The Riddle of the Natural Rate of Interest

Razzak, Weshah

School of Economics and Finance, Massey University, New Zealand

April 2020

The Riddle of the Natural Rate of Interest

W A Razzak

School of Economics and Finance,

Massey University, New Zealand

April 2020

Revised August 2020

Abstract

We rely on microeconomics theory to compute the natural rate of interest for the G7 countries from 2001 to 2017. The equilibrium natural rate of interest is determined by a parsimonious equation that is easily computed from readily observable data, hence no estimation errors. The model predicts that the natural rate of interest is equal to the consumption-leisure growth rate less the capital-labor growth rate, which is zero in the steady state, no growth. It is positive (negative) when the consumption-leisure growth gap is greater (smaller) than the capital-labor growth gap. The model predicts that fiscal expansion is an expensive policy to stimulate the economy when the Zero Lower Bound (ZLB) constraint is binding.

JEL Classification: C68, E43, E52,
Keywords: Natural rate of interest, Monetary policy

1. Introduction

Many central banks seem to base monetary policy decisions regarding the short-term nominal interest rate on Wicksell (1898). The decision for setting the short-term nominal interest rate to achieve an inflation target depends on a variable called the Natural rate of interest. Wicksell (1898) laid out the theory of the natural rate of interest, where he essentially was concerned with explaining why prices rise or fall – i.e., inflation, which he regarded as the main problem of monetary theory. Bertil Ohlin, who wrote the introduction to Wicksell's book "Interest and Prices," explains the crux of Wicksell's idea:

> "…Must not the "natural" rate of interest, governed by the marginal productivity of capital, i.e., of the roundabout methods of production which would exist if money were not used, have some connections with the rate of interest as it actually appears on the capital market? There was only one possible answer. But what was this connection? ***These two rates of interest, the natural rate and the money rate, which is quoted on the market, tend of course, to coincide. If the former differs from the latter, money can no longer be said to be "neutral," and monetary consequences in the shape of change in prices are bound to ensue. If the money rate were kept below the natural rate prices would rise, if above they would fall.*** *"* [Boldface and italics is our emphasis].

The prediction of the Wicksellian theory is that inflation occurs when the short-term nominal rate is kept lower (below) than the natural rate. If r_t^*denotes the natural rate of interest and i_t denotes the short-term nominal money market rate then inflation occurs if $r_t^* - i_t > 0$.[1]

[1] For example, if $r_t^* = 0$ and the interest rate $i_t$is kept below zero, at –2, then expected inflation would be 2 percent. If the natural rate itself is negative then a negative nominal rate that is higher than the natural rate also creates positive inflation. Let r_t^*be –1, then a nominal rate of –2 would give a 1 percent inflation, –1– (–2) = 1.

Woodford (2003) is considered the new Keynesian approach of monetary policy. The main element of this approach is the Wicksellian monetary policy transmission mechanism. Although central banks do not follow rules for making monetary policy, the design of optimal monetary policy rule in Woodford depends on the natural rate of interest as described in Wicksell (1898). The natural rate of interest is an equilibrium real interest rate that prevails in the long run. In the long run, prices are flexible (rigidity occurs in the short run). Thus, monetary policy is neutral in the long run. However, the gap between the natural rate and the actual short-term nominal interest rate is a key channel through which the central bank can affect the economy.

Making monetary policy is daunting for a number of reasons. Monetary policy is a response to shocks whose nature and permanency are hard to determine ex-ante and to identify econometrically ex-post. Random shocks nudge the macroeconomic variables away from their equilibrium levels. The equilibrium variables are unobservable, e.g., the natural rate of unemployment; the non-accelerating inflation rate of unemployment (NAIRU); the real interest rate; and potential output. The natural real interest rate is also unobservable.

Therefore, policymakers must come up with an estimate, or a guess, of the “natural” real interest rate that they think is consistent with the level of real output that is equal to potential output, and to the inflation rate that is equal to expected inflation. Both potential output and expected inflation are unobservable.

Estimation of the unobservable natural rate requires macro-econometric models. Such models are subject to specification errors because the econometrician does not know the true *data generating process*. In addition, there are estimation errors.

Moreover, the effect of monetary policy on the economy takes time; the *lags are long and variable* (Friedman, 1961). Thus, monetary policy has to be forward-looking; forecasting and projecting macroeconomic variables is an essential job for central banks. Forecasting using macro-econometric models is associated with forecast errors that could be large and variable. These errors increase around the economy's turning points and could seep into policy decisions. They are usually persistent and undoing them is costly. Macroeconomists know that the making of monetary policy is associated with significant uncertainties. Unlike risk, there is no insurance against uncertainty. Random variables have natural levels of variations that we cannot reduce.[2]

There are a few important contributions to the estimation of the natural rate of interest. See, for example, Orphanides and Williams (2002) and Laubach and Williams (2003). The determinant of the natural real rate of interest in Laubach and Williams (2003) is the growth rate of potential output; they use data for real GDP, core inflation, oil prices, import prices, the federal funds rate, and an estimate of an output gap, and use the Kalman filter to estimate the unobservable neutral rate of interest.

Hamilton *et al.* (2015) argue that the data do not lend support the abovementioned determinants. They use long-term annual data and model the real rate as non-stationary. Then, they compute the steady state as an explicit time series forecast. Their evidence points to a significant uncertainty about the steady state real interest rate.

Beyer and Wieland (2019) emphasize the uncertainty around such estimates and the problems with modeling the equilibrium rate. Orphanides and Williams (2003) use a Keynesian model, and suggest that policymakers move cautiously about changing interest rate because of such uncertainties.

---

[2] A random variable x_t has an observed mean square around its mean, $= \frac{1}{n}\sum_{i=1}^{n}(x_t - \bar{\mu}_x)^2$, where $\bar{\mu}_x$ is the mean of x_t. The expected value of this function is $(\bar{x} - \bar{\mu}_x)^2 + \sigma^2$. This is minimized if the mean of x_t, $\bar{x}$ is set equal to $\bar{\mu}_x$. Thus, the expected value of the observed mean squares cannot be reduced below the inherited natural variance σ^2.

Koenig and Armen (2015) use survey data for the long-term bond yield, long-term GDP growth, and long-term inflation to estimate the long-term real interest rate. Del Negro *et al.* (2018) use a number of key macroeconomic and financial data to estimate the natural rate at different time horizons. Belke and Klose (2019) add a variable to the Laubach and Williams (2003) model to capture the financial cycle in a number of EU countries. They show that the ex-post real interest rates are lower than the estimated natural rates. They argue against Summers' (2014) *secular stagnation* argument that monetary policy is ineffective at or near the Zero Lower Bound (i.e., the interest rate is close to zero) and, therefore, fiscal stimulus is needed. Beyer and Wieland (2019) argue against Summers' hypothesis.

Our objectives include, first, to provide a parsimonious equation to *compute* the equilibrium or the *natural real rate of interest* with minimum specifications and estimations errors. To do so we rely on a straightforward microeconomic structure with optimizing agents – household, firm, and government – that allows for a significant role for fiscal policy. This approach minimizes the specification errors, has no estimation errors, and relies fully on the observable raw data to compute the equilibrium interest rate – i.e., the natural rate.

