Pension Reforms, Population Aging, and Retirement Decision of the Elderly in a Neoclassical Growth Model

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Abstract

This study explores the linkage between the labor force participation of the elderly and the long-run performance of the economy in the context of a two-period-lived overlapping generations model. We assume that the old agents are heterogeneous in their labor efficiency and they continue working if their income exceeds the pension that can be received in the case of full retirement. We first inspect the key factors that the retirement decision of the elderly. We then examine analytically as well as numerically the long-run impact of labor participation of the elderly on capital accumulation.

Keywords: retirement decision, labor force participation, population aging, pension system, capital accumulation

JEL classification: E10, E62

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1 Introduction

In the last 30 years, the labor force participation of the elderly has been increasing in many advanced countries. For example, according to the Aging Society White Paper 2017 issued by the Japanese Cabinet Office, the labor force's share of the Japanese elderly aged 65 years and over increased from 5.9% in 1980 to 11.8% in 2016. Currently, more than 50% of the Japanese male elderly aged 65–69 years engage in full-time- or part-time jobs. Such a trend stems from rises in longevity and health status of the elderly as well as from changes in the social environment that increase the activeness of the elderly. Additionally, many researchers claim that a change in the elderly's labor supply is closely related to pension reforms. As mentioned below, several empirical studies, which use data of various countries, confirm that the recent rise in the labor force participation of the elderly had a statistically significant link to pension reforms implemented in those sampled industrialized countries after the 1990s.

In this paper, we present a model of retirement decision of the elderly that fits well to empirical observations mentioned above, We also inspect the impact of a change in the retirement decisions of the elderly on the long-run capital accumulation. To achieve those two goals in a tractable manner, we introduce our formulation of the endogenous retirement decision of the elderly into a standard neoclassical growth model with two-period lived overlapping generations. Based on this analytical framework, we investigate how changes in various factors that determine the retirement decision of the elderly affect the behaviors of key macroeconomic variables in the long run.

Specifically, in our model economy, the young agents are assumed to be homogeneous and they fully devote their available time to working. However, the old agents are heterogeneous in the sense that their labor efficiency differs from each other. We assume that the old agents draw their labor efficiency from a given distribution function at the beginning of their old age. We also assume that there is a compulsory pay-as-you-go pension plan financed by taxation on the young generation's income. Given this setting, each old agent compares the expected pension revenue in the case of full retirement with the income that can be earned from the labor force participation; this determines the agent's decision to either fully retire or participate in the labor force. Such a decision determines a cutoff level of labor efficiency—the agents who have a lower efficiency level than the cutoff select full retirement, and the agents whose efficiency exceeds the cutoff continue working in their old age. We show that the pension scheme set by the government as well as the rate of population aging directly affect the threshold level of labor efficiency, thereby determining the labor force participation of the elderly.

The production side of the economy follows the standard neoclassical growth model. Therefore, in the long-run equilibrium capital and income grow at an exogenously specified population growth rate, and per capita levels of capital and income stay constant over time. Given this setting, we analytically show that an increase in the labor force participation of the old agents may increases the steady-state levels of per capita capital stock and the per capita income. In addition to the theoretical discussion, we examine numerical examples to evaluate quantitative impacts of changes in the degree of population aging and the policy parameters on the labor force participation of the elderly and the steady-state levels of capital and per capita income. Our numerical discussion reveals that magnitudes of those impacts are sensitive to the shape of the distribution function of the labor efficiency of the old agents.

Related Literature

(i) Empirics

From the early 1960s to the late 1980s, the labor force participation of the elderly declined in many advanced countries[1]. Concerning this fact, Gruber and Wise (1999, 2004, and 2007) provide meticulous research outcomes. Among others, Gruber and Wise (1999) present detailed empirical studies on 11 industrialized countries and reveal that an increase in the generosity of the social security plan contributed towards the international trend of the decline in the labor force participation of old persons[2]. In contrast, recent studies focus on the persistent rise in the labor force participation of the elderly since the early 1990s. For example, Oshio et al. (2011) examine the impact of social security reforms on the labor force participation of the Japanese elderly for 40 years (1968-2007). They find that the labor force participation of the elderly started increasing after the 1985 reform that reduced the generosity of social security benefits, including pension plans[3]. In a similar vein, Berkel et al. (2004) study the impact of pension reforms on the retirement decision of the German elderly population, while

[1] A well-cited earlier empirical study on this topic is Krueger and Pischke (1992).
[2] Gruber and Wise (1998) summarize their main findings.
[3] Higuchi et al. (2006), Shimizutani (2013), and Shimizutani and Oshio (2013) present further evidence.

Bottazzi et al. (2008) and Bovini (2018) discuss the effect of pension reforms on the elderly's retirement decision and wealth accumulation in Italy. Additionally, Coil (2015) presents a useful survey on empirical studies on this topic[4].

Several authors point out that health status is also important for the labor force participation of the elderly. Kalwji and Vermeulen (2008) investigate the impact of health conditions on the labor force participation of the elderly in 11 European countries. They provide careful estimation results and clarify the multidimensional nature of the health status of the elderly. Overall, their results reveal that health status is one of the major factors that affect the retirement decision of the elderly. Similar studies are conducted by Mete and Schlutz (2002) on Taiwan and Cai (2007) on Australia[5].

(ii) Theory

Many authors modify Diamond's (1965) overlapping generations (OLG) model to introduce the endogenous labor supply of the old agents. The most popular formulation is to assume that young agents fully devote their available time to working, whereas old agents make the labor-leisure choice. In this setting, an increase in the labor force participation of the elderly means that old agents select a lower level of leisure time. A sample of this type of formulation includes Zhang and Zhang (2009), Gon and Liu (2012), Mizuno and Yakita (2013), and Hirazawa and Yakita (2017). Since this modeling assumes that agents are homogeneous, all the old agents select the same level of partial retirement that corresponds to the length of leisure time they choose. A deficiency of this formulation is that it fails to capture the fact that in reality, a substantial number of the elderly fully retire.

On the other hand, a few authors have examined models in which the full retirement decision of the elderly is endogenously determined. Among others, Matsuyama (2008) constructs a two-period-lived OLG model in which old agents make a discrete choice between working and retirement by comparing utilities obtained in alternative situations. In his model, agents in each cohort are homogeneous; this ensures that the decision regarding the choice between working and retirement is uniform among the elderly[6]. Aisa et al. (2015) introduce agent

[4] See also Coil and Levine (2018).

[5] Gacía-Pèrez et al. (2013) present an empirical study on the relationship between retirement incentive, pension, and employment status based on the Spanish data.

[6] Matsuyama (2008) shows that the model involves multiple steady states so that a poverty trap arises. Gon and Liu (2012) indicate that if the model allows partial retirement, then it would imply that the steady-state equilibrium is uniquely determined. Since the model discussed by Gon and Liu (2012) is a variant of the

heterogeneity. In their model, each agent is endowed with a given labor efficiency at the beginning of the agent's life, and agents whose labor efficiency exceeds an endogenously determined threshold level work in their old age. Although the basic idea of Aisa et al. (2015) is similar to our model, the authors treat a two-period model, and hence the long-run effect of the old agents' labor supply on capital accumulation is not discussed in their study[7]. Compared to the foregoing formulations mentioned above, our model, which emphasizes the heterogeneity of old agents, has an advantage— it can simultaneously determine the full retirement of some old agents as well as the aggregate level of labor force participation of the elderly in a dynamic environment.

Concerning the link between the social security and labor force participation of the elderly, Diamond and Mirrlees (1978) present an early theoretical study. These authors explore a microeconomic model of social insurance in the presence of endogenous retirement and asymmetric information. In macroeconomics research, Hu (1979) conducts one of the earliest studies on the impact of social security on the labor supply of the elderly in the context of Diamond's (1965) OLG model. In his model, the old agents make labor-leisure choice and they receive a pension for their leisure time. Given this setting, Hu (1979) explores the long-run effect of a pay-as-you-go pension system on capital accumulation. Hu's (1979) modeling is employed by Miyazaki (2017) who examines the optimal social insurance in the presence of labor-leisure choice of the old agents. Cipriani (2018) also uses a similar setting to study the macroeconomic impact of population aging[8]. Since the analytical frameworks used by Hu (1979), Miyazaki (2017), and Cipriani (2018) are essentially the same as the models of Zhang and Zhang (2009) and others mentioned above, their discussions do not depict the full retirement decision of the elderly. Again, it is worth emphasizing that our model with heterogenous elderly can highlight the relationship between pension schemes and the full retirement of the elderly in a coherent manner[9].

The rest of the paper is organized as follows. Section 2 constructs the analytical frame-

labor-leisure choice models mentioned above, it does not depict the full retirement of some of the old agents.

[7] It must be pointed out that Matsuyama (2008) briefly examines a modified model in which disutility levels of labor is heterogeneous among agents; this ensures that some old agents fully retire in equilibrium. However, he does not consider this line of formulation in detail.

[8] See also Phillippe and Pestieau (2013).

