# Corporate Debt, Endogenous Dividend Rate, Instability and Growth

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**Abstract**

In a stock-flow consistent neo-Kaleckian growth-model, we endogenize the dividend rate and debt-level in the long run and investigate the possibility of multiple equilibria and instability in the economy. We find that the economy is in a wage-led demand and debt-burdened growth regime. However, both debt-led and debt-burdened demand regimes are possible. In some instances, the speed of the adjustment parameter related to the dividend dynamics plays a crucial role in stabilizing the economy. Otherwise, the economy may lose its stability and gives birth to limit cycles. A significant rise in the interest rate may cause instability in the economy.

*Keywords: Capital Accumulation, Dividend Rate, Kaleckian Model, Instability, Limit Cycle*

*JEL classification: C62, E12, O41.*

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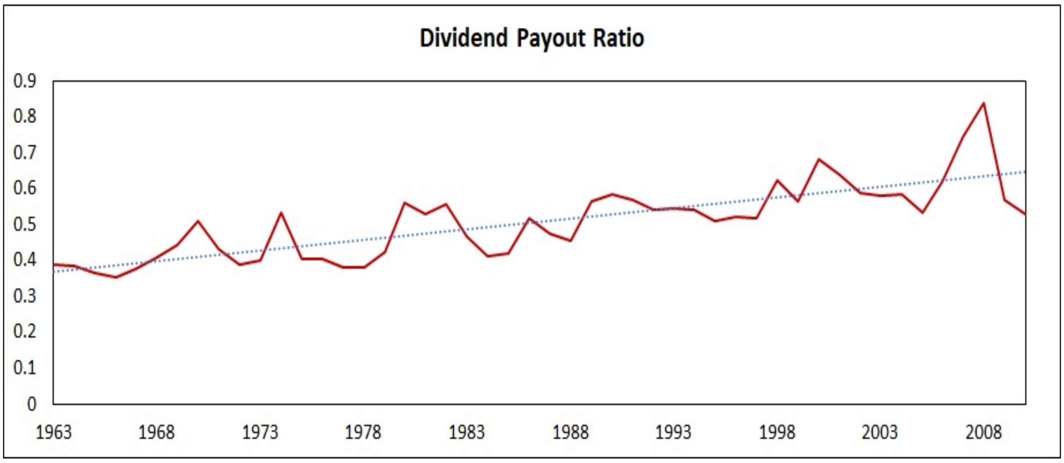
# 1 *Introduction*

Starting with mid 1980s, we observe an enormous rise in the corporate debt-capital ratio. From \$ 843.8 billion in 1979, non-financial corporate debt has risen to \$ 5,285.0 billion in 2005 (Palley, 2008). From 32.9% in 1979, non-financial corporate debt to GDP ratio has increased to 42.4% in 2005 with a peak of 46.2% in 2000 (Palley, 2008). However, Davis (2014, pp. 60) points out that rising leverage is primarily coming from large firms while there is concurrent de-leverage among small firms. We also observe a rising trend of the dividend-payout ratio (Figure 1.1) as well as the corporate interest payments (Figure 1.2), and a substantial fluctuation in the real interest rate (see Figure 1.3) for the same time period .

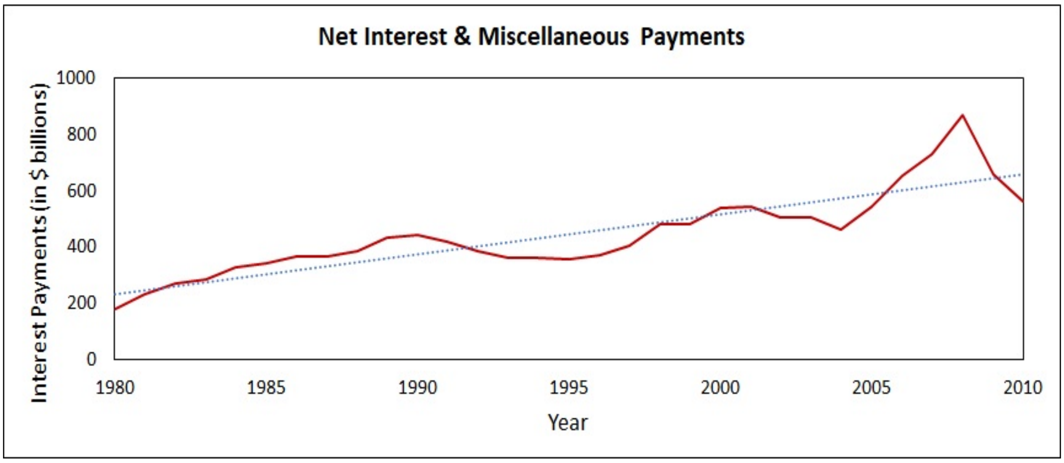



Source: Economic Report of the President (ERP), February 2012, table B-90; author's calculations.
Notes: Dividend-payout ratio = Ratio of net dividends to corporate profits after tax with inventory valuation and capital consumption adjustments.

Figure 1.1: Dividend-payout ratio, 1963–2010.

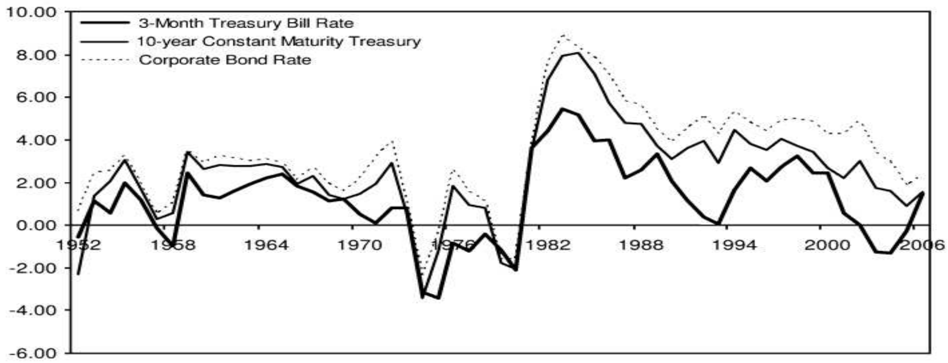
Therefore, the prime objective of this paper is to analyze how the dividend-capital ratio of shareholders and the debt-capital ratio of firms evolve through time, especially in the era of financialization in the US economy. We also want to know whether the long run interaction between the debt and dividend dynamics is capable of explaining the instability of the overall economy.

Neo-Kaleckian growth models starts with the contribution of Rowthorn (1982), and Dutt (1984, 1990). Taylor (1983, 1985), Amadeo (1986), Blecker (1989), Bhaduri and Marglin (1990), Marglin and Bhaduri (1990), Lavoie (1992) are the other contributors. However, financial variables have been introduced much later in this tradition (for example see Dutt (1995), Lavoie (1995, 2008), Hein (2006, 2007, 2008a, 2008b, 2012a, 2012b, 2012c), Charles (2008a, 2008b), Lavoie and Godley (2002), van Treeck (2008, 2009a, 2009b), Parui




Source: ERP, February 2012, table B-28; author's calculations.

Figure 1.2: Corporate net interest & miscellaneous payments, 1980-2010




Source: Skott and Ryoo (2008, pp. 832).

Figure 1.3: Real interest rates, 1952–2006.

(2020)). Most of the Kaleckian literature assumes an exogenous retention ratio even in the long run. To the best of our knowledge Charles (2008a) is the first to endogenize the retention ratio. In a neo-Kaleckian model of growth and distribution, in the long run, Charles (2008a) endogenizes firms' retention rate and level of debt. In the long run, change in retention ratio depends positively on the difference between the targeted and the actual retention ratio. For a higher debt-capital ratio, firms' managers consider themselves in a more risky situation as, *ceteris paribus,* internal funds are smaller now, and so firms try to increase the internal funds by increasing the target retention rate. Thus firms can preserve their financial autonomy and their ability to meet financial commitments. Hence, the targeted retention ratio depends positively on the debt-capital ratio. He assumes that there is no issue of new shares and so the aggregate level of investment is financed through new borrowing and retained earnings. Therefore, the dynamics of debt is governed by the difference between the level of investment and the retained earnings. While for low levels of interest rate, he finds unique and stable equilibrium, high rate of interest leads to the appearance of multiple equilibria, one of which is a saddle point. For a high initial level of interest rate and given high pressure from shareholders for higher dividend payments, a declining retention rate makes the system more financially fragile, and as a result, even small shocks can transform a stable economy into an unstable one.

Charles (2008a) assumes a positive relationship between the level of debt and retention ratio. However, in our analysis, we propose that exactly the opposite of that happens. The long run section provides a detailed discussion. Second, Charles (2008a) assumes debt-capital ratio as the only determinant of targeted retention ratio. Along with the debt-capital ratio, expected future rate of profit and expected growth rate are also significant determinants of firms' targeted dividend capital ratios, both of which we include in our analysis. On these grounds, our model provides a more fruitful explanation of dividend (or retention) dynamics than Charles (2008a).

Constructing a simple one-sector, closed economy, stock-flow consistent neo-Kaleckian growth model, our first objective is to find the main comparative static results with an exogenous dividend-capital and debt-capital ratio. Then we extend the model in the long run where we try to endogenize the dividend-capital and debt-capital dynamics. We show that the interaction between the debt and the dividend dynamics can produce instability in the system. We also show that a lower level of debt-capital ratio is neither a necessary nor a sufficient condition for the system to be stable. Under a particular condition, the speed of adjustment parameter related to the dividend dynamics plays a crucial role in stabilizing the economy. If the speed of the adjustment parameter falls below a critical level, the economy can lose its stability and give birth to the limit cycles. We argue that financialization leads to a fall in the sensitivity of targeted dividend-capital ratio to a change in expected growth rate. Same is true for the sensitivity of targeted dividend-capital ratio related to the debt-capital ratio. However, financialization leads to a rise in the sensitivity of targeted dividend-capital ratio related to the expected profit rate. All the above changes, as we show, may lead to instability in the economy. Finally, we show that a sufficient rise in the interest rate can cause instability in the economy. In that sense, our model provides an explanation for the emergence of the crisis in the economy.

The outline of the rest of the paper is as follows. Section 2 sets up the model and talks about the short run comparative statics. Section 3 discusses the long run where we endogenize the debt-capital ratio and the dividend-capital ratio. Then in section 4 we discuss possible cases which may emerge as a result of the interaction between the debt-capital and the dividend-capital dynamics. This is followed by the discussion of Hopf bifurcation, where we analyze how the interaction between debt-capital and the dividend-capital dynamics produces limit cycles. The next section (section 6) is about the discussion of some comparative statics. Section 7 offers some concluding remarks.

## 2 *The model*

We assume a simple one-sector, closed economy, stock-flow consistent neo-Kaleckian growth model in which the economy consists of workers, rentiers and firms. Neither government intervention nor technical progress is there. Income is distributed between wages and profits as

$$Y = W + R \tag{2.1}$$

where, $Y$ is nominal income, $W$ is nominal wage income and $R$ is nominal profit income. We assume excess supply of labour and under-utilization of capacity is there in the economy. For simplicity we assume there is no depreciation of capital. Workers consume whatever they earn while rentiers save a fraction $(1 - c_r)$ of their income. So,

$$C_W = W = [(1 - \pi)u]\, K \tag{2.2}$$

where, $C_W$ is consumption of workers, $K$ is the existing capital stock, $u$ is the degree of capacity utilization,[1] $\pi = \frac{R}{Y}$ is share of profit, and $r = \frac{R}{K}$ is profit rate. So, $r = \pi u$. Rentiers earn from two sources, $(i)$ from interest income on the funds they lend to the firms and $(ii)$ from dividend they get from firms. Therefore, consumption of rentiers $(C_R)$ can be represented as

$$C_R = c_r[iD + \phi K] = c_r[id + \phi]K \tag{2.3}$$

where $c_r$ is the consumption propensity of rentiers,[2] $i$ is interest rate, $D$ is total debt of firms to the rentiers, $d$ is debt to capital ratio, $\phi$ is dividend to capital ratio.

Most of the neo-Kaleckian literature assume a fraction of profit (or net profit net of interest payment) is given as a dividend to the rentiers.[3] But there is a problem with this assumption. As long as the fraction of profit that is distributed as dividend (i.e. the dividend to profit ratio) is constant,[4] we can conclude from this literature that if profit decreases in any period, dividend payment also should decrease in that very period. However literature on dividend payments starting from Lintner (1956) suggests that firms are reluctant to cut dividends. As Brav et. al. (2005, pp. 497) say "retaining the historic level of the dividend is (nearly) untouchable and is on par with initiating new investment". According to them (Brav et. al.; 2005, pp. 500-501), apart from extraordinary circumstances managers are desperate to avoid dividends cuts. As they say "...there is

[1] As long as potential output-capital ratio is fixed, actual output-capital ratio can be used as a proxy for degree of capacity utilization.

[2] $c_r \in (0, 1)$

[3] To the best of our knowledge Taylor (2012) is the only exception. It assumes dividend payment as a fraction of the existing capital stock. However Taylor (2012) does not provide any economic rationale for this assumption.

[4] At least in the short run.

Table 2.1: Balance sheet matrix

| | Workers' households | Rentiers' households | Firms | $\sum$ |
|---|---|---|---|---|
| Loans | | $+D$ | $-D$ | 0 |
| Equities | | $+P_eE$ | $-P_eE$ | 0 |
| Capital | | | $K$ | $K$ |
| $\sum$ | 0 | $D + P_eE$ | $K - (P_eE + D)$ | $K$ |

not much reward in increasing dividends but there is perceived to be a large penalty for reducing dividends." Amount of dividend payment changes only when substantial and sustainable change in earnings are there. Managers are in fact ready to sell some positive NPV (Net Present Value) investment projects in order to maintain the dividend. Skinner (2008), DeAngelo, DeAngelo and Skinner (2008) also confirm this.

So, we assume even if there is change in profit earned by firms, as long as this change is not sustainable (or as long as firms expect this change in profit is not sustainable in future), there should not be any change in dividend payment. As a result we can safely assume that in the short run firms are providing a fixed amount of dividend to the shareholders (or rentiers in our model). Therefore, Dividend provided by firms $= \phi K$. $\phi$ is fixed in the short run. As long as existing capital stock is fixed (in the short run, capital stock is fixed), for a given $\phi$, dividend earned by rentiers is also fixed. However in the long run, we assume an endogenous dividend to capital ratio.

Following Charles (2008a) we assume investment function as

$$I = [\alpha_0 + \alpha_1(\pi u - id - \phi)]K \tag{2.4}$$

where, $\alpha_0$ represents animal spirits and $\alpha_1$ the coefficient measuring the responsiveness of investment-capital ratio to the change in available internal funds. Our main purpose in this paper is to see the short run impact of debt and dividend payments on aggregate demand, economic growth, and the long run dynamics between debt and dividend. Hence, to get the model tractable and to get away from complication we assume a very simple investment function ($I$) that depends only on the level of animal spirits ($\alpha_0$), and on available internal funds ($(\pi u - id - \phi)K$).

The basic structure of the model is summarized by the balance sheet matrix in Table 2.1 and the transaction flow matrix in Table 2.2.

Table 2.2: Transaction flow matrix

| | Workers' households | Rentiers' households | Firms' current | Firms' capital | $\sum$ |
|---|---|---|---|---|---|
| Consumption | $-C_W$ | $-C_R$ | $C_W + C_R$ | | 0 |
| Investment | | | $I$ | $-I$ | 0 |
| Wages | $W$ | | $-W$ | | 0 |
| Retained profits | | | $-(R - iD - \phi K)$ | $(R - iD - \phi K)$ | 0 |
| Distributed profits (dividends) | | $\phi K$ | $-\phi K$ | | 0 |
| (Value of) Change in equities | | 0 | | 0 | 0 |
| Interest on loans | | $iD$ | $-iD$ | | 0 |
| Change in loans | | $-\dot{D}$ | | $\dot{D}$ | 0 |
| $\sum$ | 0 | 0 | 0 | 0 | 0 |

## 2.1 *Short-run Equilibrium*

In the short run, the goods market is cleared through changes in the level of output and hence through capacity utilization. In equilibrium, nominal income must be equal to aggregate demand i.e.

$$Y = C_W + C_R + I \tag{2.5}$$

$$\implies u^* = \frac{\alpha_0 + (c_r - \alpha_1)(id + \phi)}{(1 - \alpha_1)\pi} \tag{2.6}$$

The equilibrium is satisfying Keynesian stability condition if and only if the induced increase in saving as $u$ rises (i.e. $\pi$) is greater than the induced increase in investment (i.e. $\alpha_1\pi$) i.e.

$$(1 - \alpha_1)\pi > 0$$

$$\implies (1 - \alpha_1) > 0 \tag{2.7}$$

For a meaningful solution, from equation (2.6) we assume

$$\alpha_0 > (\alpha_1 - c_r)(id + \phi) \tag{2.8}$$

If $(c_r - \alpha_1) > 0$, then $\alpha_0 + (c_r - \alpha_1)(id + \phi)$ is unambiguously positive. If $(c_r - \alpha_1) < 0$ then $\alpha_0 > (\alpha_1 - c_r)(id + \phi)$ is required for a meaningful solution. Inserting the equilibrium value of degree of capacity utilization ($u^*$) into equation (2.4)[5] we get

$$g^* = \frac{\alpha_0 - \alpha_1(1 - c_r)(id + \phi)}{(1 - \alpha_1)} \tag{2.9}$$

[5] Growth rate of the economy is expressed as $g = \frac{I}{K}$

For a positive equilibrium growth rate, from equation (2.9) we assume

$$\alpha_0 > \alpha_1(1-c_r)(id+\phi) \tag{2.10}$$

[6]The equilibrium rate of profit is

$$r^* = \pi u^* = \frac{\alpha_0 + (c_r - \alpha_1)(id+\phi)}{(1-\alpha_1)} \tag{2.11}$$

## 2.2 *Comparative Statics*

Partial differentiation of $u^*$, $g^*$, and $r^*$ with respect to $\alpha_0$ yield,

$$\frac{\partial u^*}{\partial \alpha_0} = \frac{1}{(1-\alpha_1)\pi} > 0, \quad \frac{\partial g^*}{\partial \alpha_0} = \frac{1}{(1-\alpha_1)} > 0 \quad \frac{\partial r^*}{\partial \alpha_0} = \frac{1}{(1-\alpha_1)} > 0 \tag{2.12}$$

So, due to an increase in animal spirits ($\alpha_0$), equilibrium degree of capacity utilization, growth rate and rate of profit all increase. Financialization (through its preference channel)[7] affects $\alpha_0$ negatively. As a result, because of financialization, equilibrium degree of capacity utilization, accumulation rate and rate of profit all decrease in the short run.

