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Abstract

We reexamine the optimal fiscal and monetary policy in combined shopping-time monetary models with capital accumulation. Four models are constructed to examine how the production cost of money and the utility from physical capital affect the toolbox of the fiscal and monetary policy. It is shown that the optimality of the Friedman rule hinges on the producing cost of money and capital-in-utility overturns the Chamley-Judd zero capital income taxation theorem. When the production cost of money approaches zero, the Friedman rule is optimal; and when the consumer cares about the utility from capital, the limiting capital income tax is not zero in general.

Keywords: Transactions technology; Inflation tax; Capital income tax; Friedman rule; Capital in utility.

JEL Classification Numbers: E40, E52, H20, H21.

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1 Introduction

The problem of optimal fiscal and monetary policy has been analyzed in numerous studies. Most of these studies examine fiscal and monetary policies separately. Dynamic taxation theorists examine how to tax factor incomes in dynamic models without money.[1] Some researchers on monetary theory execute their analysis in monetary models without capital accumulation[2], other researchers investigate optimal monetary policies in monetary models with capital but without any considerations on dynamic fiscal policies[3]. Besides, a few researchers examine the optimal fiscal and monetary policy in monetary economies without capital.[4] The purpose of this paper is to reexamine the optimal fiscal and monetary policy in a combined monetary model with capital accumulation. We construct four models with different combinations on the production cost of capital and the utility generated by physical capital. Some of them reproduces zero norminal interest rate or zero limiting capital income tax; others generate more complex tradeoffs from which we develop interesting new insights.

Dynamic tax theory follows the standard Ramsey-Cass-Koopman (RCK) framework. The most importang result in this research agenda is the famous Chamley-Judd zero capital income taxation theorem developed by Chamley (1986) and Judd (1985). The theroem brings about an importang question in the theory of public finance: is physical capital special as a stock? In a generalized model with human capital and effective labor, Jones, Manuelli and Rossi (1997) establish that under some conditions[5] both capital and labor income taxes can be chosen to be zero in the steady state; moreover, if preferences satisfy an additional condition, all taxes can be chosen to be asymptotically zero. That is, there is nothing special for physical capital as a stock variable, and the taxation rules on factors income hinge on model specifications. A large literature working on this research line drives very different conclusions from different channels, such as Lucas (1990), Zhu (1992), Jones, Manuelli and Rossi (1993, 1997), Aiyagari (1995), Correia (1996), Golosov, Kocherlakota, and Tsyvinski (2003), etc.. On the other hand, a large literature on optimal monetary policy is motivated by Friedman (1969)'s seminal contribution in which he proposed a monetary policy rule that might generate zero nominal interest rates (on assets with a riskless nominal return). There are many supporters and opponents of the Friedman rule. Most of them bases their arguments on the uniform commodity taxation theorem developed by Atkinson

[1] See Judd (1985), Chamley (1986), Zhu (1992), Jones, Manuelli and Rossi (1993, 1997), Aiyagari (1995), Correia (1996), Golosov, Kocherlakota, and Tsyvinski (2003).

[2] See Calvo (1983), Lucas and Stokey (1983), Correia and Teles (1996, 1999), Chari, Christiano and Kehoe (1996).

[3] See Sidrauski (1967), Fischer (1979), Stockman (1981), Chamley (1985).

[4] See Lucas and Stokey (1983), Chary, Christiano, Kehoe (1991), Correia, Nicolini, and Teles (2008).

[5] Jone, Manuelli and Rossi (1997) provide these three conditions: (1) there are no profits from accumuating either capital stock, (2) the tax code is sufficiently rich, and, (3) there is no role for relative prices to reduce the value of fixed sources of income.

and Stiglitz (1972) or the intermediate good optimal taxation rules by Diamond and Mirrlees (1971). Economists come to realize that both theorems cannot apply directly and need additional preconditions, as suggested by Sidrauski (1967), Fischer (1979), Chamley (1985), Kimbrough (1986), Faig (1988), Guidotti and Vegh (1993), Chari, Christiano, and Kehoe (1996), Correia and Teles (1996), and Woodford (1990), etc.. Acturally, a simple argument of the Friedman rule is that a good that is costless to produce should be priced at zero. Correia and Teles (1996) argue that this simple rule about the production cost of money plays the key role in determining the optimality of the Friedman rule. They show that if the production cost of real money approaches zero, the Friedman rule is optimal; if not, the Friedman rule is not optimal and the optimal inflation tax relies on the degree of homogeneity of the transaction function.

