Patent Puzzle, Inflation, and Internal Financial Constraint

Suzuki, Keishun

Chiba University

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Keishun Suzuki†

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Abstract

Although Schumpeterian growth models typically predict that stronger patent protection enhances innovation-driven economic growth, the empirical evidence does not support this idea. We explore the unclear relationship at work by shedding light on the financing of R&D investment. Empirically, R&D-intensive firms preferentially rely on their internal cash flows rather than external funds. We develop a simple monetary Schumpeterian growth model in which R&D firms face an endogenous financing choice that is consistent with this evidence. In our model, the scale of R&D investment may be financially constrained by internal cash because external financing is costly. Our model shows that the relationship between patent protection and growth can be either N-shaped, inverted-U shaped, or positive depending on the inflation rate. Specifically, we find that the growth effect of the pro-patent policy is likely to be negative under a high inflation rate, while the growth effect is always positive under the Friedman rule.

Keywords: Innovation, Patent Protection, Inflation, Financing of R&D

JEL-Classification: E44, O31, O34

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†Graduate School of Social Sciences, Chiba University. 1-33, Yayoi-cho, Inage-ku, Chiba, Japan. Email: ksuzuki@chiba-u.jp.

1 Introduction

Endogenous growth theory has traditionally predicted that pro-patent policy (i.e., strengthening patent protection) increases the reward from innovation and enhances economic growth (which is called the *Schumpeterian effect*). However, the empirical literature does not support this prediction. This contradiction is called the "patent puzzle" in the literature, and many researchers have attempted to explain the mechanism as listed below.

Our paper contributes to the literature on the patent puzzle by shedding light on the financing of R&D investment. The existing literature on finance and growth has emphasized that financial accessibility is crucial to R&D-based growth (e.g., King and Levine, 1993). Generally, R&D-intensive firms can finance investment with internal cash flows (i.e., operating profit) and external funds (e.g., new equity issuance or debt). This paper attempts to connect the patent puzzle to R&D firms' financing problem.

How do actual firms finance their R&D investment? Empirical studies show that R&D-intensive firms preferentially rely on their internal cash flows rather than external funds. Hall and Lerner (2010), in a comprehensive survey, conclude that firms appear to prefer internal funds for financing R&D investments, and they manage their cash flow to ensure this. This corporate financing behavior is consistent with the *pecking order theory* of Myers and Majluf (1984). Namely, firms prefer to use internal financing first, then debt, while issuing new equity is the last resort.

Why do innovating firms tend to finance R&D investments with their internal cash flows? One reason is that external financing is more costly than internal financing.[1] For example, debt financing simply entails borrowing costs (i.e., the interest rate on a loan). Furthermore, equity financing entails costs in the sense that the stock price drops at the announcement of a new equity issue.[2] These factors make it difficult for firms to finance R&D with external funds.

If firms finance their R&D projects with internal cash flows, some firms may face an "internal financial constraint." That is, the current cash flow constrains the scale of their R&D investment, and fluctuations in their internal cash can affect R&D. Empirically, this hypothesis seems to be correct because many studies not only report a positive relationship between R&D expenditure and internal cash flows (e.g., Hall, 1992; Himmelberg and Petersen, 1994; Sasidharan et al., 2015) but also show that R&D expenditure is sensitive to cash flow fluctuations (e.g., Brown and Petersen, 2009; Weng and Söderbom, 2018).

Our paper builds an analytically tractable growth model featuring the financing choice

[1] As another reason, R&D-intensive firms may face financial frictions because R&D-intensive firms tend to have few collateralizable assets. R&D expenditures largely go to wages for researchers, and obviously, their human capital cannot be collateralized. In addition, banks may not have the necessary skills to evaluate R&D projects subject to significant uncertainty.

[2] See Eckbo et al. (2007).

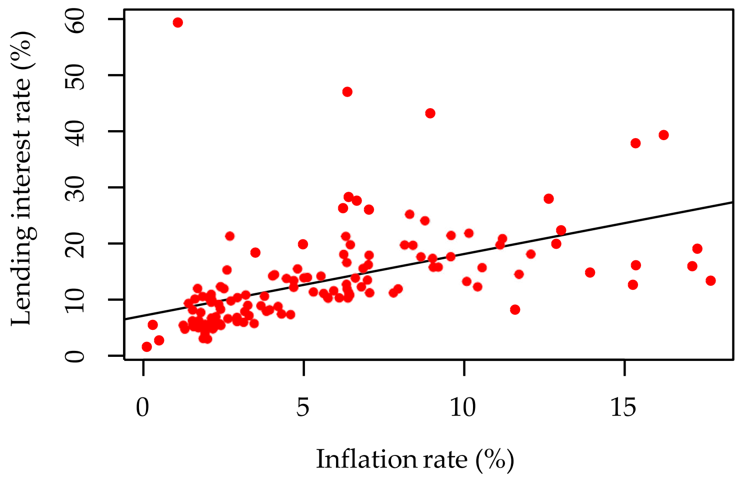

Figure 1: Scatter plots of the inflation rate (x-axis) and lending interest rate (y-axis) in 139 countries. We use the World Bank Database for both datasets. The inflation rate is measured by the inflation rate (consumer prices). The lending rate is the bank rate that usually meets the short- and medium-term financing needs of the private sector. Both variables are average values for the period 2000-2019. Both datasets are from the World Bank Database. We exclude South Sudan, Venezuela, Congo, and Argentina as outliers because their annual average inflation rate in this period is over 40%. The upward-sloping line is the approximation line. The coefficient is 1.10, the standard error is 0.16, and the t-value is 6.96.

of R&D firms that is consistent with the above stylized facts. The R&D firms finance their R&D investment using only internal cash when the incentive to innovate is weak. In this case, the scale of R&D investment may be financially constrained by internal cash because external financing is costly. However, when the incentive is sufficiently strong, the R&D firms additionally rely on external funds.

In our model, the inflation rate plays a crucial role in an R&D firm's financing choice through the interest rate. As a stylized fact, the lending interest rate is positively correlated with the inflation rate (see Fig. 1). This relationship emerges via the Fisher equation in our model. Since a higher inflation rate increases borrowing costs, an R&D firm that exhausts its internal cash is likely to face an internal financial constraint. Conversely, when the inflation rate is the lowest (i.e., the nominal interest rate is zero), an R&D firm that exhausts its internal cash can smoothly borrow money because there is no borrowing cost.

We show that the relationship between patent protection and growth can be either N-shaped, inverted-U shaped, or positive depending on the inflation rate. Specifically, we find that the pro-patent policy is likely to have a negative (positive) effect on growth when the inflation rate is (low) high. To the best of our knowledge, the relationship between the patent puzzle and inflation obtained in our paper is a new finding.

