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Abstract

We consider a patent licensing game with a capacity constrained innovator. We show that when the constraint is strong (weak), the patentee prefers licensing by means of a fixed fee (unit royalty). In the case of a two-part tariff, the innovator charges a positive fixed fee if and only if the constraint is strong enough.

JEL codes: D45

Keywords: Patent licensing; capacity constraint.

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1. Introduction

We consider optimal patent licensing when the innovator is capacity constrained. When the capacity constraint is maximum (that is, the innovator cannot produce), the model coincides with the case of an outside patentee; when the capacity constraint is not binding, the model coincides with the case of an unconstrained inside patentee. Therefore, our model provides a bridge between the two cases usually considered in the literature: outside innovator and unconstrained inside innovator.

Capacity constraint is often relevant for the innovator. Pavitt et al. (1987), Acs and Audretsch (1990), OECD (2004), Marx et al. (2014), and Scholz (2017) provide evidence of the importance of small firms in generating technological innovations which are diffused by means of licenses. For example, Scholz (2017) emphasizes that, due to the increasing scarcity of raw materials that posit severe capacity constraints especially to small firms, licensing agreements that delegate production (or part of production) to other firms in change of the innovation are becoming widespread. OECD (2004) stresses that "[innovating] *firms lack the complementary assets, such as marketing and manufacturing, which are necessary to successfully commercialise their inventions*" (p.16). Pavitt et al. (1987), by analyzing the size distribution of innovating firms in UK after the Second World War, show that smaller firms are more likely to commercialize innovations than bigger firms. More recently, McClellan et al. (2020) suggest that, when developing monoclonal antibodies as a treatment for COVID-19, total capacity is split between companies which employ an in-house manufacturing network and others that act solely as contract manufacturers. Therefore, in many cases the innovator is a small firm with limited production possibilities, that licenses its innovation to other firms.

Theoretical literature has rarely considered the role of capacity constraint in determining the licensing choice of the patentee. Scholz (2017) analyses a vertical model where the upstream firms are capacity constrained, while the patentee is an outside innovator. Alderighi (2008) proposes a licensing method consisting in maximum authorized production for the licensee. However, at the best of our knowledge, the case of a capacity constrained patentee has not been considered yet.[1] As capacity constraint is a specific form of decreasing returns to scale, our work

[1] Mukherjee (2001) considers the case where, after the licensing decision of the patentee, both firms can endogenously decide to restrict production, which is a set-up different from ours.

also relates with the literature of patent licensing under returns to scale (Sen and Stamatopoulos, 2009; 2016; 2019).

While the literature has shown that in the case of an outside (inside) innovator the fixed fee (unit royalty) is preferred by the patentee (Kamien and Tauman, 1986, Wang, 1998, Sen and Tauman, 2007), we show that fixed fee is preferred by the patentee even if the patentee competes with the licensee, provided that the capacity constraint of the patentee is strong enough, that is the patentee can produce only a small quantity. Therefore, fixed fee might be preferred to unit royalty even if the patentee is an insider. This happens both in the case of drastic and non-drastic innovation.

The rest of the paper proceeds as follows. In Section 2 we introduce the model. In Section 3 we derive some preliminary results regarding a constrained Cournot duopoly with asymmetric firms. In Section 4 we derive the equilibrium profits under licensing. In Section 5 we compare different licensing mechanisms. Section 6 concludes. Proofs are in the Appendix.

2. The model

Consider a Cournot duopoly with two firms, 1 and 2, with inverse demand $p = 1 - q_1 - q_2$. Firm 1 (the patentee) has a cost-reducing innovation. The marginal cost of a firm is 0 with the innovation and $c > 0$ without the innovation where $0 < c < 1$. Since firm 1 has the innovation, its marginal cost is 0. Firm 1 is constrained by capacity $k > 0$, firm 2 has no capacity constraint.

Next we introduce the distinction between drastic and non-drastic innovation (Arrow, 1962). Consider a monopolist facing demand $p = 1 - Q$ who is not capacity constrained and who has the cost reducing innovation, so its marginal cost is 0. The monopoly price under marginal cost 0 is $p_M \equiv 1/2$. A cost reducing innovation is *drastic* if the monopoly price p_M under the reduced cost does not exceed c (the marginal cost without innovation); otherwise the innovation is *non drastic*. Thus an innovation is drastic if $c \geq 1/2$ and non drastic if $c < 1/2$.

Remark. If firm 1 is not capacity constrained and it has a drastic innovation, it has no incentive to license the innovation to firm 2 as without the innovation firm 2 drops out of the market and firm 1 obtains the monopoly profit under the reduced cost. However, this may not be the case when firm 1 is capacity constrained.

We consider three licensing policies:

(i) *unit royalties*: If firm 1 licenses the innovation to firm 2 with unit royalty $r \geq 0$, firm 2 has the cost-reducing innovation and for every unit it produces firm 2 has to pay r to firm 1. So the effective marginal cost of firm 2 is $(0 + r) = r$. Firm 2's marginal cost without the innovation is c, so unit royalties that are acceptable to firm 2 must have $r \leq c$.

(ii) *fixed fees*: If firm 1 licenses the innovation to firm 2 with fixed fee $f \geq 0$, firm 2 has the cost-reducing innovation and it pays the fee f upfront to firm 1.

(iii) *combinations of unit royalties licensing and fixed fees*: If firm 1 licenses the innovation to firm 2 using a policy (r, f) that has unit royalty $r \geq 0$ and fixed fee $f \geq 0$, firm 2 has the cost-reducing innovation, it pays the fee f upfront to firm 1 and for every unit it produces, it has to pay r to firm 1. So the effective marginal cost of firm 2 is $(0 + r) = r$.

Since firm 1 has the cost reducing innovation, its marginal cost is 0. If firm 2 does not have the innovation, its marginal cost is c. If firm 2 has the innovation under a fixed fee policy, its marginal cost is 0. If firm 2 has the innovation under a policy that has royalty r (either a royalty policy or a policy that is a combination of royalty and fee), then the effective marginal cost of firm 2 is r.

The strategic interaction between firms 1 and 2 is modeled as the three-stage licensing game G. In stage 1 of G, firm 1 decides whether to licenses its innovation to firm 2 or not and offers a licensing policy to firm 2; in stage 2 firm 2 decides whether to accept the policy or not; in stage 3, firms 1 and 2 compete in the Cournot duopoly and payments are made according to the policy.

3. Cournot duopoly $D^k(r)$

For $0 \leq r \leq c$ and $k > 0$, denote by $D^k(r)$ the Cournot duopoly in which firm 1 has marginal cost 0 and capacity constraint k; firm 2 has marginal cost r and no capacity constraint. In particular note that with respect to marginal cost of firm 2, $r = c$ corresponds to the situation where firm 2 does not have the innovation.

Thus, if firm 2 has the innovation under a fixed fee policy, the resulting Cournot duopoly is $D^k(0)$. If firm 2 has the innovation under a policy that has royalty r, it is $D^k(r)$. If firm 2 does not have the innovation, the resulting Cournot duopoly is $D^k(c)$.

To determine optimal licensing policies for firm 1, it is therefore useful to determine equilibrium outcomes of $D^k(r)$ for all $0 \le r \le c$ and $k > 0$. When there is no capacity constraint, the quantities produced by firms 1,2 in the unique (Cournot-Nash) equilibrium are: $\bar{q}_1(r) = \begin{cases} (1+r)/3 & if \quad 0 \le r < 1/2 \\ 1/2 & if \quad r \ge 1/2 \end{cases}$ and $\bar{q}_2(r) = \begin{cases} (1-2r)/3 & if \quad 0 \le r < 1/2 \\ 0 & if \quad r \ge 1/2 \end{cases}$.

Lemma 1: *For any* $0 < c < 1$, *the Cournot duopoly* $D^k(r)$ *has a unique* (*Cournot-Nash*) *equilibrium. If the capacity* k *exceeds* $\bar{q}_1(r)$, *the equilibrium outcome is the same as the case with no capacity constraint. Otherwise, the capacity constraint is binding and firm* 1 *exhausts its capacity* (*that is,* $q_1 = k$).

Proof: See the Appendix. ■

For the Cournot duopoly $D^k(r)$, denote the equilibrium price by $p^k(r)$, quantities of firms 1,2 by $q_1^k(r)$, $q_2^k(r)$ and profits by $\varphi_1^k(r)$, $\varphi_2^k(r)$.

4. Equilibrium profits

No license. When firm 1 does not license the innovation, the resulting Cournot duopoly is $D^k(c)$, where firm 1 obtains Cournot profit $\varphi_1^k(c)$.

Unit royalty policy. When firm 1 licenses the innovation to firm 2 with unit royalty $r \ge 0$, the Cournot duopoly game $D^k(r)$ is played where the Cournot quantity of firm 2 is $q_2^k(r)$. So for firm 1, the licensing revenue from royalty is $rq_2^k(r)$. The payoff of firm 1 is the sum of its Cournot profit and licensing revenue, given by

(1) $\pi^k_R(r) = \varphi_1^k(r) + rq_2^k(r)$

Recall that no royalty with $r > c$ is acceptable to firm 2. So under unit royalty policy, the problem of firm 1 is to choose r ($0 \le r \le c$) to maximize $\pi^k_R(r)$ given in (1). We also need to compare the payoff from optimal royalty policy with $\varphi_1^k(c)$ to see whether licensing by means of royalty is superior than not licensing.

Fixed fee policy. When firm 1 licenses the innovation to firm 2 with fixed fee $f \geq 0$, the resulting Cournot duopoly is $D^k(0)$ in which firm 2 obtains the Cournot profit $\varphi_2{}^k(0)$. If firm 2 refuses to have a license, the resulting Cournot duopoly is $D^k(c)$ in which firm 2 obtains the Cournot profit $\varphi_2{}^k(c)$. Therefore the maximum fixed fee firm 1 can set is $\varphi_2{}^k(0) - \varphi_2{}^k(c)$ (provided this is non-negative), making firm 2 just indifferent between accepting and rejecting. So the payoff of firm 1 has under the fixed fee policy has two parts: (i) firm 1's Cournot profit $\varphi_1{}^k(0)$ and (ii) fixed fee $\varphi_2{}^k(0) - \varphi_2{}^k(c)$. This payoff is

(2) $\pi^k{}_F = \varphi_1{}^k(0) + \varphi_2{}^k(0) - \varphi_2{}^k(c)$

We need to compare this payoff with $\varphi_1{}^k(c)$ to see whether licensing by means of fixed fee is superior than not licensing.

Combinations of unit royalties and fixed fees policy. Suppose firm 1 licenses the innovation to firm 2 using a licensing policy (r, f) where r $(0 \leq r \leq c)$ is the unit royalty and $f \geq 0$ is the fixed fee firm 2 has to pay firm 1. If firm 2 accepts this policy, it obtains the Cournot profit $\varphi_2{}^k(r)$. If it rejects, it operates with marginal cost c and obtains the Cournot profit $\varphi_2{}^k(c)$. So for any r, the maximum fixed fee firm 1 can set is

(3) $f = \varphi_2{}^k(r) - \varphi_2{}^k(c)$

Under the licensing policy (r, f), the payoff of firm 1 has three parts: (i) its Cournot profit $\varphi_1{}^k(r)$, (ii) royalty revenue $rq_2{}^k(r)$ and (iii) fixed fee f given by (3). When f is chosen optimally as in (1), the payoff of firm 1 as function of r is

(4) $\pi^k{}_{RF}(r) = \varphi_1{}^k(r) + rq_2{}^k(r) + \varphi_2{}^k(r) - \varphi_2{}^k(c)$

As the fixed fee f is chosen optimally for any r, under combinations of unit royalties and fees the problem of firm 1 is to choose r $(0 \leq r \leq c)$ to maximize $\pi^k{}_{RF}(r)$ given in (4). We also need to compare the payoff from optimal combination with $\varphi_1{}^k(c)$ to see whether such a policy is superior than not licensing.

