

# MPRA

Munich Personal RePEc Archive

## **Implementation with foresighted agents**

Korpela, Ville and Lombardi, Michele and Vartiainen, Hannu

Turku University, University of Glasgow, University of Helsinki

2019

Online at <https://mpra.ub.uni-muenchen.de/102496/>  
MPRA Paper No. 102496, posted 26 Aug 2020 09:44 UTC

# Implementation with farsighted agents

Ville Korpela\*      Michele Lombardi†      Hannu Vartiainen‡

May 28, 2020

## Abstract

Agents are farsighted when they consider the ultimate consequences of their actions. We re-examine the classical questions of implementation theory under complete information in a setting with transfers, where farsighted coalitions are considered fundamental behavioral units, and the equilibrium outcomes of their interactions are predicted via the stability notion of the largest consistent set. The designer's exercise consists of designing a rights structure that formalizes the idea of power distribution in society. The designer's challenge lies in forming a rights structure in which the equilibrium behavior of agents always coincides with the recommendation given by a social choice rule. We show that (Maskin) monotonicity fully identifies the class of implementable single-valued social choice rules. Even though, monotonicity is not necessary for implementation in general, we show that every monotonic social choice rule can be implemented. These findings imply that the class of implementable social choice rules in core equilibria is unaltered by farsighted reasoning.

Keywords: Implementation, rights structures, largest consistent set, core, (Maskin) monotonicity.

JEL Classifications: C72; C78; CD47; D71.

---

\*Turku School of Economics, University of Turku. E-mail: vipeko@utu.fi.

†Adam Smith Business School, University of Glasgow. E-mail: michele.lombardi@glasgow.ac.uk. Department of Economics and Statistics, University of Naples.

‡University of Helsinki and Helsinki Graduate School of Economics, Finland. E-mail: hannu.vartiainen@helsinki.fi.

# 1 Introduction

The challenge of implementation lies in designing a mechanism (i.e., game form) in which the equilibrium behavior of agents always coincides with the recommendation given by a *social choice rule* (SCR). If such a mechanism exists, the SCR is implementable.

Thus, the key question is how to design an implementing mechanism so that its outcomes can be predicted through the application of game theoretic solution concepts. Most early studies of implementation focused on noncooperative solution concepts, such as the Nash equilibrium and its refinements. As demonstrated in the seminal paper by Koray and Yildiz (2018), an alternative to the noncooperative approach is to allow groups of agents to coordinate their behaviors in a mutually beneficial manner. To move away from non-cooperative modeling, the details of coalition formation are left unspecified. Consequently, coalitions—not individuals—become the basic decision-making units. Here, the role of the solution concept is to explain why, when, and which coalition forms and what it can achieve.

More importantly, the chosen coalitional solution concept is independent of the physical structure under which coalition formation takes place (e.g., Chwe, 1994). This structure, often defined by an effectivity relationship, specifies which coalitions can form given a status quo outcome, and what they can achieve when they form (i.e., what new status quo outcomes they can induce). From an implementation viewpoint, the effectivity relationship is the design variable, playing the role of the mechanism.

Koray and Yildiz (2018) formalize this idea and study its implications. In their framework, the implementation of an SCR is achieved by designing a generalization of the effectivity relationship, introduced by Sertel (2001), called a *rights structure*.<sup>1</sup> A rights structure  $\Gamma$  consists of a state space  $S$ , an outcome function  $h$  that associates every state with an outcome, and a code of rights  $\gamma$ . A code of rights specifies, for each pair of states  $(s, t)$ , a collection of coalitions  $\gamma(s, t)$  that are effective at moving from  $s$  to  $t$ . The rights structure is more flexible than the effectivity function, as it allows the strategic options of coalitions to depend on how the status quo outcome is reached (i.e., on the current state).

As a coalitional solution, Koray and Yildiz (2018) adopt a version of the *core*.<sup>2</sup> State  $t$  *directly* dominates state  $s$  if a coalition  $K$  exists that is effective at moving from  $s$  to  $t$ , and each member of  $K$  receives a larger payoff under  $t$  than they receive under  $s$ . State  $s$  is a core state under a given rights structure and agents' preferences if no state that directly dominates it exists.

---

<sup>1</sup>McQuillin and Sugden (2011) propose a similar notion, named the *game in transition function form*, as a generalization of effectivity functions.

<sup>2</sup>Korpela et al. (2020) study the implementation in core by a rights structure  $\Gamma = (S, h, \gamma)$ , where  $S$  is the set of outcomes and  $h$  is the identity map.

This classical solution is based on a myopic notion of dominance, which creates inevitable problems. Ray and Vohra (2015) illustrate this point clearly in the following example using two agents and three states. Suppose that only agent 1 is effective in moving from  $s$  to  $t$ , i.e.,  $s \xrightarrow{\{1\}} t$ , and only agent 2 is effective in moving from  $t$  to  $s'$ , i.e.,  $t \xrightarrow{\{2\}} s'$ . Figure 1 depicts this example, where the payoffs to the agents in each of the states are in parentheses.

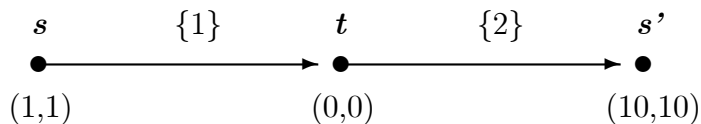


Figure 1.

The core consists of states  $s$  and  $s'$ . Although agent 1 has the power to move from  $s$  to  $t$ , agent 1 has no incentive to do so:  $t$  does not directly dominate  $s$ . However, the stability of  $s$  is based on myopic reasoning. If agent 1 was farsighted, the agent should move to  $t$  because agent 2 (who is rational) will in turn move to  $s'$ . Thus, farsighted agents do not necessarily move because they have a direct objection but because their moves can trigger further changes, eventually leading to a *better* outcome. Clearly, the classic notion of core does not incorporate any farsightedness.

To address this gap, two questions must be answered. Where does the objection process lead? Can we be sure that the end state of the process creates an effective deterrence for the deviating coalition? These questions, which form the scope of an expanding body of literature on farsighted coalition formation, do not have a clear answer in the current context.<sup>3</sup> However, as noted by Koray and Yildiz (2018), the notion of equilibrium by these authors' is shortsighted.

Harsanyi (1974), in his critique of the vNM stable set (von Neumann and Morgenstern, 1947), suggests replacing the notion of direct dominance with “indirect dominance”. In defining his *largest consistent set* (LCS), Chwe (1994) formalizes a version of Harsanyi’s indirect dominance. State  $t$  *indirectly* dominates  $s$  if  $t$  can replace  $s$  via a sequence of “moves” such that, at each move, the effective moving coalition prefers the outcome associated with  $t$  (the final state) to the outcome it would obtain if it decided not to move (for a formal definition, see Definition 3). Figure 1 shows that  $s'$  indirectly dominates  $s$ . This is so because agent 1 can move from  $s$  to  $t$ , and agent 1’s payoff at  $s'$  is larger than the payoff at  $t$ , and agent 2 can move from  $t$  to  $s'$ . Additionally, agent 2’s payoff at  $s'$  is larger than the payoff at  $t$ . Thus, indirect dominance captures the fact that farsighted agents consider the final states to which their moves may lead.

<sup>3</sup>See, for example, Chwe (1994), Vartiainen (2011), Vohra and Ray (2019), and Dutta and Vohra (2017).

Based on this notion of indirect dominance, Chwe (1994) suggests a new concept of stability, namely, the largest consistent set (LCS), which has the advantages of “ruling out with confidence” and being non-empty under weak conditions.<sup>4</sup> To check whether state  $s$  is stable, suppose that coalition  $K$  deviates to state  $t$ . Further deviations from  $t$  may occur, which end up at  $s'$ , where  $s'$  indirectly dominates  $t$ . Alternatively, no further deviations from  $t$  may occur, making  $t = s'$  the final state. In either case, the final state,  $s'$ , should itself be stable. If a member of the deviating coalition does not prefer  $s'$  to the original state,  $s$ , then the *deviation is deterred*. State  $s$  is *stable* if all deviations are deterred. Since whether a state is stable depends on whether other stable states exist, a set of stable states is called a *consistent set*. Although many consistent sets may exist, the LCS uniquely exists; that is, a consistent set that includes all others. If state  $s$  is not contained in the LCS, the interpretation is that  $s$  cannot be stable; there is no consistent story behind  $s$ .

For this reason, and given the lack of clear theoretical guidance as to which farsighted stability solution to follow, we adopt the LCS as a coalitional solution. The implementation problem consists of designing a rights structure,  $\Gamma$ , with the property that, for each profile of agents’ preferences, the outcome associated with the LCS always coincides with the recommendation of the given SCR. If such a rights structure exists, the SCR is LCS-implementable by a rights structure.