Related literature

Our paper relates to several strands of the literature. First, many existing studies have addressed the patent puzzle (e.g., Furukawa, 2007; Iwaisako and Futagami, 2013; Suzuki, 2015, 2019; Chu, Cozzi, Fan, Pan, and Zhang, 2019 (hereafter referred to as CCFPZ (2019)); Chu, Lai, and Liao, 2019; Klein, 2020). These papers study the issue in many different ways. However, to the best of our knowledge, CCFPZ (2019) is the only study that addresses the patent puzzle in an endogenous growth model with financial constraints. They emphasize that pro-patent policy has a negative (positive) effect on growth when the financial constraint is (not) binding. As a result, their model shows an inverted-U shaped relationship between patent protection and innovation.

Although CCFPZ (2019) overlaps with our paper, our contributions are notably different from theirs. First, our model analyzes the impact of the inflation rate on the growth effect of pro-patent policy by considering a monetary authority that can control the interest rate on loans by targeting the inflation rate, while there is no such monetary authority in CCFPZ (2019). Therefore, our paper provides new insights into the interaction of monetary policy and pro-patent policy. Second, the mechanisms that generate the nonmonotonic relationship between patent protection and innovation are different. CCFPZ (2019) consider an "external" financial constraint whereby the amount of money that entrepreneurs can borrow from banks is limited due to imperfect information. In contrast, we consider an internal financial constraint; that is, the scale of R&D investment is constrained by the R&D firm's internal cash due to the borrowing cost. Third, we show that the relationship between patent protection and innovation can be either N-shaped, inverted-U shaped, or positive. Therefore, the inverted-U shaped relationship in CCFPZ (2019) emerges as a special case in our model.

In addition, Chu, Lai, and Liao (2019) is also related to our paper because they also investigate the interaction between patent policy and monetary policy.[3] While they analyze the impact of patent protection on the growth effect of monetary policy, we focus on the converse direction as already explained. Therefore, our paper complements their paper. Furthermore, the results are notably different. Although the direction of the growth effect of patent policy is independent of monetary policy in Chu, Lai, and Liao (2019)[4], our paper finds that the growth effect of patent policy can be positive or negative

[3] Huang et al. (2017) also analyze the interaction of patent policy and monetary policy. They assume that entrepreneurs must finance an exogenous fraction of their R&D investment with external debt, while we consider an endogenous financing choice for R&D firms. Therefore, our paper complements their paper.

[4] See equation (25) in their study.

depending on monetary policy.

Second, this paper relates to recent endogenous growth models with financial constraints. These studies consider exogenous financial constraints on external financing.[5] For example, Aghion et al. (2019) assume that an innovating firm cannot invest more than μ times the current firm value. Similarly, Aghion et al. (2010) and Aghion et al. (2012) assume that an entrepreneur cannot borrow more than μ times the short-run profit flow. In contrast, we do not consider such exogenous financial constraints on external financing. Instead, we consider the internal financial constraint explained above.

Finally, this paper also relates to recent monetary Schumpeterian growth models in Chu and Cozzi (2014), Chu et al. (2015), Chu et al. (2017), Chu, Cozzi, Fan, Furukawa, and Liao (2019), and Zheng et al. (2019). They assume that potential firms must finance an exogenous fraction of their R&D investment with external debt.[6] Although their models help us to understand the effect of borrowing costs on innovation, they do not consider firms' financing choice, which is endogenized in our model. Therefore, there is a notable difference between their models and our model.

A stylized fact

We consider the empirical relationship among patent protection, the inflation rate, and economic growth by using cross-country panel data on 45 countries for the period 1998-2012.[7] The patent protection data come from Papageorgiadis et al. (2014), who provide an index of patent system strength for developing countries and industrialized countries. We use the World Bank Database for the GDP growth rate, the per capita GDP growth rate, and the inflation rate.[8]

We consider a similar specification to CCFPZ (2019). We estimate

$$\begin{aligned} g_{i,t+1} \quad = \quad & \beta_0 + \beta_1 \text{Patent}_{i,t} + \beta_2 \text{Inflation}_{i,t} + \beta_3 \text{Patent}_{i,t} \cdot \text{Inflation}_{i,t} \\ & + \Gamma \mathbf{X}_{i,t} + \text{Country}_i + \text{Year}_t + \varepsilon_{i,t}. \end{aligned}$$

The dependent variable $g_{i,t+1}$ is the growth rate of GDP or the growth rate of per capita GDP in country i in year $t+1$. The lagged value allows us to capture the time lag of R&D success. $\text{Patent}_{i,t}$ is the strength of patent protection, and $\text{Inflation}_{i,t}$ is the inflation rate in country i in year t. $\text{Patent}_{i,t} \cdot \text{Inflation}_{i,t}$ is the interaction term. $\mathbf{X}_{\mathrm{i,t}}$ is a vector of control

[5]An exception is Hori (2019). In his model, the financial constraint on external financing endogenously emerges through the borrowers' incentive to engage in moral hazard.

[6]This setup seems to be inconsistent with Brown et al. (2009). They point out that young firms in the U.S. finance R&D investment almost entirely with internal cash or external equity, not external debt.

[7]There are 48 countries in Papageorgiadis et al. (2014). However, we exclude Argentine and Taiwan because their inflation rates during the period are not available in the World Bank Database. Additionally, we drop Hong Kong because the ratio of deposit money banks' assets to GDP is not available.

[8]For the inflation rate, we use "Inflation, consumer prices (annual %)" in the World Bank Database.

	GDP growth rate		GDP per capita growth rate	
	(1)	(2)	(3)	(4)
Patent	0.70**	0.57	0.66**	0.63
	(0.32)	(0.33)	(0.32)	(0.33)
Inflation	0.05	0.09	0.08	0.11
	(0.07)	(0.07)	(0.07)	(0.07)
Patent · Inflation	−0.03	−0.04**	−0.04**	−0.05**
	(0.02)	(0.02)	(0.02)	(0.02)
Other controls	No	Yes	No	Yes
Year fixed effect	Yes	Yes	Yes	Yes
Country fixed effect	Yes	Yes	Yes	Yes
Observations	561	561	561	561
Adjusted R^2	0.58	0.59	0.55	0.56

Table 1: * Significant at 10%; ** Significant at 5%; *** Significant at 1%. Patent is the level of patent protection. Inflation is the inflation rate. Other control variables include the unemployment rate, the level of financial development, and the degree of openness. Standard errors in parentheses.

variables that consists of the unemployment rate, the level of financial development, and the degree of openness.[9] To control for omitted country characteristics and time trends, a country fixed effect (Country_i) and a year fixed effect (Year_t) are included. Table 2 in Appendix A reports the summary statics of these variables.