Second, we compute the natural rate of interest using annual data of the G7 countries from 2000 to 2017., Third, we produce a baseline projection of the natural rate is made for the period 2018 to 2024. Finally, we test Summers' (2014) *secular stagnation* argument out-of-sample by making projections of the growth rate of government spending required to stimulate aggregate demand when the ZLB is binding, i.e., $i_t = 0$, over the period 2018 to 2024.

The results indicate that, first; our estimates of the natural rates were significantly different across the G7 countries because the macroeconomic fundamentals are different across these countries. This is true for the EU countries too (namely, France, Germany, and Italy).

Second, Following Woodford (2003), the gap between the natural rate of interest and the short-term nominal interest rate is a key channel through which monetary policy affects the economy, and an indicator of past policy set up. A negative (positive) gap implies a tighter (looser) monetary policy because the policy interest rate is above (below) the natural rate. We do not know the views of these central banks about the natural rate, and it is not necessarily optimal to have a zero gap, Walsh (2005). However, given our computed natural rates for the period 2001-2017, Canada's gap was mostly negative, and so were France, Italy, and Japan. Germany, the U.K. and the U.S. gaps were very similar; the gaps were negative before 2009 and positive after 2009. Third, fiscal policy is effective when the ZLB constraint is binding. Our projections over the period 2018 to 2024 show that the increase in government spending increases total consumption and can stimulate aggregate demand; however, it is very costly. It requires a significant increase in the growth rate of government spending.

Next, we present our standard model. In section 3, we compute the natural rate of interest. We make projections in section 4. Section 5 is a conclusion.

2. The Model

We follow standard theory used in quantitative studies of business cycle. See Cooley (1995) and Cole and Ohanian (1999). In the depression literature, see, Kehoe and Prescott (2002); in public finance literature, see, Christaino and Eichenbaum (1992) and Baxter and King (1993); in the stock market literature see, McGrattan and Prescott (2003), and Boldrin, Christian and Fisher (2001).

Our model is a a standard microeconomic model, whereby the household maximizes a discounted log-linear and time – separable utility function in order to make decisions about consumption-savings and consumption-leisure choice.[3]

2.1 The Household

The household holds bonds and stocks, owns the capital stock, and rents it to the firm. The firm combines capital and labor to produce real output using a constant return to scale Cobb-Douglas production function. The household also pays taxes on the consumption good, on investments, on labor income, and on capital income. All tax revenues, except those used to finance the pure consumption good are given back to households in the form of transfers. The transfers are lump sum (independent of household income). Public expenditures are generally substitutes for private consumption in the U.S. Prescott (2004) assumed that they substitute on a one-to-one basis for private consumption with the exception of military expenditures. The goods and services in question consist mostly of publicly provided education, health care, protection services, and judiciary services. The government budget constraint holds all the time. It follows from Prescott's argument above that the model's consumption, c_t (lowercase) is not the same as SNA measure rather that equals to $C_t + G_t - G_t^M - t_t^i$, where C_t is the household consumption, G_t is government consumption, G_t^M is military spending, and t_t^i is the indirect tax on consumption.

The utility function is in a log-linear form helps in the computation.

$$E\{\sum_{t=s}^{\infty} \beta^{t+s}(ln\, c_{t+s} + \alpha\, ln\, (l_{t+s})\}, \quad (1)$$

[3] There is a large literature criticizing this, von Neumann-Morgenstern, utility function on the basis that it does not fit the data, see Campbell and Cochrane (1999) who argue that introducing a habit formation parameter in time-non-separable utility function resolves many of the empirical irregularities such as the equity premium puzzle of Mehra and Prescott (1985). See for example Constantinides (1990) on the same issues.

where l is leisure. The parameter $\alpha > 0$ measures the nonmarket productive time of the household (e.g., the relative value of leisure). We assume that a person has 100 hours of productive time a week. The nonmarket productive time i.e., leisure is $100-L_t$, where labor is average weekly hours worked per worker.[4] The household owns the capital stock, and rents it to the firm.

$$K_{t+1} = (1-\delta)K_t + I_t \ , \qquad (2)$$

where, K_t is the initial stock of physical capital, δ is the depreciation rate, and I_t is investment.

Next, we set the Lagrange multiplier optimization problem with the Lagrange multiplier λ and the discount factor β. In the budget constraint, we introduce a simple tax system similar to that in Nickell 2003 and Prescott, 2004. Let τ_c be the consumption tax rate; τ_I is the investment tax rate; τ_L is the marginal labor tax rate; w_t is the real wage rate; τ_k is the capital income tax rate; r_t is the rental price of capital, and TR_t is transfers. Tax revenues, except those used to finance pure public good consumption, are returned to households as lump-sum transfer payments – i.e., independent of the household's income.[5]

Taxes could affect the prices of consumption and investment goods (e.g., investment tax credit).[6] The household owns bonds B_t and stocks S_t, where their prices are P_t^b and P_t^s respectively (the superscript b denote bonds, and s stocks).

---

[4] Production of goods and services during leisure time is untaxed.

[5] The majority of public expenditures in G7 (i.e., education, health etc.) are perfect substitutes for private consumption, except for military spending. This is especially true for the U.S., Christaino and Eichenbaum (1992). However, this assumption could be highly restrictive for other countries as explained in Prescott (2004).

[6] This is a significantly simpler tax system than the systems used by the G7 countries. An accelerated depreciation and investment tax credits would affect the price of the investment good relative to the consumption good, but would not alter the inference drawn in this case. Similarly, introducing a corporate sector, with dividends not taxed, as is generally the case in the EU, or taxed as ordinary income, as they are in

The budget constraint BC_t is:

$$(1-\tau_L)w_t L_t + (1-\tau_k)(r_t-\delta)K_t + \delta K_t + B_{t-1} + (P_t^s + d_t)S_{t-1} + TR_t = (1+$$

$$\tau cPtct{+}1{+}\tau IIt{+}PtbBt{+}PtsSt \qquad (3)$$

The multiple periods Lagrange multiplier is:

$$[ln\mathrm{c_t} + \alpha\, ln(100-\mathrm{L_t}) + \beta u(ln\mathrm{c_{t+1}} + \alpha\, ln(100-\mathrm{L_{t+1}}) + \beta^2 u(ln\mathrm{c_{t+2}} + \alpha ln(100-\mathrm{L_{t+2}})]$$
$$+\beta^3 u(\dots) + \lambda_t[\mathrm{BC_t}] + \beta\lambda_{t+1}[\mathrm{BC_{t+1}}] + \beta^2\lambda_{t+2}[\dots] \qquad (4)$$

2.2 The Firm

We assume a firm producing output, Y_t, using capital $K_t$and labor L_t in a Cobb-Douglas production function that exhibits a constant return to scale with the shares of capital and labor, θ and (1-θ) respectively. The variable A_t is labor-augmenting technical progress, which we assume to be exogenous for simplicity.

$$Y_t = A_t K_t^{\theta} L_t^{1-\theta}, \qquad (5)$$

Solving from the time-sequential Lagrange multiplier problems, and focusing on the variables, consumption-leisure ratio c_t / $(100 - L_t)$, and capital-labor ratio K_t / L_t, the MRS between leisure and consumption is:

$$\frac{\frac{\alpha}{100-L_t}}{\frac{1}{c_t}} = \frac{1-\tau_t}{1+\tau_c}\frac{w_t}{P_t}. \qquad (6)$$

[7]

To simplify further, we introduce the tax rate τ:

Let $\frac{\tau_c+\tau_L}{1+\tau_c} = \tau$. (7)

---

the U.S., would not alter any conclusion significantly because in this model, the most important parameters are the factor shares and the relative value of leisure. See McGrattan and Prescott (2002).

