Whom Should I Merge With? How product substitutability affects merger profitability

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Abstract. The paper presents a simple model of oligopoly, in which three firms supply differentiated products. The degree of product substitutability is not uniform across goods. We investigate the merger profitability, and we show that profitability depends on the degree of good differentiation. Contrary to what seems to emerge from different models, we find that merger between firms that supply "more similar" product is more profitable as compared to merger between firms supplying more differentiated goods.

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1. Introduction

Merger profitability is investigated by a large body of theoretical literature, starting from the seminal contributions by Salant et al. (1983), Deneckere and Davidson (1985), and Perry and Porter (1985). A striking result is that merger can be individually unprofitable for merging firms, as they reduce their output production, in front of non-merging firm(s) increasing output. As shown by subsequent literature, the occurrence of this outcome, known as "the merger paradox", is due to strategic interdependence. The reasons of the paradox are well known: two countervailing effects are in operation, when firms decide to merge, in a oligopolistic market. On the one side, there is a positive profit effect, due to the elimination of rival firm(s); on the other hand, non-merging firms have an incentive to increase production – if competition is in quantity, and reaction curves are downward sloping. The relative size of these two effects drives the final result concerning the merger profitability. A number of factors matter, including the initial and final (pre- and post- merger) market shares, the firms' choice variable(s), that is, the price or quantity competition (see McElroy, 1993), the competition along one or more dimensions (Pinto and Sibley, 2016; Brekke et al., 2017), the cost structure and the cost nature (Farrel and Shapiro, 1990; Fanti and Meccheri, 2012; Majumdar, 2019), and the product characteristics (e.g., Hsu and Wang, 1990). A literature review is provided by Faulì-Oller and Sandonis (2018).

What has induced the present investigation is the debate following the proposal of merger between FCA and Renault, in Summer 2019. The merger proposal had very short life –as both parties, substantially, withdraw the proposal in some days. During those days, however, several observers, especially in Italy and France, cast doubts about the profitability of such a merger, in front of the fact that these firms supply very similar products in the car market (substantially, both FCA and Renault produce "traditional" cars of "medium level", and they are absent from the market segment of innovative electric cars). Some months later, the merger between FCA and PSA (Peugeot-Citroen) has successfully occurred. In this case, columnists and policy-makers generally expressed positive opinions, also basing on the fact that the two groups have different points of strength. These pieces of news lead to ask whether, in a market of differentiated products, a firm finds it more profitable to merge with a more similar or more different product supplier.

Though very huge, the theoretical body of economic literature on mergers overlooks this specific point, namely whether mergers are more profitable if they occur between firms providing similar or different products. To the best of my knowledge, I can mention only two theoretical papers dealing with this point; they reach opposite conclusions. Ebina and Shimizu (2009) analyse a framework made by four firms, and merger incentives are shown to be stronger for firms producing closely related goods than more differentiated goods. The main interest of Ebina and Shimizu (2009) is in the occurrence of sequential mergers, and they also show that either a sequence of mergers occurs, or no mergers occur in equilibrium. Hsu and Wang (2010) consider a uniform product substitution rate among varieties, and show that merging emerges to be profitable if varieties are sufficiently distant substitutes: however, that paper considers symmetric substitutability across all pairs of goods, and the more distant goods are, the more profitable the merger between two firms is.

In this article, I propose a very simple model specifically focussing on the point of merger profitability as related to the degree of differentiation of products. The present model differs from Hsu and Wang (2010), as long as substitutability between goods is not homogeneous. It differs from Ebina and Shimizu (2009), as it is simpler, more focussed on the role of the degree of product differentiation, and does not consider sequential merger. Thus, I believe that the economic reason driving the result can emerge more neatly. Roughly speaking, the result from the present model is that merging with a more similar product is more profitable than merging with the firm supplying more different product variety. Thus, the conclusion is more in line with the outcome from Ebina and Shimizu (2009) than Hsu and Wang (2010). The intuition rests on the fact that when two firms producing similar goods merge, they internalise a strong (negative) externality, which overcomes the negative effect deriving from the non-merging firm, which finds it optimal to increase production. On the contrary when the merger occurs between firms producing highly differentiated goods, the internalised externality is limited, and not able to compensate the negative effect due to the increased production of the non-merging firm.

The present model is very simple indeed: it is focussed on the market structure; it is static; it does not take into account product and process innovation; it omits to consider the effects of merger upon inputs' cost; it disregards the multidimensional nature of competition; and so on. Nevertheless, it permits to understand pros and cons of merger, for a specific firm, from a market structure perspective. It also permits to draw some insights about static market social welfare. The merger paradox may occur, as previous models have documented. In the present model we simply show that, under a wide range of sensible parameter configuration, the paradox typically occurs when strongly differentiated goods providers do merge; the paradox is harder to emerge if merger occurs between similar goods providers. The economic reason is simple: when merging firms offer similar products, the negative externality from competition (which is internalised after merger) is strong, and it is larger than the competitive pressure deriving from the outside (non-merging) firm. On the opposite, if two very different firms do merge, they internalise a tiny externality, and the competitive pressure deriving from the outside (non-merging) firm prevails.