We investigate the LCS-implementation of SCRs in environments with transfers in which the initial state is exogenously given. There are important situations where the initial state is naturally determined.<sup>5</sup> For instance, the initial state is the no trade allocation in a house allocation problem, no production in a Cournot oligopoly market, or when agents can together produce one unit of output but no agent can independently produce any output, and so on. We also assume that agents’ preferences are continuous and money monotonic (following Morimoto and Serizawa, 2015).

Although (Maskin) monotonicity is generally not necessary for LCS-implementation via

---

<sup>4</sup>A growing body of literature studies farsighted stability in coalitional games, which includes Aumann and Myerson (1988), Xue (1998), Diamantoudi and Xue (2003), Herings et al. (2004), Jordan (2006), Ray (2007), Mauleon et al. (2011), Vartiainen (2011), Kimya (2015), Ray and Vohra (2015), Bloch and van den Nouweland (2020), Dutta and Vohra (2017), Dutta and Vartiainen (2019), and Vohra and Ray (2019).

<sup>5</sup>In case in which the initial state is not naturally determined, the problem of convergence from unstable states to stable states is particularly important. Indeed, Koray and Yildiz (2018) show that the convergence to core equilibria can be assured by imposing an external myopic stability property on the implementing rights structure: each equilibrium state can be reached from any nonequilibrium state via a sequence of states such that the passage from each state in the sequence to the next one is justified both rights- and preference-wise. In this paper, rather than imposing an external stability property, we study implementation problems in which the initial state is naturally determined and the task of the designer is in some sense easier because we ask him to devise a rights structure in which any stable state can be reached from the initial state.

rights structures, we show that it is sufficient. This result is obtained by designing a rights structure satisfying the following *convergence property*: Every stable state directly dominates the initial state. Therefore, we establish a direct convergence from the initial (unstable) state to the stable states, which is particularly important in our design framework. This result relies on the domain assumption that each agent considers the outcome corresponding to the initial state to be worse than any outcome in the range of the SCR. For example, in a house allocation problem, this would be satisfied by requiring traders to be (discernibly) strictly better off when they trade.

This is surprising because monotonicity, in the sense of Maskin (1999), is at the heart of the characterization results of Koray and Yildiz (2018). Korpela et al. (2020) show that monotonicity fully characterizes the class of implementable SCRs via a rights structure in a setting with transfers in which agents have money monotonic and continuous preferences.<sup>6</sup> Thus, the desire to design rights structures that are immune to farsighted behavior leaves the class of SCRs that are implementable in core equilibria unchanged.<sup>7</sup>

Finally, we also show that monotonicity fully characterizes the class of LCS-implementable social choice functions (SCFs)—an SCF is a single-valued SCR. This result is both interesting per se and also useful in providing a full characterization of various refinements and modifications to which the definition of LCS has led (for further discussion see subsection 3.2). Indeed, although there is no clear theoretical guidance as to which farsighted stability solution to follow, it is well accepted in the literature on coalition formation that any farsighted solution concept needs to pass the minimal test of the LCS (see, e.g., Dutta and Vartiainen, 2020).

The remainder of the paper is divided into four sections. Section 2 sets out the theoretical framework and outlines the basic model. Section 3 provides our characterization results. Section 4 concludes.

## 2 Preliminaries

We consider an *environment with transfers*, which consists of a collection of  $n$  agents (we write  $N$  for the set of agents), a set of possible types  $\Theta$ , and a (nonempty) set of outcomes  $Z \equiv D \times \mathbb{R}^n$ .  $D$  is the set of potential social decisions, with  $d \in D$  as a typical element.  $\mathbb{R}^n$  is the set of transfers to the agents, with  $t = (t_1, \dots, t_n) \in \mathbb{R}^n$  as a typical transfer profile. For notational simplicity, sometimes we write  $d$  for outcome  $(d, 0, \dots, 0) \in Z$ . For any agent  $i$ 's

---

<sup>6</sup>More precisely, Korpela et al (2020) show that monotonicity, when combined with unanimity, is necessary and sufficient for implementation by rights structures.

<sup>7</sup>The rights structure devised in Theorem 1 also implements  $F$  in core equilibria.

transfer  $t_i$ ,  $(0_{-i}, t_i)$  denotes a transfer profile that assigns  $t_i$  to agent  $i$  and zero to everyone else.

In this environment, there is an *initial outcome*  $\sigma \in Z$ , which is exogenously given. As discussed in the introduction, there are important economic situations in which the initial state is naturally determined, and it cannot be chosen by the designer. For instance, the initial state is the no trade allocation in a house allocation problem.

Agent  $i$ 's preferences are represented by a utility function:  $u_i : Z \times \Theta \rightarrow \mathbb{R}$ .  $u_i(x, \theta)$  is agent  $i$ 's utility at type  $\theta$  when the outcome is  $x$ . Given type  $\theta$  and outcome  $x$ , let agent  $i$ 's *lower contour set* of  $u_i(\cdot, \theta)$  at  $x$  be defined by  $L_i(x, \theta) \equiv \{y \in Z | u_i(x, \theta) \geq u_i(y, \theta)\}$ , and agent  $i$ 's *strict lower contour set* of  $u_i(\cdot, \theta)$  at  $x$  be defined by  $SL_i(x, \theta) = \{y \in Z | u_i(x, \theta) > u_i(y, \theta)\}$ .

Fix any  $i \in N$ , any  $\theta, \theta' \in \Theta$  and any  $x \in Z$ . We say that  $\theta'$  is a monotonic transformation of  $\theta$  at  $x$  for agent  $i$  if  $L_i(x, \theta) \subseteq L_i(x, \theta')$ . If  $\theta'$  is a monotonic transformation of  $\theta$  at  $x$  for each agent  $i \in N$ , we say that  $\theta'$  is a monotonic transformation of  $\theta$  at  $x$ .

For each agent  $i \in N$ , agent  $i$ 's utility function  $u_i : Z \times \Theta \rightarrow \mathbb{R}$  is assumed to satisfy the following properties.

**Definition 1** Agent  $i$ 's utility function  $u_i : Z \times \Theta \rightarrow \mathbb{R}$  is *money monotonic* provided that for each  $\theta \in \Theta$ , each  $d \in D$ , each  $t_{-i} \in \mathbb{R}^{n-1}$  and each  $t_i, t'_i \in \mathbb{R}$ , if  $t_i < t'_i$ , then  $u_i(d, (t_{-i}, t'_i), \theta) > u_i(d, (t_{-i}, t_i), \theta)$ .

**Definition 2** Agent  $i$ 's utility function  $u_i : Z \times \Theta \rightarrow \mathbb{R}$  is *continuous* provided that for each  $\theta \in \Theta$  and each  $x \in Z$ , the sets  $L_i(x, \theta)$  and  $U_i(x, \theta)$  are closed, where  $U_i(x, \theta) = \{y \in Z | u_i(y, \theta) \geq u_i(x, \theta)\}$ .

We focus on complete information environments in which the true type is common knowledge among agents but unknown to the designer. The power set of  $N$  is denoted by  $\mathcal{N}$ , and  $\mathcal{N}_0 \equiv \mathcal{N} - \{\emptyset\}$  is the set of all nonempty subsets of  $N$ . Each group of agents,  $K$  (in  $\mathcal{N}_0$ ), is a coalition.

The goal of the designer is to implement a *social choice rule* (SCR)  $F : \Theta \rightarrow Z$  defined by  $\emptyset \neq F(\theta) \subseteq Z$  for every  $\theta \in \Theta$ . We refer to  $x \in F(\theta)$  as the  $F$ -optimal outcome at  $\theta$ .  $F$  is said to be a *social choice function* (SCF) if  $F(\theta) \in Z$  for every  $\theta \in \Theta$ .

Throughout the paper, we make the following assumption.

**Assumption 1** The initial outcome  $\sigma \in Z$  is such that for all  $\theta \in \Theta$ ,  $u_i(x, \theta) > u_i(\sigma, \theta)$  for all  $i \in N$  and all  $x \in F(\theta)$ .

Assumption 1 is a requirement that  $\sigma$  is such that each agent considers it to be worse than any  $F$ -optimal outcome at  $\theta$ . For example, in a house allocation problem, this would be satisfied by requiring all agents to gain from trade (at each state).

To implement his goal, the designer devises rights structure  $\Gamma$ , which is a triplet,  $(S, h, \gamma)$ , where:

- $S$  is the state space;
- $h : S \rightarrow Z$  is the outcome function; and
- $\gamma$  is a code of rights, which is a (possibly empty) correspondence  $\gamma : S \times S \rightarrow \mathcal{N}$ .

Code of rights  $\gamma$  specifies, for each pair of states  $(s, t)$ , a family of coalitions  $\gamma(s, t)$  entitled to approve a change from state  $s$  to  $t$ .