Partial differentiation of $u^*$, $g^*$, and $r^*$ with respect to $(id+\phi)$ yield,

$$\frac{\partial u^*}{\partial (id+\phi)} = \frac{(c_r - \alpha_1)}{(1-\alpha_1)\pi} \gtrless 0 \text{ depending on } c_r \gtrless \alpha_1 \tag{2.13}$$

$$\frac{\partial g^*}{\partial (id+\phi)} = -\frac{\alpha_1(1-c_r)}{(1-\alpha_1)} < 0 \tag{2.14}$$

$$\frac{\partial r^*}{\partial (id+\phi)} = \frac{(c_r - \alpha_1)}{(1-\alpha_1)} \gtrless 0 \text{ depending on } c_r \gtrless \alpha_1 \tag{2.15}$$

As for a rise in either of $i$, $d$, and $\phi$, $(id+\phi)$ rises, the result for a rise in either of $i$, $d$ and $\phi$ is qualitatively similar to a rise in $(id+\phi)$. Note that if $(c_r-\alpha_1) > 0$, the economy is always in a debt-led demand regime i.e. $\frac{\partial u^*}{\partial d} = \frac{(c_r-\alpha_1)i}{(1-\alpha_1)\pi} > 0$. But if $(c_r-\alpha_1) < 0$ then

[6]Keynesian stability condition implies $\alpha_1 < 1$. Also $c_r \in (0,1)$. Therefore, $\alpha_1 c_r < c_r$. Further, $\alpha_1(1-c_r) > 0$ and $(\alpha_1 - c_r) \gtrless 0$.

$$\text{But, } \alpha_1(1-c_r) = \alpha_1 - \alpha_1 c_r > (\alpha_1 - c_r) \;\; (\because \alpha_1 c_r < c_r)$$

$$\implies \alpha_1(1-c_r)(id+\phi) > (\alpha_1 - c_r)(id+\phi).$$

Therefore, if equation (2.10) is satisfied equation (2.8) is also satisfied.

[7]'Shareholder value orientation' influences managers' (here firms') to shift their preference from retaining profit and reinvesting it to enhance the rate of capital accumulation to downsizing the labour force and distributing the profit to shareholders. "The preference for growth, and hence the willingness to invest in capital stock, therefore suffers, too" (Hein ; 2012b, pp. 39). This route through which shareholder power works is called the 'preference channel'.

the economy is in a debt-burdened demand regime i.e. $\frac{\partial u^*}{\partial d} < 0$. However, the economy is always in a debt-burdened growth regime because $\frac{\partial g^*}{\partial d} = -\frac{\alpha_1(1-c_r)i}{(1-\alpha_1)} < 0$. In Charles (2008a), a rise in debt-capital ratio[8] has unambiguous negative effect on equilibrium degree of capacity utilization, profit rate and growth. But unlike Charles (2008a), here, the impact of a rise in debt-capital ratio on equilibrium degree of capacity utilization is ambiguous. We obtain an ambiguous result for the profit rate as well.

An increase in $d$, by reducing the available internal fund, reduces investment demand by $\alpha_1 i$ amount. But consumption demand of rentiers increases by $c_r i$ amount. If the latter is higher than the former then for a given amount of capital, the aggregate demand and hence the degree of capacity utilization rises.

An increase in $d$ affects the equilibrium growth rate in two ways. First, by reducing the available internal fund it directly negatively affects the growth rate. On the other hand, through its effect on equilibrium degree of capacity utilization, it indirectly affects the equilibrium growth rate. The latter effect is ambiguous and is dominated by the former, and so the overall effect is unambiguously negative.

Unlike Charles (2008a) we get ambiguous results on the impact of interest rate on equilibrium degree of capacity utilization and profit rate. *Ceteris paribus*, an increase in $i$, by reducing the available internal fund, reduces investment demand by $\alpha_1 d$ amount. But consumption demand of rentiers increases by $c_r d$ amount. If the latter is higher than the earlier then for a given amount of capital, the aggregate demand and hence the degree of capacity utilization rises. However due to an increase in $i$, similar to Charles (2008a), $g^*$ falls unambiguously.

We get ambiguous results on the impact of dividend to capital ratio on the equilibrium degree of capacity utilization and the profit rate. An increase in $\phi$, by reducing the available internal fund, reduces investment demand by $\alpha_1$ amount. But consumption demand of rentiers increases by $c_r$ amount. If the latter is higher than the former, then for a given amount of capital, the aggregate demand and hence the degree of capacity utilization rises.

Differentiating partially $u^*$, $g^*$, and $r^*$ with respect to $\pi$ we get,

$$\frac{\partial u^*}{\partial \pi} = -\frac{\alpha_0 + (c_r - \alpha_1)(id + \phi)}{(1-\alpha_1)\pi^2} = -\frac{u^*}{\pi} < 0, \quad \frac{\partial g^*}{\partial \pi} = \frac{\partial r^*}{\partial \pi} = 0 \tag{2.16}$$

Equation (2.16) tells that the economy is in a wage-led demand regime. This is because

[8] As long as in the short run existing stock of capital is fixed, an increase in debt leads to a rise in debt to capital ratio.

Table 2.3: Impact of changes in various parameters on $u^*, g^*$ and $r^*$

| | $u^*$ | $g^*$ | $r^*$ |
|---|---|---|---|
| $\alpha_0$ | $+$ | $+$ | $+$ |
| $i$ | $+/-$ | $-$ | $+/-$ |
| $d$ | $+/-$ | $-$ | $+/-$ |
| $\phi$ | $+/-$ | $-$ | $+/-$ |
| $\pi$ | $-$ | 0 | 0 |

a rise in wage share (or a fall in profit share) distributes income to the wage earners (who have a higher propensity to consume ($c_w = 1$)).[9]

A rise in $\pi$, for a given value of $u^*$, raises the investment rate by $\alpha_1 u^*$ unit whereas a rise in $\pi$, through its effect on $u^*$, reduces the investment rate by exactly the same unit (i.e. by $\alpha_1 \pi \frac{\partial u^*}{\partial \pi} = \alpha_1 u^*$ unit). Consequently, a rise in profit share has no impact on equilibrium growth rate.

The above discussed short run comparative static results are encapsulated in Table 2.3. In the next section, we proceed to the long run dynamics.

# 3 *Long Run*

In this section, we analyse the long-run dynamics of the debt-capital ratio and dividend-capital ratio. We assume that the short run equilibrium values are always attained in the long run, i.e. equilibrium values of $u^*, r^*$ and $g^*$ are always attained. The long run equilibrium is defined as where the debt-capital ratio ($d$) and the dividend-capital ratio ($\phi$) remain constant over time.

## 3.1 *Dynamics of the debt-capital ratio*

We know that for business flows of funds, sources of funds must be equal to the uses of funds. Sources of funds consists of retained earnings, funds from new borrowings and funds from issuance of new equities whereas use of funds consists of investment demand. For simplicity we assume there is no issuance of new equities. So, retained earnings and new borrowings must be equal to investment demand. This in turn implies the following equation,

$$(\pi u - id - \phi) + \dot{d} + dg = g \tag{3.1}$$

[9]Note that consumption by rentiers does not depend on the profit share in the short run. On the other hand firms save their entire retain profit which is equal to $\{R - (id + \phi)K\}$. So, higher the profit share higher is the value of $\frac{R}{Y}$ and as a result higher is the savings by firms.

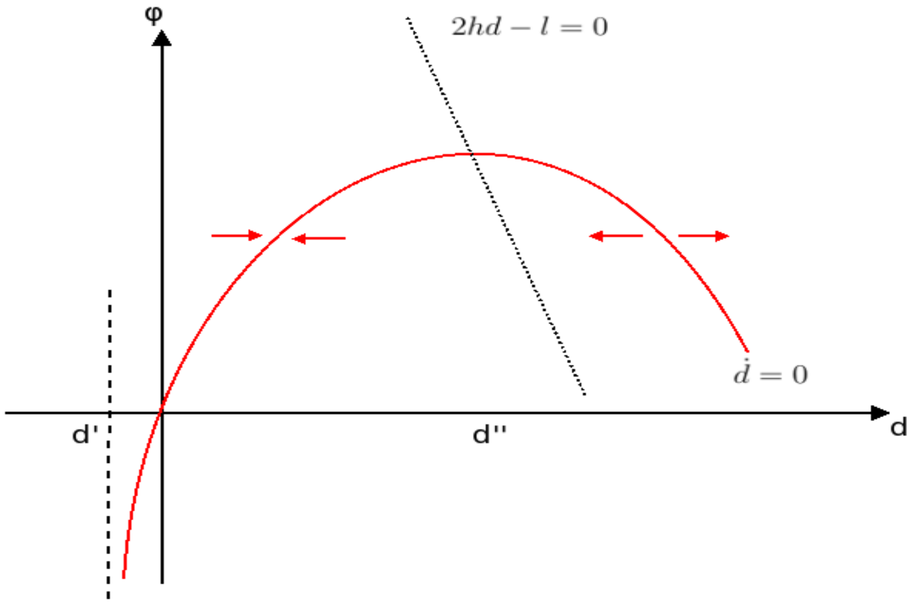



Figure 3.1: $\dot{d} = 0$ isocline.

In equilibrium, $\dot{d} = 0$. This in turn implies,

$$\phi\Big|_{\dot{d}=0} = \frac{\alpha_0 d}{(1-c_r)(1-\alpha_1+\alpha_1 d)} - id \tag{3.2}$$

Inserting $d = 0$ in equation (3.2) we get the vertical intercept as $\phi\Big|_{\dot{d}=0}^{d=0} = 0$. Differentiating equation (3.2) with respect to $d$ we get the slope of the $\dot{d} = 0$ isocline as,

$$\frac{d\phi}{dd}\Big|_{\dot{d}=0} = \frac{(1-c_r)(1-\alpha_1)\alpha_0}{[(1-c_r)(1-\alpha_1+\alpha d)]^2} - i \tag{3.3}$$

$$\frac{d\phi}{dd}\Big|_{\dot{d}=0} > 0 \text{ provided that } \left\{\frac{(1-c_r)(1-\alpha_1)\alpha_0}{[(1-c_r)(1-\alpha_1+\alpha_1 d)]^2} - i\right\} > 0$$

$$\implies \frac{d\phi}{dd}\Big|_{\dot{d}=0} > 0 \text{ provided that } d < \left(1 - \frac{1}{\alpha_1} + \frac{1}{\alpha_1}\sqrt{\frac{\alpha_0(1-\alpha_1)}{(1-c_r)i}}\right) = d'' \tag{3.4}$$

So, point of inflection is at $d = d''$.[10] Thus $\forall d \in (0, d'')$, $\frac{d\phi}{dd}\Big|_{\dot{d}=0} > 0$, and $\forall d \in (d'', \infty)$ $\frac{d\phi}{dd}\Big|_{\dot{d}=0} < 0$. $\dot{d} = 0$ isocline is vertically asymptotic at $(1 - \alpha_1 - \alpha_1 d) = 0$ (i.e. at $d = 1 - \frac{1}{\alpha_1} = d'$). Note that (from equation (2.7)) $\alpha_1 < 1$ ensures $d' < 0$. Figure 3.1 represents the diagram of $\dot{d} = 0$ isocline.

Rearranging equation (3.1) we get,

$$\dot{d} = \frac{(1-c_r)\alpha_1 i d^2 - [\alpha_0 - (1-c_r)\{(1-\alpha_1)i + \alpha_1\phi\}]\, d + (1-\alpha_1)(1-c_r)\phi}{(1-\alpha_1)} \tag{3.5}$$

[10] Note that if $d'' \leq 0$ then for every positive value of $d$ we are unable to get a positive value of $\phi$. So we assume $d'' > 0$. Justification of this assumption will be provided very soon.

$$\implies \dot{d} = \frac{hd^2 - ld + m}{(1-\alpha_1)} \tag{3.6}$$

where $h = (1-c_r)\alpha_1 i > 0$ and $m = (1-\alpha_1)(1-c_r)\phi > 0$ (as long as $\phi > 0$). We assume $l = [\alpha_0 - (1-c_r)\{(1-\alpha_1)i + \alpha_1\phi\}] > 0$.[11]

Partial differentiation of equation (3.6) w.r.t. $d$ yields,

$$J_{11} = \frac{\partial \dot{d}}{\partial d} = \frac{2hd - l}{(1-\alpha_1)} = \frac{M}{(1-\alpha_1)} \tag{3.7}$$

So, $J_{11} \gtreqless 0$ depending on the value of $M \gtreqless 0$. But $M \gtreqless 0$ depending on whether $d \gtreqless \frac{l}{2h}$. Thus, $J_{11} \gtreqless 0$ depending on whether $d \gtreqless \frac{l}{2h}$.

Partial differentiation of equation (3.6) w.r.t. $\phi$ yields,

$$J_{12} = \frac{\partial \dot{d}}{\partial \phi} = \frac{(1-c_r)(1-\alpha_1+\alpha_1 d)}{(1-\alpha_1)} = \frac{N}{(1-\alpha_1)} > 0 \tag{3.8}$$

Thus the slope of the $\dot{d} = 0$ isocline can be represented as $\left.\frac{d\phi}{dd}\right|_{\dot{d}=0} = -\frac{\frac{\partial \dot{d}}{\partial d}}{\frac{\partial \dot{d}}{\partial \phi}} = -\frac{J_{11}}{J_{12}} = -\frac{M}{N}$.

Remember that $2hd - l = 0$ implies $\phi = \frac{\alpha_0 - (1-c_r)(1-\alpha_1)i}{(1-c_r)\alpha_1} - 2id$. Therefore, we get a negatively sloped straight line for $2hd - l = 0$. For the meaningful case, we assume the intercept is positive i.e. we assume $\alpha_0 > (1-c_r)(1-\alpha_1)i$ which in turn ensures that $\left(1 - \frac{1}{\alpha_1} + \frac{1}{\alpha_1}\sqrt{\frac{\alpha_0(1-\alpha_1)}{(1-c_r)i}}\right) = d'' > 0$.

Now we explain equations (3.7) and (3.8) respectively. $J_{11}$ shows the effect of an increase in the debt-capital ratio on a change in the debt-capital ratio itself. For a given $\phi$, a rise in $d$ by one unit decreases the investment rate by $\frac{\alpha_1(1-c_r)i}{(1-\alpha_1)}$ unit (see equation (2.14)). Retained earning, on the other hand, falls by $\frac{(1-c_r)i}{(1-\alpha_1)}$ unit (as $\frac{\partial}{\partial d}((\pi u^* - id - \phi)) = -\frac{(1-c_r)i}{(1-\alpha_1)}$). Hence, for a rise in $d$, the finance through the retained earning falls more than the fall in investment rate (as $\frac{(1-c_r)i}{(1-\alpha_1)} > \frac{\alpha_1(1-c_r)i}{(1-\alpha_1)}$), and as a consequence the debt level (normalized by the capital stock) increases by $\frac{(1-c_r)(1-\alpha_1)i}{(1-\alpha_1)}$ amount (as $\frac{\partial\left(\frac{\dot{D}}{K}\right)}{\partial d} = \frac{(1-c_r)(1-\alpha_1)i}{(1-\alpha_1)} > 0$). Further, as $\dot{d} = \frac{\dot{D}}{K} - dg^*$, we get $\frac{\partial \dot{d}}{\partial d} = \frac{\partial\left(\frac{\dot{D}}{K}\right)}{\partial d} - \{g^* + d\frac{\partial g^*}{\partial d}\}$. As the economy is always in a debt-burdened growth regime, a rise in $d$ reduces $g^*$, and therefore, the sign of $\{g^* + d\frac{\partial g^*}{\partial d}\}$ is ambiguous. Moreover, the sign of $\frac{\partial \dot{d}}{\partial d}$ depends on the level of $d$. If $d < \frac{l}{2h}$, a rise in $d$ negatively affects the change in the debt-capital ratio i.e. $J_{11} < 0$. On the other hand, a higher level of $d$ ($d > \frac{l}{2h}$) has a positive effect on the change in the debt-capital ratio, and hence, $J_{11} > 0$.

[11] The assumption that $l = [\alpha_0 - (1-c_r)\{(1-\alpha_1)i + \alpha_1\phi\}] > 0$ is consistent with the short run as well. Rearranging equation (3.2) we get $(id + \phi) = \frac{\alpha_0 d}{(1-c_r)(1-\alpha_1+\alpha_1 d)}$. Inserting this value in equation (2.10) we get $\alpha_0 > \alpha_1(1-c_r)\left\{\frac{\alpha_0 d}{(1-c_r)(1-\alpha_1+\alpha_1 d)}\right\}$ which in turn implies $1 > \alpha_1$. But this is true from equation (2.7). Hence the assumption that $l > 0$ is consistent with the short run as well.

$J_{12}$ shows the effect of an increase in the dividend-capital ratio on the change in the debt-capital ratio. For given $d$, a unit rise in $\phi$ decreases the investment rate by $\frac{\alpha_1(1-c_r)}{(1-\alpha_1)}$ unit (see equation 2.14). Retained earning, on the other hand, falls by $\frac{(1-c_r)}{(1-\alpha_1)}$ unit (as $\frac{\partial}{\partial \phi}((\pi u^* - id - \phi)) = -\frac{(1-c_r)}{(1-\alpha_1)}$). So, for a rise in $\phi$, the finance through the retained earning falls more than the fall in investment rate (as $\frac{(1-c_r)}{(1-\alpha_1)} > \frac{\alpha_1(1-c_r)}{(1-\alpha_1)}$), and as a consequence, the debt level (normalized by the capital stock) increases by $\frac{(1-c_r)(1-\alpha_1)}{(1-\alpha_1)}$ unit (as $\frac{\partial\left(\frac{\dot{D}}{K}\right)}{\partial \phi} = \frac{(1-c_r)(1-\alpha_1)}{(1-\alpha_1)} > 0$). Further, as $\dot{d} = \frac{\dot{D}}{K} - dg^*$, we get $\frac{\partial \dot{d}}{\partial \phi} = \frac{\partial\left(\frac{\dot{D}}{K}\right)}{\partial \phi} - d\frac{\partial g^*}{\partial \phi}$. As $\frac{\partial g^*}{\partial \phi} < 0$, the sign of $\frac{\partial \dot{d}}{\partial \phi}$ (or $J_{12}$) is unambiguously positive.

