A note on exit and inflation bias in a currency union

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1 August 2020

1 Introduction

This note shows how the threats of exits affect the consequences of collective policymaking and outcomes in a currency union. In particular, we examine how a member's exit influences the possibility of implementing the optimal monetary policy that does not generate inflation bias.

Our departure point is the canonical model of discretionary monetary policymaking (Barro and Gordon, 1983a), which is extended to a monetary union scenario with an exit option. In the initial stage, there are N homogenous members in the monetary union, each of which will experience heterogeneous within-country output shocks. After observing the individual shocks, each member chooses whether to remain the currency union or not. If one leaves the union, it has to pay a fixed cost regarding the exit, but obtains a domestic currency. On the other hand, the members which chose to remain collectively select the common monetary policy by majority voting. In this setting, members that experience extremely high (or low) output shocks have a strong incentive to exit.

The main findings are as follows. First, the inflation bias arises if more than one members exits the monetary union. Intuitively, the country that leaves the union discretionarily chooses its own monetary policy, which hikes the expected inflation and actual inflation, under rational expectation. Second, to implement the optimal policy without inflation bias the median voter has to be the one which experiences a positive output shock. Intuitively, it decreases the equilibrium (expected) inflation, since a positive shock makes the median voter prefer lower inflation.

This paper is related to the literature on time-inconsistency problems, and collective policymaking in monetary unions. The seminal work by Kydland and Prescott (1977) shows the major insights into the time-inconsistency problem: when the government has the flexibility to alternate the policy, the rational individuals anticipate such behavior and the outcome becomes worse for both the government and the people. Barro and Gordon (1983a) point out the time-inconsistency problem induces the inflation bias in monetary policymaking, and highlighted the benefit of commitment rules rather than undertaking discretionary policymaking. Since then, a considerable amount of research has investigated ways to relax the inflationary bias (Barro and Gordon, 1983b; Stokey, 1989; Chari and Kehoe, 1990; Rogoff, 1985; Walsh, 1995; Dal Bo, 2006; Riboni, 2010).

Several papers have studied the collective policymaking in a monetary union. Von Hagen and Süppel (1994) studied collective monetary policymaking under different institutional frameworks without exit options. Farvaque and Matsueda (2009) investigated the sustainability of a monetary union on external shocks. Saito (2018) studied the optimal design of a monetary union where the monetary policy is determined by Nash bargaining, and finds that the more bargaining power that the richer country has reduced the inflation bias.

The remainder of the article is organized as follows. Section 2 describes our economy. Section 3 describes the implications of the currency union without an exit option. Section 4 studies the model with exit option and discusses the main result. Section 5 concludes the article.

2 Setting

We extend Barro and Gordon (1983a) to a currency union setting. In the beginning, the currency union consists of N identical member countries. An arbitrary country i has the following identical loss function:

$$L_i = \frac{b}{2}\left(y_i - k\right)^2 + \frac{1}{2}\pi_i^2$$

where y_i is the output, π_i is the inflation, and $k > 0$ is the target level of output. The output y_i follows the Lucas supply function:

$$y_i = a\left(\pi_i - \pi^e\right) + \epsilon_i \tag{1}$$

where π^e is the expected inflation, and ϵ_i is the i.i.d. country-specific supply shock with mean zero and variance σ^2. For simplicity, we suppose that the shocks are not correlated, offset each other in aggregate, i.e. $\sum_i \epsilon_i = 0$, and all states are equally likely, i.e. $Prob(\epsilon_i) = \frac{1}{N}$ for all i. The private sector rationally forms its expectation: $\pi^e = E[\pi]$.

Before studying the collective policymaking case, let us quickly review the cases of a single country setting, and if a monetary union with a social planner. If a country has own currency and discretionary chooses its monetary policy, as per Barro and Gordon (1983a), it holds that

$$\pi_i = ab\left(k - \frac{\epsilon_i}{1+a^2b}\right),\ y_i = \frac{\epsilon_i}{1+a^2b}.$$

Instead, let us suppose there is a utilitarian social planner with social loss $\mathcal{L} \coloneqq \frac{1}{N}\sum_{i=1}^{N} L_i$. By plugging $\pi_i = \pi_{union}\ \forall i$ and 1 into $\mathcal{L}$, we obtain that:

$$\mathcal{L} = \frac{b}{2}\left[(\pi_{union} - \pi^e - k)^2 + \sigma^2\right] + \frac{\pi_{union}^2}{2}. \tag{2}$$