Our model in subsequent sections predicts that the growth effect of pro-patent policy is likely to be negative under a higher inflation rate. This implies that $\beta_3 < 0$.

Table 1 shows the results. As shown in columns (2) and (4), the coefficient of the interaction term is significantly negative even after including some control variables. In line with our theoretical prediction, this result shows that a higher inflation rate has a negative impact on the growth effect of pro-patent policy. Furthermore, in columns (2) and (4), the coefficient of patent protection is positive but not significant. This seems to be consistent with the unclear relationship between patent protection and economic growth that we obtained in the model.

[9]Following CCFPZ (2019), we use the ratio of deposit money banks' assets to GDP as the level of financial development. The degree of openness is defined as the sum of exports and imports as a share of GDP. We use the World Bank Database for the unemployment rate and the degree of openness. For the level of financial development, we use the financial structure dataset (September 2019 version) in the Global Financial Development Report 2019/2020 of the World Bank.

Roadmap

Our paper is organized as follows. Section 2 develops a monetary Schumpeterian growth model and solves for the long-run equilibrium. Section 3 shows the relationship between inflation and the growth effect of pro-patent policy. Section 4 concludes the paper.

2 The model

Our model is based on Grossman and Helpman (1991, Ch.4).[10] We extend their model by (a) introducing debt financing to raise funding for R&D investment, (b) assuming that the duopolistic industry consists of a leader and a follower, and (c) assuming Cournot competition instead of Bertrand competition.

2.1 Households

The economy consists of identical and infinitely lived households. The population size in the economy is $L > 0$, and there is no population growth. Time is continuous. Each household inelastically supplies one unit of labor and earns a wage in every period. A representative household has the following intertemporal utility function:

$$u_t = \int_0^\infty \exp(-\rho t) \ln c_t dt,$$

where ρ is the subjective discount rate and c_t denotes the consumption of the final good at time t. The budget constraint (expressed in real terms) is given by

$$\dot{a}_t + \dot{m}_t = r_t a_t + \tau_t + i_t b_t + w_t - \pi_t m_t - c_t.$$

a_t is the real value of assets (equities), and r_t is the real interest rate. τ_t is a lump-sum transfer from the government. b_t is the amount of real money lent to firms, and i_t is the rate of return. w_t is the real wage rate. m_t is the amount of money held by this household, and it entails the opportunity cost of the inflation rate $\pi_t \equiv \dot{P}_t / P_t$, where P_t is the price level of final goods. The representative household can lend money until it reaches m_t. Therefore, $b_t \leq m_t$ holds. We do not impose a cash-in-advance constraint on consumption because, as Chu and Cozzi (2014) showed, it does not affect the balanced growth path when the households inelastically supply labor.

From standard optimization, when the households hold both money and assets, the Fisher equation holds: $i_t = \pi_t + r_t$. This is a no-arbitrage condition and shows that i_t is

[10] In their model, only potential firms that earn zero profit conduct R&D by financing the investment with external equity. In contrast, our model assumes that existing firms that earn a positive profit invest in R&D by financing it with internal cash and debt.

also the nominal interest rate.

Each household decides c_t in each period to maximize the intertemporal utility function, u_t, subject to the intertemporal budget constraint. From standard dynamic optimization, the household's optimal time path of consumption is represented by

$$\frac{\dot{c}_t}{c_t} = r_t - \rho. \tag{1}$$

2.2 Final good industry

The final good is produced by perfectly competitive firms that use a composite of intermediate goods as inputs. The economy has a continuum of intermediate goods industries indexed by $\ell \in [0,1]$. The production function is given by

$$X = \exp\left[\int_0^1 \ln\left(\sum_{k=0}^{\tilde{k}(\ell)} q_k(\ell) y_{k,t}^d(\ell)\right) d\ell\right], \tag{2}$$

where $y_{k,t}^d(\ell)$ is the input of an intermediate good with quality $q_k(\ell)$ in industry ℓ at time t. There are $\tilde{k}(\ell)+1$ generations of goods $(k = 0, 1, ..., \tilde{k}(\ell))$ in industry ℓ. We assume that the quality of each generation $q_k(\ell)$ is represented as an integer k power of $\lambda > 1$, which means that the quality of the new generation is λ times higher than that of the previous generation. Then, $q_k(\ell) = \lambda q_{k-1}(\ell)$ holds. We assume that the initial quality is one: $q_0 = 1$. Then, $q_k = \lambda^k$ holds. By the additive specification in the abovementioned production function, the $\tilde{k}(\ell)+1$ generations of intermediate goods are perfect substitutes. As discussed below, intermediate goods firms produce only the latest generation of goods. From profit maximization, the conditional demand function for the intermediate good in industry ℓ is given by

$$y_{\tilde{k}}^d(\ell) = \frac{X}{p_{\tilde{k}}(\ell)}.$$

2.3 Monetary authority

Following Chu and Cozzi (2014) and subsequent studies, we assume that the monetary authority can keep the nominal interest rate i_t constant ($i_t = i \geq 0$). Then, the inflation rate is endogenously determined according to the Fisher equation such that $\pi_t = i - r_t$. The growth rate of the nominal money supply is given by $\mu_t = \pi_t + \dot{m}_t/m_t$. The monetary authority returns the seigniorage revenue to households through a lump-sum transfer $\tau L = \mu_t m_t$.

2.4 Intermediate goods industry

Basic setup

Each intermediate goods industry is a duopolistic market in which there is a leader and a follower that imitates the leader's good. They engage in Cournot competition. Following Goh and Olivier (2002), we assume that the unit cost of producing imitative goods is increasing in patent breadth. Specifically, while the leader can produce one state-of-the-art good by using one unit of labor, the follower must devote $\chi \in [1, \lambda)$ units of labor to produce one unit of the same quality good, where χ is the level of patent breadth. For example, $\chi = 1$ implies that the follower can perfectly imitate the leader's production technology (no protection case).

The follower invests in R&D with the aim of developing the next generation of the good. After the follower succeeds at innovation, it leapfrogs the leader and becomes the new leader.

When $\chi < \lambda$ holds, the old leader (the current follower) chooses to imitate the new leader's good rather than produce its own good, which is one generation behind the new leader's good. This means that old generations of intermediate goods are never produced by firms. The reason is as follows. Since each generation of goods consists of perfect substitutes, the final goods firms purchase the intermediate good that has the lowest price per quality. Therefore, the follower chooses to imitate the leader's good if the unit production cost per quality is decreased by doing so. The quality of the leader's good is $\lambda^{\tilde{k}}$, and the follower can produce a unit of imitation good by paying χw. Therefore, the unit production cost per quality is $\chi w/\lambda^{\tilde{k}}$. On the other hand, the quality of the previous latest good is $\lambda^{\tilde{k}-1}$. The follower (the old leader) can produce a unit of good by paying w. Therefore, the unit production cost per quality is $w/\lambda^{\tilde{k}-1}$. If $\chi < \lambda$, then $\chi w/\lambda^{\tilde{k}} < w/\lambda^{\tilde{k}-1}$ holds. As a result, the follower chooses to imitate the new leader's good rather than produce its own good, which is one generation behind the new leader's good.