## 3.2 *Dynamics of the dividend-capital ratio*

Now, let's focus on the conflict between firms and shareholders regarding the distribution of profits in terms of dividends. Using the following equation we explain the long run dividend dynamics. We assume,

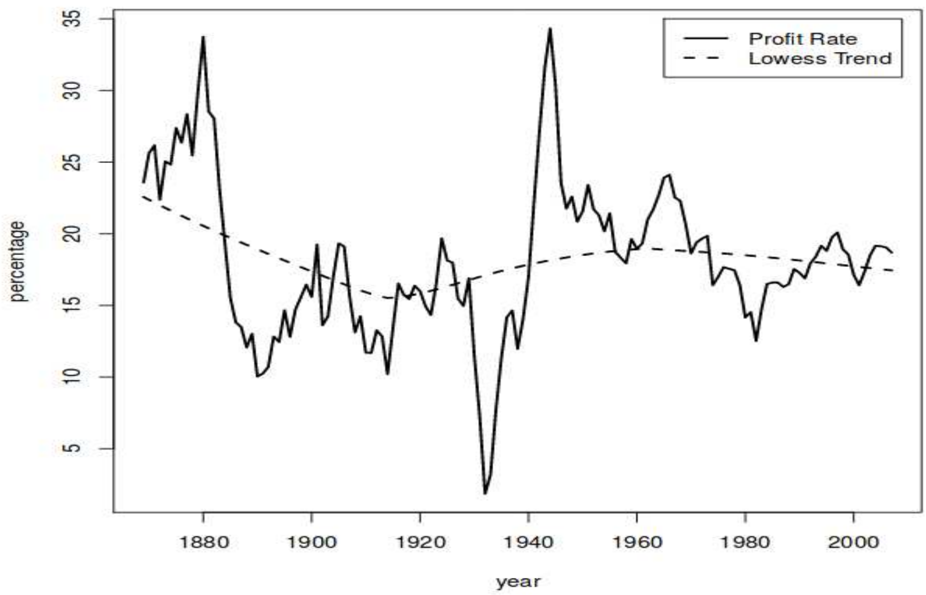
$$\dot{\phi} = \rho[\phi^d - \phi]; \quad \phi^d \in [0,1], \;\; \phi \in [0,1], \;\; \rho > 0 \tag{3.9}$$

$$\phi^d = \varepsilon_0 + \varepsilon_1 r - \varepsilon_2 g + \varepsilon_3 d \tag{3.10}$$

where $\varepsilon_0 > 0, \varepsilon_1 > 0, \varepsilon_2 > 0, \varepsilon_3 > 0$ and $\phi^d$ represents firms' targeted dividend-capital ratio. However actual dividend capital ratio does not adjust to the targeted one instantaneously. It depends on the speed of adjustment $\rho$. $\rho$ captures the conflict between shareholders (or rentiers) and managers (firms). Higher the value of $\rho$, higher the speed of adjustment. The advantage of this specification is that it takes care of the lags between the moment when expectations related to the dividends (i.e. the targeted dividend-capital ratio) are formed and the moment when they are realized. For the analysis to be meaningful we assume $\phi \in [0,1]$ and $\phi^d \in [0,1]$. These are followed by the assumption that $\rho > 0$. Rationale behind the assumption $\phi \in [0,1]$ is as follows. For any firm, for a particular period, it is quite possible that the dividend it pays to its shareholders exceeds the profit it earns for that period. However, this phenomenon cannot be sustained for a long time period. As a result, we can safely assume that the dividend-capital ratio for the firm is less than the rate of profit. For the US economy, we see that the rate of profit for the economy as a whole did not exceed 35% since 1869 (Basu and Manolakos; 2013. See Figure 3.2 as well). As a consequence, we can assume $\phi < r < 1$.

Charles (2008a) assumes a positive relationship between the level of debt and retention ratio. According to him in case of a higher level of debt, "to preserve their financial autonomy and their ability to meet financial commitments",[12] firms reduce dividend pay-

[12]Charles (2008a, pp. 786)




Source: Basu and Manolakos (2013).

Figure 3.2: Long Waves in the U.S. Profit Rate, 1869-2007.

ment and increase retention ratio. However, in the era of financialization, we can expect the exact opposite to happen. A higher level of debt results in a higher level of risk and financial fragility. As a consequence, shareholders demand a higher level of dividend in order to compensate the high level of risk. Dividends at the same time are perceived (by shareholders) as a signal of profitability and financial strength and stability of firms (Baker et. al., 2016). So, in case of a higher level of risk caused by a higher level of debt, firms like to provide a higher level of dividend as a signal of financial stability to the shareholders. So, unlike Charles (2008a), we assume $\phi^d$ to be positively related to the debt-capital ratio. $\varepsilon_3$ represents the sensitivity of targeted dividend-capital ratio to a change in debt-capital ratio. Under financialization, shareholders become very powerful and can influence the managers by influencing their remuneration schemes (Hein; 2012b, pp. 37). As a consequence, more the pressure of shareholders (rentiers) on firms, higher will be the value of $\varepsilon_3$.

Another problem with Charles' analysis is that it (Charles, 2008a) assumes debt-capital ratio to be the only determinant of targeted retention ratio.[13] Along with the debt-capital ratio, expected future rate of profit and expected growth rate may also be important determinants of firms' targeted dividend capital ratios.

If firms have optimistic expectations about the future, they would like to invest more in coming years and therefore they would like to have more retained earnings. Thus there is a negative relationship between expected growth rate and dividend payment. This analysis is also backed by the "free cash flow hypothesis" of Gul (1999), Jensen (1986)

[13] Or targeted payout ratio.

and the "pecking order theory" of Myers and Majluf (1984). The studies of Rozeff (1982) and Lloyd et. al. (1985) also confirm the negative relationship between the expected growth rate and dividend payments. We assume that firms take the current growth rate as a proxy for expected future growth rate of the economy. Thus, we assume $\phi^d$ to be negatively dependent on $g$. $\varepsilon_2$ measures the sensitivity of targeted dividend-capital ratio to a change in growth rate. More the pressure of shareholders (rentiers) on firms, lower will be the value of $\varepsilon_2$.

Anticipated future profitability is another (and probably the most influential) determinant of firms' dividend policies. Starting from Lintner (1956) many contributions to the literature including Fama and Babiak (1968), Brav et. al (2005), Fama and French (2002) and Demirgunes (2015) confirm the relation. When firms expect a rise in the rate of profit to sustain for a longer time period, there targeted dividend capital ratio is influenced positively by the expected rate of profit. We assume that firms take the current rate of profit as a proxy for expected future profit rate of the economy. Thus, we assume $\phi^d$ positively depends on $r$. Hence $\varepsilon_1$, which is a measure of the sensitivity of targeted dividend-capital ratio to a change in rate of profit, is positive. It is noteworthy that more the pressure of shareholders (rentiers) on firms, higher will be the value of $\varepsilon_1$.

$\varepsilon_0$ represents the floor level of the targeted dividend-capital ratio. Consider an economy which is stagnant (i.e. there is no growth), firms are not indebted and they earn zero profit. Even in such a situation, to get the confidence of rentiers, firms can think of giving more dividends to the shareholders (even if they have to borrow for the same) and as a result, there is a positive targeted dividend-capital ratio. Higher the value of $\varepsilon_0$, more the confidence of shareholders and lenders (here rentiers are playing both the roles) on the firms. So, $\varepsilon_0$ can be thought as financial investments by firms which send signals (regarding the level of confidence related to firms' future) to the rentiers and as a consequence, higher the level of $\varepsilon_0$, *ceteris paribus*, brighter the future of firms as perceived by rentiers.

Replacing the value of $\phi^d$ from equation (3.10) into equation (3.9) we get,

$$\dot{\phi} = \rho[\varepsilon_0 + \varepsilon_1 r - \varepsilon_2 g + \varepsilon_3 d - \phi] \tag{3.11}$$

Now putting the equilibrium values of $r^*$ and $g^*$ from (2.11) and (2.9) respectively in the above equation we get,

$$\begin{aligned}\dot{\phi} = \; & \rho\left[\frac{\{\varepsilon_0(1-\alpha_1)+\alpha_0(\varepsilon_1-\varepsilon_2)\}-\{(1-\alpha_1)-\varepsilon_1(c_r-\alpha_1)-\alpha_1\varepsilon_2(1-c_r)\}\phi}{(1-\alpha_1)}\right] \\ & +\rho\left[\frac{[(1-\alpha_1)\varepsilon_3+\{\varepsilon_1(c_r-\alpha_1)+\alpha_1\varepsilon_2(1-c_r)\}i]d}{(1-\alpha_1)}\right]\end{aligned} \tag{3.12}$$

Partial differentiation of equation (3.12) w.r.t. $d$ and $\phi$ yields the following two equations respectively,

$$J_{21} = \frac{\partial \dot{\phi}}{\partial d} = \rho \left[ \frac{(1-\alpha_1)\varepsilon_3 + \{\varepsilon_1(c_r - \alpha_1) + \alpha_1\varepsilon_2(1-c_r)\}i}{(1-\alpha_1)} \right] = \frac{\rho P}{(1-\alpha_1)} \tag{3.13}$$

$$J_{22} = \frac{\partial \dot{\phi}}{\partial \phi} = -\rho \left[ \frac{\{(1-\alpha_1) - \varepsilon_1(c_r - \alpha_1) - \alpha_1\varepsilon_2(1-c_r)\}}{(1-\alpha_1)} \right] = -\frac{\rho Q}{(1-\alpha_1)} \tag{3.14}$$

where $P = [(1-\alpha_1)\varepsilon_3 + \{\varepsilon_1(c_r - \alpha_1) + \alpha_1\varepsilon_2(1-c_r)\}i] \gtreqless 0$;

$T = \{\varepsilon_0(1-\alpha_1) + \alpha_0(\varepsilon_1 - \varepsilon_2)\} \gtreqless 0$

and $Q = \{(1-\alpha_1) - \varepsilon_1(c_r - \alpha_1) - \alpha_1\varepsilon_2(1-c_r)\} \gtreqless 0$.

$J_{21}$ shows the effect of an increase in the debt-capital ratio on the change in the dividend-capital ratio. For every unit rise in $d$, $\phi^d$ rises by $\varepsilon_3$ unit. As the economy is always in a debt-burdened growth regime, a rise in $d$ reduces $g^*$ by $\frac{\alpha_1(1-c_r)i}{(1-\alpha_1)}$ amount and so a rise in $d$ raises $\phi^d$ by $\frac{\varepsilon_2\alpha_1(1-c_r)i}{(1-\alpha_1)}$ amount. Suppose the economy is in a debt-led demand regime i.e. $(c_r - \alpha_1) > 0$. So, a rise in $d$ raises $r^*$ by $\frac{(c_r-\alpha_1)i}{(1-\alpha_1)}$ amount and hence $\phi^d$ by $\frac{\varepsilon_1(c_r-\alpha_1)i}{(1-\alpha_1)}$ amount. Thus, when the economy is in a debt-led demand regime, the overall effect of a rise in $d$ on $\phi^d$ is unambiguously positive i.e. $J_{21} > 0$ (or $P > 0$). On the other hand when the economy is in a debt-burdened demand regime (i.e. when $(c_r - \alpha_1) < 0$), a rise in $d$ reduces $r^*$ by $\frac{(c_r-\alpha_1)i}{(1-\alpha_1)}$ amount and hence $\phi^d$ by $\frac{\varepsilon_1(c_r-\alpha_1)i}{(1-\alpha_1)}$ amount. Hence the overall effect is ambiguous and depends on the strength of $|c_r - \alpha_1|$. If $|c_r - \alpha_1|$ is too strong i.e. if the economy is in a strong debt-burdened demand regime, the overall effect of a rise in $d$ on $\phi^d$ can be negative and hence we get $J_{21} < 0$ (or $P < 0$). Otherwise (for a weak debt-burdened demand regime), the overall effect of a rise in $d$ on $\phi^d$ is positive i.e. $J_{21} > 0$ (or $P > 0$).

$J_{22}$ shows the effect of an increase in the dividend-capital ratio on the change in the dividend-capital ratio. As equation (2.14) shows, a rise in $\phi$ reduces $g^*$ by $\frac{\alpha_1(1-c_r)}{(1-\alpha_1)}$ amount and so raises the change in dividend-capital ratio by $\frac{\rho\varepsilon_2\alpha_1(1-c_r)}{(1-\alpha_1)}$ amount. Suppose the economy is in a debt-led demand regime i.e. $(c_r - \alpha_1) > 0$. Then, a rise in $\phi$ raises $r^*$ by $\frac{(c_r-\alpha_1)}{(1-\alpha_1)}$ amount and hence raises the change in dividend-capital ratio by $\frac{\rho\varepsilon_1(c_r-\alpha_1)}{(1-\alpha_1)}$ amount. The reverse is true for the debt-burdened demand regime. On the other hand, the dividend-capital ratio will erode its own change at a speed of $\rho$, holding $\phi^d$ constant. Thus the overall effect of an increase in dividend-capital ratio on the change in dividend-capital ratio itself is ambiguous.

We get the slope of the $\dot{\phi} = 0$ isocline (see Figure 3.3) as

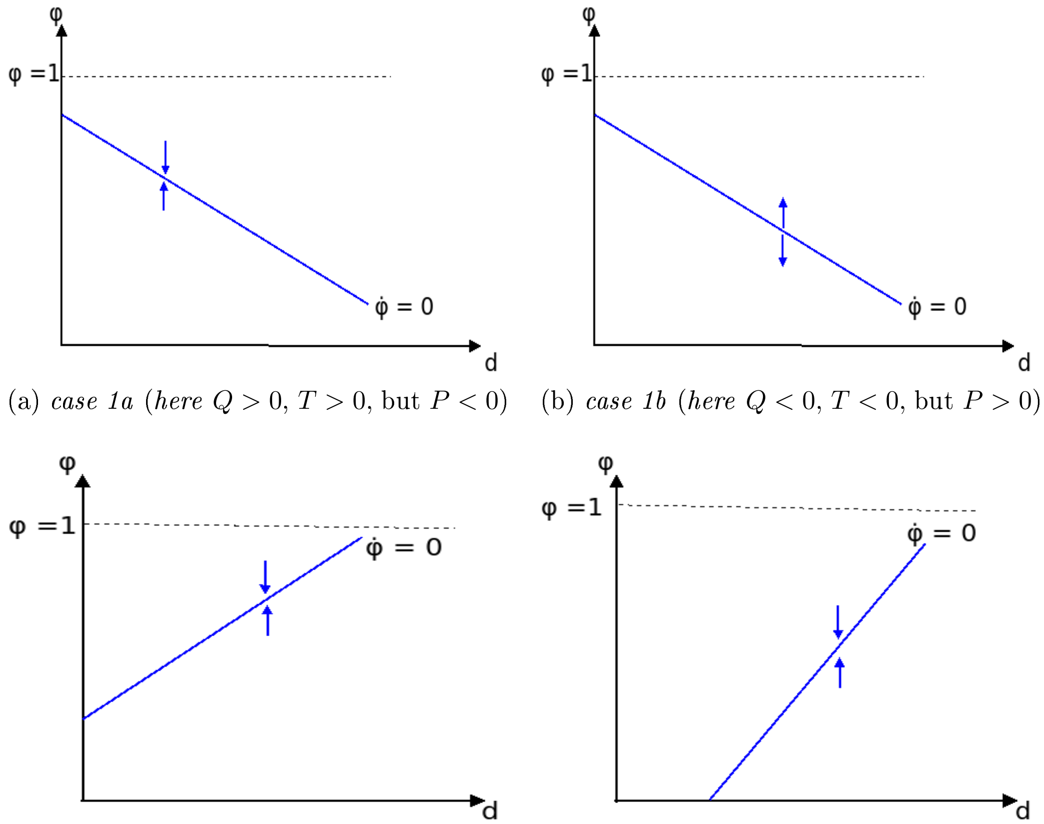
$$\left.\frac{d\phi}{dd}\right|_{\dot{\phi}=0} = -\frac{J_{21}}{J_{22}} = \frac{P}{Q} \tag{3.15}$$

Further, in the long-run equilibrium, $\dot{\phi} = 0$, which in turn implies,

$$\phi\Big|_{\dot{\phi}=0} = \left[\frac{(1-\alpha_1)(\varepsilon_o + \varepsilon_3 d) + \alpha_0(\varepsilon_1 - \varepsilon_2) + \{\varepsilon_1(c_r - \alpha_1) + \alpha_1\varepsilon_2(1-c_r)\}id}{(1-\alpha_1) - \varepsilon_1(c_r - \alpha_1) - \alpha_1\varepsilon_2(1-c_r)}\right] \tag{3.16}$$

Inserting $d = 0$ in equation (3.16) we get the vertical intercept of the $\dot{\phi} = 0$ isocline as[14]

$$\phi\Big|_{\dot{\phi}=0}^{d=0} = \frac{\varepsilon_0(1-\alpha_1) + \alpha_0(\varepsilon_1 - \varepsilon_2)}{(1-\alpha_1) - \varepsilon_1(c_r - \alpha_1) - \alpha_1\varepsilon_2(1-c_r)} = \frac{T}{Q} \gtreqless 0 \tag{3.17}$$




(a) *case 1a* (*here* $Q > 0$, $T > 0$, but $P < 0$)

(b) *case 1b* (*here* $Q < 0$, $T < 0$, but $P > 0$)

(c) *case 2* (here $Q < 0$, $T < 0$ and $P > 0$)

(d) *case 3* (*here* $Q > 0$, $T < 0$ and $P > 0$)

Figure 3.3: $\dot{\phi} = 0$ isocline.

In the next section we discuss the possible cases that may arise due to the interaction between the debt-capital ratio and dividend-capital ratio dynamics. In Charles (2008a), while a low level of interest rate exhibits an unique stable equilibrium, a high value of it (the interest rate) is a necessary precondition for the existence of multiple equilibria and the occurrence of financial fragility in the system. However, such condition is not essential for the existence of multiple equilibria in our model.

[14] If $\varepsilon_1 > \varepsilon_2$ then the numerator of equation (3.17) is unambiguously positive. However if $\varepsilon_1 < \varepsilon_2$ then the numerator is positive only if $\varepsilon_0(1-\alpha_1) > \alpha_0(\varepsilon_1 - \varepsilon_2)$.

