Fiscal policy under involuntary unemployment

Tanaka, Yasuhito

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Yasuhito Tanaka

Faculty of Economics, Doshisha University,
Kamigyo-ku, Kyoto, 602-8580, Japan.
E-mail: yatanaka@mail.doshisha.ac.jp

Abstract

We show the existence of involuntary unemployment based on consumers' utility maximization and firms' profit maximization behavior under monopolistic competition with increasing, decreasing or constant returns to scale technology using a three-periods overlapping generations (OLG) model with a childhood period as well as younger and older periods. We also analyze the effects of fiscal policy financed by tax and budget deficit (or seigniorage) to achieve full-employment under a situation with involuntary unemployment. We show the following results. 1) If the realization of full employment will increase consumers' disposable income, in order to achieve full-employment from a state with involuntary unemployment, we need a budget deficit (Proposition 1). 2) If the full-employment state has been achieved, we do not need budget deficit to maintain full-employment (Proposition 2). Additionally we present a game-theoretic interpretation of involuntary unemployment and full-employment.

Keywords: Involuntary unemployment, Three-periods overlapping generations model, Monopolistic competition.

1 Introduction

In this paper we analyze the effects of fiscal policy to achieve full-employment under a situation with involuntary unemployment. Involuntary unemployment in this paper is a situation where workers are willing to work at the market wage or just below but are prevented by factors beyond their control, mainly, deficiency of aggregate demand. Umada (1997) derived an upward-sloping labor demand curve from the mark-up principle for firms, and argued that such an upward-sloping labor demand curve leads to the existence of involuntary unemployment without wage rigidity[1]. But his model of firm behavior is ad-hoc. Otaki (2009) says that there exists involuntary unemployment for two reasons: (i) the nominal wage rate is set above the reservation nominal wage rate; and (ii) the employment level and economic welfare never improve by lowering the nominal wage rate. He assume indivisibility (or inelasticity) of individual labor supply, and has shown the existence of involuntary unemployment using efficient wage bargaining according to McDonald and Solow (1981). The arguments of this paper, however, do not depend on bargaining. If labor supply is indivisible, it may be 1 or 0. On the other hand, if it is divisible, it takes a real value between 0 and 1. As discussed by Otaki (2015) (Theorem 2.3) and Otaki (2012), if the labor supply is divisible and very small, no unemployment exists[2]. However, we show that even if labor supply is divisible, unless it is so small, there may exit involuntary unemployment. We consider consumers' utility maximization and firms' profit maximization in an overlapping generations (OLG) model under monopolistic competition according to Otaki (2007, 2009, 2011, 2015), and demonstrate the existence of involuntary unemployment without the assumption of wage rigidity.

Also we analyze the effects of fiscal policy financed by tax and budget deficit (or seigniorage). We show the following results.

1. If the realization of full employment will increase consumers' disposable income, in order to achieve full-employment from a state with involuntary unemployment, we need a budget deficit. (Proposition 1)

2. If the full-employment state has been achieved, we do not need budget deficit to maintain full-employment. (Proposition 2)

From these results we can say that in order to achieve full-employment from a state with involuntary unemployment we need budget deficit of the government. However, when full-employment is achieved, in order to maintain full-employment we need balanced budget. Therefore, additional government expenditure to achieve full-employment should be financed by seigniorage not public debt.

In the next section we analyze and show the existence of involuntary unemployment under monopolistic competition with increasing or decreasing or constant returns to scale technology using a three-periods OLG model with a childhood period as well as younger (working) and older (retired) periods. Also we consider pay-as-you go pension

[1] Lavoie (2001) presented a similar analysis.

[2] About the indivisible labor supply also please see Hansen (1985).

system for the older generation. In a simple two-periods OLG model falls in the nominal wage rate and prices of goods may increase consumption and employment by the so-called real balance effect. In such a model consumers have savings for future consumption, but no debt. In a three-periods model with childhood period they consume goods in their childhood period by borrowing money from (employed) consumers of the previous generation and/or scholarships, and must repay their debts in the next period. Real value of the debt is increased by falls in the nominal wage rate and prices, and consumptions and employment may decrease. In addition to this configuration we consider a pay-as-you go pension system for the older generation which may reduce the savings of consumers. We think our model is more general and realistic than a simple two-periods OLG model. In Section 3 we examine the effects of a fall in the nominal wage rate. In our three-periods OLG model with pay-as-you-go pension increases in consumption and employment due to falls in the nominal wage rate and prices of goods might be small or even negative. In Section 4 we study the fiscal policy financed by tax and budget deficit (or seigniorage) to achieve full-employment at a state with involuntary unemployment. Additionally we present a game-theoretic interpretation of involuntary unemployment and full-employment in Section 5.

As we will state in the concluding remarks, the main limitation of this paper is that the goods are produced by only labor and there exists no capital and investment of firms. A study of the problem of involuntary unemployment and fiscal policy in such a situation is the theme of future research.

This paper is an extension and generalization of some recent our papers, Tanaka (2020b) and Tanaka (2020a) in which we analyze the existence of involuntary unemployment and fiscal policy under perfect competition with indivisible labor supply.

Schultz (1992) showed that there does not exist involuntary unemployment in an overlapping generations model. His arguments depends on the real balance effect on consumption of the older generations consumers. Even with involuntary unemployment, the nominal wage rate does not necessarily fall. In this paper, however, we consider a three generations overlapping generations model with pay-as-you go pension to explore the possibility of avoiding the real balance effect. See Section 3.

2 Existence of involuntary unemployment

2.1 Consumers

We consider a three-periods (0: childhood, 1: younger or working, and 2: older or retired) OLG model under monopolistic competition. It is a re-arrangement and an extension of the model put forth by Otaki (2007), Otaki (2009), and Otaki (2015). The structure of our model is as follows.