For the analysis it will be convenient first to characterize optimal combinations of unit royalties and fees.

Optimal combinations of unit royalties and fixed fees. When the unit royalty is r, the resulting Cournot duopoly is $D^k(r)$ in which firm 1 has marginal cost 0 and firm 2 has (effective) marginal cost r. Therefore Cournot profits are: $\varphi_1^k(r) = p^k(r)q_1^k(r)$ and $\varphi_2^k(r) = [p^k(r) - r]q_2^k(r)$.

Using this in (4) and denoting $Q^k(r) = q_1^k(r) + q_2^k(r)$ (the total quantity), we have

$$(5)\ \pi^k_{RF}(r) = p^k(r)q_1^k(r) + rq_2^k(r) + [p^k(r) - r]q_2^k(r) - \varphi_2^k(c) = p^k(r)[q_1^k(r) + q_2^k(r)] - \varphi_2^k(c) =$$

$$= p^k(r)Q^k(r) - \varphi_2^k(c) = p^k(r)[1 - p^k(r)] - \varphi_2^k(c)$$

For any price p ($0 \leq p \leq 1$), let $\varphi_M(p) = p(1 - p)$ be the profit of the monopolist at price p who has marginal cost 0. Observe from (5) that

$$(6)\ \pi^k_{RF}(r) = \varphi_M(p^k(r)) - \varphi_2^k(c)$$

Since $\varphi_2^k(c)$ is a constant not affected by r, by (4), the problem of firm 1 is to choose r to maximize $\varphi_M(p^k(r))$. Note that $\varphi_M(p)$ is increasing for $p < 1/2$, decreasing for $p > 1/2$ and its unique maximum is attained when p equals $p_M \equiv 1/2$ (the monopoly price with marginal cost 0). Let $\varphi_M^* \equiv \varphi_M(p_M) = 1/4$ (the monopoly profit at marginal cost 0). From (4), the maximum possible payoff firm 1 can obtain is $\varphi_M^* - \varphi_2^k(c)$ (the monopoly profit with marginal cost 0 by leaving firm 2 its reservation profit $\varphi_2^k(c)$).

Proposition 1 characterizes optimal combinations of unit royalties and fixed fees.

Proposition 1 *When firm* 1 *uses combinations of unit royalties and fixed fees, the optimal licensing policies are as follows:*

(1) *Suppose the innovation is non drastic* ($c < 1/2$). *If $k < c$, the unique optimal policy for firm* 1 *is to license the innovation to firm* 2 *using unit royalty $r = k$ and positive fixed fee. If $k \geq c$, the unique optimal policy is to license the innovation to firm* 2 *using a pure royalty policy* (*zero fixed fee*) *with unit royalty $r = c$.*

(2) *Suppose the innovation is drastic* ($c \geq 1/2$). *If $k < Q_M \equiv 1/2$* (*the monopoly output with marginal cost* 0)*, the unique optimal policy for firm* 1 *to license the innovation to firm* 2 *using unit royalty $r = k$ and positive fixed fee. If $k \geq 1/2$, it is optimal for firm* 1 *to not license the innovation and use it exclusively to obtain the monopoly profit.*

Proof: See the Appendix. ■

Figure 1 presents optimal licensing policies for firm 1 in the (c, k) plane. The line OA is the 45 degree line ($k = c$); the line AH has equation $k = 1/2$. Therefore, a positive fixed fee emerges provided that k is low enough, both in the case of drastic innovation and non-drastic innovation.

Remark. Note that the set of licensing policies with combinations of fixed fees and royalties include as special cases policies that have only fixed fees or only royalties. Therefore:

(i) for cases where not licensing is superior to combinations of unit royalties and fixed fees for firm 1, not licensing must be also superior to only fixed fees or only unit royalties.

(ii) for cases where optimal combination has only royalty and no fixed fee and such a policy is also superior to not licensing, this policy must also be the optimal unit royalty policy as well and it must be also superior to pure fixed fees.

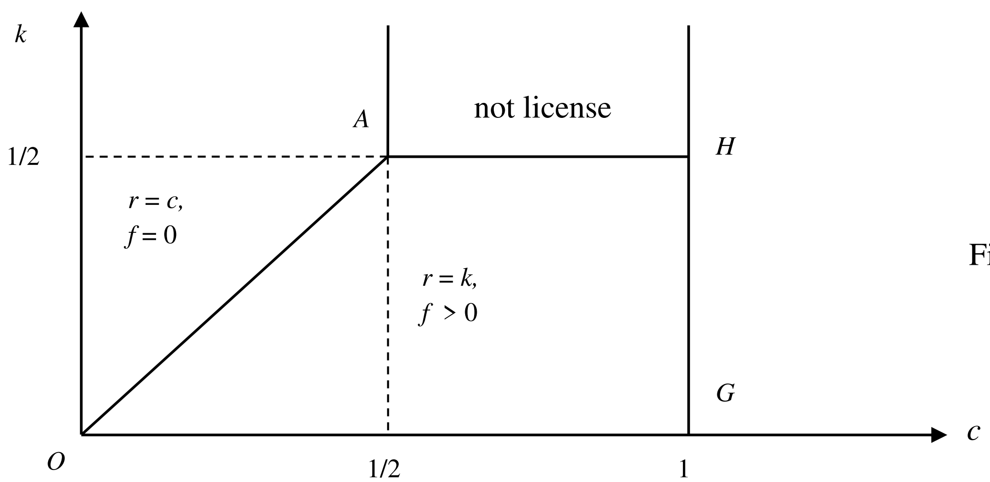

Figure 1

The following corollary is immediate from Proposition 1.

Corollary 1:

(1) *If the innovation is non drastic* ($c < 1/2$) *and* $k \geq c$, *the unique optimal unit royalty policy for firm* 1 *has* $r = c$. *This policy is superior to not licensing and any pure fixed fee policy.*

(2) *If the innovation is drastic* ($c \geq 1/2$) *and* $k \geq 1/2$*, not licensing is superior to both unit royalty policies and fixed fee policies; it is optimal for firm* 1 *to not license the innovation and use it exclusively to obtain the monopoly profit*.

Optimal unit royalty policies. In view of Corollary 1, to completely characterize optimal pure royalty policies, we need to find optimal pure royalty policies for the region where $k < 1/2$ and $k < c$ (region *OAHG* in Figure 1). The next proposition presents the result.

Proposition 2 *Optimal unit royalty policies for firm* 1 *are as follows:*

(1) *Suppose the innovation is non drastic* ($c < 1/2$). *Then the unique optimal policy for firm* 1 *to license the innovation to firm* 2 *is using unit royalty* $r = c$.

(2) *Suppose the innovation is drastic* ($c \geq 1/2$). *If* $k < 1/2$*, the unique optimal policy for firm* 1 *to license the innovation to firm* 2 *is using unit royalty* $r = 1/2$. *If* $k \geq 1/2$*, it is optimal for firm* 1 *to not license the innovation and use it exclusively to obtain the monopoly profit*.

Proof: See the Appendix. ■

Figure 2 presents optimal pure royalty policies for firm 1 in the (c, k) plane.

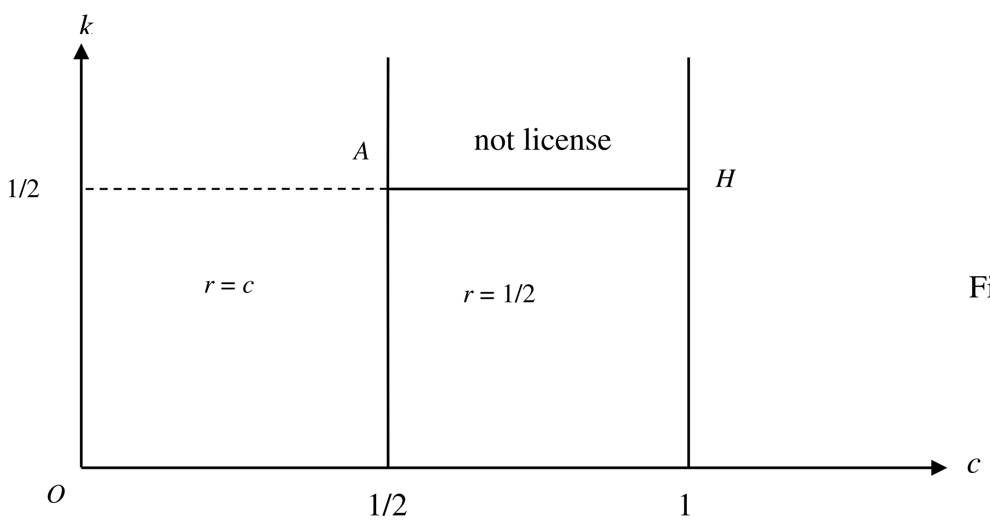

Figure 2

Optimal fixed fee policies. By Corollary 1, if $c \geq 1/2$ (drastic innovations) and $k \geq 1/2$ (the region above line *AH* in Figure 1), not licensing is superior to combinations of unit royalties and fixed fees, so not licensing is also superior to fixed fee policies. To completely characterize optimal fixed fee policies, we look at the rest of the regions.

Proposition 3 *The unique optimal fixed fee policy for firm* 1 *is to set* $f = \varphi_2^k(r) - \varphi_2^k(c)$ *and it has the following properties.*

(1) *If* $c < 2/5$, *the fixed fee policy is superior to not licensing for all k.*

(2) *If* $2/5 < c < 2/3$, *there is a decreasing function* $k_0(c)$ *such that the fixed fee policy is superior to not licensing if* $k < k_0(c)$ *and not licensing is superior to the fixed fee policy if* $k > k_0(c)$.

(3) *If* $2/3 < c < 1$, *the fixed fee policy is superior to not licensing if* $k < 1/3$ *and not licensing is superior to the fixed fee policy if* $k > 1/3$.

Proof: See the Appendix. ■

Figure 3 presents optimal fixed fee royalty policies for firm 1 in the (c, k) plane. If $c < 2/5$, fixed fee is superior to not licensing for any k. If $c \geq 2/5$, fixed fee is superior to not licensing if k is below *XFT* and not licensing is superior if k is above *XFT*. The function $k_0(c)$ in Proposition 3 is given by the curve *XF*. The line *AG* has equation $k = 1 - c$ and the line *AB* has equation $k = (1 + c)/3$. As *XF* meets line *AB* at point *X* and *AG* at point *F*, note that $k_0(2/5) = (1 + 2/5)/3 = 7/15$ and $k_0(2/3) = 1 - 2/3 = 1/3$.

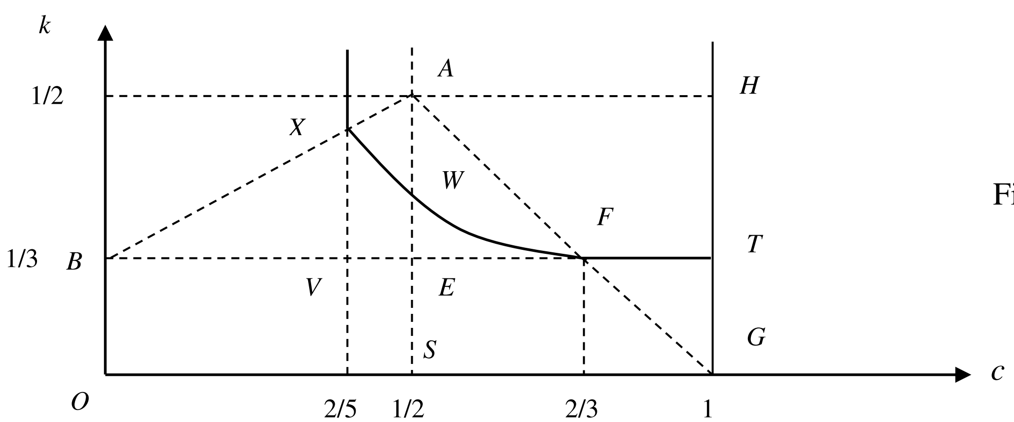

Figure 3

5. Comparing unit royalty and fixed fee policies

We can now compare optimal royalty and fixed fee policies. Consider the curve *XF* in Figure 3 (representing the function $k_0(c)$) and the 45 degree line *OA* (that has equation $k = c$). As $k_0(c)$ is decreasing, $k_0(2/5) = 7/15 > 2/5$ and $k_0(1/2) < k_0(2/5) < 1/2$, there is unique point of intersection of curve *XF* and line *OA* and it corresponds to $2/5 < c < ½$, see Figure 4.1).

