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26 August 2020

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August 26, 2020

Abstract

The developing literature on consumer information and vertical relations has yet to consider information provision via costly retail price advertising. By exploring this, we show that the double marginalisation problem exists in equilibrium despite an upstream supplier offering a two-part tariff that is common knowledge to consumers. Intuitively, the supplier elicits higher retail prices to strategically reduce retailers' advertising expenditure in order to extract additional rents. We then demonstrate how vertical restraints, such as resale price maintenance, can increase supply-chain profits and consumer welfare by lowering retail prices despite paradoxically discouraging price advertising.

Keywords: Price Advertising; Consumer Search; Double Marginalisation; Vertical Restraints; Clearinghouse

JEL Codes: D40; D83; L42

Word Count: 1996

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1 Introduction

There is a developing literature that shows how limited consumer information about prices is important for understanding vertical relations and the policy implications of vertical restraints (e.g. Asker and Bar-Isaac 2020, Janssen and Ke 2020, Janssen 2020). One strand of this literature demonstrates that when consumers acquire information about retail prices through costly search, the double marginalisation problem can become exaggerated when consumers are also uninformed of wholesale prices, leading to lower welfare (e.g. Janssen and Shelegia 2015, Garcia et al 2017).

While such information acquisition *by* consumers is undoubtedly important in understanding vertical relations, the literature has yet to consider information provision *to* consumers via retailers' costly price advertising. This omission is significant because, unlike the costs of consumer search, the costs of price advertising are borne by retailers. As such expenditures will affect the profits of the supply chain, they may not be in a supplier's interests. Consequently, a supplier may have an incentive to strategically deter retail advertising with different implications for prices and welfare.

To explore this, we introduce an upstream monopoly supplier into a clearinghouse-style model where retailers can advertise their price to consumers (e.g. Baye and Morgan 2001). We show how the double marginalisation problem exists in equilibrium when retailers' advertising expenditure is positive despite i) the supplier offering a two-part tariff, and ii) the tariff being common knowledge to consumers. Intuitively, the supplier elicits higher retail prices to reduce advertising expenditure in order to extract larger rents from retailers. We then demonstrate how vertical restraints, such as resale price maintenance (RPM), can increase supply-chain profits and consumer welfare by lowering retail prices despite paradoxically discouraging price advertising.

A loosely related paper is Asker and Bar-Isaac (2020). In a framework with *costless* price advertising, they analyse manufacturers' use of minimum-advertised-price (MAP) restrictions that enforce a price floor on retailers' advertised prices without restricting transaction prices. Given it is costless, retailers always advertise and manufacturers have no incentive to deter advertising expenditure. In contrast, rather than focussing on MAPs, we demonstrate the implications of *costly* price advertising on double marginalisation and vertical restraints more widely.

2 Model

Suppose a monopoly supplier, S, sells its product to consumers through $n \geq 2$ identical retailers, $i = \{1, ..., n\}$. The supplier's marginal cost is $c \geq 0$ and fixed costs are zero. To constrast with the previous literature (e.g. Janssen and Shelegia 2015), we deliberately focus on a simple setting, where it is common knowledge that the supplier offers all retailers the same two-part tariff with a wholesale price, w, and a fixed fee, F. Retailers have no additional production costs. Retailer i charges a unit price, p_i, and chooses whether to advertise this to consumers, $a_i = 1$, or not, $a_i = 0$, where advertising incurs a cost, $\phi \geq 0$.

There is a unit mass of consumers. At a retail price, p, each consumer demands $q(p) = \max\left\{\frac{v-p}{b}, 0\right\}$ units, where $v > c$ and $b > 0$. Consumers are uninformed about a retailer's price unless it is advertised. Consistent with Baye and Morgan (2001), consumers behave as follows. If one or more retailers advertise a price (weakly) below v, then all consumers buy at the cheapest advertised price (selecting at random amongst any tied retailers). If no retailer advertises, then consumers visit a retailer at random and buy provided the price does not exceed v.