To capture farsightedness, Chwe (1994) formalizes the following notion of “indirect dominance” relation—a notion informally introduced by Harsanyi (1974) in his criticism of the vNM stable set (von Neumann and Morgenstern, 1947), which is based on “direct dominance.” For all  $\theta \in \Theta$  and  $K \in \mathcal{N}_0$ , let  $x \succ_K^\theta y$  denote  $u_i(x, \theta) > u_i(y, \theta)$  for all  $i \in K$ .

**Definition 3** A state  $s$  is *indirectly dominated* by  $s'$  at  $(\Gamma, \theta)$ , or  $s' \gg^{(\Gamma, \theta)} s$ , if there exist  $s_0, s_1, \dots, s_J$  in  $S$  (where  $s_0 = s$  and  $s_J = s'$ ) and  $K_0, K_1, \dots, K_{J-1}$  in  $\mathcal{N}_0$  such that  $K_{j-1} \in \gamma(s_{j-1}, s_j)$  and  $h(s') \succ_{K_{j-1}}^\theta h(s_{j-1})$  for  $j = 1, \dots, J$ . A state  $s$  is *directly dominated* by  $s'$  at  $(\Gamma, \theta)$  if  $J = 1$ .

Based on this indirect dominance, the LCS of Chwe (1994) can be defined as follows.<sup>8</sup>

**Definition 4 (Chwe, 1994)** For any  $\Gamma$  and any  $\theta \in \Theta$ , a set  $T \subseteq S$  is a *consistent set* at  $(\Gamma, \theta)$  if  $s \in T$  if and only if for all  $t \in S$  and all  $K \in \mathcal{N}_0$  such that  $K \in \gamma(s, t)$ , there exists  $s' \in T$ , where  $s' = t$  or  $s' \gg^{(\Gamma, \theta)} t$ , such that not  $h(s') \succ_K^\theta h(s)$ . The *LCS* at  $(\Gamma, \theta)$ , denoted by  $LCS(\Gamma, \theta)$ , is the unique maximal consistent set at  $(\Gamma, \theta)$  with respect to set inclusion. We refer to  $s \in LCS(\Gamma, \theta)$  as a *stable state* (at  $(\Gamma, \theta)$ ).

The LCS is (*internally*) consistent in the sense that a deviation by coalition  $K$  from state  $s$  in the LCS to state  $t$  is deterred, if subsequent deviations by other coalitions from  $t$  could lead to a state  $s'$  in the LCS which indirectly dominates  $t$  and at which not all members of  $K$  are strictly better off with respect to  $s$ . Moreover, the LCS is (*externally*) consistent in the sense that a defection from a state outside the LCS cannot be deterred. When the set of states  $S$  is finite or countably infinite, the LCS is also externally stable in the sense that every state not in the LCS is indirectly dominated by another state in the LCS (Chwe, 1994;

---

<sup>8</sup>Given a game,  $(\Gamma, \theta)$ , where  $\Gamma$  is such that  $S$  is the set of outcomes and  $h$  is the identity map, Chwe shows that if  $S$  is countable and contains no infinite sequence  $s_1, s_2, \dots$  such that  $j > i$  implies that  $s_j \gg^{(\Gamma, \theta)} s_i$ , then  $LCS(\Gamma, \theta)$  is nonempty. This result has been extended by Xue (1997) by removing the countability requirement.



Proposition 2). In the context of stability, Chwe's interpretation of indirect dominance is that if  $s' \gg^{(\Gamma, \theta)} t$  and  $s'$  is presumed to be stable, then it is possible, not certain, that the coalitions  $K_0, \dots, K_{J-1}$  will move from  $t$  to  $s'$ .

Our notion of implementation can be stated as follows.

**Definition 5** A rights structure  $\Gamma$  *implements*  $F$  in the LCS, or simply *LCS-implements*  $F$ , if  $F(\theta) = h \circ LCS(\Gamma, \theta)$  for all  $\theta \in \Theta$ , where  $h \circ LCS(\Gamma, \theta) = \{h(s) \mid s \in LCS(\Gamma, \theta)\}$ . If such a  $\Gamma$  exists, then  $F$  is *LCS-implementable by a rights structure*.

### 3 Characterization results

A well-known condition in implementation theory is (Maskin) *monotonicity* (Maskin, 1999). This condition states that if  $x$  is an  $F$ -optimal outcome at  $\theta$ , and  $\theta'$  is a monotonic transformation of  $\theta$  at  $x$ , then  $x$  must be an  $F$ -optimal outcome at  $\theta'$ . Formally:

**Definition 6**  $F$  is *monotonic* provided that for all  $\theta, \theta' \in \Theta$ , if  $x \in F(\theta)$  and

$$L_i(x, \theta) \subseteq L_i(x, \theta') \text{ for all } i \in N,$$

then  $x \in F(\theta')$ .

**Remark 1** Observe that since the agents' preferences are continuous and money monotonic, we have

$$SL_i(x, \theta) \subseteq SL_i(x, \theta') \iff L_i(x, \theta) \subseteq L_i(x, \theta'),$$

for all  $\theta, \theta' \in \Theta$ , all  $x \in Z$  and all  $i \in N$ .

Monotonicity is not, in general, necessary for LCS-implementation by a rights structure. The reason is that monotonicity is a condition formulated for Nash implementation. In contrast to the definition of consistent set, which is a setwise definition, the definition of Nash equilibrium is based on a pointwise definition. To illustrate this, we construct a non-monotonic SCR that is LCS-implementable in a quasilinear environment. An environment  $\Theta$  is quasilinear when for each  $\theta \in \Theta$ , each  $i \in N$ , and each  $(d, t) \in Z$ , agent  $i$ 's utility

$$u_i((d, t), \theta) = v_i(d, \theta) + t_i$$

is linear in  $t_i$ .

**Example 1** Let  $\Theta = \{\theta, \theta'\}$  be a quasilinear environment. Suppose that  $n = 2$  and that  $D = \{v, w, x, y, z, \sigma\}$ . Agents' rankings of the outcomes in  $D$  are represented in the table below:

	$v_1(\cdot, \theta) = v_1(\cdot, \theta')$	$v_2(\cdot, \theta)$	$v_2(\cdot, \theta')$
$v$	2	1	3
$w$	0	3	2
$z$	4	4	4
$y$	3	2	1
$x$	1	-2	0
$\sigma$	-4	-4	-4

Suppose that  $u_i(d, \hat{\theta}) = v_i(d, \hat{\theta})$  for all  $d \in D$ , all  $\hat{\theta} \in \Theta$ , and all  $i \in N$ . Define  $F$  on  $\Theta$  by  $F(\theta) = \{v, w, x, z\}$  and  $F(\theta') = \{v, z\}$ . Note that Assumption 1 is satisfied because for all  $\hat{\theta} \in \Theta$ ,  $u_i(d, \hat{\theta}) > u_i(\sigma, \hat{\theta})$  for all  $i \in N$  and all  $d \in F(\hat{\theta})$ .

To see that  $F$  is not monotonic, assume, to the contrary, that it is monotonic. Since  $x \in F(\theta) \setminus F(\theta')$ , monotonicity implies that there exists agent  $i$  and outcome  $(d, t)$  such that

$$v_i(x, \theta) \geq v_i(d, \theta) + t_i$$

and

$$v_i(d, \theta') + t_i > v_i(x, \theta').$$

By construction, it must be that agent  $i$  coincides with agent 2. Combining these two inequalities and simplifying produces

$$v_2(x, \theta) - v_2(x, \theta') > v_2(d, \theta) - v_2(d, \theta').$$

However, by construction, it holds that

$$v_2(x, \theta) - v_2(x, \theta') \leq v_2(d', \theta) - v_2(d', \theta')$$

for all  $d' \in D$ , which is a contradiction. Thus,  $F$  is not monotonic.

However,  $F$  is LCS-implementable by a rights structure. To see this, let us consider the following rights structure  $\Gamma = (S, h, \gamma)$  where the state space is  $S = D$ , the outcome function  $h$  is the identity map, and the code of rights  $\gamma : S \times S \rightarrow \mathcal{N}$  is defined as follows:

$$\gamma(\sigma, x) = \gamma(x, y) = \{1\} \text{ and } \gamma(y, z) = \gamma(y, w) = \gamma(w, v) = \{2\},$$

and empty in all other cases. Figure 2 below gives a graphical illustration of the implementing rights structure.

