Fiscal Expansion, Government Debt and Economic Growth: A Post-Keynesian Perspective

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3 September 2020

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September 3, 2020

Abstract

Constructing a post-Keynesian growth model, we try to explore how the interaction between capital accumulation and government debt opens up the possibility of multiple equilibria and instability in the economy. We investigate the impact of various parameters such as different tax rates, savings propensities, interest rate etc. on the short run aggregate demand and long run equilibrium growth rate and fiscal debt-capital ratio. We explore the relationship between a progressive tax system and wage-led demand regime. We show that when there is fiscal deficit and government incurs debt, a sufficiently high government expenditure to GDP ratio is essential for achieving stability in the system. Moreover, in certain case, a low speed of adjustment of the rate of capital accumulation is required, as otherwise, the economy may lose its stability and produces the limit cycles. In case of a moderate level of a fiscal expenditure to GDP ratio, when Keynesian stability condition is satisfied, a lower rate of interest and a higher autonomous investment demand are desirable as they enhance the stable region of the economy.

Keywords: *Government deficits and debt, Post-Keynesian, Instability, Limit cycle, Growth model*

JEL codes: *C62, E12, E32, E62, O41.*

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1 Introduction

Most of the mainstream economists possess the view that expansionary fiscal policy is ineffective in stimulating the economy. According to Keefer and Knack (2007), an increase in government investment expenditure in countries with low quality of governance will have either no or little impact on growth. Cameron (1982) and Landau (1983) are in the opinion that fiscal expenditure and taxation crowd out private investment in physical and human capital and hence negatively influence the economy. Cameron (1982, pp. 51) explores that an increase in public spending to GDP ratio by 1 percentage point between the 1960s and late 1970s caused a fall in the growth rate in the late 1970s by 0.05 percentage point. Koskela and Viren (2000) suggest that a rise in government demand for labour through its positive impact on real wages crowds out private employment and output.

On the other hand, post-Keynesian economists are in the opinion that expansionary fiscal policy plays a crucial role in enhancing the aggregate demand and economic growth. You and Dutt (1996) were the first to analyze the impact of government expenditure on growth formally. According to them, the fiscal expansion has a significant effect on the government debt-capital ratio, income distribution and economic growth. The fiscal expansion has a positive effect on the economic growth in the short-run. A rise in fiscal expansion leads to a rise in aggregate demand which in turn raises the growth rate. Nevertheless, fiscal expansion has an ambiguous effect on the growth rate in the long run. The reason behind it is that while fiscal expansion raises the growth rate through an increase in aggregate demand and the degree of capacity utilization, it has an ambiguous effect on the government debt-capital ratio. Government debt-capital ratio, on the other hand, has a positive effect on growth rate. If because of a rise in fiscal expansion, the government debt-capital ratio rises, fiscal expansion has an unambiguously positive effect on the growth rate. However, for a rise in the fiscal expansion, if government debt-capital ratio falls, government expenditure in that scenario ambiguously affects the growth rate.

Considering a Kaleckian framework, Commendatore and Pinto (2011), explore the effect of different kinds of government expenditures on capacity utilization and growth. Government consumption expenditure, as they say, through its effect on effective demand increases the equilibrium capacity utilization rate, which in turn increases the equilibrium rate of capital accumulation. However, fiscal investment expenditure influences the capacity utilization rate in three ways. It increases the effective demand. Second, it crowds-in private investment. Third, fiscal investment expenditure leads to a rise in capital productivity through the rise in potential output-capital ratio. However, capital productivity itself has a negative effect on the equilibrium capacity utilization rate. Therefore, the final effect of an increase in government investment expenditure on the equilibrium capacity utilization rate and capital accumulation rate is ambiguous. However, the physical capital to output ratio is nearly constant. It is one of Kaldor's (1963) stylized facts. Moreover, public investment expenditure like expenditure on streets and highways, electricity, gas and water supply can enhance labour productivity as well. However, this is missing in Commendatore and Pinto (2011).

Impact of different kinds of government expenditure on capacity utilization and growth can be found in Commendatore *et al.* (2009), Commendatore *et al.* (2011), and Dutt (2013) as well. On the other hand, the impact of fiscal policy on government deficit, the sustainability of public debt, and the stabilization of the economy can be found in Arestis and Sawyer (2003), Setterfield (2007), Palley (2013), Skott (2016), Hein (2018), and Ribeiro and Lima (2018).

Considering a Kaleckian growth model with positive saving propensity out of wages, Parui (2020) investigates the impact of different kinds of government expenditures on aggregate demand and economic growth. He considers two types of government expenditure: consumption and investment expenditure. Certain kind of government investment expenditure influences labour productivity, which in turn affects the income distribution. In a profit-led demand regime, when there is a balanced budget, a shift in government expenditure from consumption to investment purposes causes a rise in both aggregate demand and economic growth. However, the result in the wage-led demand regime is ambivalent. Once the balanced budget assumption is relaxed, although a rise in public investment expenditure may decrease aggregate demand and growth in the wage-led demand regime, it unambiguously raises both aggregate demand and growth rate in a profit-led demand regime.

However, in this paper, our objective is not to investigate which kind of government expenditure is more effective in enhancing aggregate demand and economic growth. Instead, considering a post-Keynesian framework, we try to investigate how the interaction between capital accumulation and government debt opens up the possibility of multiple equilibria and instability in the economy. We also examine the impact of various parameters such as different tax rates, savings propensities, interest rate etc. on the short run aggregate demand and long run growth rate. We show that in case of a balanced budget with no government debt, a more progressive tax system is essential for achieving a profit-led demand regime, whereas a regressive tax system, *ceteris paribus*, suffices the wage-led demand regime. On the contrary, when there are fiscal deficit and government incurs in debt, a more regressive tax system makes the economy more likely to be in a profit-led demand regime. When the government runs in deficit and incurs a debt, a sufficiently high fiscal expenditure to GDP ratio is essential in the long run for achieving stability in the system. Moreover, when Keynesian stability condition is satisfied, for a moderate level of fiscal expenditure to GDP ratio, a rise in the interest has a destabilizing effect on the macroeconomic trajectory. A lower rate of interest and a higher autonomous investment demand are desirable as they enhance the stable region of the economy.

The outline of the rest of the paper is as follows. Section 2 sets up the basic framework. In Section 3, we consider a balanced budget with no government debt and explore short and long run impact of various parameters on aggregate demand and economic growth. The balanced budget assumption is relaxed in Section 4. Here we assume government runs in deficit and incurs debt. We explain different possible cases which may arise due to the interaction between government debt and the capital accumulation dynamics. We examine the dynamic stability of the economy. This is followed by the discussion of some comparative statics. In Section 5, we (again) consider balanced budget assumption with some past government debt and investigate long run stability and comparative statics. Section 6 offers some concluding remarks.

2 The model

A simple one-sector, closed economy, post-Keynesian growth model is assumed in which the labour supply is constant. There is no technological change in the economy. The economy consists of two classes: workers and capitalists. While workers save a fraction s_W of their wage income, capitalist's saving propensity is s_P. We assume workers have lower savings propensity (s_W) than that of capitalists. Income is distributed between profits and wages in the following way

$$pY = WL + rpK \tag{2.1}$$

where Y is real income, p is price level, L is total amount of labour employment, W is nominal wage rate, r is the real rate of profit, and K is the existing capital stock. There is excess supply of labour and no depreciation of capital in the economy. The production function is of Leontief type i.e.

$$Y = \min\{aL, bK\} = aL, \quad b = \frac{Y^P}{K} > \frac{Y}{K} \tag{2.2}$$

where, Y^P is the potential output level. So the actual output is below the potential output level. The market is oligopolistic in nature where price is determined by mark-up on prime cost. For simplicity we assume away cost of raw materials and overhead cost and assume the only cost to be the labour cost. Therefore, price is given as

$$p = (1+\lambda)\frac{WL}{Y} = (1+\lambda)\frac{W}{a} \tag{2.3}$$

where, λ is the rate of mark-up and $a = \frac{Y}{L}$ is labour productivity. $\frac{WL}{pY} = \frac{w}{a}$ represents the wage share and the profit share is $\pi = (1 - \frac{w}{a})$.

In the next section we assume that the government budget is balanced and investigate the impact of various parameters on aggregate demand and economic growth from the short run as well as long run perspective. This balanced budget assumption is dropped in Section 4. We explore the possibility of multiple equilibria and instability due to the the interaction between government debt dynamics and the capital accumulation dynamics in this section (i.e. in Section 4).

3 Balanced Budget Scenario

We assume that government expenditure is proportional to the aggregate real income i.e. $G = \theta Y$, where θ represents government expenditure-output ratio. Government raises revenue through an income tax. The aggregate government tax revenue is given as,

$$T = t_P P + t_W W = [(t_P - t_W)\pi u + t_W u]\,K \tag{3.1}$$

where, t_P and t_W are the tax rates imposed on capitalist and workers respectively. In this section we assume that the government budget is balanced and there is no existing government debt. Therefore, here

$$G = T \tag{3.2}$$

The aggregate private savings in the economy is,

$$S = (1 - t_P)s_P P + s_W(1 - t_W)W = [\{(1 - t_P)s_P - (1 - t_W)s_W\}\,\pi + s_W(1 - t_W)]\,uK \tag{3.3}$$

We assume that the investment demand is exogenously given in the short run as

$$I = gK \tag{3.4}$$

where g is the investment rate (or the ratio of investment demand to the existing capital stock).

In the short run, capital stock and the investment demand are given, and the output level adjusts to clear the goods market. In the short run equilibrium, the following equation must be satisfied,

$$\frac{S}{K} + \frac{T}{K} = \frac{I}{K} + \frac{G}{K} \tag{3.5}$$

$$\implies \frac{S}{K} = \frac{I}{K} \quad (\because G = T)$$

$$\implies u^* = \frac{g}{[\{(1-t_P)s_P - (1-t_W)s_W\}\pi + s_W(1-t_W)]} = \frac{g}{\Psi} \tag{3.6}$$

where u^* is the equilibrium degree of capacity utilization.[1] The denominator in equation (3.6) is positive so that u^* becomes positive, i.e. we assume

$$\Psi = [\{(1-t_P)s_P - (1-t_W)s_W\}\pi + s_W(1-t_W)] > 0 \tag{3.7}$$

The short run comparative statics are summarized as follows:

$$\frac{du^*}{dg} = \frac{1}{\Psi} > 0, \quad \frac{du^*}{ds_P} = \frac{-(1-t_P)\pi u^*}{\Psi} < 0, \quad \frac{du^*}{dt_P} = \frac{s_P\pi u^*}{\Psi} > 0,$$

$$\frac{du^*}{ds_W} = \frac{-(1-t_W)(1-\pi)u^*}{\Psi} < 0, \quad \frac{du^*}{dt_W} = \frac{s_W(1-\pi)u^*}{\Psi} > 0.$$

A unit rise in investment demand raises the aggregate demand by one unit and hence the equilibrium degree of capacity utilization by $\frac{1}{\Psi}$ unit. A unit rise in the savings propensity of the capitalist (s_P) increase the aggregate private savings by $(1-t_P)\pi uK$ unit. This leakage leads to a fall in aggregate demand and hence the equilibrium degree of capacity utilization (i.e. $\frac{du^*}{ds_P} < 0$). Similarly, a rise in s_W increases the aggregate savings by $(1-t_W)(1-\pi)uK$ unit. Therefore, u^* falls by $\frac{(1-t_W)(1-\pi)u^*}{\Psi}$ unit.

Both the tax rates have positive impact on the equilibrium degree of capacity utilization. This is mainly because of the balanced budget assumption. Per unit increase in tax rate on capitalists (t_P) reduces consumption of capitalists by $(1-s_P)\pi uK$ unit. But this increase in the tax rate increase the tax revenue by πuK unit. As this rise in the tax revenue is entirely spent by the government, the aggregate demand increases by πuK unit. As the increase in the government spending is higher than the reduction of consumption of the capitalists, an increase in the tax rate increases the aggregate demand by $s_P\pi uK$ unit. Therefore, a unit rise in t_P increases the equilibrium degree of capacity utilization by $\frac{s_P\pi u}{\Psi}$ unit. Similarly, for a rise in t_W, the consumption demand of the workers decreases by $(1-s_W)(1-\pi)uK$ unit, whereas the tax revenue increases by $(1-\pi)uK$ unit. As the entire rise in the tax revenue is spent by the government, the aggregate demand increases by $s_W(1-\pi)uK$ unit. Consequently, the equilibrium degree of capacity utilization rises by $\frac{s_W(1-\pi)u}{\Psi}$ unit.

The impact of a rise in profit share on the equilibrium capacity utilization rate is ambiguous. The following equation captures this.

$$\frac{du^*}{d\pi} = \frac{-g\{(1-t_P)s_P - (1-t_W)s_W\}}{[\{(1-t_P)s_P - (1-t_W)s_W\}\pi + (1-t_W)s_W]^2}$$

[1] As the potential output-capital ratio is fixed, actual output-capital ratio is used as a proxy for the degree of capacity utilisation.

$\frac{du^*}{d\pi} \gtreqless 0$ according to whether $\{(1-t_P)s_P - (1-t_W)s_W\} \lesseqgtr 0$.

Note that as $s_P > s_W$, $t_P \leq t_W$ is a sufficient condition for the economy to be in a wage-led demand regime (i.e. $\frac{du^*}{d\pi} < 0$). On the other hand, $t_P > t_W$ is a necessary condition for existence of a profit-led demand regime (i.e. $\frac{du^*}{d\pi} > 0$). Thus a more progressive tax system is required for a profit-led demand regime whereas a regressive tax system, *ceteris paribus*, ensures the wage-led demand regime. Our finding on this regard is exactly opposite of Blecker (2002, pp. 141). Note that while in Rowthorn (1981), Dutt (1984), and Taylor (1985), the economy is always in a wage-led demand regime, both wage-led and the profit-led demand regime are possible in Bhaduri and Marglin (1990). However, similar to Blecker (2002) and Ko (2018), in our analysis too, tax structure plays a crucial role in determining whether the economy is in a wage-led or in a profit-led demand regime.

3.1 The long-run dynamics

Firms adjust their actual investment rate to the desired rate of investment in the long run. Following Dutt (2006, 2012), Charles (2008), and Ko (2018), we assume

$$\dot{g} = \rho \left[g^d - g\right] \tag{3.8}$$

$\dot{g}$ captures the change in the investment rate, g^d stands for the desired investment rate, and ρ represents the speed of adjustment parameter. The desired rate of investment is expressed as

$$g^d = \gamma_0 + \gamma_1 u^* + \gamma_2(1-t_P)r^* + \gamma_3(\frac{G}{K}). \tag{3.9}$$

γ_0, γ_1, γ_2, and γ_3 are all positive parameters. γ_0 is the autonomous part of the desired rate of investment. Investment rate depends positively on the rate of capacity utilization (u), profit rate (r), and government investment to capital stock ratio ($\frac{I_G}{K}$). While γ_1 represents the responsiveness of investment to a change in capacity utilization rate, γ_2 and γ_3 indicate the responsiveness of investment to a change in profit rate and government investment to capital stock ratio respectively.

The explanation for the degree of capacity utilization as a determinant of the investment function comes from Steindl (1952). According to Steindl, as the capital equipment is indivisible, for profit-maximizing firms it is profitable to have a certain desired amount of excess capacity due to fluctuations in demand. Thus, while firms invest more in a scenario where capacity utilization rises above the desired level, firms increase utilization by dis-investing (and hence by reducing the capital stock) in response to capacity utilization falling below the desired level. The rate of profit is used as a proxy for the expected rate of return. It provides internal funding for accumulation plans. It is also easier to raise external finance while the rate of profit is higher. Following Dutt (2013), and Taylor (1991), we assume that government investment expenditure positively influence the private investment through its 'crowding in' effect. The last term captures this.

Inserting equation (3.9) into (3.8) and rearranging we get,

$$\dot{g} = \rho \left[\gamma_0 + \gamma_1 u^* + \gamma_2(1 - t_P)\pi u^* + \gamma_3 \theta u^* - g\right] \tag{3.10}$$

$$\implies \dot{g} = \rho \left[\gamma_0 + \frac{\Gamma g}{\Psi} - g\right] \tag{3.11}$$

where $\Gamma = \{\gamma_1 + \gamma_2(1 - t_P)\pi + \gamma_3\theta\} > 0$. Differentiating equation (3.11) partially w.r.t. g we get,

$$\frac{\partial \dot{g}}{\partial g} = \rho \left[\frac{\Gamma}{\Psi} - 1\right] \tag{3.12}$$

Keynesian stability condition implies $\frac{\partial \dot{g}}{\partial g} = \rho\left[\frac{\Gamma}{\Psi} - 1\right] < 0$. Let us assume the Keynesian stability condition holds i.e. we get a long run stable steady state E where $g^* = \frac{\gamma_0 \Psi}{\Psi - \Gamma}$ (see Figure 3.1a). Keynesian stability condition in turn ensures $(\Psi - \Gamma) > 0$. Note that equation (3.11) has a positive intercept (γ_0) and a negative slope ($-\frac{(\Psi-\Gamma)}{\Psi}$).