Interestingly, from the standpoint of the present model, the merger with Renault should have been for FCA more profitable than the merger with a firm offering more different cars, such as Peugeot (and vice versa).

The structure of the paper is as follows. Section 2 presents the basics of the model and find the outcome of pre-merger configuration. Section 3 and 4 find the outcome when a merger occurs between firms supplying “slightly” differentiated varieties, and “strongly” differentiated varieties, respectively; comparison of firms’ profit are examined. Section 5 takes the consumer perspective, and analyses the welfare outcome of different mergers. Implications, comments and concluding remarks are gathered in Section 6.

2. The basics of the model: a triopoly game

Three firms (1, 2, 3) serve a market, each producing a product variety that is horizontally differentiated with respect to the others. Each product variety is supplied by only one firm, so that, throughout the paper, “firm”, “product” or variety” can be taken as synonymous. The location of firms in the product space is represented by Figure 1: thus, the product variety of firm 1 is more similar to the variety of firm 2 than to the variety of firm 3 (and vice versa).[1] Just to give a simple example, borrowed from the car market, let us imagine that firm 1 offers traditional cars, firm 2 offers hybrid cars, firm 3 offers electric car.

[1] Notice that, in Figure 1, firm 2 is located in a position equi-distant to the other two firms, so that firm 1 and 3 are symmetric with respect to firm 2.

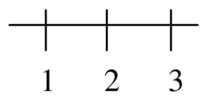

Figure 1. Location of varieties in the product space.

The inverse demand function for firm 1 is:

(1) $$p_1 = M - \beta q_1 - \gamma q_2 - \delta q_3$$

where parameter β captures the responsiveness of the price of the variety to its own produced quantity, while γ and δ are the cross- responsiveness coefficients; in particular the following assumption is made:

(1') $$\beta > \gamma > \delta > 0$$

which means that products are substitutes, and the substitutability between the varieties of firms 1 and 2 is larger than the substitutability between the varieties of firms 1 and 3.

For reason of symmetry, consistency and analytical simplicity, inverse demand functions for firms 2 and 3 are respectively:

(1'') $$p_2 = M - \beta q_2 - \gamma (q_1 + q_3)$$

(1''') $$p_3 = M - \beta q_3 - \gamma q_2 - \delta q_1$$

Cost functions are assumed to be linear, without fixed cost, and symmetric across firms, entailing constant returns to scale for all firms:

(2) $$c_i = c q_i \quad , \quad 0 < c < M \ , \quad i = 1, 2, 3.$$

As first step, we find the Nash equilibrium in this market, provided that each of the three firms individually acts as a profit-maximizer Cournot oligopolist. The individual problem faced by each of the three firms can write as follows:

$$
(3)\qquad \begin{aligned}
&Max_{q_1}: \pi_1 = (M - \beta q_1 - \gamma q_2 - \delta q_3 - c)q_1 \\
&Max_{q_2}: \pi_2 = (M - \beta q_2 - \gamma (q_1 + q_3) - c)q_2 \\
&Max_{q_3}: \pi_3 = (M - \beta q_3 - \gamma q_2 - \delta q_1 - c)q_3
\end{aligned}
$$

Let be: $A = M - c > 0$. From the FOCs one obtains the reaction functions

$$
(4)\qquad \begin{aligned}
&q_1 = (A - \gamma q_2 - \delta q_3)/(2\beta) \\
&q_2 = (A - \gamma q_1 - \gamma q_3)/(2\beta) \\
&q_3 = (A - \gamma q_2 - \delta q_1)/(2\beta)
\end{aligned}
$$

which are negatively sloped with respect to the opponents' production levels (as expected). SOCs are met, as long as all objective functions are concave. Equilibrium production levels turn out to be:

$$
(5)\qquad q_1^* = q_3^* = \frac{A(2\beta - \gamma)}{2(2\beta^2 + \beta\delta - \gamma^2)}; \qquad q_2^* = \frac{A(2\beta - 2\gamma + \delta)}{2(2\beta^2 + \beta\delta - \gamma^2)}
$$

This outcome is labelled as the 'decentralised triopoly allocation'. Given the parametric restrictions assumed above, all equilibrium quantities are positive. It is also easy to check that $q_1 = q_3 > q_2$. The fact that the production of varieties 1 and 3 are larger than the production level of variety 2 can be easily explained by observing that firm 2 faces two "close" substitute varieties, while firm 1 and 3 face only one "close" substitute, while the remaining competitor offers a more differentiated (less close) good.