The timing of the game is as follows. In Stage 1, S offers its tariff, $\{w, F\}$. In Stage 2, retailers and consumers observe $\{w, F\}$, then each retailer i simultaneously selects p_i and $a_i \in \{0, 1\}$. To allow for mixed strategies, define $\alpha_i \equiv Pr(a_i = 1) \in [0, 1]$, and $H_i^A(p)$ and $H_i^N(p)$ as retailer i's price distribution when $a_i = 1$ and $a_i = 0$, respectively. In Stage 3, consumers observe any adverts and behave as above. We analyse the symmetric subgame perfect Nash equilibria. Finally, denote i) (expected) total price advertising expenditure as $A = \phi \sum_i \alpha_i$, and ii) retailers' per-consumer profits as $\pi(p; w) \equiv (p - w)\, q(p)$, which are uniquely maximised at $p^m(w) = \frac{v+w}{2}$ so $\pi(p^m(w); w) = \frac{(v-w)^2}{4b} > 0 \;\forall\; w \leq v$.

3 Vertical Integration Benchmark

Before considering vertical separation, we outline the maximum profits the supply chain can achieve. These can be obtained under a vertically integrated monopoly which sets i) $p_i = p^m(c) = \frac{v+c}{2}\; \forall i$, and ii) price advertising expenditure to zero, $A^{VI} = \phi \sum_i \alpha_i = 0$.

Thus, industry profits and consumer surplus are

$$\Pi^{VI} = \pi\left(p^m\left(c\right);c\right) = \frac{\left(v-c\right)^2}{4b} \qquad \text{and} \qquad CS^{VI} = \int_{p^m(c)}^{v} q\left(p\right)dp = \frac{\left(v-c\right)^2}{8b}$$

and total welfare is

$$W^{VI} = \Pi^{VI} + CS^{VI} = \frac{3\left(v-c\right)^2}{8b}.$$

4 Vertical Separation

Now suppose the firms make decisions independently.

4.1 Retailers' Decisions

For a given $\{w, F\}$, the retailers' decisions follow from Baye and Morgan (2001, Proposition 3), so we just present the equilibrium and sketch the intuition below. To proceed, denote $\tilde{\phi}\left(w\right) \equiv \pi\left(p^m\left(w\right);w\right)\left(\frac{n-1}{n}\right) > 0$ and

$$\alpha^*\left(w,\phi\right) = \max\left\{0, 1-\left(\frac{\phi}{\tilde{\phi}\left(w\right)}\right)^{\frac{1}{n-1}}\right\} \tag{1}$$

Lemma 1. *For a given $\{w, F\}$, there is a unique symmetric equilibrium where:*

a) If advertising costs are high, $\phi \geq \tilde{\phi}\left(w\right)$, and $F \leq \frac{\pi(p^m(w);w)}{n}$, each retailer earns $\frac{\pi(p^m(w);w)}{n} - F \geq 0$ by never advertising, $\alpha^*\left(w,\phi\right) = 0$, and setting $p_i = p^m\left(w\right)$.

b) If advertising costs are low, $\phi < \tilde{\phi}\left(w\right)$, and $F \leq \frac{\phi}{n-1}$, each retailer earns $\frac{\phi}{n-1} - F \geq 0$ by i) not advertising and setting $p_i = p^m\left(w\right)$ with probability $1-\alpha^*\left(w,\phi\right) \in \left(0,1\right)$, and ii) advertising a price, $p_i \in \left[\underline{p}\left(w,\phi\right), p^m\left(w\right)\right]$, using distribution

$$H^A\left(p\right) = \frac{1}{\alpha^*\left(w,\phi\right)}\left(1-\left(\frac{n}{n-1}\frac{\phi}{\pi\left(p;w\right)}\right)^{\frac{1}{n-1}}\right)$$

with probability $\alpha^*\left(w,\phi\right) \in \left(0,1\right)$, where $\underline{p}\left(w,\phi\right)$ satisfies $\pi\left(p;w\right) = \frac{\phi n}{n-1}$.

c) Otherwise, the retailers are inactive.