Cournot equilibrium

From $y^d = X/p$, the inverse demand function for intermediate goods in an industry is $p = X/y^d$. Given the inverse demand function and the wage rate of one unit of labor, firm j maximizes its profit, Π_j. $j = L$ if firm j is the leader, and $j = F$ if firm j is the follower.

The profit maximization problem of firm j is

$$\max_{y_j} \Pi_j = \frac{X}{y^d} \cdot y_j - \gamma_j w \cdot y_j, \tag{3}$$

where y_j is the output level and γ_j is the unit cost of production. Note that $\gamma_L = 1$ and $\gamma_F = \chi$.

In market equilibrium, the demand for intermediate goods equals the aggregate output in the intermediate goods industry. Then, the market clearing condition is

$$y^d = y_L + y_F \equiv Y, \tag{4}$$

where y_L is the leader's output, y_F is the follower's output, and Y is the aggregate output in the intermediate goods industry.

By solving (3), we obtain the output of firm j as follows:

$$\begin{aligned} \frac{\partial \Pi_j}{\partial y_j} = 0 \quad &\Leftrightarrow \quad \frac{X}{y^d} - \frac{X}{(y^d)^2} y_j - \gamma_j w = 0 \\ &\Leftrightarrow \quad y_j = y^d - \frac{\gamma_j w}{X} (y^d)^2. \end{aligned} \tag{5}$$

Using (4) and (5), we can derive the industry's aggregate output in the Cournot equilibrium Y^C as follows:

$$\begin{aligned} Y &= y_L + y_F \\ &= Y - \frac{w}{X} Y^2 + Y - \chi \frac{w}{X} Y^2 \end{aligned}$$

$$\Leftrightarrow Y^C = \frac{1}{1+\chi} \frac{X}{w}. \tag{6}$$

Then, from $p = X/y^d$, $y^d = Y$, and (6), the Cournot equilibrium price is $p^C = (1+\chi)\,w$. Using (4)-(6), we obtain the Cournot equilibrium output of each firm as follows:

$$y_F = \frac{1}{(1+\chi)^2} \frac{X}{w}, \tag{7}$$

$$y_L = \frac{\chi}{(1+\chi)^2} \frac{X}{w}. \tag{8}$$

The follower and leader's profits are given by

$$\Pi_F(\chi) = \left(\frac{1}{1+\chi} \right)^2 X, \tag{9}$$

$$\Pi_L(\chi) = \left(\frac{\chi}{1+\chi} \right)^2 X. \tag{10}$$

2.5 R&D and financing

The success of R&D investment follows a Poisson process. The follower succeeds at performing R&D with probability aZ by employing Z units of workers ($a > 0$).

The follower can finance the cost of R&D investment, Zw, with internal cash and external debt. [11] We assume that the follower must pay an exogenous fraction $(1-\zeta) \in [0,1]$ of the profit as the dividend. In other words, $(1-\zeta)$ is the minimum *dividend payout ratio* of the follower.[12] Let z_I be the number of researchers whose cost is financed with internal cash. Then, the follower faces the following internal financial constraint:

$$z_I w \leq \zeta \Pi_F. \tag{11}$$

Let z_D be the number of researchers whose cost is financed by borrowing money from the households. $Z = z_I + z_D$ naturally holds. The follower who borrows $z_D w$ must repay $(1+i)z_D w$.

As in Aghion et al. (1997) and Aghion et al. (2001), we assume that potential firms do not perform R&D activities.[13] Recent empirical studies have found that existing firms' own-product improvement, rather than creative destruction by market entrants, is a major source of economic growth. For example, Garcia-Macia et al. (2019) report that 80.2% of TFP growth in the U.S. for the period 2003-2013 is attributable to innovation by existing firms. Therefore, in the quality-improvement innovation model, it seems that existing firms' R&D activities should be highlighted rather than potential firms' R&D activities.

Note that, as in Grossman and Helpman (1991, Ch.4) and other subsequent studies, because of *Arrow's replacement effect*, the leader in each industry does not perform R&D. Even if the current leader succeeds at performing R&D, their firm's value does not increase because the latest good is instantaneously imitated by the follower. Empirically, although it has the ability to innovate, the leader firm tends to lack the incentive to do so (e.g., Igami (2017)).

2.6 The labor market

In the economy, labor is allocated to production and R&D. In labor market equilibrium, aggregate labor demand must equal labor supply L. The condition for labor market

[11] To avoid complexity, we do not consider *seasoned equity offerings*, which are new equity issues by an existing publicly traded firm. Instead, we focus on these two financing methods.

[12] The dividend payout ratio is the proportion of a firm's profits that are paid out as a dividend to shareholders.

[13] The assumption can be justified by assuming that the research productivity of existing firms is higher than that of potential firms because manufacturing experience gives the producer essential clues about further innovations.

equilibrium is

$$y_L + \chi y_F + z_I + z_D = L. \tag{12}$$

2.7 Bellman equations

Let V_L and V_F be the firm value of the leader and the firm value of the follower, respectively. Consider the returns from holding the leader's stock. The leader earns Π_L in every period, and it is perfectly distributed to the stockholders (households). However, when the follower succeeds at innovating, the leader's firm value V_L falls to V_F. We assume that there is a perfectly risk-free asset market and that the interest rate on safe assets is equal to r. Therefore, the following equation holds as a no-arbitrage condition (NAC) in the asset market.

$$rV_L = \Pi_L(\chi) + \dot{V}_L - a(z_I + z_D)(V_L - V_F), \tag{13}$$

where $\dot{V}_L$ is the capital gain of the stock and $a(z_I + z_D)$ is the probability of the follower's R&D success.

Next, consider the returns from holding the follower's stock. The follower earns Π_F in every period. When the follower succeeds at innovating, the firm value V_F rises to V_L. The follower decides z_I and z_D under constraint (11). Therefore, the NAC for the follower's stock is as follows:

$$rV_F - \dot{V}_F = \max_{z_I, z_D} \{\Pi_F(\chi) - z_I w + a(z_I + z_D)(V_L - V_F) - (1+i) z_D w\}. \tag{14}$$

Then, from (14), the follower's optimal z_I and z_D are determined as follows:

- Case 0. $a(V_L - V_F) < w$
  - $z_I = z_D = 0$.
- Case 1. $a(V_L - V_F) = w$:
  - z_I is indeterminate in $[0, \zeta \Pi_F / w]$ and $z_D = 0$.
- Case 2. $w < a(V_L - V_F) < (1+i)w$:
  - $z_I = \zeta \Pi_F / w$ and $z_D = 0$.
- Case 3. $a(V_L - V_F) = (1+i)w$:
  - $z_I = \zeta \Pi_F / w$ and z_D is indeterminate in $[0, \infty)$.
- Case 4. $a(V_L - V_F) > (1+i)w$:

– $z_I = \zeta \Pi_F / w$ and $z_D \to \infty$.