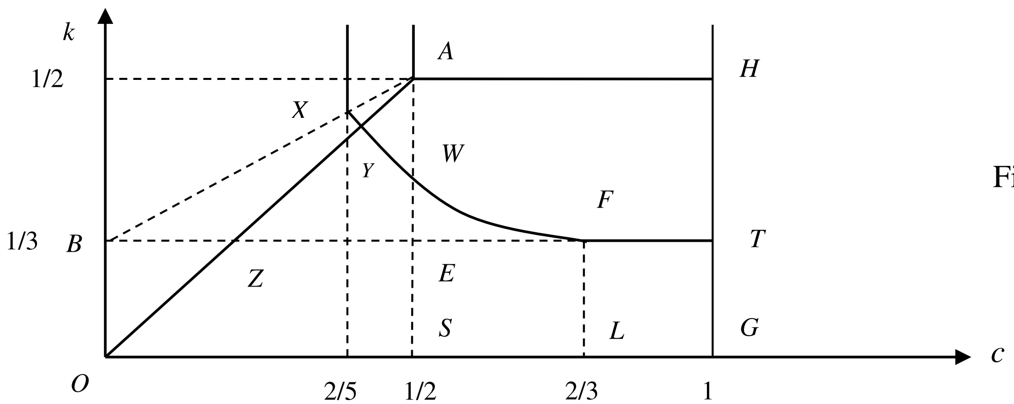

Figure 4.1

Denote by *Y* the point of intersection of *OA* and *XF* and by *Z* the point of intersection of *OA* and *BT*. From Corollary 1(1), in the region above the line *OA* ($c < 1/2$ and $k \geq c$), the optimal unit royalty policy $r = c$ is superior to both fixed fee and not licensing. From Corollary 1(2), in the region above the line *AH* ($c > 1/2$ and $k > 1/2$), not licensing is superior to both fixed fee and royalty. Consider the region *TFYAH*. In this region not licensing is superior to fixed fee (Proposition 3) and the optimal unit royalty policy $r = 1/2$ is superior to not licensing (Proposition 2), so the unit royalty policy $r = 1/2$ is superior to both fixed fee and not licensing. Therefore, the region *OYFTG* (as shown in Figure 4.2) is where we need to look at the payoffs from fixed fee and royalty policies to see which one is higher.

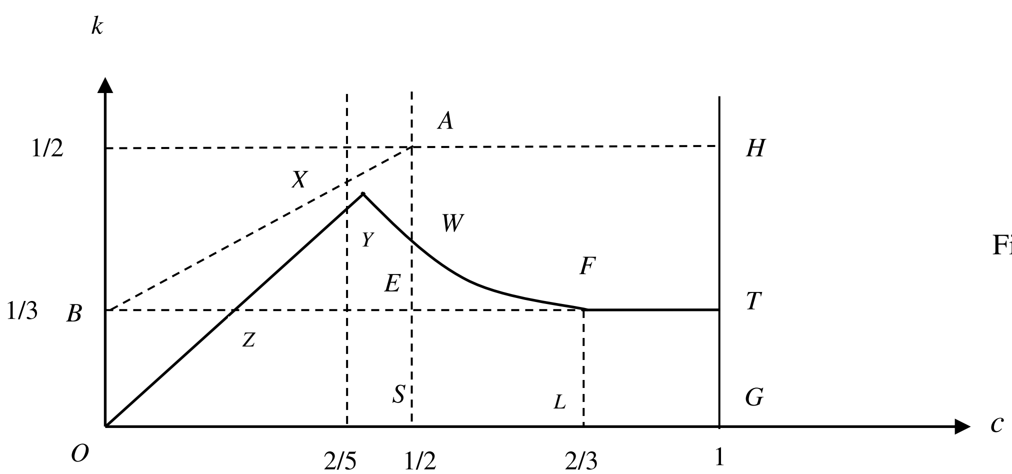

Figure 4.2

Proposition 4 *For firm* 1, *fixed fee, unit royalty and not licensing compare as follows.*

(1) *Suppose* $0 < c < 1/2$. *Unit royalty policy is superior to both fixed fee and not licensing if* $k > c/2$ *and fixed fee is superior to both unit royalty and not licensing if* $k < c/2$.

(2) *Suppose* $1/2 \leq c < 1$. *There is a function* $k_1(c)$ *such that fixed fee is superior to both unit royalty and not licensing if* $k < k_1(c)$, *unit royalty policy is superior to both fixed fee and not licensing if* $k_1(c) < k < 1/2$ *and not licensing is superior to both unit royalty policy and fixed fee if* $k > 1/2$.

Proof: See the Appendix. ■

In figure 5, the line *OM* has equation $k = c/2$ and the curve *MUV* represents the function $k_1(c)$. As shown in figure 5, there is c_0 between 2/3 and 1 (specifically, $c_0 = 1/\sqrt{2}$) such that $k_1(c)$ is increasing for $1/2 \leq c < c_0$ and decreasing for $c_0 < c < 1$.

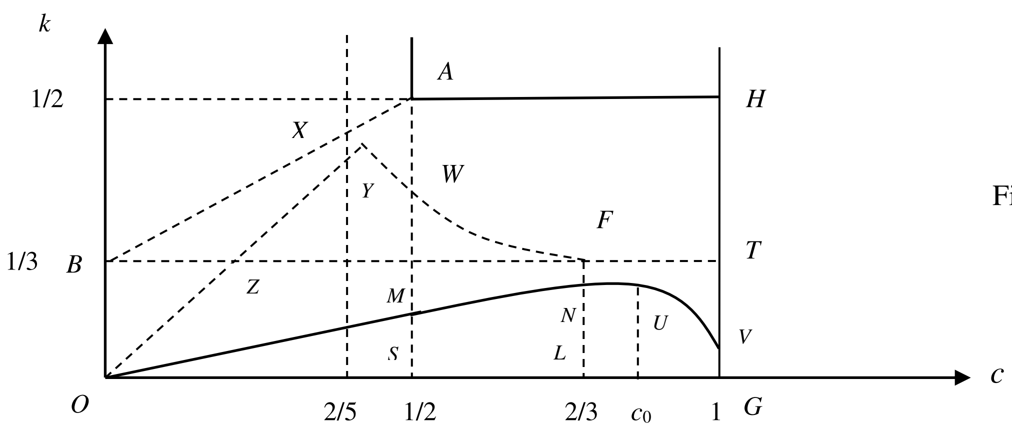

Figure 5

For non drastic innovations ($c < 1/2$), fixed fee is superior to both unit royalty and no licensing in the region *OMS* and unit royalty is superior to both fixed fee and not licensing above the line *OM*. For drastic innovations ($c \geq 1/2$), fixed fee is superior to both unit royalty and no licensing in the region *SMUVG*, unit royalty is superior to both fixed fee and not licensing in the region *MAHV* and not licensing is superior to both fixed fee and unit royalty above the line *AH*. Recall that (see Figure 1) when firm 1 uses combinations of royalties and fixed fees, not licensing is superior to licensing only above the line *AH* (that is, when $c \geq 1/2$ and $k > 1/2$); for all other cases licensing is the best choice.

Figure 6 summarizes the above discussion.

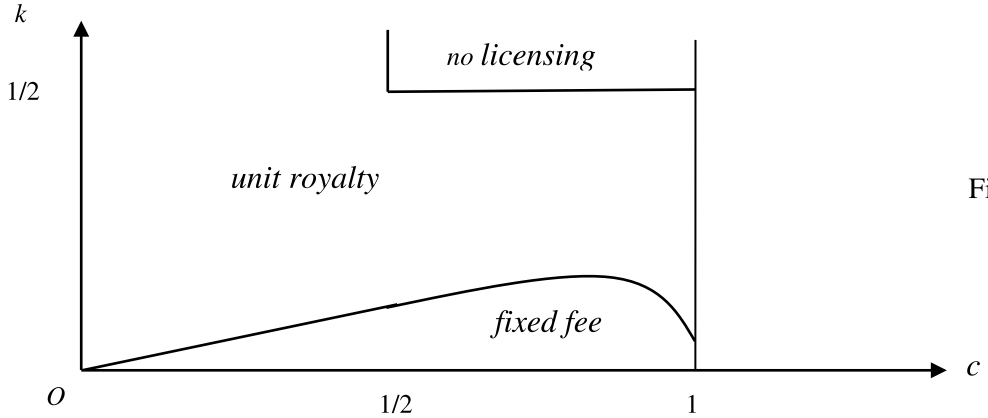

Figure 6

Therefore, Figure 6 shows that fixed fee is superior to unit royalties (and no licensing) even if the patent is incumbent, provided that the capacity constraint is strong enough, both in the case of drastic innovation and non-drastic innovation. The intuition is the following. Under pure fixed fee, the patentee and the licensee have the same marginal costs. Therefore, the revenues from licensing come from the fee required to the licensee. In contrast, under unit royalty, the patentee maintains the cost differential with the rival, and it gets additional revenues from the royalty. When the patentee is an outsider, it does not care about the cost differential with the rival (Kamien and Tauman, 1986), and fixed fee is preferred. At the opposite, when the patentee and the licensee compete in the same market (the inside case), maintaining a competitive position with respect to the rival is important for the patentee (Wang, 1998). Therefore, unit royalty is preferred. Consider now the capacity constraint. When k is low, the output produced by the patentee is low. Therefore, firm 1 does not care for its cost advantage over the licensee. Consequently, fixed fee is preferred. The opposite is true when k is high, that is, the quantity produced by firm 1 is large: in this case, the cost differential is important, and unit royalty is preferred.

Finally, even if the capacity constraint has been kept exogenous, it can be easily endogenized. Suppose firm 1 chooses, without any cost, the level of k before the game starts, anticipating that the subsequent optimal licensing policy depends on k. Consider first the case of non-drastic innovation (c<1/2). If k<c/2, by Proposition 4, the optimal licensing policy is fixed fee. In this case, the profits of firm 1 are $\varphi^k_F = (1-k)k/2 + (1-k)^2/4 - (1-c-k)^2/4$, which are

strictly increasing in k. If k>c/2, by Proposition 4, the optimal licensing policy is unit royalty. In this case, by Corollary 1 and Proposition 4, when k< 1/3, the profits of firm 1 are $\varphi^k{}_R = (1 + c - k)k/2 + c(1 - c - k)/2$, which are strictly increasing in k, and when k>1/3 the profits of firm 1 are $\pi^k{}_R = (1 + c)^2/9 + c(1 - 2c)/3$, that do not depend on k. Therefore, with non-drastic innovation, firm 1 chooses a sufficiently high k such that the capacity constraint is not binding. Consider now the case of drastic innovation (c>1/2). In this case, firm 1 cannot do better than getting the monopolistic profits. By Figure 6, monopolistic profits can be obtained by setting any k>1/2. Therefore, even in the case of drastic innovation, firm 1 chooses to be not capacity constrained.

6. Conclusions

We introduce a capacity constraint for an innovator and we discuss optimal licensing in a Cournot duopoly. Our model links the two models that are usually considered in the literature, namely outside innovator and unconstrained inside innovator. We consider unit royalties, fixed fees, and combinations of unit royalties and fixed fees. We show that a fixed fee is used if and only if the capacity constraint is sufficiently strong. Therefore, fixed fee might be preferred by the patentee even if the innovator competes with the licensee.