Then the discretionary solution is given by:

$$\pi_{union} = abk,\ Y_{union} = 0,$$

where Y_{union} is the aggregate output in the monetary union, i.e. $Y_{union} \coloneqq \sum_{i=1}^{N} y_i$. Instead, the commitment solution leads to:

$$\pi_{union}^* = 0,\ Y_{union}^* = 0.$$

Hence the commitment keeps inflation lower while realizing the same output. Notice that this commitment strategy is time-inconsistent: the policymaker has an incentive to deviate from the rule when the public sector believes it.

3 Currency Union Without Exit Option

Now we move onto the collective policymaking in the currency union. As a benchmark, this section considers the case where the members cannot exit the monetary union. The timing of events is specified as follows:

Stage 0. *The private sector forms its expectation.*

Stage 1. *The stochastic output shocks are realized.*

Stage 2. *The inflation π_{union} is chosen.*

We assume that at Stage 2 the inflation is determined by majority voting among all members. We solve the problem by backward induction.

At stage 2, an arbitrary member i has the following loss function:

$$L_i = \frac{b}{2}\left(a\left(\pi_{union} - \pi^e\right) + \epsilon_i - k\right)^2 + \frac{1}{2}\pi_{union}^2 \tag{3}$$

Since L_i is single-peaked in π_{union}, we could apply the median voter theorem: the member whose optimal inflation corresponds to the median of the bliss inflation of all the members is implemented in the equilibrium. Hence the inflation in the union is given as:

$$\pi_{union} = \frac{ab\left(a\pi^e + k - \epsilon_m\right)}{1 + a^2 b} \tag{4}$$

where ϵ_m is the median of the output shocks.

By imposing the rational expectation to Eq. 4, we obtain:

$$\pi_{union} \quad = \quad ab\left(k - \epsilon_m\right). \tag{5}$$

Notice that $sgn(\pi_{union}) = sgn(k - \epsilon_m)$. Next we investigate the committee structure which eliminates the inflation bias. Eq. 5 immediately leads to the following results:

Result 1. *The optimal outcome can be achieved if and only if $\epsilon_m = k$.*

Remember that $k > 0$, implying that the median voter has to be the one who experiences a positive output shock. The intuition is similar to Rogoff's (1985) study, which suggests that the delegation of a conservative central banker reduces the inflation bias. Since a country's bliss inflation is a decreasing function of the output shock, the positive shock makes the country prefer lower inflation. Note that the value of ϵ_m can be manipulated by the committee designer. For instance, a rule that gives more voting power on the high output members might induce higher ϵ_m than the "one person one vote" rule.

The next section allows each member's exit and investigates its effect on the outcome.

4 Exit and Inflation Bias

Suppose each member has an option to exit the monetary union. The timing of events is now given as follows.

Stage 0. *The private sector forms its expectation*

Stage 1. *The stochastic output shocks are realized*

Stage 2. *Each member i exits the monetary union by paying a fixed cost $\delta > 0$.*

Stage 3. *The inflation in the monetary union, π^{union}, is decided by majority voting amongst the remaining members. Each exited country chooses its domestic inflation by itself, if any.*

Again, we solve the model by backward induction.

We first consider the policy choice at stage 3. Consider an arbitrary member i which left the union. Its social loss, L_i^{out}, is given by:

$$L_i^{out} = \frac{b}{2}\left(a\left(\pi_i^{out} - \pi^e\right) + \epsilon_i - k\right)^2 + \frac{1}{2}\left(\pi_i^{out}\right)^2 + \delta \tag{6}$$

The first-order condition can be arranged to:

$$\pi_i^{out} = \frac{ab\left(a\pi^e + k - \epsilon_i\right)}{1 + a^2 b}. \tag{7}$$

On contrast, the inflation in the currency union is given by:

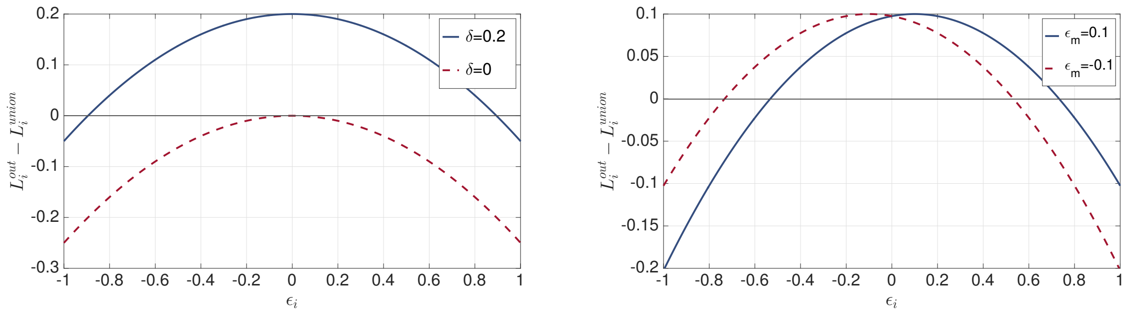
$$\pi_{union} = \frac{ab\left(a\pi^e + k - \epsilon_m\right)}{1 + a^2 b} \tag{8}$$

Notice that ϵ_m could be different from that in the previous section, if some members exited the union.

At stage 2, each country remains in the union if and only if it decreases the social loss: $L_i^{out} \geq L_i^{union}$. The next proposition characterizes the thresholds of shocks that the members decide whether to remain in the union or not.

Proposition 1. *An arbitrary member i remains in the monetary union if and only if $\epsilon_i \in [\underline{\epsilon}, \overline{\epsilon}]$ where $\underline{\epsilon} := \epsilon_m - \frac{\sqrt{2(1+a^2b)\delta}}{ab}$, $\overline{\epsilon} := \epsilon_m + \frac{\sqrt{2(1+a^2b)\delta}}{ab}$.*
Proof. *Appendix*

The result implies that a country would exit if its output is extremely high (or low) compared to the median of it. A higher ϵ_m increases both of $\underline{\epsilon}$ and $\overline{\epsilon}$; hence the remaining members prefer lower inflation on average. Additionally, an increase in the cost of exit δ upper-shifts the function $L_i^{out} - L_i^{union}$; thus it makes more members to remain in the union.



(a) Effect of δ (b) Effect of ϵ_m

Figure 1: The welfare gain by remaining the union: $L_i^{out} - L_i^{union}$

Note: Parameters of Figure (a) : $a = 1$, $b = 1$, $\epsilon_m = 0$; Parameters of Figure (b): $a = 1$, $b = 1$, $\delta = 0.1$.

Now, let us consider the private sector's rational expectation formation at stage 0.

Proposition 2. *Under the rational expectation, it holds that:*

$$E[\pi_i] = ab\left[k - (p\epsilon_m + (1-p)E\left[\epsilon_i | \epsilon_i \notin [\underline{\epsilon}, \overline{\epsilon}]\right])\right], \tag{9}$$

$$\pi^{union} = ab\left[k - \frac{(1 + a^2bp)\epsilon_m + a^2b\left((1-p)E\left[\epsilon_i | \epsilon_i \notin [\underline{\epsilon}, \overline{\epsilon}]\right]\right)}{1 + a^2b}\right], \tag{10}$$

where p is the probability that a member will choose to remain in the monetary union.
Proof. *Appendix*

We now turn to the committee design which aims to reduce the inflationary bias caused by the existence of k. Proposition 3 leads to the following result:

Corollary 1. *The optimal monetary policy is implemented if and only if the following two conditions are satisfied:*

(i) $\epsilon_m = k$,

(ii) $p = 1$.

Proof. *Appendix*

The first condition is same as **Result 1** while the second one ensures none of the members exits the union. The finding can be interpreted as follows:

Result 2. *If more than one member exits, the inflation bias arises.*

Hence the currency union has to be designed to prevent the exits in order to completely eliminate the inflation bias. How can we prevent the members' exit? One way is a higher cost of exit δ. This also could be interpreted as an increased gain from remaining in the union, i.e. low transaction cost, or a gain from the free trade, which can be at least partially designed. In addition, a smaller variance σ^2, which makes the countries more identical, could prevent the exit. Intuitively, the variance might be small if the countries with similar characteristics joined the monetary union in the first place. By explicitly considering the entering phase in the model, we might able to discuss this issue more.