Profit levels, in the decentralised triopoly equilibrium, are:

$$
(6)\qquad \pi_1^* = \pi_3^* = \frac{A^2\beta(2\beta - \gamma)^2}{4(2\beta^2 + \beta\delta - \gamma^2)^2}; \qquad \pi_2^* = \frac{A^2(2\beta - 2\gamma + \delta)^2}{4(2\beta^2 + \beta\delta - \gamma^2)^2}
$$

The following clear-cut sequence holds: $\pi_1^* = \pi_3^* > \pi_2^*$, so that one can conclude that the firm facing a larger number of close substitutes obtains smaller profit as compared to firms facing a lower number of close substitutes. This is intuitive and fully in line with the results of differentiated oligopoly literature (Spence, 1979).

3. Merging of firms producing slightly differentiated goods

Let us consider the case in which firm 1 and 2 do merge, and they aim to maximize the joint profit. We assume that the merger does not modify the number and nature of available varieties, nor cost functions. So, there are no economies of scope or economies of scale generated by the merger. These assumptions serve to rule out obvious price decreasing forces on the cost side. In other words, in the present model, merger has a pure market structure effect. Objective functions become:

$$(7)\qquad \begin{aligned} &Max_{q_1,q_2} : (\pi_1 + \pi_2) = (A - \beta q_1 - \gamma q_2 - \delta q_3)q_1 + (A - \beta q_2 - \gamma(q_1 + q_3))q_2 \\ &Max_{q_3} : \quad \pi_3 = (A - \beta q_3 - \gamma q_2 - \delta q_1)q_3 \end{aligned}$$

Notice that a unified decision maker sets jointly q_1 and q_2, while firm 3 is acts as outside (not merging) firm. From the FOCs: $\partial(\pi_1 + \pi_2)/\partial q_1 = 0,\ \partial(\pi_1 + \pi_2)/\partial q_2 = 0,$ and $\partial \pi_3 / \partial q_3 = 0$, one finds the reaction functions and hence the equilibrium levels of production; the equilibrium allocation of this setting is labelled by using the superscript M1&2 (which clearly stays for 'merging firm 1 and 2'):

$$(8)\qquad \begin{aligned} q_1^{M1\&2} &= \frac{A[4\beta^2 - 2\beta(2\gamma + \delta) + \gamma^2 + \gamma\delta)}{2[4\beta^3 - \beta(5\gamma^2 + \delta^2) + 2\gamma^2\delta]} \\ q_2^{M1\&2} &= \frac{A[4\beta^2 - 6\beta\gamma + \delta(3\gamma - \delta)]}{2[4\beta^3 - \beta(5\gamma^2 + \delta^2) + 2\gamma^2\delta]} \\ q_3^{M1\&2} &= \frac{A[2\beta^2 - \beta(\gamma + \delta) - \gamma^2 + \gamma\delta)}{4\beta^3 - \beta(5\gamma^2 + \delta^2) + 2\gamma^2\delta} \end{aligned}$$

The following parametric restriction is necessary and sufficient to avoid corner solutions:

$$(9)\qquad \gamma < \min\left[\frac{\beta(4\beta^2 - \delta^2)}{5\beta - 2\delta},\quad \frac{(4\beta^2 - \delta^2)}{6\beta - 2\delta}\right]$$

This amounts to require that a sufficiently large degree of differentiation exists, even within the pair of more similar goods.[2]

Provided that individual production levels are positive, it is easy to check that:

$q_1^{M1\&2} > q_2^{M1\&2}$

$q_1^{M1\&2} \gtrless q_1^*$,

$q_2^{M1\&2} < q_2^*$

$q_3^{M1\&2} > q_3^*$

(See Appendix 1 for proof). Verbally, as far as the merging firms are concerned, the firm that was larger before the merger, continues to produce a larger amount of good after merging; the smaller merging firm unambiguously produces a lower amount of good after merging as compared to its pre-merger production; the larger merging firm could produce a larger or smaller output level, as compared to the pre-merger output level, depending on parameter configuration; however, a contraction of aggregate production of merged firms does emerge as a consequence of the merger. As far as the non-merging (outside) firm, its equilibrium production level increases, after the opponents' merger. The outcome is in line with the main message of the theoretical literature on mergers: merging firms tend to decrease the joint production, and this drives non-merging firms to increase their production. The outcome is the reason of the so called 'merging paradox', according to which mergers could entail lower profit for merging firms and larger profit for non-merging firms. This could be the case also in the present framework, conditional on specific parameter configuration.