Throughout the paper, we consider a parameter range that satisfies $w \leq a(V_L - V_F) \leq (1+i)w$ because case 0 is uninteresting and case 4 is inconsistent with the labor market equilibrium. Therefore, we focus on case 1, case 2, and case 3.

2.8 The balanced growth path

We analyze the balanced growth path (BGP) where each variable grows at a constant growth rate. In the decentralized equilibrium, all markets clear, the firm value of two firms sums to the value of households' assets, $V_L + V_F = A$ where $A = aL$, and the amount of money borrowed by the follower is equal to the amount of money lent by the households, $z_D w = bL$. Variables with an asterisk ($*$) are the BGP values.

On the BGP, all real variables grow at g:

$$g \equiv \frac{\dot{c}_t}{c_t} = \frac{\dot{X}_t}{X_t} = \frac{\dot{w}_t}{w_t}.$$

To calculate g, we substitute $y^d = Y^C$ into (2). Then, we obtain

$$\ln X_t = \int_0^1 \ln q_t(\ell) d\ell + \ln X_t - \ln w_t - \ln(1+\chi).$$

The first term on the right-hand side can be rewritten as the product of $\ln \lambda$ and the expected number of improvements in a time interval of length t. Then,

$$\ln w_t = aZt \ln \lambda - \ln(1+\chi).$$

By differentiating this with respect to time, we obtain the economic growth rate:

$$g = aZ \ln \lambda.$$

From this and (1), the real interest rate on the BGP is given by

$$r = aZ \ln \lambda + \rho. \tag{15}$$

3 The effect of the pro-patent policy on innovation

3.1 Case 1

First, we consider the case of

$$a(V_L - V_F) = w. \tag{16}$$

By solving the equilibrium conditions, we obtain the following Lemma.

Lemma 1. *The R&D intensity in case 1 is given by*

$$z_I^* = \frac{(\chi^2 - 1)L - 2\rho\chi/a}{\chi^2 + 2\chi(1 + \ln\lambda) - 1}. \tag{17}$$

Therefore, pro-patent policy ($\chi \uparrow$) enhances innovation in case 1.

Proof. See Appendix B. □

Intuitively, pro-patent policy widens the gap between Π_L and Π_F from (9) and (10). Then, the follower has a strong incentive to be the new leader (the *Schumpeterian effect*). However, when χ becomes sufficiently high, z_I^* may reach the upper bound $\zeta\Pi_F/w$. Then, the economy shifts from case 1 to case 2.

3.2 Case 2

Second, we consider the case of $w < a(V_L - V_F) < (1+i)w$. In this case, the internal financial constraint is binding:

$$z_I = \frac{\zeta\Pi_F}{w}. \tag{18}$$

By solving the equilibrium conditions, we obtain the following Lemma.

Lemma 2. *The R&D intensity in case 2 is given by*

$$z_I^* = \frac{\zeta L}{\zeta + 2\chi}. \tag{19}$$

Therefore, pro-patent policy ($\chi \uparrow$) decreases innovation in case 2.

Proof. See Appendix C. □

The intuition is very simple. In case 2, the follower spends all its profit to finance R&D investment. In other words, the follower's R&D investment is financially constrained by its internal cash $\Pi_F(\chi)$. Since pro-patent policy widens the gap between Π_L and Π_F, the follower potentially wants to invest more. However, because external financing is costly, the follower does not borrow money (recall that $a(V_L - V_F) < (1+i)w$ holds). Since pro-patent policy decreases the follower's internal cash, it naturally stifles innovation (the *cash-shrinking effect*). This effect did not emerge in case 1 because the follower did not face the internal financial constraint.

However, when patent protection is sufficiently strong, the gap between Π_L and Π_F becomes very large. Then, the follower may have an incentive to invest in R&D by financing with external debt. This is case 3 and discussed in the next subsection.

3.3 Case 3

Third, we consider the case of

$$a(V_L - V_F) = (1+i)w. \tag{20}$$

As in case 2, the follower spends all its profit to finance R&D investment. However, in case 3, the follower starts to rely on external debt. This is the difference between case 2 and case 3.

The equilibrium conditions provide the following Lemma.

Lemma 3. *In case 3, pro-patent policy (χ ↑) increases the externally financed part of R&D investment z_D^* but decreases the internally financed part of R&D investment z_I^*. Overall, pro-patent policy increases the total R&D intensity, Z^*.*

Proof. See Appendix D. □

The intuition is as follows. Pro-patent policy increases the reward from innovation (the *Schumpeterian effect*), and it naturally stimulates the follower's incentive to invest in R&D. In contrast, as in case 2, pro-patent policy decreases the follower's profit (the *cash-shrinking effect*), and it decreases the internally financed part of R&D investment. However, unlike case 2, the follower does not face the internal financial constraint because it borrows money from the household. As a result, the Schumpeterian effect dominates the cash-shrinking effect, and therefore, pro-patent policy increases innovation.

3.4 The global relationship between patent protection and innovation

We summarize the growth effect of pro-patent policy described in the previous subsections. First, we show the parameter conditions under which each case arises. We define

$$\underline{\chi} = \frac{\rho}{aL} + \sqrt{\left(\frac{\rho}{aL}\right)^2 + 1}, \tag{21}$$

$$\chi_1 = \frac{\rho + \sqrt{\rho^2 + aL[\zeta\rho + (1+\zeta(1+\ln\lambda))aL]}}{aL}, \tag{22}$$

and

$$\chi_2 = \frac{\rho(1+i) + \sqrt{(\rho(1+i))^2 + aL\Psi}}{aL} \tag{23}$$

where $\Psi = \zeta\rho(1+i) + [1+\zeta(1+2i+(1+i)\ln\lambda)]aL$. We can easily show that $1 < \underline{\chi} < \chi_1 \leq \chi_2$ holds.

From the analyses in the previous subsections, we obtain the following result.