To outline the intuition, first denote retailer i's expected profits when all retailers advertise with probability α and use advertised price distribution $H(p)$ as:

$$\Pi_{Ri}\left(p_i, a_i; \alpha\right) = \begin{cases} \pi\left(p_i; w\right)\left(1-\alpha H\left(p_i\right)\right)^{n-1} - F - \phi & \text{if } a_i = 1 \\ \frac{\pi(p_i;w)}{n}\left(1-\alpha\right)^{n-1} - F & \text{if } a_i = 0. \end{cases}$$

This states that retailer i must always pay F. If it advertises, $a_i = 1$, retailer i incurs ϕ but expects to earn $\pi\left(p_i; w\right)$ if it has the lowest advertised price, which occurs with probability $\left(1-\alpha H\left(p_i\right)\right)^{n-1}$. If it does not advertise, $a_i = 0$, it earns $\frac{\pi(p_i;w)}{n}$ provided no other retailer advertises which occurs with probability $\left(1-\alpha\right)^{n-1}$. Hence, retailer i optimally sets $p_i = p^m\left(w\right)$ whenever $a_i = 0$.

In case a) of Lemma 1, no retailer ever advertises, $\alpha^*\left(w,\phi\right) = 0$, and so retailer i earns $\Pi_{Ri}\left(p^m\left(w\right), 0; 0\right) = \frac{\pi(p^m(w);w)}{n} - F \geq 0$. This is an equilibrium because a deviant advertising retailer would obtain, at most, $\pi\left(p^m\left(w\right); w\right) - F - \phi$ by attracting all consumers at $p^m\left(w\right)$. Hence, $\phi \geq \tilde{\phi}\left(w\right) \equiv \pi\left(p^m\left(w\right); w\right)\left(\frac{n-1}{n}\right)$ ensures the advertising cost always exceeds the profits from gaining an extra $\left(\frac{n-1}{n}\right)$ consumers.

In case b), retailers have an incentive to advertise since $\phi < \tilde{\phi}\left(w\right)$. The resulting equilibrium involves the retailers mixing over advertising, $\alpha^*\left(w,\phi\right) \in (0,1)$, and mixing over advertised prices with $H^A(p)$. This balances the incentives to price high without advertising and to compete with lower advertised prices. Thus, each retailer must be indifferent over their equilibrium actions so $\alpha^*\left(w,\phi\right)$ is derived from $\Pi_{Ri}\left(p^m\left(w\right), 0; \alpha\right) = \Pi_{Ri}\left(p^m\left(w\right), 1; \alpha\right)$ and the other equilibrium expressions ensure $\Pi_{Ri}\left(p^m\left(w\right), 0; \alpha^*\left(w,\phi\right)\right) = \frac{\phi}{n-1} - F = \Pi_{Ri}\left(p, 1; \alpha^*\left(w,\phi\right)\right) \geq 0 \ \forall\ p \in \left[\underline{p}\left(w,\phi\right), p^m\left(w\right)\right]$.

Finally, case c) applies when F is so high that the retailers would make negative profits in a) or b).

Proposition 1. *When retailers are active, the expected minimum retail price equals*

$$\widehat{p}\left(w,\phi\right) = \begin{cases} p^m\left(w\right) & \text{if } \phi \geq \tilde{\phi}\left(w\right) \\ \underline{p}\left(w,\phi\right) + \displaystyle\int_{\underline{p}(w,\phi)}^{p^m(w)} G\left(p; w,\phi\right)^{\frac{n}{n-1}} dp \in \left(w, p^m\left(w\right)\right) & \text{if } \phi < \tilde{\phi}\left(w\right), \end{cases} \tag{2}$$