1. There is one factor of production, labor, and there is a continuum of perishable goods indexed by $z \in [0, 1]$. Good z is monopolistically produced by firm z with increasing or decreasing or constant returns to scale technology.

2. Consumers consume the goods during the childhood period (Period 0). This consumption is covered by borrowing money from (employed) consumers of the younger generation and/or scholarships. They must repay these debts in their Period 1. However, unemployed consumers cannot repay their own debts. Therefore, we assume that unemployed consumers receive unemployment benefits from the government, which are covered by taxes on employed consumers of the younger generation.

3. During Period 1, consumers supply l units of labor, repay the debts and save money for their consumption in Period 2. They also pay taxes for the pay-as-you go pension system for the older generation.

4. During Period 2, consumers consume the goods using their savings carried over from their Period 1 earnings, and receive the pay-as-you go pension, which is a lump-sum payment. It is covered by taxes on employed consumers of the younger generation.

5. Consumers determine their consumptions in Periods 1 and 2 and the labor supply at the beginning of Period 1. We assume that their consumption during the childhood period is constant.

We use the following notation.

C_i^e: consumption basket of an employed consumer in Period i, $i = 1, 2$.
C_i^u: consumption basket of an unemployed consumer in Period i, $i = 1, 2$.
$c_i^e(z)$: consumption of good z of an employed consumer in Period i, $i = 1, 2$.
$c_i^u(z)$: consumption of good z of an unemployed consumer in Period i, $i = 1, 2$.
D: consumption basket of an individual in the childhood period, which is constant.
P_i: the price of consumption basket in Period i, $i = 1, 2$.
$p_i(z)$: the price of good z in Period i, $i = 1, 2$.
$\rho = \frac{P_2}{P_1}$: (expected) inflation rate (plus one).
W: nominal wage rate.
R: unemployment benefit for an unemployed individual. $R = D$.
$\hat{D}$: consumption basket in the childhood period of a next generation consumer.
Q: pay-as-you-go pension for an individual of the older generation.
Θ: tax payment by an employed individual for the unemployment benefit.
$\hat{Q}$: pay-as-you-go pension for an individual of the younger generation when he retires.
Ψ: tax payment by an employed individual for the pay-as-you-go pension.
Π: profits of firms which are equally distributed to each consumer.
l: labor supply of an individual.
$\Gamma(l)$: disutility function of labor, which is increasing and convex.
L: total employment.
L_f: population of labor or employment in the full-employment state.
$y(Ll)$: labor productivity, which is increasing or decreasing or constant
with respect to "employment $\times$ labor supply" (Ll).

We assume that the population L_f is constant.

We consider a two-step method to solve utility maximization of consumers such that:

1. Employed and unemployed consumers maximize their utility by determining consumption baskets in Periods 1 and 2 given their income over two periods:
2. Then, they maximize their consumption baskets given the expenditure in each period.

We define the elasticity of the labor productivity with respect to “employment × labor supply” as follows,

$$\zeta = \frac{y'}{\frac{y(Ll)}{Ll}}.$$

We assume that $-1 < \zeta < 1$, and ζ is constant. Increasing (decreasing or constant) returns to scale means $\zeta > 0$ ($\zeta < 0$ or $\zeta = 0$).

Since the taxes for unemployed consumers' debts are paid by employed consumers of the same generation, D and Θ satisfy the following relationship.

$$D(L_f - L) = L\Theta.$$

This means

$$L(D + \Theta) = L_f D.$$

The price of the consumption basket in Period 0 is assumed to be 1. Thus, D is the real value of the consumption in the childhood period of consumers.

Also, since the taxes for the pay-as-you-go pension system are paid by employed consumers of younger generation, Q and Ψ satisfy the following relationship:

$$L\Psi = L_f Q.$$

The utility function of employed consumers of one generation over three periods is written as

$$u(C_1^e, C_2^e, D) - \Gamma(l).$$

We assume that $u(\cdot)$ is a homothetic utility function. The utility function of unemployed consumers is

$$u(C_1^u, C_2^u, D).$$

The consumption baskets of employed and unemployed consumers in Period i are

$$C_i^e = \left(\int_0^1 c_i^e(z)^{\frac{\sigma-1}{\sigma}} dz\right)^{\frac{\sigma}{\sigma-1}},\ i = 1, 2,$$

and

$$C_i^u = \left(\int_0^1 c_i^u(z)^{\frac{\sigma-1}{\sigma}} dz\right)^{\frac{\sigma}{\sigma-1}},\ i = 1, 2.$$

σ is the elasticity of substitution among the goods, and $\sigma > 1$.

The price of consumption basket in Period i is

$$P_i = \left(\int_0^1 p_i(z)^{1-\sigma} dz\right)^{\frac{1}{1-\sigma}},\ i = 1, 2.$$

The budget constraint for en employed consumer is[3]

$$P_1 C_1^e + P_2 C_2^e = Wl + \Pi - D - \Theta + \hat{Q} - \Psi.$$

The budget constraint for en unemployed consumer is

$$P_1 C_1^u + P_2 C_2^u = \Pi - D + R + \hat{Q}$$

Since $R = D$,

$$P_1 C_1^u + P_2 C_2^u = \Pi + \hat{Q}.$$

Let

$$\alpha = \frac{P_1 C_1^e}{P_1 C_1^e + P_2 C_2^e},\ 1 - \alpha = \frac{P_2 C_2^e}{P_1 C_1^e + P_2 C_2^e}. \tag{1}$$ al1

