Status Signalling with Luxury and Cultural Goods

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As the world shifts towards greater consumerism, there is an increasing tendency for individuals to distinguish themselves from the others through the goods that they purchase – which assert the society of their taste and implicit social status. While the pursuit of social status has been well-explored, most studies in this area tend to focus on luxury consumption, limited research investigated the inclusion of cultural goods (books, etiquette classes, etc.) as another signaling tool. The main objective of the paper, therefore, seeks to understand and provide a theoretical grounding for individuals’ choices between the status goods, and its implications for policy formulations and social mobility.

The choice to signal status by purchasing either one of the goods is evaluated under two scenarios: when wage-enhancing benefits of cultural goods are not revealed and when they are. Individuals are found to always prefer luxury goods in the first instance, but upon satisfying certain conditions, there is possibility of those endowed with high cultural and social capital to consume cultural goods instead of luxury goods under the second scenario. Given equilibrium choices, the paper proposes policy measures for welfare-maximizing social planner to drive greater consumption of cultural goods, which also has positive impact on intergenerational mobility.

While the paper attempts to illustrate the choices made under more realistic settings by relaxing a few assumptions, it mainly serves to lay the groundwork for the incorporation of cultural goods in future analysis of social status signalling.

Keywords Social Status · Signalling · Conspicuous Consumption · Inconspicuous Consumption · Cultural Goods

JEL Classifications: C700, D110
1 Introduction

The pursuit of social status is an age-old question that is embedded in the core of evolutionary theory by natural selection, where individuals are propelled to race to the top to gain better access to resources that are arbitrarily governed by status hierarchy. Since no man is an island, the concept of status is a social construction, determined by one’s relative positioning in the society, and often measured by other people’s beliefs about one’s income or wealth. In order to shape societal expectations and thereby attain status, individuals engage in visible consumption.

Veblen has introduced the term “conspicuous consumption” in reference to expenditure on visible goods that mainly encompass luxury merchandises and services. This topic has received extensive research as a way to signal one’s social status. (Veblen (1899)) However, with “inconspicuous consumption”, which includes spending on art appreciation classes and award-winning books that can be more accurately classified as cultural goods, garnering greater attention, thanks to the prevalence of social media, the rich and affluent are able to identify themselves with this additional bundle of goods which would otherwise be hidden. This area has yet to be fully explored in relation to status signalling, and the paper hopes to shine a light on it. (Currid-Halkett (2017b), Kinetics (2010))

While Smith believes that vying for “conspicuous” display of status through luxury goods could stimulate savings and wealth accumulations, Veblen identified such expenditure to be fundamentally wasteful. It is an arms race where one competes in terms of possessions of branded bags and jewelries, taking away resources that could otherwise be allocated to education and healthcare. Given this potential impact, a proliferation of literature has explored the relationship between poverty and luxury consumption, and found that the poors’ purchase of “temptation goods” has led to a trade-off in human capital accumulation, thus causing the persistence of poverty. (Wisman (2019), Banerjee and Mullainathan (2007), Moav and Neeman (2008)) In contrast, cultural goods has a different story, apart from signalling status, it also enhances one’s cultural and social capital — chance to be part of the “in” circle, building connections that could advance one’s career. (Carter (2003), Alderighi and Lorenzini (2012), Throsby (2001)) As a result, it brought to the attention that since both luxury and cultural goods are available as status goods, why would one still choose to purchase “useless” luxury goods over “intrinsically useful” cultural goods that are deemed to have higher productive value.

On a more private note, I was inspired by Currid-Halkett (2017b)’s book on the shifting consumption pattern of the U.S. consumers from “conspicuous” to “inconspicuous” goods. In relation to my everyday observations, whilst newspapers reported growing number of Chinese consumers flocking abroad to purchase luxury goods, the “Crazy Rich Asians”-labelled white collars are allocating more money to cultural goods, such as engaging in yoga classes and purchasing kale. This disparity in expenditure of individuals from similar income background has led to a personal interest in exploring under what circumstances would individuals choose either one of the goods to signal status and how would that impact the society.

As a result, this study came about, seeking to examine the choice between cultural and luxury goods in the pursuit of status. By constructing a signalling game under a set of assumptions, individual’s selection is analyzed when the wage-enhancing benefits of cultural goods are not internalized, and when they are. The paper found that social welfare can be maximized under the second scenario if certain conditions are met naturally or by intervention from the social planner. Furthermore, since the worker’s payoff in the game is built upon Moav and Neeman (2008)’s overlapping generation model, the paper is also able to offer a new narrative about intergenerational mobility given the additional status good. It is found that for individuals endowed with high cultural and social capital, upon satisfying conditions to invest in cultural goods, have a chance to jump to a higher income class, while those endowed with low cultural and social capital over a few generations and invest solely in luxury goods remain stagnant in income and experience downward mobility.

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2 UNESCO defined cultural goods to be those that have symbolic meanings, expressing certain ideas or lifestyle, such as organic food, popular novels, music lessons, etc. (UNESCO (2009))
The paper does not aim at providing an empirical approach, but mainly acts as a starting point for the investigation of relationship between consumption of luxury and cultural goods and the endowment of cultural and social capital. With this in mind, the remainder of the paper follows the structure: Section 2 comprises of literature review; Section 3 proposes the game setting; Section 4 analyzes the equilibrium under 2 different scenarios as highlighted previously; Section 5 discusses the implications of the results; Section 6 explores the additional information derived from relaxing a few assumptions; and lastly, Section 7 concludes and provides potential future extensions.


2 Literature Review

2.1 Status Signalling

Individuals’ desire for status has entailed research attention across many decades. It first stems from the evolutionary perspective, where higher social status would permit one to be better off in the battle of survival and competition for mates. This mentality carries on to modern times, where acquiring higher status allows one to obtain more resources, and also, derive positive life satisfaction from being respected, while low status can lead to depression, lack of self-worth and even broader negative health impact. These materialistic and psychological needs incentivize individuals to engage in behaviours designated to achieve higher status, and thus higher utility. One of the ways for society to generate status perception is through ranking one’s income or wealth. Even among the rich, society can attribute higher status to those possessing inherited assets rather than self-made wealth, such as the case for old and new money. (Nelissen and Meijers (2011), Bloch et al. (2004), Trigg (2001)) But individuals do not go around carry a label of income or asset worth over their head (unless they are well-known and top the Forbes’ rich list), for ordinary people, a way to generate social perception about their relative income positioning is through their expenditure. The catch-phrase “you are what you buy” is accurate in highlighting the idea of status signalling through purchasing expensive goods and services. (Paskov et al. (2013), Todd (2012), Anderson et al. (2015)) In ancient times, high status recognition are often associated with those who spend lavishly on both leisure activities and possessions. However, with globalization, the consumption of material goods becomes the mainstream mechanism to signal status. This is because as people move around, they are less-informed about the activities done by others to be able to form expectations based on those, and while normal consumption are assumed to be not observable, the main purpose of flamboyant luxury goods is to display and solicit societal expectations. Therefore, research on status signalling has generally been focusing on “conspicuous” goods, especially luxury goods with “loud” branding, and by purchasing them, individuals derive utility from the perceived social status that society afforded them. (DiMaggio and Mohr (1985), Spence (1978), Pesendorfer (1995), Moav and Neeman (2008), Pesendorfer (1995), Leibenstein (1950))

Increasingly, however, there has been decoupling of “conspicuous” and status. It is suggested that those who can actually afford high status commodities have little need to signal, and are willing to pay more for “inconspicuous” goods with less pervasive branding. (Han et al. (2010)) There are also research indicating that spending on “conspicuous” consumption is related to the stereotypes of income differences, where Blacks and Hispanics have greater propensity to spend conspicuously in an attempt to elevate their perceived social status, while the Whites, often being viewed favourably in terms of racial income stereotype, engage in “inconspicuous” consumption. (Mukherjee (2006)) Even among the rich, there are diversions, the “tuhaos” of China, similar to the nouveau riche, prefers “conspicuous” luxury goods, while the old money attempts to differentiate themselves by consuming goods with subtle branding. (Lu (2013), Carbajal et al. (2015)) This highlights the maneuver towards the inclusion of “inconspicuous” luxury goods, that allude to luxury products that lacks the flash, but are still capable of reflecting the owners’ “connoisseurship” in fashion. (Bergstrom (2012), Eckhardt et al. (2015))

While the rise of “inconspicuous” consumption flags the need for a more robust status signalling game that incorporates such phenomenon, this notion has some flaws. The mere intention of signalling status through purchasing and displaying status goods is already conspicuous on its own, therefore, it would be more accurate to denote “inconspicuous” consumption as a type of expenditure on goods that do not have obvious branding. For the interest of this paper, the “conspicuous” and “inconspicuous” consumption are redefined, where “conspicuous” goods refers to luxury products in general, while “inconspicuous” goods refers to cultural goods that encompasses books, prime gym membership, piano lessons, etc., which embodied certain cultural and intellectual value. Since both types of goods are purchased to make an impression on the public and reveal one’s social standing, signalling of status through cultural goods is an additional element this study will provide.
2.2 Cultural Capital and Cultural Goods

Bourdieu has once proposed that disparities in cultural and social capital would lead to status inequalities that are stratified along different lines as compared to those arise from differences in productivity. Cultural and social capital are defined to include competencies such as intellectual skills, social practices and refined taste that allow effective interaction between people of different culture backgrounds. Individuals possessing high cultural and social capital is capable of creating exclusive circles and generating influences to their benefits. (Trigg (2001), Edgerton and Roberts (2014), Lareau and Weininger (2003), Chaudhuri and Majumdar (2006)) As per OECD definition, such endowment constitutes as part of human capital\(^3\) that contributes to one’s income. While previous study by Moav and Neeman (2008) modelled income to include an unobserved \(\pi\), depicting one’s skills and talent, in isolation from observed human capital, it remains vague of what exactly constitutes \(\pi\), thus I redefine a nature-selected \(\alpha\) which reflect one’s cultural and social capital endowment, that forms a portion of human capital unobserved to the society.