where $G(p;w,\phi) = \frac{n}{n-1}\frac{\phi}{\pi(p;w)}$, and expected total retailer profits, $n\Pi_R\left(F,\phi\right)$, equal

$$\pi\left(\widehat{p}\left(w,\phi\right); w\right) - \phi n \alpha^*\left(w,\phi\right) - nF = \begin{cases} \pi\left(p^m\left(w\right); w\right) - nF & \text{if } \phi \geq \tilde{\phi}\left(w\right) \\ n\left(\frac{\phi}{n-1} - F\right) & \text{if } \phi < \tilde{\phi}\left(w\right) \end{cases} \tag{3}$$

4.2 Supplier's Decision

S wishes to maximise its expected profits subject to each retailer earning non-negative expected profits:

$$\max_{w,F} \Pi_S\left(w,F;\widehat{p}\left(w,\phi\right)\right) = \left(w-c\right)q\left(\widehat{p}\left(w,\phi\right)\right)+nF$$

$$\text{s.t. } \Pi_R\left(F,\phi\right) = \tfrac{\pi\left(\widehat{p}\left(w,\phi\right);w\right)}{n} - \phi\alpha^*\left(w,\phi\right) - F \geq 0$$

Or, since the constraint is clearly binding:

$$\max_{w} \Pi_S\left(w,F^*;\widehat{p}\left(w,\phi\right)\right) = \left(\widehat{p}\left(w,\phi\right)-c\right)q\left(\widehat{p}\left(w,\phi\right)\right) - n\phi\alpha^*\left(w,\phi\right) \tag{4}$$

where the equilibrium fixed fee, F^*, sets $\Pi_R\left(F^*,\phi\right)=0$ such that

$$F^* \equiv \frac{\pi\left(\widehat{p}\left(w,\phi\right);w\right)}{n} - \phi\alpha^*\left(w,\phi\right) \tag{5}$$

Differentiating (4) with respect to w yields

$$\frac{d\Pi_S\left(w,F^*;\widehat{p}\left(w,\phi\right)\right)}{dw} = \frac{\partial\widehat{p}(w,\phi)}{\partial w}\left[q\left(p\right)+\left(p-c\right)\frac{\partial q}{\partial p}\right]_{p=\widehat{p}(w,\phi)} - n\phi\frac{\partial\alpha^*(w,\phi)}{\partial w}, \tag{6}$$

where (6) is used later to analyse the equilibrium wholesale price, w^*.

4.2.1 Outcomes under Zero Advertising Expenditure

Before detailing our main results, note the supply chain obtains the benchmark vertically-intergrated profits whenever there is zero equilibrium advertising expenditure, $A^* \equiv n\phi\alpha^*\left(w,\phi\right)=0$. This arises in two situations. First, when advertising costs are zero, $\phi=0$, retailers advertise with certainty, $\alpha^*\left(w,\phi\right)=1$, so the model converges to a simple setting with a fully competitive retail sector.

Proposition 2. *When advertising costs are zero, $\phi=0$, the supplier sets $w^*=p^m\left(c\right)>c$ and $F^*=0$. There is no double marginalisation, $\widehat{p}\left(w^*,0\right)=p^m\left(c\right)$, and the vertically-intergrated profits are obtained, $\Pi_S\left(p^m\left(c\right),0;p^m\left(c\right)\right)=\Pi^{VI}$.*

Proof. Substituting $\widehat{p}\left(w,0\right)=w$ and $\phi=0$ into (4)-(6) yields $w^*=\frac{v+c}{2}\equiv p^m\left(c\right)$ and $F^*=0$. $\square$

Second, when $\phi \geq \tilde{\phi}(w)$, retailers refrain from advertising completely, $\alpha^*(w,\phi) = 0$, and so retailers are local monopolies.