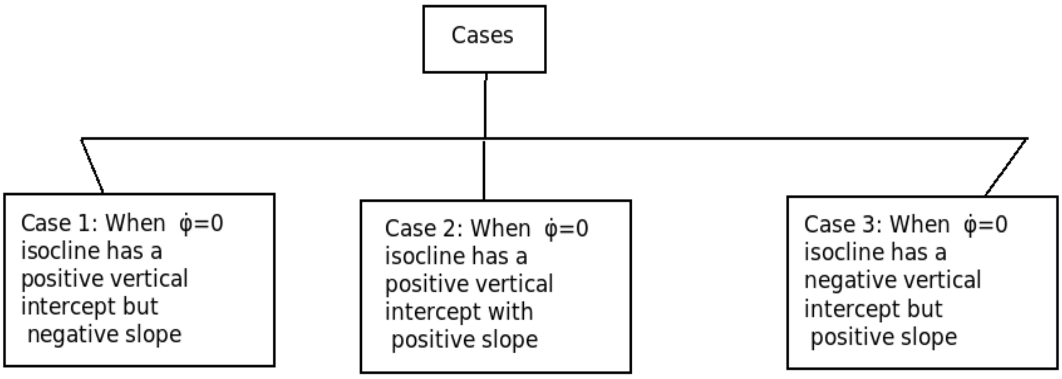



Figure 4.1: Flowchart of various cases

# 4 *Possible Cases*

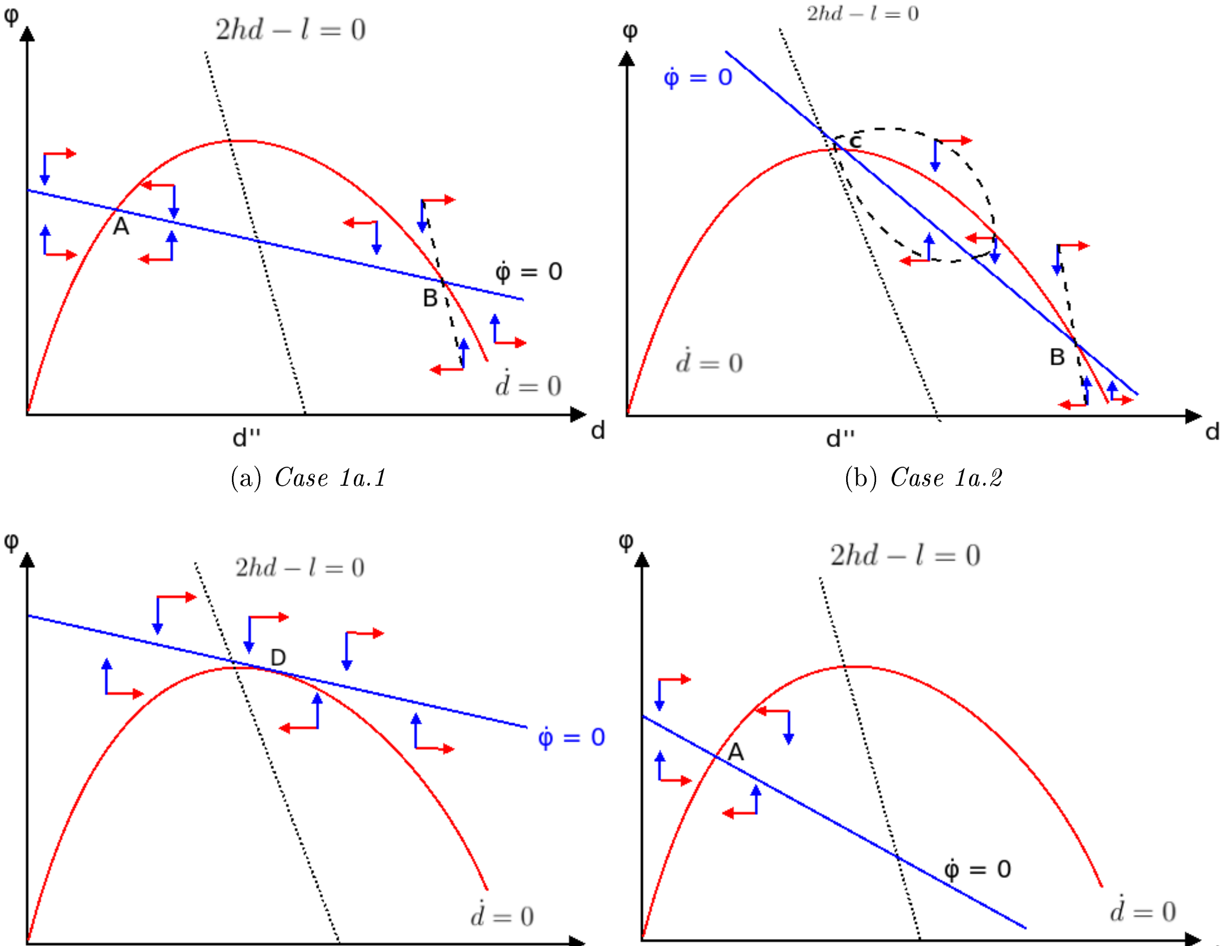
We can have three different possible cases. *Case 1*- where the $\dot{\phi} = 0$ isocline has a positive vertical intercept but the $\dot{\phi} = 0$ isocline is negatively sloped. Equation (3.17) suggests that positive vertical intercept is possible when either $(i)$ both $Q$ and $T$ are positive or $(ii)$ $Q$ and $T$ are simultaneously negative. Equation (3.15) suggests that $\dot{\phi} = 0$ isocline has a negative slope if either $(i)$ $Q > 0$ but $P < 0$ or $(ii)$ $Q < 0$ and $P > 0$. When $Q > 0$, $T > 0$ but $P < 0$ we call it as *case 1a.* So, here the $\dot{\phi} = 0$ isocline has a positive vertical intercept and negative slope. In case 1b, $Q < 0$, $T < 0$ and $P > 0$. Here too, the $\dot{\phi} = 0$ isocline has positive vertical intercept and negative slope. In *Case 2* the $\dot{\phi} = 0$ isocline has a positive vertical intercept but this time the $\dot{\phi} = 0$ isocline is positively sloped. Positively sloped $\dot{\phi} = 0$ isocline is possible when either $(i)$ both $Q$ and $P$ are positive or $(ii)$ $Q$ and $P$ are simultaneously negative. Therefore in *case 2* we assume $Q < 0$, $T < 0$ and $P > 0$. However, $Q < 0$, $T < 0$ and $P > 0$ together cannot hold. We will show this in Section 4.3. *Case 3* is the scenario where the $\dot{\phi} = 0$ isocline has a negative vertical intercept but positive slope. For a negative vertical intercept either $(i)$ $Q > 0$ but $T < 0$ have to hold or $(ii)$ $Q < 0$ and $T > 0$ must be satisfied. Therefore in *case 3* we assume $Q > 0$, $T < 0$ and $P > 0$. However, $Q < 0$, $T > 0$ and $P < 0$ together cannot hold. It will be clear in Section 4.4. Figure 4.1 represents the flowchart related to all these three possible cases.

## 4.1 *Case 1a*

In *case 1a* we assume $Q > 0, T > 0$, and $P < 0$. $P < 0$ implies (from equation (3.13)) that debt-capital ratio $(d)$ has an overall negative effect on the change in the dividend-capital ratio whereas $Q > 0$ ensures (from equation (3.14)) that the dividend-capital ratio $(\phi)$ has an overall negative effect on the change in the dividend-capital ratio itself. As $(1 - \alpha_1)\varepsilon_3$ is always positive, for $P$ to be negative, the responsiveness of investment demand to a change in the internal source of fund ($\alpha_1$) must exceed the consumption propensity of rentiers ($c_r$) by a significantly large amount. But that implies (from equation (2.13)) that the economy is in a strong debt-burdened demand regime. Therefore, *case 1a* is associated with a ***strong debt-burdened demand regime***. We consider three different sub-cases here: *cases 1a.1, 1a.2,* and *1a.3* respectively.

### 4.1.1 *Case 1a.1*

Two equilibria, $A$ and $B$, are possible here. Here, $Q > 0$ and equation (3.14) imply $J_{22} < 0$, whereas $P < 0$ and equation (3.13) imply $J_{21} < 0$.




(a) *Case 1a.1* (b) *Case 1a.2*

(c) *Case 1a.3* (d) *Case 1a.4*

Figure 4.2: Multiple cases of the long run equilibria: *case 1a*

**Consider Point** $A$**:** $d < \frac{l}{2h}$ ensures $J_{11}$ to be negative, whereas $J_{12}$ is always positive. Therefore at $A$, the determinant of the Jacobian matrix Det(J) $= (\overbrace{J_{11}}^{-}\overbrace{J_{22}}^{-} - \overbrace{J_{12}}^{+}\overbrace{J_{21}}^{-}) >$

0 and the trace of the Jacobian matrix $\text{tr(J)} = (\overbrace{J_{11}}^{-} + \overbrace{J_{22}}^{-}) < 0$. Hence point $A$ is a stable steady state. Figure 4.2a captures the diagrammatic illustration.

Let us explain the stability of the steady state $A$ intuitively. Because of some exogenous shock, let us assume that the debt-capital ratio deviates from its steady state value. Suppose that the debt-capital ratio is higher than its steady state value. First, as long as $d < \frac{l}{2h}$, $d > d^*$ implies $d$ must fall (due to $\frac{\partial \dot{d}}{\partial d} = J_{11} < 0$). This is the direct effect. Second, as the debt-capital ratio is higher than its steady state value, the dividend-capital ratio decreases due to $J_{21} < 0$. This fall in $\phi$ in turn leads to a fall in the debt-capital ratio due to $J_{12} > 0$. This is the indirect effect. Here both the direct and indirect effects are stable. As a result, if the debt-capital ratio rises from the steady state value, it again comes back to its steady state. Hence, the steady state is stable.

**Consider point $B$:** Here $d > \frac{l}{2h}$ i.e. $J_{11} > 0$. Here the slope of the $\dot{\phi} = 0$ isocline is greater than the slope of the $\dot{d} = 0$ isocline i.e.

$$0 > \left.\frac{d\phi}{dd}\right|_{\dot{\phi}=0} = -\frac{J_{21}}{J_{22}} > \left.\frac{d\phi}{dd}\right|_{\dot{d}=0} = -\frac{J_{11}}{J_{12}}$$

$$\implies J_{11}J_{22} - J_{12}J_{21} < 0 \quad (\because J_{22} < 0 \text{ and } J_{12} > 0)$$

Hence the determinant of the Jacobian matrix $\text{Det(J)} = J_{11}J_{22} - J_{12}J_{21} < 0$. So point $B$ is a saddle point (see Figure 4.2a).

Let us discuss it intuitively. Debt-capital ratio, suppose due to some reason, deviates from the steady state and is now higher than its steady state value. There exist two opposite effects near the steady state $B$. First, as the debt-capital ratio is higher than its steady state value, it must rise further due to $J_{11} > 0$ (as $d > \frac{l}{2h}$ near $B$, $J_{11} > 0$). This is the direct unstable effect. Second, the rise in debt-capital ratio leads to a fall in the dividend-capital ratio due to $J_{21} < 0$. As $J_{12} > 0$, this fall in dividend-capital ratio leads to a fall in the debt-capital ratio. This second effect is an indirect stable effect. However, as slope of the $\dot{\phi} = 0$ isocline is greater than slope of the $\dot{d} = 0$ isocline, absolute value of $J_{21}$ is relatively weak (as both the slopes are negative and $J_{21} < 0$). As a result, a rise in $d$ through equation (3.13) leads to a negligible amount of fall in the dividend-capital ratio which in turn through equation (3.8) leads to a negligible amount of fall in debt-capital ratio. Therefore, the direct unstable effect dominates the indirect stable effect and results the steady state to be unstable. There is only one stable arm that reaches to the equilibrium point $B$. Hence $B$ emerges as a saddle point.

### 4.1.2 *Case 1a.2*

Here two equilibria are possible. These are $C$ and $B$ respectively. For point $B$ the analysis is the same as in point $B$ of *case 1a.1*. At the steady state $C$, $d > \frac{l}{2h}$, and therefore $J_{11} > 0$. Here, the slope of the $\dot{d} = 0$ isocline is greater than slope of the $\dot{\phi} = 0$ isocline (see Figure 4.2b) i.e.

$$0 > \left.\frac{d\phi}{dd}\right|_{\dot{d}=0} = -\frac{J_{11}}{J_{12}} > \left.\frac{d\phi}{dd}\right|_{\dot{\phi}=0} = -\frac{J_{21}}{J_{22}}$$

$$\implies J_{11}J_{22} - J_{12}J_{21} > 0 \quad (\because J_{22} < 0 \text{ and } J_{12} > 0)$$

Hence, the determinant of the Jacobian matrix Det(J) $= J_{11}J_{22} - J_{12}J_{21} > 0$ and the trace of the Jacobian matrix is $tr(J) = J_{11} + J_{22} = \frac{1}{(1-\alpha_1)}[M - \rho Q] \gtreqless 0$. As a result, point $C$ can be either stable or unstable equilibrium depending on the speed of adjustment parameter $\rho > \hat{\rho} = \frac{M}{Q}$ or $\rho < \hat{\rho} = \frac{M}{Q}$ respectively. If $\rho = \hat{\rho}$, limit cycles occur due to Hopf-bifurcation. More discussion regarding Hopf-bifurcation is provided in section 5.

Intuition behind the stability at point $C$ is as follows. First, when the debt-capital ratio rises above its steady state value, as $J_{11} > 0$, the positive self-feedback effect leads to a further rise in the debt-capital ratio. This is the direct unstable effect. Second, a rise in $d$ through equation (3.13) leads to a fall in the dividend-capital ratio ($\because J_{21} < 0$ here) which in turn through equation (3.8) causes a fall in the debt-capital ratio ($\because J_{12} > 0$). This is the indirect stable effect. When the speed of adjustment parameter of the dividend-capital ratio ($\rho$) is sufficiently high, the dynamics of the system could become stable because the negative indirect-feedback mechanism of the debt-capital ratio becomes strong and dominates the unstable self-feedback effect.

### 4.1.3 *Case 1a.3*

As illustrated in Figure 4.2c, $D$ is a saddle point unstable steady state.

From the above mentioned analysis of *case 1a* one can conclude the following proposition.[15]

[15] Note that the above discussed three sub-cases are in no sense exhaustive. However, the characters of the steady states, we discussed, are entirely exhaustive. For example, we can get a new sub-case where the $\dot{\phi} = 0$ isocline intersects and positively sloped part of the $\dot{d} = 0$ isocline, and the slope of the $\dot{\phi} = 0$ isocline is so steep that it does not intersect the $\dot{d} = 0$ isocline twice. In that scenario, we can get a unique steady state $A$ (see Figure 4.2d). Similarly, we can get a sub-case where a unique steady state $C$ exists. However, in all of those sub-cases, the characters of the steady states would be either of the character of $A$, or $B$, or $C$ or of $D$. In that sense, the characters of the steady states, we discuss in *case 1a,* are completely exhaustive. The same kind of story is applicable for *case 1b* as well.

**Proposition 1.** *In case 1a, as long as steady state exists, a lower value of debt-capital ratio* ($d < \frac{l}{2h}$) *is sufficient to ensure the stability of the steady state.*

*Proof.* Suppose ($d < \frac{l}{2h}$). This ensures $J_{11} < 0$. For *case 1a*, $J_{22} < 0$ (by assumption), $J_{21} < 0$ (by assumption) and $J_{12} > 0$ (always holds). Therefore we see, the determinant of the Jacobian matrix is positive and the trace is negative. As a result, the steady state is a stable steady state. □

Note that for a given value of $\phi$, as $l = [\alpha_0 - (1 - c_r)\{(1 - \alpha_1)i + \alpha_1\phi\}]$, the value of $l$ depends on $\alpha_0$. Strong 'shareholder value orientation' is associated with the shift in the preference of firms from long-term growth perspective to short term profitability (Hein; 2012b, pp. 70). As a consequence, there is a negative pressure on the animal spirit of firms (encapsulated by $\alpha_0$). A reduction in the level of $\alpha_0$, *ceteris paribus*, leads to a reduction in $l$ and given the value of $h$, $\frac{l}{2h}$ falls. So, more strong the 'shareholder value orientation' (or more powerful the shareholder are), it is more likely that the existing debt-capital ratio is crossing the threshold level ($\frac{l}{2h}$) and as a result the economy loses the unconditional stability and becomes either unstable (point $B$) or conditionally stable[16] (point $C$).

## 4.2 *Case 1b*

In *case 1b* we assume $Q < 0$, $T < 0$, and $P > 0$. Here the vertical intercept of the $\dot{\phi} = 0$ isocline is positive (as $Q < 0$ and $T < 0$) whereas the slope is negative (as $Q < 0$ and $P > 0$). In *case 1b,* the debt-capital ratio ($d$) and the dividend-capital ratio ($\phi$) both have positive effects on the change in dividend-capital ratio ($\because P > 0$ and $Q < 0$).