[7] The price level could be set to 1, but we kept it as it will become clear at the end why we did that.

Add 1 to both sides,

$$1 - \frac{\tau_c + \tau_L}{1 + \tau_c} = 1 - \tau. \tag{8}$$

We arrive at:

$$\left(\frac{1 - \tau_L}{1 + \tau_c}\right) = 1 - \tau. \tag{9}$$

So the MRS becomes

$$\frac{\frac{\alpha}{100 - L_t}}{\frac{1}{c_t}} = (1 - \tau)\frac{w_t}{P_t}. \tag{10}$$

Our model is a simple macro model, which has no specific equation for the financial market. We assume no financial market friction; therefore, we do not model the banking system, leverages, and default rate. That said, the model captures the linkages between the financial market and the rest of the economy via bond, stock, and the general price level.

The stock-pricing equation is:

$$-\lambda P_t^s + \beta \lambda_{t+1}(P_{t+1}^s + d_{t+1}) = 0, \tag{11}$$

And for the bond price

$$-\lambda_t P_t^b + \beta \lambda_{t+1} = 0. \tag{12}$$

From finance theory and (12), the price of the bond is:

$$P_t^b = \frac{\beta \lambda_{t+1}}{\lambda_t}. \tag{13}$$

The term $\frac{\beta \lambda_{t+1}}{\lambda_t}$ is the "pricing kernel" of the economy. Thus, the price of the bond is equal to the pricing kernel × 1. The one is the payoff of the nominal bond assuming the face value of the bond = 1. The price of bonds, stocks, and the aggregate price are linked via the pricing kernel.

The stock price and the bond price are linked:

$$P_t^s = \frac{\beta\lambda_{t+1}}{\lambda_t}(P_{t+1}^s + d_{t+1}). \tag{14}$$

In real terms, divide by the CPI,

$$\frac{P_t^s}{P_t} = P_t^b \frac{(P_{t+1}^s + d_{t+1})}{P_t}. \tag{15}$$

Although it does not impinge on the solution of our model, this relationship demonstrates, implicitly that the pricing kernel binds the three prices, which represent the relationship between the macro economy and financial markets.

The FOCs from the firm side, basically, the marginal products whose ratio gives the MRTS between capital and labor equal to the factor input price ratio.

$$\theta\left(\frac{Y_t}{K_t}\right) = \lambda_t(1-\tau_k)r_t + \tau_k\delta, \tag{16}$$

and,

$$(1-\theta)\left(\frac{Y_t}{L_t}\right) = \lambda_t(1-\tau_L)w_t, \tag{17}$$

Solving for w_t:

$$w_t = \frac{1-\theta}{\theta}\frac{(1-\tau_k)r_t+\tau_k\delta_k}{(1-\tau_L)}\frac{\frac{Y_t}{L_t}}{\frac{Y_t}{K_t}}, \tag{18}$$

or,

$$w_t = (\frac{1-\theta}{\theta})\frac{(1-\tau_k)r_t+\tau_k\delta_k}{(1-\tau_L)}\frac{K_t}{L_t}. \tag{19}$$

And from equation (10):

$$w_t = \frac{\alpha P_t c_t}{(1-\tau)(100-L_t)}. \tag{20}$$

Equate equations (19) and (20), we get:

$$(1-\tau_k)r_t + \tau_k\delta = \left(\frac{\theta}{1-\theta}\right)\left(\frac{1-\tau_L}{1-\tau}\right)\left(\frac{K_t}{L_t}\right)^{-1}\left(\frac{\alpha P_t c_t}{100-L_t}\right) \tag{21}$$

Subtract $\tau_k\delta$ from both sides, divide both sides by $1-\tau_k$, then divide by P_t:

$$\frac{r_t}{P_t} = \left(\frac{\theta}{1-\theta}\right)\left(\frac{1-\tau_L}{(1-\tau)(1-\tau_k)}\right)\left(\frac{K_t}{L_t}\right)^{-1}\left(\frac{\alpha c_t}{(100-L_t)}\right) - \frac{\tau_k\delta}{1-\tau_k} \tag{22}$$

Let the constant terms be

$$\Gamma_1 = \frac{\theta}{1-\theta}\frac{\alpha(1-\tau_L)}{(1-\tau)(1-\tau_k)}, \tag{23}$$ [8]

and

$$\Gamma_2 = \frac{\tau_k\delta}{1-\tau_k}. \tag{24}$$

Then let

$$l_t = 100 - L_t. \tag{25}$$

Thus (21) reduces to,

$$\frac{r_t}{P_t} = \Gamma_1 * \left(\frac{K_t}{L_t}\right)^{-1} * \frac{c_t}{l_t} - \Gamma_2. \tag{26}$$

[8] There are either small or no changes in taxes. However, if changes are significant the tax rates could be kept in the model. The shares of capital and labor are almost constant.

Write the interest rate $(1 + r_t)$ and take log,

$$ln(1 + r_t) - ln(P_t) = ln(\Gamma_1) - ln(K_t) + ln(L_t) + ln(c_t) - l\,n(l_t) - ln(\Gamma_2)\,. \qquad (27)$$

Lag the equation once and subtract from the above, and note that the Taylor Series expansion approximates the growth rate of $(1+r_t) \approx \Delta\ln\,(1 + r_t) \;\approx r_t$, and $\Delta\ln(P_t) = \pi_t$, we get:

$$r_t - \pi_t = r_t^* = (\Delta lnc_t - \Delta lnl_t) - (\Delta lnK_t - \Delta lnL_t). \qquad (28)$$

Equation (28) is a parsimonious equation for computing the natural rate of interest. The RHS variables are the growth rates of four observable variables that can be computed easily. To remind you of the variables Δlnc_t is the rate of growth of consumption. Consumption is measured by household consumption plus government consumption less military spending, minus the indirect tax on consumption. $\Delta lnl_t$is the rate of growth of leisure, where leisure is 100-h_t, and $h_t$is average weekly hours worked per worker. $\Delta lnK_t$is the rate of growth of the stock of capital, and ΔlnL_t is the growth rate of labor, which we measure using working age population because changes in average weekly hours worked is very small. It predicts the natural rate to be zero in the steady state because the growth rates are zero The natural rate increases when the growth rate of consumption exceeds the growth rate of leisure and decreases when the growth rate of capital is faster than the growth rate of labor.[9] It would be positive (negative) if the consumption-leisure growth rates gap is greater (smaller) than the capital-labor growth rates gap.

2.2 Computing the natural rate

We use data for the G7 countries from 2000 to 2017. The data are described in the data appendix. Canada, France, Germany, Italy, and the U.K. have formal inflation targeting

---

[9] Doing the same thing for asset prices in equation (16) gives us $R_t^S - \pi = R_t^B - \pi_t + R_{P_{t+1}^S + d_{t+1}}$, where the LHS, the real return from the stock market is equal to the real return from safe bonds plus the return from future price and dividend movements.

regimes. Japan and the U.S. do not have a formal inflation targeting regimes, but they seem to have an inflation target. The three EU countries share a common inflation target and monetary policy; i.e., have the same short-term nominal policy interest rate. However, their natural rates of interest are country-specific because the real determinants of r_t^* are different across countries.