While luxury goods are commonly known to signal one’s unobserved income, cultural goods have rarely been used as a signalling tool. However, researches show that cultural expenditures are strongly and positively correlated with income, and individuals of higher cultural and social capital endowment behave differently by having greater preference for cultural goods. Hence, this provides the potential for cultural goods to act as a signalling tool reflective of one’s cultural and social capital endowment. (Swidler (1986), Holt (1998))

Apart from its potential to signal status, cultural goods have the additional benefit of enhancing habitus qualities, which is a set of habits and skills in developing taste and judgment, that constitutes as part of cultural and social capital. In the cultural capital accumulation approach, individuals who have invested more in cultural goods consumption will end up with a larger cultural stock. Research have tested the combined effects of cultural consumption and habitus variables, such as reading books and going to theatre, on income, and found them to be statistically significant, despite having a smaller impact than education. As a result, this differentiate cultural goods from luxury goods. However, this positive effect is rarely internalized, thus leading to the potential for individuals to act differently in selection between luxury and cultural goods when income benefits of cultural goods are revealed or hidden. (Böröcz and Southworth (1996), Diniz and Machado (2011), Eckhardt et al. (2015), Cheng (2006), Stigler and Becker (1977), Brito and Barros (2005))

It is intuitive to think that cultural goods should be consume all the time if individuals are aware of the benefits, thus rendered the investigation of choice between the status goods redundant. This is far from the case because of the need for individuals to go through a “learning-by-consuming” or “cultivation-of-taste” process in order to appreciate cultural goods. (Stigler and Becker (1977), Brito and Barros (2005), Currid-Halkett (2017a)) While purchasing a “Shakespeare and Company” tote bag could cost as little as €10, it requires one to first know about the significance of the bookstore, and possibly be in Paris to buy it. Consequently, there exists a non-monetary cost barrier in addition to the actual price of the products, which includes the time commitment to develop certain taste. This makes cultural goods more expensive than luxury goods, and the decision between the two goods more ambiguous.

The current prevalence of social media and the culture of sharing have also weakened Veblen’s argument on the importance of luxury goods as the sole status indicator. Technology has made it easier for individuals to display and stay informed about status good expenditures others have made that might not involve obvious branding, for example, individuals engaging in yoga classes are able to display their yoga poses on social media, and such consumption potentially generates cultural exclusivity and labels the consumers as individuals of higher social status.

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\(^3\) Human capital is “the knowledge, skills, competencies and attributes that one possesses to facilitate the creation of personal, social and economic well-being.” (OECD (2007))
3 Model Construction

3.1 Defining Variables

With the background provided, a model where selection between luxury and cultural goods will be constructed to evaluate individuals’ choices in signalling status. But before elaborating on the details, the table below defines the variables used:

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha$</td>
<td>Unobserved human capital, indicating cultural and social capital endowment, private information to individuals and firms, unobserved to the society</td>
</tr>
<tr>
<td>$P(\alpha) = p$</td>
<td>Probability of $\alpha$ being high</td>
</tr>
<tr>
<td>$P(\alpha) = 1 - p$</td>
<td>Probability of $\alpha$ being low</td>
</tr>
<tr>
<td>$h$</td>
<td>Observed human capital, public information</td>
</tr>
<tr>
<td>$e$</td>
<td>Units of education, public information</td>
</tr>
<tr>
<td>$y$</td>
<td>Income, partially observable to society</td>
</tr>
<tr>
<td>$c$</td>
<td>Consumption, private information, observed only to individuals</td>
</tr>
<tr>
<td>$b$</td>
<td>Bequests, private information, observed only to individuals</td>
</tr>
<tr>
<td>$x$</td>
<td>Luxury goods</td>
</tr>
<tr>
<td>$P_x$</td>
<td>Price of luxury goods</td>
</tr>
<tr>
<td>$m$</td>
<td>Cultural goods</td>
</tr>
<tr>
<td>$P_m$</td>
<td>Price of cultural goods</td>
</tr>
<tr>
<td>$U(\cdot)$</td>
<td>Workers’ utility function</td>
</tr>
<tr>
<td>$S$</td>
<td>Status component</td>
</tr>
<tr>
<td>$h_1$</td>
<td>Information set 1 where $x$ has been chosen</td>
</tr>
<tr>
<td>$h_2$</td>
<td>Information set 2 where $m$ has been chosen</td>
</tr>
<tr>
<td>$\mu_1$</td>
<td>Probability of reaching the node where $x$ has been chosen given $\alpha$</td>
</tr>
<tr>
<td>$\mu_2$</td>
<td>Probability of reaching the node where $m$ has been chosen given $\alpha$</td>
</tr>
</tbody>
</table>

3.2 2-period Overlapping Generation Model

This paper adopts Moav and Neeman (2008)’s 2-period overlapping generation model (OLG) as the baseline to determine the variables in use, construct workers’ payoffs, and imply the potential relationship between one period decision that carries on to another.

1. “Childhood” period: While individuals do not work in this period, they are endowed with bequests from parents, and they decide the units of education to invest in, which contributes to the observed portion of human capital.

2. “Adulthood” period: Individuals start working and earn an income. At the beginning of the period, workers make decision whether to signal their unobserved human capital component through expenditure on either luxury or cultural goods, and firms will offer wages based on the overall human capital endowment.

It is common knowledge to all that the formulation of income is based on the worker’s overall human capital. Human capital is completely known to workers and firms, but only partially to the society. The observed portion arises from previous period expenditure on education, and the unobserved portion from cultural and social capital endowment given by nature. Only the unobserved component requires signalling.

In the case where individuals and firms internalize the enhancement to production from one’s cultural goods investment, purchasing cultural goods would increase human capital through a multiplying factor on the initial cultural and social capital stock. (Trigg (2001)) It is expected that for the high type, they would end up with even higher human capital and thus, income level. The multiplying factor takes the form of $ln(1 + m)$ in consideration of diminishing returns from additional units of cultural goods on income, but this property is only useful in Section 6.3 when individuals are allowed to choose the units of status goods. In current setting, individuals can only decide on the type of goods — luxury or cultural goods.
Utility of an individual is constructed based on normal consumption, bequests, and social perceived status. Normal consumption does not include spending on luxury and cultural goods, and is privately known to workers, same as the case for bequests. Since they are not observable to the society, individuals have to shift expenditure towards more visible status goods, such as luxury and cultural goods in order to generate status perception, and these go into improving one’s utility through the status component. (Moav and Neeman (2008))

Utility when luxury goods are chosen,

\[ u_x(c, b, S) = [c^{1-\lambda}b^\lambda]^{1-\psi}S_x^\psi \text{ where } S_x = E(\alpha|x) \]

\[ \text{s.t. } c + b + P_x x \leq y \]

\[ 0 < \lambda < 1, 0 < \psi < 1 \]

Utility when cultural goods are chosen,

\[ u_m(c, b, S) = [c^{1-\lambda}b^\lambda]^{1-\psi}S_m^\psi \text{ where } S_m = E(\alpha|m) \]

\[ \text{s.t. } c + b + P_m m \leq y \]

\[ 0 < \lambda < 1, 0 < \psi < 1 \]

3.3 “Luxury-Cultural” Game

In contrast to the traditional signalling game, an individual’s choice of status goods does not aim at inspiring any response from firms, it is a signal sent to the society, where society reacts with a belief of individual’s cultural and social capital type that enters directly into one’s utility function.

To emphasize, workers and firms have complete knowledge about one’s human capital. Society has incomplete information. While it knows the observed human capital component (education), it does not observe cultural and social capital, which is classified into two types, \( \alpha = [\alpha, \overline{\alpha}] \). It can only make predictions about high and low types based on worker’s selection of status goods.

As previously mentioned, after learning their types, workers can only choose either luxury or cultural goods, but not both because they serve the same signalling purpose. Also, given that even the poorest individual desires status, it is rationale to assume workers would not purchase zero amount of both goods either. (Van Kempen (2003a))
$((y - P_m m)^{1-\psi}(E(\pi(m)))^\psi, 0)$

$((y - P_m m)^{1-\psi}(E(\pi(m)))^\psi, 1)$

$((y - P_m m)^{1-\psi}E(\alpha(m))^\psi, 1)$

$((y - P_m m)^{1-\psi}E(\alpha(m))^\psi, 0)$

Figure 1: Extensive Form “Luxury-Cultural” Game

Table 1: Normal Form “Luxury-Cultural” Game

The set of possible actions for society and worker are:

Society = \{HH, HL, LH, LL\}

\(^4\)First letter denotes action taken at \(h_1\)
Worker = \{xx, xm, mx, mm\}^5

Depending on the good chosen, worker’s payoff is given by utility functions (3) or (4), where \(\pi\) indicates income of those endowed with \(\alpha\) and \(y\) is the income for \(\alpha\), the formulation of \(y\) is dependent on whether income-enhancing benefits from cultural goods are internalized or not. Society as a player will be “_rewarded_” when it accurately predicts the individual’s type given observed expenditure, and resource allocation is correctly stratified based on status hierarchy.

From hereon, variables will be denoted as defined in Section 3.1.

\footnote{First letter denotes action taken given \(\pi\)}
4 Equilibrium Analysis

To evaluate the equilibrium actions taken by the workers, the following assumptions were made in advance, some of them will be relaxed later to take into account more realistic scenarios:

Assumption 1. Nature can choose $\pi$ with probability $p \in [0, 1]$. However, the general assumption of the entire model is $p > 0.5$.

Assumption 2. Individuals are classified into either $\alpha$ or $\bar{\alpha}$.

Assumption 3. The cost of $m$ and $x$ satisfies the relationship $P_m > P_x$.

Assumption 4. A number is not denoted to $x$ and $m$, they are only indicative of the type of status goods chosen. Individuals cannot select both goods at the same time, neither can they select none.

Assumption 5. $E(\pi|x) = E(\pi|m) > E(\alpha|x) = E(\alpha|m)$. If society expects $\pi$, status utility generated is the same regardless of whether $x$ or $m$ is purchased, but it is always higher than when society expected $\alpha$.

Assumption 6. Equilibrium analysis is carried out on the current generation, where $h$ is assumed to be the same for all individuals, only $\alpha$ differs.

4.1 No Income Updating

Even though researches suggest that $m$ increases wages, workers and firms rarely take this positive externality into account. If consumers who purchase $m$ only concern about improving utility through status acquisition and do not expect any future increases in income, then the income formulation would be $y = h + \alpha$, regardless of the good purchased. I label this as “no income updating” game. (Brito and Barros (2005), Cheng (2006))

Proposition 1. Holding assumptions 1 to 6 true, when there are no income updating effects from purchasing $m$, pooling equilibria. $(xx, HH, \mu_1 > 0.5, \mu_2 > 0.5)$ and $(xx, HL, \mu_1 > 0.5, \mu_2 \leq 0.5)$ exist.