Since the utility functions $u(C_1^e, C_2^e, D)$ and $u(C_1^u, C_2^u, D)$ are homothetic, α is determined by the relative price $\frac{P_2}{P_1}$, and do not depend on the income of the consumers. Therefore, we have

$$\alpha = \frac{P_1 C_1^e}{P_1 C_1^e + P_2 C_2^e} = \frac{P_1 C_1^u}{P_1 C_1^u + P_2 C_2^u},$$

$$1 - \alpha = \frac{P_2 C_2^e}{P_1 C_1^e + P_2 C_2^e} = \frac{P_2 C_2^u}{P_1 C_1^u + P_2 C_2^u},$$

From the first order conditions and the budget constraints for employed and unemployed consumers we obtain the following demand functions for consumption baskets.

$$C_1^e = \alpha \frac{Wl + \Pi - D - \Theta + \hat{Q} - \Psi}{P_1},\ C_2^e = (1-\alpha) \frac{Wl + \Pi - D - \Theta + \hat{Q} - \Psi}{P_2},$$

and

$$C_1^u = \alpha \frac{\Pi + \hat{Q}}{P_1},\ C_2^u = (1-\alpha) \frac{\Pi + \hat{Q}}{P_2}.$$

Lagrange functions in the second step for employed and unemployed consumers are

$$\begin{aligned} \mathscr{L}_1^e = &\left(\int_0^1 c_1^e(z)^{\frac{\sigma-1}{\sigma}} dz\right)^{\frac{\sigma}{\sigma-1}} \\ &- \lambda_1^e \left[\int_0^1 p_1(z) c_1^e(z) dz - \alpha(Wl + \Pi - D - \Theta + \hat{Q} - \Psi)\right], \end{aligned} \tag{2}$$ cal1

[3] Employed consumers of the younger generation lend money to consumers in the childhood period of the next generation. It is repaid in the next period.

$$\mathscr{L}_2^e=\left(\int_0^1 c_2^e(z)^{\frac{\sigma-1}{\sigma}}dz\right)^{\frac{\sigma}{\sigma-1}}$$
$$-\lambda_2^e\left[\int_0^1 p_2(z)c_2^e(z)dz-(1-\alpha)(Wl+\Pi-D-\Theta+\hat{Q}-\Psi)\right],$$

$$\mathscr{L}_1^u=\left(\int_0^1 c_1^u(z)^{\frac{\sigma-1}{\sigma}}dz\right)^{\frac{\sigma}{\sigma-1}}-\lambda_1^u\left[\int_0^1 p_1(z)c_1^u(z)dz-\alpha(\Pi+\hat{Q})\right],$$

and

$$\mathscr{L}_2^u=\left(\int_0^1 c_2^u(z)^{\frac{\sigma-1}{\sigma}}dz\right)^{\frac{\sigma}{\sigma-1}}-\lambda_2^u\left[\int_0^1 p_2(z)c_2^u(z)dz-\alpha(\Pi+\hat{Q})\right].$$

λ_1^e, λ_2^e, λ_1^u and λ_2^u are Lagrange multipliers. Solving these maximization problems, the following demand functions of employed and unemployed consumers are derived[4].

$$c_1^e(z)=\left(\frac{p_1(z)}{P_1}\right)^{-\sigma}\frac{\alpha(Wl+\Pi-D-\Theta+\hat{Q}-\Psi)}{P_1},$$

$$c_2^e(z)=\left(\frac{p_2(z)}{P_2}\right)^{-\sigma}\frac{(1-\alpha)(Wl+\Pi-D-\Theta+\hat{Q}-\Psi)}{P_2},$$

$$c_1^u(z)=\left(\frac{p_1(z)}{P_1}\right)^{-\sigma}\frac{\alpha(\Pi+\hat{Q})}{P_1},$$

and

$$c_2^u(z)=\left(\frac{p_2(z)}{P_2}\right)^{-\sigma}\frac{(1-\alpha)(\Pi+\hat{Q})}{P_2}.$$

From these analyses we obtain the indirect utility functions of employed and unemployed consumers as follows:

$$V^e=u\left(\alpha\frac{Wl+\Pi-D-\Theta+\hat{Q}-\Psi}{P_1},(1-\alpha)\frac{Wl+\Pi-D-\Theta+\hat{Q}-\Psi}{P_2},D\right)-\Gamma(l),$$

and

$$V^u=u\left(\alpha\frac{\Pi+\hat{Q}}{P_1},(1-\alpha)\frac{\Pi+\hat{Q}}{P_2},D\right).$$

Let

$$\omega=\frac{W}{P_1},\ \rho=\frac{P_2}{P_1}.$$

Then, since the real value of D in the childhood period is constant, we can write

$$V^e=\varphi\left(\omega l+\frac{\Pi-D-\Theta+\hat{Q}-\Psi}{P_1},\rho\right)-\Gamma(l),$$

[4]About some calculations of these maximization problems please see Appendix

$$V^u = \varphi\left(\frac{\Pi + \hat{Q}}{P_1}, \rho\right),$$

ω is the real wage rate. Denote

$$I = \omega l + \frac{\Pi - D - \Theta + \hat{Q} - \Psi}{P_1}. \qquad (3)\ \boxed{\mathtt{i}}$$

The condition for maximization of V^e with respect to l given ρ is

$$\frac{\partial \varphi}{\partial I}\omega - \Gamma'(l) = 0, \qquad (4)\ \boxed{\mathtt{ve}}$$

where

$$\frac{\partial \varphi}{\partial I} = \alpha \frac{\partial u}{\partial C_1^e} + (1-\alpha)\frac{\partial u}{\partial C_2^e}.$$

Given P_1 and ρ the labor supply is a function of ω. From (4) we get

$$\frac{dl}{d\omega} = \frac{\frac{\partial \varphi}{\partial I} + \frac{\partial^2 \varphi}{\partial I^2}\omega l}{\Gamma''(l) - \frac{\partial^2 \varphi}{\partial I^2}\omega^2}. \qquad (5)\ \boxed{\mathtt{ve2}}$$

If $\frac{dl}{d\omega} > 0$, the labor supply is increasing with respect to the real wage rate ω.

