Optimal Resource Allocation in the Brain and the Capital Asset Pricing Model

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Abstract

Using recent findings from brain sciences, we relax the implicit CAPM assumption of sufficient brain resources, and model human brain as solving two optimization problems instead of one, which are: 1) Optimal resource allocation in the brain. 2) Mean-variance optimization. A security market line with varying slopes (flat, upwards, and downwards) arises depending on the resource allocation decisions in the brain. Size, value, and momentum effects also emerge in this enriched framework. This suggests that the classical CAPM is not misspecified. Rather, what appears as misspecification may be the result of ignoring the optimal resource allocation problem in the brain.

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Optimal Resource Allocation in the Brain and the Capital Asset Pricing Model

The CAPM of Sharpe (1964), Lintner (1965), and Mossin (1966) is the most widely used model of risk-return trade-off in finance (Levy and Welch 2017).[4] It posits that expected returns and betas should be positively related. However, in violation of this basic prediction, the observed security market line (SML) is generally too flat (see Fama and French (2004) for a comprehensive review).[5] Intriguingly, there are specific times when this relationship is positive such as months when inflation is low or negative (Cohen, Polk, and Vuolteenaho 2005), days when news about inflation, unemployment, or Federal Open Markets Committee (FOMC) interest rate decisions are scheduled to be announced (Savor and Wilson 2014), periods of pessimistic investor sentiment (Antoniou et al 2015), periods when margin requirements are relaxed by the Federal Reserve (Jylha 2018), and overnight (Hendershott et al 2019). Why is this relationship only positive at specific times, while flat or even downward sloping at other times? In this article, we show that relaxing the implicit CAPM assumption of sufficient brain resources provides a unified explanation for the varying SML slopes, along with providing explanations for size, value, and momentum effects. Our results suggest that CAPM may not be misspecified. Rather, what appears as misspecification may be the result of ignoring the optimal resource allocation problem in the brain.

Brain architecture (see Alonso et al (2014) and references therein) suggests an optimal resource allocation mechanism in the brain. We incorporate this mechanism into CAPM and model investors as solving, instead of one, two optimization problems which are as follows: 1) Optimal resource allocation in the brain. 2) Mean-Variance Maximization. We

[4] Some studies (such as Murphy (1990), Kim (1997), Jostova and Philipov (2005), Fu, Murphy, and Benzschawel (2015), and Wu (2018)) report favorable outcomes for CAPM.

[5] Fama and French (2016) find deviations from the implications of the model, such as related to beta, size, value and momentum that have persisted in varying degrees since early studies by Black, Jensen, and Scholes (1972), Stoll and Whaley (1983), Fama and French (1993), and Jegadeesh and Titman (1993) among others. Based on this poor empirical record, it has been suggested that there is misspecification in CAPM, and additional risk factors have been suggested that improve the model (Fama and French 2016, 2011, 1993). Over the last decade, only the momentum factor has persisted in generating average returns that are abnormally high relative to those expected by the CAPM (Blitz 2020).

show that this enrichment to the standard CAPM framework explains why the SML slope is positive at specific times only, while staying flat or even downward sloping at other times. The enriched framework also explains the major CAPM anomalies such as size, value, and momentum.

A sufficient condition for CAPM to apply is for investors to be mean-variance maximizers. Recent studies in neuroscience have found evidence that the human brain separately encodes these two moments of investment payoffs (see Bossaerts (2009) and references therein, Fukunaga et al 2018). Expected reward is encoded in the subcortical projection areas of the dopamine neurons, in particular, the ventral striatum, whereas brain regions involved in risk (variance) encoding include right and left insula, and thalamus. The executive part of the brain then constructs value from the statistics of gambles (Bossaerts 2009).[6]

Research in brain sciences has established that when there are multiple tasks for a person to conduct, each task is assigned to a particular system of neurons that coordinate their electrical firings to process information inputs that enable carrying out that task. Each of these systems of neurons, which require energy and other biological resources to carry out their tasks via neural firings, compete for the scarce resources with other systems in the brains. The 'central executive system' (CES) located in the lateral prefrontal cortex of the brain allocates finite resources to different systems of neurons with task performance dependent on resource allocation (see Alonso et al (2014) and references there in). With respect to the valuation of an asset, or a gamble, separate encoding of reward and risk are conducted in the brain, which are then combined to generate an integrated value in the executive part of the brain (Bossaerts 2009). It follows then that the two tasks involved in asset valuation (estimating expected cashflows and the risk of the cashflows) are performed by distinct systems of neurons that compete for scarce brain resources. In this article, we allow the resource constraint in the brain to bind. That is, the possibility that sufficient brain resources may not be allocated to either or both tasks has been considered here.

[6] Evidence from decision neuroscience indicates that certain/immediate rewards are disproportionately favoured over long-term/risky rewards (McClure 2004). Andreoni and Sprenger (2012) find that such direct preference for certainty explains the findings in their experiments well. Siddiqi (2017) and Siddiqi and Anwar (2020) explore the implications of such direct certainty preference for financial innovations.

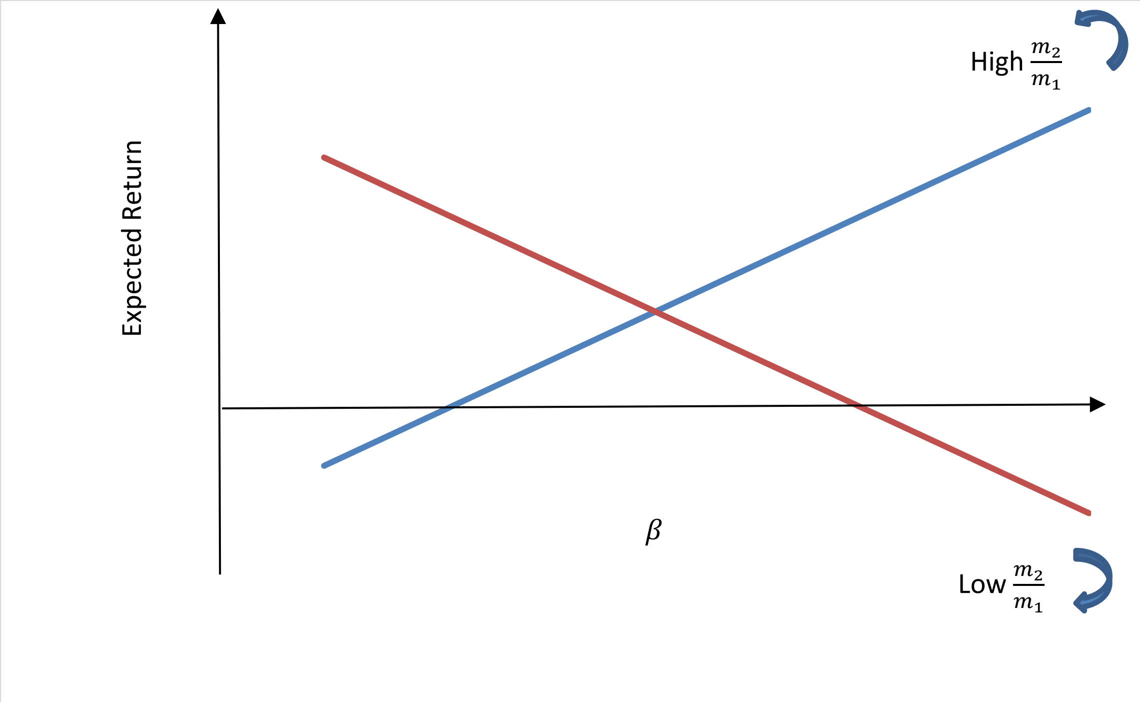

Figure 1 - Slope and Intercept of SML

Slope and intercept of SML changes with the ratio $\frac{m_2}{m_1}$ where m_1 is the fraction of required brain resource allocated to estimating expected cashflows, and m_2 is the fraction of required brain resources allocated to cashflow risk estimation. When optimal resource allocation in the brain shifts towards risk, $\frac{m_2}{m_1}$ rises, and SML rotates in the counter clockwise direction. When optimal resource allocation shifts towards expected cashflows, $\frac{m_2}{m_1}$ falls, and SML rotates in the clockwise direction.

Figure 1 illustrates the main result in the paper regarding SML slopes (proposition 1). The fraction of required brain resources allocated to expected cashflow estimation is denoted by m_1, whereas the fraction of required brain resources allocated to cashflow risk estimation is denoted by m_2. Figure 1 shows that when optimal resource allocation in the brain shifts towards risk, that is when $\frac{m_2}{m_1}$ rises, SML rotates in the counter clockwise direction, and when optimal resource allocation shifts in favor of expected cashflow estimation, SML rotates in the clockwise direction. We argue that specific periods when the empirically observed SML slope is positive are periods with high $\frac{m_2}{m_1}$ (see section 2.2).