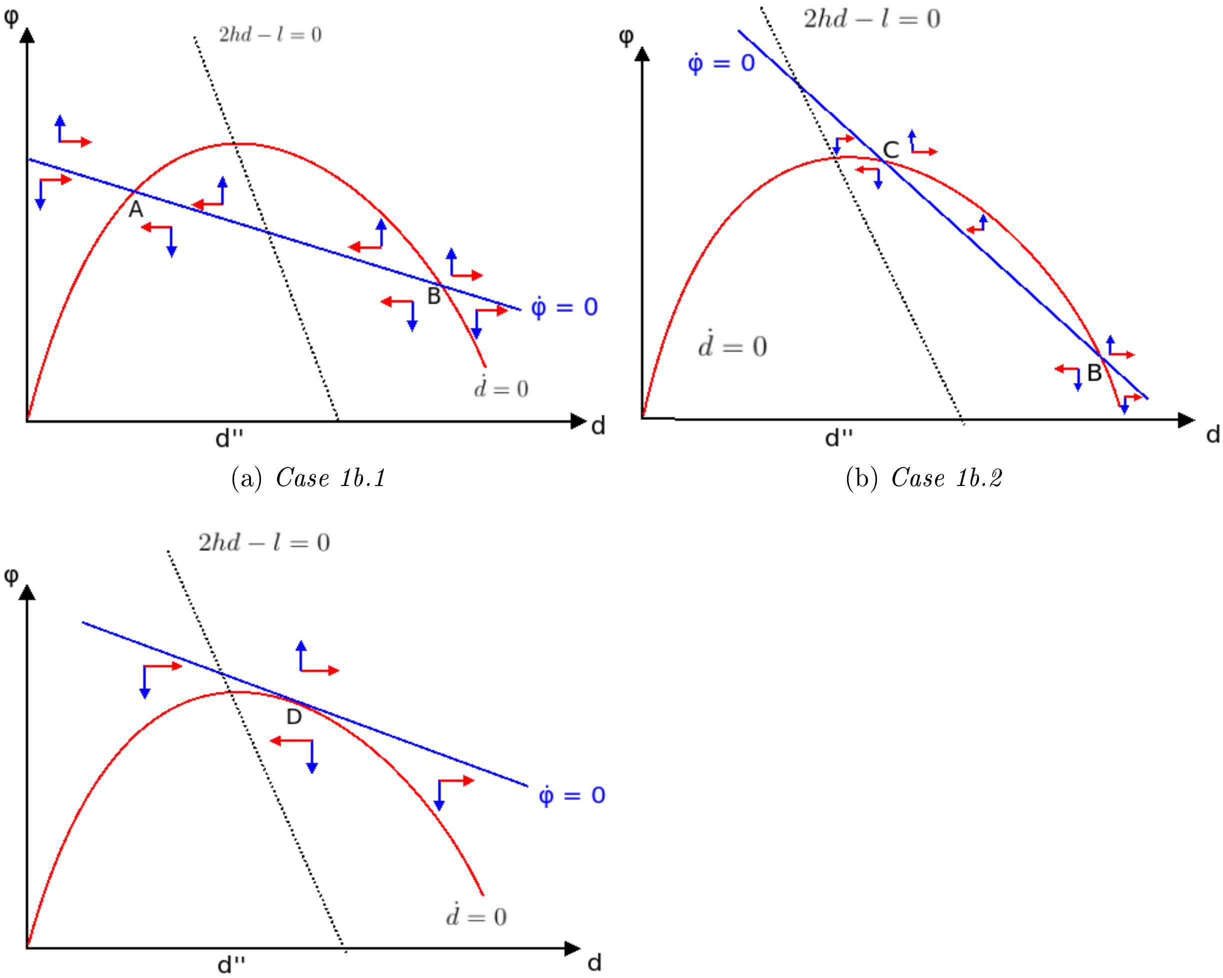
Note that $Q < 0$ & $P > 0$ together can hold only if $\{\varepsilon_1(c_r - \alpha_1) + \alpha_1\varepsilon_2(1 - c_r)\}$ is positive. As $\varepsilon_1 > 0$, $\varepsilon_2 > 0$, $\alpha_1 \in (0, 1)$ and $c_r \in (0, 1)$, therefore $\{\varepsilon_1(c_r - \alpha_1) + \alpha_1\varepsilon_2(1 - c_r)\} > 0$ implies ***either the economy is in a debt-led demand regime*** (i.e. $c_r > \alpha_1$) ***or in a weak debt-burdened demand regime***.

We consider three different sub-cases here: *cases 1b.1, 1b.2,* and *1b.3* respectively.

### 4.2.1 *Case 1b.1*

*Case 1b.1* consists of two equilibria: $A$ and $B$ respectively. In both the steady states, $Q < 0$ and equation (3.14) imply $J_{22} > 0$, $P > 0$ and equation (3.13) imply $J_{21} > 0$. $J_{12}$ is always positive.

[16] We are calling $C$ as conditionally stable because its stability depends on the condition whether $\rho > \hat{\rho}$ or not.




(a) *Case 1b.1*

(b) *Case 1b.2*

(c) *Case 1b.3*

Figure 4.3: Multiple cases of the long run equilibria: *case 1b*

**Consider Point** $A$**:** $d < \frac{l}{2h}$ ensures $J_{11}$ to be negative. Therefore at $A$, the determinant of the Jacobian matrix is negative ($\text{Det(J)} = (\overbrace{J_{11}}^{-}\ \overbrace{J_{22}}^{+} - \overbrace{J_{12}}^{+}\ \overbrace{J_{21}}^{+}) < 0$). Hence point $A$ is a saddle point unstable steady state. Figure 4.3a represents the diagrammatic illustration.

**Consider point** $B$**:** As $d > \frac{l}{2h}$ here, $J_{11} > 0$. Here the slope of the $\dot{\phi} = 0$ isocline is greater than the slope of the $\dot{d} = 0$ isocline i.e.

$$0 > \left.\frac{d\phi}{dd}\right|_{\dot{\phi}=0} = -\frac{J_{21}}{J_{22}} > \left.\frac{d\phi}{dd}\right|_{\dot{d}=0} = -\frac{J_{11}}{J_{12}}$$

$$\implies J_{11}J_{22} - J_{12}J_{21} > 0 \quad (\because J_{22} > 0 \text{ and } J_{12} > 0)$$

The determinant of the Jacobian matrix $\text{Det(J)} = J_{11}J_{22} - J_{12}J_{21} > 0$ and the trace $tr(J) = J_{11} + J_{22} > 0$. Thus point $B$ is an unstable steady state (see Figure 4.3a).

### 4.2.2 *Case 1b.2*

Here two equilibria- $C$ and $B$ are possible. The analysis for $B$ is the same as in point $B$ of *case 1b.1*. At the steady state $C$, $d > \frac{l}{2h}$, and therefore $J_{11} > 0$. Here, the slope of the $\dot{d} = 0$ isocline is greater than slope of the $\dot{\phi} = 0$ isocline (see Figure 4.2b) i.e.

$$0 > \left.\frac{d\phi}{dd}\right|_{\dot{d}=0} = -\frac{J_{11}}{J_{12}} > \left.\frac{d\phi}{dd}\right|_{\dot{\phi}=0} = -\frac{J_{21}}{J_{22}}$$

$$\implies J_{11}J_{22} - J_{12}J_{21} < 0 \quad (\because J_{22} > 0 \text{ and } J_{12} > 0)$$

As the determinant is negative (Det(J) $= J_{11}J_{22} - J_{12}J_{21} < 0$), point $C$ emerges as a saddle point unstable steady state (see Figure 4.3b).

### 4.2.3 *Case 1b.3*

As illustrated in Figure 4.3c, $D$ is a saddle point unstable steady state.

## 4.3 *Case 2*

In *case 2*, we assume $Q > 0$, $T > 0$, and $P > 0$. The $\dot{\phi} = 0$ isocline is positively sloped (as $Q > 0$ and $P > 0$) with positive vertical intercept (as $Q > 0$ and $T > 0$). *Cases 2.1, 2.2* and *2.3* are possible here. In *case 2*, $Q > 0$ implies $J_{22} < 0$ and $P > 0$ implies $J_{21} > 0$. $Q > 0$ and $P > 0$ together imply absolute value of $\{\varepsilon_1(c_r - \alpha_1) + \alpha_1\varepsilon_2(1 - c_r)\}$ is small. Therefore either $(c_r - \alpha_1) > 0$ or $(c_r - \alpha_1) < 0$ is possible but, the absolute value of $(c_r - \alpha_1)$ has to be small. As a result, $Q > 0$ and $P > 0$ together ensure that ***the economy is either in a weak debt-burdened or in a weak debt-led demand regime***.

### 4.3.1 *Case 2.1*

Two equilibria: $A$ and $B$ can occur here.

**Consider Point** $A$**:** Here $d < \frac{l}{2h}$ which in turn ensures $J_{11} < 0$. Here, the slope of the $\dot{d} = 0$ isocline is greater than the slope of the $\dot{\phi} = 0$ isocline i.e.

$$\left.\frac{d\phi}{dd}\right|_{\dot{d}=0} = -\frac{J_{11}}{J_{12}} > \left.\frac{d\phi}{dd}\right|_{\dot{\phi}=0} = -\frac{J_{21}}{J_{22}} > 0$$

$$\implies J_{11}J_{22} - J_{12}J_{21} > 0 \quad (\because J_{22} < 0 \text{ and } J_{12} > 0)$$

Therefore the determinant is positive ($\text{Det(J)} = J_{11}J_{22} - J_{12}J_{21} > 0$) and the trace is negative ($\text{tr(J)} = J_{11} + J_{22} < 0$). Thus point $A$ is a stable equilibrium (see Figure 4.4a).

Suppose that due to some exogenous shock the debt-capital ratio deviates from its steady state value and after deviation it is now higher than its initial steady state value. First, near $A$, as $d > d^*$, $d$ must fall due to $J_{11} < 0$. This is the direct stable adjustment process. On the other hand, a rise in $d$ increases $\phi$ ($\because J_{21} > 0$). This in turn increases the debt-capital ratio due to $J_{21} > 0$. This is the indirect unstable effect. However, as slope of the $\dot{\phi} = 0$ isocline is less than slope of the $\dot{d} = 0$ isocline, absolute value of $J_{21}$ is relatively weak. As a result, a rise in $d$ through equation (3.13) leads to a small amount of rise in the dividend-capital ratio which in turn through equation (3.8) leads to a small amount of rise in debt-capital ratio. Therefore, the direct stable effect dominates the indirect unstable effect and results the steady state to be stable.

**Consider point B:** $d > \frac{l}{2h}$ ensures $J_{11}$ to be positive here. Therefore at $B$, $\text{Det(J)} = (\overbrace{J_{11}}^{+}\overbrace{J_{22}}^{-} - \overbrace{J_{12}}^{+}\overbrace{J_{21}}^{+}) < 0$. Hence point $B$ is a saddle point unstable steady state (see Figure 4.4a).

### 4.3.2 *Case 2.2*

Two equilibria, $A$ and $C$ are possible here. For $A$, the analysis is the same as in point $A$ of *case 2.1*. At $C$ we get $d < \frac{l}{2h}$ i.e. $J_{11} < 0$. Here the slope of the $\dot{d} = 0$ isocline is less than slope of the $\dot{\phi} = 0$ isocline i.e.

$$0 < \left.\frac{d\phi}{dd}\right|_{\dot{d}=0} = -\frac{J_{11}}{J_{12}} < \left.\frac{d\phi}{dd}\right|_{\dot{\phi}=0} = -\frac{J_{21}}{J_{22}}$$

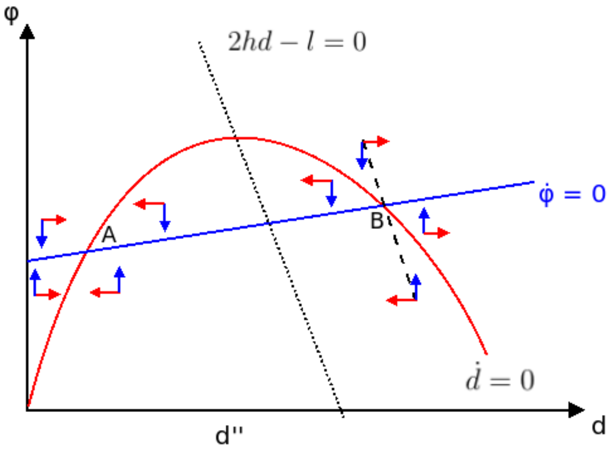
$$\implies J_{11}J_{22} - J_{12}J_{21} < 0 \quad (\because J_{22} < 0 \text{ and } J_{12} > 0)$$

As the determinant $\text{Det(J)} = J_{11}J_{22} - J_{12}J_{21} < 0$, point $C$ is a saddle point unstable steady state (see Figure 4.4b).

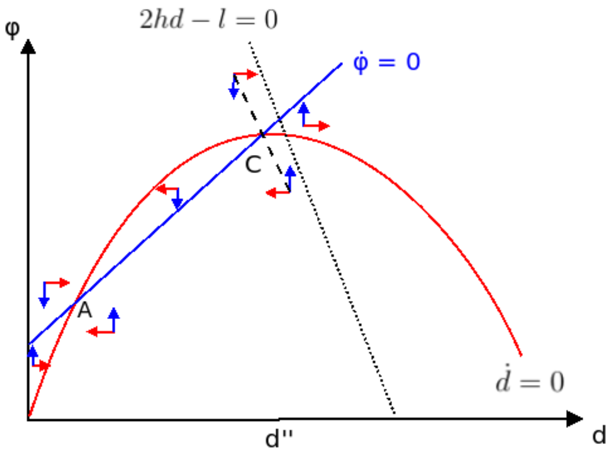
### 4.3.3 *Case 2.3*

As illustrated in Figure 4.4c, point $D$ is a saddle point unstable equilibrium.

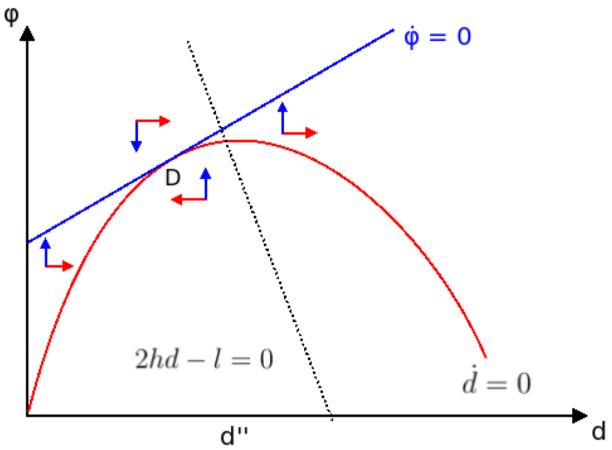
Note that $Q < 0$, $T < 0$, and $P < 0$ also imply the vertical intercept as well as the slope of $\dot{\phi} = 0$ isocline to be positive. However, both $Q < 0$ and $P < 0$ cannot hold simultaneously. $(1-\alpha_1)\varepsilon_3 > 0$, $i > 0$ and $P < 0$ together imply $\varepsilon_1(c_r - \alpha_1) + \alpha_1\varepsilon_2(1 - c_r)\} < 0$. Therefore $Q = (1-\alpha_1) - \{\varepsilon_1(c_r - \alpha_1) + \alpha_1\varepsilon_2(1-c_r)\} > 0$. But we already assumed $Q < 0$. So there




(a) *Case 2.1*




(b) *Case 2.2*



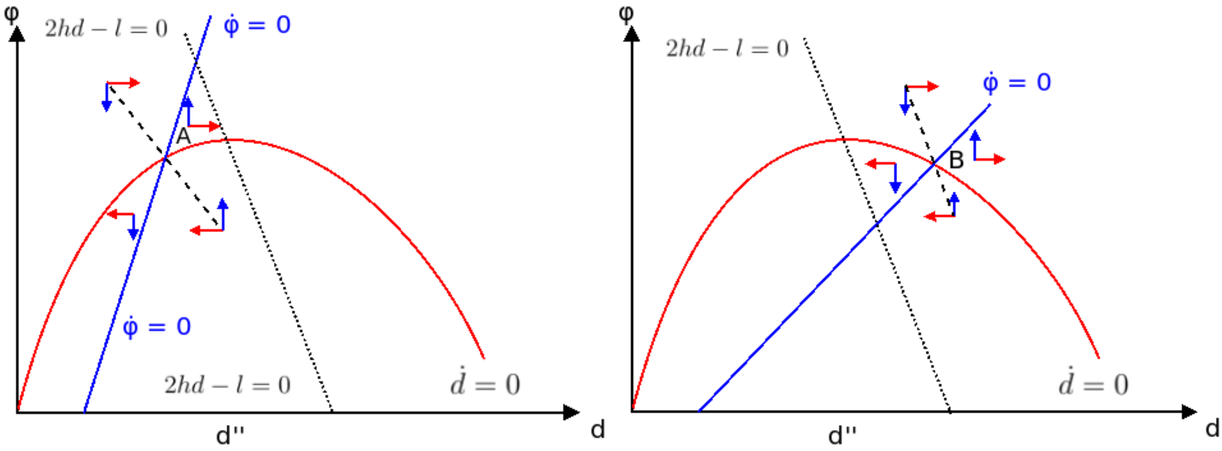

(c) *Case 2.3*

Figure 4.4: Multiple cases of the long run equilibria: *case 2*

is a logical contradiction. Hence both $Q < 0$ and $P < 0$ cannot hold simultaneously. As a result, we cannot get a scenario where $Q < 0$, $T < 0$, and $P < 0$ occur.

## 4.4 *Case 3*

For *case 3,* we assume $Q > 0$, $T < 0$, and $P > 0$. In *case 3,* $Q > 0$ implies $J_{22} < 0$, and $P > 0$ implies $J_{21} > 0$. $Q > 0$ and $P > 0$ together imply absolute value of $\{\varepsilon_1(c_r - \alpha_1) + \alpha_1\varepsilon_2(1 - c_r)\}$ is small. Therefore either $(c_r - \alpha_1) > 0$ or $(c_r - \alpha_1) < 0$ but, the absolute value of $(c_r - \alpha_1)$ is small. As a result, $Q > 0$ and $P > 0$ together ensure ***the economy to be either in a weak debt-burdened or in a weak debt-led demand regime.*** Note that here the slope of the $\dot{\phi} = 0$ isocline is positive (as $Q > 0$ and $P > 0$) whereas the vertical intercept is negative (as $Q > 0$ and $T < 0$). We consider two different cases here: *Case 3.1,* and *Case 3.2.*




(a) *Case 3.1* (b) *Case 3.2*

Figure 4.5: Multiple cases of the long run equilibria: *case 3*

### 4.4.1 *Case 3.1*

As illustrated in Figure 4.5a, only a unique equilibrium $A$ is possible. At $A$, $d < \frac{l}{2h}$ i.e. $J_{11} < 0$, and the slope of the $\dot{\phi} = 0$ isocline is greater than the slope of the $\dot{d} = 0$ isocline i.e.

$$\left.\frac{d\phi}{dd}\right|_{\dot{\phi}=0} = -\frac{J_{21}}{J_{22}} > \left.\frac{d\phi}{dd}\right|_{\dot{d}=0} = -\frac{J_{11}}{J_{12}} > 0$$

$$\implies J_{11}J_{22} - J_{12} < 0 \quad (\because J_{22} < 0 \text{ and } J_{12} > 0)$$

Hence, the determinant is negative $(\text{Det(J)} = J_{11}J_{22} - J_{12}J_{21} < 0)$. Consequently, point $A$ emerges as a saddle point unstable steady state.

### 4.4.2 *Case 3.2*

As illustrated in Figure 4.5b, at the unique equilibrium point $B$, $d > \frac{l}{2h}$, and therefore at $B$, $\text{Det(J)} = (\overbrace{J_{11}}^{+}\overbrace{J_{22}}^{-} - \overbrace{J_{12}}^{+}\overbrace{J_{21}}^{+}) < 0$. Hence point $B$ is a saddle point unstable steady state.