3.2 Comparative Statics

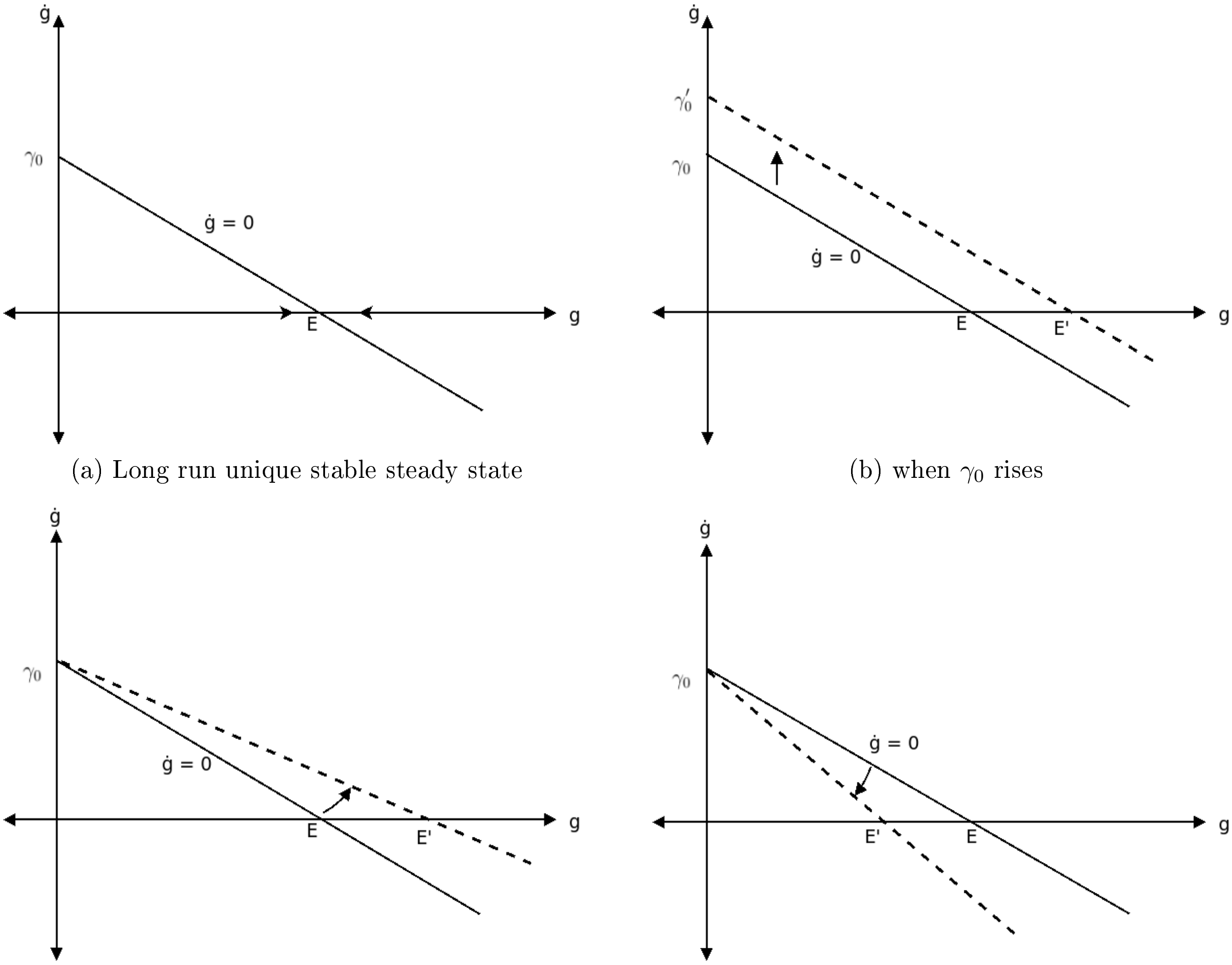
As illustrated in Figure 3.1b, for a rise in γ_0, there is a parallel upward shift in the $\dot{g} = 0$ isocline. Consequently, we get a new equilibrium E' where the equilibrium rate of capital accumulation increases. Intuitively speaking, a rise in the autonomous investment raises the desired rate of investment and thereby increases the equilibrium growth rate.[2] For an increase in the government expenditure to income ratio θ, the desired investment rate rises. Therefore, the equilibrium growth rate increases.[3] As depicted in Figure 3.1c, here the slope of the $\dot{g} = 0$ isocline becomes flatter. As a result, a new steady state E' with higher g^* is achieved.

A rise in the savings propensity of the capitalists (s_p), *ceteris paribus,* decreases the capacity utilization rate. Consequently, the desired investment rate falls. Therefore, there is a fall in the equilibrium growth rate (see Figure 3.1d). Mathematically, $\frac{dg^*}{ds_P} = \frac{d}{ds_P}\left(\frac{\gamma_0\Psi}{\Psi-\Gamma}\right) = \frac{-\gamma_0\Gamma(1-t_P)\pi}{(\Psi-\Gamma)^2} < 0$. Similarly, a rise in s_W leads to a fall in the equilibrium growth rate (here $\frac{dg^*}{ds_P} = \frac{-\gamma_0\Gamma(1-t_W)(1-\pi)}{(\Psi-\Gamma)^2} < 0$). A rise in t_W leads to a rise in g^*. This is mainly because of the balanced budget assumption. A rise in t_W leads to a rise in u^* which in turn raises the desired investment rate. Therefore g^* rises.[4] However, the effect

[2] $\frac{dg^*}{d\gamma_0} = \frac{d}{d\gamma_0}\left(\frac{\gamma_0\Psi}{\Psi-\Gamma}\right) = \frac{\gamma_0}{\Psi-\Gamma} > 0.$

[3] $\frac{dg^*}{d\theta} = \frac{\gamma_0\gamma_3\Psi}{(\Psi-\Gamma)^2} > 0.$

[4] $\frac{dg^*}{dt_W} = \frac{\gamma_0\Gamma(1-\pi)s_W}{(\Psi-\Gamma)^2} > 0.$



(a) Long run unique stable steady state

(b) when γ_0 rises

(c) A rise in either of θ, t_P

(d) A rise in either of s_P, s_W

Figure 3.1: Long run unique stable steady state and comparative static results

of a rise in t_p on g^* is ambiguous. This is because, a rise in t_P through its effect on u^* raises the desired investment rate by $\Gamma\frac{du^*}{dt_P}$ unit. On the other hand, as t_P rises, because of its negative effect on the third term of the right hand side of on equation (3.10), desired investment rate falls by $\gamma_2\pi u^*$ unit. Therefore, the final effect is ambiguous.[5]

4 No Balanced Budget Scenario

In this section, we assume that there is a budget deficit, and the government incurs debt. The government borrows from capitalists at the interest rate i. Capitalists earn profit income (P) as well as the interest income (iD) whereas workers have only one source of

[5] $\frac{dg^*}{dt_P} = \frac{\gamma_0\pi(\Gamma s_P - \gamma_2\Psi)}{(\Psi-\Gamma)^2} \gtreqless 0.$

income- wages. The aggregate government tax revenue is given as,

$$T = t_P(P + iD) + t_W W = [(t_P - t_W)\pi u + t_W u + t_P i\delta] K \tag{4.1}$$

where, D is the real stock of government debt, and $\delta = \frac{D}{K}$ is the debt-capital ratio. For simplicity, we ignore monetary and other assets. Government debt finances the entire government deficit. Therefore, the change in debt with respect to time is given as,

$$\dot{D} = G - T + iD \tag{4.2}$$

Aggregate private saving in the economy is,

$$\begin{aligned} S &= [(1 - t_P)s_P(P + iD) + s_W(1 - t_W)W] \\ &= [\{(1 - t_P)s_P - (1 - t_W)s_W\}\pi u + s_W(1 - t_W)u + (1 - t_P)s_P i\delta] K \end{aligned} \tag{4.3}$$

Inserting the values in equation (3.5) we get the equilibrium capacity utilization rate as

$$u^* = \frac{g - \{(1 - t_P)s_P + t_P\}i\delta}{[\{(1 - t_P)s_P - (1 - t_W)s_W + (t_P - t_W)\}\pi + s_W(1 - t_W) + t_W - \theta]} = \frac{g - \zeta i\delta}{\Lambda} \tag{4.4}$$

The denominator and the numerator in equation (4.4) are both positive so that u^* becomes positive, i.e. we assume

$$\Lambda = [\{(1 - t_P)s_P - (1 - t_W)s_W + (t_P - t_W)\}\pi + s_W(1 - t_W) + t_W - \theta] > 0 \tag{4.5}$$

$$\text{and } g > \zeta i\delta = \{(1 - t_P)s_P + t_P\}i\delta. \tag{4.6}$$

The short run comparative statics are summarized as follows:

$$\frac{du^*}{dg} = \frac{1}{\Lambda} > 0, \ \frac{du^*}{di} = \frac{-\zeta\delta}{\Lambda} < 0, \ \frac{du^*}{d\delta} = \frac{-\zeta i}{\Lambda} < 0, \frac{du^*}{d\theta} = \frac{g - \zeta i\delta}{\Lambda^2} > 0,$$

$$\frac{du^*}{ds_P} = \frac{-(1 - t_P)\left[\{(1 - t_W)s_W(1 - \pi) + t_W(1 - \pi) - \theta\} i\delta + g\pi\right]}{\Lambda^2} = \frac{-(1 - t_P)(\pi u^* + i\delta)}{\Lambda} < 0,$$

$$\frac{du^*}{dt_P} = \frac{-(1 - s_P)\left[\{(1 - t_W)s_W(1 - \pi) + t_W(1 - \pi) - \theta\} i\delta + g\pi\right]}{\Lambda^2} = \frac{-(1 - s_P)(\pi u^* + i\delta)}{\Lambda} < 0,$$

$$\frac{du^*}{ds_W} = \frac{-(g - \zeta i\delta)(1 - t_W)(1 - \pi)}{\Lambda^2} = \frac{-(1 - t_W)(1 - \pi)u^*}{\Lambda} < 0,$$

$$\frac{du^*}{dt_W} = \frac{-(g - \zeta i\delta)(1 - s_W)(1 - \pi)}{\Lambda^2} = \frac{-(1 - s_W)(1 - \pi)u^*}{\Lambda} < 0.$$

A rise in investment demand raises the equilibrium degree of capacity utilization ($\frac{du^*}{dg} > 0$). Same is true for a rise in government expenditure (i.e. $\frac{du^*}{d\theta} > 0$). An increase in δ decreases the equilibrium degree of capacity utilization. Due to one unit increase in

δ, the ratio of private saving to capital stock increases by $(1-t_P)s_P i$ unit while the ratio of government revenue income to capital stock increases by it_P unit. Thus due to one unit increase in δ, consumption demand (normalized by capital stock) decreases by $\{(1-t_P)s_P + t_P\}i = \zeta i$ unit. Thus aggregate demand and hence the equilibrium degree of capacity utilization decreases. Similarly, for a rise in the interest rate, private savings increases by $(1-t_P)s_P\delta K$ unit while government revenue income increases by $t_P\delta K$ unit. Thus, for a unit rise in the interest rate, aggregate demand decreases by $\{(1-t_P)s_P+t_P\}\delta K = \zeta\delta K$ unit. Hence the equilibrium capacity utilization rate decreases by $\frac{\zeta\delta}{\Lambda}$ unit. Note that the results that an increase in the government debt-capital ratio or a rise in the interest rates leads to a fall in the the rate of capacity utilization (i.e. $\frac{du^*}{d\delta} < 0$, and $\frac{du^*}{di} < 0$) are opposite to You and Dutt (1996).

A rise in the tax rate on capitalists leads to a fall in the equilibrium degree of capacity utilization.[6] Per unit increase in tax rate on capitalists reduces consumption demand of capitalists and hence the aggregate demand by $(1-s_P)(\pi u + i\delta)K$ unit. Therefore, an increase in the tax rate on capitalists decreases the equilibrium degree of capacity utilization (i.e. $\frac{du^*}{dt_P} < 0$). Similarly, for a rise in the tax rate on workers, consumption demand of the workers and hence the aggregate demand decreases by $(1-s_W)(1-\pi)uK$ unit. Hence, the equilibrium degree of capacity utilization decreases by $\frac{-(1-s_W)(1-\pi)u^*}{\Lambda}$ unit. Our results in this regard are in sharp contrast with Ko (2018) (where in Ko (2018) $\frac{du^*}{dt_P} > 0$ and $\frac{du^*}{dt_W} = 0$).

A unit rise in the savings propensity of the capitalist (s_P) increase the aggregate private savings by $(1-t_P)(\pi u + i\delta)K$ unit. This leakage leads to a fall in aggregate demand and hence the equilibrium degree of capacity utilization (i.e. $\frac{du^*}{ds_P} < 0$). Similarly, a rise in s_W increases the aggregate savings by $(1-t_W)(1-\pi)u^*K$ unit. Therefore, u^* falls by $\frac{(1-t_W)(1-\pi)u^*}{\Lambda}$ unit.

Now we focus on the impact of a change in profit share on the equilibrium capacity utilization rate. Differentiating u^* w.r.t. π we get,

$$\frac{du^*}{d\pi} = \frac{-\left[g - \{(1-t_P)s_P + t_P\}i\delta\right]\left[(1-t_P)s_P - (1-t_W)s_W + (t_P - t_W)\right]}{\left[\{(1-t_P)s_P - (1-t_W)s_W + (t_P - t_W)\}\pi + s_W(1-t_W) + t_W - \theta\right]^2}$$

[6] Note that $[\{(1-t_P)s_P - (1-t_W)s_W + (t_P-t_W)\}\pi + s_W(1-t_W) + t_W - \theta] = [\{(1-t_W)s_W(1-\pi) + t_W(1-\pi) - \theta\} + \{(1-t_P)s_P + t_P\}\pi]$. From equation (4.5) we get,

$$[\{(1-t_W)s_W(1-\pi) + t_W(1-\pi) - \theta\} + \{(1-t_P)s_P + t_P\}\pi] > 0$$

$$\implies \{(1-t_W)s_W(1-\pi) + t_W(1-\pi) - \theta\}\, i\delta + \{(1-t_P)s_P + t_P\}\pi i\delta > 0$$

This and equation (4.6) together imply,

$$\{(1-t_W)s_W(1-\pi) + t_W(1-\pi) - \theta\}\, i\delta + g\pi > 0$$

Therefore, $\frac{du^*}{ds_P} = \frac{-(1-t_P)[\{(1-t_W)s_W(1-\pi)+t_W(1-\pi)-\theta\}i\delta + g\pi]}{\Lambda^2} < 0$.

$\frac{du^*}{d\pi} \gtreqless 0$ according to whether $[(1-t_P)s_P - (1-t_W)s_W + (t_P - t_W)] \lesseqgtr 0$.

Note that $\frac{d\{(1-t_P)s_P-(1-t_W)s_W+(t_P-t_W)\}}{dt_P} = (1-s_P) > 0$, $\frac{d\{(1-t_P)s_P-(1-t_W)s_W+(t_P-t_W)\}}{dt_w} = -(1-s_W) < 0$, $\frac{d\{(1-t_P)s_P-(1-t_W)s_W+(t_P-t_W)\}}{ds_P} = (1-t_P) > 0$, and $\frac{d\{(1-t_P)s_P-(1-t_W)s_W+(t_P-t_W)\}}{ds_W} = -(1-t_W) < 0$. Therefore, *ceteris paribus,* a combination of a sufficiently high s_W, a sufficiently low s_P (so that the difference between s_P and s_W becomes very small), and a regressive tax system (so that $t_W > t_P$) makes $[(1-t_P)s_P - (1-t_W)s_W + (t_P - t_W)] < 0$. Consequently, $\frac{du^*}{d\pi} > 0$ i.e. the economy is in a profit-led demand regime. Otherwise, the economy is in a wage-led demand regime. Thus, *ceteris paribus,* a more regressive tax system makes the economy more likely to be in a profit-led demand regime, whereas a more progressive tax system makes the economy more likely to be in a wage-led demand regime. Our finding in this regard is similar to Blecker (2002, pp. 141) and opposite to the finding in Section 3. Note that for a uniform tax rate and a uniform savings propensity (i.e. if $s_P = s_W$ and $t_P = t_W$), a change in income distribution will have no impact on the equilibrium degree of capacity utilization i.e. $\frac{du^*}{d\pi} = 0$.