Appendix

Proof of Lemma 1 Lemma A1 below characterizes equilibrium of $D^k(r)$. Lemma 1 follows from Lemma A1. ■

Figure A1 presents different regions of Lemma A1 in the (c, k) plane. The line BA has equation $k = (1 + c)/3$; the line BE has equation $k = 1/3$; the line AG has equation $k = 1 - c$; the line AH has equation $k = 1/2$. The line OA is the 45 degree line ($k = c$).

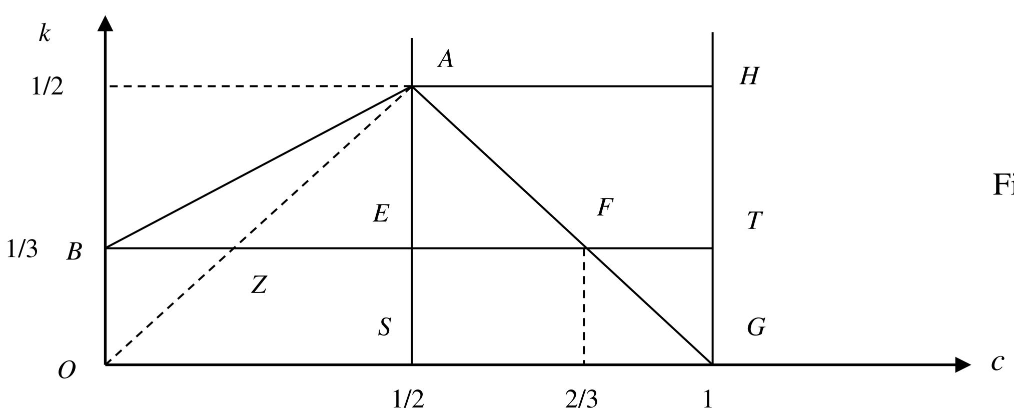

Figure A1

Lemma A1: *For any* $0 < c < 1$, *The Cournot duopoly* $D^k(r)$ *has a unique (Cournot-Nash) equilibrium. The equilibrium price* $p^k(r)$, *quantities* $q_1{}^k(r)$, $q_2{}^k(r)$ *and profits* $\varphi_1{}^k(r)$, $\varphi_2{}^k(r)$ *are as follows.*

(1) *The following holds if the innovation is non drastic* ($c < 1/2$):

(i) *If* $k \geq (1 + c)/3$ [*(c, k) above line BA*] *then* $k \geq \bar{q}_1(r)$ *for all* $0 \leq r \leq c$. *For all* $0 \leq r \leq c$: $q_1{}^k(r) = \bar{q}_1(r) = (1 + r)/3$, $q_2{}^k(r) = \bar{q}_2(r) = (1 - 2r)/3$, $\varphi_1{}^k(r) = (1 + r)^2/9$, $\varphi_2{}^k(r) = (1 - 2r)^2/9$ *and* $p^k(r) = (1 + r)/3$.

(ii) *If* $k \leq 1/3$ [*region OBES*] *then* $k \leq \bar{q}_1(r)$ *for all* $0 \leq r \leq c$. *For all* $0 \leq r \leq c$: $q_1{}^k(r) = k$, $q_2{}^k(r) = (1 - r - k)/2$, $\varphi_1{}^k(r) = (1 + r - k)k/2$, $\varphi_2{}^k(r) = (1 - r - k)^2/4$, $p^k(r) = (1 + r - k)/2$.

(iii) *If* $1/3 < k < (1 + c)/3$ [*region BAE*] *there is* $0 < r_0 < c$ *such that the equilibrium outcome is the same as* (1)(i) *for* $0 \leq r \leq r_0$ *and it is the same as* (1)(ii) *for* $r_0 \leq r \leq c$.

(2) *The following holds if the innovation is drastic* ($c \geq 1/2$):

(i) *If* $k \geq 1/2$ [*(c, k) above line AH*] *then* $k \geq \bar{q}_1(r)$ *for all* $0 \leq r \leq c$. *The equilibrium outcome is the same as* (1)(i) *for all* $0 \leq r < 1/2$. *For* $1/2 \leq r \leq c$: $q_1{}^k(r) = 1/2$, $q_2{}^k(r) = 0$, $\varphi_1{}^k(r) = 1/4$, $\varphi_2{}^k(r) = 0$, $p^k(r) = 1/2$ (*firm* 1 *obtains the monopoly profit, firm* 2 *drops out*).

(ii) *If* $k < \min\{1 - c, 1/3\}$[*region SEFG*] *then* $k \leq \bar{q}_1(r)$ *for all* $0 \leq r \leq c$. *The equilibrium outcome is same as* (1)(ii).

(iii) *If* $1/3 \leq k < 1 - c$ [*region EAF*], *there is* $0 < r_0 < 1/2$ *such that the equilibrium outcome is the same as* (1)(i) *for* $0 \leq r < r_0$ *and it is the same as* (1)(ii) *for* $r_0 \leq r \leq c$.

(iv) *If* $1 - c \leq k < 1/3$ [*region FTG*] *then there is* $1/2 < r_1 < c$ *such that the equilibrium outcome is the same as* (1)(ii) *for* $0 \leq r < r_1$. *For* $r_1 \leq r \leq c$: $q_1{}^k(r) = k$, $q_2{}^k(r) = 0$, $\varphi_1{}^k(r) = (1 - k)k$, $\varphi_2{}^k(r) = 0$, $p^k(r) = 1 - k$.

(v) *If* $\max\{1 - c, 1/3\} < k < 1/2$ [*region FAHT*], *there is* $0 < r_0 < 1/2$ *and* $1/2 < r_1 < c$ *such that the equilibrium outcome is the same as* (1)(i) *for* $0 \leq r < r_0$, *the same as* (1)(ii) *for* $r_0 \leq r < r_1$ *and the same as last part of* (2)(iv) *for* $r_1 \leq r \leq c$.

Proof: To prove the lemma, first consider the standard case of a Cournot duopoly with two firms 1,2 in which firm 1 has unit cost 0 and firm 2 has unit cost r and there is no capacity constraint. In that case, profit functions of 1,2 are:

$\varphi_1(q_1, q_2) = (1 - q_1 - q_2)\, q_1$ and $\varphi_2(q_1, q_2) = (1 - q_1 - q_2)\, q_2 - r\, q_2$

The (unique) best response of firm 1 to q_2 is: choose $q_1 = (1 - q_2)/2$ if $q_2 < 1$ and $q_1 = 0$ if $q_2 \geq 1$. The (unique) best response of firm 2 to q_1 is: choose $q_2 = (1 - r - q_1)/2$ if $q_1 < 1 - r$ and $q_1 = 0$ if $q_2 \geq 1 - r$.

Denote by BR_1, BR_2 the best response functions of 1,2.

Case 1: If $r < 1/2$, then $1/2 < 1 - r$ and BR_1, BR_2 are drawn below:

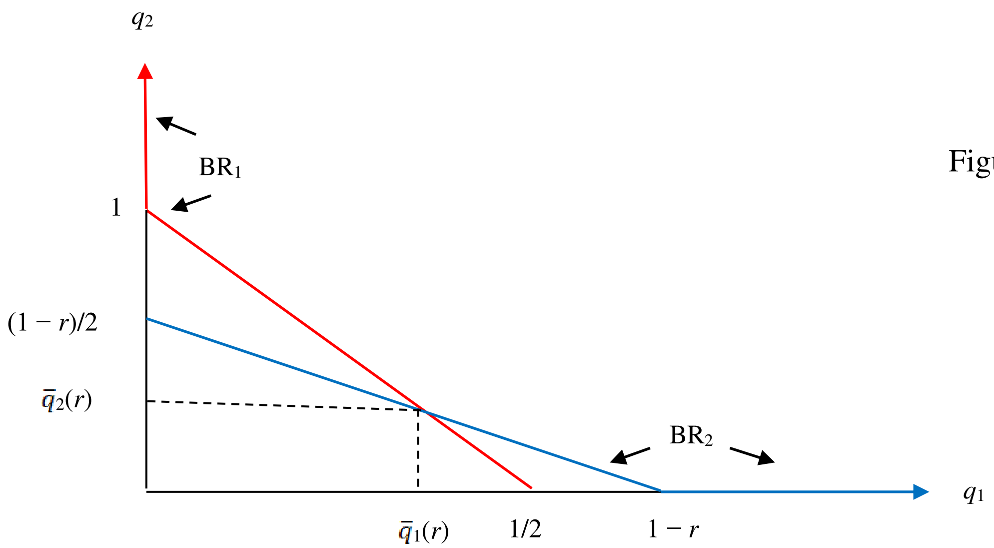

Figure I

Thus, when there is no capacity constraint, the unique equilibrium is $(\bar{q}_1(r), \bar{q}_2(r))$. Solving $q_1 = (1 - q_2)/2$ and $q_2 = (1 - r - q_1)/2$, we have $\bar{q}_1(r) = (1 + r)/3$ and $\bar{q}_2(r) = (1 - 2r)/3$.

Case 2: If $r \geq 1/2$, then $1/2 \geq 1 - r$ and BR_1, BR_2 are drawn below:

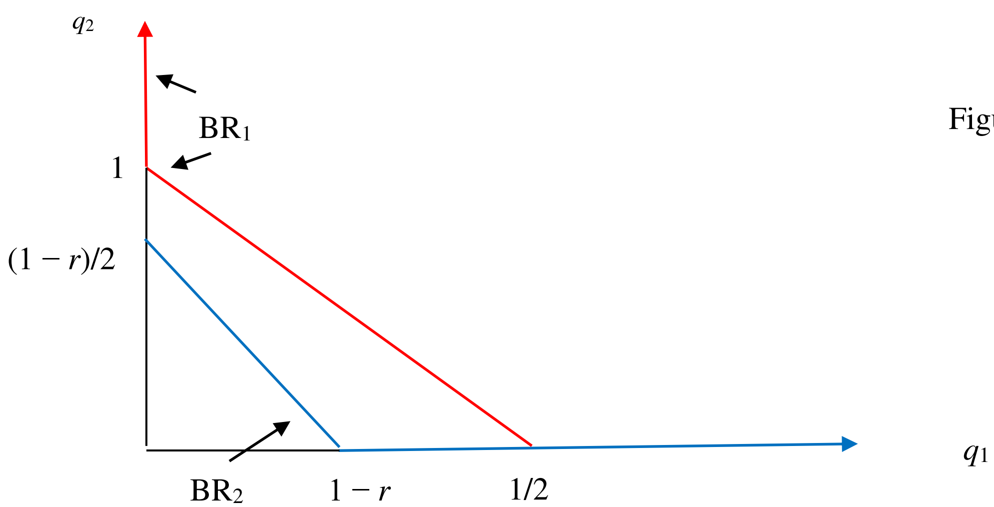

Figure II

Thus, when there is no capacity constraint, the unique equilibrium is $\bar{q}_1(r) = 1/2$ and $\bar{q}_2(r) = 0$.

Capacity constraint: when firm 1 has a capacity constraint $k > 0$: The (unique) best response of firm 1 to q_2 is: choose $q_1 = \min\{(1 - q_2)/2, k\}$ if $q_2 < 1$ and $q_1 = 0$ if $q_2 \geq 1$. The (unique) best response of firm 2 to q_1 is: choose $q_2 = (1 - r - q_1)/2$ if $q_1 < 1 - r$ and $q_1 = 0$ if $q_2 \geq 1 - r$.

For the case $r < 1/2$, there are two possibilities: (i) $k \geq \bar{q}_1(r)$ and (ii) $k < \bar{q}_1(r)$.