In the paper, we utilize the shopping-time monetary model examined by Kimbrough (1986), Faig (1988), Guidotti and Vegh (1993), Chari, Christiano, and Kehoe (1996), Correia and Teles (1996), and Woodford (1990) and focus on examining how two important channels (i.e., the production cost of money and the utility from physical capital) affect the optimal fiscal and monetary policies in four models with different combinations. Many classical results are recovered in the generalized models while many new insights are also developed. In the costless-money model without CIU (Model 1) we recover the famous results of both zero limiting capital income taxation and zero nominal interest rate that developed by Chamley (1986) and Correia and Teles (1996) respectively. In model 2, once money is costly produced, the Friedman rule is not optimal and the optimal inflation rate relies on the optimal tax rate on the labor force employed in the money sector. Meanwhile, the tax structure for capital income is changed accordingly. When the consumer cares about the utility from the capital stock, just as in model 3, the Chamley-Judd zero capital income taxation theorem will not hold, i.e., the limiting capital income tax is not zero in general, even though the Friedman rule is still optimal. Incorporating Capital-in-utility generates a non-pecuniary return (i.e., $u_k(t+1)/(u_c(t+1)-u_l(t+1)H_c(t+1))$) in the asset pricing equaiton (i.e., consumption Euler equation or the no-arbitrage condition), which contradicts the negative effect of capital taxes on the pecuniary return (i.e., $\left(1-\tau_{t+1}^k\right)r_{t+1}+1-\delta$) and makes the sign of the limiting capital tax ambiguous. In model 4, with costly money and CIU, neither the Friedman rule nor the Chamley-Judd zero capital taxation theorem is true in general. The production cost of money and CIU interact in determining the optimal fiscal and monetary policy. The main results of the paper are summerized in Table 1.

	money	ciu	inflation tax	captial income tax
Model 1	costless	without	$I_t = 0$	$\tau^k = 0$
Model 2	costly	without	$I_t \begin{matrix} < \\ = \\ > \end{matrix} (1-\tau^n_{2t})$, if $v \begin{matrix} > \\ = \\ < \end{matrix} 1$	$\tau^k_1 = 0$, $\tau^k_2 = \frac{(r_2-\delta_2)-(r_1-\delta_1)}{r_2}$
Model 3	costless	with	$I_t = 0$	$\tau^k \neq 0$
Model 4	costly	with	$I_t \begin{matrix} < \\ = \\ > \end{matrix} (1-\tau^n_{2t})$, if $v \begin{matrix} > \\ = \\ < \end{matrix} 1$	$\tau^k_1 \neq 0$, $\tau^k_2 = \frac{(r_2-\delta_2)-(r_1-\delta_1)}{r_2} + \frac{r_1}{r_2}\tau^k_1$

The remainder of the paper is organized as follows. A baseline costless-money model without capital-in-utility is examined in cection 2, in which both the inflation tax and the limiting capital income tax are zero. Section 3 investigates a costly-money model without capital-in-utility and finds out that the Friedman rule does not hold. In section 4, we introduce the physical capital stock into the utility function of models in sections 2 and 3 and explore how capital-in-utility changes the results on the optimal monetary policy. Section 5 gives the concluding remarks.

2 The model with costless money (Model 1)

2.1 Model setup

In this section, we consider a monetary economy with capital accumuation and costless money. An infinitely lived representative household likes consumption, leisure streams $\{c_t, l_t\}_{t=0}^{\infty}$ that give higher values of

$$\sum_{t=0}^{\infty} \beta^t u(c_t, l_t), \tag{1}$$

where $\beta \in (0,1)$, $c_t \geq 0$ and $l_t \geq 0$ are consumption and leisure at time t, respectively, and $u_c, u_l > 0$, $u_{cc}, u_{ll} < 0$, and $u_{cl} \geq 0$.[6] The household is endowed with one unit of time per period that can be used for leisure l_t, labor n_t, and shopping s_t, and the time allocation equation is

$$l_t + n_t + s_t = 1. \tag{2}$$

To acquire the consumption good, the household allocates time to shopping. The amount of shopping time s_t is positively related to the consumption level c_t and negatively related to