Indeed, by simple substitutions it is easy to obtain the maximum profit level for each firm, under the equilibrium emerging after the merger between firms 1 and 2:

[2] Limiting our attention to the parameter space where $0 < \delta < \gamma < \beta$, it can be analytically proved that the numerator of (8') is strictly positive, as long as $\beta > (\gamma + \delta)/2$, while the denominator is positive if $0 < \gamma < \beta(4\beta^2 - \delta^2)/(5\beta - 2\delta)$. The numerator of (8'') is positive if $0 < \gamma < (4\beta^2 - \delta^2)/(6\beta - 2\delta)$. The numerator of (8''') is positive as long as $0 < \gamma < \beta$. Hence, condition (9) follows.

$$\text{(10)} \qquad \pi_1^{M1\&2} = \frac{A^2[4\beta^2 - 2\beta(2\gamma+\delta) + \gamma(\gamma+\delta)][4\beta^3 - 2\beta^2\delta + \beta\gamma(\delta - 5\gamma) + \delta\gamma(3\gamma-\delta)]}{4[4\beta^3 - \beta(5\gamma^2+\delta^2) + 2\gamma^2\delta]^2}$$

$$\pi_2^{M1\&2} = \frac{A^2[4\beta^2 - 6\beta\gamma + \delta(3\gamma-\delta)][4\beta^3 - 2\beta^2\delta - \beta(4\gamma^2 - \gamma\delta + \delta^2) + \gamma^2(\gamma+\delta)]}{4[4\beta^3 - \beta(5\gamma^2+\delta^2) + 2\gamma^2\delta]^2}$$

$$\pi_3^{M1\&2} = \frac{A^2\beta[2\beta^2 - \beta(\gamma+\delta) - \gamma^2 + \gamma\delta]^2}{(4\beta^3 - \beta(5\gamma^2+\delta^2) + 2\gamma^2\delta)^2}$$

One can show that:

$\pi_1^{M1\&2} > \pi_1^*$

$\pi_2^{M1\&2} \gtrless \pi_2^*$

$(\pi_1^{M1\&2} + \pi_1^{M1\&2}) > (\pi_1^* + \pi_2^*)$

$\pi_3^{M1\&2} + > \pi_3^*$

The non-merging firm benefits from the merger between merging firms, as usual in this kind of models. The larger merging firm (firm 1), in this case, obtains a gain from the merger, while the smaller merging firm (firm 2) may obtain a gain or a loss from the merger, depending on parameter configuration. The joint profits of firms 1 and 2 after the merger is in any case larger than the sum of individual profits before the merger. (See Appendix 2 for details; Appendix 2 also provides the outcome of numerical simulation, to identify the parameter space where the merger paradox occurs).

4. The merger of firms producing highly differentiated goods

Let us consider what outcome derives from the merger between firm 1 and firm 3, whose goods are more differentiated as compared to the pair of goods produced by firm 1 and 2.

Repeating the steps of the procedure adopted in the previous case, we consider the problem faced by the firms in this setting, where firms 1 and 3 act as a unified decision-maker:

$$\text{(11)} \qquad Max_{q_1,q_3} : (\pi_1 + \pi_3) = (A - \beta q_1 - \gamma q_2 - \delta q_3)q_1 + (A - \beta q_3 - \gamma q_2 + \delta q_1)q_3$$

$$Max_{q_2} : \quad \pi_2 = (A - \beta q_2 - \gamma(q_1 + q_3))q_2$$

From the FOCs: $\partial(\pi_1+\pi_3)/\partial q_1 = \partial(\pi_1+\pi_3)/\partial q_3 = 0$, and $\partial \pi_2 / \partial q_2 = 0$, one finds the reaction functions and hence the equilibrium levels of production; the equilibrium allocation of this setting is labelled by using the superscript M1&3 (which stays for 'merging firm 1 and 3'):

$$(12) \qquad q_1^{M1\&3} = q_3^{M1\&3} = \frac{A(2\beta-\gamma)}{2(2\beta^2+2\beta\delta^2-\gamma^2)}$$

$$q_2^{M1\&3} = \frac{A(\beta-\gamma+\delta)}{2\beta^2+2\beta\delta-\gamma^2}$$

Given our starting symmetry assumptions, the merging firms share here the same individual production level, after the merger; moreover, in this case, no further parametric restrictions are required. It is easy to prove analytically that the individual production of the merging firms drops, following the merger, while the production level of the outside firm increases.

$$(13) \qquad q_1^{M1\&3} - q_1^* = q_3^{M1\&3} - q_3^* = \frac{A\beta\delta(\gamma-2\beta)}{2(2\beta^2+2\beta\delta^2-\gamma^2)(2\beta^2+\beta\delta-\gamma^2)} < 0$$

$$q_2^{M1\&3} - q_2^* = \frac{A\beta\delta(2\beta-\gamma)}{2(2\beta^2+2\beta\delta^2-\gamma^2)(2\beta^2+\beta\delta-\gamma^2)} > 0$$