Case 1(a): $k \geq \bar{q}_1(r)$ In this case, modifying Figure I, BR$_1$, BR$_2$ are as follows (BR$_1$ is drawn below for $\bar{q}_1(r) \leq k < 1/2$; for $k > 1/2$, BR$_1$ will be the same as in Figure I).

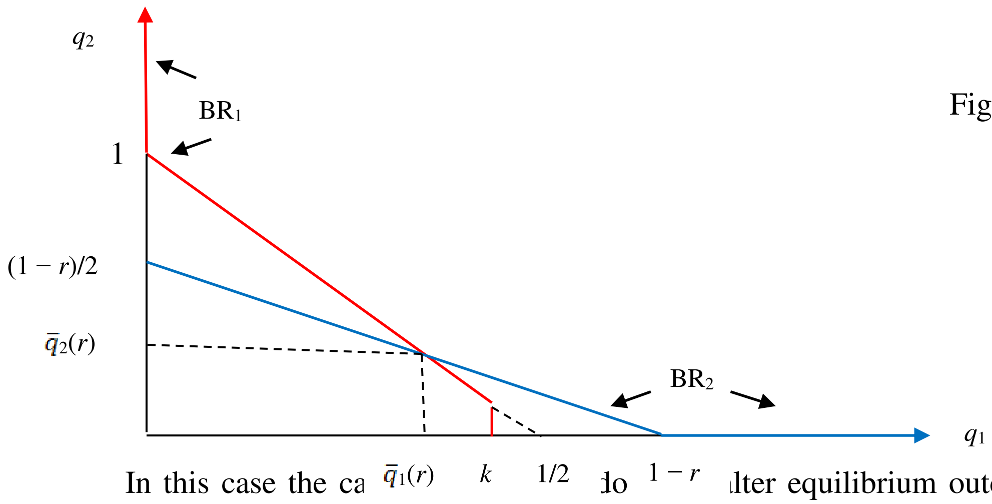

Figure I(a)

In this case the ca do lter equilibrium outcome. The equilibrium is the same as in the case of no constraints.

Case 1(b): $k < \bar{q}_1(r)$ In this case, modifying Figure 1, BR$_1$, BR$_2$ are as follows.

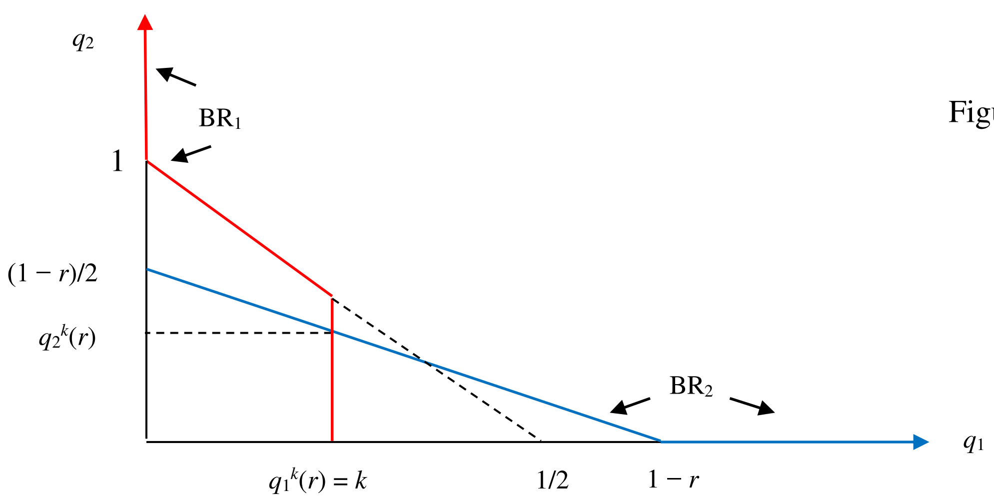

Figure I(b)

As seen from Figure 1(b), in this case the unique equilibrium has $q_1^k(r) = k$, $q_2^k(r) = (1 - r - k)/2$.

For the case $r \geq 1/2$, there are three possibilities: (i) If $k \geq 1/2$, (ii) $1 - r \leq k < 1/2$ and (iii) $k < 1 - r$. If $k \geq 1/2$, from Figure 2, the equilibrium is the same as in with no capacity constraint: $q_1^k(r) = 1/2$, $q_2^k(r) = 0$. If $1 - r \leq k < 1/2$, the situation is as in Figure II(a).

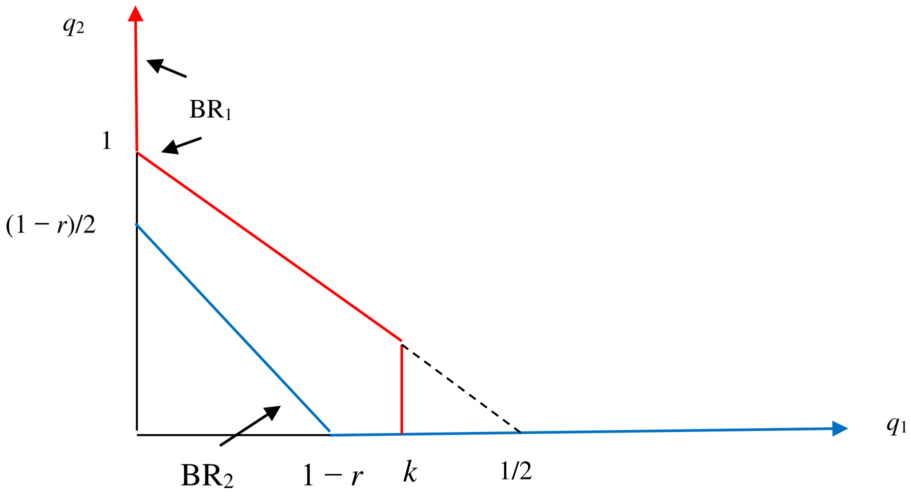

Figure II(a)

Observe from Figure 2(a) in this case, the equilibrium is $\bar{q}_1(r) = k$ and $\bar{q}_2(r) = 0$.

If $k < 1 - r$, the situation is as in Figure II(b).

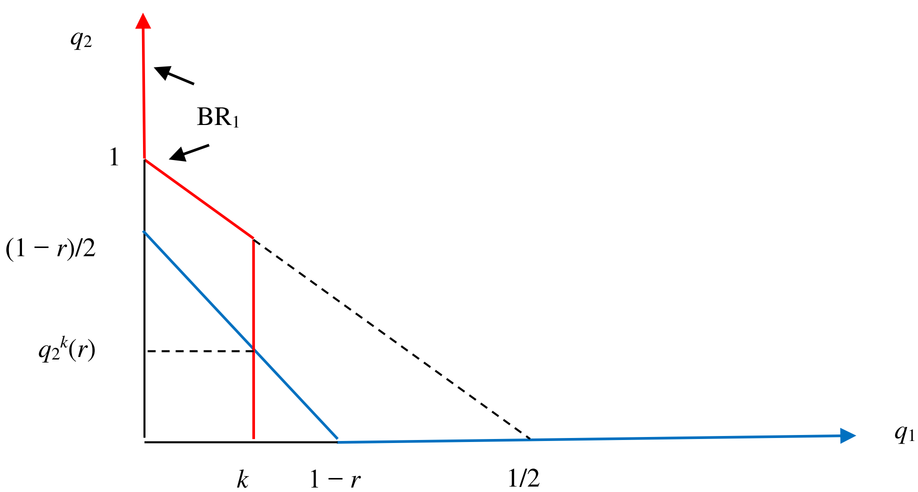

Figure II(b)

Observe from Figure II(b) in this case, the equilibrium is $\bar{q}_1(r) = k$ and $\bar{q}_2(r) = (1 - r - k)/2$.

Now we are in a position to prove the lemma.

Proof of part (1) Suppose the innovation is non drastic, that is, $c < 1/2$. Since $0 \leq r \leq c$, in this case $r < 1/2$ so that $1 - r > 1/2$ and figures I(a)-(b) apply. Note that $\bar{q}_1(r) = (1 + r)/3$ is increasing in r. Since $0 \leq r \leq c$, We have $\bar{q}_1(0) = 1/3 \leq \bar{q}_1(r) \leq \bar{q}_1(c) = (1 + c)/3$.

(i) If $k \geq (1 + c)/3$, then $k \geq \bar{q}_1(r)$ for all $0 \leq r \leq c$ and so Figure I(a) applies for all $0 \leq r \leq c$. This proves part (1)(i).

(ii) If $k \leq 1/3$, then $k \leq \bar{q}_1(r)$ for all $0 \leq r \leq c$ and so Figure I(b) applies for all $0 \leq r \leq c$. This proves part (1)(ii).

(iii) If $\bar{q}_1(0) = 1/3 < k < \bar{q}_1(c) = (1 + c)/3$, then there is $0 < r_0 < c$ such that $k \geq \bar{q}_1(r)$ for $0 \leq r \leq r_0$ and $k < \bar{q}_1(r)$ for $r_0 < r \leq c$. So figure I(a) applies for $0 \leq r \leq r_0$ and figure I(b) applies for $r_0 < r \leq c$. This proves part (1)(iii).

Proof of part (2) Suppose the innovation is drastic, that is, $c \geq 1/2$. In this case, if $0 \leq r < 1/2$, then $1/2 < 1 - r$ and if $1/2 \leq r \leq c$, then $1/2 \geq 1 - r$.

If $0 \leq r < 1/2$, then figures I(a)-(b) apply. Thus, if $k \geq \bar{q}_1(r)$, then $q_1^k(r) = \bar{q}_1(r)$, $q_2^k(r) = \bar{q}_2(r)$. If $k < \bar{q}_1(r)$, then equilibrium has $q_1^k(r) = k$, $q_2^k(r) = (1 - r - k)/2$.

If $1/2 \leq r \leq c$, then figures II(a)-(b) apply. Thus, if $k \geq 1/2$, then $q_1^k(r) = 1/2$ and $q_2^k(r) = 0$. If $1 - r \leq k < 1/2$, then $q_1^k(r) = k$ and $q_2^k(r) = 0$. If $k < 1 - r$, then $q_1^k(r) = k$ and $q_2^k(r) = (1 - r - k)/2$.

(i) If $k \geq 1/2$, then the conclusion for $1/2 \leq r \leq c$ is immediate. For $0 \leq r < 1/2$, we have $\bar{q}_1(r) < \bar{q}_1(1/2) = 1/2$, so $k \geq \bar{q}_1(r)$ and figure I(a) applies. This proves part 2(i).

(ii) If $k < \min\{1 - c, 1/3\}$, then for $0 \leq r < 1/2$, we have $\bar{q}_1(r) > \bar{q}_1(0) = 1/3$, so $k < \bar{q}_1(r)$ and figure 1(b) applies. For $1/2 \leq r \leq c$, we have $1 - r \geq 1 - c$, so $k \leq 1 - r$ and figure II(b) applies. This proves part (2)(ii).

(iii) If $1/3 \leq k < 1 - c$, then for $1/2 \leq r \leq c$, we have $1 - r \geq 1 - c$, so $k \leq 1 - r$ and figure II(b) applies. Consider $0 \leq r < 1/2$. Note that $\bar{q}_1(0) = 1/3$ and $\bar{q}_1(1/2) = 1/2$. Since $c \geq 1/2$, we have $1/2 \geq 1 - c$. Thus $1/3 \leq k < 1/2$, that is, $\bar{q}_1(0) \leq k < \bar{q}_1(1/2)$. As $\bar{q}_1(r)$ is increasing in r, it follows that there is $0 \leq r_0 < 1/2$ such that for $0 \leq r \leq r_0$ we have $k \geq \bar{q}_1(r)$ and figure 1(a) applies and for $r_0 \leq r \leq 1/2$, we have $k < \bar{q}_1(r)$ and figure I(b) applies. This proves part (2)(iii).