Acknowledging that brain resources are scarce gives rise to a resource-rational view of the brain in which cognition is viewed as arising from the optimal allocation of limited brain resources (see Alonso et al (2014) and references therein, Leider and Griffiths 2020). Research in cognitive science has established that relying on informative starting points and then attempting to adjust them appropriately is a robust strategy consistent with optimal allocation of limited brain resources, and this strategy is universally employed in the brain (see Leider et al (2018) and references therein). In the context of cashflow analysis, it follows that when analyzing a firm, a resource-rational brain, leverages the cashflow analysis of a similar firm that has been analyzed earlier, and then makes appropriate adjustments instead of starting from scratch for each firm. For example, if one has already analyzed the impact of a potential new entrant on a firm q, then while analyzing the impact on another similar firm s, the resource-rational brain leverages the analysis for q and makes adjustments for differences between q and s. We also consider the possibility that the starting point may not come from a specific firm. Rather, investors have a sector or industry schema which provides the starting point. In that case, q is interpreted as representing an industry or sector average.

If the resource constraint in the brain does not bind, then full adjustment is reached, and the cashflow analysis of q and s do not get entangled. In other words, when the resource constraint in the brain does not bind, then starting points do not matter as there are no traces of q left in the cashflow analysis for firm s. That is, even though a resource-rational brain relies on informative starting points, the implicit assumption here is that cashflow analysis involves simple enough tasks so that full adjustments away from the starting points are reached; hence, rational expectations are formed. In this article, we relax this implicit assumption and allow the resource constraint in the brain to bind. It follows that the cashflow analyses of s and q are entangled. As the two tasks in cashflow analysis are estimating expected cashflows and the risk of cashflows with each task performed by a separate brain system, it follows that expected cashflows of s are entangled with the expected cashflows of q, and the cashflow risk of s is entangled with the cashflow risk of q.

The intuition behind figure 1 is now easy to see. When m_2 rises, the influence of the starting point (q) in the estimated risks of s firms diminish; hence, the cross-sectional variation in estimated risks across firms rises. This allows betas to explain the cross-sectional variation in expected returns better; hence, the slope of SML rises, and the intercept falls (counter clockwise rotation of SML). When m_1 falls, the influence of the starting point (q) in the expected returns of s firms get stronger, which diminishes the cross-sectional variation in expected returns, making it easier for a given variation in betas to explain the variation in expected returns. Hence, the slope of SML rises, and the intercept falls (counter clockwise rotation of SML) when m_1 falls. In other words, a rise in $\frac{m_2}{m_1}$ rotates SML in the counter clockwise direction. The opposite happens (SML rotates in the clockwise direction) when $\frac{m_2}{m_1}$ falls.

When optimal resource allocation in the brain favors expected cashflows over risk of cashflows, that is, when $m_1 > m_2$, then the cross-sectional variations in the impacts of cashflow entanglements automatically give rise to both size and value effects (see section 2.3). Combining this with the implications for the slope of SML, it follows that size and value effects should only be observed when the SML is flat or downward sloping. This is consistent with the empirical findings in Hendershott et al (2019). The resource-rational view of the brain provides an explanation for the momentum effect as well, which is explained by temporary shifts in relative resource allocation in the brain concerning momentum winners and losers (section 2.5).

Overall, by viewing human cognition as the optimal use of limited computational resources (Leider and Griffiths 2020), we integrate the 'bottom-up' understanding of cognitive architecture as established by research in brain sciences (see Alonso et al (2014) and references therein) with the top-down view of functional rationality (expected utility or mean-variance maximization) as developed and typically applied in economics and finance. It follows that, instead of one, the human brain solves two optimization problems: 1) Optimal resource allocation in the brain. 2) Mean-Variance maximization. The most surprising aspect of this enrichment is that major CAPM anomalies are reconciled suggesting that CAPM is not misspecified after all. An advantage of this approach is that instead of taking biases as given and studying their implications[7], we dig deeper into the neurobiological underpinnings of choice. In this view, biases emerge due to the way finite brain resources are optimally allocated in the brain.

[7] In particular, Siddiqi (2018) and Siddiqi (2019) assume anchoring bias and study its implications for CAPM and option pricing respectively. This article bridges the gap between these articles and the neurobiology of choice.

In section 1, we summarize evidence from decision neuroscience pertaining to how the resource-rational brain processes information, and how this evidence is broadly consistent with the notion of functional rationality (mean-variance analysis or expected utility maximization) as applied in economics and finance. In section 2, we adjust the classical CAPM model for optimal resource allocation in the brain and study its properties. We show how varying SML slopes as well as size, value, high-alpha-of-low-beta, and momentum arise in the adjusted framework. Section 3 concludes with suggestions for further research.

1. Information Processing in the Resource-Rational Brain

What happens when information reaches the human brain? In economics, a black-box approach to information absorption is typically taken with an implicit assumption that information, when it reaches the brain, is accurately processed. In terms of actual brain processes, research in brain sciences has established that when information reaches the brain, a mental template or schema, is first activated, which influences information absorption.[8] Brain imaging studies show that schemas lead to rapid assimilation of schema-consistent information, which makes reliance on schemas critically important for the resource-rational brain.[9]

A schema can be conceived as a scaffold or a blueprint, representing a higher-level knowledge structure integrating lower-level units. Neurologically, it is a brain template that involves systems of neurons across various brain regions talking to each other, with each system constituting a particular unit in the schema. That is, schemas contain units as well as relationships between these units. For example, for a car schema, units could be car body and wheel, with the relationship that car body contains four wheels. For a firm schema,

[8] Schemas have long been known to be used by the brain in memory formation and recall (Bartlett 1932, Bransford and Johnson 1972, Anderson and Pearson 1984). See Hampson and Morris (1996), Anderson (2000), and Pankin (2013) for an overview of schema theory. A sample of recent literature in neuroscience that explores the neural basis of schemas and their role in information absorption includes Tse et al (2007), van Kesteren et al (2010), Tse et al (2011), van Kesteren et al (2012), Ghosh and Gilboa (2014), Ghosh et al (2014), Brod et al (2015), Spalding et al (2015), Sweegers et al (2015), Gilboa and Marlatte(2017), and Ohki and Takei (2018).

[9] Tse et al 2007, Gilboa and Marlatte 2017. See Ohki and Takei (2018) and references therein.

units could be expected cashflows and risk of cashflows with an integration of these units creating value. Schemas, by only containing the essential details, simplify the world. They provide useful starting points, and in the process, speed-up processing of relevant information. Several demonstrations of how schemas shape our thought processes exist in the cognitive science literature.[10]

We posit that when information about a firm arrives, a relevant schema is activated that provides a useful starting point to process that information. If it's a prominent firm then a scheme dedicated to it may already exist in the brain. However, if it is an average or a typical firm then there may not be a dedicated schema for it. In that case, a related schema may be activated. There are two strong possibilities: 1) The schema of a prominent firm in the same sector is activated with adjustments made for the differences between the two firms. 2) The schema is not based on any specific firm. Rather, it represents the average behavior in the sector. Adjustments are then made for differences between average behavior in the sector and firm behavior. Note that irrespective of the nature of activated schemas, they provide useful starting points to the resource-rational brain.

Relying on a starting point (supplied by a schema) and then spending brain resources to appropriately adjust it to suit a particular situation is consistent with the resource rational view of the brain (Leider et al 2018, Leider and Griffiths 2020). We have been doing this throughout our lives. For example, a child may initially only have a schema for a horse (large with four legs, hair, and a tail). However, when she encounters a cow, she may make sense of that by accessing and appropriately modifying the horse schema. She may eventually integrate horse, cow and other animal schemas to form an overarching schema for four-legged animals with each animal type a specific instance of the generic animal schema, obtained by applying appropriate modifications. Similarly, investors are expected to have clustered similar firms together and have created a schema for them, either based on a prominent firm in the sector, or may have a generic schema for the cluster that captures the average behavior without representing a specific firm.

Given that schemas are critically important for the resource-rational brain, the existence of asset valuation schemas in the brain is expected. Neuroscience research points

[10] For example, see chapter 2 in Stangor (2011)

to ventromedial prefrontal cortex (vmPFC) as the key area involved in processing information through a schema where information from multiple regions in the brain is integrated.[11] So, if there are asset-valuation schemas in the brain, then the involvement of vmPFC is necessary for their working. Intriguingly, research in brain sciences has established that vmPFC is the key brain region involved in constructing willingness-to-pay, and that vmPFC does so by integrating information from multiple sources (other brain regions) (Sescousse et al 2013, Levy and Glimcher 2012, Peters and Buchel 2010, Rangel and Hare 2010, Hare et al 2008, Wallis 2007, Plassmann et al 2007). The finding that multiple sources are integrated to construct willingness-to-pay in vmPFC points to the existence of asset-valuation schemas.

As schemas are higher-level structures that integrate lower-level units, what are the associated lower-level units in an asset-valuation schema? Bossaerts (2009) discusses neuroscience evidence showing that brain separately encodes expected reward and reward variance[12] when confronted with a gamble, and these statistics are constantly re-evaluated, suggesting that vmPFC in the brain constructs value based on the statistics of gambles. This indicates that brain has architecture for mean-variance analysis. So, the functional or top-down view of rationality in economics and finance is supported by the 'bottom-up' evidence from neuroscience.