[9] Here, we focus on theoretical studies based on the two-period lived OLG models. There are several quantitative studies on the relationship between social security programs and labor supply in the context of calibrated multi-period lived OLG models: see, for example, Kitao (2014 and 2015).

work for our discussion. Section 3 inspects the existence and stability of the steady-state equilibrium of the model economy. Section 4 characterizes the relationships between the steady-values of macroeconomic variables and the key parameters involved in the model. Section 5 concludes.

2 Model

2.1 Households

Consider an overlapping generations economy in which each agent lives for two periods, young and old. We assume that young agents live for one period with certainty, but they face a probability of surviving to the old age. We denote the probability of surviving as $\pi \in (0, 1]$. Each cohort consists of a continuum of agents, and the mass of the cohort grows at a constant rate of n. The population of the cohort born at the beginning of the period t is N_t, so that $N_t = (1 + n) N_{t-1}$. Since the population share of the old generation in period t is $\pi N_{t-1}/(N_t + \pi N_{t-1}) = \pi/(1 + \pi + n)$, population aging means that a decline in the population growth rate, n and/or a rise in the probability of surviving, π.

When young agents are homogeneous and each agent inelastically supplies one unit of hours worked. It is assumed that young agents' one unit of working time provides one unit of labor service. On the other hand, old agents are heterogeneous in the sense that labor efficiencies differ from each other. Such a difference in labor efficiency may stem from differences in each agent's health status and the motivation for labor force participation, among others. In this paper, we consider that the labor efficiency of an old agent mainly depends on her health status. We assume that the labor efficiency, denoted by h, follows a cumulative distribution function $F(h)$, which satisfies

$$F'(h) > 0, \quad F(\eta) = 0, \quad F(1) = 1, \quad 0 < \eta < 1.$$

Namely, there is a minimum level of efficiency of the old person's labor, η, and the most able old agents have the same level of labor efficiency as that of young agents. We assume that the distribution of h is stationary, and h is i.i.d over time as well as across agents. It is also assumed that one unit of hours worked of an old agent with labor efficiency h, provides h

unit of labor service.

Under our assumption, all the young agents born in period t earn the same amount of wage income, w_t. On the other hand, the old agents born in period $t-1$ who select full retirement receive a pension, p_t, whereas the agents who stay in the labor force receive $\mu p_{t+1} + (1-\xi) h w_t$, where $\mu,\ \xi \in (0,1)$. This assumption means that if an old agent works, she receives a part of the pension, μp_t plus the after-tax wage, $(1-\xi) h w_t$, where ξ denotes the rate of income tax on the working old, and $h w_t$ is the competitive wage paid for an old agent whose labor efficiency is h. In this formulation, we may consider that the pension for a working old is deducted by $\xi h w_t$: a working old who earns a higher wage income receive a smaller amount of pension[10]. For simplicity, we assume that the old agents do not make labor-leisure choice, and labor does not yield disutility. Thus, the old agents' retirement decision is based on the comparison of incomes in alternative situations. This means that the income of an old agent with a labor efficiency, h, is given by

$$y_{h,t} = \max\ \{p_t,\ \mu p_t + (1-\xi) h w_t\}.$$

As a result, the cutoff level of labor efficiency in period $t+1$ is determined by

$$h_t^* = \frac{1-\mu}{1-\xi}\frac{p_t}{w_t}. \tag{1}$$

The old agents born in period $t-1$ whose labor efficiency is lower than h_t^* fully retire, while the agents whose efficiency exceeds h_t^* stay in the labor force. We assume that the agents with $h = h_{t+1}^*$ continue working.

When setting up the lifetime optimization problem of an agent, we may use two alternative formulations. The first is to assume that agents draw h at the beginning of their young age. In this case, each young agent born at the beginning of period t exactly knows her health status when she becomes old. In such a perfect-foresight environment, the optimization problem of the agent who draws h is to maximize the discounted sum of expected life-time utilities

$$U_{h,t} = \log c_{h,t} + \pi\beta \log x_{h,t+1} \qquad 0 < \beta,\ \pi < 1$$

[10] Our formulaton of the pension for the old agents who stay in the labor force is close to the pension scheme applied to the working elderly in Japan.

subject to the intertemporal budget constraint

$$c_{h,t} + \frac{x_{h,t+1}}{(1 + r_{t+1})\frac{1}{\pi}} = w_t - \tau_t + \frac{y_{h,t+1}}{(1 + r_{t+1})\frac{1}{\pi}}. \tag{2}$$

In the above, $c_{h,t}$ and $x_{h,t+1}$,respectively, denote the type h agent's consumption in the agent's young and old ages, r_{t+1} is the risk-free rate of real interest, τ_t is an income tax, and β is a given discount factor. As usual, we assume that the competitive interest rate on financial assets incorporates the risk involved due to the agents' uncertain life time, meaning that the gross interest rate in period $t + 1$ is given by $(1 + r_{t+1})/\pi$. The budget constraint (2) reflects this assumption.

We find that the optimal levels of consumption, $c_{h,t}$ and $x_{h,t+1}$ and saving, $s_{h,t}$, are, respectively, given by

$$c_{h,t} = \frac{1}{1 + \beta\pi}\left(w_t - \tau_t + \frac{\pi y_{h,t+1}}{1 + r_{t+1}}\right), \tag{3}$$

$$x_{h,t+1} = \frac{\beta(1 + r_{t+1})}{1 + \beta\pi}\left(w_t - \tau_t + \frac{\pi y_{h,t+1}}{1 + r_{t+1}}\right), \tag{4}$$

$$s_{h,t} = w_t - \tau_t - c_{h,t} = \frac{\beta\pi}{1 + \beta\pi}(w_t - \tau_t) - \frac{\pi y_{h,t+1}}{(1 + \beta\pi)(1 + r_{t+1})}. \tag{5}$$

By our assumption, the income in the old age is given by

$$y_{h,t+1} = \begin{cases} p_{t+1}: & \text{for } \eta \leq h < h^*_{t+1}, \\ \mu p_{t+1} + (1 - \xi) h w_{t+1} & \text{for } h \geq h^*_{t+1}. \end{cases} \tag{6}$$

The second formulation is to assume that agents draw h at the beginning of their old age. In the real world, young people do not possess perfect information about their health status when they become old. Thus, the second formulation would be more realistic than the first one. If this is the case, the agents are ex-ante identical at the beginning of their young age, so that agent's optimization problem is to maximize

$$U_t = E_t\left[\log c_{,t} + \pi\beta \log x_{t+1}\right] \qquad 0 < \beta, \pi < 1$$

subject to

$$c_t + \frac{x_{t+1}}{(1 + r_{t+1})\frac{1}{\pi}} = w_t - \tau_t + \frac{y^e_{t+1}}{(1 + r_{t+1})\frac{1}{\pi}}.$$

Here, y^e_{t+1} is the old agent's income expected in period t, which is given by

$$y^e_{t+1} = F\left(h^*_{t+1}\right)p_{t+1} + \left[1 - F\left(h^*_{t+1}\right)\right]\mu p_{t+1} + (1-\xi)\int_{h^*_{t+1}}^{1} hw_{t+1}dF(h). \tag{7}$$

Since $\Pr\left(h \leq h^*_{t+1}\right) = F\left(h^*_{t+1}\right)$ and $\Pr\left(h \geq h^*_{t+1}\right) = 1 - F\left(h^*_{t+1}\right)$, the first two terms in the right-hand side of the above is the expected amount of pension received in period $t+1$. The third term represents the expected after-tax wage income. The optimal choice gives the following:

$$c_t = \frac{1}{1+\beta\pi}\left(w_t - \tau_t + \frac{\pi y^e_{t+1}}{1+r_{t+1}}\right), \tag{8}$$

$$x_{t+1} = \frac{\beta(1+r_{t+1})}{1+\beta\pi}\left(w_t - \tau_t + \frac{\pi y^e_{t+1}}{1+r_{t+1}}\right), \tag{9}$$

$$s_t = \frac{\beta\pi}{1+\beta\pi}(w_t - \tau_t) - \frac{\pi y^e_{t+1}}{(1+\beta\pi)(1+r_{t+1})}. \tag{10}$$

It is to be noted that those two alternative formulations yield the same macroeconomic equilibrium conditions. To see this, let us drive the average consumption of the young agents in the first formulation in which agents draw h at the beginning of their young age. We find

$$\int_{\eta}^{1} c_{h,t}dF(h) = \frac{1}{1+\beta\pi}\left[w_t - \tau_t + \frac{\pi}{1+r_{t+1}}\int_{\eta}^{1} y_{h,t+1}dF(h)\right],$$

where

$$\begin{aligned}\int_{\eta}^{1} y_{h,t+1}dF(h) &= \int_{\eta}^{h^*_{t+1}} p_{t+1}dF(h) + \int_{h^*_{t+1}}^{1}\mu p_{t+1}dF(h) + \int_{h^*_{t+1}}^{1} hw_{t+1}dF(h)\\ &= F\left(h^*_{t+1}\right)p_{t+1} + \left[1 - F\left(h^*_{t+1}\right)\right]\mu p_{t+1} + \int_{h^*_{t+1}}^{1} hw_{t+1}dF(h).\end{aligned}$$

Therefore, in the rational expectations equilibrium, the first and the second formulations give the identical level of young agents' average consumption. This property holds for consumption in old age as well. Consequently, the alternative formulations establish the same macroeconomic equilibrium conditions. For simplicity of exposition, we use the second formulation in the following discussion.