(iv) If $1 - c \leq k < 1/3$, then for $0 \leq r < 1/2$, we have $\bar{q}_1(r) > \bar{q}_1(0) = 1/3$, so $k < \bar{q}_1(r)$ and figure I(b) applies. For $1/2 \leq r \leq c$, we have $1 - c \leq 1 - r \leq 1/2$. Since $k < 1/3$, we have $k < 1/2$. As $1 - r$ is decreasing in r, it follows that there is $1/2 < r_1 \leq c$ such that for $0 \leq r < r_1$ we have $k < 1 - r$ and figure II(b) applies. For $r_1 \leq r \leq 1/2$, we have $1 - r \leq k < 1/2$ and figure II(b) applies. This proves part (2)(iv).

(v) If $\max\{1 - c, 1/3\} < k < 1/2$, then for $1/2 \leq r \leq c$, we have $1 - c \leq 1 - r \leq 1/2$. Since $1 - c < k < 1/2$ and $1 - r$ is decreasing in r, it follows that there is $1/2 < r_1 \leq c$ such that for $0 \leq r < r_1$ we have $k < 1 - r$ and figure II(b) applies; for $r_1 \leq r \leq 1/2$, we have $1 - r \leq k < 1/2$ and figure II(a) applies.

Consider $0 \leq r < 1/2$. As $\bar{q}_1(0) = 1/3$, $\bar{q}_1(1/2) = 1/2$, we have $\bar{q}_1(0) < k < \bar{q}_1(1/2)$. As $\bar{q}_1(r)$ is increasing in r, it follows that there is $0 < r_0 < 1/2$ such that for $0 \leq r \leq r_0$ we have $k \geq \bar{q}_1(r)$ and figure I(a) applies and for $r_0 \leq r \leq 1/2$, we have $k < \bar{q}_1(r)$ and figure I(b) applies. This proves part (2)(v). ■

Proof of Proposition 1

Proof of part (1) Consider a non drastic innovation ($c < 1/2$). The result is proved by looking at the following cases.

Case (i): $k \geq (1 + c)/3$: By Lemma A1(1)(i): $q_1^k(r) = \bar{q}_1(r)$, $q_2^k(r) = \bar{q}_2(r)$ and the Cournot price $p^k(r) = (1 + r)/3$ is at most $p^k(c) = (1 + c)/3 < 1/2$ (since $c < 1/2$). As $p^k(r)$ is increasing in r and $p^k(r) < 1/2$, it follows that in this case $\varphi_M(p^k(r))$ is increasing in r and so is $\pi^k(r)$. In this case it is optimal for firm 1 to set $r = c$; in that case by (1), the fixed fee is zero. So the optimal licensing policy for firm 1 the pure royalty policy (zero fixed fee) with $r = c$.

Case (ii): $k \leq 1/3$: By Lemma A1 (1)(ii): $q_1^k(r) = k$, $q_2^k(r) = (1 - r - k)/2$ and the Cournot price $p^k(r) = (1 + r - k)/2$. As $\varphi_M(p)$ is maximum at $p = 1/2$, the choice of r that gives $p^k(r) = 1/2$ will maximize $\varphi_M(p^k(r))$. Setting $p^k(r) = 1/2$ gives $r = k$. As $p^k(r)$ is increasing in r, $\varphi_M(p^k(r))$ is increasing for $r < k$ and is maximum at $r = k$. Since $r \leq c$, the optimal choice is (a) $r = k$ if $k < c$ and (b) $r = c$ if $k \geq c$. By (1), the fixed fee is positive if $r = k$ and it is zero if $r = c$.

Case (iii): $1/3 < k < (1 + c)/3$: By Lemma A1(1)(iii): there is $0 < r_0 < c$ such that if $0 \leq r \leq r_0$, then we have the standard Cournot outcome. Then as in Case (i), $\varphi_M(p^k(r))$ is increasing for $0 \leq r \leq r_0$, so it is sufficient to consider $r_0 \leq r \leq c$, in which case by Lemma 1(iii), the outcome is the same as in Case (ii). So the optimal choice is (a) $r = k$ if $k < c$ and (b) $r = c$ if $k \geq c$.

Proof of part (2) Consider a drastic innovation ($c \geq 1/2$). The result is proved by looking at the following cases.

Case (i): $k \geq 1/2$: By Lemma A1(2)(i): if $0 \leq r \leq 1/2$, then the equilibrium outcome is the standard Cournot outcome. Then as in Case 1(i), $\varphi_M(p^k(r))$ is increasing for $0 \leq r \leq r_0$, so it is sufficient to consider $1/2 \leq r \leq c$, in which case firm 2 drops out of the market and firm 1 obtains the monopoly profit. Setting any r with$1/2 \leq r \leq c$ is an optimal policy. However, these are redundant licensing policies as they give the same outcome (monopoly profit for firm 1) as not licensing. In this case not licensing is optimal for firm 1.

Case (ii): $k < \min\{1 - c, 1/3\}$: By Lemma A1(2)(ii): for any $0 \leq r \leq c$, the outcome is the same as in Lemma A1(1)(ii). So the Cournot price is $p^k(r) = (1 + r - k)/2$. As $\varphi_M(p)$ is maximum at $p = 1/2$, the choice of r that gives $p^k(r) = 1/2$ will maximize $\varphi_M(p^k(r))$. Setting $p^k(r) = 1/2$ gives $r = k$. As $k < 1/3$ and $c \geq 1/2$, We have $k \leq c$. So the optimal choice for firm 1 is $r = k$.

Case (iii): $1/3 \leq k < 1 - c$: By Lemma A1(2)(iii): there is $0 < r_0 < 1/2$ such that the for $0 \leq r < r_0$, we have the standard Cournot outcome. Then as in Case 1(i), $\varphi_M(p^k(r))$ is increasing for $0 \leq r \leq r_0$, so it is sufficient to consider $r_0 \leq r \leq c$, in which case by Lemma 1(2)(iii), the outcome is the same as in Case (ii). Therefore as in Case (ii), the optimal choice is $r = k$.

Case (iv): $1 - c \leq k < 1/3$: By Lemma A1(2)(iii): there is $1/2 < r_1 < c$ such that for $0 \leq r < r_1$, the outcome is the same as in Lemma A1(1)(ii). So the Cournot price is $p^k(r) = (1 + r - k)/2$. As $\varphi_M(p)$ is maximum at $p = 1/2$, $\varphi_M(p^k(r))$ is maximum when $p^k(r) = 1/2$. Setting $p^k(r) = 1/2$ gives $r = k$. Since $k < 1/3$ and $1/2 < r_1$, it follows that $k < r_1$. So for $0 \leq r < r_1$, $\varphi_M(p^k(r))$ is maximum at $r = k$. By Lemma A1(2)(iii): $\varphi_M(p^k(r))$ is a constant for $r_1 \leq r \leq c$, it follows that the optimal choice in this case is $r = k$.

Case (v): $\max\{1 - c, 1/3\} < k < 1/2$: By Lemma A1(2)(v): there is $0 < r_0 < 1/2$ and $1/2 < r_1 < c$ such that the outcome is the same as (1)(i) for $0 \leq r < r_0$, same as in (1)(ii) for $r_0 \leq r < r_1$ and the same as last part of (2)(iv) for $r_1 \leq r \leq c$. So $\pi^k(r)$ is increasing for $0 \leq r < r_0$ and constant for $r_1 \leq r \leq c$. For $r_0 \leq r < r_1$ $\pi^k(r)$ is maximum at $r = k$. Note that $(1 + r_0)/3 = k$, so $r_0 = 3k - 1 < k$ (since $k < 1/2$). Also note that $k < 1/2 < r_1$. Thus $r_0 < k < r_1$. So in this case also the optimal choice is $r = k$. ■

Proof of Proposition 2

In view of Corollary 1, to completely characterize optimal pure royalty policies, we need to find optimal pure royalty policies for the region where $k < 1/2$ and $k < c$ (region *OAHG* in figure A1).

Proof of part (1) Consider the case where $c < 1/2$ (non drastic innovation) and $k < c$ (region *OAS* in figure A1). Recall that $\pi^k_R(r) = \varphi_1^k(r) + rq_2^k(r)$.

Case 1: $k \leq 1/3$ [region *OZES*]. By Lemma A1(1)(ii), we have

(2.1) $\pi^k_R(r) = (1 + r - k)k/2 + r(1 - r - k)/2$

As $\pi^k_R(r)$ is increasing for all $r < 1/2$ and $c < 1/2$, its unique maximum over $0 \leq r \leq c$ is attained at $r = c$ and firm 1 obtains $\pi^k_R(r) = \varphi_1^k(c) + cq_2^k(c)$. If firm 1 does not license, it would obtain $\varphi_1^k(c)$. Since $q_2^k(c) > 0$, we conclude that the unit royalty policy with $r = c$ is superior to not licensing.

Case 2: $1/3 < k < (1 + c)/3$ [region *ZAE*]. By Lemma A1(1)(iii), there is $0 < r_0 < c$ such that

(2.2) $\pi^k_R(r) = (1 + r)^2/9 + r(1 - 2r)/3$

for $0 \leq r \leq r_0$ and $\pi^k_R(r)$ is given by (2.1) for $r_0 \leq r \leq c$. Note that the expression in (2.2) is increasing for all $r < 1/2$. As $r_0 < c < 1/2$, it follows that $\pi^k_R(r)$ is increasing for $0 \leq r \leq r_0$. As the expression in (2.1) is also increasing, we conclude that the unique maximum of $\pi^k_R(r)$ is attained at $r = c$. As in Case 1, the unique optimal unit royalty policy is $r = c$ and this policy is superior to not licensing.

Together with the conclusion of Corollary 1(1), this completes the proof of part (1).

Proof of part (2) Consider the case where $c \geq 1/2$ (drastic innovation) and $k < 1/2$ (region *SAHG* in figure A1).

Case 1: $k < \min\{1 - c, 1/3\}$[region *SEFG*]. Then by Lemma A1(2)(ii), $\pi^k_R(r)$ is given by (2.1) for all $0 \leq r \leq c$ and its unique maximum is attained at $r = 1/2$. By not licensing, firm 1 obtains $\varphi_1^k(c) < \varphi_1^k(c) + rq_2^k(c) = \pi^k_R(c) \leq \pi^k(1/2)$. So the unique optimal unit royalty policy has $r = 1/2$ and it is superior to not licensing.

Case 2: $1/3 \leq k < 1 - c$ [region *EAF*]. Then by Lemma A1(2)(iii)), there is $0 < r_0 < 1/2$ ($r_0 = 3k - 1$) such that $\pi^k_R(r)$ is given by (2.2) for $0 \leq r \leq r_0$ and it is given by (2.1) for $r_0 \leq r \leq c$. As the expression in (2.2) is increasing for $r < 1/2$, it follows that $\pi^k_R(r)$ is increasing for $0 \leq r \leq r_0$. As the unique maximum for the expression in (2.1) is attained at $r = 1/2$, we conclude that the unique optimal unit royalty policy has $r = 1/2$. By not licensing, firm 1 obtains $\varphi_1^k(c) < \varphi_1^k(c) + cq_2^k(c) = \pi^k_R(c) \leq \pi^k(1/2)$, so this policy is superior to not licensing.

Case 3: $1 - c \leq k < 1/3$ [region *FTG*]. Then by Lemma A1(2)(iv), there is $1/2 < r_1 < c$ such that for $0 \leq r < r_1$, $\pi^k_R(r)$ is given by (2.2) and for $r_1 \leq r \leq c$, the payoff from unit royalty is the same as not licensing. Since $1/2 < r_1$, the unique maximum of the payoff at (2.2) is attained at $r = 1/2$ and this payoff is higher than the payoff at not licensing. We conclude that the unique optimal pure royalty policy has $r = 1/2$ and this policy is superior to not licensing.