Note that both $Q < 0$ and $P < 0$ cannot hold simultaneously. $(1-\alpha_1)\varepsilon_3 > 0$, $i > 0$ and $P < 0$ together imply $\varepsilon_1(c_r - \alpha_1) + \alpha_1\varepsilon_2(1 - c_r)\} < 0$. Therefore $Q = (1-\alpha_1) - \{\varepsilon_1(c_r - \alpha_1) + \alpha_1\varepsilon_2(1-c_r)\} > 0$. But we already assumed $Q < 0$. So there is a logical contradiction. Hence both $Q < 0$ and $P < 0$ cannot hold simultaneously. As a result, we cannot get a scenario where $Q < 0$, $T > 0$, and $P < 0$ occur.

From the analysis of several possible cases we can infer the following remarks.

Table 4.1: *Summary of stability of the steady states*

| *Cases* | equilibria | Sign of the elements of $J$ | Nature of the steady state |
|---|---|---|---|
| *1a* | $A$ | $J_{11} < 0,\ J_{12} > 0,\ J_{21} < 0,\ J_{22} < 0$ | stable |
| | $B$ | $J_{11} > 0,\ J_{12} > 0,\ J_{21} < 0,\ J_{22} < 0$ | saddle point unstable |
| | $C$ | $J_{11} > 0,\ J_{12} > 0,\ J_{21} < 0,\ J_{22} < 0$ | stable/unstable/limit cycle |
| | $D$ | $J_{11} > 0,\ J_{12} > 0,\ J_{21} < 0,\ J_{22} < 0$ | saddle point unstable |
| *1b* | $A$ | $J_{11} < 0,\ J_{12} > 0,\ J_{21} > 0,\ J_{22} > 0$ | saddle point unstable |
| | $B$ | $J_{11} > 0,\ J_{12} > 0,\ J_{21} > 0,\ J_{22} > 0$ | unstable |
| | $C$ | $J_{11} > 0,\ J_{12} > 0,\ J_{21} > 0,\ J_{22} > 0$ | saddle point unstable |
| | $D$ | $J_{11} > 0,\ J_{12} > 0,\ J_{21} > 0,\ J_{22} > 0$ | saddle point unstable |
| *2* | $A$ | $J_{11} < 0,\ J_{12} > 0,\ J_{21} > 0,\ J_{22} < 0$ | stable |
| | $B$ | $J_{11} > 0,\ J_{12} > 0,\ J_{21} > 0,\ J_{22} < 0$ | saddle point unstable |
| | $C$ | $J_{11} < 0,\ J_{12} > 0,\ J_{21} > 0,\ J_{22} < 0$ | saddle point unstable |
| | $D$ | $J_{11} < 0,\ J_{12} > 0,\ J_{21} > 0,\ J_{22} < 0$ | saddle point unstable |
| *3* | $A$ | $J_{11} < 0,\ J_{12} > 0,\ J_{21} > 0,\ J_{22} < 0$ | saddle point unstable |
| | $B$ | $J_{11} > 0,\ J_{12} > 0,\ J_{21} > 0,\ J_{22} < 0$ | saddle point unstable |

**Remark 1.** *Given $d > \frac{l}{2h}$, as long as a steady state exists, whenever the slope of the $\dot{\phi} = 0$ isocline is greater than the slope of the $\dot{d} = 0$ isocline, a stable steady state cannot be achieved.*

**Remark 2.** *$d < \frac{l}{2h}$ is neither a necessary nor a sufficient for the steady state to be stable.*

Consider *case 1b* (see Figure 4.3a). Here, for point $A$, although $d < \frac{l}{2h}$ but point $A$ is not a stable steady state. Hence, $d < \frac{l}{2h}$ is not a sufficient condition for a stable steady state to exist. Now consider *case 1a* (see Figure 4.2b). Here, for point $C$, although $d > \frac{l}{2h}$ but point $C$ is a stable steady state (provided $\rho$ is sufficiently high). Hence, $d < \frac{l}{2h}$ is not a necessary condition for a steady state to be stable.

Note that $d = \frac{l}{2h}$ is the upper limit of the unconditional stability. So here, for a given $\phi$, this upper limit ($d = \frac{l}{2h}$) of the unconditional stability can be enhanced by higher level of animal spirits ($\alpha_0$) and lower level of interest rate ($i$).[17]

Table 4.1 summarizes the results of the stability related to various steady states.

In the next section we investigate the possibility of existence of Andronov-Hopf bifurcation and occurrence of cycle in the economy.

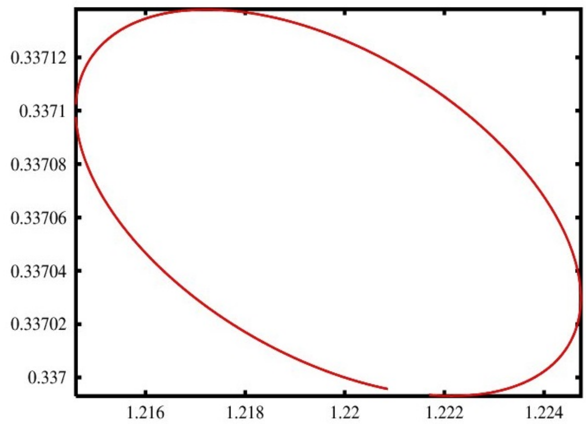
# 5 *Hopf Bifurcation*

Consider the steady state $C$ of *case 1a*.2.

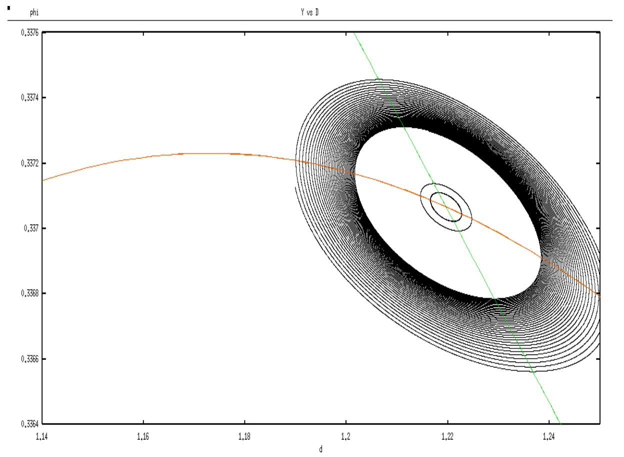
[17] As $\frac{\partial\left(\frac{l}{2h}\right)}{\partial \alpha_0} = \frac{1}{2(1-c_r)\alpha_1 i} = \frac{1}{2h} > 0$; and $\frac{\partial\left(\frac{l}{2h}\right)}{\partial i} = \frac{-\alpha_0+(1-c_r)\alpha_1\phi}{2(1-c_r)\alpha_1 i^2} < 0$.

**Proposition 2.** *For an appropriate value of the speed of adjustment parameter, $\rho$, the characteristic equation to (3.5) & (3.12) evaluated at the steady state $C$ of the case 1a has purely imaginary roots and for the same dynamical system, $\rho = \hat{\rho} = \frac{-\alpha_0+(1-c_r)(1-\alpha_1+2\alpha_1 d)i+(1-c_r)\alpha_1\phi}{(1-\alpha_1)-\varepsilon_1(c_r-\alpha_1)-\alpha_1\varepsilon_2(1-c_r)}$ provides a point of Hopf bifurcation.*

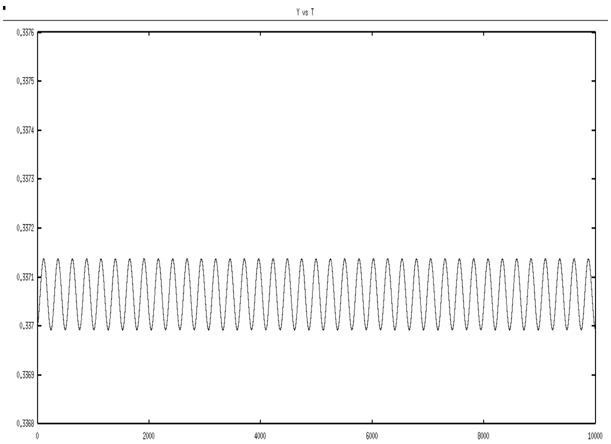
*Proof.* See Appendix A.1. □



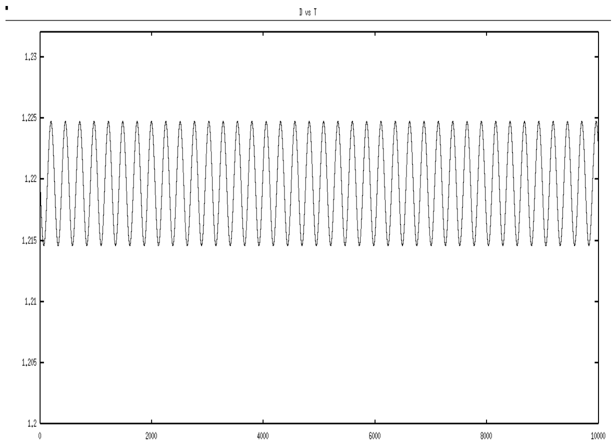

(a) *Cycle of debt-capital and dividend-capital ratio*



(b) *Solution paths in $(d, \phi)$ plane: an unstable limit cycle due to sub-critical Hopf bifurcation*



(c) *Transitional dynamics of dividend-capital ratio*



(d) *Transitional dynamics of debt-capital ratio*

Figure 5.1: *Diagram of limit cycle*

Note that the limit cycle occurs only in the strong debt-burdened demand regime. Using XPPAUT software, we find that the Hopf bifurcation is sub-critical in nature. In other words, an unstable limit cycle exists. We draw the solution path from $t = 0$ to $t = 10000$, and we find that the solution path is not a perfect closed orbit in the sense that for an initial condition close to the long-run equilibrium the solution path converges to the equilibrium whereas for the initial condition further away from the long-run equilibrium, the solution path diverges from the equilibrium (see Figure 5.1b). As a result, we confirm that in this numerical example, the sub-critical Hopf bifurcation occurs and the periodic solution is unstable. The primary objective of this numerical study is not to calibrate a real economy. Instead, the primary purpose is to confirm whether the model produces the limit cycle and to observe its basic properties. Therefore, we introduce the values so that we obtain economically meaningful outcomes. For the simulation we set $\alpha_0 = 0.28$, $\alpha_1 = 0.65$, $\varepsilon_0 = 0.15$, $\varepsilon_1 = 0.42$, $\varepsilon_2 = 0.003$, $\varepsilon_3 = 0.001$, $i = 0.132$, $c_r = 0.4$, $\hat{\rho} = 0.01047$. We get the equilibrium values of $d$ and $\phi$ as $d^* = 1.2196$ and $\phi^* = 0.33707$ for the steady state $C$ of the *case 1a*.[18] As illustrated in Figure 5.1a, for the initial values $d(0) = 1.22$ and $\phi(0) = 0.337$, a clockwise limit cycle emerges in the $(d, \phi)$-plane. Figure 5.1c shows the transitional dynamics of the dividend-capital ratio and Figure 5.1d shows the transitional dynamics of the debt-capital ratio.

In what follows, we explain the reason behind the occurrence of a limit cycle at steady state $C$ of *case 1a*. First, as $d > \frac{l}{2h}$, the self-feedback effect of the debt–capital ratio is positive, i.e. $J_{11} = \frac{\partial \dot{d}}{\partial d} = \frac{2hd-l}{(1-\alpha_1)} = \frac{M}{(1-\alpha_1)} > 0$. Besides, here the self-feedback effect of the dividend-capital ratio is negative, i.e. $J_{22} = \frac{\partial \dot{\phi}}{\partial \phi} = -\frac{\rho Q}{(1-\alpha)} < 0$. When the speed of adjustment parameter $\rho$ is small, the self-feedback effect of the debt-capital ratio dominates the self-feedback effect of the dividend–capital ratio and so the economy loses stability (As the trace becomes positive here). On the contrary, when the opposite happens, the economy achieves stability. Thus, limit cycle occurs in the boundary between the unstable and the stable feedback effect i.e. when $\rho$ reaches its critical value $\hat{\rho}$.

In the next section we investigate how various parameters influence the equilibrium values of debt-capital and the dividend-capital ratios.

# 6 *Comparative Statics*

As *case 1a* and *case 2* are the only cases with stable equilibria/equilibrium, for comparative statics we stick to only those two cases.[19]

Hein (2012b) says "In fact, with financialization, various mechanisms have been designed, on the one hand, to impose restrictions on management's ability to seek expansion, and, on the other hand, to change management's preferences themselves and align them to shareholders' profit maximization objective. Management's desire for growth is constrained through, in particular, higher dividend payouts demanded by shareholders, a

[18]Note that here equations (2.7), (2.8), (2.10), $d'' > 0$ i.e. $\alpha_0 > (1-c_r)(1-\alpha_1)i$, $l = [\alpha_0 - (1 - c_r)\{(1-\alpha_1)i + \alpha_1\phi\}] > 0$, $M > 0$, $N > 0$, $Q > 0$, $T > 0$, $P < 0$ all are satisfied.

[19]Depending on the size of the parameter $\rho$, steady state $C$ can be either stable or unstable or limit cycles can emerge. However, for comparative statics analysis we assume $C$ to be a stable steady state.

weaker ability of firms to obtain new equity finance through stock issues (which tend to decrease share prices), a larger dependence on leverage, and an increased threat of hostile takeovers in a liberalized market for corporate control. Simultaneously, financial market-oriented remuneration schemes have been developed to align management's preferences to shareholders' objectives." Therefore financialization is associated with a rise in $\varepsilon_0$, $\varepsilon_1$, and $\varepsilon_3$ and a fall in $\varepsilon_2$. We observe that there was an increase in interest rates and interest payments, especially in the 1980s (Hein, 2012b, pp. 26; also see Figures 1.3 and 1.2). From 1980 to 1989 interest payments rose from 43.78 per cent of profits to 61.2 per cent, reflecting a combination of both $(i)$ the composition of payments to capital and $(ii)$ the prevailing high interest rates of 1980s. However, thereafter the ratio has fallen drastically till 2005 where it came down to 20.37 per cent. This happened mainly due to the low interest rates policy of the Federal Reserve. Therefore, discussion of $\varepsilon_0$, $\varepsilon_1$, $\varepsilon_2$, $\varepsilon_3$ and $i-$these five parameters would be interesting. However, due to lack of space, we elaborately discuss only $\varepsilon_0$ and $i-$ these two parameters. Results of a change in $\varepsilon_1$ and $\varepsilon_3$ are similar to that of $\varepsilon_0$ and a change in $\varepsilon_2$ is opposite to that of a change in $\varepsilon_0$. Impact of all these parameters are summarized in Table 6.1.

The total differentiation of equations (3.6) and (3.12) shows the effects of parametric changes in the economy which imply

$$\begin{bmatrix} J_{11} & J_{12} \\ J_{21} & J_{22} \end{bmatrix} \begin{bmatrix} dd \\ d\phi \end{bmatrix} = \begin{bmatrix} 0 \\ -\rho \end{bmatrix} d\varepsilon_0 + \begin{bmatrix} 0 \\ -\rho r \end{bmatrix} d\varepsilon_1 + \begin{bmatrix} 0 \\ \rho g \end{bmatrix} d\varepsilon_2 + \begin{bmatrix} 0 \\ -\rho d \end{bmatrix} d\varepsilon_3 + \begin{bmatrix} -\frac{(1-c_r)(1-\alpha_1+\alpha_1 d)d}{(1-\alpha_1)} \\ -\frac{\rho\{\varepsilon_1(c_r-\alpha_1)+\alpha_1\varepsilon_2(1-c_r)\}d}{(1-\alpha_1)} \end{bmatrix} di \tag{6.1}$$

Frome equation (6.1) we get, $\frac{dd^*}{d\varepsilon_0} = \frac{\rho J_{12}}{(J_{11}J_{22}-J_{12}J_{21})}$, $\frac{d\phi^*}{d\varepsilon_0} = \frac{-\rho J_{11}}{(J_{11}J_{22}-J_{12}J_{21})}$; $\frac{dd^*}{d\varepsilon_1} = \frac{\rho r J_{12}}{(J_{11}J_{22}-J_{12}J_{21})}$, $\frac{d\phi^*}{d\varepsilon_1} = \frac{-\rho r J_{11}}{(J_{11}J_{22}-J_{12}J_{21})}$; $\frac{dd^*}{d\varepsilon_2} = \frac{-\rho g J_{12}}{(J_{11}J_{22}-J_{12}J_{21})}$, $\frac{d\phi^*}{d\varepsilon_2} = \frac{\rho g J_{11}}{(J_{11}J_{22}-J_{12}J_{21})}$; $\frac{dd^*}{d\varepsilon_3} = \frac{\rho d J_{12}}{(J_{11}J_{22}-J_{12}J_{21})}$, $\frac{d\phi^*}{d\varepsilon_3} = \frac{-\rho d J_{11}}{(J_{11}J_{22}-J_{12}J_{21})}$; $\frac{dd^*}{di} = \frac{-\frac{(1-c_r)(1-\alpha_1+\alpha_1 d)d}{(1-\alpha_1)}J_{22}+\frac{\rho\{\varepsilon_1(c_r-\alpha_1)+\alpha_1\varepsilon_2(1-c_r)\}d}{(1-\alpha_1)}J_{12}}{(J_{11}J_{22}-J_{12}J_{21})}$,

and $\frac{d\phi^*}{di} = \frac{-\frac{\rho\{\varepsilon_1(c_r-\alpha_1)+\alpha_1\varepsilon_2(1-c_r)\}d}{(1-\alpha_1)}J_{11}+\frac{(1-c_r)(1-\alpha_1+\alpha_1 d)d}{(1-\alpha_1)}J_{21}}{(J_{11}J_{22}-J_{12}J_{21})}$.