2.2 Firms

The production side of our model is the standard one. The final good and factor markets are competitive. There is a representative firm that produces a homogeneous good according to the following Cobb-Douglas production technology:

$$Y_t = AK_t^{\alpha} L_t^{1-\alpha}, \qquad 0 < \alpha < 1, \tag{11}$$

where Y_t is the output, K_t is the aggregate capital stock, and L_t is the input of labor service. The firm employs both young- and old-workers' labor services, L_t^y, and L_t^o, respectively, so that

$$L_t = L_t^y + L_t^o. \tag{12}$$

Since we have assumed that one unit of hours of work of a young agent yields one unit of labor service, L_t^y also denotes the total hours of work of the young agents. We have also assumed that one unit of hours worked provided by an old agent with the labor efficiency h yields h amount of labor service. Thus, denoting hours worked of old agents with h by $l_{h,t}$ the aggregate input of old workers' labor service is

$$L_t^o = \int_{h_t^*}^{1} h l_{h,t} dF(h). \tag{13}$$

The profit of the firm is given by

$$\Pi_t = AK_t^{\alpha} \left(L_t^y + \int_{h_t^*}^{1} h l_{h,t} dF(h) \right)^{1-\alpha} - r_t K_t - w_t L_t^y - \int_{h_t^*}^{1} (w_t h) l_{h,t} dF(h).$$

The firm maximizes profit by selecting K_t, L_t^y and $l_{h,t}$, and the first-order conditions for an optimum are given by the following:

$$\alpha A K_t^{\alpha-1} L_t^{1-\alpha} - r_t = 0, \tag{14}$$

$$(1-\alpha) A K_t^{\alpha} L_t^{-\alpha} - w_t = 0, \tag{15}$$

$$h(1-\alpha) A K_t^{\alpha} L_t^{-\alpha} - h w_t = 0 \quad \text{for } h \in [h_t^*, 1]. \tag{16}$$

Since conditions (15) and (16) are symmetric, the total demand of labor service satisfies

$$L_t = [(1-\alpha) A]^{\frac{1}{\alpha}} w_t^{-\frac{1}{\alpha}} K_t. \tag{17}$$

2.3 Government

We assume that there is a pay-as-you-go pension plan in which the tax for social security is levied on the young agents' wage income as well as on the wage income of the working elderly. The per-capita tax on the young agents given by $\tau_t = \tau w_t$, where $\tau \in (0,1)$ the flat rate of payroll tax. We have also assumed that a flat rate of income tax, $\xi \in (0,1)$, is applied to the wage income of working old. We assume that all the tax revenue received by the government is spent for pension, and there is no government debt. Hence, the government's budget constraint in period t is

$$\tau w_t N_t + \xi \pi N_{t-1} \int_{h_t^*}^{1} h w_t dF(h) = [F(h_t^*) + \mu(1 - F(h_t^*))] \, p_t \pi N_{t-1}. \tag{18}$$

The left-hand side of (18) is the aggregate tax revenue, and the right-hand side expresses the aggregate pension for the elderly. It is to noted that, by our assumption, the mass of the old agents in period t is πN_{t-1}.

2.4 Market Equilibrium Conditions

Since we have assumed that one unit of hours worked offered by a young agent yields one unit of labor service, the total supply of young agents' labor service in period t is N_t. On the other hand, the working population of the elderly in period t is $[1 - F(h_t^*)] \pi N_{t-1}$. and the average labor survive of the working old is $\int_{h_t^*}^{1} h dF(h)$, so that the aggregate labor service provided by the old is $\pi N_{t-1} [1 - F(h_t^*)] \int_{h_t^*}^{1} h dF(h)$. As a result, the total supply of labor service denoted by N_t^s is given by

$$N_t^s = N_t \left\{ 1 + [1 - F(h_t^*)] \int_{h_t^*}^{1} h dF(h) \frac{\pi}{1+n} \right\}. \tag{19}$$

The labor market equilibrium condition is

$$N_t^s = L_t. \tag{20}$$

Since only young agents save, the market equilibrium condition of asset market is

$$K_{t+1} = N_t s_t. \tag{21}$$

3 Equilibrium Dynamics

3.1 Determinants of Labor Supply of the Elderly

To derive a complete dynamic system, we first note that constraint (18) and $N_t = (1+n)\,N_{t-1}$ present

$$\frac{p_t}{w_t} = \frac{\frac{\tau}{\pi}(1+n) + \xi \int_{h_t^*}^{1} hdF\,(h)}{(1-\mu)\,F\,(h_t^*) + \mu}. \tag{22}$$

Combining this equation with (1) leads to

$$h_t^* = \left(\frac{1-\mu}{1-\xi}\right)\left[\frac{\frac{\tau}{\pi}(1+n) + \xi \int_{h_t^*}^{1} hdF\,(h)}{(1-\mu)\,F\,(h_t^*) + \mu}\right]. \tag{23}$$

Inspecting (23) , we find the following:

Proposition 1 *There exists a unique, stationary cutoff level of h^*, if the following conditions are fulfilled:*

$$\left(\frac{1-\mu}{1-\xi}\right)\left[\frac{\frac{\tau}{\pi}(1+n) + \xi \int_{\eta}^{1} hdF\,(h)}{(1-\mu)\,F\,(\eta) + \mu}\right] > \eta, \qquad \left(\frac{1-\mu}{1-\xi}\right)(1+n)\,\frac{\tau}{\pi} < 1. \tag{24}$$

Proof. Let us express

$$M\,(h_t^*) = \left(\frac{1-\mu}{1-\xi}\right)\frac{\frac{\tau}{\pi}(1+n) + \xi \int_{h_t^*}^{1} hdF\,(h)}{(1-\mu)\,F\,(h_t^*) + \mu}.$$

Note that

$$\int_{h_t^*}^{1} hdF(h) = [hF(h)]_{h_t^*}^{1} - \int_{h_t^*}^{1} F(h)\,dh,$$

so that

$$\frac{d}{dh_t^*}\int_{h_t^*}^{1} hdF(h) = -F(h_t^*) - h_t^* F'(h_t^*) + F(h_t^*) = -h_t^* F'(h_t^*) < 0$$

This means that $B(h_t^*)$ monotonically decreases with h_t^*. It is also noted that

$$M(\eta) = \left(\frac{1-\mu}{1-\xi}\right)\left[\frac{\frac{\tau}{\pi}(1+n) + \xi\int_{\eta}^{1} hdF(h)}{(1-\mu)F(\eta)+\mu}\right], \quad M(1) = \left(\frac{1-\mu}{1-\xi}\right)(1+n)\frac{\tau}{\pi}.$$

Hence, as illustrated Figure 1, under (24), the graphs the left-hand side of (23) (45 degree line) and the left-hand side of of it has a unique intersection for $h_t^* \in (\eta, 1)$. ■

[Figure 1]

From (23) 1, it is easy to see that the equilibrium level of h_t^* depends only on the parameter values involved in the model, and thus the equilibrium level of cutoff, h_t^*, stays constant over time, that is, $h_t^* = h^*$ for all $t \geq 0$. Moreover, inspecting Figure 1 reveals that h^* responds to changes in the key parameter values in the following manner:

$$\frac{\partial h^*}{\partial n} > 0, \quad \frac{\partial h^*}{\partial \pi} < 0, \quad \frac{\partial h^*}{\partial \tau} > 0, \quad \frac{\partial h^*}{\partial \xi} > 0, \quad \frac{\partial h^*}{\partial \mu} < 0.$$

For example, population aging (a rise in π and/or a decline in n) shifts the graph of $M(h_t^*)$ downward, which lowers h^*. Since the labor participation rate of the elderly is $\frac{1-h_t^*}{1-\eta}$, a decrease in h^* means that a larger fraction of old agents stay in the labor market. Consequently, we obtain the following proposition:

Proposition 2 *The rate of labor force participation of the elderly rises with a higher longevity, π, a lower population growth rate, n, a lower payroll tax rate, τ, a lower tax rate on the old agents' wage income, ξ, and a higher rate of pension payments for the working old, μ.*

The intuition behind those comparative statics outcomes is easy to obtain. For example, (22) means that a rise in π (or a decrease in n) lowers the per-capita level of pension, p_t, under given levels of h^* and w_t. This makes the selection of full retirement less attractive for the elderly, and thus a larger number of the old agents want to stay in the labor force. As a consequence, h^* will decline and the labor force participation rate of the elderly increases.