Case 4: $\max\{1 - c, 1/3\} < k < 1/2$ [region *FAHT*]. Then by Lemma A1(2)(iv), there are $0 < r_0 < 1/2 < r_1$ such that $\pi^k_R(r)$ is given by (2.2) for $0 \leq r \leq r_0$, given by (2.1) for $r_0 \leq r \leq r_1$ and the payoff is the same as not licensing for $r_1 \leq r \leq c$. So the unique maximum is attained at $r = 1/2$ and this payoff is higher than the payoff at not licensing. We conclude that the unique optimal pure royalty policy has $r = 1/2$ and this policy is superior to not licensing.

Together with the conclusion of Corollary 1(2), this completes the proof of part (2). ■

Proof of Proposition 3

Recall that the maximum fixed fee that firm 1 can set is $f = \varphi_2^k(0) - \varphi_2^k(c)$ (provided this fee is non-negative) and the payoff of firm 1 under this fixed fee is $\pi^k_F = \varphi_1^k(0) + \varphi_2^k(0) - \varphi_2^k(c)$.

Non drastic innovations First consider the case of non drastic innovations, that is, $c < 1/2$.

Case I: $k \leq 1/3$ [region *OBES* in figure 3]. By Lemma A1(1)(ii), $\varphi_1^k(0) = (1 - k)k/2$, $\varphi_2^k(0) = (1 - k)^2/4$, $\varphi_1^k(c) = (1 + c - k)k/2$. Since $\varphi_2^k(0) > \varphi_2^k(c)$, the fee is positive. So we have

(3.1) $\pi^k_F = (1 - k)k/2 + (1 - k)^2/4 - (1 - c - k)^2/4$

By not licensing, firm 1 obtains $\varphi_1^k(c)$. Note that $\pi^k_F - \varphi_1^k(c) = c(2 - 4k - c)/4$. Since $k \le 1/3$, We have $2 - 4k \ge 2 - 4/3 = 2/3 > 1/2 > c$. Hence $\pi^k_F > \varphi_1^k(c)$, so in the region *OBES*, licensing by fixed fee is superior to not licensing.

Case II: $k \ge (1 + c)/3$ [(c, k) above line *BA* in figure 3]. By Lemma A1(i), $\varphi_1^k(0) = 1/9$, $\varphi_2^k(0) = 1/9$, $\varphi_1^k(c) = (1 + c)^2/9$ and $\varphi_2^k(c) = (1 - 2c)^2/9$. Note that $\varphi_2^k(0) > \varphi_2^k(c)$ for all $0 < c < 1/2$. So

(3.2) $\pi^k_F = 1/9 + 1/9 - (1 - 2c)^2/9 = 2/9 - (1 - 2c)^2/9$

By not licensing, firm 1 obtains $\varphi_1^k(c) = (1 + c)^2/9$. Note that $\pi^k_F - \varphi_1^k(c) = c(2 - 5c)/9$. Thus licensing by fixed fee is superior to not licensing if $0 < c \le 2/5$ and not licensing is superior to licensing by fixed fee if $2/5 < c < 1/2$.

Thus, in figure 3, for $c < 2/5$, fixed fee is superior to not licensing for the region above the line *BX*, while for $2/5 < c < 1/2$, not licensing is superior to fixed fee for the region above the line *XA*.

Case III: $1/3 < k < (1 + c)/3$ [region *BAE* in figure 3]. By Lemma A1(1)(iii), $\varphi_1^k(0) = 1/9$, $\varphi_2^k(0) = 1/9$, $\varphi_1^k(c) = (1 + c - k)c/2$ and $\varphi_2^k(c) = (1 - c - k)^2/4$. Note that $\varphi_2^k(0) - \varphi_2^k(c)$ is positive if and only if $c + k > 1/3$. Since $c + k > k > 1/3$, we have $\varphi_2^k(0) > \varphi_2^k(c)$. So we have

(3.3) $\pi^k_F = 1/9 + 1/9 - (1 - c - k)^2/4 = 2/9 - (1 - c - k)^2/4$

By not licensing, firm 1 gets $\varphi_1^k(c)$. Let $m(k, c) = \pi^k_F - \varphi_1^k(c)$. Note that $m(1/3, c) = c(2 - 3c)/12 > 0$ (since $c < 1/2$). Also note that $\partial m(k, c)/\partial k = k/2 - c$.

Case 1: $c \le 1/6$. In this case $k/2 > c$ for any $k > 1/3$, so $m(k, c)$ is increasing in k. Since $m(1/3, c) > 0$, in this case $m(k, c) > 0$ for all $1/3 < k < (1 + c)/3$. So licensing by pure fixed fee is superior to not licensing.

Case 2: $1/6 < c < 1/5$. In this case $1/3 < 2c < (1 + c)/3$.

Case 2(a): $1/3 < k < 2c$. Then $m(k, c)$ is decreasing in k. Note that $m((1 + c)/3, c) = c(2 - 5c)/9$, which is positive for $1/6 < c < 1/5$. So in this case $m(k, c) > 0$ for all $1/3 < k < (1 + c)/3$.

Case 2(b): $2c \le k < (1 + c)/3$. Then $m(k, c)$ is non-decreasing or increasing in k. Since $m(1/3, c) > 0$, in this case $m(k, c) > 0$ for all $1/3 < k < (1 + c)/3$.

From cases 2(a) and 2(b), we conclude that if $1/6 < c < 1/5$, then licensing by fixed fee is superior to not licensing.

Case 3: $1/5 \le c < 1/2$. In this case $2c \ge (1 + c)/3$, so $k/2 < c$ for any $k < (1 + c)/3$. So $m(k, c)$ is decreasing in k. Note that $m((1 + c)/3, c) = c(2 - 5c)/9$.

Case 3(a): $1/5 \le c \le 2/5$. Then $m((1 + c)/3, c) \ge 0$. Since $m(k, c)$ is decreasing in k, in this case $m(k, c) > 0$ for all $1/3 < k < (1 + c)/3$. So licensing by fixed fee is superior to not licensing.

Proof of part (1) From cases 1, 2(a)-(b) and 3(a) we conclude that fixed fee is superior to not licensing in the region *BXV* in figure 3. As fixed fee is also superior in the region above the line *BX* (Case II) and below the line *BV* (Case I), we conclude that for any k, fixed fee is superior to not licensing when $c < 2/5$. This proves part (1).

Case 3(b): $2/5 < c < 1/2$. Then $m((1 + c)/3, c) < 0 < m(1/3, c)$. Since $m(k, c)$ is decreasing in k, there is $k_0(c)$ such that $m(k_0(c), c) = 0$, $m(k, c) > 0$ for $1/3 < k < k_0(c)$ and $m(k, c) < 0$ for $k_0(c) < k < (1 + c)/3$.

So licensing by fixed fee is superior to not licensing if $1/3 < k < k_0(c)$ and not licensing is superior to licensing by fixed fee if $k_0(c) < k < (1 + c)/3$. We note that

(3.4) $k_0(c) = 2c - (\sqrt{45c^2 - 18c + 1})/3$

Note that $k_0(c)$ is decreasing in c.

Proof of part (2) when $2/5 < c < 1/2$ The curve *XW* in figure 3 presents $k_0(c)$ when $2/5 < c < 1/2$. Thus licensing by fixed fee is superior to not licensing in the region *VXWE* and not licensing is superior to licensing by fixed fee in the region *XAW*. As fixed fee is superior to not licensing below the line *VE* (case I) and not licensing is superior to licensing by fixed fee above the line *XA* (case II), it follows that when $2/5 < c < 1/2$, licensing by fixed fee is superior to not licensing in the region below *XW* and not licensing is superior to licensing by fixed fee in the region above *XW*. This proves part (2) for $2/5 < c < 1/2$.

Drastic innovations

Now we consider the case of drastic innovations, that is, $c \geq 1/2$. It is already shown in Corollary 1(2) that if $c \geq 1/2$ and $k \geq 1/2$, not licensing is superior to licensing by fixed fee. So consider $c \geq 1/2$ and $k < 1/2$ (the region *SAHG* in figure 3).

Case I: $k < \min\{1 - c, 1/3\}$[region *SEFG*]. By Lemma A1(2)(ii), π^k_F is given by (3.1) and as there, licensing by fixed fee is superior to not licensing.

Case II: $1 - c \leq k < 1/3$ [region *FTG*]. By Lemma A1(2)(iv), $\varphi_1^k(0) = (1 - k)k/2$, $\varphi_2^k(0) = (1 - k)^2/4$, $\varphi_1^k(c) = (1 - k)k$ and $\varphi_2^k(c) = 0$. Note that $\varphi_2^k(0) > \varphi_2^k(c)$ and

$\pi^k_F = (1 - k)k/2 + (1 - k)^2/4 - 0 = (1 - k)(1 + k)/4$

By not licensing, firm 1 obtains $\varphi_1^k(c)$. Noting that $\pi^k_F - \varphi_1^k(c) = (1 - k)(1 - 3k) > 0$ (since $k < 1/3$), in this region fixed fee is superior to not licensing.

Case III: $\max\{1 - c, 1/3\} < k < 1/2$ [region *FAHT*]. By Lemma A1(2)(v), $\varphi_1^k(0) = 1/9$, $\varphi_2^k(0) = 1/9$, $\varphi_1^k(c) = (1 - k)k$ and $\varphi_2^k(c) = 0$. Note that $\varphi_2^k(0) > \varphi_2^k(c)$ and $\pi^k_F = 1/9 + 1/9 - 0 = 2/9$. By not licensing, firm 1 obtains $\varphi_1^k(c)$. Note that $\pi^k_F - \varphi_1^k(c) = (3k - 1)(3k - 2)/9$. Since in this region, $1/3 < k < 2/3$, we have $\pi^k_F < \varphi_1^k(c)$, so not licensing is superior to licensing by fixed fee.

Proof of part (3) From cases (I)-(III) it follows that $c > 2/3$, licensing by fixed fee is superior to not licensing below the line FT (which has equation $k = 1/3$) and not licensing is superior to above the line FT. This proves part (3).

Case IV: $1/3 \leq k < 1 - c$ [region EAF]. By Lemma A1(2)(iii), $\varphi_2^k(0) > \varphi_2^k(c)$ and π^k_F is given by (3.3). Let $m(k, c) = [\pi^k_F - \varphi_1^k(c)]$. Note that $m(1/3, c) = c(2 - 3c)/12 > 0$ (since in this region $c < 2/3$). Also recall $\partial m(k, c)/\partial k = k/2 - c$. Since in this region $c > 1/3$, we have $1 - c < 2c$. As $k < 1 - c$, we have $k/2 < c$, so $m(k, c)$ is decreasing in k.

Note that $m(1 - c, c) = (c - 1/3)(c - 2/3) < 0$ (since $1/3 < c < 2/3$). Thus $m(1 - c, c) < 0 < m(1/3, c)$. Since $m(k, c)$ is decreasing in k, there is $k_0(c)$ such that $m(k_0(c), c) = 0$, $m(k, c) > 0$ for $1/3 < k < k_0(c)$ and $m(k, c) < 0$ for $k_0(c) < k < 1 - c$ (note that $k_0(c)$ is given by (3.3)).

So licensing by fixed fee is superior to not licensing if $1/3 < k < k_0(c)$ and not licensing is superior to licensing by pure fixed fee if $k_0(c) < k < (1 + c)/3$. In this region, we have $1/2 < c < 2/3$. The curve WF in figure 3 presents $k_0(c)$. Licensing by fixed fee is superior to not licensing in the region WEF and not licensing is superior to licensing by pure fixed fee in the region WAF.

Proof of part (2) when $1/2 < c < 2/3$ From cases (I)-(IV) and Corollary 1(2) it follows that if $1/2 < c < 2/3$, licensing by fixed fee is superior to not licensing below the curve WF and not licensing is superior to licensing by fixed fee above the curve WF. ■

Proof of Proposition 4

In view of the discussion just preceding Proposition 4, to complete the proof of the proposition it remains to see how fixed fee and unit royalty compare in the region $OYFTG$ in the figure below.