5 Conclusion

Although threats of exits have been a huge political issue in the European Union, not much has been studied about the resulting effect on policymaking. In terms of monetary policymaking, exit from the monetary union grants the country its own monetary authority. By building a simple model, this paper showed that a member's exit eliminates the possibility of implementing the optimal monetary policy, which would be another new negative effect caused by exits.

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Acknowledgement

I am grateful to Antoine Camous for his comments and advices. I have also benefited from the discussions with the participants of the monetary economics workshop at Kobe University. Any remaining errors or shortcomings are my sole responsibility.

Appendix

Proof of Proposition 1

By substituting (7) into (6), we obtain:

$$L_i^{out} = \frac{b\left(a\pi^e + k - \epsilon_i\right)^2}{2(1+a^2b)} + \delta \tag{11}$$

By plugging (4) into (3) , we obtain:

$$L_i^{union} = \frac{b}{2}\left[\epsilon_i^2 - 2\left(\frac{a\pi^e + k + a^2b\epsilon_m}{1+a^2b}\right)\epsilon_i + \frac{\left(a\pi^e + k\right)^2 + a^2b\epsilon_m^2}{1+a^2b}\right]. \tag{12}$$

From (11) and (12), we obtain $L_i^{out} - L_i^{union} = \frac{a^2b^2\left[-\epsilon_i^2 + 2\epsilon_m\epsilon_i - \epsilon_m^2\right]}{2(1+a^2b)} + \delta$. *The result follows by solving* $L_i^{out} - L_i^{union} \geq 0$ *for* ϵ_i.$\square$

Proof of Proposition 2

By taking the expectation of π_i, *we obtain that*

$$E[\pi_i] = p\pi^{union} + (1-p)E[\pi_i^{out}|\epsilon_i \notin [\underline{\epsilon}, \bar{\epsilon}]] \tag{13}$$

By substituting (4) and (7) *in* (13) *, we have that:*

$$E[\pi_i] = \frac{ab\left[a\pi^e + k - \left(p\epsilon_m + (1-p)E\left[\epsilon_i|\epsilon_i \notin [\underline{\epsilon}, \bar{\epsilon}]\right]\right)\right]}{1+a^2b}. \tag{14}$$

By imposing $\pi^e = E[\pi_i]$ to (14), *we obtain the desired result.* $\square$

Proof of Corollary 1.

We need to show: $\pi^{union} \wedge E[\pi_i] = 0\,\forall i \iff p = 1 \wedge \epsilon_m = k$.

Step 1. $\pi^{union} = 0$ *&* $E[\pi_i] = 0\,\forall i \implies p = 1$ *&* $\epsilon_m = k$.

Given Eqs.(9) and (10), *by supposing* $\pi^{union} = 0$ *&* $E[\pi_i] = 0\,\forall i$ and obtain that:

$$
\begin{cases}
k - \frac{(1+a^2bp)\epsilon_m + a^2b((1-p)E[\epsilon_i|\epsilon_i \notin [\underline{\epsilon},\overline{\epsilon}]])}{1+a^2b} = 0 \\
k - (p\epsilon_m + (1-p)E\left[\epsilon_i|\epsilon_i \notin [\underline{\epsilon},\overline{\epsilon}]\right]) = 0.
\end{cases} \tag{15}
$$

By solving the simultaneous equations (15)*, we obtain* $\epsilon_m = k$*. Substitute* $\epsilon_m = k$ *into the second equation* in (15) *and obtain that:*

$$
(1-p)\underbrace{\left\{ \underbrace{k}_{>0} - \underbrace{E\left[\epsilon_i|\epsilon_i \notin [\underline{\epsilon},\overline{\epsilon}]\right]}_{<0} \right\}}_{>0} = 0. \tag{16}
$$

Notice that $\epsilon_m = k > 0$ *implies* $E\left[\epsilon_i|\epsilon_i \notin [\underline{\epsilon},\overline{\epsilon}]\right] < 0$*, by Proposition 1. Hence Eq.*(16) *hold if and only if* $p = 1$.

Step 2. $\pi^{union} = 0$ *&* $E[\pi_i] = 0 \, \forall i \Longleftarrow p = 1$ *&* $\epsilon_m = k$ *.*

The result immediately follows by substituting ($p = 1$ *&* $\epsilon_m = k$*) into Eqs.*(9)-(10).

□