Proposition 3. *When advertising costs are high, $\phi \geq \tilde{\phi}(c)$, the supplier sets $w^* = c$ and $F^* = \frac{\pi(p^m(c);c)}{n}$. There is no double marginalisation, $\widehat{p}(w^*,\phi) = p^m(c)$, and the vertically-intergrated profits are obtained, $\Pi_S\left(c, \frac{\pi(p^m(c);c)}{n}; p^m(c)\right) = \Pi^{VI}$.*

Proof. Substituting $\widehat{p}(w,\phi) = p^m(c)$ and $\alpha^*(w,\phi) = 0$ into (4)-(6) yields $w^* = c$ and $F^* = \frac{\pi(p^m(c);c)}{n}$. $\square$

In both Propositions, the supplier uses w to incentivise retailers to set $p^m(c)$ and extracts any remaining rents through F. These two specific cases follow standard results where a monopoly supplier can use a two-part tariff to overcome the double marginalisation problem.

4.2.2 Outcomes under Positive Advertising Expenditure

For our main result, we now show that when advertising expenditure is positive, $A^* \equiv n\phi\alpha^*(w,\phi) > 0$, S will surprisingly incentivise the expected minimum retail price above $p^m(c)$.

Proposition 4. *When advertising costs are low, $\phi \in (0, \tilde{\phi}(c))$, the supplier sets $w^* > c$ and $F^* < \frac{\pi(p^m(c);c)}{n}$ in any symmetric equilibrium. There is double marginalisation, $\widehat{p}(w^*,\phi) > p^m(c)$, and the vertically-intergrated profits are not obtained, $\Pi_S(w^*, F^*; \widehat{p}(w^*,\phi)) \in \left(0, \Pi^{VI}\right)$.*

Intuitively, despite offering a two-part tariff, S optimally incentivises double marginalisation through a higher w in order to lower retailers' advertising expenditure. This enables it to use F to extract more rents that retailers would have otherwise spent on advertising. To understand in more detail why this tradeoff between double marginalisation and lower retail advertising is beneficial for S, consider a small increase in w at the level where $\widehat{p}(w,\phi) = p^m(c)$. From (6), this raises the expected minimum retail price, $\frac{\partial \widehat{p}(w,\phi)}{\partial w} > 0$, but there is no first-order effect on S's profit, $\left[q(p) + (p-c)\frac{\partial q}{\partial p}\right]_{p=p^m(c)} = 0$; however, there is a first-order increase in profit due to the lower advertising probability, $\frac{\partial \alpha^*(w,\phi)}{\partial w} < 0$. This fall in $\alpha^*(w,\phi)$ arises because a small increase in w reduces each retailer's maximum gain from advertising, represented by $\tilde{\phi}(w) \equiv \pi(p^m(w); w)\left(\frac{n-1}{n}\right)$ in (1).

Next, consider the welfare effects.

Proposition 5. *When $\phi \in (0, \tilde{\phi}(c))$, relative to the vertical integration benchmark, the expected minimum retail price and total advertising expenditure are higher, $\widehat{p}(w^*, \phi) > p^m(c)$ and $A^* \equiv n\phi\alpha^*(w, \phi) > A^{VI} = 0$, while consumer surplus and total welfare are lower,*

$$CS^* = \int_{\widehat{p}(w^*,\phi)}^{v} q(p)\, dp < CS^{VI} \qquad \textit{and} \qquad W^* = \Pi^* + CS^* < W^{VI}.$$

This implies that vertical integration would benefit the firms and consumers despite paradoxically eliminating price advertising.

5 Vertical Restraints

Now consider how vertical restraints can also be used to achieve the vertically-integrated benchmark profits when $\phi \in (0, \tilde{\phi}(c))$. We focus on resale price maintenance (RPM) but other vertical restraints that restrict downstream competition can also achieve the same outcome (e.g. quantity rationing, exclusive terrorities, advertising restrictions including MAPs).