4.1 The long-run dynamics

Now we proceed for the long run dynamics. Instead of equation (3.9), we assume the desired investment rate as $g^d = \gamma_0 + \gamma_1 u^* + \gamma_2(1-t_P)r^* + \gamma_3(\frac{G}{K}) - \gamma_4\delta$. Change in the investment rate, therefore, is

$$\dot{g} = \rho\left[(\gamma_0 - \gamma_4\delta) + \frac{\Gamma\left[g - \zeta i\delta\right]}{\Lambda} - g\right] \tag{4.7}$$

where $\Gamma = \{\gamma_1 + \gamma_2(1-t_P)\pi + \gamma_3\theta\} > 0$, and γ_4 is the coefficient measuring responsiveness of investment due to a change in δ. Here the fifth term entering in the desired investment rate, represents the financial crowding out effect.[7] Partial differentiation of equation (4.7) w.r.t. g and δ respectively yields,

$$J_{11} = \frac{\partial \dot{g}}{\partial g} = \rho\left[\frac{\Gamma}{\Lambda} - 1\right] \gtreqless 0 \tag{4.8}$$

$$J_{12} = \frac{\partial \dot{g}}{\partial \delta} = \rho\left[-\gamma_4 - \frac{\Gamma\zeta i}{\Lambda}\right] < 0 \tag{4.9}$$

When Keynesian stability condition is satisfied, we get $J_{11} < 0$, otherwise J_{11} is positive. In the long run equilibrium $\dot{g} = 0$ which yields $g\big|_{\dot{g}=0} = \left(\frac{\gamma_0\Lambda}{\Lambda-\Gamma}\right) - \left(\frac{\gamma_4\Lambda+\Gamma\zeta i}{\Lambda-\Gamma}\right)\delta$. Therefore,

[7] Following Dutt (2013), we introduce it. The purpose of introduction of it is to show that even if we consider the neo-classical argument of financial crowding-out of private investment for a rise in public debt, government debt-capital ratio does not necessarily rise without bound. The model also does not necessarily become unstable.

when $(\Lambda - \Gamma) > 0$, the slope of the $\dot{g} = 0$ isocline is $\frac{dg}{d\delta}\Big|_{\dot{g}=0} = -\left(\frac{\gamma_4\Lambda + \Gamma\zeta i}{\Lambda - \Gamma}\right) < 0$, and the vertical intercept of the $\dot{g} = 0$ isocline is $g\Big|_{\dot{g}=0}^{\delta=0} = \left(\frac{\gamma_0\Lambda}{\Lambda-\Gamma}\right) > 0$. On the other hand, for $(\Lambda - \Gamma) < 0$, the $\dot{g} = 0$ isocline is a positively sloped straight line with a negative vertical intercept.

Now we analyze the dynamics of the government debt. We know, $\delta = \frac{D}{K}$. So, $\dot{\delta} = \frac{\dot{D}}{K} - \frac{D\dot{K}}{K^2} = \frac{\dot{D}}{K} - \delta g$. Further,

$$\dot{D} = G - T + iD \tag{4.10}$$

Inserting the value of T from equation (4.1), and inserting $G = \theta Y$ in equation (4.10) and rearranging we get,

$$\dot{\delta} = [(\theta - t_W) - (t_P - t_W)\pi]\, u^* + (1 - t_P)i\delta - \delta g$$

$$\implies \dot{\delta} = \Omega\left(\frac{g - \zeta i\delta}{\Lambda}\right) + (1 - t_P)i\delta - \delta g \tag{4.11}$$

where, $\Omega = [(\theta - t_W) - (t_P - t_W)\pi] \gtreqless 0$. Partial differentiation of equation (4.11) w.r.t. g and δ respectively yields,

$$J_{21} = \frac{\partial\dot{\delta}}{\partial g} = \frac{\Omega}{\Lambda} - \delta \gtreqless 0 \tag{4.12}$$

$$J_{22} = \frac{\partial\dot{\delta}}{\partial\delta} = -\frac{\Omega\zeta i}{\Lambda} + (1 - t_P)i - g \gtreqless 0 \tag{4.13}$$

Let us derive the slope of the $\dot{\delta} = 0$ isocline. In the long run equilibrium $\dot{\delta} = 0$. This implies,

$$g\big|_{\dot{\delta}=0} = \frac{[\zeta\Omega - (1 - t_P)\Lambda]\, i\delta}{\Omega - \Lambda\delta} \tag{4.14}$$

Therefore, the slope of the $\dot{\delta} = 0$ isocline is

$$\frac{dg}{d\delta}\Big|_{\dot{\delta}=0} = \frac{[\zeta\Omega - (1 - t_P)\Lambda]\, i\Omega}{[\Omega - \Lambda\delta]^2}. \tag{4.15}$$

The vertical intercept of the $\dot{g} = 0$ isocline is $g\Big|_{\dot{\delta}=0}^{\delta=0} = 0$, and the vertical asymtopte is at $\delta = \frac{\Omega}{\Lambda}$. The horizontal asymptote is at $g = \frac{[\zeta\Omega - (1-t_P)\Lambda]i}{-\Lambda} = -\frac{\Sigma}{\Lambda}$, where $\Sigma = [\zeta\Omega - (1 - t_P)\Lambda]\, i \gtreqless 0$.

Depending on the signs of Ω and $[\zeta\Omega - (1 - t_P)\Lambda]$ we get three possible cases. *Case I* where both the signs are positive. Ceteris paribus, a very high government expenditure to GDP ratio is required to achieve *Case I*.[8] In *Case II* government expenditure to GDP ratio is moderately high so that Ω positive but $[\zeta\Omega - (1 - t_P)\Lambda]$ is negative. In other

[8]Note that $\Omega > 0$ implies $\theta > [t_P\pi + t_W(1 - \pi)]$. Moreover, $[\zeta\Omega - (1 - t_P)\Lambda] > 0$ implies $\theta > \frac{(1-t_P)[(1-t_P)s_P\pi + (1-t_W)(1-\pi)s_W]}{(1+s_P-t_Ps_P)} + [t_P\pi + t_W(1 - \pi)]$.

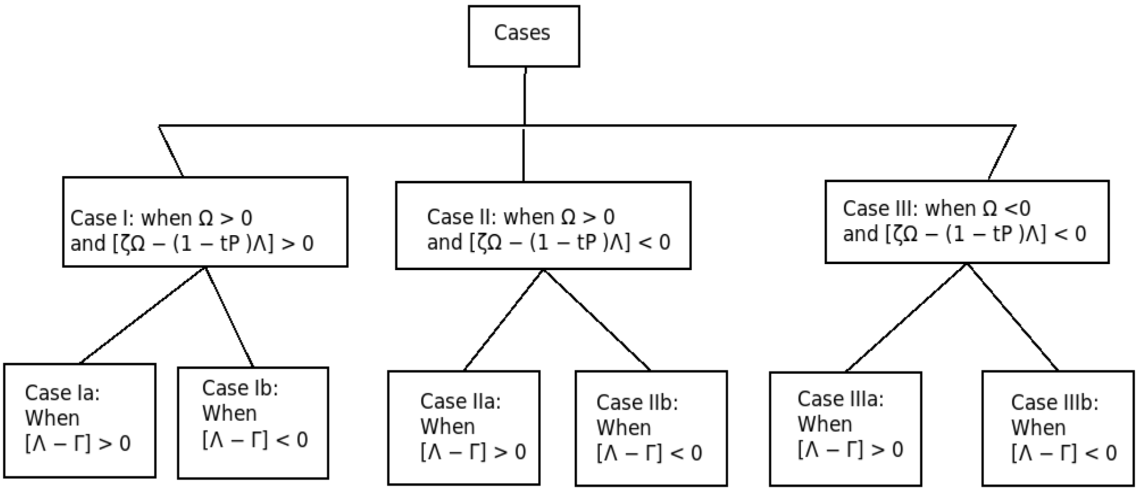


Figure 4.1: Flowchart of all possible cases

words, *Case II* is possible when $\frac{(1-t_P)[(1-t_P)s_P\pi+(1-t_W)(1-\pi)s_W]}{(1+s_P-t_Ps_P)} + [t_P\pi + t_W(1-\pi)] > \theta > [t_P\pi + t_W(1-\pi)] > 0$ holds. The last case is *Case III* where government expenditure to GDP ratio is so low that both Ω and $[\zeta\Omega - (1-t_P)\Lambda]$ are negative. For each of these cases, we get two sub-cases: when $(\Lambda - \Gamma)$ is positive (which in turn implies $J_{11} < 0$), and when $(\Lambda - \Gamma) < 0$ (which in turn ensures $J_{11} > 0$). Some post-Keynesian economists like Dallery (2007), and Skott (2010, 2012), however, have shown their doubt on whether the Keynesian stability condition holds. Therefor, along with $J_{11} < 0$ (which is ensured by the Keynesian stability condition), we also deal with $J_{11} > 0$.[9]

Figure 4.1 illustrates the flowchart related to all these cases.

Case Ia: Figure 4.2 illustrates the presence of the long-run equilibrium at E_1. $\dot{g} = 0$ isocline is a negatively sloped line with a positive vertical intercept, whereas the $\dot{\delta} = 0$ isocline is a hyperbolic curve with a positive slope that increases with δ.[10] Here, the vertical asymptote of the $\dot{\delta} = 0$ isocline is positive (i.e. $\frac{\Omega}{\Lambda} > 0$) and the horizontal asymptote is negative.[11]

As $\delta < \frac{\Omega}{\Lambda}$ at E_1, from equation (4.12) we get $J_{21} > 0$. Therefore, J_{22} is negative at E_1.[12]

At point E_1, the determinant of the Jacobian matrix $\text{Det(J)} = (\overbrace{J_{11}}^{-}\overbrace{J_{22}}^{-} - \overbrace{J_{12}}^{-}\overbrace{J_{21}}^{+}) > 0$.

[9] A reply on this issue can be found in Lavoie (2010) and Hein *et al.* (2012). Lavoie (2014, pp. 377-410) provides a summary of this controversy.

[10] As here $\Omega > 0$, and $[\zeta\Omega - (1-t_P)\Lambda] > 0$, therefore from equation (4.15), the slope of the $\dot{\delta} = 0$ isocline is $\left.\frac{dg}{d\delta}\right|_{\dot{\delta}=0} > 0$.

[11] As $[\zeta\Omega - (1-t_P)\Lambda] > 0$.

[12] Slope of the $\dot{\delta} = 0$ isocline can be represented as $\left.\frac{dg}{d\delta}\right|_{\dot{\delta}=0} = -\frac{\frac{\partial\dot{\delta}}{\partial\delta}}{\frac{\partial\dot{\delta}}{\partial g}} = -\frac{J_{22}}{J_{21}}$. As at E_1, $J_{21} > 0$ and $\left.\frac{dg}{d\delta}\right|_{\dot{\delta}=0} > 0$, J_{22} must be negative.

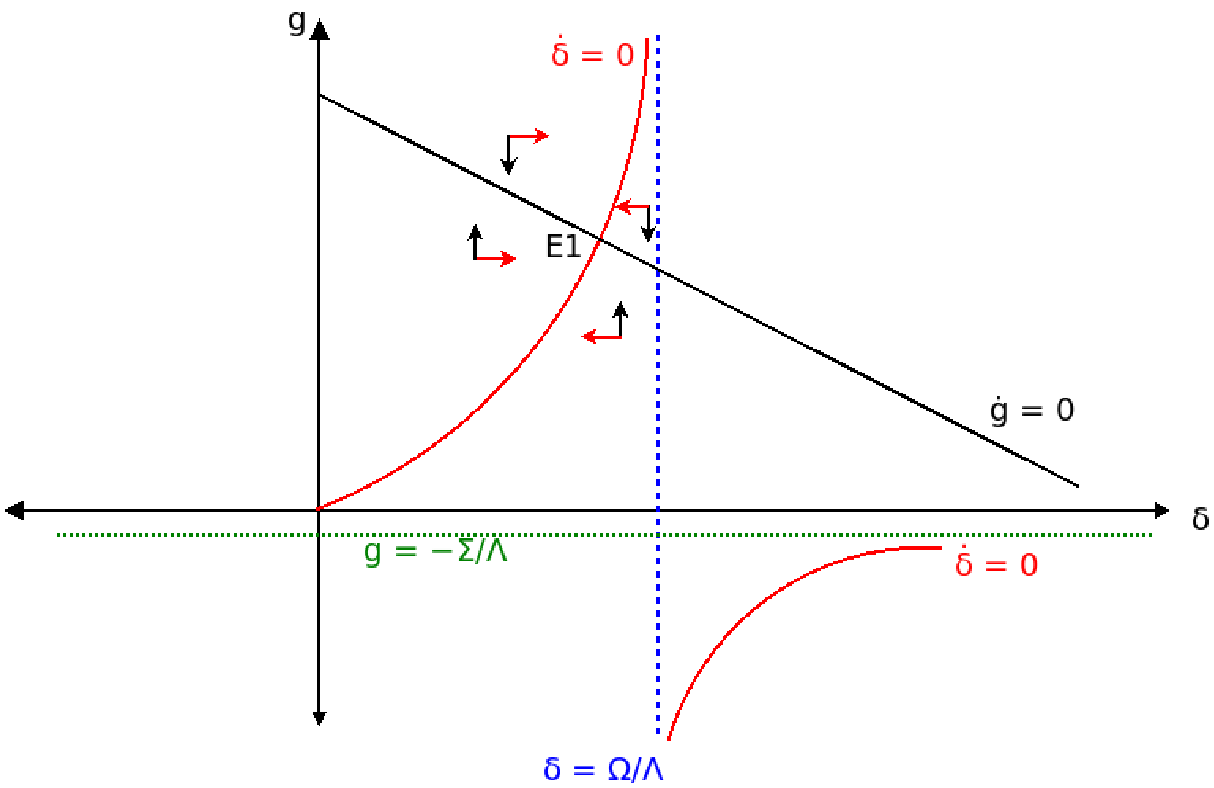


Figure 4.2: *Case Ia*

Trace of the matrix tr(J) $= J_{11} + J_{22} < 0$. As a result, point E_1 emerges as a stable steady state.[13]

Let us discuss the stability of the steady state intuitively. Suppose because of some exogenous shock, government debt-capital ratio deviates and now is above its steady state value (i.e. now $\delta > \delta^*$). From equation (4.13), the debt-capital ratio must fall under the direct stable effect. On the other hand, the rise in debt-capital ratio leads to a decrease in the rate of capital accumulation due to equation (4.9). As a result, from equation (4.12) the debt-capital ratio falls. This is the indirect stable effect. As both the effects are stable, if the debt-capital ratio rises from the steady state level, it again comes back to its steady state. Hence, the steady state is stable.

Case Ib: In *Case Ib,* as $(\Lambda - \Gamma) < 0$, the $\dot{g} = 0$ isocline has a negative vertical intercept and a positive slope. Note that $(\Lambda - \Gamma) < 0$ *ensures* $J_{11} > 0$. As illustrated in Figure 4.3, we get two steady state E_5 and E_6. At E_5, slope of the $\dot{g} = 0$ isocline is greater than the slope of the $\dot{\delta} = 0$ isocline i.e.

$$\left.\frac{dg}{d\delta}\right|_{\dot{g}=0} = -\frac{J_{12}}{J_{11}} > \left.\frac{dg}{d\delta}\right|_{\dot{\delta}=0} = -\frac{J_{22}}{J_{21}} > 0$$

[13]Here as $J_{12} < 0$ and $J_{21} > 0$ so, $tr(J)^2 - 4Det(J) = (J_{11} - J_{22})^2 + 4J_{12}J_{21} \gtreqless 0$ and hence the steady state can be either a stable node or a stable spiral.