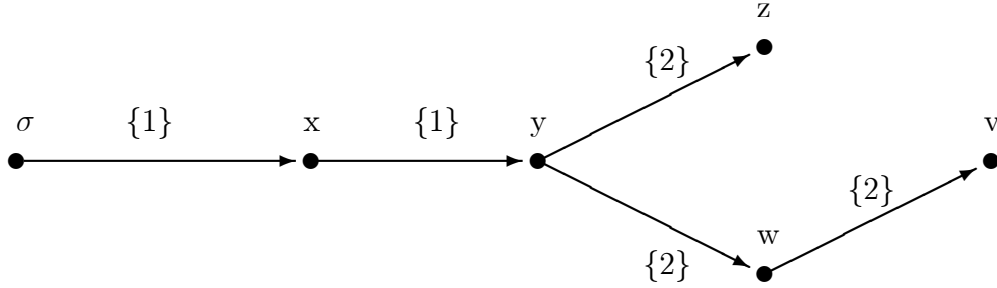


Figure 2.

We can now check that  $LCS(\Gamma, \theta) = F(\theta) = \{v, w, x, z\}$  and  $LCS(\Gamma, \theta') = F(\theta') = \{v, z\}$ . Note that states  $v$  and  $z$  are stable since no coalition can deviate from these states, regardless of type.

Let us first consider type  $\theta$ . State  $w$  is stable since agent 2 does not benefit from deviating to stable state  $v$ . State  $y$  is not stable since it is dominated both by stable state  $z$  and by stable state  $w$ — $y$  cannot be dominated by  $v$  because  $v$  does not dominate  $w$ . State  $x$  is stable since agent 1's deviation to  $y$  is deterred by the further deviation of agent 2 to stable state  $w$ .  $\sigma$  cannot be stable since it is dominated by  $x$ ,  $z$  and  $w$ —it cannot be dominated by  $v$  because  $v$  does not dominate  $w$ . We conclude that  $LCS(\Gamma, \theta) = \{v, w, x, z\}$ .

Next, let us consider type  $\theta'$ . In this case, given that every outcome in  $D \setminus \{v, z\}$  is indirectly dominated by both  $z$  and  $v$ , we conclude that  $LCS(\Gamma, \theta') = \{v, z\}$ .

### 3.1 Monotonicity is sufficient for LCS-implementation of SCRs

Though monotonicity is not, in general, a necessary condition, we show that it is sufficient for LCS-implementation of SCRs. This is an interesting result. The reason is that the implementation of SCRs in core equilibria by a rights structure can be made robust to farsighted reasoning. This is because monotonicity fully characterizes the class of SCRs that are implementable in core equilibria via rights structure in a setting with transfers in which agents have money monotonic and continuous preferences (Koray and Yildiz, 2018; Korpela et al., 2020) and in our implementing rights structure the set of core equilibria coincides with the LCS at each state. Thus, the desire to design rights structures that are immune to farsighted behavior leaves the class of SCRs that are implementable in core equilibria unchanged.

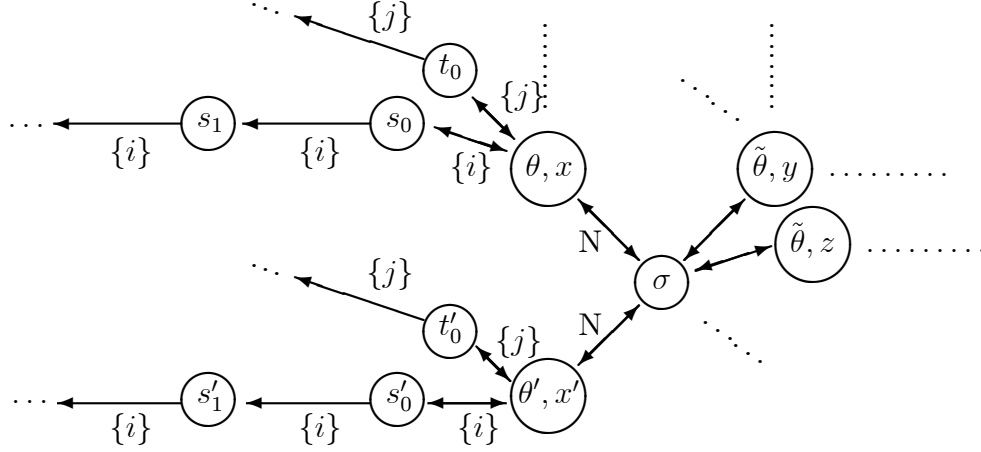


Figure 3. An illustration of the implementing rights structure.

Let us give an intuitive explanation of the implementing rights structure  $\Gamma$ , which is shown in Figure 3.

Recall that  $\sigma$  is the initial outcome, which is exogenously given. For this reason, we set  $\sigma$  as the origin of our "star graph." The outcome function  $h$  maps  $\sigma$  into itself. The code of rights,  $\gamma$ , simply allows only grand coalition  $N$  to move away from  $\sigma$  to pairs of type  $(\theta, x)$  with  $x \in F(\theta)$ , and from pairs of type  $(\theta, x)$  back to  $\sigma$ . Put differently, only the grand coalition can move from  $\sigma$  to a point of the *graph of  $F$* , i.e.,  $\{(\theta, x) \in \Theta \times Z | x \in F(\theta)\}$ , and from any point of this set back to  $\sigma$ . The outcome function  $h$  maps point  $(\theta, x)$  of the graph of  $F$  into outcome  $x$ .

An important property of the designed rights structure is that no coalition has the power to directly move from a point of the graph of  $F$  to another point of the graph. However, any two points of the graph of  $F$  are connected via state  $\sigma$ . For instance, suppose that state  $(\theta, x)$  is reached. State  $(\theta', x')$  can be reached indirectly by passing through  $\sigma$  because the rights structure  $\gamma$  allows grand coalition  $N$  to move from  $(\theta, x)$  to  $\sigma$  and from  $\sigma$  to  $(\theta', x')$ —see Figure 3.

Let us consider state  $(\theta, x)$ . Suppose that outcome  $(d, t)$  is in agent  $i$ 's strict lower contour set of  $u_i(\cdot, \theta)$  at  $x$ . Given that  $u_i(x, \theta) > u_i((d, t), \theta)$  and that  $u_i$  is continuous, there exists an arbitrarily small transfer  $\hat{t}_i$  such that

$$u_i(x, \theta) > u_i((d, t + (0_{-i}, \hat{t}_i)), \theta) > u_i((d, t), \theta).$$

These inequalities allow us to construct an infinite sequence of states  $s_0, s_1, \dots, s_k, \dots$  such

that the outcome function  $h$  maps each state  $s_k$  into  $(d, t + (0_{-i}, \frac{k}{k+1} \hat{t}_i))$ . This implies that agent  $i$ 's payoff at state  $s_k$  is  $u_i((d, t_i + \frac{k}{k+1} \hat{t}_i), \cdot)$ .<sup>9</sup> By money monotonicity, the sequence  $s_0, s_1, \dots, s_k, \dots$  is a strictly increasing sequence of payoffs for agent  $i$ , regardless of the true type. This is an important feature because it allows us to design code of rights  $\gamma$  in a way that no state of type  $s_k$  is stable.

Indeed, the devised code of rights  $\gamma$  allows only agent  $i$  to move from  $s_k$  to  $s_{k+1}$ , for  $k = 0, 1, 2, \dots$ , from  $(\theta, x)$  to  $s_0$  and from  $s_0$  back to  $(\theta, x)$ , and it does not allow any coalition to move from  $s_{k+1}$  to  $s_k$ , for  $k = 0, 1, 2, \dots$ —see Figure 3.

Since agent  $i$  has the incentive and power to move the state from  $s_k$  to  $s_{k+1}$ , for  $k = 0, 1, 2, \dots$ , it follows that no state of type  $s_k$ , for  $k = 0, 1, 2, \dots$ , is stable. That is, no state of the sequence  $s_0, s_1, \dots, s_k, \dots$  can be part of the LCS, irrespective of the true type. This construction is repeated for any outcome in agent  $i$ 's strict lower contour set of  $u_i(\cdot, \theta)$  at  $x$ , for any agent  $i$  and any state  $(\theta, x)$  in the graph of  $F$ .

Let us now briefly discuss why the rights structure illustrated in Figure 3 LCS-implements  $F$ .

Suppose that  $\theta$  is the true type. Our objective is to show that  $F(\theta) = h \circ LCS(\Gamma, \theta)$ .

The LCS of the game  $(\Gamma, \theta)$  is not empty because the set  $\{(\theta, x)\}$  is a consistent set of  $(\Gamma, \theta)$ . Indeed, if the grand coalition moves from  $(\theta, x)$  to  $\sigma$ , the grand coalition has both the power and incentive to go back to  $(\theta, x)$ , by Assumption 1. Moreover, if agent  $i$  moves from  $(\theta, x)$  to  $s_0$ , agent  $i$  has both the power and incentive to go back to state  $(\theta, x)$ . Since this reasoning holds for any state of type  $s_0$ , it follows that  $\{(\theta, x)\}$  is a consistent set of  $(\Gamma, \theta)$ . This shows that  $\{(\theta, x) \mid x \in F(\theta)\}$  is contained in  $LCS(\Gamma, \theta)$ , and so  $F(\theta) \subseteq h \circ LCS(\Gamma, \theta)$ .

