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Price-setting mixed duopoly, subsidization and the order of firms' moves: an irrelevance result

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Abstract

This paper examines price-setting duopoly games with production subsidies and shows that the optimal production subsidy, profits and economic welfare are identical irrespective of whether (i) a public firm and a private firm simultaneously and independently set prices, (ii) the public firm acts as a Stackelberg leader, or (iii) both firms behave as profit-maximizers.

Keywords: Mixed duopoly model; Price competition; Subsidization
JEL classification: C72; D21; L32

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1. Introduction

The analysis by White (1996) investigates the role that production subsidies play in a quantity-setting mixed market and how they may influence the privatization decision. He presents the following three main results. First, if subsidies are used before and after privatization, then privatization does not change economic welfare. Second, if subsidies are used only before privatization, then privatization always lowers economic welfare. Third, the subsidy contributes to overall efficiency in a mixed market due to cost distribution effects. Poyago-Theotoky (2001) shows that the optimal production subsidy is identical irrespective of whether (i) a public firm moves simultaneously with n private firms, (ii) it acts as a Stackelberg leader, or (iii) all firms behave as profit-maximizers. In addition, Ohnishi (2012) considers the role that production subsidies play in a Bertrand mixed market and shows that the results are the same as those of Cournot mixed market games.

We study the role that production subsidies play in a price-setting duopoly comprising a public firm and a private firm. We consider the following three regimes: (i) the public firm moves simultaneously with the private firm, (ii) the public firm acts as a Stackelberg leader, and (iii) both firms behave as profit-maximizers. We solve and compare the three games.

The remainder of this paper proceeds as follows. In Section 2, we describe the model. Section 3 presents the result of this study. Finally, Section 4 concludes the paper.

2. Model

Let us consider a model composed of a welfare-maximizing public firm and a profit-maximizing private firm producing imperfectly substitutable goods. In the remainder of this paper, subscripts 0 and 1 denote the public firm and the private firm, respectively. In addition, when i and j are used to refer to firms in an expression, they should be understood to refer to 0 and 1 with $i \neq j$. There is no possibility of entry or exit. On the consumption side, there is a continuum of consumers of the same type whose utility function is linear. Following Barcena-Ruiz and Garzón (2007), we assume that the representative consumer maximizes $U(q_0, q_1) - p_0 q_0 - p_1 q_1$, where q_i denotes the amount of good i and p_i is its price. The function $U(q_0, q_1)$ is quadratic, strictly concave and symmetric in q_0 and q_1: $U(q_0, q_1) = a(q_0 + q_1) - (q_0^2 + 2bq_0q_1 + q_1^2)/2$, where $a > 0$ and $0 < b < 1$. The demand function is given by

$$q_i = \frac{a(1-b) - p_i + bp_j}{1-b^2}, \tag{1}$$

where b denotes a measure of the degree of substitutability among products.

Each firm's profit is given by

$$\pi_i = (p_i - c + s)q_i, \tag{2}$$

where c denotes the total cost for each unit of output and s is the subsidy for each unit of output. We assume $0 < c < a$ to assure that the production levels of firms are positive. Economic welfare, defined as the sum of producer surplus and consumer surplus, is given by

$$W = CS + \pi_0 + \pi_1 - s(q_0 + q_1), \tag{3}$$

where $CS = [p_0^2 - 2bp_0p_1 + p_1^2 + 2a(1-b)(a - p_0 - p_1)]/2(1-b^2)$. We use subgame perfection as the equilibrium concept.

3. Main result

In this section, we consider the following three price-setting regimes: (a) mixed market, (b) private market, and (c) Stackelberg mixed market.

(a) Mixed market

There are two stages: in the first stage the government sets the production subsidy to maximize economic welfare; in the second stage both firms simultaneously and independently choose their prices conditional on the production subsidy. The game is solved by backward induction to obtain a subgame perfect equilibrium. Maximizing (2) and (3) simultaneously, we arrive at the second-stage equilibrium prices in terms of s:

$$p_0^N = \frac{ab(1-b) + c(2-b) - bs}{2-b^2}, \qquad p_1^N = \frac{a(1-b) + c(1+b-b^2) - s}{2-b^2}. \tag{4}$$

We now solve the first stage of the game. In the first stage, taking into account how firms will react to the subsidy, the government determines the welfare-maximizing subsidy:

$$s^* = \frac{(1-b-b^2+b^3)(a-c)}{1-b^2}. \tag{5}$$

Since $a > c$, s^* is strictly positive, so that the government will always grant a positive subsidy.