## 6.1 *Effect of a change in $\varepsilon_0$, the floor level of the targeted dividend-capital ratio*

***Case 1a:*** As illustrated in Figure 6.1a, for a rise in $\varepsilon_0$, $\dot{\phi} = 0$ isocline has a parallel upward shift.[20] However, there is no change in $\dot{d} = 0$ isocline (as $\frac{\partial\phi}{\partial\varepsilon_0}\Big|_{\dot{d}=0} = 0$). Thus,

[20]Partial differentiation of equation (3.17) (the intercept of the $\dot{\phi} = 0$ isocline) w.r.t. $\varepsilon_0$ yields $\frac{\partial}{\partial\varepsilon_0}\left(\phi\Big|_{\dot{\phi}=0}^{d=0}\right) = \frac{(1-\alpha_1)}{(1-\alpha_1)-\varepsilon_1(c_r-\alpha_1)-\alpha_1\varepsilon_2(1-c_r)} = \frac{(1-\alpha_1)}{Q} > 0$ ($\because Q > 0$). Slope of the $\dot{\phi} = 0$ isocline is

Table 6.1: *Summary of comparative statics results for change in various parameters*

(a) *Summary of comparative statics results for a change in* $\varepsilon_0$

| Case | steady state | Sign of elements of Jacobian Matrix | Effect on $\phi^*$ | Effect on $d^*$ | Effect on $g^*$ |
|---|---|---|---|---|---|
| *1a* | $A$ | $J_{11} < 0,\ J_{12} > 0,\ J_{21} < 0,\ J_{22} < 0$ | positive | positive | negative |
| | $C$ | $J_{11} > 0,\ J_{12} > 0,\ J_{21} < 0,\ J_{22} < 0$ | negative | positive | ambiguous |
| *2* | $A$ | $J_{11} < 0,\ J_{12} > 0,\ J_{21} > 0,\ J_{22} < 0$ | positive | positive | negative |

(b) *Summary of comparative statics results for a change in* $\varepsilon_1$

| Case | steady state | Sign of elements of Jacobian Matrix | Effect on $\phi^*$ | Effect on $d^*$ | Effect on $g^*$ |
|---|---|---|---|---|---|
| *1a* | $A$ | $J_{11} < 0,\ J_{12} > 0,\ J_{21} < 0,\ J_{22} < 0$ | positive | positive | negative |
| | $C$ | $J_{11} > 0,\ J_{12} > 0,\ J_{21} < 0,\ J_{22} < 0$ | negative | positive | ambiguous |
| *2* | $A$ | $J_{11} < 0,\ J_{12} > 0,\ J_{21} > 0,\ J_{22} < 0$ | positive | positive | negative |

(c) *Summary of comparative statics results for a change in* $\varepsilon_2$

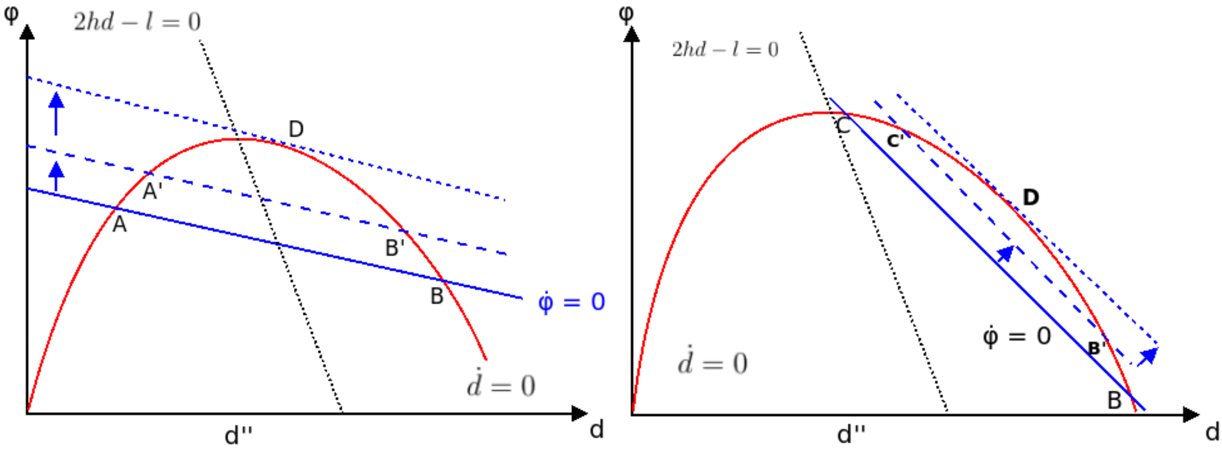
| Case | steady state | Sign of elements of Jacobian Matrix | Effect on $\phi^*$ | Effect on $d^*$ | Effect on $g^*$ |
|---|---|---|---|---|---|
| *1a* | $A$ | $J_{11} < 0,\ J_{12} > 0,\ J_{21} < 0,\ J_{22} < 0$ | negative | negative | positive |
| | $C$ | $J_{11} > 0,\ J_{12} > 0,\ J_{21} < 0,\ J_{22} < 0$ | positive | negative | ambiguous |
| *2* | $A$ | $J_{11} < 0,\ J_{12} > 0,\ J_{21} > 0,\ J_{22} < 0$ | negative | negative | positive |

(d) *Summary of comparative statics results for a change in* $\varepsilon_3$

| Case | steady state | Sign of elements of Jacobian Matrix | Effect on $\phi^*$ | Effect on $d^*$ | Effect on $g^*$ |
|---|---|---|---|---|---|
| *1a* | $A$ | $J_{11} < 0,\ J_{12} > 0,\ J_{21} < 0,\ J_{22} < 0$ | positive | positive | negative |
| | $C$ | $J_{11} > 0,\ J_{12} > 0,\ J_{21} < 0,\ J_{22} < 0$ | negative | positive | ambiguous |
| *2* | $A$ | $J_{11} < 0,\ J_{12} > 0,\ J_{21} > 0,\ J_{22} < 0$ | positive | positive | negative |

(e) *Summary of comparative statics results for a change in i*

| Case | steady state | Sign of elements of Jacobian Matrix | Effect on $\phi^*$ | Effect on $d^*$ | Effect on $g^*$ |
|---|---|---|---|---|---|
| *1a* | $A$ | $J_{11} < 0,\ J_{12} > 0,\ J_{21} < 0,\ J_{22} < 0$ | negative | ambiguous | ambiguous |
| | $C$ | $J_{11} > 0,\ J_{12} > 0,\ J_{21} < 0,\ J_{22} < 0$ | ambiguous | ambiguous | ambiguous |
| *2* | $A(\text{when } \nabla > 0)$ | $J_{11} < 0,\ J_{12} > 0,\ J_{21} > 0,\ J_{22} < 0$ | positive | positive | negative |
| | $A(\text{when } \nabla < 0)$ | $J_{11} < 0,\ J_{12} > 0,\ J_{21} > 0,\ J_{22} < 0$ | ambiguous | ambiguous | ambiguous |
| where $\nabla = \{\varepsilon_1(c_r - \alpha_1) + \alpha_1\varepsilon_2(1 - c_r)\}$ | | | | | |




(a) *case 1a.1* (b) *case 1a.2*

Figure 6.1: Effect of a rise in $\varepsilon_0$

as $\varepsilon_0$ increases, the stable steady state shifts from point $A$ to $A'$, and as a consequence, both $d^*$ and $\phi^*$ increase.

The economic intuition behind this rise in $d^*$ and $\phi^*$ is as follows. A rise in $\varepsilon_0$, *ceteris paribus*, raises the targeted dividend-capital ratio of firms and thereby pushes the $\dot{\phi} = 0$ isocline upwards. For a given $\phi$, at the old steady state $A$, the debt-capital ratio is lower than required for the new $\dot{\phi} = 0$ to be satisfied. This lower level of $d$ puts upward pressure on dividend-capital ratio through equation (3.13) (as $J_{21} < 0$ here). As a result, dividend-capital ratio ($\phi$) starts rising. As soon as $\phi$ rises, debt market deviates from its equilibrium position. Given the level of $d$, $\phi$ is now higher than required for $\dot{d} = 0$ to be satisfied. As $\frac{\partial \dot{d}}{\partial \phi} = J_{12} > 0$, debt-capital ratio must rise. Combination of higher level of debt-capital ratio and dividend-capital ratio ultimately ensures to achieve the new equilibrium point $A'$ either monotonically or spiraling around it.

At point $A$, there is an unambiguously negative effect of a rise in $\varepsilon_0$ on the long run equilibrium rate of capital accumulation, and is shown in the following equation as,

$$\frac{dg^*}{d\varepsilon_0} = \left\{ \overbrace{\frac{\partial g^*}{\partial d}}^{-} \overbrace{\frac{dd^*}{d\varepsilon_0}}^{+} + \overbrace{\frac{\partial g^*}{\partial \phi}}^{-} \overbrace{\frac{d\phi^*}{d\varepsilon_0}}^{+} \right\} < 0. \tag{6.2}$$

At point $A$, as $\varepsilon_0$ increases, $\phi^*$ rises which in turn reduces the equilibrium rate of capital accumulation (see equation (2.14)). On the other hand, a rise in $\varepsilon_0$ boosts $d^*$ which in turn decreases $g^*$ (see equation (2.14)). Hence, the final result of a rise in $\varepsilon_0$ on $g^*$ is unambiguously depressing.

$\left.\frac{d\phi}{dd}\right|_{\dot{\phi}=0} = \frac{P}{Q}$ which is invariant w.r.t. $\varepsilon_0$.

Note that for a sufficient rise in $\varepsilon_0$, two equilibria $A$ and $B$ converge to one equilibrium point $D$ which is a saddle point. Thus a sufficient rise in $\varepsilon_0$ may create instability in the economy in the sense that no stable steady state exists anymore.

At point $C$, as $\varepsilon_0$ increases, the stable steady state shifts from point $C$ to $C'$, and as a consequence, $d^*$ increases. In the era of financialization, one of the most important objective for firms is to make rentiers happy by providing them higher dividends. However, a rise in $\varepsilon_0$, instead of rising $\phi^*$ at $C$, decreases it (see Figure 6.1b). This seems to be counter-intuitive. The reason is as follows. As $\varepsilon_0$ increases, *ceteris paribus*, the targeted dividend-capital ratio of firms rises, and thereby, pushes the $\dot{\phi} = 0$ isocline upwards. For a given $\phi$, at the old steady state $C$, the debt-capital ratio is lower than required for $\dot{\phi} = 0$ to be satisfied. This lower level of $d$ puts pressure on $\phi$ through equation (3.13) (as $J_{21} < 0$ here). As a result, dividend-capital ratio starts rising initially. As soon as $\phi$ rises, debt market deviates from its equilibrium position. Given the level of $d$, $\phi$ is now higher than required for $\dot{d} = 0$ to be satisfied. As $\frac{\partial \dot{d}}{\partial \phi} = J_{12} > 0$, debt-capital ratio must rise. As the $\dot{d} = 0$ isocline is flatter than the $\dot{\phi} = 0$ isocline, the absolute value of $J_{12}$ is sufficiently high. Therefore, the debt-capital ratio rises by a significantly large amount. This rise in debt-capital ratio now leads to a decrease in the dividend-capital ration through equation (3.13). This fall in $\phi$ more than compensates the initial rise in $\phi$. Hence, because of a rise in $\varepsilon_0$, there is finally a fall in the dividend-capital ratio.

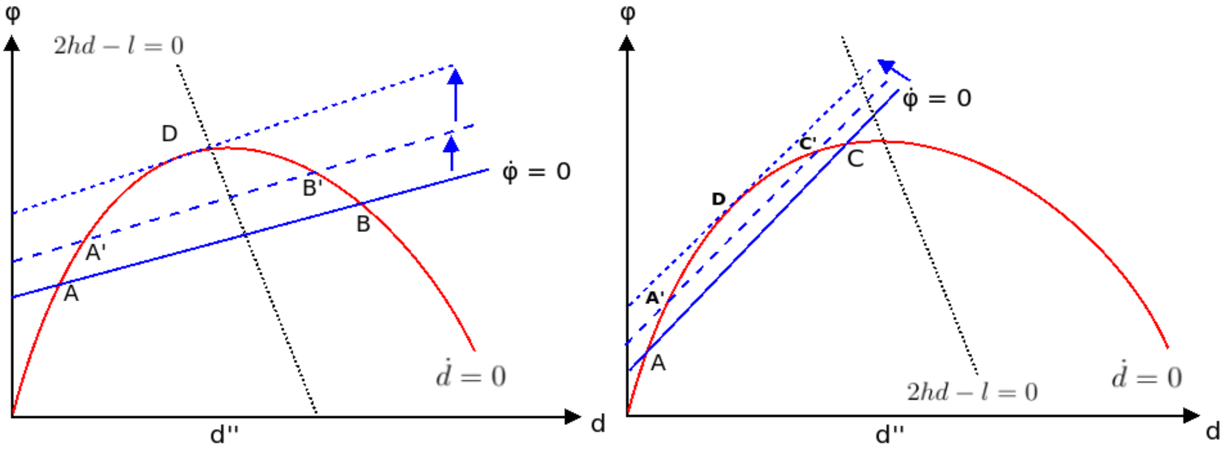
At point $C$, as $\varepsilon_0$ increases, the equilibrium value of $d^*$ increases which in turn reduces the equilibrium rate of capital accumulation. On the other hand a rise in $\varepsilon_0$ decreases $\phi^*$ and thus enhances $g^*$. Hence, the final result of a rise in $\varepsilon_0$ on $g^*$ is ambiguous (see equation 6.3).

$$\frac{dg^*}{d\varepsilon_0} = \left\{ \overbrace{\frac{\partial g^*}{\partial d}}^{-} \overbrace{\frac{dd^*}{d\varepsilon_0}}^{+} + \overbrace{\frac{\partial g^*}{\partial \phi}}^{-} \overbrace{\frac{d\phi^*}{d\varepsilon_0}}^{-} \right\} \gtreqless 0 \tag{6.3}$$

Note that for a sufficient rise in $\varepsilon_0$, two equilibria $C$ and $B$ converge to a unique saddle point equilibrium $D$. Thus for a sufficiently high rise in $\varepsilon_0$, the economy moves from a stable steady state to a situation in which no stable steady state exists.

***Case 2:*** Here, for a rise in $\varepsilon_0$, there is a parallel upward shift in $\dot{\phi} = 0$ isocline, but there is no shift in $\dot{d} = 0$ isocline.[21] As $\varepsilon_0$ increases, the economy shifts from point $A$ to a new equilibrium point $A'$ and as a result both $d^*$ and $\phi^*$ increase. Note that for a sufficiently large rise in $\varepsilon_0$, two equilibria $A$ and $B$ (or $A$ and $C$) converge to a unique saddle point steady state $D$. Figure 6.2a (6.2b) represents this diagrammatically.

[21] As $\frac{\partial}{\partial \varepsilon_0}\left(\phi\Big|_{\dot{\phi}=0}^{d=0}\right) > 0$, and slope of the $\dot{\phi} = 0$ isocline is invariant w.r.t. $\varepsilon_0$. Moreover, $\frac{\partial \phi}{\partial \varepsilon_0}\Big|_{\dot{d}=0} = 0$.




(a) *case 2.1* (b) *case 2.2*

Figure 6.2: Effect of a rise in $\varepsilon_0$

At point $A$, there is an unambiguously negative effect of a rise in $\varepsilon_0$ on the long run equilibrium rate of capital accumulation (see equation (6.4)). As $\varepsilon_0$ increases, $\phi^*$ rises which in turn reduces the equilibrium rate of capital accumulation. On the other hand, a rise in $\varepsilon_0$ boosts $d^*$ which in turn decreases $g^*$. Hence, the final result of a change in $\varepsilon_0$ on $g^*$ is negative.

$$\frac{dg^*}{d\varepsilon_0} = \left\{ \overbrace{\frac{\partial g^*}{\partial d}}^{-} \overbrace{\frac{dd^*}{d\varepsilon_0}}^{+} + \overbrace{\frac{\partial g^*}{\partial \phi}}^{-} \overbrace{\frac{d\phi^*}{d\varepsilon_0}}^{+} \right\} < 0 \tag{6.4}$$

## 6.2 *Effect of a change in the interest rate $i$*

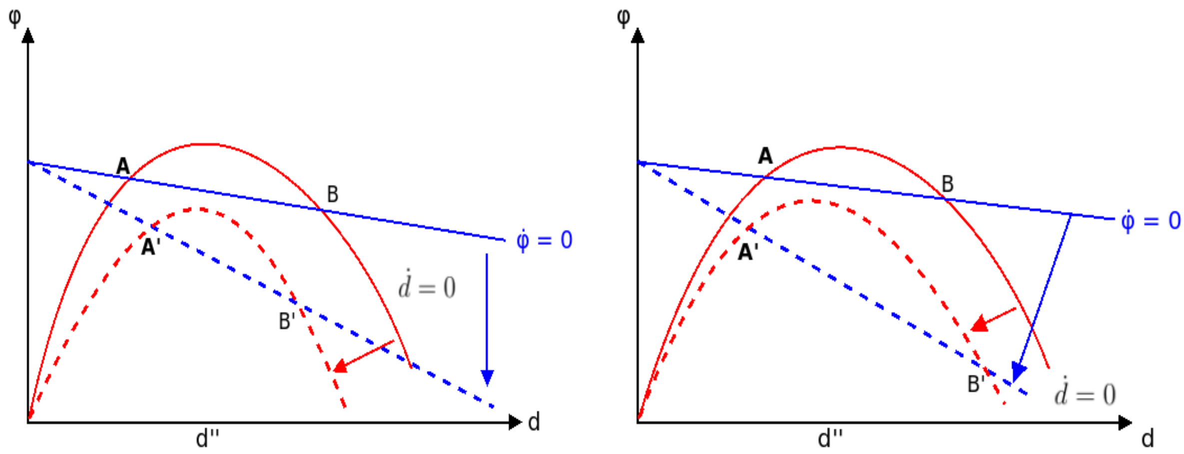
***Case 1a:*** Here $P < 0$. As $(1-\alpha_1)\varepsilon_3 > 0$, $P < 0$ implies $\{\varepsilon_1(c_r - \alpha_1) + \alpha_1\varepsilon_2(1-c_r)\} < 0$. At $A$, $\frac{\partial d}{\partial i} \gtreqless 0$ and $\frac{\partial \phi}{\partial i} < 0$. On the other hand, at $C$, $\frac{\partial d}{\partial i} \gtreqless 0$ and $\frac{\partial \phi}{\partial i} \gtreqless 0$.

Note that *ceteris paribus,* higher the value of $d$, for a same unit rise in $i$, more will be the fall in $\phi$. Hence, a rise in $i$ makes the $\dot{\phi} = 0$ isocline steeper.[22] On the other hand, *ceteris paribus,* higher the value of $d$, for a rise in $i$, more will be the fall in $\phi$ for the $\dot{\phi} = 0$ isocline. Therefore, as we move towards the right hand side in the horizontal axis, for a rise in $i$, more will be the gap between old and new $\dot{d} = 0$ isocline.[23]

Consider point $A$. Here, due to a rise in the interest rate, the equilibrium dividend-capital ratio ($\phi^*$) decreases while the effect of a rise in the interest rate on $d^*$ is ambiguous (see Figures 6.3a and 6.3b).