As expected cashflows and the risk of cashflows are the two key units involved in constructing value or willingness-to-pay, it follows that relying on starting points (either provided by a prominent firm in the sector or by a sector average schema) and making adjustments has two key tasks: appropriately modifying the expected cashflow of the starting point, and appropriately modifying the risk of the starting point. Research in brain sciences has established that, where there are multiple tasks, different brain systems (systems of neurons) are assigned to each task. These systems compete for scarce resources that are allocated by a 'central executive system' (CES) located in the lateral prefrontal cortex with relative task performance dependent on how the brain resources are allocated between them (Alonso et al 2014). It follows that the two tasks involved in valuation

[11] van Kesteren et al 2010, Tse et al 2011, Ghosh et al 2014, Brod et al 2015.

[12] Fukunaga et al (2018) present evidence that risk is primarily encoded in the brain as variance of possible outcomes.

(modifying the expected cashflows starting point, and modifying the risk starting point) are performed by distinct brain systems that compete for scarce brain resources.

In the next section, we take a modern derivation of CAPM (as in Frazzini and Pederson (2014)) and adjust it for such reliance on starting points. This transforms the decision problem underlying CAPM from just mean-variance maximization to mean-variance maximization in the context of optimal resource allocation in the resource-rational brain.

2. CAPM adjusted for Optimal Resource Allocation in the Brain

We start with a modern derivation of CAPM (such as in Frazzini and Pedersen (2014)), and add a twist that incorporates information processing via schemas in the brain. As in Frazzini and Pedersen (2014), we consider an overlapping generations (OLG) economy. Each agent lives for two periods. Agents that are born at t aim to maximize their utility of wealth at $t+1$. Their utility functions are identical and exhibit mean-variance preferences. They trade securities $s = 1, \cdots, S$ where security s pays dividends $d_t^s$and has n_s^* shares outstanding, and invest the rest of their wealth in a risk-free asset that offers a rate of r_F.

The market is described by a representative agent who maximizes:

$$\max n'\{E_t(P_{t+1} + d_{t+1}) - (1 + r_F)P_t\} - \frac{\gamma}{2} n'\Omega_t n$$

where P_t is the vector of prices, Ω_t is the variance-covariance matrix of $P_{t+1} + d_{t+1}$, and γ is the risk-aversion parameter.

It follows that the price of a security, s, is given by:

$$P_t^s = \frac{E(X_{t+1}^s) - \gamma Cov(X_{t+1}^s, X_{t+1}^M)}{1 + r_F} \tag{2.1}$$

where security s payoff is $X_{t+1}^s = P_{t+1}^s + d_{t+1}^s$

and the aggregate market payoff is:

$$X_{t+1}^M = n_1^*(P_{t+1}^1 + d_{t+1}^1) + n_2^*(P_{t+1}^2 + d_{t+1}^2) + \cdots\cdots\cdots + n_S^*(P_{t+1}^S + d_{t+1}^S).$$

2.1 Using Schemas

Bossaerts (2009) discusses evidence from decision neuroscience indicating that two key gamble statistics of expected reward and reward variance are encoded separately in the brain. Expected reward is encoded in the subcortical projection areas of the dopamine neurons, in particular, the ventral striatum, whereas brain regions involved in risk encoding include right and left insula, and thalamus. As discussed in the previous section, the key brain region involved in estimating willingness-to-pay is vmPFC, which constructs value by integrating information coming from other brain regions (Levy and Glimcher 2012). As schemas are high-level structures that integrate lower-level units and the key region involved in processing information via schemas is also vmPFC, valuation is consistent with schema reliance.

In line with 2.1, we propose that the valuation schema in the brain has the following structure:

$$Willingnes\ to\ Pay = \frac{Expected\ Cashflows - (risk\ aversion) * (Risk\ of\ Cashflows)}{1 + RiskFree\ Rate}$$

As can be seen from the above general form of a valuation-schema, in our context, the two units that need to be estimated are expected cashflows and the risk of cashflows.

Based on the evidence summarized in section 1 (see the discussion in Alonso et al (2014) for a more detailed review of neuroscience evidence), we build a resource allocation model applicable to value construction as follows. We assume that the representative agent analyses the total earnings or cashflows of a firm to estimate equity value. This analysis has two tasks. Task 1 is estimating the expected future earnings or cashflows of the firm, whereas Task 2 requires estimating the risk of future earnings or cashflows. Apart from these two tasks, we combine all other tasks that the brain may be engaged in at the time of analysis and refer to this aggregate as Task 3. Each task is performed by a separate brain system, which alone is responsible for that task. Systems are made-up of neurons, which demand resources. Resource deficit implies underperformance in the task.

We assume that the agent relies on a pre-existing schema to help with these tasks. The pre-existing schema may belong to a similar firm that the agent has analysed earlier, or it could be a generic schema for the sector. In both cases, we use q to denote the relevant starting point.

Denoting total earnings of q and s at $t+1$ by π_{t+1}^{q} and π_{t+1}^{s} respectively, Task 1 is:

$$E'(\pi_{t+1}^{s}) = E\left(\pi_{t+1}^{q}\right) - m_1 D_1 \tag{2.2}$$

where $D_1 = E\left(\pi_{t+1}^{q}\right) - E(\pi_{t+1}^{s})$ is the correct adjustment needed, and m_1 is the fraction of correct adjustment achieved.

Task 2 is:

$$Cov'(\pi_{t+1}^{s}, \pi_{t+1}^{M}) = Cov\left(\pi_{t+1}^{q}, \pi_{t+1}^{M}\right) - m_2 D_2 \tag{2.3}$$

where $D_2 = Cov\left(\pi_{t+1}^{q}, \pi_{t+1}^{M}\right) - Cov(\pi_{t+1}^{s}, \pi_{t+1}^{M})$ is the correct adjustment needed

m_2 is the fraction of correct adjustment achieved and π_{t+1}^{M} is the aggregate earnings of all firms in the market.

As in Alonso et al (2014), we assume that each system is selfish and cares only about performance in its own task. The resources that can be allocated to each system, $l \in \{1, 2, 3\}$, are in the set $\emptyset_l = [0, \overline{\varphi_l}]$. The amount of resources needed to carry out a task perfectly is denoted by $\varphi_l \in \emptyset_l$. The amount of resources a system gets is denoted by y_l. A system seeks $y_l = \varphi_l$. We assume that there is a benefit function $\vartheta_l(y_l; \varphi_l)$ associated with each task that the CES computes. The benefit function takes it maximum value when $y_l = \varphi_l$. When $y_l < \varphi_l$, there is a loss. When there are too many resources, $y_l > \varphi_l$, there is no benefit. It could even be damaging as too much attention could be counterproductive. In any case, we assume that the benefit function is non-increasing when $y_l \geq \varphi_l$.

We follow Alonso et al (2014) in defining the following benefit function (without loss of generality):

$$\vartheta_l(y_l; \varphi_l) = \begin{cases} \alpha_l u_l(y_l - \varphi_l) & if\ y_l \leq \varphi_l \\ 0 & if\ y_l > \varphi_l \end{cases} \tag{2.4}$$

where $u_l(0) = 0$, $u_l'(0) = 0$, $u_l'(z) > 0$, and $u_l''(z) < 0$ for all $z < 0$.

Under (2.4), it immediately follows that as the gap between the resources needed to successfully complete a task and the resources made available to the task $(\varphi_l - y_l)$ increases, benefit from the task falls.

We define m_1 and m_2 as follows:

$$m_1 = \frac{y_1}{\varphi_1} \tag{2.5}$$

$$m_2 = \frac{y_2}{\varphi_2} \tag{2.6}$$

So, m_1 and m_2 are fractions of required resources allocated to Task 1 (expected cashflows) and Task 2 (cashflow risk) respectively, which is taken to be the same as the fraction of correct adjustment without loss of generality. When the required resources are made available, tasks are flawlessly completed and rational expectations are formed in both Task 1 and Task 2. And, when there is a resource deficit, the adjustment process is affected in proportion with the deficit.

As in Alonso et al (2014), we assume that the optimization problem that the 'Central Executive System' (CES) in the brain solves is as follows:

$$\max_{\{y_1, y_2, y_3\}} \vartheta_1(y_1; \varphi_1) + \vartheta_2(y_2; \varphi_2) + \vartheta_3(y_3; \varphi_3)$$

$$s.t \;\; y_1 + y_2 + y_3 \leq k$$

$$y_1 \geq 0, y_2 \geq 0, y_3 \geq 0$$

Assuming a binding resource constraint ($\varphi_1 + \varphi_2 + \varphi_3 \geq k$), to characterize an interior solution, we take the simplest case of a quadratic benefit function that meets the criteria for such a function (as explained in 2.4):

$$\vartheta_l(y_l; \varphi_l) = -\alpha_l (y_l - \varphi_l)^2 \tag{2.7}$$

The interior solution is (corner solutions are also easy to characterize):

$$y_l = \varphi_l - \frac{\frac{1}{\alpha_l}}{\left(\frac{1}{\alpha_1} + \frac{1}{\alpha_2} + \frac{1}{\alpha_3}\right)} [\varphi_1 + \varphi_2 + \varphi_3 - k] \qquad for\ l \in \{1, 2, 3\} \tag{2.8}$$

For Task 1 and Task 2, plugging (2.8) in (2.5) and (2.6) leads to:

$$m_1 = \frac{\varphi_1 - \frac{\frac{1}{\alpha_1}}{\left(\frac{1}{\alpha_1}+\frac{1}{\alpha_2}+\frac{1}{\alpha_3}\right)}[\varphi_1+\varphi_2+\varphi_3-k]}{\varphi_1} \tag{2.9}$$

$$m_2 = \frac{\varphi_2 - \frac{\frac{1}{\alpha_2}}{\left(\frac{1}{\alpha_1}+\frac{1}{\alpha_2}+\frac{1}{\alpha_3}\right)}[\varphi_1+\varphi_2+\varphi_3-k]}{\varphi_2} \tag{2.10}$$

The key point of (2.9) and (2.10) is that the fraction of required resources allocated to Task 1 and Task 2 depend on relative task importance and the resources needed to successfully complete the task. For example, consider the case when all tasks are equally important: $\alpha_1 = \alpha_2 = \alpha_3 = 1$, and $\varphi_1 = 90, \varphi_2 = 60, \varphi_3 = 30$ and $k = 100$. Here, the idea is that more resources are needed for estimating expected cashflows (Task 1) than risk of cashflows (Task 2). This fits well with the observation that analysts spend most of their time in estimating cashflows (Basu et al 2013). With these parameter values, $m_1 = 0.704$ and $m_2 = 0.556$. If relative task importance changes, for example, $\alpha_1 = 2$ and $\alpha_2 = 0.5$, then $m_1 = 0.873$ and $m_2 = 0.238$.