Under RPM, retail prices and the two-part tariff are set by S, but retailers still control price advertising expenditure. However, by imposing retail prices of $p_i = p^m(c)$ $\forall i$ and setting $w = p^m(c)$ and $F = 0$, S ensures each retailer has no incentive to engage in costly advertising since retail price-cost margins are zero, such that $\phi > \tilde{\phi}(w) \equiv \pi(p^m(c); p^m(c)) \left(\frac{n-1}{n}\right) = 0$. Consequently, each retailer earns $\pi(p^m(c); p^m(c)) = 0$ and S obtains $\Pi^{VI} = \pi(p^m(c); c)$.

Proposition 6. *When advertising costs are low, $\phi \in (0, \tilde{\phi}(c))$, RPM eliminates double marginalisation, which raises profits, consumer surplus and total welfare, $\Pi^{RPM} = \Pi^{VI} > \Pi^*$, $CS^{RPM} = CS^{VI} > CS^*$ and $W^{RPM} = W^{VI} > W^*$, despite decreasing total advertising expenditure, $A^{RPM} = A^{VI} = 0 < A^*$.*

6 Discussion

Despite reducing retail price advertising, we have shown how vertical restraints that reduce intrabrand competition can decrease retail prices and increase welfare by lowering wholesale prices. This contrasts starkly to the mechanism cited by the US Supreme

Court in its landmark *Leegin* decision that overturned the longstanding per-se illegality of RPM in 2006. There, the associated reduction in intrabrand competition was supported partly because it "... encourages retailers to invest in services or promotional efforts ...".[1] This followed traditional arguments related to the underprovision of demand-increasing activities, which can impose positive externalities throughout the supply chain (e.g. Telser, 1960; Winter, 1993). Such activities include *persuasive* advertising, which raise consumers' willingness to pay for products, and *existence* advertising, which raises consumer awareness of products. In contrast, we have focussed on costly *price* advertising that has business-stealing effects, rather than demand-increasing effects. Such advertising is overprovided because it imposes a negative externality on the supplier by limiting the rent that can be extracted from retailers. Consequently, our model suggests that vertical restraints can enhance efficiency despite *discouraging* promotional efforts involving price advertising.

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Appendix

Proof of Proposition 1. If $\phi \geq \tilde{\phi}(w)$ and $F \leq \frac{\pi(p^m(w);w)}{n}$, then clearly $\widehat{p}(w,\phi) = p^m(w)$. If $\phi < \tilde{\phi}(w)$ and $F \leq \frac{\phi}{n-1}$, then $\widehat{p}(w,\phi)$ comes directly from Morgan et al (2006, p.328-9). To prove $\widehat{p}(w,\phi) \in (w, p^m(w))\ \forall\ \phi \in (0, \tilde{\phi}(w))$, note i) $\widehat{p}(w,0) = w$ because $\underline{p}(w,0) = w$ and $G(p;w,0) = 0$; ii) $\lim_{\phi \to \tilde{\phi}(w)} \widehat{p}(w,\phi) = p^m(w)$ because $\lim_{\phi \to \tilde{\phi}(w)} \underline{p}(w,\phi) = p^m(w)$; and iii) $\frac{\partial \widehat{p}}{\partial \phi} = \frac{n}{(n-1)} \int_{\underline{p}(w,\phi)}^{p^m(w)} G(p;w,\phi)^{\frac{n}{n-1}-1} \frac{\partial G(p;w,\phi)}{\partial \phi} dp > 0$. Finally, as $q(p)$ is linear, expected demand equals $q(p)$ evaluated at $\widehat{p}(w,\phi)$. Thus, $\pi(\widehat{p}(w,\phi);w) \equiv (\widehat{p}(w,\phi) - w)\, q(\widehat{p}(w,\phi))$. Together with Lemma 1, this gives (3). □

Proof of Proposition 4. First, note $\tilde{\phi}(w) = \frac{n(v-w)^2}{4b(n-1)}$ such that $\tilde{\phi}(w) < \tilde{\phi}(c)$ for any $w > c$ from $\frac{\partial \tilde{\phi}}{\partial w} < 0\ \forall w \in (c, v)$. Second, for any $\phi \in \left(0, \tilde{\phi}(c)\right)$, there exists a unique $\tilde{w}(\phi) \equiv v - 2\sqrt{\frac{\phi b n}{n-1}} \in (c, v)$ where $w \in (c, \tilde{w}(\phi)) \Longleftrightarrow \phi \in \left(0, \tilde{\phi}(w)\right)$.