We measure labor by working age population (15-64 years) instead of average weekly hours worked because the latter does not seem to vary over the sample. Leisure is 100 minus the average weekly hours worked per worker $\frac{h_t}{WAP_t}$, which is computed from $\left[\frac{h_t}{E_t} \times \frac{E_t}{WAP_t}\right]/52$, where $h_t$is *annual* hours worked, $E_t$is total employment. The OECD publishes $\frac{h_t}{E_t}$ data (average actually worked annual hours per employee). $WAP_t$is working age population (15-64 years).

The typical assumption in macroeconomic models is that the growth rates of real variables are constants in steady state. The natural rate of interest is an equilibrium real rate that does not change in the steady state. Monetary policy, therefore, should be more concerned about the value of the natural rate in the long-run than the short and the immediate runs. Hence, averages of the natural rate rather than quarterly or annual changes are more appropriate. The averages of the natural rate of interest, the short-term nominal interest rate, the ex-post real rate defined as the nominal rate less average CPI inflation (two-year moving average), the Wicksellian expected inflation arising from $r_t^* - i_t$, and average CPI inflation (two-year moving average), are reported in table (1). We also report the standard deviations. The data are reported as averages over the whole sample, and over two subsamples, 2001-2008 and

2009-2017, before and after the Great Recession. We plot the average natural rate of interest and nominal interest rates in figure (1).[10]

We found that, first; the natural rate varies across the G7 countries significantly. This is consistent with the model, where the natural rate depends on the underlying country-specific consumption, capital, labor, and leisure growth rates. None of these factors is under the control or even influenced by the central bank. On average, the natural rate was relatively lower during the period following the Great Recession (2009-2017).

For the U.S., We compare the time series that we computed with Laubach and Williams estimates because their data are available online in annual frequency. First, Woodford (2003) emphasized that the gap between the natural rate and the short-term nominal rate – the federal fund rate in the case of the U.S. – is an important indicator of the effect of monetary policy on the economy. Walsh (2005) says that a historical series of the gap could provide a useful albeit limited indicator for evaluating monetary policy. Here, we are interested in comparing our computed natural rate with Laubach and Williams (2003) estimated one. Therefore, figure (2) plots the gap $r_t^* - i_t$ and $r_t^{*LW} - i_t$, where the superscript LW denotes Laubach and Williams. There are two distinct policy periods. From 2001 to 2008, both gaps look, more or less, the same. From 2005 to 2008, both took a deep dive. A negative gap means that the federal fund rate i_t was higher than r_t^* and r_t^{LW}. Monetary policy condition was tighter. From 2009 onward, the gap $r_t^{LW} - i_t$ remained almost flat but negative; the federal fund rate was only slightly higher than the natural rate of Laubach and Williams. The picture is quite different for $r_t^* - i_t$, which was positive, and as high as 2.4 in 2015. The federal fund rate was kept too low relative to r_t^*. According to the Wicksellian theory, the Fed must have wanted some stimulate the economy after the Great Recession in 2009.

[10] We do not plot the annual time series data but they are available upon request.

We still do not know what was the Fed's view of the natural rate was during the period 2009 to 2017. We only know that the average of the gap $r_t^* - i_t$is approximately equal to the average inflation rate, and the gap$r_t^{L-W} - i_t$ is not.

However, the questions are, why $r_t^*$is positive and higher r_t^{LW}during the period from 2009 to 2017, and why r_t^{LW} became negative from 2012? The Laubach-Williams estimate of the natural rate r_t^{LW}depends on the estimates of the output among other variables, which might have a significant impact on their estimate of the natural rate. The estimate of the output gap (deviations of real GDP from an HP filter trend) turned negative from 2009 to 2014. However, in our model, r_t^*does not depend on estimated output gap. The consumption – leisure growth rates gap was higher than the capital stock – labor growth rates gap throughout the whole period from 2001 to 2017. Figure (3) illustrates. It plots an estimate of the output gap (deviation from an HP filter trend), consumption and capital stock growth rates.

Back to table (1), over the full sample from 2000 to 2017, the average short-term nominal interest rates were higher than the natural rates in all G7, except for the U.S., where the nominal rate is equal to the natural rate. In the period from 2000 to 2008, leading to the Global Financial Crisis and the Great Recession, the short-term nominal rates in the G7 were significantly higher than their natural rates. During the period after the Great Recession, both the natural rate and the short-term nominal rate fell significantly, but the nominal rates remained above their natural rates, except for Germany and the U.S. In other words, monetary conditions remained relatively tight for a long period, except for Germany and the U.S.

The standard deviations are indicative of apparent volatilities. In almost all samples and countries, the volatility of the natural rate of interest is associated with the volatility of the growth rate of consumption almost one-to-one. The variations of the natural rates and the short-term nominal rates are different across samples and across countries, and within each country. The question is which moment of the data the central bank should match when it sets policy. Should it match the nominal interest rate to the average natural rate; or match the variances, or both?

Second, the real ex-post interest rates vary across countries. On average over the full sample from 2000 to 2017 and over the subsample from 2000 to 2008, the ex-post real rate was positive in all countries. However, the ex-post real interest rates turned negative, on average, during the period 2009 to 2017; and it was lower than the natural rate in most countries, except in Japan, where the ex-post real rate and the natural rates were equal. Belke and Klose (2020) found that the ex-post real rate in the EU is lower than the natural rate. They interpreted this as evidence against Summers (2014) *secular stagnation* argument.

Third, we found no (positive) correlation between $r_t^* - i_t$ and average inflation. Figure (4) plots the correlation coefficients between actual average CPI inflation and $r_t^* - i_t$ by country. For the U.S., i_t is the effective federal fund rate (ffr), and we also have the correlation coefficients using the Laubach-Williams estimate of the natural rate. Almost all the correlations are negative.

Note that the average CPI inflation (two-year moving average), which is reported in column 8 in table (1) is not as low as is widely claimed, except for Japan. However, the inflation rates were lower than 2 percent during the period 2009 to 2017, except for the U.K., whose CPI inflation is at the target. We use CPI (all items) inflation rate.[11]

[11] There are numbers of different measures of the CPI used by central banks such as the trimmed mean, weighted median, core inflation, headline, etc.

3. Projections

We have two projection scenarios covering the period 2018 to 2024. First is a baseline projection, in which projections for r_t^* are made, and the short-term nominal interest rate i_t that is needed to achieve a CPI inflation target of 2 percent, are computed. Second is a projection under a scenario of fiscal expansion in which the ZLB constraint is binding.