Simple substitutions lead to find the maximum profit levels:

$$(14) \qquad \pi_1^{M1\&3} = \pi_3^{M1\&3} = \frac{A^2(2\beta-\gamma)(2\beta^2+\beta(2\delta-\gamma)-\delta\gamma}{4\cdot(2\beta^2+2\beta\delta-\gamma^2)^2}$$

$$\pi_2^{M1\&2} = \frac{A^2\beta[\beta-\gamma+\delta]^2}{(2\beta^2+2\beta\delta^2-\gamma^2)^2}$$

One can show (See Appendix 3):

$$\pi_i^{M1\&3} \gtrless \pi_i^*, \quad i=1,3$$

$$\pi_2^{M1\&3} > \pi_2^*$$

Thus, the merger between firms 1 and 3, can be profitable or not for the merging firms, depending on parameter configuration. In particular, numerical simulations (See Appendix 3) reveal that the merger is profitable for the merging firms if the difference $\gamma-\delta$ is limited: this means that the goods produced by firms 2 and 3 are pretty similar. Otherwise, the merger paradox does emerge, and the merging firms experience lower individual profit after the merger, as compared to the pre-merger equilibrium.

Differently, the non-merging firm unambiguously benefits from the merger, as usual in this kind of models (also in this case, the result is obtained through numerical simulations, as the analytical expression is too cumbersome to draw algebraic conclusions).

5. Whom does a firm find it profitable to merge with?

We here compare the profit of firm 1, after the merger with firm 2 or with firm 3, alternatively. The outcome is the core message of the present theoretical investigation. Unambiguously, the merger with the more similar firm is more profitable than the merger with the firm producing the more differentiated good.

Indeed, $\pi_1^{M1\&2} - \pi_1^{M1\&3}$ emerges to be always positive, in the economically meaningful parameter space (see Appendix 4). Of course, the difference is nil in the limiting case in which $\delta = \gamma$: under this circumstance, the goods produced by firms 2 and 3 are homogeneous, so that it is equivalent, for firm 1, the merger with firm 2 or firm3.

We provide an economic intuition for the clear-cut result, proving that –under our assumptions– it is more profitable for a firm to merge with the firm producing the more similar good, as compared to firm producing the more differentiated good. Indeed, merger between firms drives the merging firms to internalise the (reciprocal) negative external effect. This internalization entails "positive gains" for merging firms. On the other hand, the merged firm faces the competition from the non-merging firm, which usually sets a larger amount of production, as a reply to the output reduction by part of the merging firms. The strategic reaction of the outside (non-merging) firm is the source of the negative effect of merger upon the merging firm (and it is the source of the "merger paradox"). If the merger occurs between similar firm, the "negative reciprocal externality" that they internalise is strong. The less similar are the goods produced by the merging firms, the less strong the externality to internalize through merger. So, it is far from being surprising that the merger between firms producing more similar goods is more beneficial for the merging firms as compared to the merger between firms producing highly differentiated goods.

It is worth stressing, once again, that among our assumptions, we do not consider variation in production technology due to the merger, nor variation in the characteristics of products. In other words, we are overlooking the R&D and the innovation part of merger agreement, and we only consider the forces acting through the output market mark-up and the strategic interaction, conditional on given technology, cost structure, and product substitutability.

6. Conclusions

We have presented a very simple analytical model to deal with a simple question: in a market of differentiated products, is it more profitable, for a firm, to merge with a firm providing a similar good, or a highly differentiated good? Focussing on the incentives at the output market level, the present model provides a clear-cut answer: higher profitability is associated to the merger with the firm supplying the more similar good. The reason clearly rests on the fact that, in this case, the internalised externality through merger is larger.

The theoretical conclusion, though straightforward, can have some interest in ongoing debates concerning mergers. For instance, this investigation has been stimulated by a number of columns, especially in Italian, French and US newspapers, casting doubts about the fact that a merger between FCA and Renault could be profitable for these firms, provided that they produce quite similar cars. According to some columnists, a merger for each of these firms would be more profitable with a different firm, producing more differentiated goods (for instance, with companies which have a larger know-how in electric or hybrid motorcars). This is the case of the merger between FCA and Peugeot, which has successfully occurred some months later indeed, in the presence of a largely positive feeling of columnists and policy-makers. From a theoretical perspective, the argument of the present paper leads to the opposite conclusion: when firms produce more similar good, the externality which is internalised by the merger is larger, and hence the gain from merger is larger. Of course, the present simple model has overlooked considerations about innovation of product and process, and the different incentives for innovation efforts deriving from merger. It could be true that considerations related to incentive for product and process innovation, and a proper dynamic model, leads to reversed conclusion. However, from a static point of view, market choices related to optimal production level and mark-up in the presence of strategic interdependence clearly suggests that merger with more similar competitors is better for merging firms.