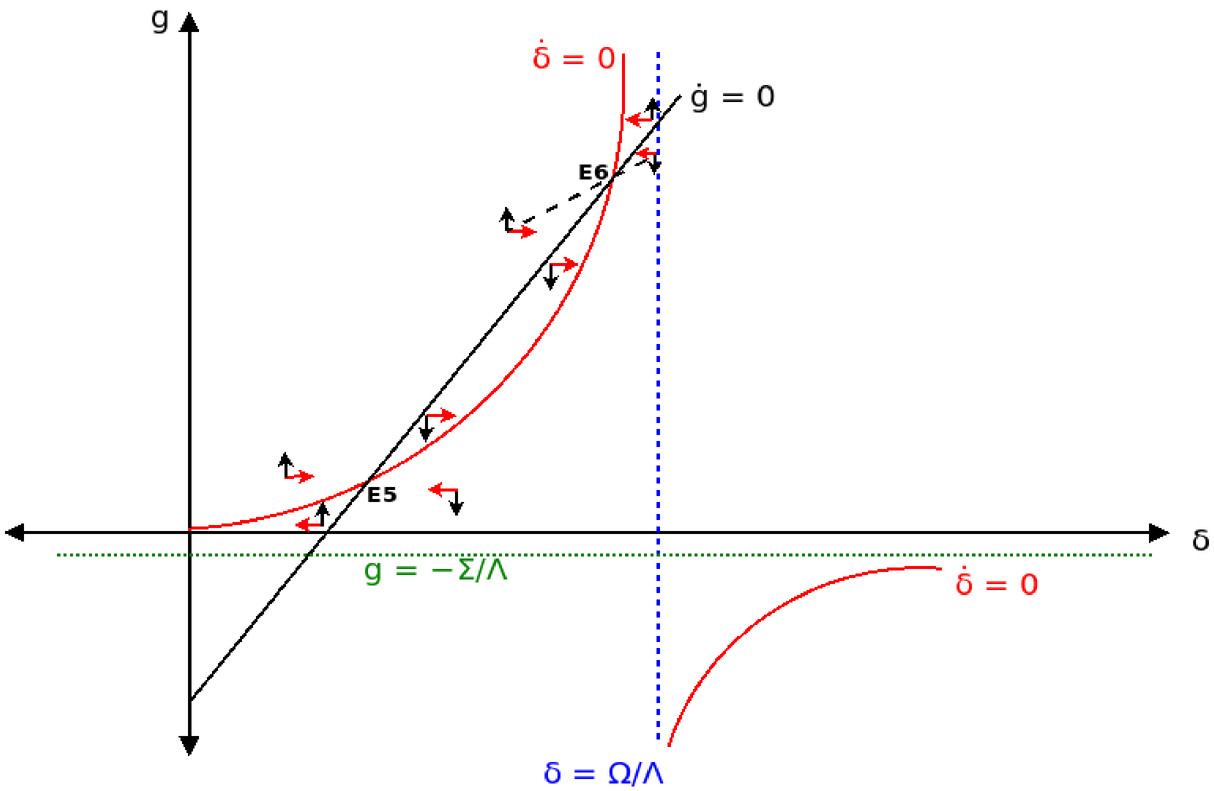


Figure 4.3: *Case Ib*

$$\Rightarrow (J_{11}J_{22} - J_{12}J_{21}) > 0 \quad (\because J_{11} > 0 \text{ and } J_{21} > 0)$$

Hence the determinant is positive. However sign of the trace is ambiguous ($\because$ tr(J) $= J_{11} + J_{22} \gtreqless 0$). Therefore, E_5 can be either a stable or an unstable equilibrium depending on the speed of adjustment parameter $\rho < \hat{\rho} = \frac{[\zeta\Omega-(1-t_P)\Lambda]i+g\Lambda}{[\Gamma-\Lambda]}$ or $\rho > \hat{\rho}$ respectively. If $\rho = \hat{\rho}$, limit cycles occur due to Hopf-bifurcation. More discussion regarding Hopf-bifurcation is provided in Section 4.2.

Intuition behind the stability at point E_5 is as follows. First, when the rate of capital accumulation rises above its steady state value, as $J_{11} > 0$, the positive self-feedback effect leads to a further rise in the capital accumulation rate. This is the direct unstable effect. Second, a rise in g through equation (4.12) leads to a rise in the debt-capital ratio ($\because J_{21} > 0$ here) which in turn through equation (4.9) causes a fall in the rate of capital accumulation ($\because J_{12} < 0$). This is the indirect stable effect. When the speed of adjustment parameter of the rate of capital accumulation is sufficiently low (i.e. when $\rho < \hat{\rho}$), the dynamics of the system could become stable because the negative indirect-feedback mechanism of the rate of capital accumulation becomes strong and dominates the unstable self-feedback effect.

On the other hand, as slope of the $\dot{g} = 0$ isocline is smaller than the slope of the $\dot{\delta} = 0$ isocline at E_6, the determinant of the Jacobian matrix is negative, and therefore E_6

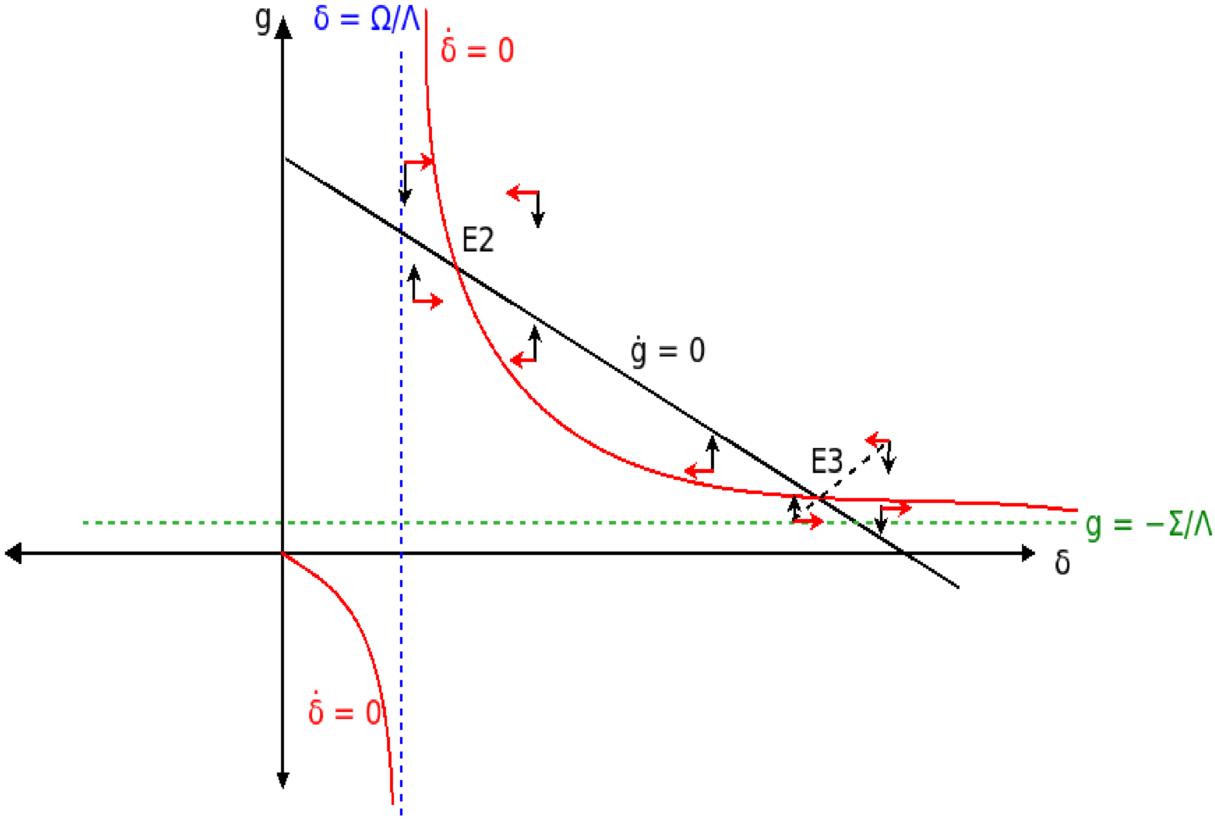


Figure 4.4: Case II

emerges as a saddle point unstable steady state.

Case IIa: Here we assume $\Omega > 0$, but $[\zeta\Omega - (1 - t_P)\Lambda] < 0$. Equation (4.15) therefore suggests that the slope of the $\dot{\delta} = 0$ isocline must be negative. Similar to *Case I*, here too the vertical asymptote is positive. As $[\zeta\Omega - (1 - t_P)\Lambda] < 0$, unlike *Case I,* here the horizontal asymptote is positive. Figure 4.4 depicts the presence of multiple equilibria: E_2 and E_3 respectively. Here too, the $\dot{g} = 0$ isocline is a negatively sloped line with a positive vertical intercept, whereas the $\dot{\delta} = 0$ isocline is a hyperbolic curve with a negative slope that flattens with δ. As $\delta > \frac{\Omega}{\Lambda}$ at E_2, from equation (4.12) we get $J_{21} < 0$. Therefore, J_{22} is negative at E_2. This is true for E_3 as well.

At point E_2, slope of the $\dot{g} = 0$ isocline is greater than the slope of the $\dot{\delta} = 0$ isocline i.e.

$$0 > \frac{dg}{d\delta}\bigg|_{\dot{g}=0} = -\frac{J_{12}}{J_{11}} > \frac{dg}{d\delta}\bigg|_{\dot{\delta}=0} = -\frac{J_{22}}{J_{21}}$$

$$\Rightarrow (J_{11}J_{22} - J_{12}J_{21}) > 0 \quad (\because J_{11} < 0 \text{ and } J_{21} < 0)$$

Hence the determinant is positive and the trace is negative ($\because$ tr(J) $= J_{11} + J_{22} < 0$). As a result, point E_2 emerges as a stable steady state.[14]

[14]Here as $J_{12} < 0$ and $J_{21} < 0$, $tr(J)^2 - 4Det(J) = (J_{11} - J_{22})^2 + 4J_{12}J_{21} > 0$. Hence the steady state is a stable node.

Let us discuss it intuitively. Government debt-capital ratio ratio, suppose due to some reason, deviates from the steady state and is now higher than its steady state value. Two opposite effects are in work near the steady state E_2. First, as the debt-capital ratio is higher than its steady state value, it must fall due to $J_{22} < 0$. This is the direct stable effect. Second, the rise in δ leads to a fall in the rate of capital accumulation due to $J_{12} < 0$. As $J_{21} < 0$, this fall in the accumulation rate leads to a rise in the debt-capital ratio. This second effect is an indirect unstable effect. As both the isocline are negatively sloped and slope of the $\dot{g} = 0$ isocline is higher than the slope of the $\dot{\delta} = 0$ isocline, absolute value of J_{21} is relatively small. As a result, a fall in the rate of capital accumulation (due to $J_{12} < 0$) leads to a small amount of rise in the debt-capital ratio. Consequently, the direct stable effect dominates the indirect unstable effect and results in the steady state to be stable.

At point E_3, slope of the $\dot{\delta} = 0$ isocline is greater than the slope of $\dot{g} = 0$ isocline i.e.

$$0 > \frac{dg}{d\delta}\Big|_{\dot{\delta}=0} = -\frac{J_{22}}{J_{21}} > \frac{dg}{d\delta}\Big|_{\dot{g}=0} = -\frac{J_{12}}{J_{11}}$$

$$\Rightarrow (J_{11}J_{22} - J_{12}J_{21}) < 0 \quad (\because J_{11} < 0 \text{ and } J_{21} < 0)$$

As the determinant is negative, point E_3 emerges as a saddle point unstable steady state.

Intuition at E_3 is similar to E_2 except that as both the isocline are negatively sloped and slope of the $\dot{g} = 0$ isocline is lower than the slope of $\dot{\delta} = 0$ isocline, absolute value of J_{21} is relatively high here. As a result, a fall in the rate of capital accumulation (due to $J_{12} < 0$) leads to a large amount of rise in the debt-capital ratio. Consequently, the indirect unstable effect dominates the direct stable effect and results in the steady state to be unstable. There is only one stable arm that reaches to the steady state E_3. Hence E_3 emerges as a saddle point unstable steady state.

Case IIb: As depicted in Figure 4.5, here we get a unique saddle point unstable steady state E_7 (As Det(J) $= (\overbrace{J_{11}}^{+}\overbrace{J_{22}}^{-} - \overbrace{J_{12}}^{-}\overbrace{J_{21}}^{-}) < 0$).

Case IIIa: Here we assume $\Omega < 0$, and therefore, $[\zeta\Omega - (1 - t_P)\Lambda] < 0$. Equation (4.15) suggests that the $\dot{\delta} = 0$ isocline is a positively sloped hyperbolic curve that decreases with δ. The vertical asymptote of the $\dot{\delta} = 0$ isocline is negative here whereas the horizontal asymptote is positive. $\dot{g} = 0$ isocline is a negatively sloped line with a positive vertical intercept. Figure 4.6 illustrates the presence of the long-run equilibrium at E_4. Here, from equation (4.12) we get $J_{21} < 0$. Therefore, J_{22} is positive at E_4. At point E_4, the determinant is negative (i.e. Det(J) $= (\overbrace{J_{11}}^{-}\overbrace{J_{22}}^{+} - \overbrace{J_{12}}^{-}\overbrace{J_{21}}^{-}) < 0$). As a result, point E_4 emerges as a saddle point unstable steady state.