3.2 Existence and Stability of the Steady-State Equilibrium

In view of (10) and (21), we see that the aggregate capital stock per population of young agents, $k_t = K_t/N_t$, follows

$$k_{t+1} = \frac{1}{1+n}\left[\frac{\beta\pi}{1+\beta\pi}(1-\tau)w_t - \frac{\pi}{(1+\beta\pi)(1+r_{t+1})}y^e_{t+1}\right]. \tag{25}$$

Here, from (7), y^e_{t+1} is expressed as

$$y^e_{t+1} = F(h^*)p_{t+1} + [1-F(h^*)]\mu p_{t+1} + (1-\xi)\int_{h^*}^{1} hw_{t+1}dF(h). \tag{26}$$

From (1), the relation between p_t and w_t is given by

$$p_t = \frac{1-\xi}{1-\mu}h^* w_t \text{ for all } t \geq 0. \tag{27}$$

Furthermore, using (17), (19) and (20), we express the labor market equilibrium condition in the following manner:

$$k_t = \Lambda(h^*)w_t^{\frac{1}{\alpha}}, \tag{28}$$

where

$$\Lambda(h^*) = \left\{1 + [1-F(h_t^*)]\int_{h_t^*}^{1} hdF(h)\frac{\pi}{1+n}\right\}[(1-\alpha)A]^{-\frac{1}{\alpha}}.$$

Combining (25), (26), and (28), we obtain the following:

$$\begin{aligned}\Lambda(h^*)w_{t+1}^{\frac{1}{\alpha}} = {} & \frac{\beta\pi}{(1+n)(1+\beta\pi)}(1-\tau)w_t \\ & - \frac{\pi}{(1+n)(1+\beta\pi)(1+r_{t+1})}\left\{[(1-\mu)F(h^*)+\mu]\frac{1-\xi}{1-\mu}h^* + (1-\xi)\int_{h^*}^{1} hdF(h)\right\}w_{t+1}.\end{aligned} \tag{29}$$

Note that $r_{t+1} = \alpha A (K_t/L_t)^{\alpha-1}$ and $w_t = (1-\alpha) A (K_t/L_t)^{-\alpha}$. Thus, we obtain

$$r_{t+1} = \alpha A \left[\frac{w_{t+1}}{(1-\alpha) A}\right]^{-\frac{1-\alpha}{\alpha}} = R(w_{t+1}).$$

This means that (29) can be written as

$$w_t = \Phi(w_{t+1}),$$

where

$$\Phi(w_{t+1}) = \frac{1}{(1-\tau)\beta[1+R(w_{t+1})]}\left\{[(1-\mu)F(h^*)+\mu]\frac{1-\xi}{1-\mu}h^* + (1-\xi)\int_{h^*}^{1} hdF(h)\right\} w_{t+1} + \frac{(1+n)(1+\beta\pi)}{(1-\tau)\beta\pi}\Lambda(h^*) w_{t+1}^{\frac{1}{\alpha}}. \quad (30)$$

We can confirm that the dynamic system given by (30) has a unique steady-state value of w_t and that it satisfies global stability.

Proposition 3 *There is a unique steady-state equilibrium that is globally stable.*

Proof. Let us express $\Phi(w_{t+1})$ given by (30) as

$$\Phi(w_{t+1}) = \frac{B_1 w_{t+1}}{1 + B_2 w_{t+1}^{1-\frac{1}{\alpha}}} + B_3 w_{t+1}^{\frac{1}{\alpha}}, \quad (31)$$

where

$$B_1 = \frac{1}{(1-\tau)\beta}\left\{[(1-\mu)F(h^*)+\mu]\frac{1-\xi}{1-\mu}h^* + (1-\xi)\int_{h^*}^{1} hdF(h)\right\} > 0,$$

$$B_2 = A[(1-\alpha)A]^{\frac{1}{\alpha}-1} > 0,$$

$$B_3 = \frac{(1+n)(1+\beta\pi)}{(1-\tau)\beta\pi}\Lambda(h^*) > 0. \quad (32)$$

Letting the steady-state value of w_t be w, it holds that $w = \Phi(w)$. Besides $w = 0$, from (31), w also satisfies

$$1 = \frac{B_1}{1 + B_2 w^{1-\frac{1}{\alpha}}} + B_3 w^{\frac{1}{\alpha}-1}. \quad (33)$$

Since the right-hand side of the above monotonically increases with w, there is a unique, positive steady-state level of real wage that establishes(33).

From (31), we find

$$\Phi'(w_{t+1}) = \frac{B_1}{1 + B_2 w_{t+1}^{1-\frac{1}{\alpha}}} + \left(\frac{1}{\alpha} - 1\right) \frac{B_1 B_2 w_{t+1}^{1-\frac{1}{\alpha}}}{\left(1 + B_2 w_{t+1}^{1-\frac{1}{\alpha}}\right)^2} + \frac{1}{\alpha} B_3 w_{t+1}^{\frac{1}{\alpha}-1} > 0.$$

Hence, $\Phi(w_{t+1})$ monotonically increases with w_{t+1}. In addition, from (33) and $\alpha \in (0,1)$, we obtain

$$\Phi'(w) = 1 + \left(\frac{1}{\alpha} - 1\right) B_3 w^{\frac{1}{\alpha}-1} + \left(1 - \frac{1}{\alpha}\right) B_3 w^{\frac{1}{\alpha}-1} + \left(\frac{1}{\alpha} - 1\right) \frac{B_1 B_2 w^{1-\frac{1}{\alpha}}}{\left(1 + B_2 w^{1-\frac{1}{\alpha}}\right)^2} > 1,$$

Moreover, It is easy to see that $\alpha \in (0,1)$ means that $\Phi(0) = 0$ and $\Phi'(0) = 0$. Since $\Phi(w_{t+1})$ is invertible, the properties of $\Phi(w_{t+1})$ function shown so far means that $\frac{d}{dw_t}\Phi^{-1}(w_{t+1}) > 0$, $\frac{d}{dw_t}\Phi^{-1}(0) = +\infty$, and $0 < \Phi^{-1}(w) < 1$. As a consequence, the phase diagram of $w_{t+1} = \Phi^{-1}(w_t)$ can be depicted as Figure 2, meaning that the w_t converges to w for all $w_0 > 0$. ■

[Figure 2]

4 Retirement Decision and Capital Accumulation

4.1 Steady-State Characterization

In the steady state, k_t and w_t stay constant over time. From (28) and (29), the steady-state conditions are summarized as the following:

$$k = \Lambda(h^*) w^{\frac{1}{\alpha}}, \tag{34}$$

$$k = \frac{\beta\pi w}{(1+n)(1+\beta\pi)} \left[(1-\tau) - \Omega(h^*) \frac{1}{1+R(w)}\right], \tag{35}$$

where

$$\Lambda(h^*) = \left\{1 + [1 - F(h_t^*)] \int_{h_t^*}^{1} h dF(h) \frac{\pi}{1+n}\right\} [(1-\alpha) A]^{-\frac{1}{\alpha}}, \tag{36}$$

$$\Omega(h^*) = [(1-\mu) F(h^*) + \mu] \frac{1-\xi}{1-\mu} h^* + (1-\xi) \int_{h^*}^{1} h dF(h). \tag{37}$$

In the above, k and w, respectively, denote the steady-state values of k_t and w_t. As mentioned before, (34) is the technological relationship between the capital stock per mass of the young generation and the real wage rate. A change in the labor force participation of the elderly changes the level of $\Lambda(h^*)$. Equation (35) expresses the per capita saving of the young generation, which is positively related to the young agents' after-tax income, $(1-\tau) w$, and negatively related to the per-capita consumption in their old age. Noting that $0 < \alpha < 1$, $\lim_{w \to 0} R(w) = +\infty$, and $\lim_{w \to +\infty} R(w) = 0$, we see that the graphs of (34) and (35) are depicted as in Figure 3.

[Figure 3]

Once k and w are fixed, the steady-state values of the key variables are determined in the following manner. The per capita level of income in the steady state is

$$\frac{AK_t^{\alpha} L_t^{1-\alpha}}{N_t + \pi N_{t-1}} = A\left(\frac{1+n}{1+n+\pi}\right) k^{\alpha} \left\{\left[1 + (1 - F(h_t^*)) \int_{h_t^*}^{1} + h dF(h) \frac{\pi}{1+n}\right]^{1-\alpha}\right\}. \tag{38}$$

Additionally, the rate of return to capital and per capita pension are, Respectively, given by

$$r = R(w) = A[(1-\alpha) A]^{\frac{1}{\alpha}-1} w^{1-\frac{1}{\alpha}}. \tag{39}$$

4.2 Comparative Statics in the Long-Run

Using the steady-state conditions derived above, we conduct some comparative statics in the long-run equilibrium.

(i) Population Aging

As mentioned earlier, in our setting, population aging means a fall in the population expansion rate, n, and/or a rise in the survival rate, π. Other things being equal, either if n

decreases or if π rises, then h^* declines, which increases the value of $\Lambda(h^*)$ in (36). Hence, the graph of (34) shifts upward. Namely, a rise in the labor force participation elderly driven by population aging makes firms select a higher level of capital under a given level of the real wage. We also find a fall in h^* decreases the first and the second terms in the right-hand side of $\Omega(h^*)$ defined by (37), while it increases the third term. If the former negative effects dominate the positive third effect, then $\Omega(h^*)$ decreases so that the graph of (10) shifts upward. In this case, the steady-state level of k_t rises: see Figure 4. Otherwise, the graph of (35) shifts downward, and thus the effect on k becomes analytically ambiguous

Concerning the impact of population aging on per capita real income given by (38), a decline in n or a rise in π leads to three effects—the two indirect effects that are represented by an increase in k and a decrease in h^*; these impacts positively affect the per capita income in the steady state. The direct effect expressed by terms $\left(\frac{1+n}{1+n+\pi}\right)$ and $\left(\frac{\pi}{1+n}\right)^{1-\alpha}$ will be negative or positive when n decreases or π increases. Since the total impact depends on the relative strength of those three effects, the effect of population aging on the steady-state level of per capita income is, again, analytically ambiguous.