2.2 Firms

Let $d_1(z)$ be the total demand for good z by younger generation consumers in Period 1. Then,

$$\begin{aligned} d_1(z) &= \left(\frac{p_1(z)}{P_1}\right)^{-\sigma} \frac{\alpha(WLl + L_f\Pi - LD - L\Theta + L_f\hat{Q} - L\Psi)}{P_1} \\ &= \left(\frac{p_1(z)}{P_1}\right)^{-\sigma} \frac{\alpha\left(WLl + L_f\Pi - L_fD + L_f\hat{Q} - L_fQ\right)}{P_1}. \end{aligned}$$

This is the sum of the demand of employed and unemployed consumers. Note that $\hat{Q}$ is the pay-as-you-go pension for younger generation consumers in their Period 2. Similarly, their total demand for good z in Period 2 is written as

$$d_2(z) = \left(\frac{p_2(z)}{P_2}\right)^{-\sigma} \frac{(1-\alpha)\left(WLl + L_f\Pi - L_fD + L_f\hat{Q} - L_fQ\right)}{P_2}.$$

Let $\overline{d_2(z)}$ be the demand for good z by the older generation. Then,

$$\overline{d_2(z)} = \left(\frac{p_1(z)}{P_1}\right)^{-\sigma} \frac{(1-\bar{\alpha})\left(\bar{W}\bar{L}\bar{l} + L_f\bar{\Pi} - L_f\bar{D} + L_fQ - L_f\bar{Q}\right)}{P_1},$$

where $\bar{W}$, $\bar{\Pi}$, $\bar{L}$, $\bar{l}$, $\bar{D}$ and $\bar{Q}$ are the nominal wage rate, the profits of firms, the employment, the individual labor supply, the debt of an individual, and the pay-as-you-go pension, respectively, during the previous period. $\bar{\alpha}$ is the value of α for the older generation. Q is the pay-as-you-go pension for consumers of the older generation themselves. Let

$$M = (1 - \bar{\alpha}) \left(\bar{W}\bar{L}\bar{l} + L_f\bar{\Pi} - L_f D + L_f Q - L_f \bar{Q} \right).$$

This is the total savings or the total consumption of the older generation consumers including the pay-as-you-go pensions they receive in their Period 2. It is the planned consumption that is determined in Period 1 of the older generation consumers. Net savings is the difference between M and the pay-as-you-go pensions in their Period 2, as follows:

$$M - L_f Q.$$

Their demand for good z is written as $\left(\frac{p_1(z)}{P_1}\right)^{-\sigma} \frac{M}{P_1}$. Government expenditure constitutes the national income as well as the consumptions of the younger and older generations. Then, the total demand for good z is written as

$$d(z) = \left(\frac{p_1(z)}{P_1} \right)^{-\sigma} \frac{Y}{P_1}, \tag{6}$$ dz

where Y is the effective demand defined by

$$Y = \alpha \left(WLl + L_f \Pi - L_f D + L_f \hat{Q} - L_f Q \right) + G + L_f \hat{D} + M.$$

Note that $\hat{D}$ is consumption in the childhood period of a next generation consumer. G is the government expenditure, except for the pay-as-you-go pensions, scholarships and unemployment benefits (see Otaki (2007), Otaki (2015) about this demand function). Now, we assume that G is financed by seigniorage similarly to Otaki (2007) and Otaki (2009). In a later section, we will consider the government's budget constraint with respect to taxes.

Let L and Ll be employment and the "employment $\times$ labor supply" of firm z. The total employment and the total "employment $\times$ labor supply" are also

$$\int_0^1 L dz = L, \quad \int_0^1 Ll dz = Ll.$$

The output of firm z is $Lly(Ll)$. At the equilibrium $Lly(Ll) = d(z)$. Then, we have

$$\frac{\partial d(z)}{\partial p_1(z)} = (y(Ll) + Lly') \frac{\partial (Ll)}{\partial p_1(z)}.$$

From (6)

$$\frac{\partial d(z)}{\partial p_1(z)} = -\sigma \frac{d(z)}{p_1(z)}.$$

The profit of firm z is

$$\pi(z) = p_1(z)d(z) - \frac{d(z)}{y(Ll)}W.$$

The condition for profit maximization is

$$\begin{aligned}\frac{\partial\pi(z)}{\partial p_1(z)} =& d(z) + \left(p_1(z) - \frac{W}{y(Ll)} + \frac{\frac{y'd(z)}{y(Ll)+Lly'}}{y(Ll)^2}W\right)\frac{\partial d(z)}{\partial p_1(z)} \\ =& d(z) + \left(p_1(z) - \frac{W}{y(Ll)} + \frac{\frac{Lly'}{y(Ll)+Lly'}}{y(Ll)}W\right)\frac{\partial d(z)}{\partial p_1(z)} \\ =& d(z) - \sigma\left(p_1(z) - \frac{W}{y(Ll)+Lly'}\right)\frac{d(z)}{p_1(z)} = 0\end{aligned}$$

Therefore, we obtain

$$p_1(z) = -\frac{\sigma}{(1-\sigma)(1+\zeta)y(Ll)}W.$$

Let $\mu = \frac{1}{\sigma}$. Then,

$$p_1(z) = \frac{1}{(1-\mu)(1+\zeta)y(Ll)}W.$$

This means that the real wage rate is

$$\omega = (1-\mu)(1+\zeta)y(Ll). \tag{7}$$ real

With increasing (decreasing or constant) returns to scale, ω is increasing (decreasing or constant) with respect to "employment $\times$ labor supply" Ll.

