, acquisitions and hostile takeovers etc. have a positive impact on the share of profit. However, in the long run, the result is need not necessarily the same. For example, in *case 2.2.b* and *2.2.e,* hostile takeovers, mergers and acquisitions have a negative influence on the profit share. A rise in the degree of mergers, acquisitions and hostile takeovers etc. causes a rise in the desired profit share of firms. *Ceteris paribus*, it leads to an initial rise in the profit share. As soon as profit share rises, through equation (3.5) it increases the debt-capital ratio (as $J_{12} > 0$ here). As the economy is in a strong debt-burdened demand regime, a rise in d leads to a remarkable fall in the capacity utilization rate which in turn reduces the desired profit share of firms significantly. Consequently, the profit share falls. This fall in profit share more than compensates the initial rise in π.

We found that under *case 2.2.c* point E_4, to achieve a higher equilibrium rate of capital accumulation and to achieve an improvement in the functional income distribution (*vis-à-vis* workers), more stringent rules regarding mergers, acquisitions and hostile takeovers are desirable.[39] On the other hand, for sufficiently high increment in the degree of mergers, acquisitions and hostile takeovers, the economy can move away from the stable steady state (E_4) to the saddle-point (E_{10}) i.e. instability emerges in the economy[40] (see Figure 5.2c).

Under a special case where firms distribute their entire profit to the rentiers and rentiers have zero consumption propensity out of wealth, only a unique stable steady state arises in the economy. Here we found that mergers, acquisitions and hostile takeovers have an unambiguously positive effect on the share of profit in the long run. However, in the long run, a rise in the degree of mergers and so on have no effect on the equilibrium level of debt-capital ratio and these mergers and acquisitions negatively affect the long run equilibrium rate of capital accumulation. As a consequence, under this particular case, to achieve a higher equilibrium rate of capital accumulation and to achieve an improvement in the functional income distribution (*vis-à-vis* workers), more stringent rules regarding mergers, acquisitions and hostile takeovers are desirable.

[39] Same is true for point E_4 of *case 2.1.2.a, 2.1.2.c, 2.2.a,* and *2.2.d* as well.

[40] Same is true in *case 2.1.2.c* and *2.2.d* as well.

The analytical framework in this paper is subject to a few limitations. First, we did not focus on the possibility of firms' indebtedness. We have also assumed that the firms' retention ratio is fixed. That can be endogenized to make the model more realistic. This, however, will be addressed in another paper. Second, in our model, banks have played a passive role. Finally, our model is that of a closed economy where there is no role for the government. These issues are, however, left for future research.

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A Appendix

A.1 Proof of Proposition 1

Proof. The characteristic equation to (3.2) & (3.10) is

$$\mu^2 + (-\mathrm{tr}(\mathrm{J}))\mu + \mathrm{Det}(\mathrm{J}) = 0.$$

A necessary condition of the Hopf bifurcation for complex roots is Det(J) > 0, which is satisfied at E_5 of *case 2.2.b*. The trace of the Jacobian matrix can be made either positive or negative by appropriately selecting the value of ρ while leaving the other parameters constant. To see this, notice that $\mathrm{tr(J)} = J_{11} + J_{22} = \frac{-2hd+l}{\psi\pi-\beta} + \rho\left[\frac{-2\psi\pi+(\psi\gamma_0+\beta)}{\psi\pi-\beta}\right]$. Hence when $\rho = \hat{\rho} = \frac{l-2hd}{2\psi\pi-(\psi\gamma_0+\beta)} > 0$ ($\because (l-2hd) > 0, \{2\psi\pi - (\psi\gamma_0+\beta)\} > 0$), the following equation holds exactly:

$$\mathrm{tr(J)} = 2 * \mathrm{Re}\mu = \frac{-2hd+l}{\psi\pi-\beta} + \rho\left[\frac{-2\psi\pi+(\psi\gamma_0+\beta)}{\psi\pi-\beta}\right] = 0$$

where tr(J) is the trace of J and Reμ is the real part of its characteristic roots. As the determinant of the Jacobian matrix is positive, the product of the roots is positive in a neighborhood of the equilibrium, assuring Im$\mu \neq 0$. Now differentiating the trace of the Jacobian matrix with respect to ρ and then evaluating it at $\rho = \hat{\rho}$ we get

$$\left.\frac{\partial(\frac{\mathrm{tr(J)}}{2})}{\partial\rho}\right|_{\rho=\hat{\rho}} = \frac{-2\psi\pi+(\psi\gamma_0+\beta)}{2(\psi\pi-\beta)} < 0 \ (\because (\psi\gamma_0+\beta) - 2\psi\pi < 0)$$

So the trace is smooth, differentiable and monotonically decreasing in the speed of adjustment parameter, ρ. The trace disappears at $\rho = \hat{\rho}$. Also note that $\mathrm{tr(J)} \gtreqless 0 \iff \rho \lesseqgtr \hat{\rho}$. From the preceding discussion, all conditions for Hopf bifurcation are satisfied at $\rho = \hat{\rho}$.[41] $\square$

[41] The method of the proof is based on Gandolfo (1997).