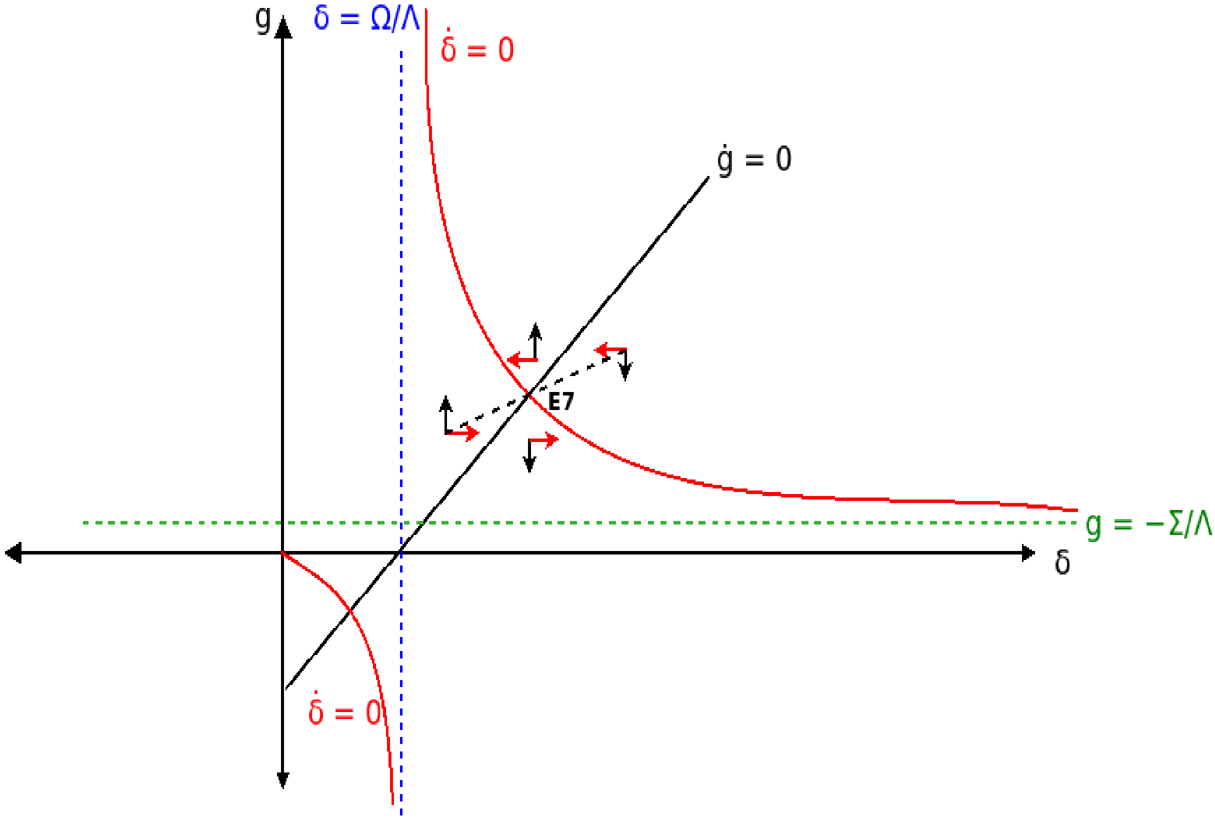


Figure 4.5: *Case IIb*

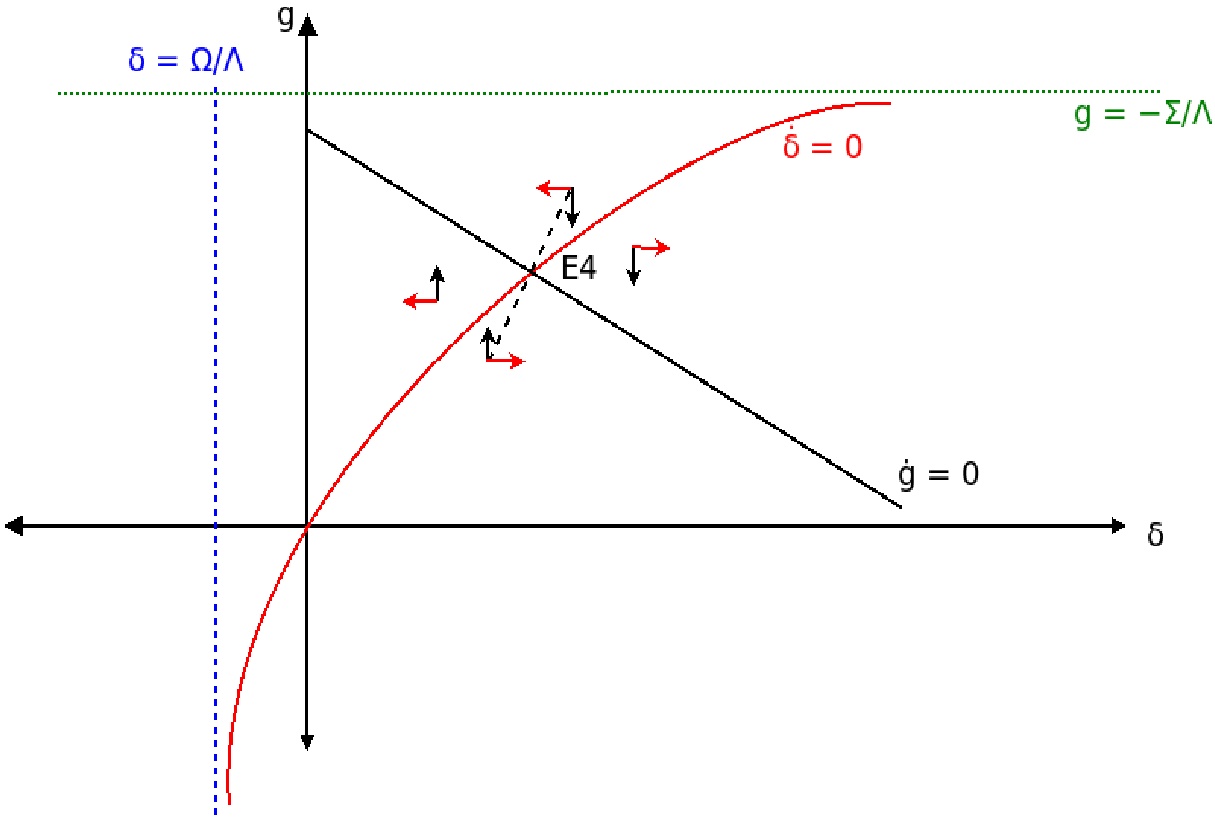


Figure 4.6: *Case IIIa*

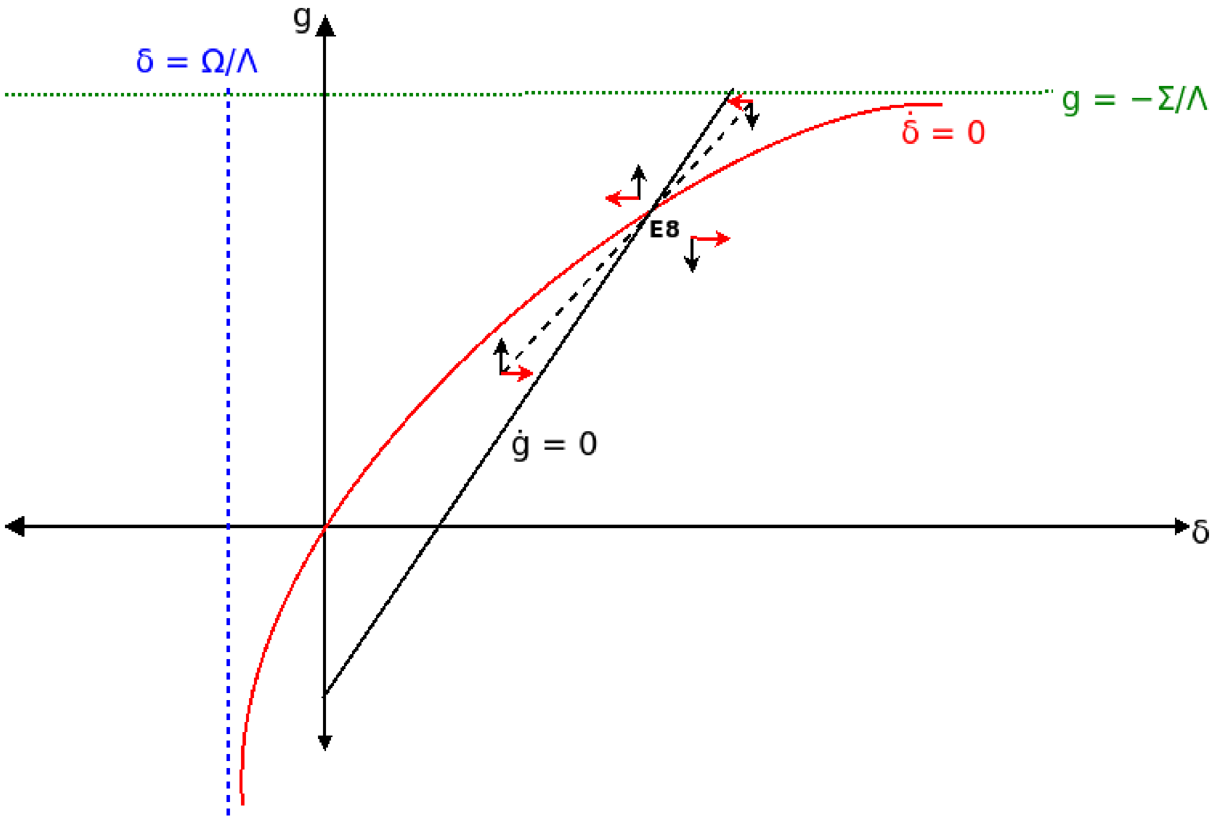


Figure 4.7: *Case IIIb*

Government debt-capital ratio, suppose due to some reason, deviates from its equilibrium position and is now higher than its steady state value. First, as δ is higher than its steady state value, it increases further due to $J_{22} > 0$. This is the direct effect. On the other hand, the rise in government debt-capital ratio leads to a fall in the rate of capital accumulation due to $J_{12} < 0$. As $J_{21} < 0$, this fall in g leads to rise in δ. This second effect is an indirect effect. As both the effects are unstable, if the debt-capital ratio rises from the steady state level, it further moves away from the steady state. There is only one stable arm (as depicted in Figure 4.6) that reaches to the equilibrium point E_4. Hence E_4 emerges as a saddle point unstable steady state.

Case IIIb: As depicted in Figure 4.7, here too we get a unique steady state E_8. Slope of the $\dot{g} = 0$ isocline is greater than the slope of $\dot{\delta} = 0$ isocline at E_8 i.e.

$$\left.\frac{dg}{d\delta}\right|_{\dot{g}=0} = -\frac{J_{12}}{J_{11}} > \left.\frac{dg}{d\delta}\right|_{\dot{\delta}=0} = -\frac{J_{22}}{J_{21}} > 0$$

$$\Rightarrow (J_{11}J_{22} - J_{12}J_{21}) < 0 \quad (\because J_{11} > 0 \text{ and } J_{21} < 0)$$

As the determinant is negative, point E_8 emerges as a saddle point unstable steady state.

From the analysis of *Cases I, II & III* we conclude that a very high government expenditure to GDP ratio is associated with a unique stable steady state (E_1) whereas a moderate government expenditure to GDP ratio is connected with multiple equilibria-one of which is saddle point unstable (E_3). The other one (E_2) is stable. However, a sufficiently low government expenditure to GDP is related to a unique saddle point unstable steady state (E_4). Therefor, when there is a budget deficit and government incurs debt, a significantly high government expenditure (to GDP ratio) is required to achieve stability in the economy.

In the next section we investigate how various parameters influence the equilibrium values of debt-capital ratio and the growth rate.

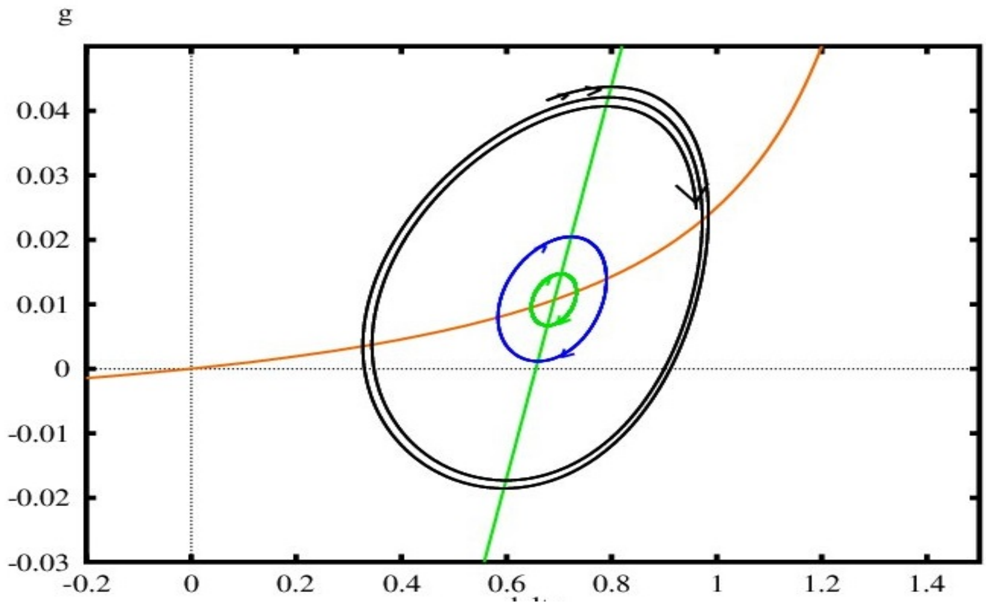
4.2 *Hopf Bifurcation*

In this sub-section, we discuss the possibilities of emergence of cycle as a solution to the dynamical systems represented by equation (4.7) and (4.11). Consider the steady state E_5 of *Case Ib*. We get the following proposition.

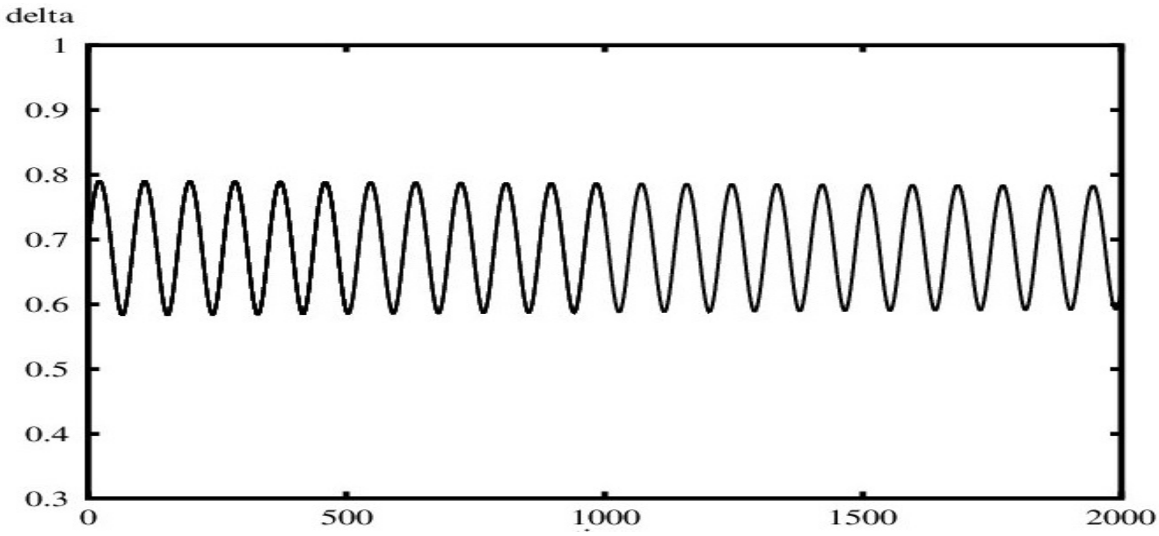
Proposition 1. *For an appropriate value of the speed of adjustment parameter ρ, the characteristic equation to (4.7) & (4.11) evaluated at the steady state E_5 has purely imaginary roots and for the same dynamical system, $\rho = \hat{\rho} = \frac{[\zeta\Omega-(1-t_P)\Lambda]i+g\Lambda}{[\Gamma-\Lambda]}$ provides a point of Hopf bifurcation.*

Proof. See Appendix A.1. □

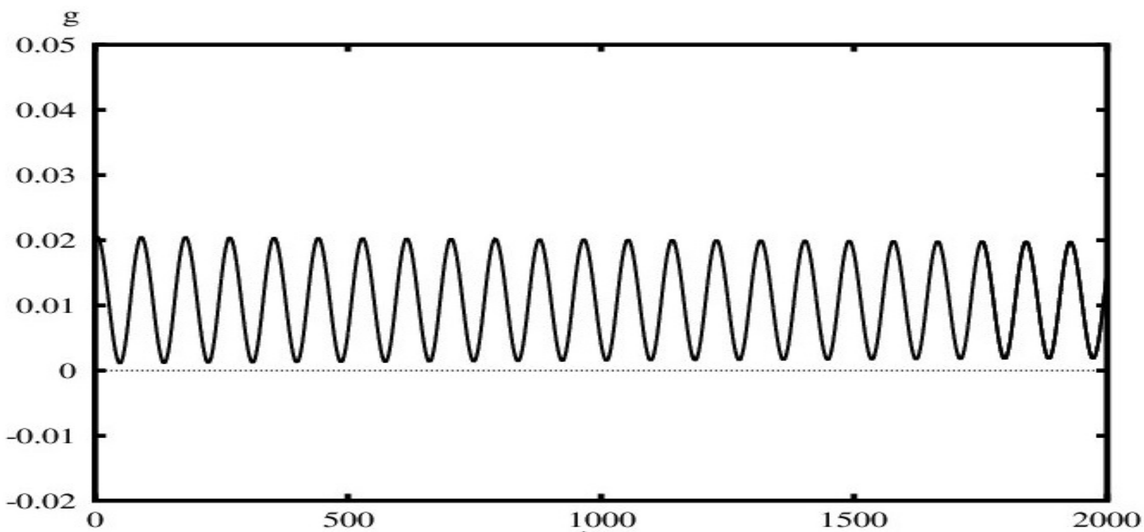
Using XPPAUT software we find that the Hopf bifurcation is super-critical in nature i.e. a stable limit cycle exists (shown by blue curve in Figure 4.8a). We draw the solution path from $t = 0$ to $t = 200$. For an initial condition close to the long-run equilibrium (eg. if it starts from $(\delta(0), g(0)) = (0.672, 0.0136)$), the solution path converges to the limit cycle (shown by green cycle in Figure 4.8a), whereas for the initial condition further away from the long-run equilibrium (eg. if it starts from $(\delta(0), g(0)) = (0.676, 0.0416)$), the solution path converges to the limit cycle (as shown by black curve in Figure 4.8a). Therefore, we conclude that in this numerical example, the super-critical Hopf bifurcation occurs and the periodic solution is stable. Instead of calibrating a real economy, the primary purpose of this numerical study is to confirm whether the model produces the limit cycle and to observe its basic properties. Therefore, we introduce the values so that we obtain economically meaningful outcomes. For the simulation we set $\gamma_0 = 0.1$, $\gamma_1 = 0.1$, $\Gamma = 0.6$, $\Lambda = 0.4$, $i = 0.05$, $\zeta = 0.7$, $t_P = 0.2$, $\Omega = 0.6$, $\hat{\rho} = 0.04636$. We get the equilibrium values $g^* = 0.010669$ and $\delta^* = 0.69072$ for the steady state E_5 of *Case Ib*. As shown in Figure 4.8a, in the (δ, g)-plane, the clockwise cycle emerges. Figure 4.8b shows the transitional



(a) *Solution paths in (δ, g) plane: a stable limit cycle due to super-critical Hopf bifurcation*



(b) *Transitional dynamics of debt-capital ratio*



(c) *Transitional dynamics of rate of capital accumulation*

Figure 4.8: Limit cycle

dynamics of the debt-capital ratio and Figure 4.8c shows the transitional dynamics of the rate of capital accumulation.[15]

In what follows, we explain the reason behind the occurrence of a limit cycle in *Case Ib*. First, the self-feedback effect of the rate of capital accumulation is positive, i.e. $J_{11} = \frac{\partial \dot{g}}{\partial g} > 0$. Besides, here the self-feedback effect of the debt-capital ratio is negative, i.e. $J_{22} = \frac{\partial \dot{\delta}}{\partial \delta} < 0$. When the speed of adjustment parameter ρ is small, the self-feedback effect of the rate of capital accumulation is dominated by the self-feedback effect of the debt–capital ratio and so the economy achieves stability (As the trace becomes negative here). On the contrary, when the opposite happens, the economy becomes unstable. Thus, limit cycle occurs in the boundary between the unstable and the stable feedback effect i.e. when ρ reaches its critical value $\hat{\rho}$.