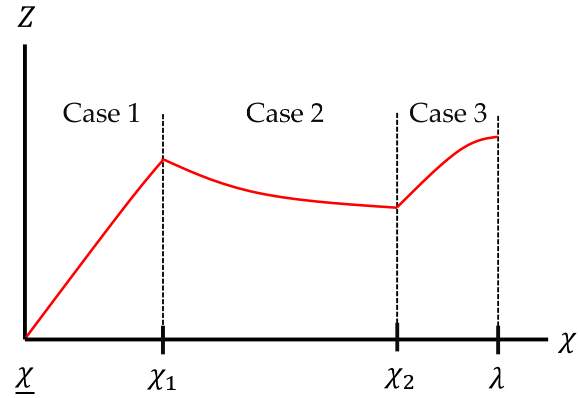

Figure 2: The effect of pro-patent policy and innovation when the inflation rate is positive ($i > 0$ and $\chi_2 < \lambda$).

Lemma 4. *Suppose that $i > 0$ holds. Then, case 1 arises when $\chi \in [\underline{\chi}, \chi_1]$, case 2 arises when $\chi \in (\chi_1, \chi_2)$, and case 3 arises when $\chi \in [\chi_2, \lambda)$.*

Proof. See Appendix E. □

Then, we can show the global relationship between patent protection and innovation. From Lemma 4 and the discussion in previous subsections, we obtain the following result.

Proposition 1. *Pro-patent policy ($\chi \uparrow$) increases innovation in $\chi \in [\underline{\chi}, \chi_1]$ (case 1) but decreases innovation in $\chi \in (\chi_1, \chi_2)$ (case 2). However, pro-patent policy increases innovation again in $\chi \in [\chi_2, \lambda)$ (case 3).*

Proposition 1 implies that the global relationship between patent protection and innovation depends on the existence of cases 2 and 3. Suppose that $i > 0$ holds. When $\chi_2 < \lambda$, the relationship between patent protection and innovation is N-shaped, as shown in Fig. 2. As can easily be predicted, when $\lambda < \chi_2$, their relationship becomes inverted-U shaped, as shown in Fig. 3. In addition, when $\lambda < \chi_1$, their relationship becomes positive, as shown in Fig. 4.

Our results suggest that pro-patent policy enhances innovation when the internal financial constraint is not binding but hinders innovation when it is binding. In the next subsection, we interpret this result by highlighting the role of the inflation rate.

3.5 The inflation rate and policy effects

We discuss the relationship between the inflation rate and the growth effect of pro-patent policy. In our model, the inflation rate π influences the growth effect of pro-patent policy

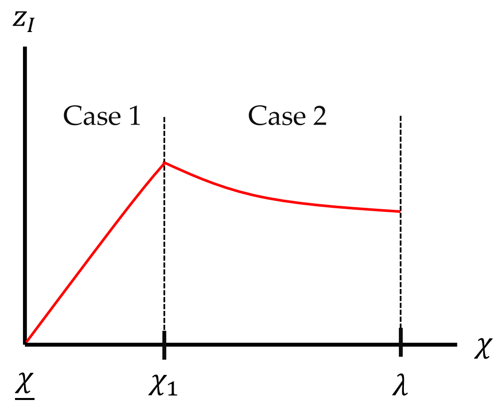

Figure 3: The effect of pro-patent policy and innovation when the inflation rate is extremely high ($i > 0$ and $\lambda < \chi_2$).

through the nominal interest rate $i = \pi + r$. In cases 1 and 2, the results are independent of the nominal interest rate because the follower does not finance R&D with external debt. Conversely, the nominal interest rate affects the results in case 3, where the follower borrows money.

Suppose that the monetary authority increases the inflation rate in case 3. Then, we obtain the following result.

Proposition 2. *A higher inflation rate decreases innovation in case 3. Therefore, a zero nominal interest rate (Friedman rule) is optimal from the perspective of growth.*

Proof. See Appendix F. □

Proposition 2 implies that a higher nominal interest rate is harmful for growth. The logic is quite simple. In our model, the nominal interest rate makes innovation costly for the follower in case 3. Therefore, the zero nominal interest rate is always optimal from the perspective of growth. To keep the nominal interest rate at zero, the monetary authority must set the inflation rate at a negative value, $\pi = -r$.

Furthermore, for the effect of the inflation rate on the optimal patent policy, we obtain the following result, which is consistent with the stylized fact in the introduction.

Proposition 3. *A higher (lower) inflation rate expands (narrows) the parameter region of case 2. Then, pro-patent policy is likely to have a negative (positive) effect on innovation. In contrast, under a zero nominal interest rate (Friedman rule), pro-patent policy always has a positive effect on innovation.*

Proof. From (23), χ_2 is increasing in the inflation rate. When the inflation rate is sufficiently high, χ_2 exceeds λ, and therefore, case 3 vanishes (see Fig. 3). When $i = 0$, we obtain

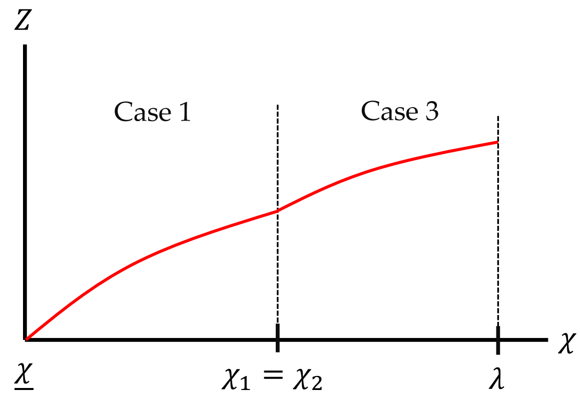

Figure 4: The effect of pro-patent policy and innovation under the Friedman rule ($i = 0$).

$\chi_1 = \chi_2$. Therefore, case 2 vanishes (see Fig. 4). □

Proposition 3 suggests that the optimal patent policy depends on the inflation rate and the current level of patent protection. If the inflation rate and the level of patent protection are sufficiently high, firms may face an internal financial constraint (e.g., case 2 in Fig. 3). In this case, the government should weaken patent protection to increase firms' internal cash. In contrast, if the inflation rate is sufficiently low, the possibility that case 2 arises is low. Specifically, under the Friedman rule, a follower that has exhausted its internal cash can smoothly rely on external debt because there is no case 2 (see Fig. 4). In this case, the government should strengthen patent protection.

4 Conclusion

Our paper analyzed the growth effect of pro-patent policy in a monetary Schumpeterian growth model with an internal financial constraint.

In our model, the follower invests in R&D in an effort to leapfrog the leader by financing R&D with internal cash (i.e., Cournot profit) and external debt. The follower endogenously chooses how to finance the R&D investment, and its financing choice depends on the patent protection level. Because debt financing entails a borrowing cost, when patent protection is weak, the follower uses only internal cash to finance R&D investment. If the borrowing cost is high, a follower that has exhausted its internal cash cannot borrow money. In this case, the follower faces a binding internal financial constraint; that is, the follower cannot invest more in R&D than its internal cash. In contrast, when patent protection is strong, the follower starts to rely on external debt in addition to internal cash because the incentive to innovate is very strong.