<table>
<thead>
<tr>
<th>Society</th>
<th>Worker</th>
</tr>
</thead>
<tbody>
<tr>
<td>$HH$</td>
<td>$EU(xx)$</td>
</tr>
<tr>
<td>$LL$</td>
<td>$EU(xx)$</td>
</tr>
<tr>
<td>$HL$</td>
<td>$EU(xx)$</td>
</tr>
<tr>
<td>$LH$</td>
<td>$EU(xx)$, if $\frac{h+\alpha-P_m m}{h+\alpha-P_x x} &gt; \left( \frac{E(\pi</td>
</tr>
</tbody>
</table>

Table 2: Worker’s Highest Payoff Given Society’s Action

Proof. For $EU(xx)$ to always be the highest given society’s actions, 2 conditions have to hold when society plays “$LH$”. Rewriting them,

\[
\frac{h+\alpha-P_m m}{h+\alpha-P_x x} < \left( \frac{E(\alpha|x)}{E(\pi|m)} \right)^{\frac{\psi_1}{\psi}}
\]

\[
\frac{h+\alpha-P_m m}{h+\alpha-P_x x} < \left( \frac{E(\alpha|x)}{E(\pi|m)} \right)^{\frac{\psi_1}{\psi}}
\]
Knowing that $0 < \psi < 1$, \( \frac{E(q|x)}{E(q|m)} \) lies between 0 and 1. For extreme case when it approaches 0, \( \bar{y} < P_m m \) and \( y < P_x x \) need to hold, and are satisfied by the budget constraints. When it tends to 1, conditions boil down to \( P_m m - P_x x > 0 \). Since this complies with assumption 3, the condition is satisfied as well. Therefore, \( xx \) will always be played in the equilibrium.

The Nash Equilibrium are \( (xx, HH), (xx, HL) \). Individuals of both types will always purchase \( x \) in spite of society’s action when observing \( m \).

To find Perfect Bayesian Equilibrium (PBE), the actions must be rational given beliefs and these beliefs need to satisfy Bayes’ rule. Holding all assumptions true, at each information set,

\[
\begin{align*}
\mu_1 &= P(\overline{\pi}|x) = \frac{P(x|\overline{\pi})P(\overline{\pi})}{P(x|\overline{\pi})P(\overline{\pi}) + P(x|\alpha)P(\alpha)} = p \\
\mu_2 &= P(\pi|m) = \frac{P(m|\pi)P(\pi)}{P(m|\pi)P(\pi) + P(m|\overline{\alpha})P(\overline{\alpha})} = 0
\end{align*}
\]

Since \( \mu_2 \) is undefined, any beliefs \( \mu_2 \in [0, 1] \) are consistent.

The payoff for the society at information set \( h_1 \) and \( h_2 \) are specified as following,

\[
EU(H|x) = \mu_1(1) + (1 - \mu_1)(0) = \mu_1 = p
\]

\[
EU(L|x) = \mu_1(0) + (1 - \mu_1)(1) = 1 - \mu_1 = 1 - p
\]

\[
EU(H|m) = \mu_2(1) + (1 - \mu_2)(0) = \mu_2
\]

\[
EU(L|m) = \mu_2(0) + (1 - \mu_2)(1) = 1 - \mu_2
\]

As per assumption 1, \( \mu_1 > 0.5 \), society will always choose \( H|x \) (high status perception given \( x \)). When \( \mu_2 > 0.5 \), society will choose \( H|m \) (high status perception given \( m \)), and \( L|m \) (low status perception given \( m \)) otherwise.

For \( \mu_1 > 0.5, \mu_2 > 0.5 \),

\[
EU(x|\overline{\pi}, (H|x, H|m)) = (h + \overline{\pi} - P_x x)^{1-\psi}(E(\overline{\pi}|x))^{\psi}
\]

\[
EU(m|\overline{\pi}, (H|x, H|m)) = (h + \overline{\pi} - P_m m)^{1-\psi}(E(\overline{\pi}|m))^{\psi}
\]

\[
EU(x|\overline{\pi}, (H|x, H|m)) > EU(m|\overline{\pi}, (H|x, H|m))
\]

\[
EU(x|\pi, (H|x, H|m)) = (h + \pi - P_x x)^{1-\psi}(E(\pi|x))^{\psi}
\]

\[
EU(m|\pi, (H|x, H|m)) = (h + \pi - P_m m)^{1-\psi}(E(\pi|m))^{\psi}
\]

\[
EU(x|\pi, (H|x, H|m)) > EU(m|\pi, (H|x, H|m))
\]

For \( \mu_1 > 0.5, \mu_2 \leq 0.5 \),

\[
EU(x|\overline{\pi}, (H|x, L|m)) = (h + \overline{\pi} - P_x x)^{1-\psi}(E(\overline{\pi}|x))^{\psi}
\]

\[
EU(m|\overline{\pi}, (H|x, L|m)) = (h + \overline{\pi} - P_m m)^{1-\psi}(E(\overline{\pi}|m))^{\psi}
\]

\[
EU(x|\overline{\pi}, (H|x, L|m)) > EU(m|\overline{\pi}, (H|x, L|m))
\]

\[
EU(x|\pi, (H|x, L|m)) = (h + \pi - P_x x)^{1-\psi}(E(\pi|x))^{\psi}
\]

\[
EU(m|\pi, (H|x, L|m)) = (h + \pi - P_m m)^{1-\psi}(E(\pi|m))^{\psi}
\]

\[
EU(x|\pi, (H|x, L|m)) > EU(m|\pi, (H|x, L|m))
\]
The results suggest that in equilibrium, workers will always purchase $x$, and society would believe those who possess $x$ to be $\bar{\pi}$, it would not matter what perception it has for those who purchase $m$. There are no profitable deviation for either to behave otherwise. This is expected since $x$ and $m$ serve the same purpose and $m$ is more costly, so both worker types will purchase $x$.

### 4.2 Income Updating

If $m$ has additional value in enhancing unobserved human capital, it is said to have “income updating” effect, and $y = h + \alpha(1 + \ln(1 + m))$. Worker’s utility payoff from consuming $m$ would need to be modified, but those purchasing $x$ remain unchanged. The inclusion of income updating effects would allow us to gain more insights on workers’ behaviour as they internalize the benefits from consuming $m$.

#### 4.2.1 Pooling Equilibrium With Income Updating ($xx$)

**Proposition 2.** Holding assumptions 1 to 6 true, pooling equilibria, $(xx, HH, \mu_1 > 0.5, \mu_2 > 0.5)$ and $(xx, HL, \mu_1 > 0.5, \mu_2 \leq 0.5)$, will exist iff

$$P_m - P_x > \bar{\pi} \ln(1 + m)$$

$$P_m - P_x > \alpha \ln(1 + m)$$

and the conditions hold strictly.

**Proof.** If both $\alpha$ types were to choose $x$, $\mu_1 = p$, $\mu_2$ is undefined. At information set $h_1$, society will choose $H|x$, and at $h_2$, $H|m$ will be selected if $\mu_2 > 0.5$ and $L|m$ otherwise.

Holding all assumptions true, same flow of steps ensues (Appendix A).

For $\mu_1 > 0.5$ (as per assumption 1), $\mu_2 > 0.5$,

$$EU(x|\bar{\alpha}, (H|x, H|m)) > EU(m|\bar{\alpha}, (H|x, H|m)) \text{ if } P_m - P_x > \bar{\pi} \ln(1 + m)$$

$$EU(x|\bar{\alpha}, (H|x, H|m)) > EU(m|\bar{\alpha}, (H|x, H|m)) \text{ if } P_m - P_x > \alpha \ln(1 + m)$$

For $\mu_1 > 0.5, \mu_2 \leq 0.5$,

$$EU(x|\bar{\alpha}, (H|x, L|m)) > EU(m|\bar{\alpha}, (H|x, L|m)) \text{ if } \frac{h + \bar{\pi} - P_x}{h + \bar{\pi} (1 + \ln(1 + m)) - P_m} > \left(\frac{E(\omega|m)}{E(\pi|x)}\right)^{\frac{1}{\psi}}$$

$$EU(x|\bar{\alpha}, (H|x, L|m)) > EU(m|\bar{\alpha}, (H|x, L|m)) \text{ if } \frac{h + \alpha - P_x}{h + \alpha (1 + \ln(1 + m)) - P_m} > \left(\frac{E(\omega|m)}{E(\pi|x)}\right)^{\frac{1}{\psi}}$$

Following the argument that $\left(\frac{E(\omega|m)}{E(\pi|x)}\right)^{\frac{1}{\psi}}$ lies between 0 and 1, in extreme case of converging to 0, the inequality satisfies budget constraints, and when the value tends to 1, conditions become

$$P_m - P_x > \alpha \ln(1 + m), \text{ where } \alpha = [\bar{\alpha}, \bar{\pi}]$$

(7)

In summary, both PBE, $(xx, HH, \mu_1 > 0.5, \mu_2 > 0.5)$ and $(xx, HL, \mu_1 > 0.5, \mu_2 \leq 0.5)$ exist iff (7) holds strictly.
The results suggest that even when workers and firms know the additional benefits of $m$, it does not impede workers from pooling over $x$, as long as income benefits from purchasing $m$ is lower than the cost differences of the goods. Research has shown that the increase in income from $m$ tends to be relatively small, thus the condition would be met with greater probability, especially when $\alpha$ is low. It can also be interpreted that the poor have lower tendency to deviate as compared to the rich. (Böröcz and Southworth (1996))

4.2.2 Pooling Equilibrium With Income Updating ($mm$)

Proposition 3. Holding assumptions 1 to 6 true, pooling equilibria, $(mm, HH, \mu_1 > 0.5, \mu_2 > 0.5)$ and $(mm, LH, \mu_1 \leq 0.5, \mu_2 > 0.5)$, will exist iff

$$\alpha \ln(1 + m) > P_m m - P_x x$$

$$\alpha \ln(1 + m) > P_n m - P_x x$$

and the conditions hold strictly.

However, given income increment from purchasing $m$, $\alpha \ln(1 + m)$, tends to be small, conditions are less likely to hold, especially for $\alpha$. Equilibrium will breakdown if poor has profitable deviation to consume $x$. (Böröcz and Southworth (1996))

Proof. If both $\alpha$ types choose $m$, $\mu_1$ would be undefined, and $\mu_2 = p$. At $h_1$, society would choose $H|x$ when $\mu_1 > 0.5$, and $L|x$ otherwise. At $h_2$, it chooses $H|m$.