[6] $u_{ii} < 0$ says that the marginal utility of any commodity decreases in its own consumption, while $u_{ij} > 0$ tells that the marginal utility of one commodity increases in the consumption of any other commodity.

the household's holdings of real money balances $m_{t+1}/p_t \equiv \widehat{m}_{t+1}$. Specifically, the shopping or transaction technology is

$$s_t = H\left(c_t, \frac{m_{t+1}}{p_t}\right), \tag{3}$$

where $H, H_c, H_{cc}, H_{m/p,m/p} \geq 0$, $H_{m/p}, H_{c,m/p} \leq 0$.[7] The shopping technology is assumed to be homogeneous of degree $v \geq 0$ in consumption c_t and real money balances m_{t+1}/p_t:

$$s_t = H\left(c_t, \widehat{m}_{t+1}\right) = c_t^v H\left(1, \frac{\widehat{m}_{t+1}}{c_t}\right), \text{ for } c_t > 0.$$

By Euler's theorem we have

$$H_c\left(c_t, \widehat{m}_{t+1}\right) c_t + H_{\widehat{m}}\left(c_t, \widehat{m}_{t+1}\right) \widehat{m}_{t+1} = vH\left(c_t, \widehat{m}_{t+1}\right). \tag{4}$$

For any consumption level c_t, we assume that a point of satiation in real money balances ψc such that

$$H\left(c_t, \widehat{m}_{t+1}\right) = H_{\widehat{m}}\left(c_t, \widehat{m}_{t+1}\right) = 0, \text{ for } \widehat{m}_{t+1} \geq \psi c.$$

It is not worthwhile to increase real balance holdings beyond this point since by doing it, it is not possilbe to save resources.

The single good is produced with labor n_t and capital k_t. Output can be consumed by households, used by the government, or used to augment the capital stock. The resource constraint is

$$c_t + k_{t+1} - (1-\delta) k_t + g_t = F\left(k_t, n_t\right), \tag{5}$$

where $\delta \in (0,1)$ is the rate at which capital depreciates and $\{g_t\}_{t=0}^{\infty}$ is an exogenous sequence of government purchases. We assume a standard increasing and concave production function that exhibits constant return to scale. By Euler's theorem on homogeneous functions, linear homogeneity of F implies $F\left(k_t, n_t\right) = F_k\left(k_t, n_t\right) k_t + F_n\left(k_t, n_t\right) n_t$.

Government. The government finances its stream of purchases $\{g_t\}_{t=0}^{\infty}$ by levying proportional factor taxes on capital and labor income, issuing new debts and printing new currency. In this case with costless money, the production of money requires no resources. The government's budget constraint is

$$g_t = \tau_t^k r_t k_t + \tau_t^n w_t n_t + \frac{B_{t+1}}{R_t} - B_t + \frac{M_{t+1} - M_t}{p_t}, \tag{6}$$

where r_t and w_t are the market-determined rental rate of capital and the wage rate for labor, τ_t^k and τ_t^n are flat-rate, time-varying taxes on earnings from capital and labor, and R_t is the gross rate

[7] $H_{m/p} < 0$ and $H_{m/p,m/p} \geq 0$ show that an increase in the real quantity of money decreases the time spent with transactions at a decreasing rate. The restriction on the second derivative of the transactions function assures that the isoquants of the production function of transactions are convec and that the demand for money depends negatively on the nominal interest rate.

of return on one-period bonds held from t to $t+1$.[8] B_t is government indebtedness to the private sector, denominated in time t goods at the beginning of period t, and M_t is the stock of currency that the government has issued as of the beginning of period t. Interest earnings on bonds are assumed to be tax exempt, which is innocuous for bond exchanges between the government and the private sector. We assume that the government can commit fully and credibly to future tax rates and thus evade the issue of time-consistency raised in Kydland and Prescott (1977).