Denoting $c_{st+1} = \frac{e_{st+1}}{\pi_{st+1}}$ and $c_{qt+1} = \frac{e_{qt+1}}{\pi_{qt+1}}$ as P/E ratios of s and q respectively (inclusive of dividends) at $t+1$, realizing that total market equity value of firm s at $t+1$ is $e_{st+1} = n_s^*(P_{t+1+}^s d_{t+1}^s)$, where n_s^* is the number of shares of firm s outstanding, and similarly for q, $e_{qt+1} = n_q^*\left(P_{t+1}^q + d_{t+1}^q\right)$, it follows from (2.2) and (2.3) that:

$$\begin{aligned} E'(P_{t+1}^s + d_{t+1}^s) \\ &= E(P_{t+1}^s + d_{t+1}^s) \\ &+ (1-m_1)\left(E\left(P_{t+1}^q + d_{t+1}^q\right)\frac{n_q^* c_{st+1}}{n_s^* c_{qt+1}} - E(P_{t+1}^s + d_{t+1}^s)\right) \end{aligned} \tag{2.11}$$

$$
\begin{aligned}
&Cov'(P^s_{t+1} + d^s_{t+1}, X^M_{t+1}) \\
&\qquad\qquad = \ Cov(P^s_{t+1} + d^s_{t+1}, X^M_{t+1}) \\
&\qquad\qquad + (1 - m_2)\left(Cov\big(P^q_{t+1} + d^q_{t+1}, X^M_{t+1}\big)\frac{n^*_q c_{st+1}}{n^*_s c_{qt+1}} - Cov(P^s_{t+1} + d^s_{t+1}, X^M_{t+1}) \right)
\end{aligned}
\tag{2.12}
$$

where the aggregate market payoff, $X^M_{t+1} = n^*_1(P^1_{t+1} + d^1_{t+1}) + n^*_2(P^2_{t+1} + d^2_{t+1}) + \cdots\cdots\cdots + n^*_S(P^S_{t+1} + d^S_{t+1})$, with m_1 and m_2 given in (2.9) and (2.10) respectively.

2.2 Generalized CAPM

A sufficient condition for classical CAPM to hold is that investors are mean-variance optimizers with an implicit assumption that the resource constraint in the brain does not bind. In this article, we relax this implicit assumption and consider what happens when the resource constraint in brain does bind. So, the decision problem an investor faces is not just how to allocate finite wealth across various assets, but also how to allocate finite brain resources to various tasks involved in mean-variance optimization. In other words, investors are solving not just one but two optimization problems. In the previous section, the optimization problem of allocating brain resources to the tasks of estimating future cashflows and the risk of cashflows is solved. In this section, we use the solution of that optimization problem as an input into the mean-variance optimization problem of wealth allocation across various assets.

The resource rational brain relies on informative starting points and optimally allocates scarce brain resources to various tasks. With a binding resource constraint in the brain, it follows that less than required resources are generally allocated, and the cashflow analysis of a firm, say s, gets entangled with the cashflow analysis of the starting point, q. Assuming that CES computes the simplest benefit function (quadratic) in allocating resources, the fraction of required resources allocated to expected cashflow estimation is given in (2.9), whereas the fraction of required resources allocated to cashflow risk estimation is given in (2.10). The estimated expected cashflows and the risk of cashflows for s are then entangled with the corresponding quantities for q, and are given in (2.11) and

(2.12) respectively. What are the implications of such entanglements for CAPM? This is the questions that we answer next.

We consider the following two cases:

1) Sector schema is based on a prominent firm. That is, the starting points come from a specific firm, q.

2) Sector schema is not based on a specific firm, rather, it is a generic schema representing the average behavior in the sector. That is, q does not represent a specific firm. Rather, it represents the average behavior in the sector.

Similar results are obtained in both cases. However, for completeness both cases are considered here.

2.2.1 Sector schema is based on a firm

We re-state the pricing relation that follows from mean-variance optimization given in (2.1) here:

$$P_t^s = \frac{E(P_{t+1}^s + d_{t+1}^s) - \gamma Cov(P_{t+1}^s + d_{t+1}^s, X_{t+1}^M)}{1 + r_F} \tag{2.13}$$

So, in this optimization problem for firm s, estimates of expected payoff, $E'(P_{t+1}^s + d_{t+1}^s)$, and the risk of payoff, $Cov'(P_{t+1}^s + d_{t+1}^s, X_{t+1}^M)$ are needed. These estimates depend on the solution to the optimal brain resource allocation problem, and are given in (2.11) and (2.12).

Plugging these estimates from (2.11) and (2.12) into (2.13), we get:

$$P_t^s = \frac{\begin{array}{c} E(P_{t+1}^s + d_{t+1}^s) + (1-m_1)\left(E\left(P_{t+1}^q + d_{t+1}^q\right)\frac{n_q^* c_{st+1}}{n_s^* c_{qt+1}} - E(P_{t+1}^s + d_{t+1}^s) \right) - \\ \gamma\left\{ Cov\left(P_{t+1}^s + d_{t+1}^s, X_{t+1}^M\right) + (1-m_2)\left(Cov\left(P_{t+1}^q + d_{t+1}^q, X_{t+1}^M\right)\frac{n_q^* c_{st+1}}{n_s^* c_{qt+1}} - Cov\left(P_{t+1}^s + d_{t+1}^s, X_{t+1}^M\right) \right) \right\} \end{array}}{1 + r_F}$$

$$(2.14)$$

where $E\left(P_{t+1}^q + d_{t+1}^q\right)$ and $Cov\left(P_{t+1}^q + d_{t+1}^q, X_{t+1}^M\right)$ are the corresponding estimates of a similar firm q.

The share price of q is given by:

$$P_t^q = \frac{E\left(P_{t+1}^q + d_{t+1}^q\right) - \gamma Cov\left(P_{t+1}^q + d_{t+1}^q, X_{t+1}^M\right)}{1 + r_F} \tag{2.15}$$

Converting (2.15) and (2.14) into expected return expressions:

$$E\left[R_{t+1}^q\right] = R_F + \frac{\gamma}{P_t^q} Cov\left(P_{t+1}^q + d_{t+1}^q, X_{t+1}^M\right) \tag{2.16}$$

$$\begin{aligned} E[R_{t+1}^s] = R_F + \frac{\gamma}{P_t^s}\Bigg\{ & Cov(P_{t+1}^s + d_{t+1}^s, X_{t+1}^M) \\ & + (1 - m_2)\left(Cov\left(P_{t+1}^q + d_{t+1}^q, X_{t+1}^M\right)\frac{n_q^* c_{st+1}}{n_s^* c_{qt+1}} - Cov(P_{t+1}^s + d_{t+1}^s, X_{t+1}^M)\right)\Bigg\} \\ & - \frac{(1 - m_1)}{P_t^s}\left(E\left(P_{t+1}^q + d_{t+1}^q\right)\frac{n_q^* c_{st+1}}{n_s^* c_{qt+1}} - E(P_{t+1}^s + d_{t+1}^s)\right) \end{aligned} \tag{2.17}$$

For simplicity, in what follows, we set $c_{st+1} \sim c_{qt+1}$. That is, q and s are expected to have the same P/E ratios at $t + 1$.