[Figure 4]

(ii) Pension Reforms

We have confirmed that if the fiscal authority raises the rate of payroll tax, τ, then h^* will increase, leading to a decline in the labor force participation of the elderly. Similarly, a fall in μ and a rise in ξ, which make the pension for the working elderly less generous, increase h^*. To cope with the expansion of public debt, the recent pension reforms implemented in advanced countries set less favorable pension schemes for the fully retired elderly. As a side effect, those reforms promote the labor force participation of the elderly. Therefore, in the context of our model. the recent pension reforms are described by falls in τ and ξ as well as by a rise in μ. Since $\Lambda(h^*)$ in (35) is independent of policy variables, a pension reform that lowers h^* rises the value of $\Lambda(h^*)$, which gives rise to an upward shift of the graph of (35). On the other hand, as well as in the case of population aging, a pension reform gives both positive and negative effects on $\Omega(h^*)$ given by (37). If the negative effects dominate, then the value of $\Omega(h^*)$ decreases, leading to an upward shift of the graph of (35). In this case, pension reform increases k. Otherwise, the long-run impact of pension reform on k, again, becomes analytically ambiguous.

(iii) Income Distribution

From (1), in the steady state, the relative income a young agent and a retired old agent is given by

$$\frac{p}{w} = \frac{1-\xi}{1-\mu} h^*, \tag{40}$$

where p denotes the steady-state level of pension for the fully retired old. Since population aging reduces h^*, a rise in π or a fall in n widens the income gap between the young agents and the fully retired elderly. On the other hand, a pension reform conducted by a decrease in ξ or an increase in μ have two opposite effects on the value of p/w. Equation (40) means that those policy changes yield a positive, direct effect on p/w as well as an indirect negative effect through a reduction in h^*. Thus, the total effect on p/w is not determined analytically.

Similarly, the relative income between the young and the working old with labor efficiency h is given by

$$\frac{\mu p + (1-\xi) hw}{w} = \frac{\mu(1-\xi)}{1-\mu} h^* + (1-\xi) h. \tag{41}$$

Thus, changes in the relative income generated by a change in n, π, and τ are essentially the same as those in the relative income between the young and the fully retired old. Additionally, the relative income between a working old with labor efficiency, h, and the fully retired old is

$$\frac{\mu p + (1-\xi) hw}{p} = \mu + (1-\xi) \frac{h}{h^*}. \tag{42}$$

This expression means that a rise in π and a fall in τ, which lowers h^*, widens the income gap between the working elderly and fully retired elderly. Also, a decrease in ξ raises the income gap, while the effect of a fall in ξ on the income distribution between the working old and the retired elderly is analytically ambiguous.

4.3 Numerical Analysis

Since manipulating the analytical model does not provide us with further information, we examine some numerical examples. To conduct a numerical analysis, we assume that the cumulative distribution function of h is specified as a truncated Pareto distribution in such a way that

$$F(h) = \frac{1 - \left(\frac{h}{\eta}\right)^{-\psi}}{1 - \left(\frac{1}{\eta}\right)^{-\psi}}, \quad \psi > 1, \quad 0 < \eta < 1 \qquad h \in [\eta, 1]. \tag{43}$$

Our specification has two merits. First, it has been pointed out that an upper tail of income distribution among the elderly follows a Pareto distribution. Since the income distribution among the working elderly depends on the distribution of h, such an empirical finding is well described if $F(h)$ follows a Pareto distribution[11]. In addition, the degree of heterogeneity among the elderly can be represented by the shape parameter, ψ. Namely, a smaller ψ means that the distribution function $\Pr(h \geq h^*) = 1 - F(h)$ has a fatter tail. Given (43), the average labor efficiency of the working old is specified as

$$\int_{h^*}^{1} h dF(h) = \frac{\psi \eta^{\psi-1}}{(1-\eta^{\psi})(\psi-1)} \left[(h^*)^{1-\psi} - 1\right],$$

which is a decreasing function of h^*. Hence, the cutoff condition (23) is expressed as

$$h_t^* = \left(\frac{1-\mu}{1-\xi}\right) \left[\frac{\frac{\tau}{\pi}(1+n) + \xi \frac{\psi \eta^{\psi-1}}{(1-\eta^{\psi})(\psi-1)} \left[(h^*)^{1-\psi} - 1\right]}{(1-\mu) \frac{1-\left(\frac{h^*}{\eta}\right)^{-\psi}}{1-\left(\frac{1}{\eta}\right)^{-\psi}} + \mu} \right]. \tag{44}$$

We first focus on the cutoff condition (44), which determines the cutoff level of labor efficiency, h^* that, in turn, determines the labor force participation rate of the elderly, $\frac{1-h^*}{1-\eta}$. We assume that one period spans across 40 years and $\pi = 0.5$; this ensures that the average lifetime of the old agents is 20 years. We also assume that the baseline population growth rate is 0.5% per year; this means that $1 + n = (1 + 0.005)^{40} \approx 1.22$, so that we set $n = 0.2$. As for the discount factor, β, we assume that the annual time discount rate of the household is 0.018, which is a conventional value used in the real business cycle models. As a result, the discount factor over one period is $(1 + 0.018)^{-40} \approx 0.49$, so we set $\beta = 0.5$. The baseline

[11] Concerning the recent study on income distribution among the Japanese elderly, see Shirahase (2015) and Seiyama (2016).

values of the other parameters are as follows:

$$A = 1.0, \quad \alpha = 0.3, \quad \psi = 1.5, \ \eta = 0.125, \ \mu = 0.5, \quad \tau = 0.3, \quad \xi = 0.2$$

In this baseline setting, we find that (44) yields $h^* = 0.325,$ and thus the equilibrium rate of labor force participation of the elderly is $\frac{L_t^0}{N_{t-1}} = \frac{1-h^*}{1-0.125} = 0.75.$

Keeping the other parameters constant, we change one of the key parameters to see how the cutoff level efficiency, h^*, and the labor participation rate of the elderly respond to such a change. Figures 5 and 6 summarize the outcomes. Figure 5 depicts the linkage between population aging and the retirement decision of the elderly. Panel (a) shows the relationship between the probability of surviving, π, and the cutoff level, h^* under alternative levels of $\psi = 1.2$, 1.5, and 3.0. Panel (b) depicts the relationships between h^* and n. Similarly, Panels (c) and (d) describe the relationship between population aging and the labor force participation rate of the elderly. As was expected from the theoretical analysis, population aging (a rise in π and a fall in n) lowers the cutoff level of efficiency, so the number of fully retired elderly decreases and the labor force participation rate of the elderly rises. The graphs in Figure 5 also reveal that the profiles of those linkages are sensitive to the magnitude of the shape parameter, ψ: under given levels of π and n, the labor force participation rate of the elderly becomes higher as ψ increases. Since the magnitude of ψ represents the degree of homogeneity of the elderly, the graphs reveal that other things being equal, the labor force participation rate is higher in the economy in which the old agents are less heterogeneous in their labor efficiency.

[Figure 5]

Figure 6 shows the impacts of changes in the policy parameters on the cutoff efficiency and the labor force participation rate of the elderly. As we have confirmed analytically, less generous pension reforms for the fully retired elderly, that is, decrease in τ and ξ, as well as a rise in μ, reduce h^*, so that the labor force participation of the elderly increases. Again, the profiles of those relationships are sensitive to the level of ψ.

[Figure 6]

Figure 7 shows the impacts of population aging and changes in the policy parameters on the steady-state level of $k_t(= K_t/N_t)$. As discussed in the previous subsection, the long-run effects of those factors are analytically ambiguous: a positive effect of a rise in the total labor supply on output would be offset by a fall in the young agents' savings. However, in our calibration with plausible parameter magnitudes, population aging (a rise in π) and less beneficial pension plans for the retired elderly (decreases in τ and ξ) have positive effects on the steady state-levels of k_t. An exception is the effect of a change in μ. As the graph shows, a higher μ, which also raises the labor force participation of the elderly, yields a negative impact on k.In addition, Figure 8 reveals that those impacts on the the steady-state level of per capita income defined by (38). Concerning changes in π, τ, and ξ, their effects on the steady-state level of per capita income are essentially the same as those on the steady-state value of k_t However, a change in μ now has a negative long-run effect on the per capita income.

[Figure 7] , [Figure 8]

Finally, Figure 9 depicts the effects of population aging and changes in key policy parameters on the relative income between a young agent and a fully retired old agent. As to the effects of a changes in π and τ, the graphs displayed in Figure 9 confirm our analytical argument. However, in our setting, the relationship between p/w and ξ exhibits a hump-shaped profile. Moreover, a rise in μ, which also reduces h^*, decreases p/w, so that the income gap between the young and the retired old becomes larger as the labor force participation of the elderly increases.