APPENDIXES

Appendix 1. Comparison of pre-merger and post-merger production levels: merger between firm 1 and 2

As to the difference:

$$q_1^{M1\&2} - q_2^{M1\&2} = \frac{A(\gamma - \delta)(2\beta + \gamma - \delta)}{2[4\beta^3 - \beta(5\gamma^2 + \delta^2) + 2\gamma^2\delta]},$$

its sign is unambiguously positive, as both the numerator and the denominator are positive, under the assumptions at hand.

As to the difference:

$$q_2^{M1\&2} - q_2^* = A\left(\frac{4\beta^2 - 6\beta\gamma + \delta(3\gamma - \delta)}{2[4\beta^3 - \beta(5\gamma^2 + \delta^2) + 2\gamma^2\delta]} - \frac{2(\beta - \gamma) + \delta}{2(2\beta^2 + \beta\delta - \gamma^2)}\right)$$

while it is unclearly signed from an analytical point of view, all numerical simulations (fixing $\beta = 1$) show that the difference is negative in the relevant parameter range.

As to the difference:

$$q_1^{M1\&2} - q_1^* = A\left(\frac{4\beta^2 - 2\beta(2\gamma + \delta) + \gamma^2 + \gamma\delta}{2[4\beta^3 - \beta(5\gamma^2 + \delta^2) + 2\gamma^2\delta]} - \frac{2\beta - \gamma}{2(2\beta^2 + \beta\delta - \gamma^2)}\right)$$

which is unclearly signed from an analytical point of view, numerical simulations show that it is usually negative, apart from the case where parameter γ is above a threshold level, which means that the differentiation between 1 and 2 is somewhat limited. The threshold level (for $\beta = 1$) is $\bar{\gamma} = \left((\delta - 3) + \sqrt{\delta^2 - 2\delta + 17}\right)/2$ For instance, if $\delta = 0.2$, the post-merger level of production of firm 1 is smaller than the pre-merger level for $0.2 < \gamma < 0.639$, while it is larger for $0.639 < \gamma < 0.733$.

Since the reaction function of firm 3 is negatively sloped with respect to opponents' production levels, we expect that its production level increases, if q_1 decreases, while the outcome could be ambiguous in the parameter space where q1 increases. In fact, numerical simulations show that firm 3's production level unambiguously increases as a consequence of the merger between of firm 1 and 2. QED.

Appendix 2. Comparison of pre-merger and post-merger profits: merger between firm 1 and 2

As to the difference: $\pi_1^{M1\&2} - \pi_1^*$, its signs is unclear form an analytical point of view. However, note that A has a pure scale effect. Fixing A=10 and $\beta = 1$, one can numerically check that the differential is positive in the whole sensible parameter space concerning γ and δ.

As to the difference: $\pi_2^{M1\&2} - \pi_2^*$, its signs is unclear form an analytical point of view; A has a pure scale effect. Fixing $\beta = 1$, numerical simulation prove that this difference can be positive or negative, depending on combination of (admissible) parameter γ and δ. For instance, Figure 2 reports the plot of the difference as a function of γ, having set $\delta = 0.2$. (Remember that, in this case, it makes sense to evaluate the outcome for $0.2 < \gamma < 0.733$); the difference is positive for $0.2 < \gamma < 0.355$, while it is negative for larger values.

As to the difference $(\pi_1^{M1\&2} + \pi_1^{M1\&2}) - (\pi_1^* + \pi_2^*)$, numerical simulations show that it is always positive in the relevant parameter space. It is interesting to report note that the difference is monotonically increasing in γ (for given values of all other parameters) and monotonically decreasing in δ, for given values of all other parameters.