To fix ideas, initially it is useful to assume that there are just two firms in the market, s and q before generalizing to N firms. Multiplying (2.16) by $w_q = \frac{n_q^* P_t^q}{P_t^M}$, which is the weight of firm q in the market portfolio (P_t^M is the price of aggregate market portfolio), multiplying (2.17) by $w_s = \frac{n_s^* P_t^s}{P_t^M}$, and adding:

$$\begin{aligned} E[R_{t+1}^M] = R_F + \frac{\gamma}{P_t^M}\{ & Var(X_M) \\ & + (1 - m_2)\left(Cov\left(P_{t+1}^q + d_{t+1}^q, X_{t+1}^M\right)n_q^* - Cov(P_{t+1}^s + d_{t+1}^s, X_{t+1}^M)n_s^*\right)\} \\ & - \frac{(1 - m_1)}{P_t^M}\left(E\left(P_{t+1}^q + d_{t+1}^q\right)n_q^* - E(P_{t+1}^s + d_{t+1}^s)n_s^*\right) \end{aligned}$$

$$\Rightarrow \gamma = \frac{(E[R_{t+1}^M] - R_F)P_t^M + (1 - m_1)\left(E\left(P_{t+1}^q + d_{t+1}^q\right)n_q^* - E(P_{t+1}^s + d_{t+1}^s)n_s^*\right)}{\left\{Var(X_M) + (1 - m_2)\left(Cov\left(P_{t+1}^q + d_{t+1}^q, X_{t+1}^M\right)n_q^* - Cov(P_{t+1}^s + d_{t+1}^s, X_{t+1}^M)n_s^*\right)\right\}} \tag{2.18}$$

Substituting (2.18) into (2.16) and (2.17) and simplifying leads to:

$$E\left[R_{t+1}^{q}\right] = R_F + \left[\left(E[R_{t+1}^{M}] - R_F\right) + (1-m_1)\left(w_q E\left(R_{t+1}^{q}\right) - w_s E\left(R_{t+1}^{s}\right)\right)\right] \cdot \beta_q \cdot \left(\frac{1}{1+(1-m_2)\left(w_q\beta_q - w_s\beta_s\right)}\right) \tag{2.19}$$

$$E[R_{t+1}^{s}] = \frac{1}{m_1}\left\{R_F + \left[\left(E[R_{t+1}^{M}] - R_F\right) + (1-m_1)\left(w_q E\left(R_{t+1}^{q}\right) - w_s E\left(R_{t+1}^{s}\right)\right)\right] \cdot \beta_s \left(\frac{1+(1-m_2)\left(\frac{w_q\beta_q}{w_s\beta_s} - 1\right)}{1+(1-m_2)\left(w_q\beta_q - w_s\beta_s\right)}\right) - (1-m_1)E\left(R_{t+1}^{q}\right)\frac{w_q}{w_s}\right\} \tag{2.20}$$

Extending the analysis to a large number of q type firms, with each q spawning multiple s type firms, then the following generalized CAPM expressions are obtained:

$$E\left[R_{t+1}^{q}\right] = R_F + \left[\left(E[R_{t+1}^{M}] - R_F\right) + (1-m_1)\sum_q\sum_s\left(w_q E\left(R_{t+1}^{q}\right) - w_s E\left(R_{t+1}^{s}\right)\right)\right] \cdot \beta_q \cdot \left(\frac{1}{1+(1-m_2)\sum_q\sum_s\left(w_q\beta_q - w_s\beta_s\right)}\right) \tag{2.21}$$

$$E[R_{t+1}^{s}] = \frac{1}{m_1}\left\{R_F + \left[\left(E[R_{t+1}^{M}] - R_F\right) + (1-m_1)\sum_q\sum_s\left(w_q E\left(R_{t+1}^{q}\right) - w_s E\left(R_{t+1}^{s}\right)\right)\right] \cdot \beta_s \left(\frac{1+(1-m_2)\left(\frac{w_q\beta_q}{w_s\beta_s} - 1\right)}{1+(1-m_2)\sum_q\sum_s\left(w_q\beta_q - w_s\beta_s\right)}\right) - (1-m_1)E\left(R_{t+1}^{q}\right)\frac{w_q}{w_s}\right\} \tag{2.22}$$

In a given cross-section of firms, the following two quantities are constant:

$$h = \sum_q\sum_s\left(w_q\beta_q - w_s\beta_s\right) \tag{2.22a}$$

$$g = \sum_q\sum_s\left(w_q E\left(R_{t+1}^{q}\right) - w_s E\left(R_{t+1}^{s}\right)\right) \tag{2.22b}$$

It follows that the generalized CAPM expression can be written as:

$$E[R_{t+1}^{s}] = \frac{1}{m_1}\left\{R_F + [(E[R_{t+1}^{M}] - R_F) + (1-m_1)g]\cdot\beta_s\left(\frac{1+(1-m_2)\left(\frac{w_q\beta_q}{w_s\beta_s}-1\right)}{1+(1-m_2)h}\right) - (1-m_1)E\left(R_{t+1}^{q}\right)\frac{w_q}{w_s}\right\} \qquad (2.22c)$$

If the resource constraint in the brain is not binding, that is, both m_1 and m_2 are equal to 1, then the above generalized CAPM expression converges to the classical CAPM expression as can be easily verified.

Given evidence that investor attention is highly asymmetric with a lion's share devoted to prominent large market-cap firms (Fang and Peress 2009), we assume that they are the q firms. It follows that $w_q \gg w_s$. So, it is possible that for some firms in a given sector, $w_q\beta_q < w_s\beta_s$; however, when aggregated across all firms in the sector and then across all sectors in the market, we expect $\sum_q \sum_s \left(w_q\beta_q - w_s\beta_s\right) = h > 0$. Similarly, even though it is possible for some firms in a given sector to be such that $w_q E\left(R_{t+1}^{q}\right) < w_s E(R_{t+1}^{s})$, when aggregated across all firms in the sector and then across all sectors in the market, we expect $\sum_q \sum_s \left(w_q E\left(R_{t+1}^{q}\right) - w_s E(R_{t+1}^{s})\right) = g > 0$.

2.2.2 Sector schema is based on average behavior

Suppose market is divided into clusters with each cluster comprising of similar firms. There is a sector schema for each cluster based on the average behavior in the cluster. That is, q does not denote a specific firm, rather the average behavior in the sector. It is easy to verify that a generalized CAPM expression very similar to (2.22c) is obtained with the only difference being that $h = g = 0$.

The generalized CAPM expression with sector average as the starting point is:

$$E[R_{t+1}^{s}] = \frac{1}{m_1}\left\{R_F + [(E[R_{t+1}^{M}] - R_F)] \cdot \beta_s \left(1 + (1 - m_2)\left(\frac{w_q \beta_q}{w_s \beta_s} - 1\right)\right) - (1 - m_1) E\left(R_{t+1}^{q}\right) \frac{w_q}{w_s}\right\} \qquad (2.22d)$$

The results do not depend on whether we take $h > 0$ and $g > 0$ or set $h = 0$ and $g = 0$. We consider both cases for completeness.

(2.22c) (or equivalently, (2.22d)) is the key equation corresponding to CAPM when resource constraint in the brain binds. As can be easily verified by plugging-in $m_1 = 1$ and $m_2 = 1$ in (2.22c) or (2.22d), when the resource constraint in the brain does not bind, the classical CAPM expression is recovered.

2.2.3 Varying SML Slopes

As can be seen from (2.22c) or (2.22 d), expected return varies with beta; however, this variation is different from the variation under the classical CAPM. Taking the partial derivative in (2.22c) with respect to beta:

$$\frac{\partial E[R_{t+1}^{s}]}{\partial \beta_s} = \left[\frac{\delta_M + (1 - m_1) g}{1 + (1 - m_2) h}\right] \cdot \frac{m_2}{m_1} \qquad (2.22e)$$

where $\delta_M = (E[R_{t+1}^{M}] - R_F)$.

It is clear from (2.22e) that the slope of SML varies positively with $\frac{m_2}{m_1}$. As this ratio rises, the slope of SML increases. It does not matter whether we take large market-cap firms to be q firms $(h > 0, g > 0)$ or take the sector averages as the hypothetical q firms $(h = 0, g = 0)$. We get the same result in both cases.

One may note that a more accurate measure of SML slope is not the partial derivative in (2.22e) but the total derivative as when beta changes, one expects the weight of the firm in the aggregate market portfolio to change (w_s) as well. After all, market price changes with beta.

The corresponding total derivative can be written as:

$$\frac{dE[R_{t+1}^s]}{d\beta_s} = \left[\frac{\delta_M + (1-m_1)g}{1+(1-m_2)h}\right] \cdot \frac{1}{m_1} \cdot \left\{m_2 - (1-m_2)\frac{w_q\beta_q}{w_s^2}\frac{\partial w_s}{\partial \beta_s} + (1-m_1)\frac{w_q}{w_s^2}\frac{\partial w_s}{\partial \beta_s}\right\} \qquad (2.22f)$$

If $\frac{\partial w_s}{\partial \beta_s} > 0$, even if quite small, then for low enough values of $\frac{m_2}{m_1}$, the slope of the SML is downward sloping. Proposition 1 follows.

Proposition 1 (Varying SML slopes) ***The slope of the Security Market Line (SML) can be upwards, flat, or downwards, depending on the relative resource allocation ($\frac{m_2}{m_1}$) in the brain. When $\frac{m_2}{m_1}$ rises, the slope steepens. It flattens (could be downward sloping) when $\frac{m_2}{m_1}$ falls.***

Next, we present a numerical example that illustrates proposition 1.