[Figure 9]

5 Conclusion

In general, the heterogeneity among the elderly is wider than that among the young people. In particular, individual differences in health status and motivation for labor force participation are more prominent among the elderly than among the young. This study relies on this simple fact to determine the retirement decision of the elderly and thereby their aggregate labor supply. In our model, the heterogeneity among the elderly is represented by a distribution function of their labor efficiency. We have shown that the aggregate level of labor force participation of the elderly is determined by pension schemes, the level of population aging, and the profile of the distribution of labor efficiency among the elderly. The analytical results reveal that both population aging and less favorable pension reform decrease the fraction of elderly who select full retirement from the labor force. Those outcomes fit well with the recent experiences in advanced countries. We have also investigated how changes in the pension scheme and the level of population aging affects the long-run performance of the aggregate economy. Our model shows that population aging and a decline in the payroll tax levied on the young generation promote labor force participation of the old generation, which increases the aggregate labor supply. While a higher labor supply enhances production and investment, a higher income of the elderly increases their consumption, which has a negative effect on capital accumulation. The long-run effect of a rise in labor supply of the elderly on economic growth hinges upon the relative strengths of these opposing effects generated by a change in the labor force participation of the elderly. To investigate the net effect of a change in the elderly's labor supply on the long-term growth, we examined some numerical examples. Our numerical experiment demonstrates that an increase in the elderly's labor force participation enhances capital accumulation.

To obtain clear analytical outcomes, we have set some restrictive assumptions. Particularly, we have assumed that payroll taxes levied on the young generation is proportional to their wage income and that all the tax revenue of the government to expenditure for the pension. Due to these restrictions, the aggregate labor participation rate of the elderly stays constant during the transition process of the economy. If we assume an alternative pension scheme in which the per capita level of pension is fixed, then it can be shown that the threshold level of labor efficiency, h_t^*, does not stay constant over time, leading to more complex aggregate dynamics than that treated in this study. Similarly, if we assume that the tax revenue of the government is spent for other purposes, in addition to pension, then the cutoff level, h_t^*, will change over time. Such an extension, again, requires us to treat a more complex dynamic analysis. This means that we should heavily rely on numerical considerations rather than analytical arguments. Finally, since we use the neoclassical growth framework, the per capita income stays constant in the steady-state equilibrium. It would be interesting to introduce our setting into an endogenous growth model in which a persisting increase of the per capita income is allowed.

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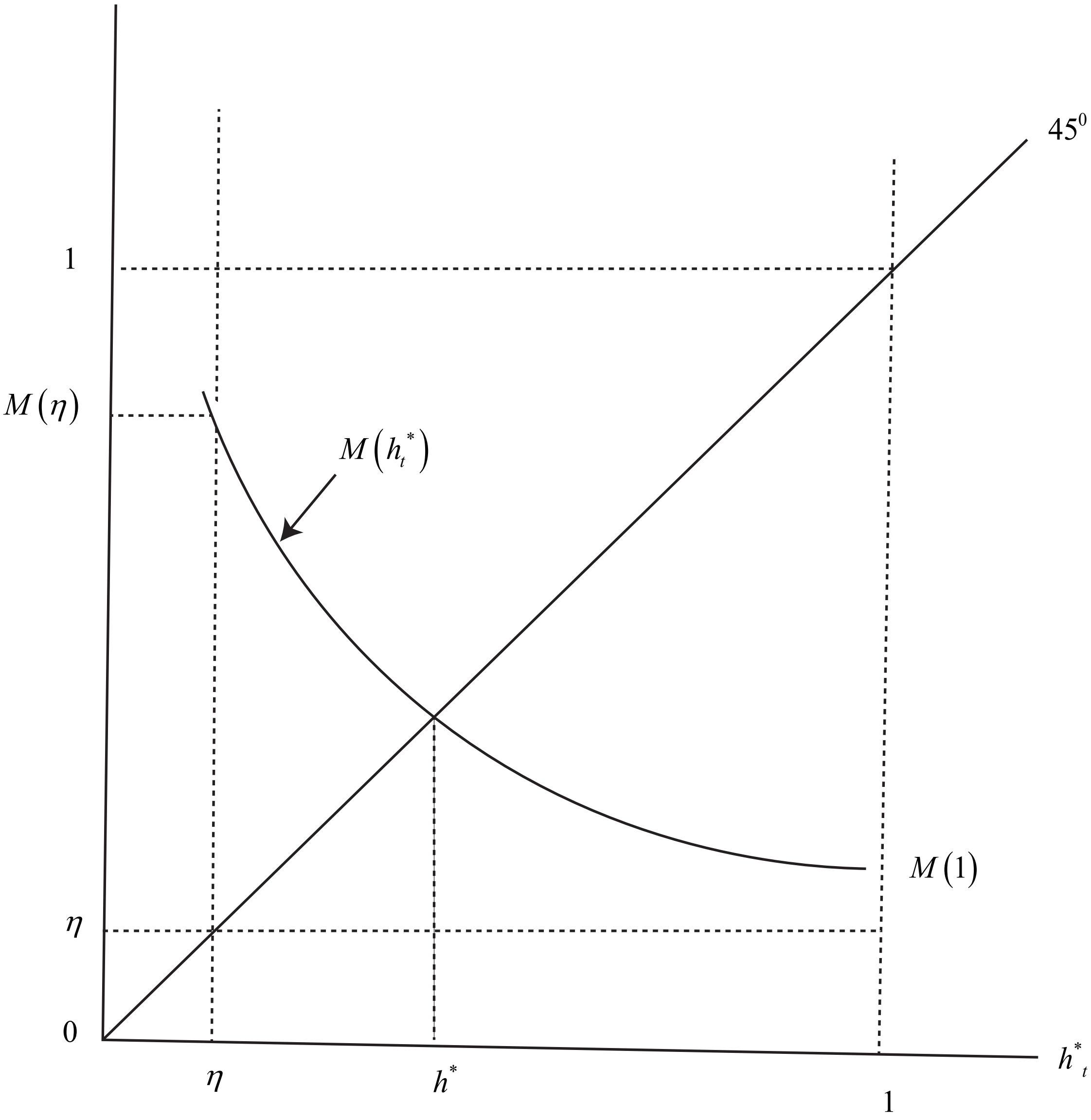