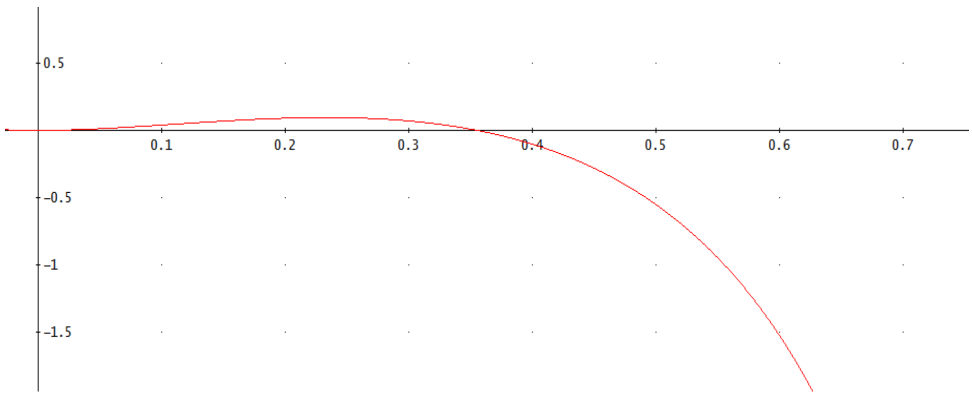

Figure 2. - Pattern of $\pi_2^{M1\&2} - \pi_2^*$ as a function of γ, having set: A=10, $\beta = 1$, $\delta = 0.2$. Outcome is economically meaningful for $0.2 < \gamma < 0.733$.

Appendix 3. Comparison of pre-merger and post-merger profits: merger between firm 1 and 3

The difference $\pi_i^{M1\&3} - \pi_i^*$, *i*=1,3, that is, the profit differential post vs pre- merger profit of each merging firm is analytically cumbersome; however, numerical simulations show that it can be positive or negative, depending on parameter configuration., and more specifically the difference is positive if the difference $\gamma - \delta$ is limited, while it is negative if the distance between δ and γ is large enough. For instance, let *A*=10, $\beta = 1$. Moreover, if $\gamma = 0.5$, the pattern of $\pi_i^{M1\&3} - \pi_i^*$ is reported by Figure 3.a, which shows that the differential is positive for $0.457 < \delta < \gamma = 0.5$. Alternatively, Figure 3.b reports the pattern of the difference $\pi_i^{M1\&3} - \pi_i^*$ as a function of γ under the parameter configuration *A*=10, $\beta = 1$ and $\delta = 0.2$; clearly, the difference is positive if $0.2 = \delta < \gamma < 0.323$, while it is negative for larger values of γ. So, verbally, the merger between 1 and 3 is profitable for the merging firms, if the goods produced by firms 2 and 3 are pretty similar, that is, the parameter capturing the degree of differentiation of good 1 with respect to good 2 (γ) and good 3 (δ) are "not too different".

Numerical simulations concerning the difference $\pi_2^{M1\&3} - \pi_2^*$ are not reported for the sake of brevity; they unambiguously show that the difference is positive, for any economically meaningful parameter configuration (normalizing *A*=10, $\beta = 1$).

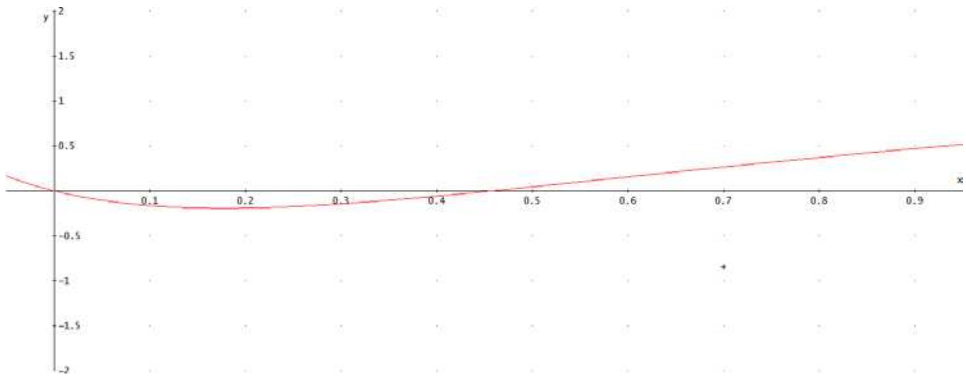

Figure 3.a Pattern of $(\pi_i^{M1\&3} - \pi_i^*), i$=1,3 as a function of δ, having set: A=10, $\beta = 1$, $\gamma = 0.5$. Outcome is economically meaningful for $0 < \delta < \gamma = 0.5$.

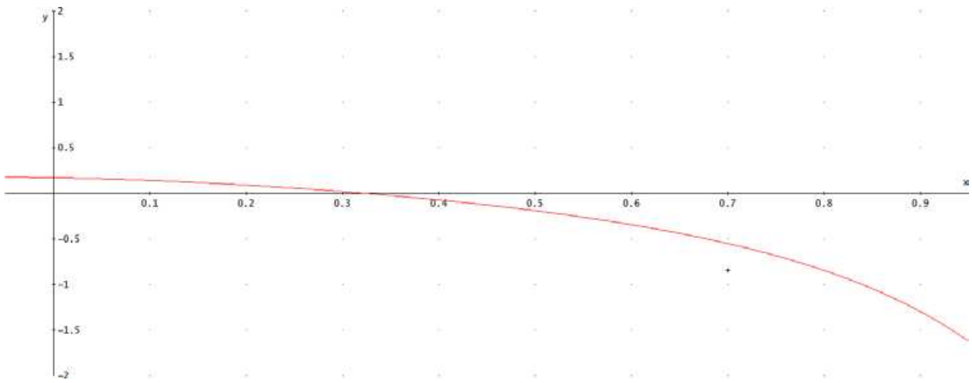

Figure 3.b Pattern of $\pi_i^{M1\&3} - \pi_i^*$ as a function of γ, having set: A=10, $\beta = 1$, $\delta = 0.2$. Outcome is economically meaningful for $0.2 < \gamma < 0.733$.