From (3), (4) and (7), we have

$$\frac{\partial\varphi}{\partial I}(1-\mu)(1+\zeta)y(Ll) - \Gamma'(l) = 0,$$

with

$$I = (1-\mu)(1+\zeta)y(Ll)l + \frac{\Pi - D - \Theta + \hat{Q} - \Psi}{P_1}.$$

Then, from (5)

$$\frac{dl}{d(Ll)} = \frac{dl}{d\omega}\frac{d\omega}{d(Ll)} = \frac{\left[\frac{\partial\varphi}{\partial I} + \frac{\partial^2\varphi}{\partial I^2}(1-\mu)(1+\zeta)y(Ll)l\right](1-\mu)(1+\zeta)y'}{\Gamma''(l) - \frac{\partial^2\varphi}{\partial I^2}[(1-\mu)(1+\zeta)y']^2}.$$

Assuming $\frac{dl}{d\omega} > 0$, with increasing (decreasing) returns to scale $y' > 0$ ($y < 0$), this is positive (negative). Since

$$\frac{d(Ll)}{dL} = l + L\frac{dl}{dL}, \tag{8}$$ lll

$$\frac{dl}{dL} = \frac{dl}{d(Ll)}\frac{d(Ll)}{dL} = \left(l + L\frac{dl}{dL}\right)\frac{dl}{d(Ll)}.$$

Thus,

$$\frac{dl}{dL} = \frac{l}{1 - L\frac{dl}{d(Ll)}}\frac{dl}{d(Ll)}.$$

Usually $\frac{dl}{dL}$ and $\frac{dl}{d(Ll)}$ have the same sign, and we assume $\frac{d(Ll)}{dL} > 0$ in (8). Also, since $-1 < \zeta < 1$, we have

$$\frac{d(Lly(Ll))}{Ll} = y(Ll) + Lly' = y(Ll)(1 + \zeta) > 0. \quad (9) \boxed{\texttt{out1}}$$

Then, the output $Lly(Ll)$ increases by an increase in L.

Since all firms are symmetric,

$$P_1 = p_1(z) = \frac{1}{(1-\mu)(1+\zeta)y(Ll)}W. \quad (10) \boxed{\texttt{price}}$$

2.3 Involuntary unemployment

Aggregate supply of the goods is equal to

$$WL + L_f\Pi = P_1Lly(Ll).$$

Aggregate demand is

$$\begin{aligned}&\alpha\left(WL + L_f\Pi - L_fD + L_f\hat{Q} - L_fQ\right) + G + L_f\hat{D} + M\\ =&\alpha\left[P_1Lly(Ll) - L_fD + L_f\hat{Q} - L_fQ\right] + G + L_f\hat{D} + M.\end{aligned}$$

Since they are equal,

$$P_1Lly(Ll) = \alpha\left[P_1Lly(Ll) - L_fD + L_f\hat{Q} - L_fQ\right] + G + L_f\hat{D} + M,$$

or

$$P_1Lly(Ll) = \frac{\alpha\left(-L_fD + L_f\hat{Q} - L_fQ\right) + G + L_f\hat{D} + M}{1-\alpha}.$$

In real terms[5]

$$Lly(Ll) = \frac{\alpha\left(-L_fD + L_f\hat{Q} - L_fQ\right) + G + L_f\hat{D} + M}{(1-\alpha)P_1}, \quad (11) \boxed{\texttt{e3}}$$

or

$$Ll = \frac{\alpha\left(-L_fD + L_f\hat{Q} - L_fQ\right) + G + L_f\hat{D} + M}{(1-\alpha)P_1y(Ll)}.$$

From (4) and (5) the individual labor supply l is a (usually increasing) function of ω. From (7) ω is a function of Ll. With increasing (decreasing or constant) returns to scale technology it is increasing (decreasing or constant) with respect to Ll or with respect

[5] $\frac{1}{1-\alpha}$ is a multiplier.

to L given l. The individual labor supply l may be increasing or decreasing in L or Ll. However, we assume that Ll is increasing in L. This requires

$$\frac{dLl}{dL} = l + \frac{dl}{dL} > 0.$$

It means $Ll < L_f l$ for $L < L_f$. The equilibrium value of Ll cannot be larger than $L_f l$. However, it may be strictly smaller than $L_f l$. Then, we have $L < L_f$ and involuntary umemployment exists.

If the government collects a lump-sum tax T from the younger generation consumers, the aggregate demand is

$$\begin{aligned}
&\alpha\left(WL + L_f\Pi - T - L_f D + L_f\hat{Q} - L_f Q\right) + G + L_f\hat{D} + M \\
=&\alpha\left[P_1 Lly(Ll) - T - L_f D + L_f\hat{Q} - L_f Q\right] + G + L_f\hat{D} + M.
\end{aligned}$$

2.4 Discussion summary

The real wage rate depends on the employment elasticity of the labor productivity and the employment level. But the employment level does not depend on the real wage rate. The real aggregate demand and the employment level are determined by the value of

$$\frac{\alpha\left(-L_f D + L_f\hat{Q} - L_f Q\right) + G + L_f\hat{D} + M}{P_1}. \qquad (12)\ \boxed{11}$$

If the employment is smaller than the labor population, then involuntary unemployment exists.