From (4) and (5), we derive the following subgame perfect equilibrium outcomes:

$$p_0^N(s^*) = \frac{c(2-3b^2+b^4)}{(1-b^2)(2-b^2)} = p_1^N(s^*), \tag{6}$$

$$q_0^N(s^*) = \frac{(2-2b-b^2+b^3)(a-c)}{(1-b^2)(2-b^2)} = q_1^N(s^*), \tag{7}$$

$$\pi_0^N(s^*) = \frac{(1-b-b^2+b^3)(2-2b-b^2+b^3)(a-c)^2}{(1-b^2)^2(2-b^2)} = \pi_1^N(s^*), \tag{8}$$

$$CS^N(s^*) = \frac{(4-4b-12b^2+12b^3+13b^4-13b^5-6b^6+6b^7+b^8-b^9)(a-c)^2}{(1-b^2)^3(2-b^2)^2}, \tag{9}$$

$$W^N(s^*) = \frac{(4-4b-12b^2+12b^3+13b^4-13b^5-6b^6+6b^7+b^8-b^9)(a-c)^2}{(1-b^2)^3(2-b^2)^2}. \tag{10}$$

Note that each firm sets a price that equals c.

(b) Private market

In stage one, the government decides the production subsidy to maximize economic welfare; in stage two, both firms simultaneously and independently choose their prices conditional on the production subsidy. The game is solved by backward induction to obtain a subgame perfect equilibrium. Maximizing (2) simultaneously, we obtain the second-stage equilibrium in terms of s:

$$p_i^P(s) = \frac{a(2-b-b^2)+(2+b)(c-s)}{(2-b)(2+b)}. \tag{11}$$

In stage one, taking into account how firms will react to the subsidy, the government determines the welfare-maximizing subsidy. It happens that the optimal subsidy, prices, outputs, profits, consumer surplus and economic welfare in this case are identical with those in the Bertrand mixed market. Therefore, expressions (6) – (10) also represent the relevant expressions for the subsidized private market.

(c) Stackelberg mixed market

We now consider the following three-stage game. In the first stage, the government chooses the production subsidy. In the second stage, the public firm chooses its price. In the third stage, the private firm chooses its price. Starting from the third stage, we obtain

$$p(p_0, s) = \frac{a(1-b) - c - s + bp_0}{2}. \quad (12)$$

In the second stage, the public firm decides its price for given subsidy anticipating how its choice affects the private firm's price decision. This results in

$$p_0^S(s) = \frac{ab(1-b-b^2+b^3) + c(4-b-6b^2+b^3+2b^4) - bs(1-b^2)}{4-7b^2+3b^4}, \quad (13)$$

and further we obtain

$$p_1^S(s) = \frac{a(2-2b-3b^2+3b^3+b^4-b^5) + c(2+2b-4b^2-3b^3+2b^4+b^5) - s(1-b^2)(2-b^2)}{4-7b^2+3b^4}.$$

(14)

In the first stage, the government anticipating how its choice of subsidy affects firms' price choices, maximizes (3). The optimal subsidy is

$$s^S = \frac{(1-b-b^2+b^3)(a-c)}{1-b^2} = s^*.$$

Prices, outputs, profits, consumer surplus and economic welfare are identical to those obtained in (a) and (b), i.e. given by expressions (6) – (10).

Now we can state the following proposition.

Proposition 1: In a price-setting model with production subsidization, the optimal subsidy is identical irrespective of whether (i) the public firm moves simultaneously with the private firm, (ii) the public firm acts as a Stackelberg leader, or (iii) both firms behave as profit-maximizers.

This proposition means that our result is the same as that of quantity-setting market games obtained by Poyago-Theotoky (2001).

4. Conclusion

We have investigated the role that production subsidies play in a price-setting duopoly comprising a public firm and a private firm. We have considered the following three regimes: (i) the public firm moves simultaneously with the private firm, (ii) the public firm acts as a Stackelberg leader, and (iii) both firms behave as profit-maximizers. We have found that the optimal subsidy, prices, outputs, profits, consumer surplus and economic welfare are identical in all three games.

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