For the converse, note that no state of type  $s_k$ , for  $k = 0, 1, 2, \dots$ , can belong to  $LCS(\Gamma, \theta)$ —as discussed in the preceding paragraphs, the reason for this is that agent  $i$  has both the incentive and power to follow the increasing sequence of payoffs generated by  $s_{k+1}, s_{k+2}, \dots$

Moreover, the initial state,  $\sigma$ , cannot belong to  $LCS(\Gamma, \theta)$  either. To see this, suppose that  $\sigma \in LCS(\Gamma, \theta)$ . Since the grand coalition has the power to move from  $\sigma$  to  $(\theta, x)$ , this move must be deterred, in the sense that the move must lead, possibly via indirect dominance, to state  $s \in LCS(\Gamma, \theta)$  such that either  $s = (\theta, x)$  or  $s$  indirectly dominates  $(\theta, x)$ , and at which not all members of  $N$  are strictly better off with respect to  $\sigma$ .

By Assumption 1,  $s$  is different from  $(\theta, x)$ , otherwise, everyone is strictly better off with respect to  $\sigma$ , which is a contradiction. Moreover, by Assumption 1,  $s$  is different from  $\sigma$ , otherwise,  $s$  cannot indirectly dominate  $(\theta, x)$ . It follows that  $s \notin \{(\theta, x), \sigma\}$ , and so it is a point of the graph of  $F$  with  $s \neq (\theta, x)$ .

By construction, as we have discussed above, states  $(\theta, x)$  and  $s$  are connected only via

---

<sup>9</sup>Note that for  $k = 0$ ,  $u_i(s_0, \cdot) = u_i((d, t_i), \cdot)$ .

$\sigma$ , and only the grand coalition has the power to move state from  $(\theta, x)$  to  $\sigma$ . Therefore, the fact that  $s$  indirectly dominates  $(\theta, x)$  implies that every agent needs to strictly prefer  $h(s)$  to  $h(\theta, x)$ . In addition, since everyone also strictly prefers  $h(\theta, x)$  to  $\sigma$ , by Assumption 1, it follows from transitivity of preferences that everyone strictly prefers  $h(s)$  to  $\sigma$ , which contradicts the fact that the move from  $\sigma$  to  $(\theta, x)$  should be deterred.

From the preceding discussion, we have that  $LCS(\Gamma, \theta)$  is contained in the graph of  $F$ , that is, in the set  $\{(\theta', x') \in \Theta \times Z \mid x' \in F(\theta')\}$ . To establish that  $h \circ LCS(\Gamma, \theta) \subseteq F(\theta)$ , we will be using the assumption that  $F$  is monotonic.

Take any  $(\theta', x') \in LCS(\Gamma, \theta)$ . Suppose that  $x' \notin F(\theta)$ . Monotonicity, in combination with Remark 1, implies that there exists an agent  $i$  that experiences a preference reversal of the form  $u_i(x', \theta') > u_i(y, \theta')$  and  $u_i(y, \theta) \geq u_i(x', \theta)$ . Since  $y$  is an element of agent  $i$ 's strict lower contour set of  $u_i(\cdot, \theta')$  at  $x'$ , it follows that there exists a state  $s'_0$  such that agent  $i$  has the power to move the state from  $(\theta', x')$  to  $s'_0$ , where  $h(s'_0) = y$ , and from  $s'_0$  back to  $(\theta', x')$ , by construction. Since  $(\theta', x') \in LCS(\Gamma, \theta)$ , the move from  $(\theta', x')$  to  $s'_0$  must be deterred, in the sense that a deviation of agent  $i$  to  $s'_0$  should lead, via indirect dominance, to another state  $t \in LCS(\Gamma, \theta)$  in which agent  $i$  does not strictly prefer  $h(t)$  to  $h(\theta', x')$ , according to his ranking at  $\theta$ .<sup>10</sup> However, by construction of the rights structure, the fact that  $t$  indirectly dominates  $s'_0$  means that the sequence of states and the sequence of coalitions leading to  $t$  are such that the grand coalition moves from  $(\theta', x')$  to  $\sigma$  and everyone strictly prefers  $h(t)$  to  $h(\theta', x')$ , which contradicts the fact that the move from  $(\theta', x')$  should be deterred.

The result can be stated as follows.

**Theorem 1** Let Assumption 1 hold. If  $F$  is *monotonic*, then  $F$  is *LCS-implementable by a rights structure*.

**Proof.** Suppose that  $F$  is monotonic. We now construct the implementing rights structure  $\Gamma = (S, h, \gamma)$ . Fix any  $\theta \in \Theta$ . State space  $S^\theta$  is

$$S^\theta = \{(\theta, x) \mid x \in F(\theta)\} \cup T^\theta,$$

where  $T^\theta$  is defined by

$$T^\theta = \{((d, t), x, \theta, i, k) \mid (d, t) \in SL_i(x, \theta) \text{ for } i \in N, k \in \mathbb{Z}_+ \text{ and } x \in F(\theta)\}, \quad (1)$$

where  $\mathbb{Z}_+$  denotes the set of non-negative integers. The outcome corresponding to  $(\theta, x)$  is  $x$ . To define the outcome corresponding to state  $((d, t), x, \theta, i, k)$ , we fix an arbitrarily small transfer  $\hat{t}_i$  such that

$$u_i(x, \theta) > u_i((d, t) + (0_{-i}, \hat{t}_i), \theta) > u_i((d, t), \theta). \quad (2)$$

---

<sup>10</sup>Note that  $t \neq s'_0$  since  $LCS(\Gamma, \theta)$  is contained in the graph of  $F$ .

This transfer exists because  $u_i$  is continuous. The outcome corresponding to  $((d, t), x, \theta, i, k)$  is  $h^\theta((d, t), \theta, i, k) = (d, t + (0_{-i}, \frac{k}{k+1}\hat{t}_i))$ , so that agent  $i$ 's outcome is  $(d, t_i + \frac{k}{k+1}\hat{t}_i)$ . This definition is important because it rules out state  $((d, t), x, \theta, i, k)$  as a stable state, irrespective of the true type. To see this, let us first define code of rights  $\gamma^\theta$  as follows:

- (1) For all  $(y, x, \theta, i, 0), (\theta, x) \in S^\theta$ ,  $\gamma^\theta((\theta, x), (y, x, \theta, i, 0)) = \gamma^\theta((y, x, \theta, i, 0), (\theta, x)) = \{i\}$ .
- (2) For all  $(y, x, \theta, i, k), (y, x, \theta, i, k+1) \in T^\theta$ ,  $\gamma^\theta((y, x, \theta, i, k), (y, x, \theta, i, k+1)) = \{i\}$ .
- (3) Otherwise, it is empty.

Let  $x \in F(\theta)$  and  $(d, t) \in SL_i(x, \theta)$ . We allow only agent  $i$  to be effective in moving from  $(\theta, x)$  to  $((d, t), x, \theta, i, 0)$ , from  $((d, t), x, \theta, i, 0)$  back to  $(\theta, x)$ , and from  $((d, t), x, \theta, i, k)$  to  $((d, t), x, \theta, i, k+1)$ . In all other cases, no coalition is effective. To see that no state of the form  $((d, t), x, \theta, i, k)$  can be a stable state, it suffices to observe that the money monotonicity of agent  $i$ 's utility function assures that

$$u_i\left(d, t_i + \frac{k+1}{k+2}\hat{t}_i, \theta'\right) > u_i\left(d, t_i + \frac{k}{k+1}\hat{t}_i, \theta'\right)$$

for every non-negative integer  $k \geq 0$  and every type  $\theta' \in \Theta$ , so that agent  $i$  always has the power as well as incentive to move from  $((d, t), x, \theta, i, k)$  to  $((d, t), x, \theta, i, k+1)$ .

Let us define rights structure  $\Gamma = (S, h, \gamma)$  as follows. We define state space  $S$  by

$$S = \cup_{\theta \in \Theta} S^\theta \cup \{\sigma\}.$$

We define outcome function  $h : S \rightarrow Z$  by  $h(s) = h^\theta(s)$  for all  $s \in S^\theta$  and all  $\theta \in \Theta$ , and  $h(\sigma) = \sigma$ . Define the code of rights  $\gamma : S \times S \rightarrow \mathcal{N}$  as follows. For all  $s, s' \in S$ ,

- (A) If  $s, s' \in S^\theta$  for some  $\theta \in \Theta$ , then  $\gamma(s, s') = \gamma^\theta(s, s')$ .
- (B) For all  $\theta \in \Theta$ , if  $s = \sigma$  and  $s' = \theta$ , then  $\gamma(\sigma, \theta) = \gamma(\theta, \sigma) = N$ .
- (C) Otherwise,  $\gamma(s, s')$  is empty.