Next, we establish that there exists a unique $w' \in (c, \tilde{w}(\phi))$ that sets $\widehat{p}(w',\phi) = p^m(c)$, where $\widehat{p}(w,\phi) \leq p^m(c)\ \forall\ w \leq w'$. Given $\phi < \tilde{\phi}(c)$ guarantees $\widehat{p}(c,\phi) < p^m(c)$ and $\widehat{p}(\tilde{w}(\phi),\phi) = p^m(\tilde{w}(\phi)) > p^m(c)$, it suffices to prove $\frac{\partial \widehat{p}}{\partial w} > 0\ \forall w \in (c, \tilde{w}(\phi))$. Differentiating (2) with respect to w yields

$$\frac{\partial \widehat{p}}{\partial w} = \frac{\partial \underline{p}(\cdot)}{\partial w} + G(p^m(w);w,\phi)^{\frac{n}{n-1}} \cdot \frac{\partial p^m(w)}{\partial w} - \frac{\partial \underline{p}(\cdot)}{\partial w} + \int_{\underline{p}(w,\phi)}^{p^m(w)} \frac{\partial}{\partial w} \left[G(p;w,\phi)^{\frac{n}{n-1}}\right] dp$$

where $\frac{\partial \widehat{p}}{\partial w} > 0$ as the second term equals $G\left(p^m\left(w\right); w, \phi\right)^{\frac{n}{n-1}} \cdot \frac{1}{2} > 0$ and

$$\frac{\partial}{\partial w}\left[G\left(p; w, \phi\right)^{\frac{n}{n-1}}\right] = \frac{n}{n-1}\left(\frac{n}{n-1}\frac{\phi}{\pi\left(p; w\right)}\right)^{\frac{n}{n-1}-1}\frac{n\phi}{n-1}\frac{q(p)}{\pi\left(p; w\right)^2}$$
$$= \frac{q(p)}{\phi}G\left(p; w, \phi\right)^{\frac{2n-1}{n-1}} > 0.$$

Next, evaluate (6) at any $w \leq w'$. The first term on the RHS is non-negative, since $\frac{\partial \widehat{p}}{\partial w} > 0$ and $\widehat{p}\left(w, \phi\right) \leq p^m\left(c\right)\ \forall\ w \leq w'$. The second term is strictly positive because $\frac{\partial \alpha^*}{\partial w} < 0$ given $\frac{\partial \tilde{\phi}}{\partial w} < 0\ \forall w \in \left(c, v\right)$. Thus, (6) is strictly positive $\forall\ w \leq w'$. Hence, $w^* > w' > c$ such that $\widehat{p}\left(w^*, \phi\right) > \widehat{p}\left(w', \phi\right) = p^m\left(c\right)$.

We know $\Pi_S\left(w^*, F^*; \widehat{p}\left(w^*, \phi\right)\right) > 0$ from $\Pi_S\left(w^*, F^*; \widehat{p}\left(w^*, \phi\right)\right) > \Pi_S\left(w', F^*; p^m\left(c\right)\right) > 0$ as $w' \in \left(c, v\right)$. Further, given $\widehat{p}\left(w^*, \phi\right) > p^m\left(c\right)$, it follows from (5) that $F^* < \frac{\pi\left(p^m\left(c\right); c\right)}{n}$ and $\Pi_S\left(w^*, F^*; \widehat{p}\left(w^*, \phi\right)\right) < \Pi^{VI}$. $\square$