4.3 Comparative Statics

For comparative statics analysis, we focus only on those steady states which are stable i.e we only focus on E_1, E_5 and E_2.[16] The effects of parametric changes can be shown by totally differentiating equations (4.7) and (4.11), which imply

$$\begin{aligned} \begin{bmatrix} J_{11} & J_{12} \\ J_{21} & J_{22} \end{bmatrix} \begin{bmatrix} dg \\ d\delta \end{bmatrix} &= \begin{bmatrix} -\rho \\ 0 \end{bmatrix} d\gamma_0 + \begin{bmatrix} -\frac{\rho(g-\zeta i\delta)(\Lambda\gamma_3+\Gamma)}{\Lambda^2} \\ -\frac{(g-\zeta i\delta)(\Lambda+\Omega)}{\Lambda^2} \end{bmatrix} d\theta \\ &+ \begin{bmatrix} \frac{\rho\Gamma\zeta\delta}{\Lambda} \\ \frac{[\zeta\Omega-(1-t_P)\Lambda]\delta}{\Lambda} \end{bmatrix} di + \begin{bmatrix} -\rho\Gamma\frac{du^*}{ds_p} \\ -\Omega\frac{du^*}{ds_p} \end{bmatrix} ds_p + \begin{bmatrix} -\rho\Gamma\frac{du^*}{ds_W} \\ -\Omega\frac{du^*}{ds_W} \end{bmatrix} ds_W \\ + \begin{bmatrix} -\rho\left[\Gamma\frac{du^*}{dt_P} - \gamma_2\pi u^*\right] \\ -\left(\Omega\frac{du^*}{dt_P} - \pi u^* - i\delta\right) \end{bmatrix} dt_P &+ \begin{bmatrix} -\rho\Gamma\frac{du^*}{dt_W} \\ -\left(\Omega\frac{du^*}{dt_W} - (1-\pi)u^*\right) \end{bmatrix} dt_W \end{aligned} \tag{4.16}$$

4.3.1 *Effect of a rise in autonomous investment,* γ_0

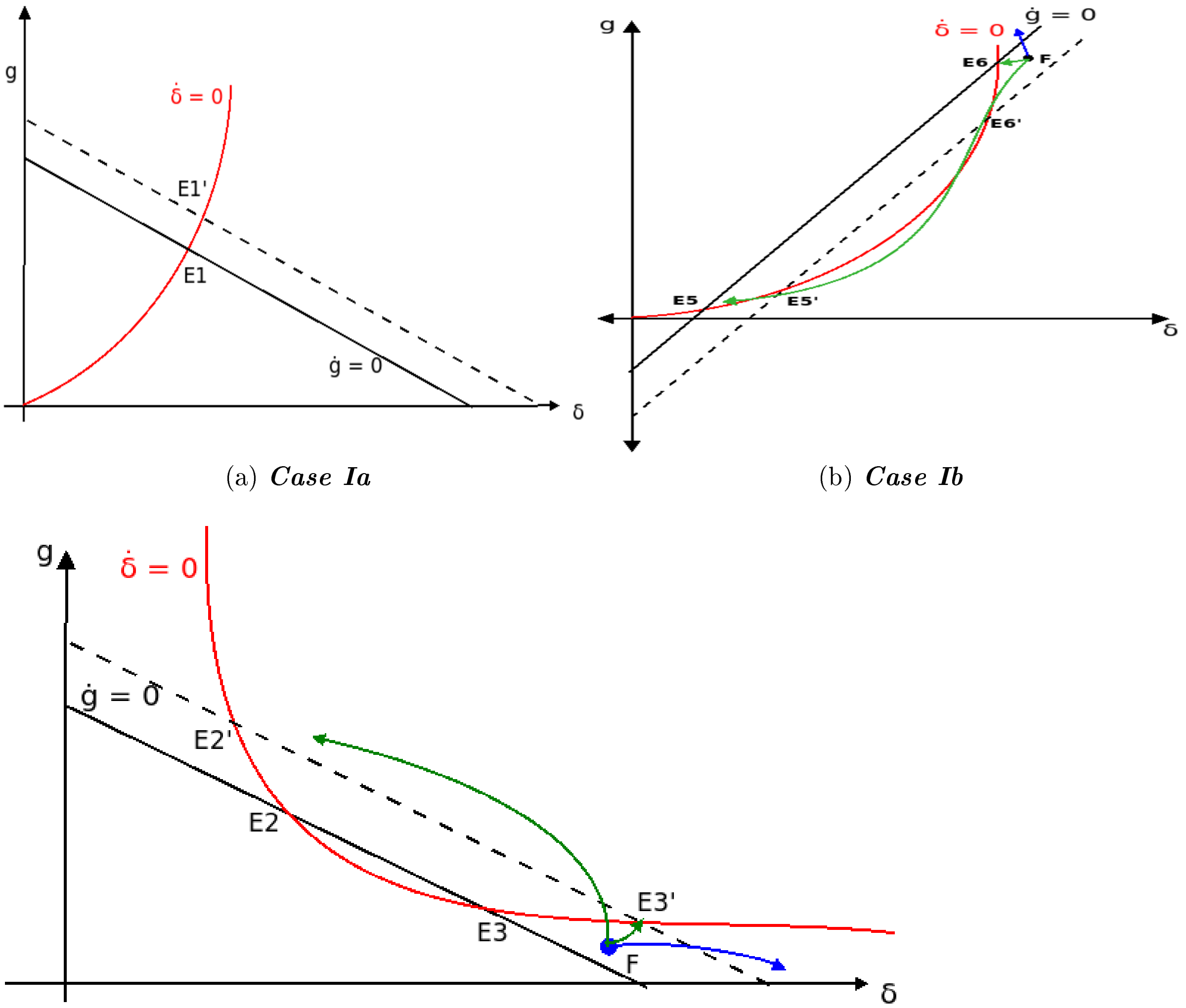
From equation (4.16) we get,

$$\frac{dg^*}{d\gamma_0} = \frac{-\rho J_{22}}{(J_{11}J_{22} - J_{12}J_{21})}; \quad \frac{d\delta^*}{d\gamma_0} = \frac{\rho J_{21}}{(J_{11}J_{22} - J_{12}J_{21})} \tag{4.17}$$

At E_1 as $J_{22} < 0$, and $J_{21} > 0$, from equation (4.17) we get $\frac{dg^*}{d\gamma_0} > 0$ and $\frac{d\delta^*}{d\gamma_0} > 0$ respectively. Thus as θ increases, both g^* as well as δ^* increase. Same is true for E_5. On the other hand, at E_2, $J_{21} < 0$, and $J_{22} < 0$. Therefore, from equation (4.17) we get $\frac{dg^*}{d\gamma_0} > 0$ and $\frac{d\delta^*}{d\gamma_0} < 0$ respectively.

[15] Note that for the transitional dynamics, we draw the solution path from $t = 0$ to $t = 2000$ where $\delta(0) = 0.69$ and $g(0) = 0.02$.

[16] Note that E_5 is stable provided that ρ is sufficiently small.



(a) ***Case Ia*** (b) ***Case Ib***

(c) ***Case IIa***

Figure 4.9: Effect of a rise in autonomous investment

For a rise in γ_0, the $\dot{\delta} = 0$ isocline does not shift. However, a rise in the autonomous investment raises the desired rate of investment and thereby pushes the $\dot{g} = 0$ isocline upwards. For a given g, at the old steady state E_1, the debt-capital ratio is lower than required for the new $\dot{g} = 0$ to be satisfied. This lower level of δ puts upward pressure on growth rate through equation (4.9) (as $J_{12} < 0$). As a result, the growth rate starts rising. As soon as g rises, debt market deviates from its equilibrium position. Given the level of δ, g is now higher than required for $\dot{\delta} = 0$ to be satisfied. As $\frac{\partial \dot{\delta}}{\partial g} = J_{21} > 0$, debt-capital ratio must rise. Combination of higher level of growth rate and debt-capital ratio ultimately ensure to achieve the new equilibrium point E_1' either monotonically or spiraling around E_1'.

We find a similar kind of mechanism in *Case IIa* as well. The only difference is that, as $J_{21} = \frac{\partial \dot{\delta}}{\partial g} < 0$ near E_2, a higher value of g causes a fall in the government debt-capital ratio. Combination of higher level of growth rate and a lower level of debt-capital ratio ultimately ensure to achieve the new equilibrium point E_2' monotonically (see Figure 4.9c).

Autonomous part of the investment plays an important role on macroeconomic stability in *Case II*. As depicted in the phase diagram in Figure 4.9c, an increase in γ_0 shifts the saddle point from E_3 to E_3' and increases the stable region. For example, initially (i.e. when E_3 is a saddle point), starting from point F, the economy would have an unstable trajectory moving further away from the steady state E_3. It is shown by the blue curve. But after a rise in γ_0 (when E_3' becomes the saddle point), starting from point F, the economy would have a stable trajectory that either converges to the steady state E_3' or to the steady state E_2. These are represented by the green curve. Therefore, a higher autonomous investment demand is desirable as the rise in γ_0 has a stabilizing effect on the macroeconomic trajectory.[17]

On the other hand, when the economy is at E_5 of *Case Ia*, as shown in in Figure 4.9b, an increase in γ_0 shifts the saddle point from E_6 to E_6' and thereby, decreases the stable region. Thus, in *Case 1a* where the Keynesian stability condition is violated (i.e. where $\Gamma > \Lambda$), a lower autonomous investment demand has a stabilizing effect on the macroeconomic trajectory.

4.3.2 *Effect of a rise in the interest rate, i*

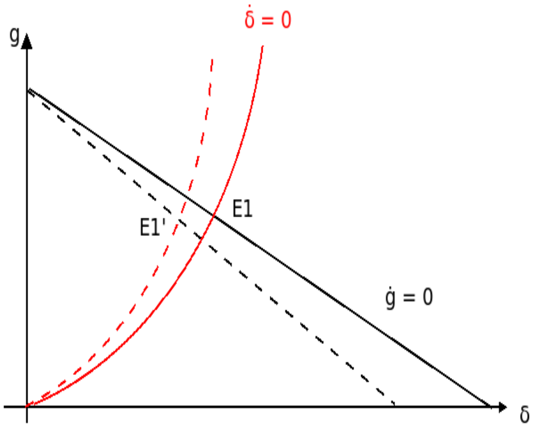
From equation (4.16) we get,

$$\frac{dg^*}{di} = \frac{\frac{\rho\Gamma\zeta\delta}{\Lambda}J_{22} - \frac{[\zeta\Omega-(1-t_P)\Lambda]\delta}{\Lambda}J_{12}}{(J_{11}J_{22} - J_{12}J_{21})}; \quad \frac{d\delta^*}{di} = \frac{\frac{[\zeta\Omega-(1-t_P)\Lambda]\delta}{\Lambda}J_{11} - \frac{\rho\Gamma\zeta\delta}{\Lambda}J_{21}}{(J_{11}J_{22} - J_{12}J_{21})} \tag{4.18}$$

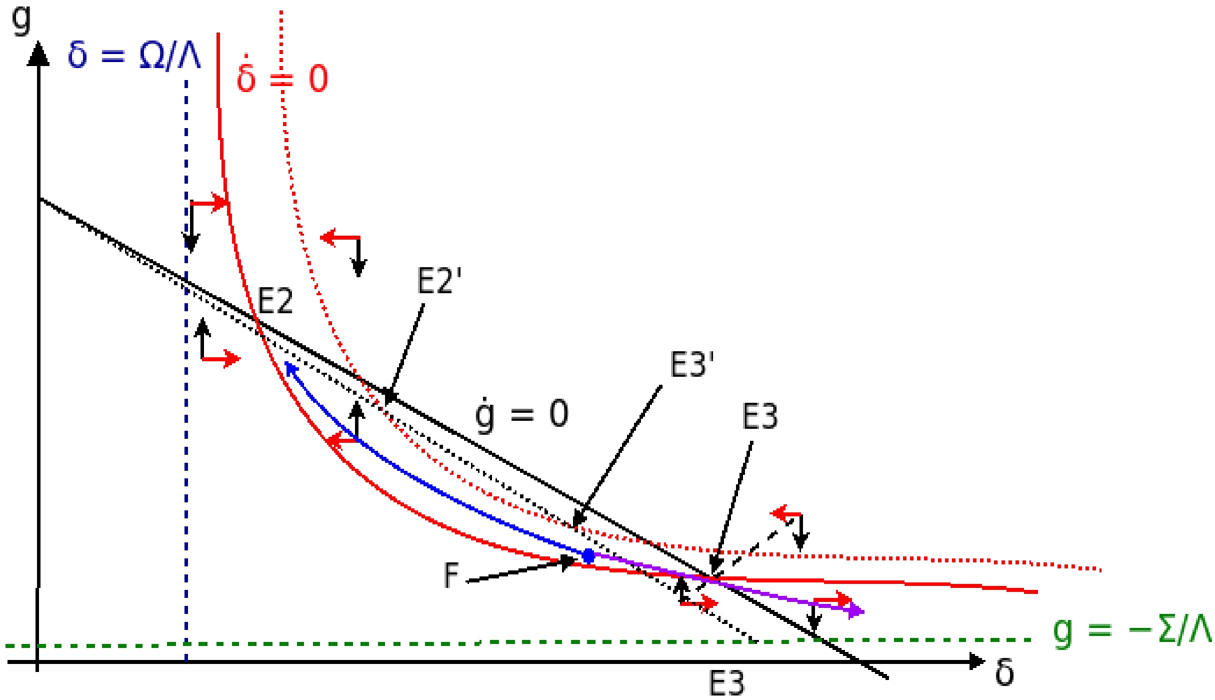
At E_1, as $J_{11} < 0$, $J_{12} < 0$, $J_{21} > 0$, $J_{22} < 0$, and $\frac{[\zeta\Omega-(1-t_P)\Lambda]\delta}{\Lambda} > 0$ from equation (4.18) we get $\frac{dg^*}{di} \gtreqless 0$ and $\frac{d\delta^*}{di} < 0$ respectively. Thus as i increases, δ^* decreases. But the effect of a change in i on g^* is ambiguous (see Figure 4.10a). On the other hand, at E_2, $J_{11} < 0$, $J_{12} < 0$, $J_{21} < 0$, $J_{22} < 0$ and $\frac{[\zeta\Omega-(1-t_P)\Lambda]\delta}{\Lambda} < 0$. Therefore, from equation (4.18) we get $\frac{dg^*}{di} < 0$ and $\frac{d\delta^*}{di} > 0$ respectively (see Figure 4.10b). However, at E_5, the effect of a rise in i on both g^* and δ^* are ambiguous.

A higher interest rate leads to a fall in the degree of capacity utilization, which in turn decreases the growth rate by negatively influencing the desired rate of investment. Therefore for a given values of g and δ, when i rises, $\dot{g}$ falls (as $\frac{\partial \dot{g}}{\partial i} = \frac{\rho\Gamma\zeta\delta}{\Lambda} < 0$). Consequently,

[17] See Isaac and Kim (2013, pp. 264-65) for more on the concept of stable region.