Figure 1: Exixtence of the cutoff level of labor efficiency

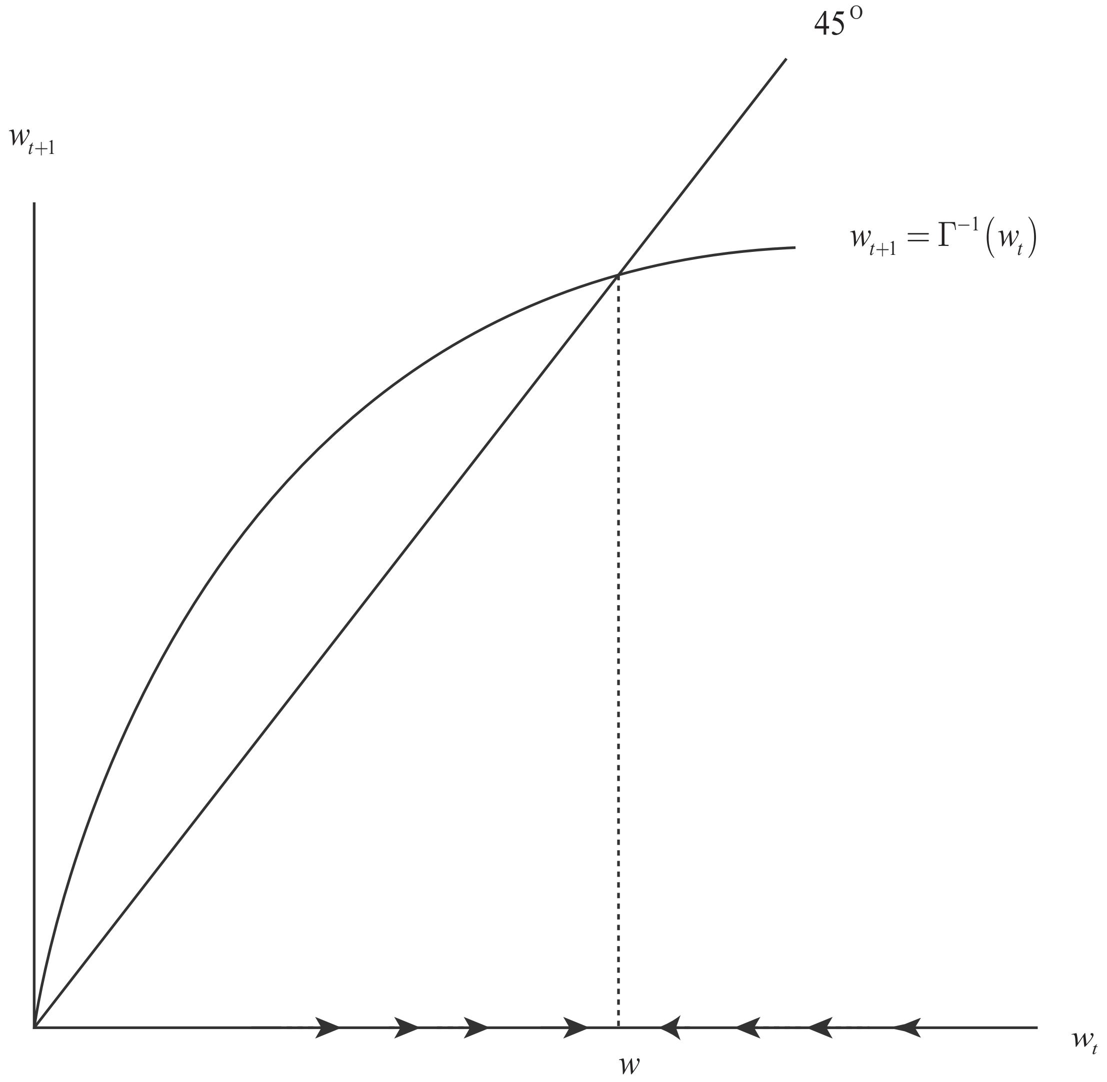

Figure 2 : Convergence to the steady state

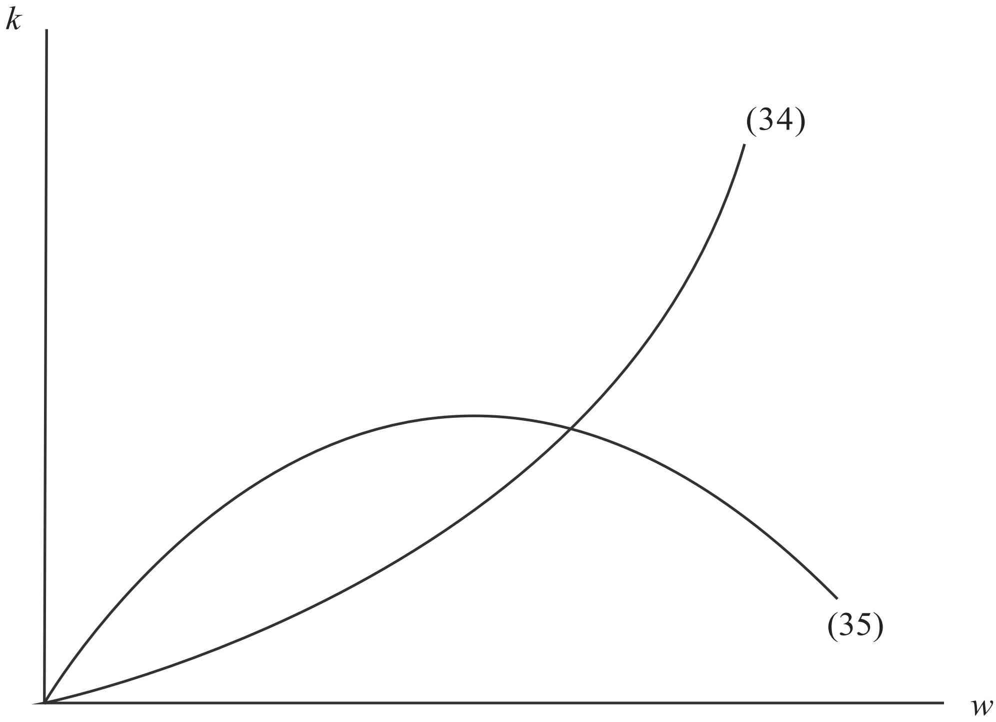

Figure 3: The steady-state conditions in w-k space

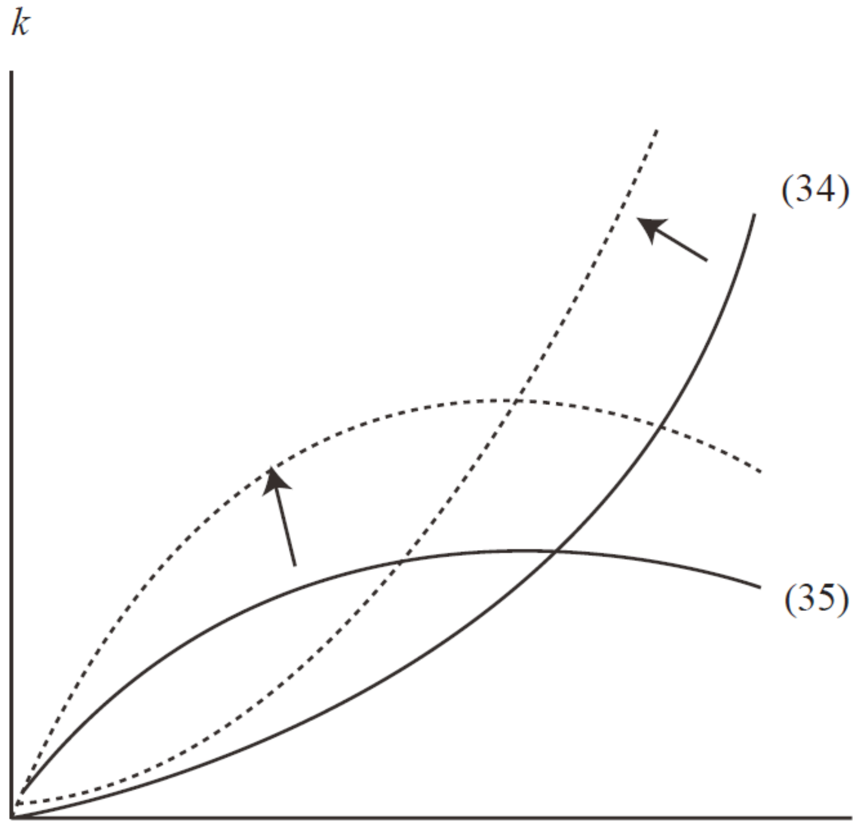

Figure 4: The effects of population aging

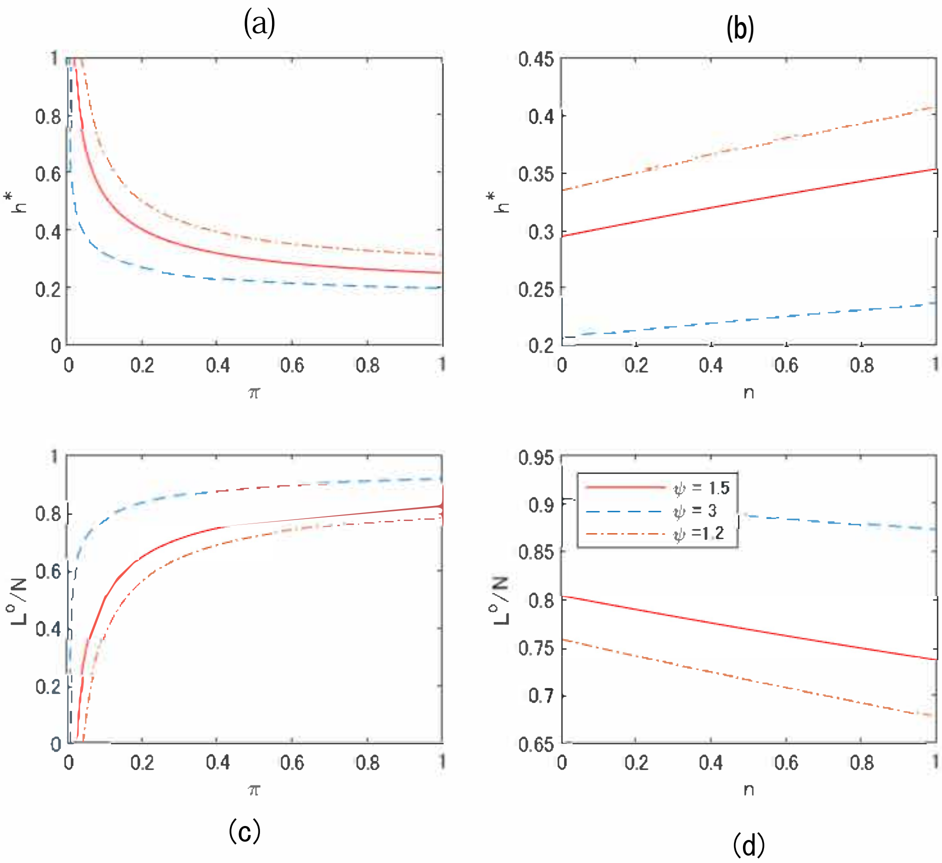

Figure 5: The effects of population aging

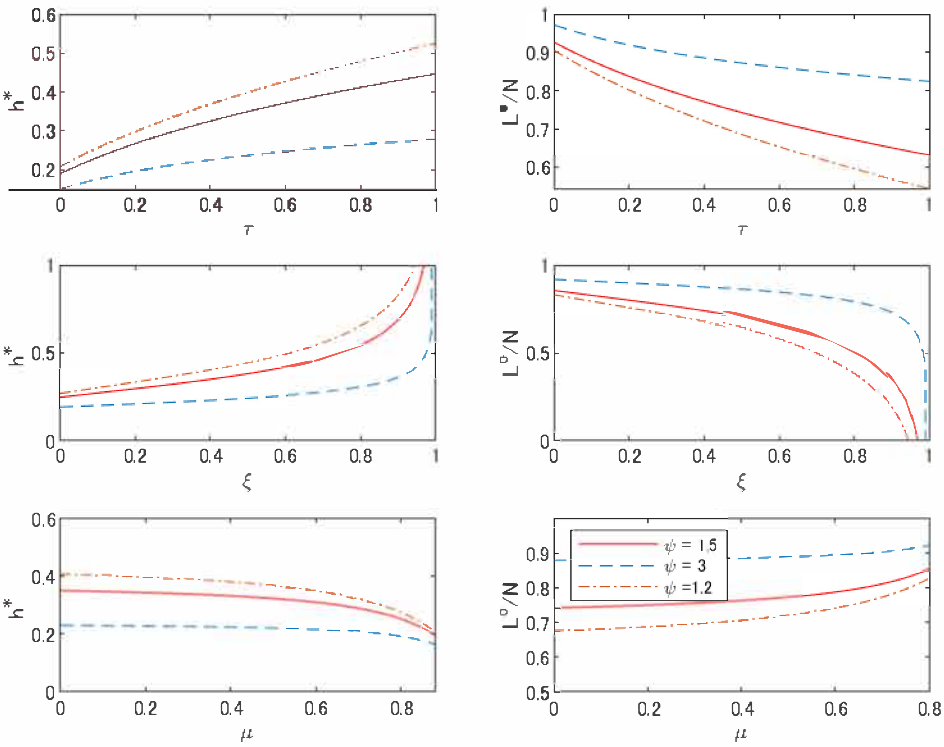

Figure 6: The effects of changes in the policy parameters