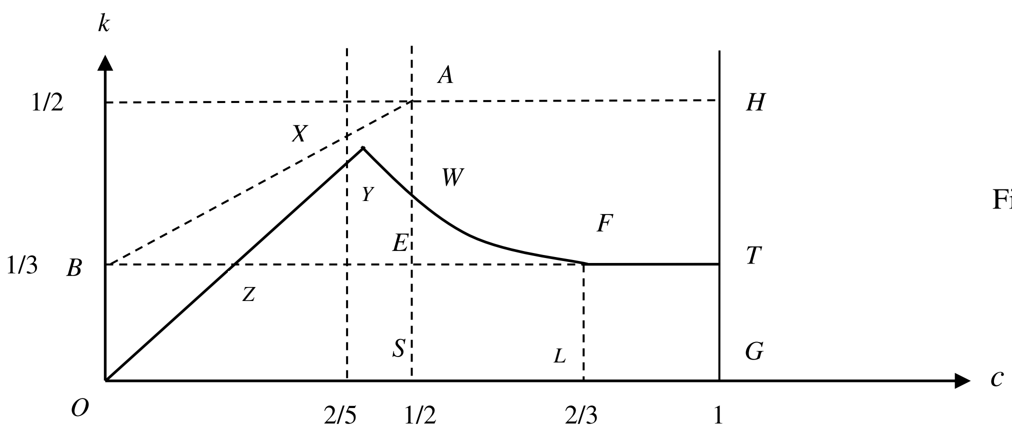

Figure A3

Non drastic innovations First consider $c < 1/2$. By Proposition 2, in this case the optimal pure royalty policy has $r = c$. Note that $1/3 < k_0(c)$, so $\min\{c, 1/3\} < k_0(c)$.

Case 1: $k \leq \min\{c, 1/3\}$ [region *OZES* in figure A3]. In this by Lemma A1(1)(ii), $\pi^k{}_F$ (the payoff of firm 1 from fixed fee policy) is given by (3.1) (see the proof of Proposition 3) and the payoff of firm 1 from the optimal pure royalty policy $r = c$ is given by

(4.1) $\pi^k{}_R = \varphi_1{}^k(c) + cq_2{}^k(c) = (1 + c - k)k/2 + c(1 - c - k)/2$

From (3.1) and (4.1), we note that $\pi^k{}_R - \pi^k{}_F = (c/2)(k - c/2)$. Since $c < 1/2$, we have $c/2 < 1/3$, so $c/2 \leq \min\{c, 1/3\}$. Thus fixed fee is superior to unit royalty if $k < c/2$ and unit royalty is superior to fixed fee if $c/2 < k \leq \min\{c, 1/3\}$. The line *OM* in figure A4 corresponds to $k = c/2$.

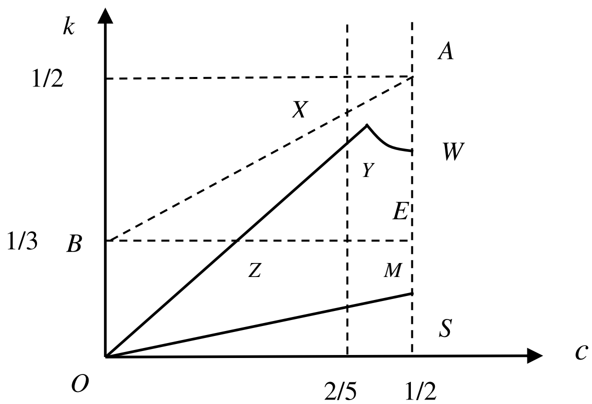

Figure A4

Case 2: $1/3 < k < \min\{c, k_0(c)\}$ [region *ZYWE* in figure A3]. In this case by Lemma A1(1)(iii), $\pi^k{}_F$ is given by (3.3) (see the proof of Proposition 3) and $\pi^k{}_R$ is given by (4.1). From (3.3) and (4.1), $\pi^k{}_R - \pi^k{}_F = (3c - 3k + 1)(3k - 3c + 1)/36$. Since in this region $k < c$, the term $3c - 3k + 1$ is positive. Since $k > 1/3$ and $c < 1/2$, we have $3k - 3c + 1 > 1 - 3/2 + 1 = 1/2 > 0$. Thus $\pi^k{}_R > \pi^k{}_F$. So pure royalty is superior to pure fixed fee in this region.

Proof of part (1) From Cases 1 and 2 it follows that for $c < 1/2$, fixed fee is superior to unit royalty in the region *OMS* and unit royalty is superior to fixed fee in the region *OYWM*. This proves part (1) of Proposition 4.

Drastic innovations Consider $c \geq 1/2$. By Proposition 2, in this case the optimal pure royalty policy has $r = 1/2$.

Case 3: $1/2 \leq c \leq 2/3$ and $k < 1/3$ [region *SEFL* in figure A3]. In this case by Lemma A1(2)(ii), $\pi^k{}_F$ is given by (3.1) and the payoff of firm 1 from the optimal pure royalty policy $r = 1/2$ is given by

(4.2) $\pi^k{}_R = \varphi_1{}^k(1/2) + (1/2)q_2{}^k(1/2) = (3 - 2k)k/4 + (1 - 2k)/8$

From (3.1) and (4.2), $\pi^k{}_R - \pi^k{}_F = ck/2 + c^2/4 - c/2 + 1/8$ and

(4.5) $\pi^k{}_R - \pi^k{}_F > 0$ if and only if $k > k_1(c)$ where $k_1(c) = 1 - 1/4c - c/2$.

Note that $\mathrm{d}k_1(c)/\mathrm{d}c = 1/4c^2 - 1/2$, so

(4.6) $dk_1(c)/dc > 0$ if and only if $c < 1/\sqrt{2}$.

As $1/\sqrt{2} > 2/3$, it follows that $k_1(c)$ is increasing for $1/2 \leq c \leq 2/3$. Note that $k_1(1/2) = 1/4 > 0$ and $k_1(2/3) = 7/24 < 1/3$. Thus $0 < k_1(c) < 1/3$ for any $1/2 \leq c \leq 2/3$. So in this case fixed fee is superior to unit royalty if $0 < k < k_1(c)$ and unit royalty is superior to pure fixed fee if $k_1(c) < k < 1/3$. The curve *MN* in figure A5 corresponds to $k_1(c)$. Note that at $c = 1/2$, we have $k_1(c) = c/2$, so the curve of $k_1(c)$ meets the line *OM* (that has equation $k = c/2$) of figure A4 at *M*.

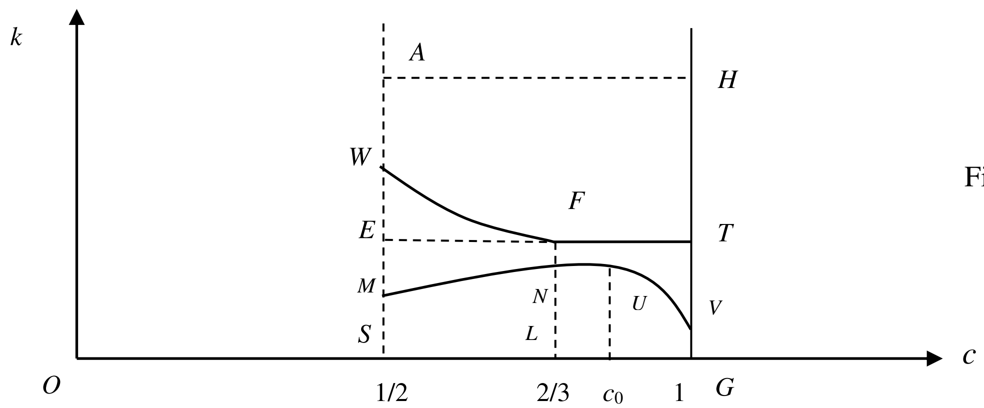

Figure A5

Case 4: $2/3 < c < 1$ and $k < 1/3$ [region *LFTG* in figure A3]. In this case by Lemma A1 (parts (2)(ii), (2)(iv)), π^k_F is given by (3.1), π^k_R is given by (4.2) and as in the previous case, (4.5) and (4.6) hold. Denote $c_0 \equiv 1/\sqrt{2}$.

Case 4(a): $2/3 < c < c_0$ and $k < 1/3$: In this case $k_1(c)$ is increasing. Note that $k_1(2/3) = 7/24 > 0$ and $k_1(c_0) = 1 - 1/\sqrt{2} < 1/3$. Thus $0 < k_1(c) < 1/3$ for any $2/3 < c < c_0$. So in this case fixed fee is superior to unit royalty if $0 < k < k_1(c)$ and unit royalty is superior to pure fixed fee if $k_1(c) < k < 1/3$. The curve *NU* in figure A5 presents the $k_1(c)$ when $2/3 < c < c_0$.

Case 4(b): $c_0 < c < 1$ and $k < 1/3$: In this case $k_1(c)$ is decreasing. Recall $k_1(c_0) < 1/3$. Also $k_1(1) = 1/4 > 0$. Thus $0 < k_1(c) < 1/3$ for any $c_0 < c < 1$. So in this case fixed fee is superior to unit royalty if $0 < k < k_1(c)$ and unit royalty is superior to pure fixed fee if $k_1(c) < k < 1/3$. The curve *UV* in figure A5 presents the $k_1(c)$ when $c_0 < c < 1$.

Proof of part (2) The curve *MUV* in Figure A5 represents $k_1(c)$ for $1/2 \leq c < 1$. Note that $k_1(c)$ is increasing for $1/2 \leq c < c_0$ and decreasing for $c_0 < c < 1$. From Cases 3, 4(a) and 4(b) it follows that for $c \geq 1/2$, fixed fee is superior to unit royalty in the region *SMVG* and unit royalty is superior to fixed fee in the region *MWFTV*. This proves part (2) of Proposition 4. ■

References

Acs, Z J. and D. B. Audretsch, 1990, *Innovation and Small Firms*, Cambridge, MA: MIT Press.

Alderighi, M, 2008, On the optimality of patent licensing with maximum production volume, *Economics Bulletin* 12: 1-7.

Kamien, M., and Y. Tauman, 1986, Fees versus royalties and the private value of a patent, *Quarterly Journal of Economics* 101: 471-491.

Marx, M., J.S. Gans, and D.H. Hsu, 2014, Dynamic commercialization strategies for disruptive technologies: evidence from the speech recognition industry, *Management Science* 60: 3103-3123.

McClellan, M.B., I. Sharma, M. Wosinska, A. Kroetsch, and H. Sullivan, 2020, COVID-19 Manufacturing for monoclonal antibodies, Policy Briefs, Duke Margolis Center for Health Policy, 22 June.

Mukherjee, A., 2001, Technology transfer with commitment, *Economic Theory* 17: 345-369.

OECD, 2004, *Patents and Innovation: Trends and Policy Challenges*, OECD Publishing, Paris,

Pavitt, K., M. Robson, and J. Townsend, 1987, The size distribution of innovating firms in the U.K.: 1945-1983, *Journal of Industrial Economics* 35: 297-316.

Scholz, E-M, 2017, Licensing downstream technology when upstream firms are capacity constrained, *CORE Discussion Papers* 2017004, Université Catholique de Louvain.

Sen, D., and G. Stamatopoulos, 2009, Technology transfer under returns to scale, *Manchester School* 77: 337-365.

Sen, D., and G. Stamatopoulos, 2016, Licensing under general demand and cost functions, *European Journal of Operational Research* 253: 673-680.

Sen, D., and G. Stamatopoulos, 2019, Decreasing returns, patent licensing, and price-reducing taxes, *Journal of Institutional and Theoretical Economics* 175: 291-307.

Sen, D., and Y. Tauman, 2007, General licensing schemes for a cost-reducing innovation, *Games and Economic Behavior* 59: 163-186.

Wang, X.H., 1998, Fee versus royalty in a Cournot duopoly model, *Economics Letters* 50: 55-62.