Holding all assumptions true, for $\mu_1 > 0.5, \mu_2 > 0.5$,

$$EU(m|\alpha, (H|x, H|m)) > EU(x|\alpha, (H|x, H|m)) \text{ if } \alpha \ln(1 + m) > P_m m - P_x x$$

$$EU(m|\alpha, (H|x, H|m)) > EU(x|\alpha, (H|x, H|m)) \text{ if } \alpha \ln(1 + m) > P_m m - P_x x$$

For $\mu_1 \leq 0.5, \mu_2 > 0.5$,

$$EU(m|\alpha, (L|x, H|m)) > EU(x|\alpha, (L|x, H|m)) \text{ if } \frac{h + \alpha(1 + \ln(1 + m)) - P_m m}{h + \alpha} - P_x x > (\frac{E(\alpha|x)}{E(\alpha|m)})^{\frac{\psi}{\psi}}$$

$$EU(m|\alpha, (L|x, H|m)) > EU(x|\alpha, (L|x, H|m)) \text{ if } \frac{h + \alpha(1 + \ln(1 + m)) - P_m m}{h + \alpha} - P_x x > (\frac{E(\alpha|x)}{E(\alpha|m)})^{\frac{\psi}{\psi}}$$

Similarly, $0 < (\frac{E(\alpha|x)}{E(\alpha|m)})^{\frac{\psi}{\psi}} < 1$. In extreme case where it tends to 1, the conditions for both equilibria to hold are the same,

$$\alpha \ln(1 + m) > P_m m - P_x x$$

(8) implies that the cost differences between the status goods have to be smaller than the additional income benefits reaped from investing in $m$. As mentioned previously, the income increment tends to be small, the condition is more binding for the poor, and it is more probable for them to have a profitable deviation in purchasing $x$ instead. As a result, it is most likely for PBE, $(mm, HH, \mu_1 > 0.5, \mu_2 > 0.5)$ and $(mm, LH, \mu_1 \leq 0.5, \mu_2 > 0.5)$ to breakdown given that the conditions are harder to satisfy.

In conclusion, pooling PBE with $mm$ is most likely to breakdown given the conditions. It would be difficult to observe a situation where all $\alpha$ types choose $m$. 

13
4.2.3 Separating Equilibrium With Income Updating ($mx$)

**Proposition 4.** Holding assumptions 1 to 6 true, separating equilibrium, ($mx, LH, \mu_1 = 0, \mu_2 = 1$), will exist as $(E(\alpha|x))^{\psi} \xrightarrow{\psi \to 0} 1$ iff

$$\alpha n(1 + m) < P_m m - P_x x < \alpha \ln(1 + m)$$

In general case, the separating equilibrium will exist iff

$$\ln(1 + m) > \frac{P_x x - P_m m}{h - P_x x}, \overline{\alpha} - \alpha > 0$$

and the conditions hold strictly.

**Proof.** If $\pi$ choose $m$ and $\alpha$ choose $x$, the belief at each information set is given by, $\mu_1 = 0, \mu_2 = 1$. Society will always choose $L|x$ and $H|m$.

Holding all assumptions true, the payoff for workers will be,

$$EU(m|\pi, (L|x, H|m)) > EU(x|\alpha, (L|x, H|m))$$

if

$$\frac{h + \pi (1 + \ln(1 + m)) - P_m m}{h + \alpha - P_x x} > \frac{E(\alpha|x)}{E(\pi|m)}$$

$$EU(x|\alpha, (L|x, H|m)) > EU(m|\pi, (L|x, H|m))$$

if

$$\frac{h + \alpha - P_x x}{h + \alpha (1 + \ln(1 + m)) - P_m m} > \frac{E(\pi|m)}{E(\alpha|x)}$$

The conditions for ($mx, LH, \mu_1 = 0, \mu_2 = 1$) to exist as PBE are

$$\frac{h + \pi (1 + \ln(1 + m)) - P_m m}{h + \alpha - P_x x} > \frac{E(\alpha|x)}{E(\pi|m)}$$

(9)

$$\frac{h + \alpha - P_x x}{h + \alpha (1 + \ln(1 + m)) - P_m m} > \frac{E(\pi|m)}{E(\alpha|x)}$$

(10)

Similarly, with $0 < (E(\alpha|x))^{\psi} \xrightarrow{\psi} 1$, in extreme case when it tends to 1, the conditions become

$$\alpha n(1 + m) < P_m m - P_x x < \alpha \ln(1 + m)$$

(11)

Separating PBE, ($mx, LH, \mu_1 = 0, \mu_2 = 1$), will exist as long as the cost differences between the status goods fall within a certain range, denoted by the potential income increment for each $\alpha$ type when they invest in $m$. And even though the income increment is likely to be small, it is more probable for the rich to meet the criteria.

In the more general case, combining (9) and (10),

$$\ln(1 + m) > \frac{P_x x - P_m m}{h - P_x x}, \overline{\alpha} - \alpha > 0$$

(12)

needs to hold for the separating PBE to exist. This inequality suggests that the multiplying effect of $m$ on income has to be greater than a fraction of the cost differences between $x$ and $m$.

It is interesting to note that the level of $h$ does matter for the equilibrium to exist based on (12). By assumption 6, $h$ is held constant for all individuals in this generation, any changes would mean for the whole population. Together with assumption 3, if $h > P_x x$, condition will be satisfied, ($mx, LH, \mu_1 = 0, \mu_2 = 1$) will always exist. Otherwise, when $h < P_x x$, condition becomes more binding, the income multiplying factor has to be greater for the separating PBE to exist. This is consistent with Moav and Neeman (2008), where increase in education (and the resultant rise in $h$), would lead some to deviate away from $x$, and in this case, condition for the separating PBE will be satisfied and $\pi$ would purchase $m$ instead.
A research conducted on a cohort of highly-educated U.S. Gen Y has also shown that in a high setting, would invoke a cultural environment that criticizes excessive spending on , and considers as a mark of sophistication. Thus given the lowering in status perception of , some will deviate to , leading to and LH being played in the equilibrium. (Danziger (2017))

4.2.4 Separating Equilibrium With Income Updating \((x_m)\)

Proposition 5. Holding assumptions 1 to 6 true, separating equilibrium, \((x_m, HL, \mu_1 = 1, \mu_2 = 0)\), will exist as \(\begin{align*}
\frac{E(\alpha|x)}{E(\alpha|m)} & \xrightarrow{\psi \to 0} 1 \text{ iff } \\
\alpha n(1 + m) & < P_m m - P_x x < \alpha n(1 + m)
\end{align*}\) \(13\)

However, the condition contradicts \(\bar{\alpha} > \alpha\), PBE breaks down.

Proof. If \(\bar{\alpha}\) choose \(x\) and \(\alpha\) choose \(m\), the belief at each information set is given by, \(\mu_1 = 1, \mu_2 = 0\). Society will choose \(H|x, L|m\).

Holding all assumptions true, workers’ payoffs are,

\[EU(x|\bar{\alpha}, (H|x, L|m)) > EU(m|\alpha , (H|x, L|m))\text{ if } \frac{h + \bar{\alpha} - P_x x}{h + \alpha (1 + ln(1 + m)) - P_m m} > \frac{E(\alpha|m)}{E(\alpha|x)} \xrightarrow{\psi} \]

\[EU(m|\alpha, (H|x, L|m)) > EU(x|\alpha , (H|x, L|m))\text{ if } \frac{h + \alpha (1 + ln(1 + m)) - P_m m}{h + \alpha - P_x x} > \frac{E(\alpha|x)}{E(\alpha|m)} \xrightarrow{\psi}
\]

As a result, the conditions for \((x_m, HL, \mu_1 = 1, \mu_2 = 0)\) to exist as PBE are

\[\frac{h + \bar{\alpha} - P_x x}{h + \bar{\alpha}(1 + ln(1 + m)) - P_m m} > \frac{E(\alpha|m)}{E(\alpha|x)} \xrightarrow{\psi} \]

\[\frac{h + \alpha (1 + ln(1 + m)) - P_m m}{h + \alpha - P_x x} > \frac{E(\alpha|x)}{E(\alpha|m)} \xrightarrow{\psi} \]

\(14\)

\(15\)

In extreme cases where \(\frac{E(\alpha|m)}{E(\alpha|x)} \xrightarrow{\psi} \) tends to 1, the condition can be rewritten as,

\[\alpha n(1 + m) < P_m m - P_x x < \alpha n(1 + m)
\]

and even in this special case, it is contradicting to \(\bar{\alpha}n(1 + m) > \alpha n(1 + m)\), therefore, this separating PBE breaks down.

4.2.5 Semi-separating Equilibrium

In summary, it is possible to eliminate \(mm\) and \(xm\) being played in the equilibrium since the conditions for them to exist are harder to fulfill or breaks down entirely as compare to playing \(xx\) and \(mx\) in the equilibrium, but depending on which conditions are fulfilled, either the pooling or separating PBE will persist.
Given the pure strategies summarized, we can investigate if there is any semi-separating equilibrium. For pooling equilibria, where workers play x, high types have greater chances of facing an income increment from investing in m that might lead them to have profitable deviation to switch away from x, thus it is more plausible to explore the possibility of high type mixing between x and m with probability q and (1 - q) respectively, while the low type only consume x. Furthermore, in reality, even though more rich are investing in m, they did not abandon x altogether, therefore, this is a situation I would like to explore further. (Hoffower (2018))

**Proposition 6.** Holding assumptions 1 to 6 true, semi-separating equilibrium, \((\sigma_w, \sigma_s)\) exist iff

\[
1 > \frac{h + \alpha(1 + \ln(1 + m)) - P_m m}{h + \alpha - P_x x} > 0
\]

given
\[
\frac{h + \alpha(1 + \ln(1 + m)) - P_m m}{h + \alpha - P_x x} < \left(\frac{E(q|x)}{E(\alpha|m)}\right)^\psi
\]

and the conditions hold strictly.

**Proof.** Holding all assumptions true, the worker’s strategy is,

\[
\sigma_w(\alpha) = \begin{cases} 
  qx + (1 - q)m, & \alpha = \bar{\alpha} \\
  x, & \alpha = \underline{\alpha} 
\end{cases} \tag{16}
\]

Society need to mix between H and L with some positive probability in order for workers to be willing to mix. Thus, for society to be indifferent between H and L following x,

\[
\mu_1(\pi|x) = \frac{qp}{qp + (1 - p)}, \mu_2(\pi|m) = 1 \tag{17}
\]

\[
\mu_1(1) + (1 - \mu_1)(0) = \mu_1(0) + (1 - \mu_1)(1)
\]

\[
q = \frac{1}{p} - 1 \tag{18}
\]

As per assumption 1, \(p > 0.5\), then \(q \in [0, 1)\).