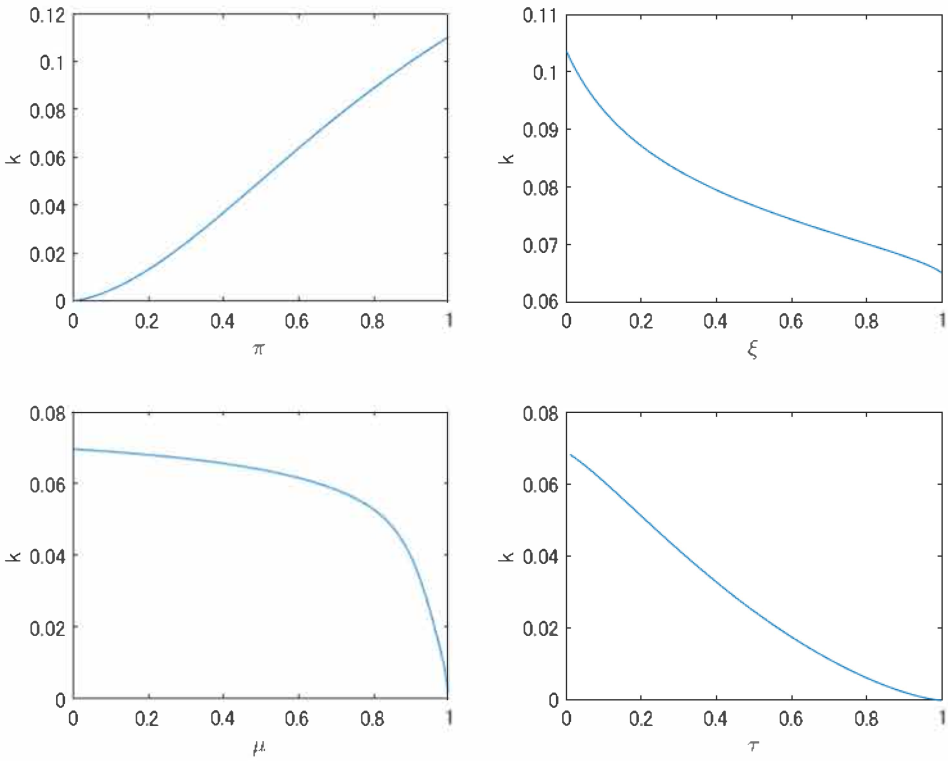

Figure 7: 'The relationships between the key parameters and the steady-state value of the capital per young agents

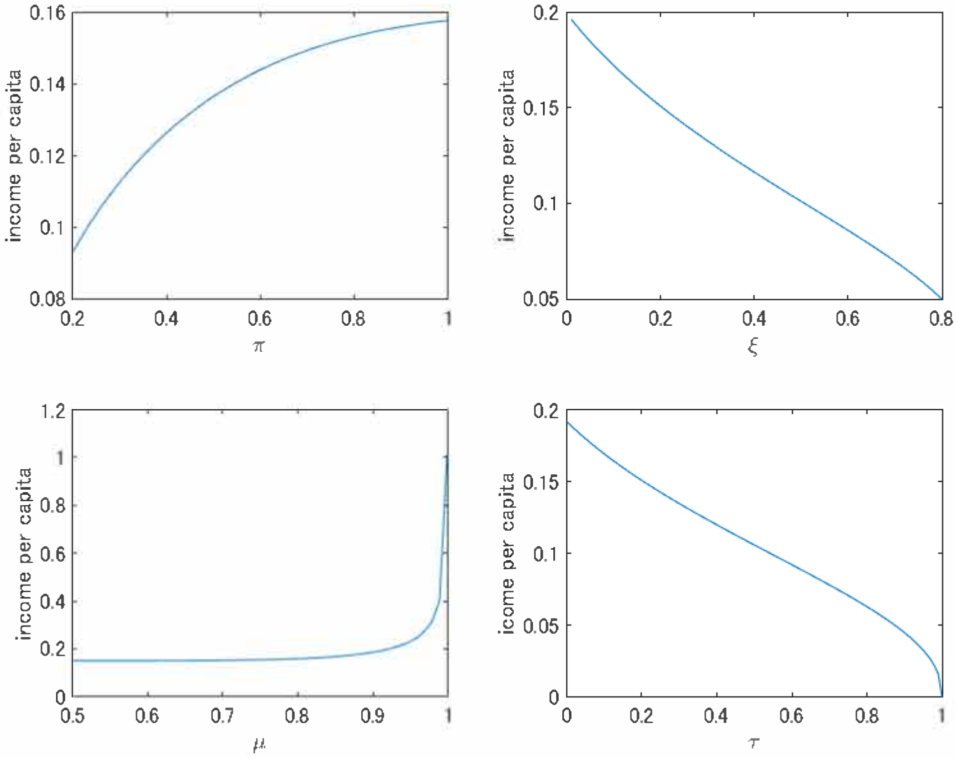

Figure 8: The relationships between the key parameters and the steady-state value of the per capita income

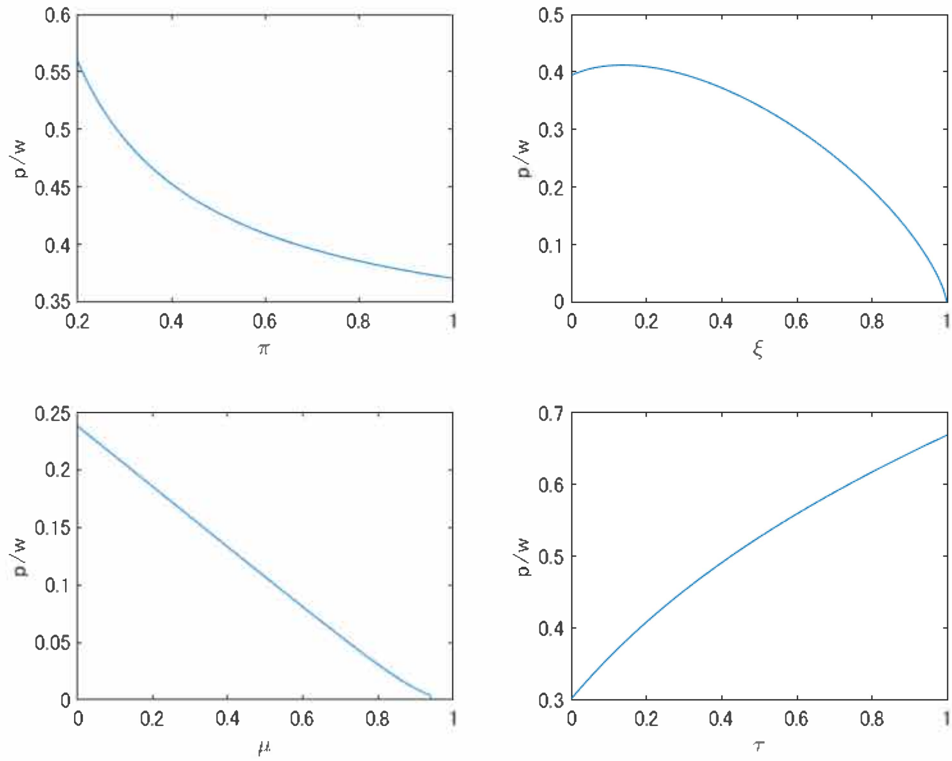

Figure 9: The relationships between the key parameters and the relative income between a young agent and a retiered old agent