3.1 Baseline Projections

We make projections of the natural rate for the period 2018 to 2024. The assumptions over the projection horizon include, first, that consumption follows a random walk process (Hall, 1978). Second, the stock of capital evolves according to equation (2). The IMF – World Economic Outlook published in October 2019 reports projections of the nominal investment-nominal GDP ratio and the projections of the GDP deflator, which we use to get real total investment. The initial stock of capital is the level in 2017, and the depreciation rate times series are taken from the Penn World Table 9.0. Third, the labor supply is the working age population (15-64 years) projections taken from OECD population projections. Fourth, leisure is measured by $100 -$ average weekly hours worked $\frac{h_t}{WAP_t}$. As shown earlier, average weekly hours worked per worker is computed from $\left[\frac{h_t}{E_t} \times \frac{E_t}{WAP_t}\right]/52$, where $h_t$is *annual* hours worked, $E_t$is total employment, and $WAP_t$is working age population (15-64). Employment is the IMF -WEO projection. Unfortunately, these projections are shorter than our projection horizon; they are only reported up to 2020. So we assume that average weekly hours worked remain unchanged in the years 2021 to 2024; in fact, average weekly hours worked do not change from 2017 to 2020. This implies that leisure does not change over the projection horizon.

In addition to the projection of the natural rate, we compute the nominal interest rate required to achieve an inflation target of 2 percent over the projection period as predicted by Wicksell. Table (2) reports the average baseline projections and figure (5) is a plot of average of the baseline projections of the natural rates and the nominal interest rates that are required to achieve a 2 percent inflation target. We also report the confidence interval, $r_t^* \pm 2\sigma$.

The projections of the natural rate differ significantly across countries because they reflect different underlying economic fundamentals. For Canada, the natural rate is projected to be negative from 2018 to 2024. Therefore, the nominal interest rate must be even more negative in order to achieve the inflation target, -1.65 and -3.65 percent respectively. For France, the natural rate is projected to be near zero, therefore, the nominal rate required to achieve 2 percent inflation is around $-$ 2 percent. Germany is quite different. We project a high positive natural rate because consumption growth is projected to be increasing in Germany while it would be declining in France.

Germany's average natural rate projection is 2.89 percent. The short-term nominal rate required to achieve a CPI inflation target of 2 percent is -0.89 on average. We project a positive natural rate for Italy. On average, the projected natural rate is about 1.25 percent, therefore, the average nominal rate required to achieve the inflation target would be -0.75 percent. It is rather more daunting for the European Central Bank than any of the other central banks to set interest rate policy based on completely different natural rates across the EU.

The projection of the natural rate is high in Japan, on average, the highest in the G7, 3.44 percent over the projection horizon. Therefore, the nominal rate required to achieve the 2 percent inflation target would be 1.44 percent on average. The projection is driven by projections of near constant growth rate of consumption and rapidly falling growth rates of the stock of capital and labor. The fact that the central bank of Japan is pursuing a negative interest rate policy contradicts this projection.

The U.K. natural rate is projected to be about 0.60 percent over the period 2017 to 2024, therefore, the nominal interest rate required to achieve a CPI inflation target of 2 percent is -1.4 percent on average. Finally, for the U.S. we project an average natural rate of −0.90 over the projection horizon, and an average federal fund rate of −2.9 percent. The Federal Reserve should be lowering rates significantly if it wants to be consistent with the Wicksellian theory of price stability. Both consumption growth and capital stock growth are projected to be falling, with the latter more sharply declining over the period 2018 to 2014.[12]

3.2 Secular stagnation, fiscal policy

There is a widespread belief that monetary policy has reached its limit, the ZLB, and it is no longer effective in stimulating aggregate demand. Summers (2014), argues for a fiscal policy expansion to get the economy out of *secular stagnation*. Increasing government-spending increases consumption thus stimulates aggregate demand and eventually increases inflation.[13]

[12] Our model has no role for population *per se* other than working age population. To test the hypothesis that aging affects the natural rate of interest; it is possible to assume an aging working age population by including the projections of the population aged 65-69 years and even 70-74 years in the calculation of the average weekly hours worked. Doing so would increase working age population. This has two effects. First, it increases labor, which increases the natural rate. If labor grows faster than capital, the natural rate increases. Second, it lowers average weekly hours worked, thereby increases leisure time. If leisure grows faster than consumption, the natural rate declines. There is another effect, which our model does not capture and that is the direct effect of ageing population on total consumption. If consumption growth declines with ageing population, the natural rate falls. Thus, the final effect is ambiguous and depends on the magnitude of the relative changes in the growth rates of consumption, labor, and leisure. We do not report the projections because this issue is beyond the scope of this paper.

[13] There are a number of careful analyses, which showed that the fiscal multiplier is rather small and that fiscal stimulus such as Obama fiscal stimulus have not been effective in stimulating aggregate demand, see Taylor (2009) and Ramey (2011) for example.

The question is, what is the level of government spending required to stimulate aggregate demand? We assume that the government aims at stimulating aggregate demand by increasing spending to a level consistent with no more than 2 percent inflation when the ZLB constraint is binding. We leave the assumptions of the baseline projections unchanged. We calculate the level of government spending in the consumption equation $c_t(g) = C_t + G_t - G_t^m - t_t^i$,such that the natural rate of interest associated with it, is $r_t^*(g) = 2 = (\Delta \ln c_t - \Delta \ln l_t) - (\Delta lnK_t - \Delta lnL_t)$.

Summers (2014) argument is correct in the sense that fiscal expansion is stimulatory and inflationary, but it is very costly. Government spending level from 2018 to 2024 under this scenario would have to be very high. Table (3) reports the average of the growth rates of consumption, capital, working age population, leisure, and the natural rate projection $r_t^*(g)$. Figure (6) plots the average growth rate of government spending over the projection horizon compared with the average growth rate over the period 2000 to 2017.

We exclude Germany and Japan because the baseline projections of the natural rates were already higher than 2 percent. Government spending must grow, on average over the period 2018 to 2024, by 10.2, 7.6, 5.2, 3.1, and 8.6 percent, in Canada, France, Italy, the U.K. and the U.S. respectively. The growth rates of government spending are significantly higher than rates that prevailed over the period 2000 to 2017. Fiscal expansion is an expensive policy to generate 2 percent inflation.[14]

[14] The Coronavirus pandemic was growing at the time of writing this paper. Now we know that the data for consumption, leisure, capital, and labor will change significantly in 2020 onwards. The natural rate of interest will change too. We do not attempt to add anything to our projections because of there is a significant amount of uncertainty, but our projections will change most probably. Consumption will decline, thus, capital (savings) to increase. The decline in consumption and the increase in capital will have a negative effect on the natural rate. Employment will only affect our measure of leisure. The decline in employment will increase leisure time.

5. Conclusions

We provided a microeconomics model with optimizing agents (a household, firm, and government) to compute the natural rate of interest for the G7 countries over the period 2000 to 2017. In this model, the natural rate is determined by a parsimonious equation of four observable growth rates of consumption, leisure, capital, and labor, which all are easily computable using raw data. The natural rate is zero in the steady state, when the growth gaps, consumption less leisure, and capital less labor, are closed. When consumption grows faster than leisure, the natural rate rises. And, when capital grows faster than labor, the natural rate declines. The natural rate of interest is positive (negative) when the consumption – leisure growth rate gap is greater (smaller) than the capital – labor growth rate gap.

Baseline projections over 2018-2024 indicate the natural rate would be negative in Canada, France, and the U.S. which implies that the short-term nominal policy rate should be more negative for monetary policy to be stimulatory. The natural rate is positive for the other G7 countries. The natural rate is positive, higher than 2 percent in Germany, and higher than 3 percent in Japan, thus nominal short-term interest rates less than these figures indicate a stimulatory monetary policy. Fiscal policy is more effective than monetary policy when the ZLB constraint is binding; however, the data showed that it is a costly policy to stimulate aggregate demand. Government spending growth rates required to achieve an inflation rate as high as 2 percent are very high.