Let us show that  $\Gamma$  LCS-implements  $F$ . Suppose that  $\theta$  is the true type.

Let us first show that  $\{(\theta, x) | x \in F(\theta)\} \subseteq LCS(\Gamma, \theta)$ . Fix any  $(\theta, x)$ . Let us show that it is a consistent set. By definition of  $\gamma$ , there are only two possible ways to move away from  $(\theta, x)$ .

First, suppose that  $N$  moves to  $\sigma$ . Since  $N \in \gamma(\sigma, (\theta, x))$ , by the definition of  $\gamma$ , and since  $u_i(x, \theta) > u_i(\sigma, \theta)$  for all  $i \in N$ , by Assumption 1, it follows that coalition  $N$  has the incentive as well as power to go back to  $(\theta, x)$ .

Second, suppose that agent  $i$  moves from  $(\theta, x)$  to  $(y, x, \theta, i, 0)$ . Since  $y \in SL_i(x, \theta)$ , by construction of  $T^\theta$ , agent  $i$  has the incentive as well as power to go back to  $(\theta, x)$ —note that  $\{i\} = \gamma((y, x, \theta, i, 0), \theta)$  by construction.

Since the choice of  $(y, x, \theta, i, 0)$  is arbitrary, it follows that  $\{(\theta, x)\}$  is a consistent set of  $(\Gamma, \theta)$ . Since the choice of  $x \in F(\theta)$  is arbitrary, it follows that  $\{(\theta, x) | x \in F(\theta)\} \subseteq LCS(\Gamma, \theta)$ .

Next, let us show that  $h \circ LCS(\Gamma, \theta) \subseteq F(\theta)$ . We already know that  $\{(\theta, x) | x \in F(\theta)\} \subseteq LCS(\Gamma, \theta)$ . Moreover, for the reasoning explained above, we also know that no state  $t \in \cup_{\bar{\theta} \in \Theta} T^{\bar{\theta}}$  can be a stable state at  $(\Gamma, \theta)$ . Consequently, it follows that

$$LCS(\Gamma, \theta) \subseteq \{(\bar{\theta}, \bar{x}) | \bar{\theta} \in \Theta \text{ and } \bar{x} \in F(\bar{\theta})\} \cup \{\sigma\}.$$

Let us show that  $\sigma \notin LCS(\Gamma, \theta)$ . Assume, on the contrary, that  $\sigma \in LCS(\Gamma, \theta)$ . Take any  $(\theta, x)$ . Since  $N \in \gamma(\sigma, (\theta, x))$ , by construction, it follows from the definition of the LCS that there exists  $s \in LCS(\Gamma, \theta)$ , where  $s = (\theta, x)$  or  $s \gg^{(\Gamma, \theta)} (\theta, x)$ , such that not  $h(s) u_N^\theta h(\sigma)$ .

An immediate contradiction of Assumption 1 is obtained if  $s = (\theta, x)$ . Thus, let us consider the case  $s \gg^{(\Gamma, \theta)} (\theta, x)$ . Again, an immediate contradiction of Assumption 1 is obtained if  $s = \sigma$ . Thus, let  $s \in \{(\bar{\theta}, \bar{x}) | \bar{\theta} \in \Theta \text{ and } \bar{x} \in F(\bar{\theta})\}$  be such that  $s \neq (\theta, x)$ . By the definition of  $\gamma$  and the fact that  $s \gg^{(\Gamma, \theta)} (\theta, x)$ , it holds that the sequence of states of  $s \gg^{(\Gamma, \theta)} (\theta, x)$  is such that  $s_{j-1} = (\theta, x)$  and  $s_j = \sigma$  for some  $j = 1, \dots, J$ , that  $h(s) u_N^\theta x$ , and that  $N \in \gamma((\theta, x), \sigma)$ . Since  $x u_N^\theta h(\sigma)$ , by Assumption 1, and since  $h(s) u_N^\theta x$ , it follows by transitivity of preferences that  $h(s) u_N^\theta h(\sigma)$ , which is a contradiction. Thus,  $\sigma \notin LCS(\Gamma, \theta)$ , and so  $LCS(\Gamma, \theta) \subseteq \{(\bar{\theta}, \bar{x}) | \bar{\theta} \in \Theta \text{ and } \bar{x} \in F(\bar{\theta})\}$ .

Finally, take any  $(\theta', x) \in LCS(\Gamma, \theta)$ . Then, by definition,  $x \in F(\theta')$ . Let us show that  $x \in F(\theta)$ . Assume, to the contrary, that  $x \notin F(\theta)$ . Since  $F$  is monotonic, it follows that there exist  $i \in N$  and  $y \in SL_i(x, \theta')$  such that  $u_i(y, \theta) \geq u_i(x, \theta)$  by following Remark 1. Then,  $(y, x, \theta', i, 0) \in T^{\theta'}$ , by definition of  $T^{\theta'}$  given in (1). Note that  $\{i\} = \gamma((\theta', x), (y, x, \theta', i, 0))$ , by definition of  $\gamma$ . Since  $(\theta', x) \in LCS(\Gamma, \theta)$ , there exists  $s \in LCS(\Gamma, \theta) \subseteq \{(\bar{\theta}, \bar{x}) | \bar{\theta} \in \Theta \text{ and } \bar{x} \in F(\bar{\theta})\}$ , where  $s \gg^{(\Gamma, \theta)} (y, x, \theta', i, 0)$ , such that not  $h(s) u_i^\theta x$ . Note that  $s \neq (\theta', x)$  since only agent  $i$  can move away from  $(y, x, \theta', i, 0)$  and  $u_i(y, \theta) \geq u_i(x, \theta)$ . By the definition of indirect dominance, it follows that there exist  $s_0, s_1, \dots, s_J$  in  $S$  (where  $s_0 = (y, x, \theta', i, 0)$  and  $s_J = s$ ) and  $K_0, K_1, \dots, K_{J-1}$  in  $\mathcal{N}_0$  such that  $K_{j-1} \in \gamma(s_{j-1}, s_j)$  and  $h(s) u_{K_{j-1}}^\theta h(s_{j-1})$  for  $j = 1, \dots, J$ . By the definition of  $\gamma$ , it follows that for some  $j = 1, \dots, J$ ,  $K_{j-1} = N$ ,  $s_{j-1} = (\theta', x)$  and  $s_j = \sigma$ . This means that  $h(s) u_N^\theta x$ , which contradicts the fact that not  $h(s) u_i^\theta x$ . We conclude that  $x \in F(\theta)$ , and so



$h \circ LCS(\Gamma, \theta) \subseteq F(\theta)$ . ■

We are left to show that the rights structure designed in the proof of Theorem 1 also implements in core equilibria. To this end, we need some additional notation. For any rights structure  $\Gamma$  and any  $\theta \in \Theta$ , a state  $s \in S$  is a *core equilibrium* at  $\theta$  if, for no  $t \in S$ , so that  $h(s) \neq h(t)$  and no  $K \in \gamma(s, t)$  is  $h(t) u_K^\theta h(s)$ . We express  $C(\Gamma, R)$  for the set of core equilibria at  $\theta$ .

**Definition 7** A rights structure  $\Gamma$  implements  $F$  in core-equilibria, or simply *core-implements*  $F$ , if and only if  $F(\theta) = h \circ C(\Gamma, \theta)$  for all  $\theta \in \Theta$ . If such a rights structure exists,  $F$  is core-implementable by a rights structure.

**Theorem 2** Let Assumption 1 hold. If  $F$  is *monotonic*, then  $F$  is *LCS-implementable* and *core-implementable* by the same rights structure.

**Proof.** Let the premises hold. Let us consider the rights structure  $\Gamma$  designed in the proof of Theorem 1. By this theorem, we already know that  $\Gamma$  LCS-implements  $F$ . Let us show that  $F(\theta) = h \circ C(\Gamma, \theta)$  for all  $\theta \in \Theta$ . Suppose that  $\theta$  is the true type.

Taking any  $x \in F(\theta)$ , we show that  $x \in h \circ C(\Gamma, \theta)$ . Since  $x \in F(\theta)$ , we have that  $(\theta, x) \in S^\theta$ . By construction,  $N$  has the power to move the state from  $(\theta, x)$  to  $\sigma$  but no agent has incentive to do so, by Assumption 1. Fix any  $i \in N$  and any  $((d, t), x, \theta, i, 0) \in T^\theta$ . By definition,  $(d, t) \in SL_i(x, \theta)$ , and so agent  $i$  does not have any incentive to move from  $(\theta, x)$  to  $((d, t), x, \theta, i, 0)$ . Since these are the only two ways that agents can move away from  $(\theta, x)$ , by construction, it follows that  $(\theta, x) \in C(\Gamma, \theta)$ .