We have identified two opposite effects of pro-patent policy on innovation-driven growth. The first is a positive effect of pro-patent policy on innovation, which comes from the fact that stronger patent protection increases the reward for innovation (the *Schumpeterian effect*). The second is a negative effect of pro-patent policy on innovation through the internal cash constraint. Stronger patent protection prevents the follower from improving its production technology by imitating the leader's technology. Then, pro-patent policy hinders innovation because it shrinks the follower's profit, which is the follower's only financing source (the *cash-shrinking effect*).

Our main results are summarized as follows. When patent protection is weak, pro-patent policy enhances innovation because the follower's investment is not financially constrained. In this case, there is only the Schumpeterian effect and no cash-shrinking effect. In contrast, when patent protection is moderate, pro-patent policy stifles innovation because the follower's investment is financially constrained. However, when patent protection is strong, pro-patent policy enhances innovation again because the follower finances R&D investment with external debt, and therefore, R&D is not financially constrained. As a result, there is an N-shaped relationship between patent protection and innovation under certain parameter conditions. Furthermore, we find that a zero nominal interest rate not only enhances innovation but also makes the growth effect of pro-patent policy always positive.

Appendix

A. The descriptive statistics

Variable	Obs	Mean	SD	Min	Max
$g_{i,t+1}$(GDP)	561	3.18	3.44	-14.76	14.53
$g_{i,t+1}$ (GDP per capita)	561	2.32	3.35	-14.38	13.64
$\text{Patent}_{i,t}$	561	6.44	2.10	2.50	9.90
$\text{Inflation}_{i,t}$	561	4.67	7.99	-4.48	85.75
$\text{Unemployment}_{i,t}$	561	7.81	4.92	0.62	33.47
$\text{FinDev}_{i,t}$	561	95.35	7.48	53.69	100.00
$\text{Open}_{i,t}$	561	85.47	59.29	16.44	437.33

Table 2: The descriptive statistics. $g_{i,t+1}$ is the economic growth rate measured by the growth rate of GDP or the growth rate of GDP per capita. $\text{Patent}_{i,t}$ is the index of patent protection. $\text{Inflation}_{i,t}$ is the inflation rate. $\text{Unemployment}_{i,t}$ is the unemployment rate. $\text{FinDev}_{i,t}$ is the ratio of deposit money banks' assets to GDP. $\text{Open}_{i,t}$ is the sum of exports and imports as a share of GDP.

B. The proof of Lemma 1

From (14), (16), and $z_D = 0$, we obtain the follower's firm value as follows:

$$V_F = \frac{\Pi_F(\chi)}{r}. \tag{24}$$

Then, from (13), (24), and $z_D = 0$, the leader's firm value is given by

$$V_L = \frac{\Pi_L(\chi) + (az_I/\rho) \cdot \Pi_F(\chi)}{r + az_I}. \tag{25}$$

Then, using (15), (16), and (25), we obtain an important equation as follows:

$$\frac{w}{X} = \frac{a}{\rho + az_I(1 + \ln \lambda)} \frac{\chi^2 - 1}{(1+\chi)^2}. \tag{26}$$

In addition, from (7), (8), and (12), we obtain another important equation.

$$\frac{w}{X} = \frac{2\chi}{(1+\chi)^2} \frac{1}{L - z_I}. \tag{27}$$

By solving (26) and (27), we obtain (17).

To complete the proof, we show that z_I^* is increasing in χ. Differentiating (17) with respect to χ yields

$$\begin{aligned}
\frac{dz_I^*}{d\chi} &= \frac{(\chi^2 + 2\chi(1+\ln\lambda) - 1)(2\chi L - 2\rho/a) - (2\chi + 2(1+\ln\lambda))[(\chi^2-1)L - 2\rho\chi/a]}{(\chi^2 + 2\chi(1+\ln\lambda) - 1)^2} \\
&= \frac{2(\chi^2+1)(L(1+\ln\lambda) + \rho/a)}{(\chi^2 + 2\chi(1+\ln\lambda) - 1)^2} > 0.
\end{aligned}$$

C. The proof of Lemma 2

From (9) and (18), we obtain

$$\frac{w}{X} = \frac{\zeta}{(1+\chi)^2 z_I}. \tag{28}$$

From (7), (8), and (12), we obtain (27) again. Using (27) and (28), we can solve for z_I^* as in (19). Furthermore, (19) is decreasing in χ.

D. The proof of Lemma 3

For convenience, we first prove the second sentence in Lemma 3. By using (14), (18), and (20), we obtain the follower's firm value as follows:

$$V_F = \frac{(1+\zeta i)\Pi_F(\chi)}{\rho + aZ\ln\lambda}. \tag{29}$$

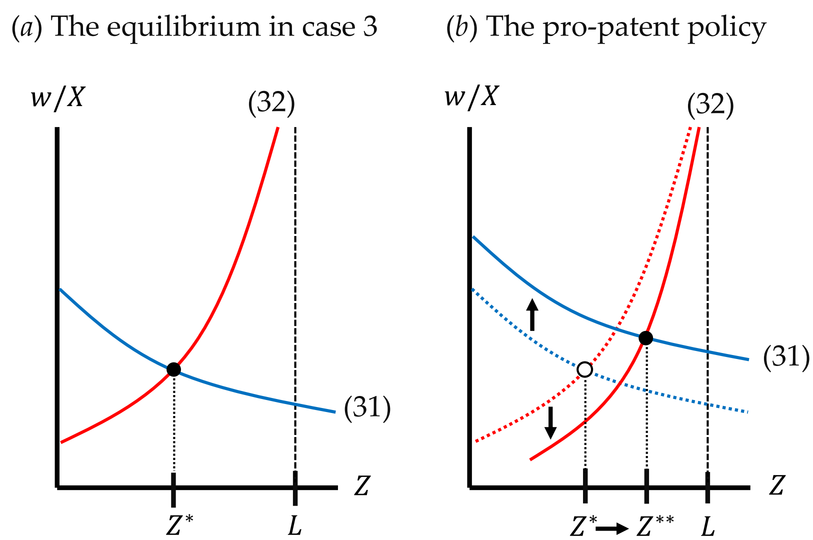

Figure 5: The equilibrium in case 3.