(a) ***Case Ia***



(b) ***Case IIa***

Figure 4.10: Effect of a rise in the interest rate

the $\dot{g} = 0$ isocline becomes steeper.[18] From equation (4.11) we get $\frac{\partial \dot{\delta}}{\partial i} = \frac{-[\zeta\Omega-(1-t_P)\Lambda]\delta}{\Lambda}$. Therefore for a given values of g and δ, when i rises, $\dot{\delta}$ falls in *Case I* and rises in *Case II*. As a result, the $\dot{\delta} = 0$ isocline shifts to the left in *Case I* and to the right in *Case II*. As δ has a negative effect on the desired investment rate, a fall in δ increases g^d in *Case I*. Therefore, the final effect of a rise in i on the equilibrium growth rate in ambiguous in *Case I*. However, in *Case II*, as $\frac{\partial \dot{\delta}}{\partial i} > 0$, for a rise in i, δ rises. This rise in δ in turn decreases the desired investment rate. Hence, a rise in the interest rate unambiguously leads to a fall in the equilibrium growth rate in *Case II*. These are illustrated in Figures 4.10a and 4.10b respectively.

Note that the interest rate can play an important role on macroeconomic stability in *Case II*. As illustrated in the phase diagram in Figure 4.10b, an increase in the interest rate shifts the saddle point from E_3 to E_3'. Therefore, a higher interest rate reduces the stable region in the positive space of (δ, g). For example, initially (i.e. when E_3 is a saddle point), starting from point F, the economy would have a stable trajectory converging to the stable steady state E_2. It is shown by the blue curve. But after a rise in the interest rate (when E_3' becomes the saddle point), starting from point F, the economy would have an unstable trajectory which is represented by the violet curve. Therefore, a lower interest rate is desirable as the rise in the interest rate has a destabilizing effect on the macroeconomic trajectory.

4.3.3 *Effect of a rise in government expenditure, θ*

From equation (4.16) we get,

$$\frac{dg^*}{d\theta} = \frac{-\rho\left[\frac{(g-\zeta i\delta)(\Lambda\gamma_3+\Gamma)}{\Lambda^2}\right]J_{22} + \frac{(g-\zeta i\delta)(\Lambda+\Omega)}{\Lambda^2}J_{12}}{(J_{11}J_{22} - J_{12}J_{21})} \tag{4.19}$$

$$\frac{d\delta^*}{d\theta} = \frac{\rho\left[\frac{(g-\zeta i\delta)(\Lambda\gamma_3+\Gamma)}{\Lambda^2}\right]J_{21} - \frac{(g-\zeta i\delta)(\Lambda+\Omega)}{\Lambda^2}J_{11}}{(J_{11}J_{22} - J_{12}J_{21})} \tag{4.20}$$

Note that $\rho\left[\frac{(g-\zeta i\delta)(\Lambda\gamma_3+\Gamma)}{\Lambda^2}\right]$ and $\frac{(g-\zeta i\delta)(\Lambda+\Omega)}{\Lambda^2}$ both are positive. At E_1, as $J_{11} < 0$, $J_{12} < 0$, $J_{21} > 0$, and $J_{22} < 0$, from equations (4.19) and (4.20) we get $\frac{dg^*}{d\theta} \gtreqless 0$ and $\frac{d\delta^*}{d\theta} > 0$ respectively. Thus as θ increases, δ^* increases. But the effect of a change in θ on g^* is ambiguous. However, at E_5, the effect of a rise in fiscal expenditure to GDP ratio on both g^* and δ^* are ambiguous. On the other hand, at E_2, $J_{11} < 0$, $J_{12} < 0$, $J_{21} < 0$,

[18] As due to a rise in i, the slope of the $\dot{g} = 0$ isocline falls i.e. $\frac{d}{di}\left(\frac{dg}{d\delta}\Big|_{\dot{g}=0}\right) = -\frac{\Gamma\zeta}{\Lambda-\Gamma} < 0$. However, the vertical intercept of the $\dot{g} = 0$ isocline i.e. $g\Big|_{\dot{g}=0}^{\delta=0} = \left(\frac{\gamma_0\Lambda}{\Lambda-\Gamma}\right)$ does not change.

and $J_{22} < 0$. Therefore, from equations (4.19) and (4.20) we get $\frac{dg^*}{d\theta} \gtreqless 0$ and $\frac{d\delta^*}{d\theta} \gtreqless 0$ respectively. Thus the effect of a change in θ on g^* as well as δ^* both are ambiguous. A further investigation shows that when the rate of capital accumulation is sufficiently high (i.e. when $g > \left\{\frac{(\Lambda+\Omega)(\Lambda\gamma_4+\Gamma\zeta i)}{\Lambda\gamma_3+\Gamma} - \frac{[\Omega\zeta-(1-t_P)\Lambda]i}{\Lambda}\right\}$), a rise in government expenditure to GDP ratio (i.e. a rise in θ) leads to a rise in the equilibrium rate of capital accumulation. $[\Omega\zeta-(1-t_P)\Lambda]$ is positive at E_1 and E_5, whereas it is negative at E_2. Therefore, relative to E_1 and E_5, a higher g is required at E_2 to achieve $\frac{dg^*}{d\theta} > 0$.

4.3.4 *Effect of a rise in the savings propensity of the capitalists, s_p*

From equation (4.16) we get,

$$\frac{dg^*}{ds_p} = \frac{-\rho\Gamma\frac{du^*}{ds_p}J_{22} + \Omega\frac{du^*}{ds_p}J_{12}}{(J_{11}J_{22} - J_{12}J_{21})}; \quad \frac{d\delta^*}{ds_p} = \frac{-\Omega\frac{du^*}{ds_p}J_{11} + \rho\Gamma\frac{du^*}{ds_p}J_{21}}{(J_{11}J_{22} - J_{12}J_{21})} \tag{4.21}$$

At E_1, as $J_{11} < 0$, $J_{12} < 0$, $J_{21} > 0$, and $J_{22} < 0$, from equation (4.21) we get $\frac{dg^*}{ds_p} \gtreqless 0$ and $\frac{d\delta^*}{ds_p} < 0$ respectively. Thus for a rise in s_P, δ^* decreases. But the effect of a change in s_P on g^* is ambiguous. At E_5, however, we get $\frac{dg^*}{ds_p} \gtreqless 0$ and $\frac{d\delta^*}{ds_p} \gtreqless 0$. On the other hand, at E_2, $J_{11} < 0$, $J_{12} < 0$, $J_{21} < 0$, and $J_{22} < 0$. Therefore, from equation (4.21) we get $\frac{dg^*}{ds_p} \gtreqless 0$ and $\frac{d\delta^*}{ds_p} \gtreqless 0$ respectively.

4.3.5 *Effect of a rise in the savings propensity of the workers, s_W*

From equation (4.16) we get,

$$\frac{dg^*}{ds_W} = \frac{-\rho\Gamma\frac{du^*}{ds_W}J_{22} + \Omega\frac{du^*}{ds_W}J_{12}}{(J_{11}J_{22} - J_{12}J_{21})}; \quad \frac{d\delta^*}{ds_W} = \frac{-\Omega\frac{du^*}{ds_W}J_{11} + \rho\Gamma\frac{du^*}{ds_W}J_{21}}{(J_{11}J_{22} - J_{12}J_{21})} \tag{4.22}$$

At E_1, as $J_{11} < 0$, $J_{12} < 0$, $J_{21} > 0$, and $J_{22} < 0$, from equation (4.22) we get $\frac{dg^*}{ds_W} \gtreqless 0$ and $\frac{d\delta^*}{ds_W} < 0$ respectively. Thus for a rise in s_W, δ^* decreases. But the effect of a change in s_W on g^* is ambiguous. At E_5, $\frac{dg^*}{ds_W} \gtreqless 0$ and $\frac{d\delta^*}{ds_W} \gtreqless 0$. On the other hand, at E_2, $J_{11} < 0$, $J_{12} < 0$, $J_{21} < 0$, and $J_{22} < 0$. Therefore, from equation (4.22) we get $\frac{dg^*}{ds_W} \gtreqless 0$ and $\frac{d\delta^*}{ds_W} \gtreqless 0$ respectively.

4.3.6 *Effect of a rise in the tax rate on capitalists, t_P*

From equation (4.16) we get,

$$\frac{dg^*}{dt_P} = \frac{-\rho\left[\Gamma\frac{du^*}{dt_P} - \gamma_2\pi u^*\right]J_{22} + \left[\Omega\frac{du^*}{dt_P} - \pi u^* - i\delta\right]J_{12}}{(J_{11}J_{22} - J_{12}J_{21})} \tag{4.23}$$

$$\frac{d\delta^*}{dt_P} = \frac{-\left[\Omega\frac{du^*}{dt_P} - \pi u^* - i\delta\right] J_{11} + \rho\left[\Gamma\frac{du^*}{dt_P} - \gamma_2\pi u^*\right] J_{21}}{(J_{11}J_{22} - J_{12}J_{21})} \tag{4.24}$$

At E_1, as $J_{11} < 0$, $J_{12} < 0$, $J_{21} > 0$, and $J_{22} < 0$, from equations (4.23) and (4.24) we get $\frac{dg^*}{dt_P} \gtreqless 0$ and $\frac{d\delta^*}{dt_P} < 0$ respectively. Thus for a rise in s_W, δ^* decreases. But the effect of a change in s_W on g^* is ambiguous. However, at E_5 we get $\frac{dg^*}{dt_P} \gtreqless 0$ and $\frac{d\delta^*}{dt_P} \gtreqless 0$. On the other hand, at E_2, $J_{11} < 0$, $J_{12} < 0$, $J_{21} < 0$, and $J_{22} < 0$. Therefore, from equations (4.23) and (4.24) we get $\frac{dg^*}{dt_P} \gtreqless 0$ and $\frac{d\delta^*}{dt_P} \gtreqless 0$ respectively.

4.3.7 *Effect of a rise in the tax rate on workers,* t_W

From equation (4.16) we get,

$$\frac{dg^*}{dt_W} = \frac{-\rho\Gamma\frac{du^*}{dt_W}J_{22} + \left[\Omega\frac{du^*}{dt_W} - (1-\pi)u^*\right] J_{12}}{(J_{11}J_{22} - J_{12}J_{21})} \tag{4.25}$$

$$\frac{d\delta^*}{dt_W} = \frac{-\left[\Omega\frac{du^*}{dt_W} - (1-\pi)u^*\right] J_{11} + \rho\Gamma\frac{du^*}{dt_W}J_{21}}{(J_{11}J_{22} - J_{12}J_{21})} \tag{4.26}$$

At E_5 we get $\frac{dg^*}{dt_W} \gtreqless 0$ and $\frac{d\delta^*}{dt_W} \gtreqless 0$. At E_1,on the other hand, as $J_{11} < 0$, $J_{12} < 0$, $J_{21} > 0$, and $J_{22} < 0$, from equations (4.25) and (4.26) we get $\frac{dg^*}{dt_W} \gtreqless 0$ and $\frac{d\delta^*}{dt_W} < 0$ respectively. Thus for a rise in s_W, δ^* decreases. But the effect of a change in s_W on g^* is ambiguous. On the other hand, at E_2, $J_{11} < 0$, $J_{12} < 0$, $J_{21} < 0$, and $J_{22} < 0$. Therefore, from equations (4.25) and (4.26) we get $\frac{dg^*}{dt_W} \gtreqless 0$ and $\frac{d\delta^*}{dt_W} \gtreqless 0$ respectively.

5 Primary Balanced Budget

In this section we assume that the government already has incurred debt, but now onward government wants to balance the budget i.e. $G = T$ holds. Therefore the short run equilibrium can be expressed as,

$$\frac{S}{K} + \frac{T}{K} = \frac{I}{K} + \frac{G}{K}$$

$$\implies \frac{S}{K} = \frac{I}{K} \quad \text{(as budget is balanced here)}$$

$$\implies u^* = \frac{g - (1-t_P)s_P i\delta}{[\{(1-t_P)s_P - (1-t_W)s_W\}\pi + s_W(1-t_W)]} = \frac{g - (1-t_P)s_P i\delta}{\Psi} \tag{5.1}$$

7The denominator and the numerator in equation (5.1) are both positive so that u^* becomes positive, i.e. we assume

$$\Psi = [\{(1-t_P)s_P - (1-t_W)s_W\}\pi + s_W(1-t_W)] > 0 \tag{5.2}$$

$$\text{and } g > (1-t_P)s_P i\delta \tag{5.3}$$

The short run comparative statics are summarized as follows:

$$\frac{du^*}{dg} = \frac{1}{\Psi} > 0, \quad \frac{du^*}{di} = \frac{-(1-t_P)s_P\delta}{\Psi} < 0, \quad \frac{du^*}{d\delta} = \frac{-(1-t_P)s_P i}{\Psi} < 0,$$

$$\frac{du^*}{ds_P} = \frac{-(1-t_P)[s_W(1-t_W)(1-\pi)i\delta + g\pi]}{\Psi^2} = \frac{-(1-t_P)(\pi u^* + i\delta)}{\Psi} < 0,$$

$$\frac{du^*}{dt_P} = \frac{s_P[s_W(1-t_W)(1-\pi)i\delta + g\pi]}{\Psi^2} = \frac{s_P(\pi u^* + i\delta)}{\Psi} > 0,$$

$$\frac{du^*}{ds_W} = \frac{-(1-t_W)(1-\pi)u^*}{\Psi} < 0, \quad \frac{du^*}{dt_W} = \frac{s_W(1-\pi)u^*}{\Psi} > 0.$$

A unit rise in investment demand raises the aggregate demand and hence the equilibrium degree of capacity utilization. For a rise in δ by one unit, the ratio of private saving to capital stock increases by $(1-t_P)s_P i$ unit whereas the rise in government revenue income is balanced by a rise in government expenditure. Therefore the aggregate demand and hence the equilibrium degree of capacity utilization decreases. Similarly, for a rise in the interest rate, private savings increases by $(1-t_P)s_P\delta K$ unit while the rise in government revenue income is balanced by a rise in government expenditure. Therefore, aggregate demand and hence the equilibrium capacity utilization rate decreases. Explanation of a rise in s_P or s_W is same as in Section 4.

Unlike Section 4, here both the tax rates have positive impact on the equilibrium degree of capacity utilization. This is mainly because of the balanced budget assumption. Per unit increase in tax rate on capitalists (t_P) reduces consumption of capitalists by $(1-s_P)(\pi u+i\delta)K$ unit. But this increase in the tax rate increase the tax revenue by $(\pi u+i\delta)K$ unit. As this rise in the tax revenue is entirely spent by the government, the aggregate demand increases by $(\pi u + i\delta)K$ unit. As the increase in the government spending is higher than the reduction of consumption of the capitalists, an increase in t_P increases the aggregate demand by $s_P(\pi u + i\delta)K$ unit. Therefore, a rise in t_P by a unit, increases the equilibrium degree of capacity utilization by $\frac{s_P(\pi u+i\delta)}{\Psi}$ unit. Similarly, for a rise in t_w, the consumption demand of the workers decreases by $(1-s_W)(1-\pi)uK$ unit, whereas the tax revenue increases by $(1-\pi)uK$ unit. As the entire rise in the tax revenue is spent by the government, the aggregate demand increases by $s_W(1-\pi)uK$ unit. Consequently, the equilibrium degree of capacity utilization rises by $\frac{s_W(1-\pi)u}{\Psi}$ unit.

Similar to Sections 3 and 4, here also the impact of a rise in profit share on the equilibrium capacity utilization rate is ambiguous. The following equation captures this.

$$\frac{du^*}{d\pi} = \frac{-\left[g-(1-t_P)s_P i\delta\right]\{(1-t_P)s_P-(1-t_W)s_W\}}{\left[\{(1-t_P)s_P-(1-t_W)s_W\}\pi+(1-t_W)s_W\right]^2}$$

$\frac{du^*}{d\pi} \gtrless 0$ according to whether $\{(1-t_P)s_P-(1-t_W)s_W\} \lesseqgtr 0$. A more progressive tax system is required for a profit-led demand regime whereas a regressive tax system, *ceteris paribus*, ensures the wage-led demand regime.