2.2.2 A Numerical Example

To illustrate proposition 1, we take a numerical example with a total of 6 firms in the market belonging to the same sector or cluster. Their expected firm-level payoffs and covariance of payoffs with the aggregate market payoff are given in Table 1. We assume that the sector average is used as a starting point. That is, the q firm is a hypothetical firm. We set risk-aversion coefficient at $\gamma = 0.1$, and the risk-free rate at 3%. By definition, the variance of aggregate market payoff, $Var(X_{Mt+1})$, is the sum of all firm-level covariances, and aggregate market value at t, P_M, is the sum of all market values at t.

Total market value of a firm's equity at t follows (from 2.14 slightly modified):

$$V_s = n_s^* P_t^s = \frac{E\left(n_s^*(P_{t+1}^s + d_{t+1}^s)\right) + (1-m_1)\left(E\left(n_q^*(P_{t+1}^q + d_{t+1}^q)\right) - E\left(n_s^*(P_{t+1}^s + d_{t+1}^s)\right)\right) - \gamma\left\{Cov\left(n_s^*(P_{t+1}^s + d_{t+1}^s), X_{t+1}^M\right) + (1-m_2)\left(Cov\left(n_q^*(P_{t+1}^q + d_{t+1}^q), X_{t+1}^M\right) - Cov\left(n_s^*(P_{t+1}^s + d_{t+1}^s), X_{t+1}^M\right)\right)\right\}}{1+r_F}$$

The equity value of each firm under rational expectations, $m_1 = 1$ and $m_2 = 1$, as well as with insufficient resource allocation to risk, $m_1 = 1$ and $m_2 = 0.1$, are given in Table 1. The beta of each firm can be inferred from:

$$\beta_s = \frac{Cov(n_s^*(P_{t+1}^s + d_{t+1}^s), X_{Mt+1})P_M}{Var(X_M)n_s^* P_t^s}$$

Table 1

	Expected Payoffs at $t+1$ $E\left(n^*(P_{t+1} + d_{t+1})\right)$	Risk of Payoffs at $t+1$ $Cov(n^*(P_{t+1} + d_{t+1}), X_{Mt+1})$	Equity Value at t $n^* P_s$ $m_1 = 1, m_2 = 1$	Equity Value at t $n^* P_s$ $m_1 = 1, m_2 = 0.1$
S1	100	20	95.15	94.05
S2	105	25	99.51	98.86
S3	110	30	103.88	103.67
S4	120	35	113.11	113.33
S5	125	40	117.48	118.13
S6	130	45	121.84	122.94
$Var(X_{Mt+1})$	-	195		
P_M			650.97	650.97
Average (q firm)	115	32.5		

Figure 2. The slope of SML varies with relative resource allocation in the brain. When resource allocation in the brain favors risk estimation ($\frac{m_2}{m_1}$ rises), SML rotates in the counter clockwise direction. When the resource allocation in the brain favors estimation of expected cashflows ($\frac{m_2}{m_1}$ falls), SML rotates in the clockwise direction.

Expected returns vs beta are plotted in figure 2 for the following combinations of m_1 and m_2: $\frac{m_2}{m_1} = 0.1\ (m_1 = 1, m_2 = 0.1)$, $\frac{m_2}{m_1} = 0.5\ (m_1 = 1, m_2 = 0.5)$, $\frac{m_2}{m_1} = 0.75\ (m_1 = 1, m_2 = 0.75)$, $\frac{m_2}{m_1} = 1\ (m_1 = 1, m_2 = 1)$, and $\frac{m_2}{m_1} = 1.05\ (m_1 = 0.95, m_2 = 1)$. As can be seen from figure 2, when $\frac{m_2}{m_1}$ rises, SML tilts in the counter clockwise direction.

2.2.3 Empirical Evidence on Varying Slopes

Empirical evidence suggests that at specific times, the slope of SML is steeper, whereas the slope is flat or downward sloping at other times. The times when the observed slope is steeper are consistent with resource allocation in the brain favoring risk. That is, $\frac{m_2}{m_1}$ is higher at such times:

1) Cohen et al (2005) find that months of low or negative inflation correspond to the positive slope of SML. Such months are times of low or depressed aggregate demand; which naturally are periods of heightened risk for businesses. So, one expects both α_2, and φ_2 to rise. It follows that optimal resource allocation in the brains of investors is expected to shift towards risk in such months. Hence, $\frac{m_2}{m_1}$ rises, which steepens SML.

2) Savor and Wilson (2014) find that SML is steeper on days of major macroeconomic announcements (unemployment, inflation, and interest rate decisions by FOMC). As this is macro risk news being released on such days, more brain resources are needed to accurately estimate the risk of cashflows. That is, φ_2 rises. This shifts the optimal resource allocation in the brain towards risk. Consequently, $\frac{m_2}{m_1}$ rises and SML steepens.

3) Antoniou et al (2015) report that periods of pessimistic investor sentiment correspond to a steeper SML slope. As pessimistic sentiment means that risk consideration in the CES is stronger, α_2 rises. It follows that optimal resource allocation in the brain shifts towards risk, $\frac{m_2}{m_1}$ rises and SML steepens.

4) Jylha (2018) finds that tightening of margin requirements by Federal Reserve on stocks corresponds with flattening of SML. Federal Reserve was given the mandate to monitor and adjust margin requirements on stocks after the 1929 stock market crash which triggered the Great Depression. The idea was to check the optimistic sentiment when markets are booming unsustainably to make any subsequent fall less damaging. It follows that margin requirement tightening tends to correspond to periods of optimistic sentiment (booming stock market) when the optimal resource allocation in the brain shifts away from risk. That is, α_2 falls. Hence, $\frac{m_2}{m_1}$ falls in such periods, which flattens SML.

5) Hendershott et al (2019) find that SML has a positive slope with a negative intercept overnight (close-to-open) whereas it has a negative slope with a positive intercept during the day (open-to-close). At-open, due to the break of 16-18 hours in trading overnight, there could be a large deviation between the previous day's close and this morning's open. Hence,

risk consideration is more important at open. Furthermore, at open, more brain resources are needed to incorporate global risks as reflected by performances in other markets. So, one expects both α_2, and φ_2 to rise at such times. It follows that, at open, optimal resource allocation in the brain shifts towards risk estimation. It follows that $\frac{m_2}{m_1}$ is higher at open, which rotates the SML in counter clockwise direction. This temporary increase in $\frac{m_2}{m_1}$ is gradually reversed as the day progresses, causing the SML to rotate back in the clockwise direction.

2.3 Special Case 1: Cashflow-Schema CAPM

In this section, we consider the case when $m_1 \sim 1$. This captures the case when substantially more resources are allocated to estimating expected cashflows when compared with the risk of the cashflows. From (2.9), this corresponds to a situation when the relative importance of estimating expected cashflows is significantly greater: $\alpha_1 > \alpha_2$. Given the importance given to earnings-level news (Basu et al 2013), we conjecture that this is the case which is typically observed. We call this special case, the cashflow-schema CAPM. In terms of SML slope, having resource allocation in the brain favoring earnings estimation implies that $\frac{m_2}{m_1}$ is small, which flattens the SML slope.

The corresponding generalized CAPM expression when q is a prominent large-cap firm in the sector is obtained by plugging $m_1 = 1$ in (2.22c):

$$E[R_{t+1}^{s}] = R_F + (E[R_{t+1}^{M}] - R_F) \cdot \beta_q \cdot \left(\frac{1 + (1 - m_2)\left(\frac{w_q \beta_q}{w_s \beta_s} - 1\right)}{1 + (1 - m_2)h} \right) \tag{2.23}$$

where $h = \sum_q \sum_s \left(w_q \beta_q - w_s \beta_s\right)$.

The corresponding generalized CAPM expressions when q is the sector average:

$$E[R_{t+1}^{s}] = R_F + (E[R_{t+1}^{M}] - R_F) \cdot \beta_q \cdot \left(1 + (1 - m_2)\left(\frac{w_q \beta_q}{w_s \beta_s} - 1\right)\right) \tag{2.23a}$$

It is intriguing to note that the above generalized CAPM expressions have the same form as the classical CAPM with only one difference: a factor that multiplies β appears. For further analysis, it is useful to write (2.23) and (2.23a) in the following equivalent form:

$$E[R_{t+1}^{s}] - R_F = \alpha_s + (E[R_{t+1}^{M}] - R_F) \cdot \beta_s \tag{2.24}$$

where

$$\alpha_s = \left(\frac{w_q\beta_q}{w_s} - \beta_s(1+h)\right)\frac{(1-m_2)\delta_M}{1+(1-m_2)h} \tag{2.24a}$$

$h = \sum_q \sum_s \left(w_q\beta_q - w_s\beta_s\right)$ and $\delta_M = E[R_{t+1}^{M}] - R_F$

(Add and subtract $\beta_s(E[R_M] - R_F)$ from the right-hand-side in 2.23 and re-arrange to get 2.24). Note, that if q is the sector average then $h = 0$.

Writing the generalized CAPM as in (2.24) is useful as it highlights that the impact of a binding resource constraint in the brain is to give rise to an additional term alpha in the classical CAPM expression. One can directly see from (2.24a) that this additional term or alpha is bigger from small- beta and small-size stocks in a given cross-section of stocks.