Next, let us show  $h \circ C(\Gamma, \theta) \subseteq F(\theta)$ . Observe that for the same reasoning given in the proof of Theorem 1, no state  $t \in \cup_{\bar{\theta} \in \Theta} T^{\bar{\theta}}$  can be a core equilibrium at  $(\Gamma, \theta)$ . Consequently, it follows that

$$C(\Gamma, \theta) \subseteq \{(\bar{\theta}, \bar{x}) \mid \bar{\theta} \in \Theta \text{ and } \bar{x} \in F(\bar{\theta})\} \cup \{\sigma\}.$$

Let us show that  $\sigma \notin C(\Gamma, \theta)$ . Assume, on the contrary, that  $\sigma \in C(\Gamma, \theta)$ . Take any  $(\theta, x)$ . Since  $N \in \gamma(\sigma, (\theta, x))$ , by construction, and since Assumption 1 holds, it follows that  $\sigma \notin C(\Gamma, \theta)$ , which is a contradiction. Thus,  $C(\Gamma, \theta) \subseteq \{(\bar{\theta}, \bar{x}) \mid \bar{\theta} \in \Theta \text{ and } \bar{x} \in F(\bar{\theta})\}$ .

Take any  $(\theta', x) \in C(\Gamma, \theta)$ , so that  $x \in F(\theta')$ . Assume, to the contrary, that  $x \notin F(\theta)$ . Monotonicity implies that there exist  $i$  and  $y \in L_i(x, \theta')$  such that  $u_i(y, \theta) > u_i(x, \theta)$ . Since  $u_i$  is continuous and money monotone it follows that there exists  $y' \in SL_i(x, \theta')$  such that  $u_i(y', \theta) > u_i(x, \theta)$ . Since by construction of  $\gamma$  it holds that  $i \in \gamma((x, \theta'), y')$ , and since  $u_i(y', \theta) > u_i(x, \theta)$ , it follows that  $(x, \theta') \notin C(\Gamma, \theta)$ , which is a contradiction. ■

### 3.2 Monotonicity fully characterizes the class of LCS-implementable SCFs

Our next result is that the class of LCS-implementable SCFs by a rights structure coincides with the class of monotonic SCFs. This result has two main implications in the context of our analysis.

First, comparing this result with the class of SCFs that are implementable in core equilibria by a rights structure, we obtain that, in our environment, the class of LCS-implementable SCFs coincides with the class of SCFs that are implementable in core equilibria. As already mentioned, this is important because the desire to design rights structures that are robust to farsighted behavior leaves the class of SCFs that can be implemented in core equilibria unchanged.

Second, the LCS has led to various refinements and modifications, which are applied in a variety of settings. For instance, the vNM FSS is derived from the classical vNM solution by replacing direct dominance with indirect dominance—see Definition 3. Chwe (1994) introduced the vNM FSS and showed that it is a refinement of the LCS. Refinements and modifications of the LCS and vNM FSS can be found in various studies on coalition formation, such as Mauleon and Vannetelbosch (2004), Nagarajan and Sošić (2007), Dutta and Vohra (2017), and Dutta and Vartiainen (2020), and on network stability, such as Page et al. (2005) and Herings et al. (2009). Although Konishi and Ray’s (2003) approach to farsightedness is different from the reasoning leading to the LCS, one of the features of their equilibrium (dynamic) process of coalition formation (EPCF) is that the set of all absorbing states under all deterministic absorbing EPCFs is a proper refinement of the LCS when the discount factor is large enough. Sošić (2006) and Nagarajan and Sošić (2007) have used this result in their analyses of various models of operations management. Since monotonicity fully characterizes the class of LCS-implementable SCFs in our setup with transfers, monotonicity is necessary and sufficient for the implementation by rights structures of every refinement of the LCS provided that the refinement is not empty whenever the LCS is not empty.

**Theorem 3** Let Assumption 1 hold. An SCF  $F$  is *monotonic* if and only if  $F$  is *LCS-implementable by a rights structure*.

**Proof.** Let the premises hold. The proof of the "only if" part follows from the proof of Theorem 1. To complete the proof, we need to show that monotonicity is necessary for LCS-implementation. To this end, suppose that  $\Gamma$  LCS-implements  $F$ . Fix any  $\theta, \theta' \in \Theta$ . Suppose that  $L_i(F(\theta), \theta) \subseteq L_i(F(\theta'), \theta')$  for all  $i \in N$ . We show that  $F(\theta) = F(\theta')$ .

By Remark 1, we have that  $SL_i(F(\theta), \theta) \subseteq SL_i(F(\theta'), \theta')$  for all  $i \in N$ . By the LCS-implementation of  $F$ , we have that  $h \circ LCS(\Gamma, \theta) = F(\theta)$ . Therefore, there exists  $s \in$

$LCS(\Gamma, \theta)$  such that  $h(s) = F(\theta)$ . Let us first show that the set  $LCS(\Gamma, \theta)$  is a consistent set at  $(\Gamma, \theta')$ .

Fix any  $t \in S$  and any  $s \in LCS(\Gamma, \theta)$  such that  $h(s) = F(\theta)$ . Nothing must be proved if  $\gamma(s, t) = \emptyset$ . Then, suppose that  $K \in \gamma(s, t)$  for some  $K \in \mathcal{N}_0$ . Since  $LCS(\Gamma, \theta)$  is a consistent set at  $(\Gamma, \theta)$ , it follows that there exists  $s' \in LCS(\Gamma, \theta)$  such that either  $s' = t$  or  $s' \gg^{(\Gamma, \theta)} t$ , and not  $h(s') u_K^\theta h(s)$ . Note that  $h(s') = h(s) = F(\theta)$ . Then, we are done if  $s' = t$ . Suppose that  $s' \neq t$ . Thus,  $s' \gg^{(\Gamma, \theta)} t$ . Since  $SL_i(h(s'), \theta) \subseteq SL_i(h(s'), \theta')$  for all  $i \in N$ , it follows that  $s' \gg^{(\Gamma, \theta')} t$ . Since  $t \in S$ ,  $K \in \mathcal{N}_0$  and  $s \in LCS(\Gamma, \theta)$  have been chosen arbitrarily, we have proved that  $LCS(\Gamma, \theta)$  is a consistent set at  $(\Gamma, \theta')$ . Since  $LCS(\Gamma, \theta')$  is the LCS at  $(\Gamma, \theta')$  with respect to set inclusion and  $LCS(\Gamma, \theta)$  is a consistent set at  $(\Gamma, \theta')$ , it follows that  $LCS(\Gamma, \theta) \subseteq LCS(\Gamma, \theta')$ . Therefore,  $F(\theta) = F(\theta')$  by the LCS-implementability of  $F$ . Thus,  $F$  is monotonic. ■

## 4 A necessary condition for SCRs

In this section, we introduce a new condition, called *set-quasimonotonicity*, which we show to be necessary for the implementation of SCRs. Formally:

**Definition 8**  $F$  is *set-quasimonotonic* provided that for all  $\theta, \theta' \in \Theta$ , if

$$SL_i(x, \theta) \subseteq SL_i(x, \theta') \text{ and } L_i(x, \theta) \cap F(\theta) \subseteq L_i(x, \theta')$$

for all  $x \in F(\theta)$  and all  $i \in N$ , then  $F(\theta) \subseteq F(\theta')$ .

Set-quasimonotonicity coincides with monotonicity when  $F$  is an SCF. It requires that if for each agent  $i$ , his or her strict lower contour set at each optimal outcome at  $\theta$  does not shrink when the state changes from  $\theta$  to  $\theta'$ , as well as his or her set  $L_i(x, \theta) \cap F(\theta)$  at each optimal outcome  $x \in F(\theta)$  does not shrink, then the set  $F(\theta')$  of optimal outcomes at  $\theta'$  is a superset of the set  $F(\theta)$  of optimal outcomes at  $\theta$ . This condition is stronger than the weak set monotonicity condition of Mezzetti and Renou (2012), which is a necessary and almost sufficient condition for implementation in mixed Nash equilibria. It is stronger because weak set monotonicity requires that  $F(\theta) \subseteq F(\theta')$  whenever for all  $x \in F(\theta)$ , the following two conditions are satisfied for every agent  $i \in N$ : (i)  $SL_i(x, \theta) \subseteq SL_i(x, \theta')$  and (ii)  $L_i(x, \theta) \subseteq L_i(x, \theta')$ .

**Theorem 4** If  $F$  is *LCS-implementable by a rights structure*, then it is *set-quasimonotonic*.

**Proof.** Suppose that  $\Gamma$  LCS-implements  $F$ . Take any  $\theta, \theta' \in \Theta$ . Suppose that  $SL_i(x, \theta) \subseteq SL_i(x, \theta')$  and  $L_i(x, \theta) \cap F(\theta) \subseteq L_i(x, \theta')$  for all  $x \in h \circ LCS(\Gamma, \theta)$  and all  $i \in N$ . The

statement follows if we show that  $LCS(\Gamma, \theta) \subseteq LCS(\Gamma, \theta')$ . To this end, it suffices to show that the set  $LCS(\Gamma, \theta)$  is a consistent set of  $(\Gamma, \theta')$ .