References

Beyer, R. C. M and V. Wieland (2019). Instability, Imprecision, and Inconsistent Use of Equilibrium Real Interest Rate Estimates, *Journal of International Money and Finance*, 94, 1-14.

Belke, A. and J. Klose (2020). Equilibrium Real Interest Rate and the Financial Cycle: Empirical Evidence for Euro Area Member Countries, *Economic Modeling*, 84, 357-366.

Baxter, M. and R. King (1993). Fiscal Policy in General Equilibrium, *American Economic Review,* 83 (June), 315-334.

Boldrin, M., L. J. Christiano and J. D. M. Fisher (2001). Habit Persistence, Asset Returns, and the Business Cycle, *American Economic Review,* 91 (March), 149-166.

Campbell, J. Y. and J. H. Cochrane (1999). By force of habit: A consumption-based explanation of aggregate stock market behavior, *Journal of Political Economy,* 107(2), 205-251.

Christiano, L. J., and M. Eichenbaum (1992). Current Real Business Cycle Theories and Aggregate Labor Market Fluctuations, *American Economic Review*, 82 (June), 430-450.

Cole, H. L., and L.E. Ohanian (1999). The Great Depression in the United States from a Neoclassical Perspective, *Federal Reserve Bank of Minneapolis Quarterly Review,* 23 (Winter), 2-24.

Cole, H. L., and L.E. Ohanian (2002). The Great UK Depression: A Puzzle and a Possible Resolution, *Review of Economic Dynamics,* 5 (January), 19-44.

Constantinides, G. (1990), Habit Formation: A Resolution of the Equity Premium Puzzle, *Journal of Political Economy*. 98(3), 519-43.

Cooley, T., (ed.) (1995). Frontiers of Business Cycle Research, Princeton, Princeton University Press.

Del Negro, M., D. Giannone, M. Giannoni and A. Tambalotti (2018). Global Trends in Interest Rates, Working Paper no. 1812, Federal Reserve Bank of Dallas.

Friedman, M. (1961). The Lag in Effect of Monetary Policy, *Journal of Political Economy*, 69, 447-447.

Hamilton, J D, E S Harris, J Hatzius and K D West (2015). The Equilibrium Real Funds Rate: Past, Present and Future, *Proceedings of the US Monetary Policy Forum.*

Hall, R. E. (1978). Stochastic Implications of the Life Cycle – Permanent Income Hypothesis: Theory and Evidence, *Journal of Political Economy,* 86, 971-987.

Koenig, E. F. and A. Armen (2015). Assessing Monetary Accommodation: A Simple Empirical Model of Monetary Policy and Its Implications for Unemployment and Inflation, Federal Reserve Bank of Dallas *Staff Papers*, No. 23, 2015.

Kehoe, T. J. and E. C. Prescott (2002). Great Depressions of the 20th Century, *Review of Economic Dynamics,* 5 (January) 1-18.

Kydland, F. E. and E. C. Prescott (1982). Time to Build and Aggregate Fluctuations, *Econometrica* 50 (November), 1345-1370.

Laubach, T and J C Williams (2003). Measuring the Natural Rate of Interest, *The Review of Economics and Statistics,* 85(4), 1063-1070

McGrattan, E. R. and E. C. Prescott (2003). Taxes, Regulations, and the Value of US Corporations: A General Equilibrium Analysis, *Research Department Staff Report 309, Federal Reserve Bank of Minneapolis* (August).

McGrattan, E. R. and E. C. Prescott (2003). Average Debt and Equity Returns: Puzzling? *American Economic Review,* 93 (May), 392-397.

Mehra, R. and Edward C.Prescott (1985). The Equity Premium: A Puzzle, *Journal of Monetary Economics*, 15(2), 145-161.

Nickell, S. (2003). Employment and Taxes, *CESIFO* Working Paper No.1109, (December).

Orphanides, A and J C Williams. (2002). Robust Monetary Policy Rules with Unknown Natural Rates", *Brookings Papers on Economic Activity,* 2002(2), 63-145.

Prescott, E. C. (2004). Why Do Americans Work So Much More Than Europeans? *Federal Reserve Bank of Minneapolis Quarterly Review*, 28(1), 2-3.

Ramey, V., (2011). Can Government Purchases Stimulate the Economy? *American Economic Review*, 49(3), 673-85.

Summers, L. (2014). U.S. Economic Prospect: Secular Stagnation, Hysteresis, and the Zero Lower Bound, *Business Economics*, 49 (2), S65-73.

Taylor, J. B. (2009). The Lack of an Empirical Rationale for a Revival of Discretionary Fiscal Policy, *American Economic Review*. 99(2), 550-55.

Walsh, C. E. 2005. Interest and prices: foundations of a theory of monetary policy: Book Review. Macroeconomic Dynamics 9, 462-468.

Wicksell, K. (1898). Interest and Prices: A Study of the Causes Regulating the Value of Money. Translated by R. F. Khan with an Introduction by Bertil Ohlin. The Library of the Congress Card Number 65-16993. Sentry Press, New York, U.S.

Woodford, M. 2003. Interest and prices: foundations of a theory of monetary policy, Princeton University Press.

Table (1)