2.3.1 High-alpha-of-low-beta, value, and size effects

(2.23) and (2.23a) show that in the generalized (cashflow-schema) CAPM, there is an additional multiplicative factor, which multiplies β. For a firm s whose schema is created by modifying the schema of a similar firm q, this additional multiplicative factor is equal to:

$$f = \left(\frac{1+(1-m_2)\left(\frac{w_q\beta_q}{w_s\beta_s} - 1\right)}{1+(1-m_2)h}\right) \tag{2.25}$$

However, if sector averages are used as starting points then:

$$f = \left(1+(1-m_2)\left(\frac{w_q\beta_q}{w_s\beta_s} - 1\right)\right) \tag{2.25a}$$

Proposition 2 shows the emergence of high-alpha-of-low-beta in the cashflow-schema CAPM

Proposition 2 (High-alpha-of-low-beta) ***In a given cross-section of stocks, a stock with low beta outperforms a stock with large beta on a risk-adjusted basis, all else equal.***

Proof

Suppose there are two stocks s and s' such that $\beta_s < \beta_{s'}$. Risk-adjusted return on s is given by:

$$\frac{E[R^s_{t+1}] - R_F}{\beta_s} = \left\{1 + (1 - m_2)\left(\frac{w_q\beta_q}{w_s\beta_s} - 1\right)\right\} \times \frac{1}{v} \times (E[R^M_{t+1}] - R_F)$$

where v is a constant in a given cross-section: $v = 1 + (1 - m_2)h$

Risk-adjusted return on s' is given by:

$$\frac{E\left[R^{s'}_{t+1}\right] - R_F}{\beta_{s'}} = \left\{1 + (1 - m_2)\left(\frac{w_q\beta_q}{w_s\beta_{s'}} - 1\right)\right\} \times \frac{1}{v} \times (E[R^M_{t+1}] - R_F)$$

As β_s and $\beta_{s'}$ appear in the denominator on R.H.S, it follows that:

$$\frac{E[R^s_{t+1}] - R_F}{\beta_s} > \frac{E\left[R^{s'}_{t+1}\right] - R_F}{\beta_{s'}}$$

Similar proof follows for the case when q represents the sector average by setting $h = 0$

■

The high-alpha-of-low-beta effect can also be directly seen in (2.24a) as with h being a constant or 0 in a given cross-section:

$$\frac{\partial\alpha_s}{\partial\beta_s} = -\frac{(1 - m_2)\delta_M(1 + h)}{1 + (1 - m_2)h} < 0 \qquad (2.26)$$

From (2.26), one can also directly see that high-alpha-of-low-beta is a stronger effect when the market risk-premium, δ_M, is larger.

Proposition 3 (Size effect) ***In a given cross-section of stocks, a stock with a lower weight in the market portfolio outperforms a stock with a higher weight on a risk-adjusted basis, all else equal***

Proof

Suppose there are two stocks s and s' such that $w_s < w_{s'}$. Following the same steps as in the proof of proposition 2, it is easy to see that $\frac{E[R^s_{t+1}]-R_F}{\beta_s} > \frac{E\left[R^{s'}_{t+1}\right]-R_F}{\beta_{s'}}$.

■

One can also see size-effect directly in (2.24a):

$$\frac{\partial \alpha_s}{\partial w_s} = -\left(\frac{w_q \beta_q}{w_s^2}\right)\frac{(1-m_2)\delta_M}{1+(1-m_2)h} < 0 \tag{2.27}$$

As with high-alpha-of-low-beta, size is a stronger effect when market risk-premium, δ_M is larger, and does not depend on whether h is positive or 0.

Intriguingly, in cashflow-schema CAPM, an effect similar to value effect is seen as well. Value effect refers to the finding that a stock with low price to fundamentals tends to outperform a stock with high price to fundamentals. Suppose there are two stocks s and s' that have similar fundamentals (expected payoff and the risk of payoff). That is, $E(P^s_{t+1} + d^s_{t+1}) \approx E(P^{s\prime}_{t+1} + d^{s\prime}_{t+1})$, and $Cov(P^s_{t+1} + d^s_{t+1}, X^M_{t+1}) \approx Cov(P^{s\prime}_{t+1} + d^{s\prime}_{t+1}, X^M_{t+1})$. Assume that $P_s < P_{s'}$. That is, stock s is cheaper with the same fundamentals; hence, is a value stock.

If there is a value effect, then it must be so that

$$\frac{E[R^s_{t+1}]-R_F}{\beta_s} > \frac{E\left[R^{s'}_{t+1}\right]-R_F}{\beta_{s'}}$$

In other words, the risk-adjusted return on a low price-to-fundamentals stock should be greater if there is a value effect.

To see if the above is true, start from:

$P_s = \frac{E(P_{t+1}^s + d_{t+1}^s) - \gamma\left\{Cov(P_{t+1}^s + d_{t+1}^s, X_{t+1}^M) + (1-m_2)\left(Cov(X_{t+1}^q, X_{t+1}^M)\frac{n_q^*}{n_s^*} - Cov(P_{t+1}^s + d_{t+1}^s, X_{t+1}^M)\right)\right\}}{1+r_F} < P_{s'} = \frac{E(P_{t+1}^{s'} + d_{t+1}^{s'}) - \gamma\left\{Cov(P_{t+1}^{s'} + d_{t+1}^{s'}, X_{t+1}^M) + (1-m_2)\left(Cov\left(P_{t+1}^{q'} + d_{t+1}^{q'}, X_{t+1}^M\right)\frac{n_{q'}^*}{n_{s'}^*} - Cov(P_{t+1}^s + d_{t+1}^s, X_{t+1}^M)\right)\right\}}{1+r_F}$. Assuming the same fundamentals across the two stocks, $E(P_{t+1}^s + d_{t+1}^s) \approx E(P_{t+1}^{s'} + d_{t+1}^{s'})$, and $Cov(P_{t+1}^s + d_{t+1}^s, X_{t+1}^M) \approx Cov(P_{t+1}^{s'} + d_{t+1}^{s'}, X_{t+1}^M)$, it follows that:

$$Cov\left(P_{t+1}^q + d_{t+1}^q, X_{t+1}^M\right)\frac{n_q^*}{n_s^*} > Cov\left(P_{t+1}^{q'} + d_{t+1}^{q'}, X_{t+1}^M\right)\frac{n_{q'}^*}{n_{s'}^*}$$

$$\Rightarrow \frac{Cov\left(P_{t+1}^q + d_{t+1}^q, X_{t+1}^M\right)}{Cov(P_{t+1}^s + d_{t+1}^s, X_{t+1}^M)}\frac{n_q^*}{n_s^*} > \frac{Cov\left(P_{t+1}^{q'} + d_{t+1}^{q'}, X_{t+1}^M\right)}{Cov(P_{t+1}^s + d_{t+1}^s, X_{t+1}^M)}\frac{n_{q'}^*}{n_{s'}^*}$$

$$\Rightarrow \frac{w_q\beta_q}{w_s\beta_s} > \frac{w_{q'}\beta_{q'}}{w_{s'}\beta_{s'}} \tag{2.28}$$

It follows immediately from (2.23) and (2.23a) that:

$$\frac{E[R_{t+1}^s] - R_F}{\beta_s} > \frac{E\left[R_{t+1}^{s'}\right] - R_F}{\beta_{s'}}$$

Proposition 4 follows.

Proposition 4 (Value effect) ***In a given cross-section of stocks, a stock with low price to fundamentals outperforms a stock with high price to fundamentals on a risk-adjusted basis.***

One can also see an effect similar to value in (2.24a):

$$\frac{\partial\alpha_s}{\partial w_q\beta_q} = \frac{(1-m_2)\delta_M}{\{1+(1-m_2)h\}w_s} > 0 \tag{2.29}$$

So, alpha is larger for a stock whose schema is spawned by a firm that has a larger market-weighted beta or alpha is larger for a stock that belongs to a sector with high average risk. This phenomenon is similar to value effect because two stocks could be otherwise identical except for the fact that one's schema is spawned by a firm with a larger market-weighted beta or it belongs to a sector with high average risk. Such a stock is likely to have a lower price. As with size and high-alpha-of-low-beta effects, this effect is also stronger when the market risk-premium, δ_M, is larger.

A common theme across high-alpha-of-low-beta, size and value effects as they arise here is that all three effects are weaker in a booming stock market (presumably when market-wide risk-premium is low). This can be directly seen from (2.26), (2.27), and (2.29). Intriguingly, Blitz (2020) find that high-alpha-of-low-beta, size and value are substantially weaker in the stock market boom decades of the 1990-1999 and 2010-2019**.** To our knowledge, there is no other approach that makes this prediction, pointing to a promising area for future research seeking empirical validation of the model developed here.

2.4 Special Case 2: Risk-Schema CAPM

Here, we set $m_2 \sim 1$. We refer to this as risk-schema CAPM. This corresponds to a situation where substantially more resources are devoted to estimating risk when compared with the expected payoff estimation. Such a situation may arise in specific times when risk considerations are particularly important, for example, when macroeconomic announcement regarding unemployment, inflation, and monetary policy are made, or at-open, when there is a risk of opening prices being substantially different from the previous days close, and one needs to consider what has happened in other markets when this particular market was closed. The complexity of the risk task is higher at such times as well as the importance of the risk task. That is, in (2.9) and (2.10), $\varphi_2 > \varphi_1$ and $\alpha_2 > \alpha_1$. With these values, it follows that $m_2 > m_1$. It follows that SML has steeper slope as well at such times.