Take any  $s \in LCS(\Gamma, \theta)$ . Fix any  $t \in S$ . Nothing has to be proved if  $\gamma(s, t) = \emptyset$ . Then, suppose that  $K \in \gamma(s, t)$  for some  $K \in \mathcal{N}_0$ . Since  $s \in LCS(\Gamma, \theta)$ , there exists  $s' \in LCS(\Gamma, \theta)$ , where  $t = s'$  or  $s' \gg^{(\Gamma, \theta)} t$ , such that not  $h(s') u_K^\theta h(s)$ .

Since not  $h(s') u_K^\theta h(s)$ , it follows that  $u_i(h(s), \theta) \geq u_i(h(s'), \theta)$  for some  $i \in K$ . Since  $s' \in LCS(\Gamma, \theta)$ , it follows that  $h(s') \in L_i(h(s), \theta) \cap h \circ LCS(\Gamma, \theta)$ . Since this intersection is contained in  $L_i(h(s), \theta')$ , by our initial supposition, we have that not  $h(s') u_K^{\theta'} h(s)$ . We proceed according to whether  $s' = t$  or not.

Suppose that  $s' = t$ . Since not  $h(s') u_K^{\theta'} h(s)$ , we have that there exists  $s' \in LCS(\Gamma, \theta)$  such that not  $h(s') u_K^{\theta'} h(s)$ .

Suppose that  $s' \neq t$ , and so  $s' \gg^{(\Gamma, \theta)} t$ . Since  $SL_i(h(s'), \theta) \subseteq SL_i(h(s'), \theta')$  for all  $i \in N$ , it follows that  $s' \gg^{(\Gamma, \theta')} t$ . Thus, we established that there exists  $s' \in LCS(\Gamma, \theta)$ , where  $s' \gg^{(\Gamma, \theta')} t$ , such that not  $h(s') u_K^{\theta'} h(s)$ .

Since  $s \in LCS(\Gamma, \theta)$ ,  $t \in S$  and  $K \in \mathcal{N}_0$  have been chosen arbitrarily, one can see that  $LCS(\Gamma, \theta)$  is a consistent set at  $(\Gamma, \theta')$ . Thus,  $F$  is set-quasimonotonic. ■

**Remark 2** Set-quasimonotonicity is a necessary condition for LCS-implementation in any environment.

## 5 Conclusions

This paper extends the analysis of implementation through rights structures to farsighted agents. We adopt Chwe's (1994) LCS as the cooperative solution concept used to predict the outcome of a rights structure. The study analyzes the implementation problems in a setting with transfers in which there is a given, exogenous, initial state, and in which each agent has a money monotonic and continuous preference. We show that any SCR is LCS-implementable through rights structures if it is monotonic in the sense of Maskin (1999). Monotonicity fully characterizes the class of LCS-implementable SCFs.

As a coalitional solution, Koray and Yildiz (2018) adopt a version of the core, which is based on myopic reasoning: State  $s$  is an equilibrium state under a given rights structure and agents' preferences if no effective coalition can guarantee each of its members a utility level higher than the one they received under  $s$ . At the heart of their characterization result is (Maskin) monotonicity. Korpela et al. (2020) show that monotonicity fully characterizes the class of implementable SCRs in core equilibria via a rights structure in a setting with transfers in which each agent has a money monotonic and continuous preference. Our results imply that the class of SCRs that are implementable in core equilibria can be made immune

to farsighted reasoning. We conjecture that this insight extends to other farsighted stability solutions such as the farsighted stable set (Ray and Vohra, 2015).

Finally, let us remark that the full characterization of the class of SCRs that are LCS-implementable by a rights structure is an important and difficult topic that is left for future research. To this end, we have presented a necessary condition for LCS-implementation, called set-quasimonotonicity, which is similar to the set-monotonicity condition of Mezzetti and Renou (2012), which is a necessary and almost sufficient condition for implementation in mixed Nash equilibria.

## References

- [1] Aumann R, Myerson R. Endogenous Formation of Links Between Players and of Coalitions, An Application of the Shapley Value, in *The Shapley Value: Essays in Honor of Lloyd Shapley*, Alvin Roth, ed., 175–191. Cambridge: Cambridge University Press, 1988.
- [2] Bloch F, van den Nouweland A. Farsighted stability with heterogeneous expectations. *Games Econ Behav* (2020), forthcoming.
- [3] Chwe M S-Y. Farsighted coalitional stability. *J Econ Theory* 63 (1994) 299–325.
- [4] Diamantoudi E, Xue L. Farsighted Stability in Hedonic Games. *Soc Choice Welf* 21 (2003) 39–61.
- [5] Dutta B, Vohra R. Rational Expectations and Farsighted Stability. *Theoretical Econ* 12 (2017) 1191–1227.
- [6] Dutta B, Vartiainen H. Coalition Formation and History Dependence. *Theoretical Econ* 15 (2020) 159–197.
- [7] Granot D, Sošić G. Formation of Alliances in Internet Based Supply Exchanges. *Manag Sci* 51 (2005) 92–105.
- [8] Harsanyi, J. An equilibrium-point interpretation of stable sets and a proposed alternative definition. *Manag Sci* 20 (1974) 1472–1495.
- [9] Herings P J-J, Mauleon A, Vannetelbosch V. Rationalizability for social environments. *Games Econ Behav* 49 (2004) 135–156.
- [10] Herings P J-J, Mauleon A, Vannetelbosch V. Farsightedly stable networks. *Games Econ Behav* 67(2009) 526–541.

- [11] Jordan JS. Pillage and Property. *J Econ Theory* 131 (2006) 26–44.
- [12] Kimya M. Equilibrium Coalitional Behavior. Mimeo, Brown University (2015).
- [13] Konishi H, Ray D. Coalition formation as a dynamic process. *J Econ Theory* 110 (2003) 1–41.
- [14] Koray S, Yildiz K. Implementation via rights structures. *J Econ Theory* 176 (2018) 479–502.
- [15] Korpela V, Lombardi M, Vartiainen H. Do Coalitions Matter in Designing Institutions? *J Econ Theory* 185 (2020).
- [16] Mauleon A, Vannetelbosch V. Farsightedness and Cautiousness in Coalition Formation Games with Positive Spillovers. *Theory and Decision* 56 (2004) 291–324.
- [17] Mauleon A, Vannetelbosch V, Vergote W. von Neumann-Morgenstern farsighted stable sets in two-sided matching. *Theoretical Econ* 6 (2011) 499–521.
- [18] Maskin E. Nash equilibrium and welfare optimality. *Rev Econ Studies* 66 (1999) 23–38.
- [19] Mcquillin B, Sugden R. The representation of alienable and inalienable rights: games in transition function form. *Soc Choice Welf* 37 (2011) 683–706.
- [20] Mezzetti C, Renou L. Implementation in mixed Nash equilibrium. *J Econ Theory* 147 (2012) 2357–2375.
- [21] Morimoto S, Serizawa S. Strategy-proofness and efficiency with non-quasi-linear preferences: A characterization of minimum price Walrasian rule. *Theoretical Econ* 10 (2015) 445–487.
- [22] Nagarajan M, Sošić G. Stable Farsighted Coalitions in Competitive Markets. *Manag Sci* 53 (2007) 29–45.
- [23] Nagarajan M, Sošić G. Coalition Stability in Assembly Models. *Operations Research*, 57 (2009) 131–145.
- [24] von Neumann J, Morgenstern O. *Theory of games and economic behavior*. Princeton Univ. Press, Princeton, NJ, 1947.
- [25] Page FH, Wooders MH, Kamat S. Networks and Farsighted Stability. *J Econ Theory* 120 (2005) 257–269.

- [26] Ray D. *A Game-Theoretic Perspective on Coalition Formation*. Oxford University Press, 2007.
- [27] Ray D, Vohra R. The Farsighted Stable Set. *Econometrica* 83 (2015) 977–1011.
- [28] Sertel MR. Designing rights: Invisible hand theorems, covering and membership, mimeo, 2001, Bogazici University.
- [29] Sošić G. Transshipment of Inventories Among Retailers: Myopic vs. Farsighted Stability. *Manag Sci* 52 (2006) 1493–1508.
- [30] Vartiainen H. Dynamic coalitional equilibrium. *J Econ Theory* 146 (2011) 672–698.
- [31] Vohra R, Ray D. Maximality in the Farsighted Stable Set. *Econometrica* (2019), forthcoming.
- [32] Xue L. Nonemptiness of the Largest Consistent Set. *J Econ Theory* 73 (1997) 453–459.
- [33] Xue L. Coalitional Stability under Perfect Foresight. *Econ Theory* 11 (1998) 603–627.