Country	Samples	Stats	$\Delta\ln(c_t)$	$\Delta\ln(K_t)$	$\Delta\ln(L_t)$	$\Delta\ln(l_t)$	r_t^*	i_t	r_t	$r_t^* - i_t$	$\bar{\pi}_t$
Canada	01-17	Mean	2.43	2.62	0.89	-0.08	0.78	1.98	0.14	-1.20	1.84
		std	1.20	0.38	0.35	0.59	1.50	1.30	1.15	1.16	0.61
	01-08	Mean	3.35	2.86	1.19	-0.19	1.86	3.26	1.01	-1.40	2.26
		std	1.05	0.31	0.11	0.50	1.01	0.85	1.19	0.94	0.36
	09-17	Mean	1.71	2.42	0.65	0.00	-0.05	0.98	-0.53	-1.03	1.52
		std	0.75	0.32	0.27	0.66	1.29	0.20	0.48	1.35	0.57
France	01-17	Mean	1.32	1.66	0.29	0.00	-0.04	1.37	-0.01	-1.41	1.39
		std	0.75	0.33	0.40	0.44	1.02	1.51	1.12	1.46	0.73
	01-08	Mean	1.93	2.01	0.69	0.00	0.62	2.90	1.00	-2.29	1.91
		std	0.57	0.11	0.14	0.70	1.14	0.79	0.84	1.74	0.28
	09-17	Mean	0.85	1.38	-0.01	0.00	-0.55	0.18	-0.80	-0.73	0.98
		std	0.47	0.10	0.20	0.00	0.58	0.43	0.49	0.73	0.72
Germany	01-17	Mean	1.95	0.97	-0.21	-0.15	0.92	1.55	0.12	-0.63	1.43
		std	1.21	0.21	0.60	0.60	1.38	1.64	1.38	2.64	0.51
	01-08	Mean	1.41	1.06	-0.36	-0.15	0.14	3.09	1.41	-2.95	1.68
		std	1.29	0.21	0.18	0.43	1.30	0.90	0.70	1.48	0.37
	09-17	Mean	2.43	0.89	-0.07	-0.14	1.61	0.18	-1.03	1.43	1.21
		std	0.96	0.18	0.81	0.75	1.10	0.43	0.49	1.36	0.52
Italy	01-17	Mean	0.06	1.18	0.06	-0.07	-0.98	1.55	-0.24	-2.53	1.79
		std	1.56	0.76	0.37	0.69	1.84	1.64	1.25	2.45	0.92
	01-08	Mean	0.76	1.91	0.07	-0.31	-0.77	3.09	0.75	-3.86	2.34
		std	0.75	0.15	0.29	0.58	0.84	0.90	0.88	1.23	0.29
	09-17	Mean	-0.56	0.52	0.05	0.14	-1.17	0.18	-1.11	-1.36	1.30
		std	1.86	0.34	0.44	0.75	2.47	0.43	0.80	2.72	1.02
Japan	01-17	Mean	0.95	0.68	-0.76	-0.08	-0.41	0.18	0.14	-0.59	0.04
		std	0.90	0.49	0.40	1.00	1.42	0.22	0.75	1.49	0.75
	01-08	Mean	1.04	1.08	-0.57	0.00	-0.61	0.24	0.44	-0.85	-0.20
		std	0.86	0.33	0.21	0.72	0.72	0.29	0.33	0.92	0.51
	09-17	Mean	0.87	0.33	-0.92	-0.15	-0.23	0.12	-0.13	-0.35	0.25
		std	0.99	0.30	0.46	1.24	1.88	0.10	0.93	1.90	0.90
UK	01-16	Mean	1.80	1.50	0.59	-0.08	0.97	2.77	0.77	-1.80	2.00
		std	1.95	0.38	0.33	0.73	1.91	2.19	2.35	2.38	0.76
	01-08	Mean	2.77	1.81	0.83	0.00	1.79	4.83	2.96	-3.03	1.86
		std	1.59	0.14	0.17	0.68	1.69	0.73	0.51	2.12	0.57
	09-16	Mean	0.82	1.19	0.35	-0.16	0.14	0.72	-1.42	-0.58	2.13
		std	1.85	0.28	0.28	0.81	1.85	0.25	0.80	2.05	0.93
US	02-17	Mean	2.46	1.74	0.72	0.09	1.34	1.33	-0.71	0.11	2.04
		std	1.12	0.68	0.36	0.60	1.08	1.73	1.59	1.99	0.88
	02-08	Mean	3.18	2.49	1.08	0.00	1.78	2.93	0.43	-1.38	2.51
		std	0.68	0.18	0.15	0.00	1.22	1.75	1.52	2.24	0.45
	09-17	Mean	1.97	1.25	0.48	0.15	1.06	0.26	-1.48	1.09	1.73
		std	1.12	0.29	0.21	0.79	1.04	0.29	1.15	1.00	0.98

$c_t$is total consumption;$K_t$is the stock of capital, $L_t$is WAP(15-64), $l_t$is leisure (100-average weekly hours worked), ; $r_t^* = [(\Delta lnc_t - \Delta lnl_t) - (\Delta lnK_t + \Delta lnL_t)]$, $i_t$is 3-month nominal interest rate (ffr for the U.S.), $r_t = i_t - \bar{\pi}_t$and $\bar{\pi}_t$two-year moving average CPI inflation.

Table (2)
Average Baseline Projections 2018 – 2024*

	$\Delta ln\ (c_t)$	$\Delta ln\ (K_t)$	$\Delta ln\ (L_t)$	$\Delta ln\ (l_t)$	$r_t^* - 2\sigma$	r_t^*	$r_t^* + 2\sigma$	i_t
Canada	1.43	3.36	0.19	-0.09	-2.17	-1.65	-1.13	-3.65
France	0.51	0.63	0.05	0.09	-0.50	-0.16	0.18	-2.16
Germany	4.23	0.99	-0.32	0.02	0.98	2.89	4.80	0.89
Italy	0.28	-1.11	-0.19	-0.06	-0.55	1.26	3.07	-0.74
Japan	0.26	-3.39	-0.73	-0.53	1.94	3.44	4.95	1.44
UK	-0.01	-0.26	0.23	-0.14	-0.41	0.62	1.65	-1.38
US	1.45	2.66	0.22	-0.08	-1.43	-0.91	-0.37	-2.91

c_t is total real consumption, $K_t$is the projection of real stock of capital; $L_t$is the projection of working age population (15-64); $l_t$is the projection of leisure measured by 100-average weekly hours worked; $r_t^* = [\Delta lnc_t - \Delta lnK_t + \Delta lnL_t - \Delta lnl_t] * 100$; and $i_t$is the short-term nominal rate **required to achieve an inflation target of 2** percent based on the Wicksellian theory. * The U.K. sample is 2017 to 2024. σ is the standard deviation.

Table (3)
Average Projections under the Fiscal Expansion Scenario 2018-2024*

	$\Delta \ln\ c_t(g)$	$\Delta \ln\ K_t$	ΔlnL_t	Δlnl_t	$r_t^*(g)$
Canada	4.76	2.90	0.18	0.00	2.03
France	2.60	0.63	0.05	0.00	2.02
Italy	1.35	−1.11	−0.19	0.00	2.27
U.K.	1.42	−0.26	0.23	−0.14	2.05
U.S.	4.53	2.66	0.22	0.00	2.10

$c_{t\ (g)}$ is the projection of total real consumption with a level of government spending that is required to achieve 2 percent inflation target; $K_t$is the baseline projection of the real stock of capital; $L_t$is baseline projection of working age population (15-64); $l_t$is baseline projection of leisure measured by 100-average weekly hours worked; $r_t^* = [\Delta lnc_t - \Delta lnK_t + \Delta lnL_t - \Delta lnl_t] * 100$ * The U.K. sample is 2017 to 2024.

Figure (1)
Average Natural Rate

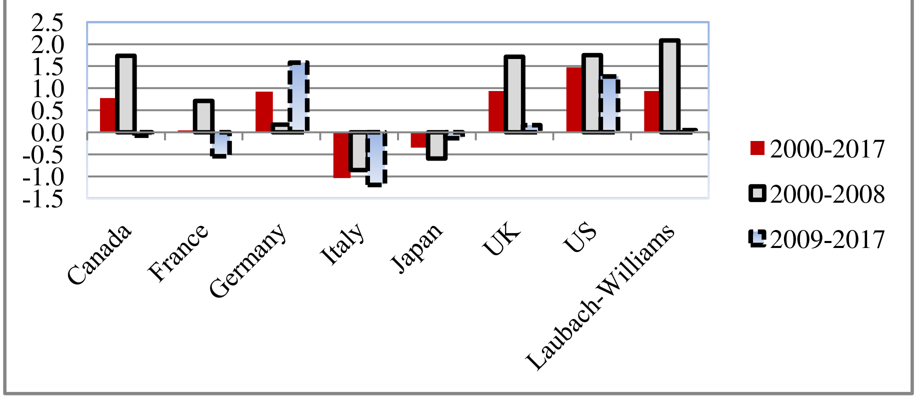
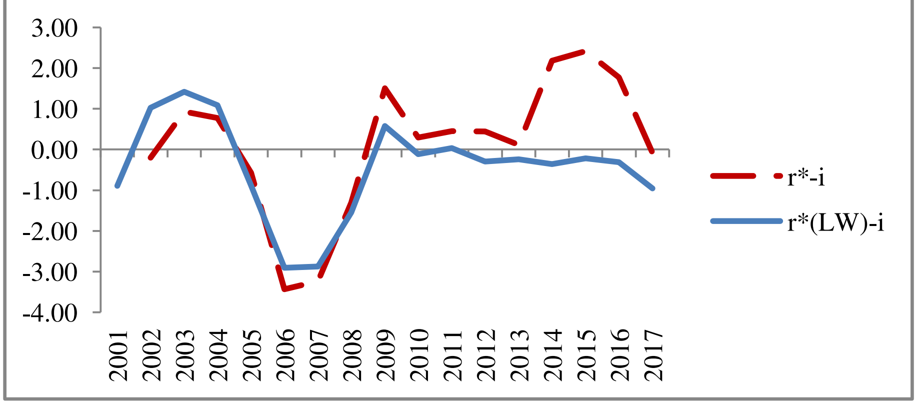