The following generalized CAPM expression is obtained by setting $m_2 = 1$ in (2.22c):

$$E[R_{t+1}^s] = \frac{1}{m_1}\left\{R_F + [(E[R_{t+1}^M] - R_F) + (1 - m_1)g] \cdot \beta_q - (1 - m_1)E(R_{t+1}^q)\frac{w_q}{w_s}\right\} \quad (2.30)$$

If q represents the sector average, then the corresponding CAPM expression is obtained by setting $g = 0$ above:

$$E[R_{t+1}^s] = \frac{1}{m_1}\left\{R_F + [(E[R_{t+1}^M] - R_F)] \cdot \beta_q - (1 - m_1)E(R_{t+1}^q)\frac{w_q}{w_s}\right\} \quad (2.31)$$

It is immediately obvious that, in risk-schema CAPM, the relationship between beta and excess stock return is steeper than what the classical CAPM predicts as beta is multiplied by a factor larger than excess market return. Larger the beta, bigger the improvement over classical CAPM prediction. Furthermore, the implied risk-free rate is smaller than what the classical CAPM predicts and is likely negative:

$$R_F' = R_F - (1 - m_1)E(R_{t+1}^q)\frac{w_q}{w_s} \quad (2.32)$$

It is straightforward to see that large size (market capitalization) stocks do better in this version as the implied risk-free rate is larger for them

Proposition 5 presents the key differences between the two versions of CAPM.

Proposition 5 (Differences between the two versions) ***CAPM when substantially more brain resources are allocated to expected cashflows estimation (Cashflow-Schema CAPM) differs from the CAPM when substantially more brain resources are allocated to risk estimation (Risk-Schema CAPM) in the following ways:***

1) ***The former has a flatter relationship between beta and expected returns, whereas the latter has a steeper relationship between beta and expected returns.***
2) ***The implied risk-free rate is smaller in the latter and is likely negative.***
3) ***Small size, and low beta stocks do better in the former whereas large size, and high beta stocks do better in the latter.***

2.5 The Momentum Effect as Underreaction/Overreaction to News

Momentum effect arises in the generalized CAPM as an underreaction/overreaction to news phenomenon. Given that earnings estimation and risk estimation are the two tasks in constructing value, and the accuracy of these estimates depend on relative resource allocation to these tasks, any change in relative resource allocation matters. Assuming an interior solution to the optimal resource allocation problem with a binding resource constraint, and a quadratic benefit function as discussed in section 2.1, it follows from (2.9) and (2.10):

$$m_1 = \frac{\varphi_1 - \frac{\frac{1}{\alpha_1}}{\left(\frac{1}{\alpha_1} + \frac{1}{\alpha_2} + \frac{1}{\alpha_3}\right)}[\varphi_1 + \varphi_2 + \varphi_3 - k]}{\varphi_1} \qquad (2.33a)$$

$$m_2 = \frac{\varphi_2 - \frac{\frac{1}{\alpha_2}}{\left(\frac{1}{\alpha_1} + \frac{1}{\alpha_2} + \frac{1}{\alpha_3}\right)}[\varphi_1 + \varphi_2 + \varphi_3 - k]}{\varphi_2} \qquad (2.33b)$$

For momentum winners, one expects risk-consideration to become temporarily less important. That is, we expect $\frac{\alpha_1}{\alpha_2}$ to rise for momentum winners. For momentum losers, one expects risk consideration to become temporarily more important. That is, $\frac{\alpha_1}{\alpha_2}$ falls for such stocks. From (2.33a) and (2.33b):

$$\frac{\partial m_1}{\partial\left(\frac{\alpha_1}{\alpha_2}\right)} > 0 \; and \; \frac{\partial m_2}{\partial\left(\frac{\alpha_1}{\alpha_2}\right)} < 0$$

Hence, m_1 rises and m_2 falls for momentum winners. These changes create underreaction to good news and overreaction to bad news among momentum winners. For momentum losers, m_1 falls and m_2 rises, creating underreaction to bad news and overreaction to good news among momentum losers. When these temporarily changes are gradually reversed, momentum winners see a further price appreciation, whereas momentum losers see a further price decline.

News can take either of the following forms: a change in expected earnings or a change in risk. We consider both.

$$\frac{\partial P_t^s}{\partial E(P_{t+1}^s + d_{t+1}^s)} = \frac{m_1}{1+r}$$

Adding the change in $\frac{\alpha_1}{\alpha_2}$ to the above:

$$\frac{dP_t^s}{dE(P_{t+1}^s + d_{t+1}^s)}$$
$$= \frac{m_1}{1+r} - \frac{\partial m_1}{\partial\left(\frac{\alpha_1}{\alpha_2}\right)}\left\{\frac{n_q^*}{n_s^*}E\left(P_{t+1}^q + d_{t+1}^q\right) - E(P_{t+1}^s + d_{t+1}^s)\right\}$$
$$+ \frac{\partial m_2}{\partial\left(\frac{\alpha_1}{\alpha_2}\right)}\left\{\frac{n_q^*}{n_s^*}Cov\left(P_{t+1}^q + d_{t+1}^q, X_{t+1}^M\right) - Cov(P_{t+1}^s + d_{t+1}^s, X_{t+1}^M)\right\} \quad (2.34)$$

Similarly, a change in risk with a change in $\frac{\alpha_1}{\alpha_2}$ also considered is:

$$\frac{dP_t^s}{dCov(P_{t+1}^s + d_{t+1}^s)}$$
$$= -\frac{\gamma m_2}{1+r} - \frac{\partial m_1}{\partial\left(\frac{\alpha_1}{\alpha_2}\right)}\left\{\frac{n_q^*}{n_s^*}E\left(P_{t+1}^q + d_{t+1}^q\right) - E(P_{t+1}^s + d_{t+1}^s)\right\}$$
$$+ \frac{\partial m_2}{\partial\left(\frac{\alpha_1}{\alpha_2}\right)}\left\{\frac{n_q^*}{n_s^*}Cov\left(P_{t+1}^q + d_{t+1}^q, X_{t+1}^M\right) - Cov(P_{t+1}^s + d_{t+1}^s, X_{t+1}^M)\right\} \quad (2.35)$$

2.5.1 Differential Impact of Good vs Bad News

From (2.34) and (2.35), the differential impact of good news vs bad news follows both for momentum winners as well as for momentum losers. Consider momentum winners first. For such stocks, m_1 rises and m_2 falls because $\frac{\alpha_1}{\alpha_2}$ temporarily rises. If good news arrives in the form of an increase in earnings, it follows from (2.34) that the price does not rise as much as it otherwise would have in the absence of changes in m_1 and m_2. Note, that we have assumed that $\frac{n_q^*}{n_s^*}E\left(P_{t+1}^q + d_{t+1}^q\right) - E(P_{t+1}^s + d_{t+1}^s) > 0$ and

$\frac{n_q^*}{n_s^*}Cov\left(P_{t+1}^q + d_{t+1}^q, X_{t+1}^M\right) - Cov(P_{t+1}^s + d_{t+1}^s, X_{t+1}^M) > 0$, which follow if q firm is a prominent large market-cap firm. Similar effects are seen if the good news arrives in the form of a reduction in risk. It is straightforward to see that there is overreaction to bad news among momentum winners. When the temporary changes in m_1 and m_2 are gradually reversed, momentum winners see a price appreciation.

For momentum losers, m_2 rises and m_1 falls as $\frac{\alpha_1}{\alpha_2}$ temporarily falls. It is clear from (2.34) and (2.35) that when bad news arrives for such stocks (either earnings reduction or risk increase), the fall in price is moderated by these changes. However, when good news arrives, the same effects cause an overreaction. When the temporary changes in m_1 and m_2 are gradually reversed, momentum losers see a price reduction.

3. Conclusions

A sufficient condition for CAPM to hold is that investors are mean-variance maximizers with an implicit assumption that the resource constraint in the brain does not bind. In this article, we have relaxed this implicit assumption and have considered what happens when the resource constraint in the brain does bind. With a binding resource constraint, human brain needs to solve two rather than just one optimization problem, which are: 1) Optimal resource allocation in the brain. 2) Mean-variance optimization. We show that with a binding resource constraint, a generalized CAPM expression is obtained, which contains the classical CAPM as a special case. This special case is only obtained if the resource constraint in the brain does not bind. Varying SML slopes follow depending on the relative resource allocation in the brain. When the resource allocation in the brain favors risk estimation, SML steepens, and when the resource allocation in the brain favors earnings estimation, SML flattens and could even be downward sloping. Features akin to size, value, and high-alpha-of-low-beta, are observed in the generalized CAPM when SML is flat or downward sloping. Momentum effect also arises as an underreaction/overreaction to news phenomenon due to temporary shifts in relative resource allocation in the brain. Overall, the results in this article suggest that the classical CAPM is not misspecified. Rather what appears as misspecification may be the result of ignoring the optimal resource allocation problem in the brain.

An intriguing prediction of the approach developed in this article is that high-alpha-of-low-beta, size, and value effects are weaker in booming stock markets (when market-wide risk-premium is low). A closer examination of this prediction is a natural subject for future research.

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