Let \(r, r \in (0, 1)\), be the probability society plays H|x. It is assumed to not be 0 or 1, so society will always mix to induce mixing on worker’s side. It will play H|m because those who reach information set h2 are always the high type. For \(\pi\) to be indifferent between x and m,

\[
r = \frac{\left(\frac{h + \alpha(1 + \ln(1 + m)) - P_m m}{h + \alpha - P_x x}\right)^{1-\psi} - \left(\frac{E(q|x)}{E(\pi|x)}\right)^\psi}{1 - \left(\frac{E(q|x)}{E(\pi|x)}\right)^\psi} \tag{19}
\]
Given society’s strategy, the best response for \( \alpha \) is \( x \) with probability 1 if \( EU(x) > EU(m) \), expressing in terms of \( r \),
\[
r > \frac{(h + \alpha(1 + \ln(1 + m))) - P_x m}{h + \alpha - P_x x} 1 - \psi \frac{(E(\alpha|x)}{E(\bar{\alpha}|m)} \psi
\]
(20)

In order for semi-separating PBE to exist, \( r \) in (19) has to lie between 0 and 1, as well as satisfying (20). As a result, the conditions for that to happen are
\[
1 > \frac{h + \alpha(1 + \ln(1 + m)) - P_x m}{h + \alpha - P_x x} \quad \text{given} \quad h + \alpha(1 + \ln(1 + m)) - P_x m > 0
\]
(21)

Based on previous arguments, \( \alpha ln(1 + m) > P_x m - P_x x \) tends to breakdown because \( \alpha ln(1 + m) \) is small. As a result, (22) is the only condition that has to hold strictly for equilibrium to exist, and as \( \psi \to 0 \), (22) becomes
\[
P_x m - P_x x > \alpha ln(1 + m) > P_x m - h - \bar{\alpha}
\]
(22)

It is similar to that of pooling PBE with \( xx \). Combine with previous results, this suggests that pure pooling strategy \( xx \) will be played in equilibrium as long as income increment cannot cover the cost differences, but if society decides to mix, as shown in this case, individuals would purchase \( m \) with positive possibility as long as they can afford \( m \) with their entire salary.

If condition (22) is satisfied, a semi-separating PBE with \( (\sigma_w, \sigma_s) \) would exist, where the strategy for workers \( (\sigma_w) \) is to choose \( x \) given \( \alpha \), and mix between \( x \) and \( m \) with probability \( q \) and \( (1 - q) \) given \( \bar{\alpha} \). The strategy for society \( (\sigma_s) \) is to choose \( H|x \) with probability \( r \) and \( L|x \) with probability \( (1 - r) \), and it will always choose \( H|m \).

This section highlights the potential for \( \bar{\alpha} \) to mix as long as society decides to mix. Intuitively, while \( x \) is perceived as a high status marker, its designs are easily imitated, thus leading to the existence of counterfeits that could induce society to attribute positive probability in believing one to be \( \alpha \) when observing \( x \). (Van Kempen (2003b)) When society are indifferent playing \( H|x \) and \( L|x \), \( \bar{\alpha} \) will also randomize between purchasing \( x \) and \( m \). The conditions for this to happen is when both \( \alpha \) types are unable to cover the cost differences between the status goods with investment in \( m \) alone, therefore, it is interesting that if this semi-separating PBE exists, \( \bar{\alpha} \) is willing to sacrifice more normal consumption and bequests in order to pay for \( m \) with positive probability.

It is hard to believe in real world circumstances that the randomization in society’s and workers’ strategies, within this incomplete information game, will follow specified probability distribution. As a result, for discussion purpose, I will only be focusing on pure PBE strategies. (Harsanyi (1973))
5 Discussion

5.1 Social Welfare

Herein, consider a social planner who always know the benefits of $m$, and concerns about the social welfare, defined as the aggregate workers’ utility in the economy. Comparing welfare yielded between pooling equilibrium, where $xx$ is played, and separating equilibrium, where $mx$ is played,

$$W_p = p(h + \alpha - P_x x) \ln(E(\bar{\pi}|x)) + (1 - p)(h + \alpha - P_x x) \ln(E(\alpha|x))$$  

$$W_s = p(h + \alpha(1 + ln(1 + m)) - P_m m) \ln(E(\bar{\pi}|m)) + (1 - p)(h + \alpha - P_x x) \ln(E(\alpha|x))$$  

For $W_p > W_s$, this will bring the question back to the condition,

$$\frac{h + \bar{\pi} - P_x x}{h + \bar{\pi}(1 + ln(1 + m)) - P_m m} > \left(\frac{E(\bar{\pi}|m)}{E(\alpha|x)}\right)$$  

holding all assumptions true, the condition will be satisfied as long as $P_m m - P_x x > \bar{\pi}ln(1 + m)$, meaning high types cannot cover cost differences between the status goods based on income increment from investing in $m$ alone, and will be better off just purchasing $x$. In this case, social planner and the workers have the same preference, and both prefer $x$ in the equilibrium whether or not information about the benefits of $m$ is revealed to the workers and the firms.

However, social planner can also seek to improve social welfare by maximizing the sum of income (utilitarian social welfare function). Then, he/she will always prefer separating PBE ($mx$) than pooling PBE ($xx$).

$$p[h + \bar{\pi}(1 + ln(1 + m))] + (1 - p)(h + \alpha) > p(h + \bar{\pi}) + (1 - p)(h + \alpha)$$  

This case is more interesting since it implies that social planner has different preferences as compared to workers in the equilibrium.

Given that the purchase of $x$ does not contribute to production growth, it is rather a relatively “wasteful” consumption in the eyes of social planner, in comparison to $m$ that has positive wage-enhancing externality.

Based on the equilibrium analysis, it was found that if workers do not internalize the benefits from $m$, as in the “no income updating” game, the different $\alpha$ types behave similarly by pooling resources on $x$, thus spending “wastefully”. However, if workers and firms learn about the positive impact of $m$ on income, $\bar{\pi}$ type workers will have incentive to deviate and purchase $m$ instead. Under this scenario, social welfare would be higher. This possibility does not exist under “no income updating”. Therefore, to encourage the high types, who has the greatest propensity to signal by purchasing $m$ at the equilibrium, to act differently than pooling over $x$, it would be in the best interest for policymakers to reveal information about wage-enhancing benefits of $m$, possibly through campaigns. This brought about another interesting point regarding the welfare benefit of gaining access to more information.

However, such separating PBE only exists when income increment from purchasing $m$ for the high type is greater than the cost differences between the status goods. Thus there lies potential for social planner to subsidize the purchase of $m$. It was found in previous studies that a decrease in tax on $m$ would increase welfare and efficiency, it is plausible for a subsidy to achieve the same purpose. (Nelissen and Meijers (2011)) Compared to “no income updating” case, lower amount of subsidies is required to induce high type individuals to purchase $m$ when there is income updating. For instance, based on the conditions,

$$Subsidy = \begin{cases} 
    P_m m - P_x x, & \text{No Income Updating} \\
    P_m m - P_x x - \bar{\pi}ln(1 + m), & \text{With Income Updating}
\end{cases}$$
In line with the argument, it seems possible to induce all individuals to purchase $m$ if the subsidy is high enough, however, social planner has to break-even, tax and subsidy have to equalize in this hypothetical economy. Therefore, not everyone can enjoy the subsidy. Furthermore, to enforce a separating equilibrium where $\sigma$ chooses $m$ and $\alpha$ chooses $x$, a regressive subsidy will need to be imposed, and the rich benefits at an expense of the poor in order to improve the aggregate social welfare.

5.2 Intergenerational Mobility

Just like the old saying goes, “the loudest one in the room are usually the weakest”, and in this case, researches noted that individuals with lesser income have the greatest tendency to purchase “conspicuous” goods in an attempt to attain social status, and as a result, they save lesser for human capital investment. Empirical studies were conducted in India, where 4 out of 5 are living in poverty, yet individuals spend lavishly on wedding celebrations and devote 15% of their yearly income to status goods consumption, depriving themselves of high return investments such as education. (Moav and Neeman (2008), Gupta and Srivastav (2016), Bloch et al. (2004), Duesenberry et al. (1949), Dynan et al. (2004), Bellet and Colson-Sihra (2018), Khamis et al. (2012)) While this illustrates the potential impact of $x$ on the persistence of poverty, given the additional option of wage-enhancing $m$, this issue can be re-examined.

There are many forms of social mobility, in this case, I focus on the intergenerational changes in income positioning. In the model proposed, observed human capital is based upon education choices in first period, $e_t$, which is again dependent on the bequests given by their parents, $b_t$. Therefore,

$$h_{t+1} = \gamma e_t(b_t)$$  \hspace{1cm} (27)

$$y_{t+1} = h_{t+1} + \alpha(1 + \ln(1 + m))$$  \hspace{1cm} (28)

With income updating, individuals investing in $m$ not only experience utility gain from status, but also direct increase in their income, denoted as a multiplying factor of their $\alpha$ types. Since bequests are perceived as normal goods, so as education, the increase in income will lead to consequential increase in bequests and education for the next generation. Their income can be increased through the observed human capital component, and they will potentially be elevated into a higher income class.

Even though the income increment from investing in $m$ tends to be small, the impact could become more elaborate across a few generations. The potential trends across a 4-generation period with income updating are illustrated in the following graphs, only pooling equilibrium with $xx$ and separating equilibrium with $mx$ are depicted because, as previously evaluated, these equilibria are most likely to arise holding all assumptions true. Also, the case where $xx$ is played in the equilibrium under income updating is the same as for “no income updating”.

19
Figure 2: Income Trends Across a 4-Generation Period Under Separating Equilibrium ($mx$)

Figure 3: Income Trends Across a 4-Generation Period Under Pooling Equilibrium ($xx$)
Provided individuals in all generations have the same preferences, and in generation $t_1$, individuals are endowed with the same $h$, the only difference in income is a result of variations in $\alpha$ types. For each new generation, a new $\alpha = [\underline{\alpha}, \bar{\alpha}]$ will be assigned.

For both graphs that map across a period of 4 generations, the income range fall in between the 2 tick lines. Evidently, greater polarization of income happen under separating equilibrium. Those lucky enough to be endowed with $\bar{\alpha}$ across all 4 generations will experience a much higher income growth when they select $m$ in equilibrium than choosing $x$. Even for those who are endowed with $\bar{\alpha}$ in $t_2$ and $\underline{\alpha}$ afterwards, the effect of higher bequests from second generation will carry on with a diminishing impact for the next few generations, and it will sustain longer for those invested in $m$ instead of $x$. Based on this, it can be seen that $\bar{\alpha}$ has more chances of moving up the income ladder under separating equilibrium, while it remains the same for $\underline{\alpha}$ if they are selected into the type for all 4 generations.