By using (13), (18), and (20), we obtain the leader's firm value as follows:

$$V_L = \frac{\Pi_L(\chi) + aZV_F}{\rho + aZ(1 + \ln \lambda)}. \tag{30}$$

Then, from (9), (10), (20), (29), and (30), we obtain an important equation:

$$\begin{aligned} & w = \frac{a}{1+i}(V_L - V_F) \\ \Leftrightarrow \quad & w = \frac{a}{1+i}\frac{\Pi_L - (1+\zeta i)\Pi_F}{\rho + aZ(1+\ln\lambda)} \\ \Leftrightarrow \quad & \frac{w}{X} = \frac{a}{1+i}\left(\frac{1}{1+\chi}\right)^2 \frac{\chi^2 - (1+\zeta i)}{\rho + aZ(1+\ln\lambda)}. \end{aligned} \tag{31}$$

Furthermore, from (7), (8), and (12), we obtain another important equation as follows:

$$\frac{w}{X} = \frac{2\chi}{(1+\chi)^2(L-Z)}. \tag{32}$$

(31) is a decreasing function of Z, and (32) is an increasing function of Z. In Figure 5, Z^* is determined by the intersection of (31) and (32), and it is given by

$$Z^* = \frac{(\chi^2 - (1+\zeta i))L - (1+i)2\chi\rho/a}{(\chi^2 - (1+\zeta i)) + (1+i)2\chi(1+\ln\lambda)}.$$

From (9) and (10), pro-patent policy shifts the downward-sloping curve (31) upward.

We define

$$f(\chi) \equiv \frac{2\chi}{(1+\chi)^2}.$$

By differentiating this with respect to χ, we obtain

$$f'(\chi) = \frac{2(1-\chi)}{(1+\chi)^3} < 0.$$

Therefore, pro-patent policy shifts the upward-sloping curve (32) downward. As a result, pro-patent policy raises Z^*.

Next, we prove the first part of Lemma 3. From (13) and (20), V_L can also be written as

$$V_L = \frac{\Pi_L(\chi) - Z(1+i)w}{\rho + aZ\ln\lambda}.$$

Substituting this, (18), and (29) into (20) yields

$$\begin{aligned} w &= \frac{\Pi_L(\chi) - [\zeta(1+i)(1+\ln\lambda) + 1 + \zeta i]\Pi_F(\chi)}{(1+i)(\rho/a + (1+\ln\lambda)z_D)} \\ \Leftrightarrow \quad \frac{w}{X} &= \frac{1}{(1+\chi)^2}\frac{\chi^2 - [\zeta(1+i)(1+\ln\lambda) + 1 + \zeta i]}{(1+i)(\rho/a + (1+\ln\lambda)z_D)}. \end{aligned} \tag{33}$$

In addition, using (9), (18), and (32), we obtain

$$\frac{w}{X} = \frac{2\chi + \zeta}{(1+\chi)^2(L - z_D)}. \tag{34}$$

The intersection of (33) and (34) gives z_D^*. Pro-patent policy shifts the downward-sloping curve (33) upward. We define

$$g(\chi) \equiv \frac{2\chi + \zeta}{(1+\chi)^2}.$$

Differentiating this with respect to χ yields

$$g'(\chi) = \frac{2(1-\chi-\zeta)}{(1+\chi)^3} < 0.$$

Therefore, pro-patent policy shifts the upward-sloping curve (34) downward. As a result, pro-patent policy increases z_D^*. In addition, from (9), (18) and (32), we obtain

$$z_I^* = \frac{\zeta(L - z_D^*)}{2\chi + \zeta}.$$

As already shown, pro-patent policy increases z_D^*. Therefore, pro-patent policy decreases z_I^*.

E. The proof of Lemma 4

First, we derive the lower bound of χ in case 1 ($z_I^* \in [0, \zeta\Pi_F/w]$). From (17), z_I^* is zero when $(\chi^2-1)L - 2\rho\chi/a = 0$. By solving this, we obtain

$$\underline{\chi} = \left(\frac{\rho}{aL}\right) + \sqrt{\left(\frac{\rho}{aL}\right)^2 + 1}.$$

Second, we derive the upper bound of χ in case 1. On the boundary of case 1 and case 2, z_I^* is equal to $\zeta\Pi_F/w$. From (9), (17), and (27),

$$\begin{aligned}
\zeta\frac{\Pi_F}{w} &= \zeta\frac{X}{(1+\chi)^2 w} \\
&= \zeta\frac{L - z_I}{2\chi} \\
&= \zeta\frac{L(1+\ln\lambda)+\rho/a}{\chi^2+2\chi(1+\ln\lambda)-1}.
\end{aligned}$$

Then, substituting this into (18) yields

$$\begin{aligned}
z_I^* = \frac{\zeta\Pi_F}{w} \;&\Leftrightarrow\; (\chi^2-1)L - \;2\rho\chi/a = \zeta[L(1+\ln\lambda)+\rho/a] \\
&\Leftrightarrow\; aL\chi^2 - 2\rho\chi - [\zeta\rho + (1+\zeta(1+\ln\lambda))aL] = 0.
\end{aligned}$$

By solving this, we obtain the threshold between case 1 and case 2 as follows:

$$\chi_1 = \frac{\rho + \sqrt{\rho^2 + aL[\zeta\rho + (1+\zeta(1+\ln\lambda))aL]}}{aL},$$

Finally, we derive the upper bound of χ in case 2. From (13) and (14), we obtain

$$rV_L = \Pi_L - az_I^*(V_L - V_F),$$

and

$$rV_F = (1-\zeta)\Pi_F + az_I^*(V_L - V_F).$$

Then, using (19),

$$a(V_L - V_F) = \frac{a[\Pi_L - (1-\zeta)\Pi_F]}{\rho + a(\ln\lambda+2)\zeta L/(\zeta+2\chi)}.$$

On the boundary of case 2 and case 3, $a(V_L - V_F)$ is equal to $(1+i)w$. Therefore,

$$\begin{aligned}
\frac{a[(\chi/(1+\chi))^2 - (1-\zeta)(1/(1+\chi))^2]}{\rho + a(\ln\lambda+2)\zeta L/(\zeta+2\chi)} &= (1+i)\frac{w}{X} \\
&= (1+i)\frac{\zeta+2\chi}{(1+\chi)^2 L}
\end{aligned}$$

$$\Leftrightarrow \quad a[\chi^2 - (1-\zeta)] = [\rho + a(\ln\lambda + 2)\frac{\zeta L}{\zeta + 2\chi}](1+i)\frac{\zeta + 2\chi}{L}$$
$$\Leftrightarrow \quad aL\chi^2 - 2\rho(1+i)\chi - \Psi = 0.$$

where $\Psi = \zeta\rho(1+i) + [1 + \zeta(1 + 2i + (1+i)\ln\lambda)]aL$. Then, the threshold between cases 2 and 3 is given by

$$\chi_2 = \frac{\rho(1+i) + \sqrt{(\rho(1+i))^2 + aL\Psi}}{aL}.$$

F. The proof of Lemma 4

Z^* is determined by the intersection of (31) and (32). Since the curve of (31) shifts downward, Z^* falls.

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