2.5 The case of full-employment

If $Ll = L_f l$, full-employment is achieved. Then, (11) is re-written as

$$L_f l y(L_f l) = \frac{\alpha\left(-L_f D + L_f\hat{Q} - L_f Q\right) + G + L_f\hat{D} + M}{(1-\alpha)P_1}. \qquad (13)\ \boxed{\text{e4}}$$

Since L_f and $L_f l$ are constant (if $L = L_f$, ω is constant), this is an identity not an equation. On the other hand, (11) is an equation not an identity. (13) should be written as

$$\frac{\alpha\left(-L_f D + L_f\hat{Q} - L_f Q\right) + G + L_f\hat{D} + M}{(1-\alpha)P_1} \equiv L_f l y(L_f l).$$

This yields:

$$P_1 = \frac{1}{(1-\alpha)L_f l y(L_f l)}\left[\alpha\left(-L_f D + L_f\hat{Q} - L_f Q\right) + G + L_f\hat{D} + M\right].$$

Then, the nominal wage rate is determined by:

$$W = (1-\mu)(1+\zeta)y(L_f l)P_1.$$

3 Effects of a decrease in the nominal wage rate

In the model of this paper, no mechanism determines the nominal wage rate except at the full-employment state. For example, when the nominal value of G increases, the nominal aggregate demand and supply increase. If the nominal wage rate rises, the prices also rise. If, when G increases, the prices rise considerably, then the outputs might not increase and involuntary unemployment might not decrease. If the prices do not rise or rise only slightly, involuntary unemployment decreases.

Let us examine the effects on employment of a decrease in the nominal wage rate. A decrease in the nominal wage rate induces decreases in the prices of the goods (see (10)), and it does not directly rescue involuntary unemployment. Proposition 2.1 in Otaki (2016) says

> Suppose that the nominal wage sags. Then, as far as its indirect effects on the aggregate demand are negligible, this only results in causing a proportionate fall in the price level. In other words, a fall in the nominal wage never rescues workers who are involuntarily unemployed.

However, *indirect effects* on aggregate demand due to a fall in the nominal wage rate may exist. We assume that falls in the nominal wage rate and the prices are not predicted by consumers. If the prices of the goods fall, the real value of the older generation's savings increases. But, at the same time, decreases in the prices of the goods increase the real value of the younger generation consumers' debts.

The real values of the following variables will be maintained even when both the nominal wage rate and the prices fall.

G/P_1: the government expenditure.

$\hat{D}/P_1$: consumption in the childhood period of a next generation consumer.

Q/P_1: pay-as-you-go pension for an older generation consumer.

$\hat{Q}/P_1$: pay-as-you-go pension for a younger generation consumer when he retires.

On the other hand, the nominal value of D and that of $M - L_f Q$, which is the older generation's net savings, do not change. Therefore, from (12), whether a fall in the nominal wage rate increases or decreases the effective demand depends on whether

$$M - L_f Q - \alpha L_f D \tag{14}$$

mq

is positive or negative. This is the so-called real balance effect. If D or Q is large, (14) is negative, and a fall in the nominal wage rate increases involuntary unemployment[6].

4 Analysis of fiscal policy

4.1 Steady state with constant employment under constant prices

First consider a steady state where the employment is constant. With constant employment the real wage rate and the labor supply do not change, thus the output also does

[6]The discussion in this section is from the different perspectives of the real balance effect for which the argument was fought by Pigou (1943) and Kalecki (1944).

not change. We assume also $\rho = 1$, that is, the constant prices of the goods. Consumers correctly predict that the prices are constant. Let T be the tax revenue. Then,

$$P_1 Lly(Ll) = \alpha\left[P_1 Lly(Ll) - T - L_f D + L_f \hat{Q} - L_f Q\right] + G + L_f \hat{D} + M. \qquad (15)\ \text{ss1}$$

At the steady sate it must be that $\hat{D} = D$ and $\hat{Q} = Q$. Thus,

$$P_1 Lly(Ll) = \alpha\left[P_1 Lly(Ll) - T - L_f D\right] + G + L_f D + M. \qquad (16)\ \text{gt}$$

The savings of the younger generation including the pay-as-you-go pension is equal to M. Therefore,

$$(1-\alpha)\left[P_1 Lly(Ll) - T - L_f D\right] = G - T + M = M. \qquad (17)\ \text{fis1}$$

This means

$$G - T = 0.$$

Thus, to maintain a sate with constant employment and prices we need balanced budget.

4.2 Fiscal policy for full-employment under constant prices

Next, consider a fiscal policy to achieve full-employment from the state with involuntary unemployment. The employment L and the output $Lly(Ll)$ increase by fiscal policy. We assume constant prices of the goods again. Consumers correctly predict that the prices are constant. If the employment L increases, the labor supply l, the real wage rate ω and the labor productivity $y(Ll)$ increase in the case of increasing returns to scale. However, in the case of decreasing returns to scale, the labor supply, the real wage rate and the labor productivity may decrease. In the former (latter) case the rate of increase in the output is probably larger (smaller) than the rate of increase in the employment. By (9) we can assume that both are positive.

Let G' and T' be the government expenditure and the tax to achieve full-employment. Then, (16) is written as

$$P_1 L_f ly(L_f l) = \alpha\left[P_1 L_f ly(L_f l) - T' - L_f D\right] + G' + L_f D + M. \qquad (18)\ \text{td1}$$

From this

$$(1-\alpha)\left[P_1 L_f ly(L_f l) - T' - L_f D\right] = G' - T' + M. \qquad (19)\ \text{fis2}$$

Suppose $P_1 L_f ly(L_f l) - T' > P_1 Lly(Ll) - T$, that is, the realization of full employment will increase consumers' disposable income. Then, from (17) and (19) we get

$$G' - T' > 0. \qquad (20)\ \text{?gt2?}$$

From this we obtain the following proposition.