The model also offers a potential explanation for the shrinking middle class phenomenon. The connotation of the concept was proposed by many, but the definition of the middle class is varied. One frequently used method is to stratify individuals based on income using cumulative distribution method, but that limits the middle class to a fixed percentage and prevents any analysis about changes in its size, thus using OECD definition, the middle class indicates those earning between 75% and 200% of the median national income. (Brookings (2018), Gornick and Jäntti (2014), OECD (2019))

While research relates this phenomenon with self-selection into high and low-skilled occupations in order to avoid fierce competition in the middle-income jobs. (Blanchard and Willmann (2016)) This model provides an alternative possibility that the individuals are sorted based on different $\alpha$ types. Take the graphs to be individuals that are categorized to be in the same class in generation $t_1$, when separating PBE with $mx$ ensues, over a few generations, polarization within the class happens, those endowed with $\bar{\alpha}$ and invest in $m$ over the time would potentially be classified into a higher income class. As per definition, the range of income that fall in the middle class changes, thus those endowed with $\underline{\alpha}$ for a few generations and did not experience much change in income would potentially be pushed down the income ladder as the median rises.
6 Extensions

6.1 Relaxing Assumption 2: Continuous $\alpha$ types

The model classified $\alpha$ into only high and low types, but in reality, $\alpha$ is likely to be drawn from a continuous distribution that has support $[\alpha; \bar{\alpha}]$. Thus, to model a more realistic scenario, assumption 2 is relaxed while holding all other assumptions.

When there is no income updating, the cost differences between $x$ and $m$ will always propel consumers to choose $x$, thus this is not an interesting phenomenon I would elaborate. However, when there is income updating, there is potential for some to invest in $x$ and some in $m$, and this is an aspect I am interested in exploring.

To construct a two message equilibrium with continuous $\alpha$'s that conveys information, it needs to be the case that society’s reaction to $x$ and $m$ are different. Based on the separating PBE with $m_x$ that is likely to arise amongst the other PBEs in previous income updating game, assume without loss of generality that society plays $(L|x, H|m)$, and suppose $\alpha$ follows uniform distribution. There exists a threshold, $\tilde{\alpha}$, where individuals with $\alpha$'s below which will invest in $x$ and those beyond invest in $m$. If the threshold is so high that even the highest $\alpha$ invests in $x$, then $\alpha < \tilde{\alpha}$, and if the threshold is so low, even the lowest $\alpha$ would want to consume $m$, then $\alpha = \tilde{\alpha}$.

Suppose society generates perceived status as follows,

$$E(\alpha|x) = \begin{cases} \frac{1}{2}(\bar{\alpha} + \alpha), & \text{if } \alpha < \tilde{\alpha} \\ \frac{1}{2}(\tilde{\alpha} + \alpha), & \text{if } \alpha \geq \tilde{\alpha} \end{cases}$$

$$E(\alpha|m) = \begin{cases} \text{not defined}, & \text{if } \alpha < \tilde{\alpha} \\ \frac{1}{2}(\tilde{\alpha} + \bar{\alpha}), & \text{if } \alpha \geq \tilde{\alpha} \end{cases}$$

For the case where some choose $x$ and some choose $m$, the utility functions are rewritten as,

$$U(y, x) = \left(\frac{1}{2}\right)^{\psi}(h + \alpha - P_x x)^{1-\psi}$$

$$U(y, m) = \left(\frac{1}{2}\right)^{\psi}(h + \alpha(1 + \ln(1 + m)) - P_m m)^{1-\psi}(\tilde{\alpha} + \bar{\alpha})^{\psi}$$

For incentive compatibility to hold, the high types need $U(y, m) > U(y, x)$ for it to be rationale for them to choose $m$, and the marginal worker that is indifferent between the status goods will experience,

$$\frac{h + \alpha(1 + \ln(1 + m)) - P_m m}{h + \alpha - P_x x} = \left(\frac{\tilde{\alpha} + \alpha}{\tilde{\alpha} + \bar{\alpha}}\right)^{\psi}$$  \hspace{1cm} (29)

The set of belief for this PBE to hold would be,

$$P(\alpha < \tilde{\alpha}|x) = 1$$  \hspace{1cm} (30)

$$P(\alpha > \tilde{\alpha}|m) = 1$$  \hspace{1cm} (31)

We can use (29) to analyze the impact of changes in variables on $\tilde{\alpha}$ while holding assumptions other than 2 true.

(1) As $\psi$ tends to 1, $\tilde{\alpha}$ would decrease. This implies that as individuals treasure social status more, more of them will be willing to invest in $m$ as it generates higher status utility.

(2) While restricting $\tilde{\alpha}$ to be always positive, increase in $h$ could cause $\tilde{\alpha}$ to decrease so much so that even the lowest $\alpha$ can afford $m$. This is similar to Moav and Neeman (2008), where increasing education would lead to decreasing $x$ consumption, and in this case, they will switch to consuming $m$. 
(3) Increase in $P_m$ relative to $P_x$ will lead to a decrease in demand for $m$ since it becomes more expensive to consume $m$, up till a point that even the highest $\alpha$ would not be able afford it, then everyone will consume $x$ instead.

(4) Increase in $P_x$ relative to $P_m$ works in the opposite direction. However, as long as assumption 3 holds, there will always be some individuals that consume $x$.

(5) A decrease in $\alpha$ will lead to a decrease in $\tilde{\alpha}$, and a greater proportion of individuals will invest in $m$ to differentiate themselves from the extreme low. An increase in its value will lead to increase in $\tilde{\alpha}$, but if $\alpha$ is lifted close enough to $\tilde{\alpha}$, then a pooling equilibrium where everyone buys $m$ will arise.

For the upper bound, a reduction would cause an increase in $\tilde{\alpha}$, less individuals would be able to afford $m$. And if $\tilde{\alpha}$ is sufficiently low, everyone will be purchasing $x$.

In summary, a reduction in $\alpha$ and an increase in $\tilde{\alpha}$ will decrease $\tilde{\alpha}$, inducing more individuals to purchase $m$. Also, $\alpha < \tilde{\alpha} < \tilde{\alpha}$ would hold true more often if the range of $\alpha$ is broader, and separating equilibrium would ensue.

This section models the possibility of having different $\alpha$ types in the economy, and it offers some insights into individuals’ choices between $x$ and $m$ given changes in a few variables. In relation to maximizing social welfare, policymakers can nudge more consumers into purchasing $m$ by altering certain variables, such as increasing the importance of status, improving education for the whole population, augmenting $P_x$, or heightening the highest bundle of cultural and social capital that one can be selected into having. It is also interesting to note that decreasing $\alpha$ can induce more individuals to consume $m$, which could potentially explain for the rationale to bring in migrants with lower $\alpha$s, and push individuals around the previous threshold to switch from consuming $x$ to $m$ instead.

Since this modelling has restricted the choice between $x$ and $m$, one is unable to choose zero units of both goods. If we were to include possibility of choosing between {zero units of both goods, $x$, $m$}, then there will be incentive for some individuals with low $\alpha$s to not consume any goods, the existence of two $\alpha$ thresholds has to be further tested. Since this is not expected to bring much new information, it will not be explored in this paper.

6.2 Relaxing Assumptions 3 & 5: Consider $P_x = P_m$, $P_x > P_m$ and Different Perceived Status For Same $\alpha$

With the provision of social media and greater information circulation, $m$ has not only become more visible as an additional status signalling tool, its cost has also significantly decreased. Previously, the non-monetary barrier includes search cost of finding the right $m$ that can generate status perception, the increase in the accessibility of such information through social media sharing has lowered this cost, along with the effort cost of displaying consumption of $m$, which becomes less reliant on face-to-face interactions. This highlights the rationale to explore the effects of decreasing $P_m$ relative to $P_x$ while holding other assumptions true.

Exploring the relaxation of assumption 3, when $P_m = P_x$, under “no income updating”, workers are indifferent between consuming $x$ or $m$, but there will not be any separating equilibrium, because any deviation away from pooling on the same item would cause the individual to be perceived as poor. With income updating, individuals will always prefer $m$ since it can generate higher income without additional cost.
Under “no income updating”, individuals need to exist as the conditions boil down to

Table 4: PBE with No Income Updating with $P_x > P_m$

Investigating the case where $P_m$ decreases so much that $P_x > P_m$. Under “no income updating”, individuals need to meet conditions stated in Table 4 for each equilibrium to exist. It was found that only pooling PBE with $mm$ exist as the conditions boil down to $P_{mm} - P_x x < 0$.

Table 5: PBE with Income Updating with $P_x > P_m$
Same for the case with income updating, but the conditions to be met for pooling PBE with \( m \) to exist condense to:
\[ P_x - P_m m > -\alpha n(1 + m), \]
where \( \alpha = [\alpha, \bar{\alpha}] \), and it is always fulfilled.

Knowing \( m \) will always be played in the equilibrium when relaxing assumption 3, if I were to drop assumption 5 as well, allowing perceived status generated from the same \( \alpha \) type to be different for \( x \) and \( m \). This means that \( E(\pi|x) \neq E(\pi|m) \), \( E(\alpha|x) \neq E(\alpha|m) \). It would be interesting to explore when would individuals choose \( x \) over \( m \) in this scenario.

Under “no income updating”, for pooling PBE with \( xx \) played, given \( \mu_1 > 0.5, \mu_2 > 0.5 \),
\[
EU(x|\alpha, (H|x, H|m)) > EU(m|\alpha, (H|x, H|m)) \text{ if } \frac{h + \alpha - P_x x}{h + \alpha - P_m m} > \left( \frac{E(\alpha|m)}{E(\pi|x)} \right)^{\psi}
\]

\( (E(\pi|m) - E(\alpha|m)) \) needs to be strictly smaller than 1 for the equilibrium conditions to be satisfied. For all individuals to still choose \( x \), perceived status from \( m \) has to be lower than \( x \) given same \( \pi \) type is expected. This could happen if “brand prominence” of \( x \) has additional effect in elevating the perceived social status. (Han et al. (2010)) The expectations could be more accurately presented as following,
\[
E(\pi|x) > E(\alpha|m), \text{ where } \pi > \alpha > \alpha
\]
\[
E(\alpha|x) > E(\bar{\alpha}|m), \text{ where } \alpha > \bar{\alpha}
\]

With income updating, for pooling PBE with \( xx \) played, given \( \mu_1 > 0.5, \mu_2 > 0.5 \),
\[
EU(x|\bar{\pi}, (H|x, H|m)) > EU(m|\bar{\pi}, (H|x, H|m)) \text{ if } \frac{h + \bar{\pi} - P_x x}{h + \bar{\pi}(1 + ln(1 + m)) - P_m m} > \left( \frac{E(\bar{\pi}|m)}{E(\bar{\pi}|x)} \right)^{\psi}
\]
\[
EU(x|\bar{\alpha}, (H|x, H|m)) > EU(m|\bar{\alpha}, (H|x, H|m)) \text{ if } \frac{h + \bar{\alpha} - P_x x}{h + \bar{\alpha}(1 + ln(1 + m)) - P_m m} > \left( \frac{E(\bar{\alpha}|m)}{E(\bar{\alpha}|x)} \right)^{\psi}
\]

For individuals to still pool over \( x \) in equilibrium with \( P_m = P_x \) or \( P_x > P_m \), society needs to attribute even higher status to those purchasing \( x \) than \( m \) when same \( \pi \) type is predicted because of the additional income increment factor.