Appendix 4. Comparison of firm 1's payoff, in case of merger with firm 2 or firm 3, alternatively.

The difference $\pi_1^{M1\&2} - \pi_1^{M1\&3}$ can not be analytically signed. For this reason, we resort to numerical simulation. Having normalised A=10 and $\beta = 1$, three-dimensional graphs show positive values for the difference at hand, for any possible economically meaningful combination of δ, γ. More clearly, we report two two-dimension graphs, having added, respectively a numerical value concerning γ and a numerical value concerning δ, alternatively. Under A=10, $\beta=1$, and having set $\gamma=0.5$, then the difference $\pi_1^{M1\&2}-\pi_1^{M1\&3}$ as a function of δ (with $0<\delta<\gamma=0.5$) turns out to be always positive and monotonically decreasing in δ, over the economically relevant parameter space (See figure 4.a); obviously, the difference is zero in the limiting case $\delta=\gamma$, for the reason that the products of firms 2 and 3 are homogeneous in this case, so that it makes no difference for firm 1 to merge with firm 2 or 3. Similarly, under A=10, $\beta=1$, $\delta=0.2$ the difference is always positive and increasing in γ (figure 4.b) over the whole relevant parametrical space (remember that, in this case, the parameter space is economically meaningful for $0.2<\gamma<0.733$). Also in this case, the difference is zero in the limiting case in which $\delta=0.2=\gamma$, that corresponds to the case of homogeneous productions of firms 2 and 3. Thus, the more differentiated are the goods of firms 2 and 3, the more profitable is for firm1 to merge with the closer firm, as compared to the merger with the firm providing the more differentiated good.

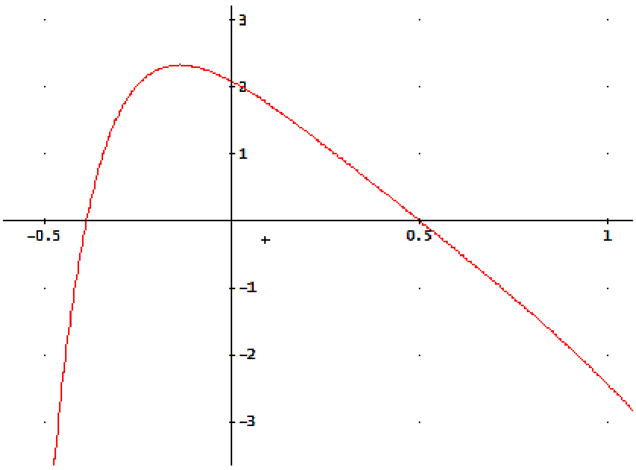

Figure 4.a Pattern of $\pi_1^{M1\&2}-\pi_1^{M1\&3}$ as a function of δ, having set: A=10, $\beta=1$, $\gamma=0.5$. Outcome is economically meaningful for $0<\delta<\gamma=0.5$.

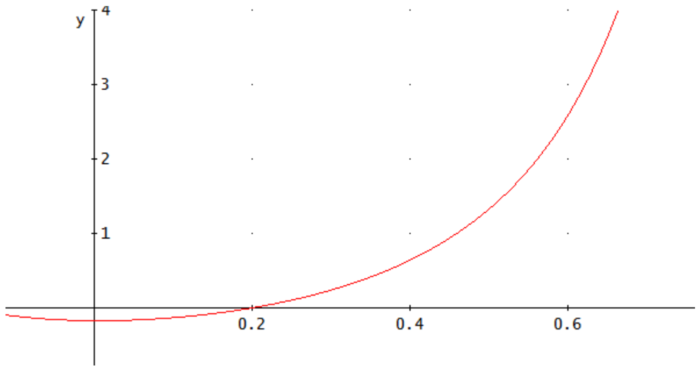

Figure 4.b. Pattern of $\pi_1^{M1\&2} - \pi_1^{M1\&3}$ as a function of γ, having set: A=10, $\beta = 1$, $\delta = 0.2$. Outcome is economically meaningful for $0.2 < \gamma < 0.733$.

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