Proposition 1. *If the realization of full employment will increase consumers' disposable income, in order to achieve full-employment from a state with involuntary unemployment, we need a budget deficit.*
⟨p1⟩

Let G'', T'' and M' be the government expenditure, the tax revenue and the savings of the younger generation consumers in the next period after realization of full-employment. (16) is written as

$$P_1 L_f l y(L_f l) = \alpha \left[P_1 L_f l y(L_f l) - T'' - L_f D \right] + G'' + L_f D + M'.$$

To maintain full-employment the savings of the younger generation including the pay-as-you-go pension must be equal to M'. Then, we have

$$(1-\alpha)[P_1 L l y(L l) - T'' - L_f D] = G'' - T'' + M' = M'.$$

Therefore,

$$G'' - T'' = 0.$$

This means that to maintain full-employment, budget deficit is not required. Thus, we obtain the following proposition.

Proposition 2. *If the full-employment state has been achieved by fiscal policy, we do not need budget deficit to maintain full-employment.*

⟨p2⟩

A simple example

Assume $M = 0$ and $T' = 0$ in (18). Then,

$$P_1 L_f l y(L_f l) = \alpha \left[P_1 L_f l y(L_f l) - L_f D \right] + G' + L_f D.$$

This means

$$G' = (1-\alpha) \left[P_1 L_f l y(L_f l) - L_f D \right].$$

This is the government expenditure necessary to achieve full employment, and it is equal to the savings of the younger generation. Let denote it by M'.

In the next period the following relation holds.

$$P_1 L_f l y(L_f l) = \alpha \left[P_1 L_f l y(L_f l) - T'' - L_f D \right] + G'' + L_f D + M'.$$

To maintain full-employment with $T'' = 0$ we need

$$(1-\alpha)[P_1 L l y(L l) - L_f D] = G'' + M' = M'.$$

Thus,

$$G'' = 0.$$

Demand and supply of money

Demand for money carried over from Period 1 to Period 2 by consumers of the younger generation is equal to the savings by consumers of the younger generation. It is equal to "consumption by consumers of the younger generation in the next period" − "pay-as-you-go pension for the younger generation in the next period" − "repayment of the debt by consumers of the next generation".

On the other hand, supply of money is equal to

"consumption by consumers of the older generation" − "pay-as-you-go pension for the older generation" − "repayment of the debt by consumers of the younger generation" + "government expenditure" − "taxes for government expenditure".

Under constant prices we have

"pay-as-you-go pension for the younger generation in the next period" = "pay-as-you-go pension for the older generation",

and

"repayment of the debt by consumers the next generation" = "repayment of the debt by consumers of the younger generation".

Scholarships are offset by supply and repayment.

Then, (19) means

"consumption by consumers of the younger generation in the next period" − "consumption by consumers of the older generation" = "government expenditure" − "taxes for government expenditure".

Thus, demand for money and supply of money are equal, and money supply increases by

"government expenditure" − "taxes for government expenditure".

4.3 Realization of full-employment under inflation or deflation

First we assume that the output and the employment are constant, and the prices of the goods rise or fall at the rate $\rho - 1$. If $\rho > 1(< 1)$, consumers correctly predict that the prices rise (fall). Let T be the tax revenue. With $\rho \neq 0$, $\hat{D} = \rho D$ and $\hat{Q} = \rho Q$. Thus, (15) is written as

$$P_1 L l y(Ll) = \alpha \left[P_1 L l y(Ll) - T - L_f D + (\rho - 1) L_f Q\right] + G + \rho L_f D + M. \quad (21) \ \texttt{gt3}$$

The savings of the younger generation including the pay-as-you-go pension must be equal to ρM. Therefore,

$$(1 - \alpha) \left[P_1 L l y(Ll) - T - L_f D + (\rho - 1) L_f Q\right] = G - T + (\rho - 1) L_f (D + Q) + M = \rho M. \quad (22) \ \texttt{fis4}$$

This means that:

$$G - T = (\rho - 1)(M - L_f D - L_f Q). \quad (23) \ \texttt{gt-1}$$

If $M > L_f D + L_f Q$, in order to maintain a state where the output and the employment are constant with rising prices ($\rho > 1$) (falling prices ($\rho < 1$)) a budget deficit (surplus) is required. If $M < L_f D + L_f Q$, we obtain the inverse results.

Let G' and T' be the government expenditure and the tax to achieve full-employment. Then, (21) is written as

$$P_1 L_f l y(L_f l) = \alpha \left[P_1 L_f l y(L_f l) - T' - L_f D + (\rho - 1) L_f Q\right] + G' + \rho L_f D + M.$$

From this

$$(1-\alpha)\left[P_1 L_f l y(L_f l) - T' - L_f D + (\rho - 1) L_f Q\right] = G' - T' + (\rho - 1) L_f (D + Q) + M. \tag{24}$$

fis3

Suppose $P_1 L_f l y(L_f l) - T' > P_1 L l y(L l) - T$, that is, the realization of full employment will increase consumers' disposable income. Then, from (22) and (24) we get

$$G' - T' > (\rho - 1)(M - L_f D - L_f Q).$$

Therefore, in order to achieve full-employment under inflation or deflation we need budget deficit which is larger than (23).

Let G'', T'', M' and P_1' be the government expenditure, the tax revenue, the savings of the younger generation consumers and the price of the consumption basket in the next period after realization of full-employment. (21) is written as

$$P_1' L_f l y(L_f l) = \alpha \left[P_1' L_f l y(L_f l) - T'' - L_f D + (\rho - 1) L_f Q\right] + G'' + \rho L_f D + M'.$$

To maintain full-employment, the savings of the younger generation including the pay-as-you-go pension must be equal to $\rho M'$. Then, we have

$$(1-\alpha)[P_1' L_f l y(L_f l) - T'' - L_f D + (\rho - 1) L_f Q] = G'' - T'' + (\rho - 1) L_f (D + Q) + M' = \rho M'.$$

Therefore,

$$G'' - T'' = (\rho - 1)(M' - L_f D - L_f Q).$$

This means that to maintain full-employment, budget deficit larger than (23) is not required.