Based on these evaluations, it can be concluded that workers will more likely choose \( m \) over \( x \) in equilibrium with or without income updating when price of \( m \) decreases relative to \( x \). However, if \( x \) can generate sufficiently high perceived status, potentially due to the value of branding, then PBE with \( xx \) will emerge.

This provides potential explanation for why we still observe \( x \) being purchased despite \( m \) is becoming relatively cheaper. From the perspective of the policymaker, this could render price adjustment as a policy measure less effective in compelling greater consumption of \( m \), but it serves as a good news for the luxury producers, that by sustaining societal valuation of branding, individuals will still choose \( x \) over \( m \).

### 6.3 Relaxing Assumption 2 & 4: \( x \) and \( m \) Denotes Number of Units Chosen

If individuals can purchase different units of \( x \) and \( m \), and their perceived status is dependent on the units of status goods they chose instead of solely on the type, then they will want to maximize their utility with respect to each good given their budget constraint.
The utility function experienced by the worker is reformulated as,

\[ u(c, b, S) = \left[ e^{1-\lambda} b^\lambda \right]^{1-\psi} E(\alpha|x, m)^\psi \]

s.t. \[ c + b + P_x x + P_m m \leq y \]
\[ 0 < \lambda < 1, 0 < \psi < 1 \]  

(32)

For illustration of utility in different scenarios, assuming society weighs \( x \) and \( m \) similarly, and perceived status takes the form \( E(\alpha|x, m) = (m^{0.5} x^{0.5}) \). The more units of \( x \) and \( m \) are bought, the higher the perceived \( \alpha \) type the individual possesses. And \( \alpha \) in this case, is drawn from a continuous distribution, so \( \alpha \in [\alpha_0, \alpha_1] \).

As depicted in Diagram 1 & 2, individuals will consume more \( x \) and \( m \) to maximize utility under income updating, and the bundle would be higher for those with higher \( \alpha \). However, if individuals do not care as much about status as compared to consumption and bequests, then \( \psi \) tends to 0, individuals will consume more \( m \) compared to \( x \) to maximize utility, and those who care a lot will consume more of both goods (Diagram 3 & 4). Any point on the surface
plot indicates a combination of \(x\) and \(m\), but the maximum point denotes a bundle of the goods that contributes to the highest utility for a certain \(\alpha\) type. For individuals with utility-maximizing objective, they would not deviate from consuming the optimal bundle, or else they will be perceived as having lower status, and therefore lower utility. In this setting, consumption of zero units of both goods is possible, but that will cause the total utility to be zero, and this complies with the fact that even the poorest will purchase some status goods. (Van Kempen (2003a))

Lagrangian for utility under no income updating,

\[
L = [c^{1-\lambda b^\lambda}]^{1-\psi}E(\alpha|x,m)^\psi + \zeta(y - P_m m - P_x x - c - b)
\]

\[
c : (1 - \psi)(1 - \lambda)\frac{[c^{1-\lambda b^\lambda}]^{1-\psi}}{c}E(\alpha|x,m)^\psi = \zeta
\]

\[
b : \lambda(1 - \psi)\frac{[c^{1-\lambda b^\lambda}]^{1-\psi}}{b}E(\alpha|x,m)^\psi = \zeta
\]

\[
x : \psi[c^{1-\lambda b^\lambda}]^{1-\psi} \frac{\delta E(\alpha|x,m)}{\delta x}E(\alpha|x,m)^{\psi-1} = \zeta P_x
\]

\[
m : \psi[c^{1-\lambda b^\lambda}]^{1-\psi} \frac{\delta E(\alpha|x,m)}{\delta m}E(\alpha|x,m)^{\psi-1} = \zeta P_m
\]

\[
\zeta : y = P_m m - P_x x - c - b = 0\quad\text{(Budget Constraint)}
\]

Constraints and complementary slackness hold.

Combining (34) and (35),

\[
\frac{b}{c} = \frac{\lambda}{1 - \lambda}
\]

(36) and (37),

\[
\frac{P_m}{P_x} = \frac{\frac{\delta E(\alpha|x,m)}{\delta m}}{\frac{\delta E(\alpha|x,m)}{\delta x}}
\]

Under "no income updating", the marginal rate of substitution (MRS) between perceived status derived from \(m\) and \(x\) is determined by the price ratio of the goods. \(m\) and \(x\) exhibit characteristics of perfect substitutes, and as per assumption 3, \(\frac{P_m}{P_x} > 1\), workers will trade more units of \(m\) for one unit of \(x\) while maintaining the same utility.

When there is income updating, (40) becomes,

\[
\frac{P_m - \alpha \frac{1}{m+1}}{P_x} = \frac{\frac{\delta E(\alpha|x,m)}{\delta m}}{\frac{\delta E(\alpha|x,m)}{\delta x}}
\]

Changes in \(m\) will cause variations in the new price ratio,

\[
\frac{P_m}{P_x} = \frac{\frac{\delta E(\alpha|x,m)}{\delta m}}{\frac{\delta E(\alpha|x,m)}{\delta x}}, \quad \text{where } P_m = P_m - \alpha \frac{1}{m+1}
\]

Given MRS is equal to the market trade-off, \(\frac{P_m}{P_x}\) (40), a few cases will arise. In case 1, when \(MRS < \frac{P_m}{P_x}\), workers would decrease consumption of \(m\) in order to consume more \(x\). In case 2, \(MRS = \frac{P_m}{P_x} x\), is the same as (40). In case 3, \(MRS > \frac{P_m}{P_x}\), marginal utility from consuming \(m\) exceeds that of \(x\), thus more of \(m\) will be consumed.

Assuming \(\alpha\) is always positive, everyone will first fall into case 3, but as they increase consumption of \(m\), the price ratio would increase, along with changes in ratio of marginal utility from status perception of consuming \(m\) and \(x\), up to a point, the equilibrium will be restored. The difference between higher \(\alpha\)s and lower ones is that equilibrium will be reached faster for lower \(\alpha\). This also suggests case 1 will not happen.
In this section, by relaxing assumption 2 & 4, individuals are judged based on the amount of $x$ and $m$ they bought, the more they buy, the higher status they attain. For each type, they will consume an optimal bundle of both $x$ and $m$ in the equilibrium. Individuals are also found to always prefer more $x$ than $m$ when there is no income updating effect, but with income updating, everyone will consume more $m$ as compared to the previous case, especially those endowed with high $\alpha$s. Thus it remains valid for policymakers to reveal wage-enhancing benefits of $m$ as a way to induce greater consumption of it.
7 Paving The Way

7.1 Conclusion

With the proliferation of papers surrounding conspicuous display of status through luxury goods, this study hopes to serve as a starting point in exploring status signalling that encompasses a broader range of status goods, and herein, the inclusion of cultural goods. In order to understand more about individuals’ choice between luxury and cultural goods, a signalling game was constructed to examine the circumstances under which individuals, who are endowed with different cultural and social capital, select either good to reveal their type to the society as a way to attain status.

Based on the model, individuals will choose luxury goods over cultural goods when information of wage-enhancing benefits from cultural goods investment is not taken into account by both workers and firms; and when cost differences between the status goods are higher than the potential income increment. These could inspire welfare-maximizing social planner to implement policies to stimulate greater consumption of cultural goods through revelation of cultural goods’ wage-enhancing benefits and/or by reducing its cost. Furthermore, the model also offers a view on intergenerational mobility, where those endowed with high cultural and social capital and engaged in cultural consumption shown better chances of moving up the income ladder over the generations.

Given that the model was setup with a lot of assumptions, a few of them were relaxed to provide a more realistic backdrop under which the choices were made. By having many types of individuals that possess different amounts of cultural and social capital, it was found that policy instruments such as education, price adjustments of status goods, as well as changing upper and lower bound of cultural and social capital endowment in the society, can be used to influence the number of individuals consuming cultural goods. Owing to the ubiquity of social media, we are already observing some cost reduction in cultural goods relative to luxury goods via a decrease in information and revelation cost, but individuals will still choose luxury over cultural goods if value of branding do induce a higher status perception, rendering price adjustments measure less useful. Lastly, if individuals were allowed to choose the units of status goods to consume, utility-maximizing individuals will consume a bundle of both goods, but in general, those aware of income increment from cultural goods investment will consume comparably more cultural goods in the equilibrium.

Even though dropping more assumptions would provide a more holistic view of the issue, my results are already able to capture the generic cases and shine a light on the circumstances under which individuals choose luxury goods over cultural goods, as well as providing some implications of the choice.

7.2 Future Extensions

The model offers a basic framework for evaluating the selection between the status goods, there are potentials for it to incorporate effects other than wage-enhancing benefits from purchasing cultural goods.

In China, the “tuhaos”, often referred to as the new money that lacks the cultural refinement, are looked down upon by many because of their extravagant behaviour in purchasing branded products. As a result, increasingly, the possession of luxury goods has a negative connotation, where instead of insinuating one to possess high cultural and social capital, it is the direct opposite, and individuals could derive disutility from associating with luxury goods. Furthermore, research has shown that people usually do not want to befriend with someone who appears rich, while this might be specific to the viewpoint of those who lack self-confidence, it is indicative of the potential that individuals who purchase luxury goods might suffer in terms of their social network, thus adversely affecting their income. (Sethi (2018), Garcia et al. (2019))

Recent studies have also pointed out the potential correlation between personality and luxury expenditure, where indigent extroverts tend to spend more. Putting this into context, extroverts often engage in greater information sharing, and thus they would have lower non-monetary barrier towards consuming and displaying cultural goods. This could
lead to different equilibrium actions carried out by individuals of disparate personality traits. (Economist (2017)) Such are areas that can be further explored by modifying the current model.

Nonetheless, the concept of cultural goods is rapidly evolving, in the interest of this paper, they are defined to encompass books, piano classes, even tote bags with slogans, which displays some form of knowledge and sophistication that can be more exclusive to garner respect from certain social circle. However, the prevalence of social media and the uprisings of the “Wanghong” economy⁶, which is transforming young consumers’ consumption pattern, might lead to re-classification of cultural goods to include taking part in recreational activities, such as “checking in” into an Instagram-worthy restaurant recommended by famous vloggers. With the aid of social media sharing, cultural goods of this sort could more “conspicuously” display one’s purchasing power, nonetheless, it can also garner higher social status perception by showcasing one’s knowledge of the in-trend recreational activities and be at the forefront of the cultural trend. Even though it is not obvious how such evolution in cultural goods would have similar effect in increasing income as the current bundle, knowing the perfect in-trend place to bring one’s business partners or potential clients could certainly help an individual to impress and expand their social circles.