5.1 The long-run dynamics

Here the change in investment can be expressed as,

$$\dot{g} = \rho\left[\gamma_0+\gamma_1 u^*+\gamma_2(1-t_P)\pi u^*+\gamma_3\theta u-\gamma_4\delta-g\right] \tag{5.4}$$

$$\implies \dot{g} = \rho\left[(\gamma_0-\gamma_4\delta)+\frac{\Gamma\{g-(1-t_P)s_P i\delta\}}{\Psi}-g\right] \tag{5.5}$$

where $\Gamma = \{\gamma_1+\gamma_2(1-t_P)\pi+\gamma_3\theta\} > 0$. Differentiating equation (5.5) partially w.r.t. g we get,

$$J_{11} = \frac{\partial\dot{g}}{\partial g} = \rho\left[\frac{\Gamma}{\Psi}-1\right] \tag{5.6}$$

Similarly, differentiating equation (5.5) partially w.r.t. δ we get,

$$J_{12} = \frac{\partial\dot{g}}{\partial\delta} = \rho\left[-\gamma_4-\frac{\Gamma(1-t_P)s_P i}{\Psi}\right] < 0 \tag{5.7}$$

Keynesian stability condition implies $\left[\frac{\Gamma}{\Psi}-1\right] < 0$. Consequently, $J_{11} < 0$. On the other hand, when the Keynesian stability condition is not satisfied, we get $J_{11} > 0$. In the long run equilibrium $\dot{g} = 0$ which yields $g\big|_{\dot{g}=0} = \left(\frac{\gamma_0\Lambda}{\Lambda-\Gamma}\right)-\left(\frac{\gamma_4\Lambda+\Gamma\zeta i}{\Lambda-\Gamma}\right)\delta$. Therefore, similar to Section 4.1, here too, for $(\Lambda-\Gamma) > 0$, the slope of the $\dot{g} = 0$ isocline is $\frac{dg}{d\delta}\Big|_{\dot{g}=0} = -\left(\frac{\gamma_4\Lambda+\Gamma\zeta i}{\Lambda-\Gamma}\right) < 0$, and the vertical intercept of the $\dot{g} = 0$ isocline is $g\Big|_{\dot{g}=0}^{\delta=0} = \left(\frac{\gamma_0\Lambda}{\Lambda-\Gamma}\right) > 0$. On the other hand, for $(\Lambda-\Gamma) < 0$, the $\dot{g} = 0$ isocline is a positively sloped straight line with a negative vertical intercept.

Now we analyze the dynamics of the government debt. We know, $\delta = \frac{D}{K}$. So, $\dot{\delta} = \frac{\dot{D}}{K}-\frac{D\dot{K}}{K^2} = \frac{\dot{D}}{K}-\delta g$. Further,

$$\dot{D} = G-T+iD = iD \tag{5.8}$$

Hence,

$$\dot{\delta} = \frac{\dot{D}}{K}-\delta g = (i-g)\delta \tag{5.9}$$

Differentiating equation (5.9) partially w.r.t. g we get,

$$J_{21} = \frac{\partial \dot{\delta}}{\partial g} = -\delta < 0 \tag{5.10}$$

Similarly, differentiating equation (5.9) partially w.r.t. δ we get,

$$J_{22} = \frac{\partial \dot{\delta}}{\partial \delta} = (i - g) \gtreqless 0 \tag{5.11}$$

Note that the slope of the $\dot{\delta} = 0$ isocline is either the vertical axis itself or it is parallal to the horizontal axis at $g = i$. As illustrated in Figure 5.1, when the Keynesian stability condition is satisfied, there exist two equilibria- A where $(\delta_A^*, g_A^*) \equiv (0, \frac{\gamma_0 \Psi}{\Psi - \Gamma})$ and B where $(\delta_B^*, g_B^*) \equiv \left(\frac{\Psi\gamma_0 - (\Psi - \Gamma)i}{(1-t_P)s_P\Gamma i + \Psi\gamma_4}, i\right)$. Note that at B, both δ_B^* as well as g_B^* are positive. At the steady state B as $g_B^* = i$, $J_{22} = (i - g) = 0$. Therefore the determinant of the Jacobian matrix is $Det(J) = (\overbrace{J_{11}}^{-}\overbrace{J_{22}}^{0} - \overbrace{J_{12}}^{-}\overbrace{J_{21}}^{-}) = J_{12}J_{21} < 0$. As a result, steady state B emerges as a saddle point unstable steady state. On the other hand, at steady state A, $\delta_A^* = 0$ and therefore $J_{21} = -\delta = 0$. If the interest rate is sufficiently high (i.e. if $i > g$), J_{22} becomes positive and therefore, the determinant of the Jacobian matrix is $Det(J) = (\overbrace{J_{11}}^{-}\overbrace{J_{22}}^{+} - \overbrace{J_{12}}^{-}\overbrace{J_{21}}^{0}) = J_{11}J_{22} < 0$. Therefore, A becomes a saddle point unstable steady state (see Figure 5.1a). On the other hand a sufficiently low rate of interest makes J_{22} negative and consequently, the determinant of the Jacobian matrix $Det(J) = (\overbrace{J_{11}}^{-}\overbrace{J_{22}}^{+} - \overbrace{J_{12}}^{-}\overbrace{J_{21}}^{0}) = J_{11}J_{22} > 0$. Moreover, the trace of the Jacobian matrix $tr(J) = J_{11} + J_{22} < 0$. As a result, A emerges as a stable steady state (see Figure 5.1b).

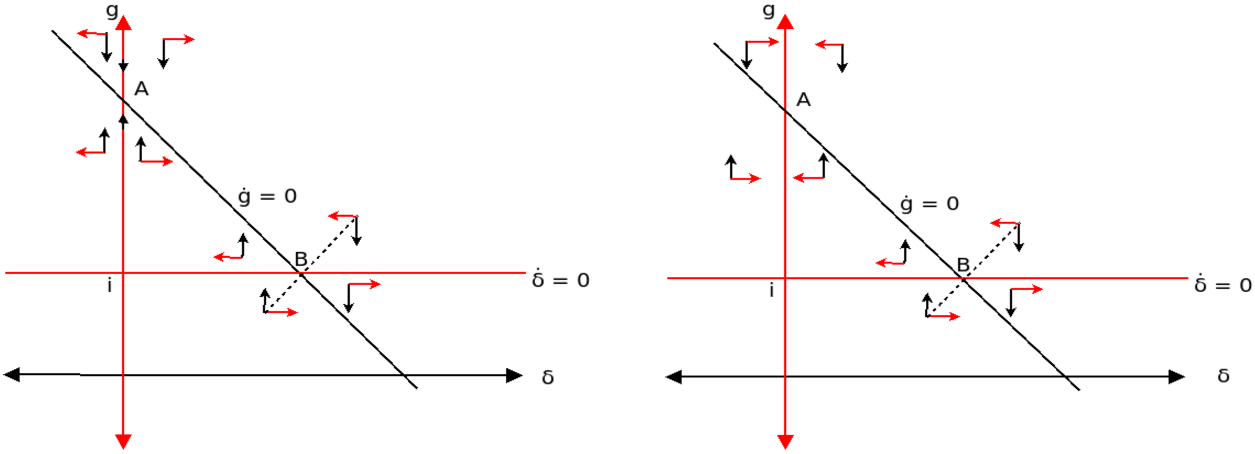
However, when the Keynesian stability condition is not satisfied, we get a unique saddle point unstable steady state C(as here $Det(J) = (\overbrace{J_{11}}^{+}\overbrace{J_{22}}^{0} - \overbrace{J_{12}}^{-}\overbrace{J_{21}}^{-}) = J_{12}J_{21} < 0$). This is shown in Figure .

5.2 Comparative Statics

For comparative statics analysis, we focus only on the stable steady state. Therefore, we focus on steady state A and assume the interest rate is low enough so that $i < g$ holds.

A rise in the autonomous investment raises the desired rate of investment and thereby increases the equilibrium growth rate.[19] However, a change in the interest rate has no effect on g_A^*. As at the steady state A, the debt-capital ratio (δ) is zero, a rise in i does

[19] $\frac{dg_A^*}{d\gamma_0} = \frac{d}{d\gamma_0}\left(\frac{\gamma_0\Psi}{\Psi-\Gamma}\right) = \frac{\gamma_0}{\Psi-\Gamma} > 0.$



(a) when $i \geq g$ (b) when $i \leq g$

Figure 5.1: Long run steady states and stability (when Keynesian stability condition is satisfied)

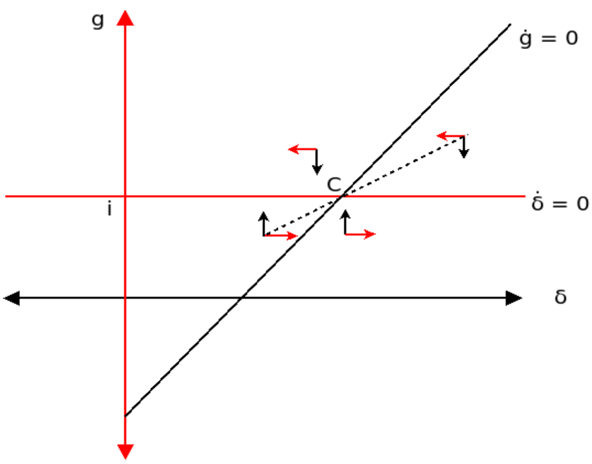


Figure 5.2: Long run steady states and stability (when Keynesian stability condition is not satisfied)

Table 5.1: Effects of changes in various parameters on u^* in the short run

	$\frac{du^*}{dg}$	$\frac{du^*}{di}$	$\frac{du^*}{d\delta}$	$\frac{du^*}{d\theta}$	$\frac{du^*}{ds_P}$	$\frac{du^*}{ds_W}$	$\frac{du^*}{dt_P}$	$\frac{du^*}{dt_W}$
Balanced budget with no government debt	$+$	0	0	0	$-$	$-$	$+$	$+$
No balanced budget, positive government debt	$+$	$-$	$-$	$+$	$-$	$-$	$-$	$-$
Budget is balanced, but government already has incurred debt	$+$	$-$	$-$	0	$-$	$-$	$+$	$+$

Table 5.2: Effects of changes in various parameters on g^* and δ^* in the long run

	Balanced budget with no government debt	No balanced budget, government incurs debt						Balanced Budget, government incurred debt	
		At E_1		At E_5		At E_2		At A	
	g^*	g^*	δ^*	g^*	δ^*	g^*	δ^*	g^*_A	δ^*_A
γ_0	positive	positive	positive	positive	positive	positive	negative	positive	no effect
i	no effect	ambiguous	negative	ambiguous	ambiguous	negative	positive	no effect	no effect
θ	positive	ambiguous	positive	ambiguous	ambiguous	ambiguous	ambiguous	positive	no effect
s_P	negative	ambiguous	negative	ambiguous	ambiguous	ambiguous	ambiguous	negative	no effect
s_W	negative	ambiguous	negative	ambiguous	ambiguous	ambiguous	ambiguous	negative	no effect
t_P	ambiguous	ambiguous	negative	ambiguous	ambiguous	ambiguous	ambiguous	ambiguous	no effect
t_W	positive	ambiguous	negative	ambiguous	ambiguous	ambiguous	ambiguous	positive	no effect

not have any impact on the desired investment rate. Therefore, there is no change in the equilibrium growth rate. For an increase in the government expenditure to income ratio θ, the desired investment rate rises. Therefore, the equilibrium growth rate increases.[20] Rest of the comparative statics results are same as in Section . Only difference is that, instead of $\delta = 0$, here we get a positive debt-capital ratio.

6 Conclusion

We presented a post-Keynesian model of growth and distribution that examines the short run and the long run effects of various fiscal policies and the change in interest rate on the economy. The main findings are as follows.

1. When the government budget is balanced, a regressive tax system is sufficient to make the economy to be in a wage-led demand regime, while a progressive tax system is a necessary condition for existence of a profit-led demand regime. This result is in sharp contrast with Blecker (2002). However, when government runs in deficits and incurs debt, *ceteris paribus,* a more regressive tax system makes the

[20] $\frac{dg^*_A}{d\theta} = \frac{\gamma_0\gamma_3\Psi}{(\Psi-\Gamma)^2} > 0$.

economy more likely to be in a profit-led demand regime. Our finding on this regard is similar to Blecker (2002).

2. In case of a balanced budget, a rise in tax rates (t_P and t_W) have expansionary effect on short run aggregate demand. However, when the balanced budget assumption is relaxed, the the tax rates have contractionary effect on short run aggregate demand.

3. When government runs in deficit and incurs debt, for a high fiscal expenditure to GDP ratio (θ), if the capacity utilization rate responsiveness of investment demand is higher than that of savings rate, and there is low level of δ^* and g^* (i.e. if the economy is at E_5 of *Case Ib*), whenever the speed of adjustment parameter of the rate of capital accumulation ρ rises to $\hat{\rho}$, the economy loses its stability and produces a stable limit cycle. For a rise in ρ above $\hat{\rho}$, the economy loses its stability.

4. When government runs in deficit and incurs debt, fiscal expenditure to GDP ratio (θ) has a positive effect on the short run aggregate demand and hence the equilibrium degree of capacity utilization. In the long run, a sufficiently high government expenditure to GDP ratio is required to achieve a stable steady state. Otherwise, the economy will reach to a unique saddle point unstable steady state (E_4). For a significantly high fiscal expenditure to GDP ratio (so that the economy is at E_1), a rise in θ leads to an unambiguous rise in the public debt-capital ratio. However, at E_5, the result is ambiguous. On the other hand, for a moderately high government expenditure to GDP ratio (i.e. when the economy is at E_2), θ has an ambiguous effect on the public debt-capital ratio. However, in all the cases (i.e. at E_1, E_5, and at E_2) the impact of θ on the long run equilibrium rate of capital accumulation is ambiguous and depends on whether $g \gtrless \left\{\frac{(\Lambda+\Omega)(\Lambda\gamma_4+\Gamma\zeta i)}{\Lambda\gamma_3+\Gamma} - \frac{[\Omega\zeta-(1-t_P)\Lambda]i}{\Lambda}\right\}$.

5. When government runs in deficit and incurs debt, interest rate has an ambiguous effect on the long run equilibrium rate of capital accumulation and a negative effect on the equilibrium debt-capital ratio at E_1. On the contrary, a rise in the interest rate leads to a fall in the equilibrium rate of capital accumulation and a rise in debt-capital ratio at E_2. Moreover, a rise in the interest has a destabilizing effect on the macroeconomic trajectory in *Case II*a. Therefore, in *Case II*a, a fall in the interest rate is not only desirable for achieving higher growth rate and a lower level of debt (at E_2), but it also enhances the stable region of the economy. Similarly, a higher autonomous investment demand (γ_0) is desirable in *Case IIa* as it enhances the stable region of the economy as well as increases the long run economic growth and reduces the debt-capital ratio at E_2. γ_0 has a positive effect on the equilibrium rate of capital accumulation at E_1 as well. However, unlike E_2, here (at E_1) the debt-capital ratio also increases.

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A Appendix

A.1 Proof of Proposition 1

Proof. The characteristic equation to (4.7) & (4.11) is

$$\mu^2 + (-\mathrm{tr(J)})\mu + \mathrm{Det(J)} = 0.$$

A necessary condition of the Hopf bifurcation for complex roots is $\mathrm{Det(J)} > 0$, which is satisfied at E_5 of *Case Ib*. The trace of the Jacobian matrix can be made either positive or negative by appropriately selecting the value of ρ while leaving the other parameters constant. To see this, notice that $\mathrm{tr(J)} = J_{11} + J_{22} = \rho\left[\frac{\Gamma}{\Lambda} - 1\right] + \left[-\frac{\Omega\zeta i}{\Lambda} + (1-t_P)i - g\right]$. Hence when $\rho = \hat{\rho} = \frac{[\zeta\Omega-(1-t_P)\Lambda]i+g\Lambda}{[\Gamma-\Lambda]} > 0$ $(\because J_{11} > 0, J_{22} < 0)$, the following equation holds exactly:

$$\mathrm{tr(J)} = 2 * \mathrm{Re}\mu = \rho\left[\frac{\Gamma}{\Lambda} - 1\right] + \left[-\frac{\Omega\zeta i}{\Lambda} + (1-t_P)i - g\right] = 0,$$

where $\mathrm{tr(J)}$ is the trace of J and $\mathrm{Re}\mu$ is the real part of its characteristic roots. As the determinant of the Jacobian matrix is positive, the product of the roots is positive in a neighborhood of the equilibrium, assuring $\mathrm{Im}\mu \neq 0$. Now differentiating the trace of the Jacobian matrix with respect to ρ and then evaluating it at $\rho = \hat{\rho}$ we get

$$\left.\frac{\partial(\frac{tr(J)}{2})}{\partial\rho}\right|_{\rho=\hat{\rho}} = \left[\frac{\Gamma}{\Lambda} - 1\right] > 0$$

So the trace is smooth, differentiable and monotonically increasing in the speed of adjustment parameter, ρ. The trace disappears at $\rho = \hat{\rho}$. Also note that $\mathrm{tr(J)} \gtreqless 0 \iff \rho \gtreqless \hat{\rho}$. From the preceding discussion, all conditions for Hopf bifurcation are satisfied at $\rho = \hat{\rho}$. $\square$