5 Game-theoretic interpretation of involuntary unemployment and full-employment

⟨game⟩ A steady state under balanced budget with involuntary unemployment is in a Nash equilibrium of a game with firms and consumers.

1. Given the government expenditure and tax, and the strategies of consumers and other firms, each firm maximizes its profit. Consumers' strategies are labor supply and consumption. Firms' strategies are employment and production.

2. Given the government expenditure and tax, and the strategies of other consumers and firms, each employed consumer and each unemployed consumer maximize their utility. Each unemployed consumer determines his strategy given a state where he is not employed.

Further we present three more results.

1. Increases in employment and production by firms and increases in labor supply and consumption by the younger generation consumers take the state out of the Nash equilibrium because consumption of the older generation consumers is insufficient.

2. If the government increases its expenditure keeping taxes intact, the full-employment state may be in a Nash equilibrium. The budget deficit makes up for deficiency of consumption of the older generation consumers.

3. Then, in the next period we can achieve full-employment without budget deficit because consumption of the older generation consumers, who work when they are young, is larger than consumption of the older generation consumers in the previous period. This is a property of a dynamic OLG model.

6 Discussion and Concluding Remarks

From Propositions 1 and 2 we can say that in order to achieve full-employment from a state with involuntary unemployment we need budget deficit of the government. However, when full-employment is achieved, in order to maintain full-employment we need balanced budget. Therefore, additional government expenditure to achieve full-employment should be financed by seigniorage not public debt.

We have examined the existence of involuntary umemployment and the effects of fiscal policy using a three-generation OLG model under monopolistic competition with increasing, decreasing or constant returns to scale. We considered the case of a divisible labor supply, and we assumed that the goods are produced only by labor.

In future research, we want to analyze involuntary unemployment and fiscal policy in a situation where goods are produced by capital and labor, and there exist investments of firms.

Appendix: Some calculations

The first order condition for (2) is

$$\left(\int_0^1 c_1^e(z)^{\frac{\sigma-1}{\sigma}}dz\right)^{\frac{1}{\sigma-1}} c_1^e(z)^{-\frac{1}{\sigma}} - \lambda_1^e p_1(z) = 0. \tag{A.1}$$

ap1

From this

$$\left(\int_0^1 c_1^e(z)^{\frac{\sigma-1}{\sigma}}dz\right)^{-1} c_1^e(z)^{\frac{\sigma-1}{\sigma}} = \left(\lambda_1^e\right)^{1-\sigma} p_1(z)^{1-\sigma}.$$

Then,

$$\left(\int_0^1 c_1^e(z)^{\frac{\sigma-1}{\sigma}}dz\right)^{-1} \int_0^1 c_1^e(z)^{\frac{\sigma-1}{\sigma}}dz = \left(\lambda_1^e\right)^{1-\sigma} \int_0^1 p_1(z)^{1-\sigma}dz = 1,$$

It means

$$\lambda_1^e \left(\int_0^1 p_1(z)^{1-\sigma}dz\right)^{\frac{1}{1-\sigma}} = 1,$$

and so

$$P_1 = \frac{1}{\lambda_1^e}.$$

From (A.1)

$$\left(\int_0^1 c_1^e(z)^{\frac{\sigma-1}{\sigma}}dz\right)^{\frac{1}{\sigma-1}} c_1^e(z)^{\frac{\sigma-1}{\sigma}} = \lambda_1^e p_1(z) c_1^e(z).$$

Then,

$$\left(\int_0^1 c_1^e(z)^{\frac{\sigma-1}{\sigma}}dz\right)^{\frac{1}{\sigma-1}} \int_0^1 c_1^e(z)^{\frac{\sigma-1}{\sigma}}dz = \left(\int_0^1 c_1^e(z)^{\frac{\sigma-1}{\sigma}}dz\right)^{\frac{\sigma}{\sigma-1}}$$

$$=C_1^e = \lambda_1^e \int_0^1 p_1(z) c_1^e(z)dz = \frac{1}{P_1} \int_0^1 p_1(z) c_1^e(z)dz.$$

Therefore,

$$\int_0^1 p_1(z) c_1^e(z)dz = P_1 C_1^e.$$

Similarly,

$$\int_0^1 p_2(z) c_2^e(z)dz = P_2 C_2^e.$$

Thus,

$$\int_0^1 p_1(z) c_1^e(z)dz + \int_0^1 p_2(z) c_2^e(z)dz = P_1 C_1^e + P_2 C_2^e = Wl + \Pi - D - \Theta + \hat{Q} - \Psi.$$

From (1)

$$P_1 C_1^e = \alpha(Wl + \Pi - D - \Theta + \hat{Q} - \Psi).$$

By (A.1)

$$\left(\int_0^1 c_1^e(z)^{\frac{\sigma-1}{\sigma}}dz\right)^{\frac{\sigma}{\sigma-1}} c_1^e(z)^{-1} = C_1^e c_1^e(z)^{-1} = \left(\lambda_1^e\right)^{\sigma} p_1(z)^{\sigma} = \left(\frac{p_1(z)}{P_1}\right)^{\sigma}.$$

From this we get

$$c_1^e(z) = \left(\frac{p_1(z)}{P_1}\right)^{-\sigma} \frac{\alpha(Wl + \Pi - D - \Theta + \hat{Q} - \Psi)}{P_1}.$$

$c_2^e(z)$, $c_1^u(z)$ and $c_2^u(z)$ are similarly obtained.

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