In conclusion, the incorporation of cultural goods can bring about new exciting prospects for status signalling studies. This paper hopes to encourage a re-look at the atavistic issue of status in this digital age, and pave the way for future analysis of cultural goods in the game of status.

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⁶Economy driven by online influencer marketing.
A Appendix A Detailed Proof

Since exploration of equilibrium for “no income updating” case has already been done in details in the main paper, it will not be repeated here. Appendix A only includes detailed proof for pooling equilibrium with income updating \((xx)\) [Section 4.2.1] and separating equilibrium with income updating \((mx)\) [Section 4.2.3]. The flow of steps is the same for pooling equilibrium with income updating \((mm)\) [Section 4.2.2] and separating equilibrium with income updating \((xm)\) [Section 4.2.4], therefore, these will not be elaborated to prevent repetition.

A.1 Income Updating: Pooling Equilibrium \((xx)\)

Holding all assumptions true.

Belief at each information set:
\[
\mu_1 = P(\pi|x) = \frac{P(x|\pi)P(\pi)}{P(x|\pi)P(\pi) + P(x|\alpha)P(\alpha)} = \frac{p}{p + q}
\]
\[
\mu_2 = P(\pi|m) = \frac{P(m|\pi)P(\pi)}{P(m|\pi)P(\pi) + P(m|\alpha)P(\alpha)} = \frac{0}{p + q}
\]

Payoffs for society:

At information set \(h_1\):
\[
EU(H|x) = \mu_1(1) + (1 - \mu_1)(0) = \mu_1 = p
\]
\[
EU(L|x) = \mu_1(0) + (1 - \mu_1)(1) = 1 - \mu_1 = 1 - p
\]
\[
EU(H|x) > EU(L|x)
\]

As per assumption 1, \(p > 0.5\). Society will always choose \(H|x\) at \(h_1\).

At information set \(h_2\):
\[
EU(H|m) = \mu_2(1) + (1 - \mu_2)(0) = \mu_2
\]
\[
EU(L|m) = \mu_2(0) + (1 - \mu_2)(1) = 1 - \mu_2
\]
\[
EU(H|m) > EU(L|m)\text{ if }\mu_2 > 0.5, EU(L|m) > EU(H|m)\text{ if }\mu_2 \leq 0.5
\]

Society will choose \(H|m\) at \(h_1\) if \(\mu_2 > 0.5\) and \(L|m\) if \(\mu_2 \leq 0.5\).

Payoffs for workers:

For \(\pi, \mu_1 > 0.5, \mu_2 > 0.5\), society will play \(H|m\),
\[
EU(x|\pi, (H|x, H|m)) = (h + \pi - P_x x)^{1 - \psi(E(\pi|x))} \psi
\]
\[
EU(m|\pi, (H|x, H|m)) = (h + \pi(1 + ln(1 + m)) - P_m m)^{1 - \psi(E(\pi|m))} \psi
\]
\[
EU(x|\pi, (H|x, H|m)) > EU(m|\pi, (H|x, H|m)) \text{ if } P_m m - P_x x > \pi m(1 + m)
\]

For \(\alpha, \mu_1 > 0.5, \mu_2 > 0.5\),
\[
EU(x|\alpha, (H|x, H|m)) = (h + \alpha - P_x x)^{1 - \psi(E(\alpha|x))} \psi
\]
\[
EU(m|\alpha, (H|x, H|m)) = (h + \alpha(1 + ln(1 + m)) - P_m m)^{1 - \psi(E(\alpha|m))} \psi
\]
\[
EU(x|\alpha, (H|x, H|m)) > EU(m|\alpha, (H|x, H|m)) \text{ if } P_m m - P_x x > \alpha m(1 + m)
\]
For $α,μ_1 > 0.5, μ_2 ≤ 0.5$, society choose $L|m$,

$$EU(x|\alpha, (H|x, L|m)) = (h + \alpha - P_x x)^{1-\psi}(E(\alpha|x))^\psi$$

$$EU(m|\alpha, (H|x, L|m)) = (h + \alpha(1 + \ln(1 + m)) - P_m m)^{1-\psi}(E(\alpha|m))^\psi$$

$$EU(x|\alpha, (H|x, L|m)) > EU(m|\alpha_H, (H|x, L|m)) \text{ if } \frac{h + \alpha - P_x x}{h + \alpha(1 + \ln(1 + m)) - P_m m} > \frac{E(\alpha|m)}{E(\alpha|x)}^{1-\psi}$$

For $α, μ_1 > 0.5, μ_2 ≤ 0.5$,

$$EU(x|\alpha, (H|x, L|m)) = (h + \alpha - P_x x)^{1-\psi}(E(\alpha|x))^\psi$$

$$EU(m|\alpha, (H|x, L|m)) = (h + \alpha(1 + \ln(1 + m)) - P_m m)^{1-\psi}(E(\alpha|m))^\psi$$

$$EU(x|\alpha, (H|x, L|m)) > EU(m|\alpha_x, (H|x, L|m)) \text{ if } \frac{h + \alpha - P_x x}{h + \alpha(1 + \ln(1 + m)) - P_m m} > \frac{E(\alpha|m)}{E(\alpha|x)}^{1-\psi}$$

As a result, ($xx, μ_1 > 0.5, HH, μ_2 > 0.5$) and ($xx, μ_1 > 0.5, HL, μ_2 ≤ 0.5$) are both PBE iff

$$\frac{h + \alpha - P_x x}{h + \alpha(1 + \ln(1 + m)) - P_m m} > \frac{E(\alpha|m)}{E(\alpha|x)}^{1-\psi}$$

$$P_m m - P_x x > αln(1 + m)$$

$0 < \frac{E(\alpha|m)}{E(\alpha|x)}^{1-\psi} < 1$, thus the conditions boil down to only 2. Both PBE exist iff conditions below hold strictly.

$$P_m m - P_x x > αln(1 + m)$$

A.2 Income Updating: Separating Equilibrium ($mx$)

Holding all assumptions true.

Belief at each information set:

$$\mu_1 = P(\alpha|x) = \frac{P(x|\alpha)P(\alpha)}{P(x|\alpha)P(\alpha) + P(x|\alpha)P(\alpha)} = 0$$

$$\mu_2 = P(\alpha|m) = \frac{P(m|\alpha)P(\alpha)}{P(m|\alpha)P(\alpha) + P(m|\alpha)P(\alpha)} = 1$$

Payoffs for society:

At information set $h_1$:

$$EU(H|x) = 0, EU(L|x) = 1$$

$$EU(L|x) > EU(H|x)$$

Society will always choose $L|x$ at $h_1$.

At information set $h_2$:

$$EU(H|m) = 1, EU(L|m) = 0$$
As a result, \( PBE \)

\[
EU(H|m) > EU(L|m)
\]

Society will always choose \( H|m \) at \( h_2 \).

Payoffs for workers:

For \( \pi \),

\[
EU(m|\pi, (L|x, H|m)) = (h + \pi(1 + ln(1 + m)) - P_m m)^{1-\psi}(E(\pi|m))^{\psi}
\]

\[
EU(x|\pi, (L|x, H|m)) = (h + \pi - P_x x)^{1-\psi}(E(\alpha|x))^{\psi}
\]

\[
EU(m|\pi, (L|x, H|m)) > EU(x|\pi, (L|x, H|m)) \text{ if } \frac{h + \pi(1 + ln(1 + m)) - P_m m}{h + \pi - P_x x} > \left( \frac{E(\alpha|x)}{E(\pi|m)} \right)^{\frac{1}{\psi}}
\]

For \( \alpha \),

\[
EU(m|\alpha, (L|x, H|m)) = (h + \alpha(1 + ln(1 + m)) - P_m m)^{1-\psi}(E(\alpha|m))^{\psi}
\]

\[
EU(x|\alpha, (L|x, H|m)) = (h + \alpha - P_x x)^{1-\psi}(E(\alpha|x))^{\psi}
\]

\[
EU(x|\alpha, (L|x, H|m)) > EU(m|\alpha, (L|x, H|m)) \text{ if } \frac{h + \alpha - P_x x}{h + \alpha(1 + ln(1 + m)) - P_m m} > \left( \frac{E(\alpha|m)}{E(\alpha|x)} \right)^{\frac{1}{\psi}}
\]

As a result, \( PBE \) \( (m_x, \mu_1 = 0, LH, \mu_2 = 1) \) exists iff

\[
\frac{h + \pi(1 + ln(1 + m)) - P_m m}{h + \pi - P_x x} > \left( \frac{E(\alpha|x)}{E(\pi|m)} \right)^{\frac{1}{\psi}}
\]

\[
\frac{h + \alpha - P_x x}{h + \alpha(1 + ln(1 + m)) - P_m m} > \left( \frac{E(\alpha|m)}{E(\alpha|x)} \right)^{\frac{1}{\psi}}
\]

\[0 < \left( \frac{E(\alpha|x)}{E(\pi|m)} \right)^{\frac{1}{\psi}} < 1, \text{ conditions boil down to the following in extreme case when the ratio tends to 1,}
\]

\[
\frac{\alpha}{\pi}ln(1 + m) < P_m m - P_x x < \frac{\pi}{\alpha}ln(1 + m)
\]

In general case, combine the conditions,

\[
\frac{h + \pi(1 + ln(1 + m)) - P_m m}{h + \pi - P_x x} > \frac{h + \alpha(1 + ln(1 + m)) - P_m m}{h + \alpha - P_x x}
\]

\[
(h + \pi(1 + ln(1 + m)) - P_m m)(h + \alpha - P_x x) > (h + \alpha(1 + ln(1 + m)) - P_m m)(h + \pi - P_x x)
\]

\[
h\alpha + \pi(1 + ln(1 + m))h - \pi(1 + ln(1 + m))P_x x - P_m m\alpha > h\alpha + \alpha(1 + ln(1 + m))h - \alpha(1 + ln(1 + m))P_x x - P_m m\pi
\]

\[
(\pi - \alpha)(1 + ln(1 + m))(h - P_x x) + P_m m - h > 0
\]

\[
[ln(1 + m)(h - P_x x) - P_x x + P_m m](\pi - \alpha) > 0
\]

\[
ln(1 + m) > \frac{P_x x - P_m m}{h - P_x x}, \pi - \alpha > 0
\]

The separating \( PBE \), \( (m_x, LH, \mu_1 = 0, \mu_2 = 1) \), exists iff this set of conditions holds strictly.
References


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35