[22]Partial differentiation of equation (3.16) w.r.t. $i$ yields $\frac{\partial \phi}{\partial i}\Big|_{\dot{\phi}=0} = \frac{\{\varepsilon_1(c_r-\alpha_1)-\alpha_1\varepsilon_2(1-c_r)\}d}{(1-\alpha_1)-\varepsilon_1(c_r-\alpha_1)-\alpha_1\varepsilon_2(1-c_r)} < 0$ (as in *case 1a* $\{\varepsilon_1(c_r-\alpha_1) - \alpha_1\varepsilon_2(1-c_r)\} < 0$).

[23]Partial differentiation of equation (3.2) w.r.t. $i$ yields $\frac{\partial \phi}{\partial i}\Big|_{\dot{d}=0} = -d < 0$.




(a) Effect of a rise in $i$ (here $\phi^*$ decreases but $d^*$ rises)

(b) Effect of a rise in $i$ (here $\phi^*$ and $d^*$ both decrease)

Figure 6.3: Effect of a rise in $i$

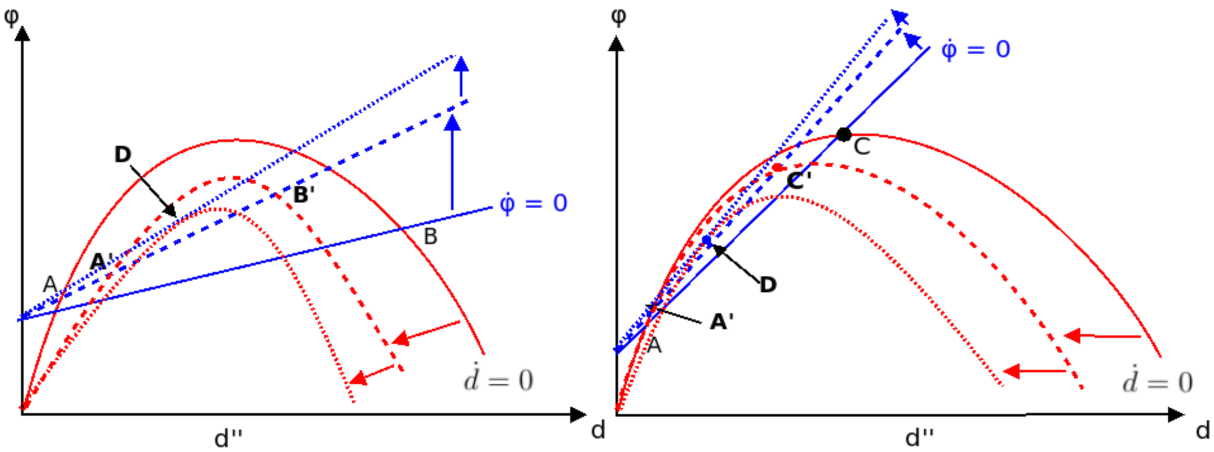
At point $A$, there is an ambiguous effect of a rise in $i$ on the long run equilibrium rate of capital accumulation (see equation (6.5)). The reason is as follows. As $i$ increases, the equilibrium value of $d$ may increase or decrease and so the effect of a rise in $i$ on $g^*$ through its effect on $d$ is ambiguous. On the other hand, a rise in $i$ boosts $\phi$ which in turn decreases $g^*$. Finally, a rise in the interest rate directly decreases $g^*$. Hence the final result is ambiguous.

$$\frac{dg^*}{di} = \left\{ \overbrace{\frac{\partial g^*}{\partial d}}^{-} \overbrace{\frac{dd^*}{di}}^{+/-} + \overbrace{\frac{\partial g^*}{\partial \phi}}^{-} \overbrace{\frac{d\phi^*}{di}}^{+} + \overbrace{\frac{\partial g^*}{\partial i}}^{-} \right\} \gtreqless 0 \tag{6.5}$$

At point $C$, there is an ambiguous effect of a rise in $i$ on the long run equilibrium rate of capital accumulation (see equation (6.6)). As $i$ increases, the equilibrium value of $d$ may increase or decrease and so the effect of a rise in $i$ on $g^*$ through its effect on $d$ is ambiguous. So is the case for a rise in $i$ on $g^*$ through its effect on $\phi$. However, a rise in the interest rate directly decreases $g^*$. Hence the final result is ambiguous.

$$\frac{dg^*}{di} = \left\{ \overbrace{\frac{\partial g^*}{\partial d}}^{-} \overbrace{\frac{dd^*}{di}}^{+/-} + \overbrace{\frac{\partial g^*}{\partial \phi}}^{-} \overbrace{\frac{d\phi^*}{di}}^{+/-} + \overbrace{\frac{\partial g^*}{\partial i}}^{-} \right\} \gtreqless 0 \tag{6.6}$$

***Case 2:*** Here $Q > 0$, $P > 0$, and $N > 0$. Here the absolute value of $\{\varepsilon_1(c_r - \alpha_1) + \alpha_1\varepsilon_2(1 - c_r)\}$ is small. However, the sign of $\{\varepsilon_1(c_r - \alpha_1) + \alpha_1\varepsilon_2(1 - c_r)\}$ can be anything: either negative or positive. When $\{\varepsilon_1(c_r - \alpha_1) + \alpha_1\varepsilon_2(1 - c_r)\} > 0$, at point $A$ we get, $\frac{\partial d}{\partial i} > 0$ and $\frac{\partial \phi}{\partial i} > 0$. However, if $\{\varepsilon_1(c_r - \alpha_1) + \alpha_1\varepsilon_2(1 - c_r)\} < 0$, at point $A$ we get $\frac{\partial d}{\partial i} \gtreqless 0$




(a) *case 2.1* (b) *case 2.2*

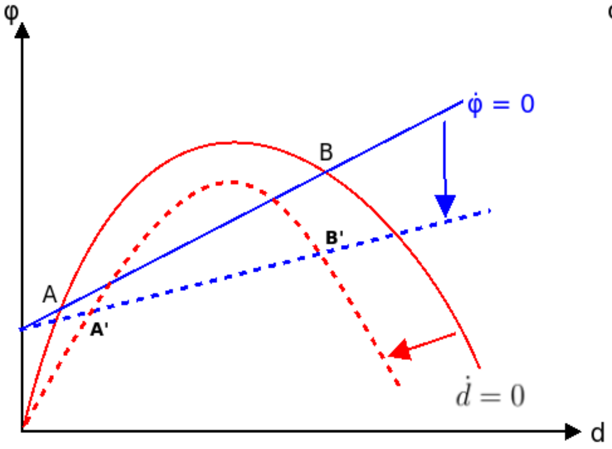
Figure 6.4: Effect of a rise in $i$ when $\{\varepsilon_1(c_r-\alpha_1)+\alpha_1\varepsilon_2(1-c_r)\}>0$

and $\frac{\partial\phi}{\partial i}\gtreqless 0$. If we consider $\{\varepsilon_1(c_r-\alpha_1)+\alpha_1\varepsilon_2(1-c_r)\}>0$, $\left.\frac{\partial\phi}{\partial i}\right|_{\dot\phi=0}>0$ and its magnitude depends on $d$. Hence the $\dot\phi=0$ isocline becomes steeper (see Figures 6.4a and 6.4b). However, if $\{\varepsilon_1(c_r-\alpha_1)+\alpha_1\varepsilon_2(1-c_r)\}<0$, $\left.\frac{\partial\phi}{\partial i}\right|_{\dot\phi=0}=\frac{\{\varepsilon_1(c_r-\alpha_1)-\alpha_1\varepsilon_2(1-c_r)\}d}{(1-\alpha_1)-\varepsilon_1(c_r-\alpha_1)-\alpha_1\varepsilon_2(1-c_r)}<0$. *Ceteris paribus,* higher the value of $d$, for a rise in $i$, more will be the fall in $\phi$. Hence the $\dot\phi=0$ isocline becomes flatter (see Figures 6.5a, 6.5b, and 6.5c). Partial differentiation of equation (3.2) w.r.t. $i$ yields $\left.\frac{\partial\phi}{\partial i}\right|_{\dot d=0}=-d<0$. Here too, *ceteris paribus,* higher the value of $d$, for a rise in $i$, more will be the fall in $\phi$. Therefore, as we move towards the right hand side in the horizontal axis, for a rise in $i$, more will be the gap between the initial $\dot d=0$ isocline and the new $\dot d=0$ isocline.

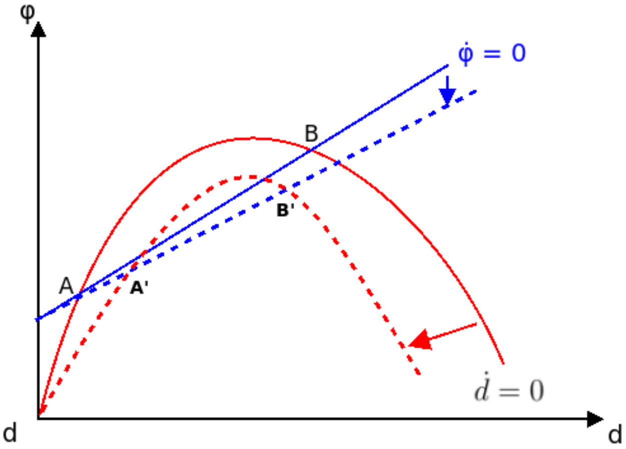
First, we consider $\{\varepsilon_1(c_r-\alpha_1)+\alpha_1\varepsilon_2(1-c_r)\}>0$. Here, at point $A$, due to a rise in $i$, $\phi^*$ and $d^*$ both increase (see Figure 6.4a). Note that for a large increment in $i$, the stable equilibrium may loose its stability and converge to a saddle-point unstable steady state $D$. We get an identical analysis for Figure 6.4b as well. At point $A$, there is an unambiguous negative effect of a rise in $i$ on the long run equilibrium rate of capital accumulation (see equation (6.7)). The reason is as follows. As $i$ increases, the equilibrium value of $d$ rises and so $g^*$ falls. On the other hand, a rise in $i$ boosts $\phi$ which in turn decreases $g^*$. Finally, a rise in the interest rate directly decreases $g^*$. Hence the final result is unambiguously negative.

$$\frac{dg^*}{di}=\left\{\overbrace{\frac{\partial g^*}{\partial d}}^{-}\overbrace{\frac{dd^*}{di}}^{+}+\overbrace{\frac{\partial g^*}{\partial\phi}}^{-}\overbrace{\frac{d\phi^*}{di}}^{+}+\overbrace{\frac{\partial g^*}{\partial i}}^{-}\right\}<0. \tag{6.7}$$

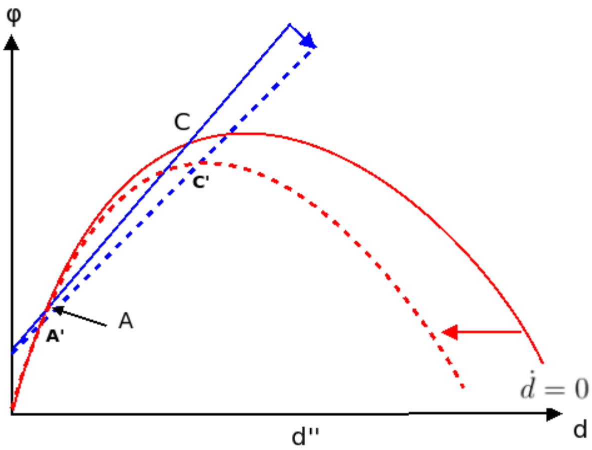
Now consider $\{\varepsilon_1(c_r-\alpha_1)+\alpha_1\varepsilon_2(1-c_r)\}<0$. Here, because of a rise in $i$, $\phi^*$ and $d^*$ are affected ambiguously (see Figures 6.5a, 6.5b, and 6.5c). Here, at point $A$, there is an ambiguous effect of a rise in $i$ on the long run equilibrium rate of capital accumulation.




(a) *case 2.1 (when $d^*$ rises but $\phi^*$ falls)*




(b) *case 2.1 (when both $d^*$ and $\phi^*$ rise)*




(c) *case 2.2 (here $d^*$ and $\phi^*$ both fall)*

Figure 6.5: Effect of a rise in $i$ when $\{\varepsilon_1(c_r - \alpha_1) + \alpha_1\varepsilon_2(1 - c_r)\} < 0$

The following equation shows this.

$$\frac{dg^*}{di} = \left\{ \overbrace{\frac{\partial g^*}{\partial d}}^{-} \overbrace{\frac{dd^*}{di}}^{+/-} + \overbrace{\frac{\partial g^*}{\partial \phi}}^{-} \overbrace{\frac{d\phi^*}{di}}^{+/-} + \overbrace{\frac{\partial g^*}{\partial i}}^{-} \right\} \gtreqless 0. \tag{6.8}$$

# 7 Conclusion

In this paper we dealt with a neo-Kaleckian growth model in which in the long run, the dividend-capital ratio of rentiers and the debt-capital ratio of firms evolve endogenously. We examined the short-run stability condition and analysed some comparative statics. We proposed a different way of distributing dividends to the rentiers by firms than the conventional neo-Kaleckian literature and provided economic rationale for it. We reached to the conclusion that the economy is in a wage-led demand and debt-burdened growth regime. However, the demand regime (in terms of debt) can be both debt-burdened or debt-led. We also found that the equilibrium rate of capital accumulation is invariant with respect to the change in income distribution. We found that an increase in dividend payment to the rentiers is not necessarily contractionary for the aggregate demand of the economy. Rather, if rentiers' consumption out of a rise in dividend is sufficiently high, it can overcompensate the loss in investment demand due to a reduction in the available internal funds of firms. However, an increase in dividend payment to the rentiers is always contractionary for the equilibrium rate of capital accumulation of the economy. Same result is achieved for a rise in the interest rate.

In the long run, we endogenized the dividend and debt dynamics and investigated the possibility of instability in the economy. Note that in Charles (2008a), while a low level of interest rate exhibits an unique stable equilibrium, a high value of it (the interest rate) is a necessary precondition for the existence of multiple equilibria and the occurrence of financial fragility of the system. However, such condition is not essential for the existence of multiple equilibria in our model.

We found that a lower value of debt-capital ratio is neither a necessary nor a sufficient condition for the system to be stable. In a particular scenario (at point $C$ of *case 1a*), the speed of adjustment parameter related to the dividend dynamics plays a crucial role for stabilizing the economy. If the speed of adjustment parameter falls to a critical level, the economy loses its stability and gives birth to the limit cycles. There is an unstable self-feedback effect of the debt-capital ratio near point $C$ (of *case 1a*). But, the self-feedback effect of the dividend-capital ratio is stable here. When the speed of adjustment parameter of the dividend dynamics ($\rho$) is sufficiently high (i.e. $\rho > \hat{\rho} = \frac{M}{Q}$), the stable self-feedback effect of the dividend-capital ratio becomes strong and dominates the unstable self-feedback effect of the debt-capital ratio. Hence the economy becomes stable. On the contrary, when the opposite happens, the economy loses its stability. Thus, limit cycle occurs in the boundary between the unstable and the stable feedback effect i.e. when $\rho$ reaches its critical value $\hat{\rho}$.

It is to be noted that because of financialization $\varepsilon_0$, $\varepsilon_1$ and $\varepsilon_3$ rise while $\varepsilon_2$ falls in the long run. Any of them if changes significantly, under certain circumstances, can lead to instability in the economy. We also found that financialization may be associated with a rise in the interest rate, a significant rise of which, under a certain conditions, may cause instability in the economy. In that sense our model provides an explanation for the emergence of financial crisis in the economy.

Needless to say, our model has a few limitations as well. First, in our model, for simplicity we assumed away the possibility of issuance of new equities by firms. Second, we assumed that there is no borrowing by the workers' households. This is a very strong assumption, particularly in the context of the US economy where workers' borrowing played a crucial role for the emergence of financial fragility and crisis. Third, banks have played a passive role in our model. Fourth, our model is that of a closed economy where there is no role of the government. These issues are, however, left for future research.

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# A Appendix

## A.1 Proof of Proposition 2

*Proof.* The characteristic equation to (3.5) & (3.12) is $\{\mu^2 + (-tr(J))\mu + Det(J) = 0\}$. A necessary condition of the Hopf bifurcation for complex roots is Det(J) $> 0$, which is satisfied at $C$ of *case 1a.2*. The trace of the Jacobian matrix can be made either positive or negative by appropriately selecting the value of $\rho$ while leaving the other parameters constant. To see this, notice that $\text{tr(J)} = J_{11} + J_{22} = \frac{2hd-l}{(1-\alpha_1)} - \rho\left[\frac{\{(1-\alpha_1)-\varepsilon_1(c_r-\alpha_1)-\alpha_1\varepsilon_2(1-c_r)\}}{(1-\alpha_1)}\right]$. Hence when $\rho = \hat{\rho} = \frac{M}{Q} = \frac{-\alpha_0+(1-c_r)(1-\alpha_1+2\alpha_1 d)i+(1-c_r)\alpha_1\phi}{(1-\alpha_1)-\varepsilon_1(c_r-\alpha_1)-\alpha_1\varepsilon_2(1-c_r)} > 0 \ \ (\because Q > 0,\ M > 0)$, the following equation holds exactly:

$$\text{tr(J)} = 2 * \text{Re}(\mu) = \frac{2hd-l}{(1-\alpha_1)} - \rho\left[\frac{\{(1-\alpha_1)-\varepsilon_1(c_r-\alpha_1)-\alpha_1\varepsilon_2(1-c_r)\}}{(1-\alpha_1)}\right] = 0$$

where tr(J) is the trace of $J$ and Re($\mu$) is the real part of its characteristic roots. As the determinant of the Jacobian matrix is positive, the product of the roots is positive in a neighborhood of the equilibrium, assuring Im($\mu$) $\neq 0$. Now differentiating the trace of the Jacobian matrix with respect to $\rho$ and then evaluating it at $\rho = \hat{\rho}$ we get

$$\left.\frac{\partial(\frac{\text{tr(J)}}{2})}{\partial\rho}\right|_{\rho=\hat{\rho}} = -\frac{Q}{2(1-\alpha_1)} = -\frac{(1-\alpha_1)-\varepsilon_1(c_r-\alpha_1)-\alpha_1\varepsilon_2(1-c_r)}{2(1-\alpha_1)} < 0 \ (\because Q > 0)$$

So the trace is smooth, differentiable and monotonically decreasing in the speed of adjustment parameter, $\rho$. The trace disappears at $\rho = \hat{\rho}$. Also note that $tr(J) \gtreqless 0 \iff \rho \lesseqgtr \hat{\rho}$. From the preceding discussion, all conditions for Hopf bifurcation are satisfied at $\rho = \hat{\rho}$.[24] □

[24] The method of the proof is based on Gandolfo (1997).