Figure (2)
Natural Rate - Nominal Interest Rate Gap



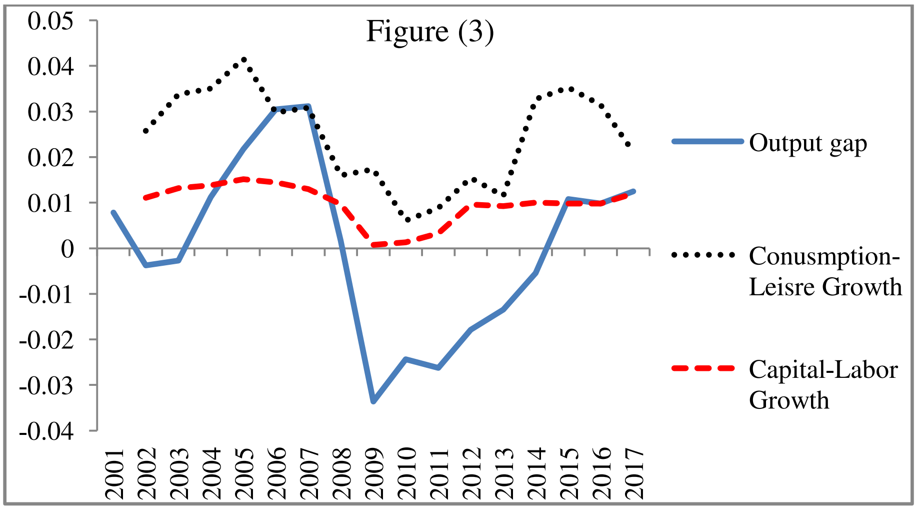


Figure (3)

Figure (4)
Correlation Coefficient between Actual Average CPI Inflation and r*-i

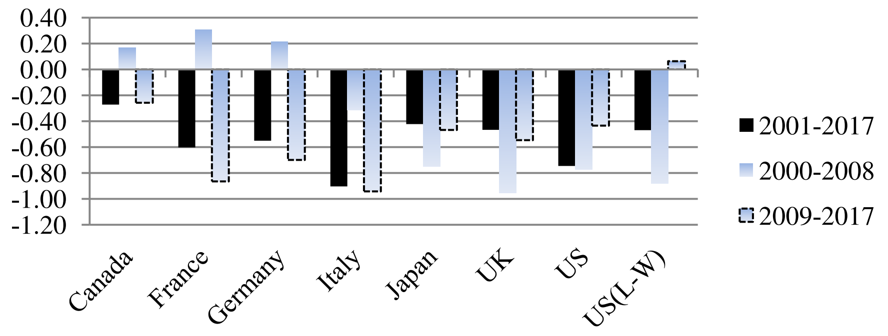
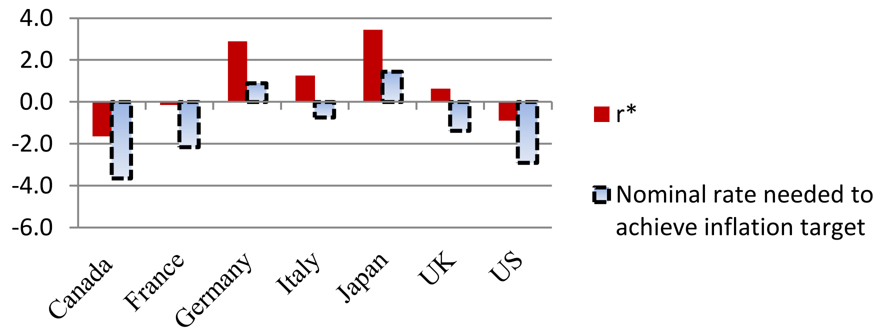


Figure (5)
Average Projections
2018-2014



Data Appendix

Data are annual from 2000 to 2017. Australia and the UK data are 2000-2016. All data are in national currencies.

WAP_t	Working age population (15-64).	OECD Statistics
E_t	Total employment (full time plus part time).	OECD Statistics
$\frac{h_t}{E_t}$	Hours worked per worker	OECD Statistics
L_t	$(\frac{E_t}{WAP_t} \times \frac{h_t}{E_t})/52$ is average Weekly Hours Worked – labor supply	
C_t	Real consumption at constant 2011 national prices (in mil. 2011US$),	Penn WT 9.1
G_t	Total Government Expenditures in billion Of local currencies converted into real using the price level of government consumption, price level 2011=1, Penn WT 9.1	IMF
G_t^M	Real Military Spending	World Bank Data
T_t^i	Indirect tax rate on consumption consists of sales tax plus tax on use of goods plus Custom duties collected for the EU plus custom duty plus excise plus tax on specific goods plus tax on specific services plus.	OECD Statistics
$c_t = C_t + G_t - G_t^M - t_t^i$	Model consumption	
K_t	Capital stock at constant 2011 national prices (in mil. 2011US$)	Penn World Table 9.1
I_t	Total investments	IMF-WEO
Price deflator	Implicit deflator	IMF-WEO
P_t^K	The capital price deflator	Penn World Table 9.1
θ_t and $1 - \theta_t$	Share of labor compensation in GDP at current national prices	Penn World Table
τ_{tL}	Tax rate on labor income	OECD Statistics
τ_t	The Tax wedge as proxy for effective marginal income tax rate. **The tax wedge** is the combined	OECD Statistics

	central and sub-central government income tax plus employee and employer social security contribution taxes, as a percentage of labour costs defined as gross wage earnings plus employer social security contributions. The tax wedge includes cash transfers. **TheTax wedge** is defined as the ratio between the amount of **taxes** paid by an average single worker (without children) and the corresponding total labor cost for the employer. The average **tax wedge** measures the extent to which **tax** on labor income discourages employment.	
τ_{tK}	The tax rate on capital income	OECD Statistics
δ	Average depreciation rate of the capital stock	Penn World Table 9.1
i_t	3-month interest rate. For the US, the effective federal funds rate.	FRED and OECD
P_t	CPI (all items, base year 2015=1)	OECD Statistics
Inflation	CPI inflation ΔlnP_t	OECD Statistics
Average Inflation	Two-year moving average inflation	