Households. A representative household chooses $\{c_t, l_t, k_{t+1}, b_{t+1}, m_{t+1}\}_{t=0}^{\infty}$ to maximizes expression (1) subject to the transaction technology (3), the time allocation constraint (2) and the sequence of budget constraints

$$c_t + k_{t+1} + \frac{b_{t+1}}{R_t} + \frac{m_{t+1}}{p_t} = \left(1-\tau_t^k\right) r_t k_t + \left(1-\tau_t^n\right) w_t n_t + \left(1-\delta\right) k_t + b_t + \frac{m_t}{p_t}, \tag{7}$$

for $t \geq 0$, given k_0, b_0 and m_0. Here, $m_{t+1} \geq 0$[9] is nominal money balances held between times t and $t+1$; p_t is the price level; b_t is the real value of one-period government bond holdings that mature at the beginning of period t, denominated in units of time t consumption. Substituting the shopping technology (3) and the time allocation equation (2) into (7), introducing the Lagrange multiplier λ_t, and constructing the Lagrangian, we solve following the first-order conditions:

$$c_t : u_c\left(c_t, l_t\right) = \lambda_t \left[\left(1-\tau_t^n\right) w_t H\left(c_t, \widehat{m}_{t+1}\right) + 1\right], \tag{8}$$

$$k_{t+1} : \lambda_t = \beta \lambda_{t+1} \left[\left(1-\tau_{t+1}^k\right) r_{t+1} + 1 - \delta\right], \tag{9}$$

$$l_t : u_l\left(c_t, l_t\right) = \lambda_t \left(1-\tau_t^n\right) w_t, \tag{10}$$

$$b_{t+1} : \frac{\lambda_t}{R_t} = \beta \lambda_{t+1}, \tag{11}$$

$$m_{t+1} : \left[\left(1-\tau_t^n\right) w_t H_{m/p}\left(c_t, \widehat{m}_{t+1}\right) + 1\right] \frac{\lambda_t}{p_t} = \beta \frac{\lambda_{t+1}}{p_{t+1}}. \tag{12}$$

From equation (8) and (10), we have

$$\frac{u_l\left(c_t, l_t\right)}{u_c\left(c_t, l_t\right) - u_l\left(c_t, l_t\right) H_c\left(c_t, \widehat{m}_{t+1}\right)} = \left(1-\tau_t^n\right) w_t, \tag{13}$$

which displays that the marginal rate of substitution of consumption and leisure equals their (after-tax) price ratio. Substituting equation (8) and the after-tax wage $\left(1-\tau_t^n\right) w_t$ in (13) into (9) leads to the consumption Euler equation

$$\left[\begin{array}{c} u_c\left(c_t, l_t\right) - \\ u_l\left(c_t, l_t\right) H_c\left(c_t, \widehat{m}_{t+1}\right) \end{array}\right] = \beta \left[\begin{array}{c} u_c\left(c_{t+1}, l_{t+1}\right) - \\ u_l\left(c_{t+1}, l_{t+1}\right) H_c\left(c_{t+1}, \widehat{m}_{t+2}\right) \end{array}\right] \left[\left(1-\tau_{t+1}^k\right) r_{t+1} + 1 - \delta\right]. \tag{14}$$

[8] One-period government bond cannot be accumulated like the private capital. Hence we do not introduce the government bond into the utility function of the representative consumer in Models 3 and 4 with capital-in-utility.

[9] Maximization of expression (1) is subject to $m_{t+1} \geq 0$ for all $t \geq 1$, since households cannot issue money; however, no restrictions on the sign of b_{t+1} for $t \geq 1$.

Equations (9) and (11) implies the no-arbitrage condition for trades in capital and bonds that ensures that these two assets have the same rate of return, namely,

$$R_t = \left(1 - \tau_{t+1}^k\right) r_{t+1} + 1 - \delta. \tag{15}$$

By substituting equation (15) into equation (14), we obtain an expression for the real interest rate,

$$R_t = \frac{\left[u_c\left(c_t, l_t\right) - u_l\left(c_t, l_t\right) H_c\left(t\right)\right]}{\beta\left[u_c\left(c_{t+1}, l_{t+1}\right) - u_l\left(c_{t+1}, l_{t+1}\right) H_c\left(t+1\right)\right]}. \tag{16}$$

The combination of equations (11) and (12) yields

$$\frac{R_t - R_{mt}}{R_t} = -\left(1 - \tau_t^n\right) w_t H_{m/p}\left(t\right) \left(= \frac{i_t}{1 + i_t} \equiv I_t\right), \tag{17}$$

which sets the cost equal to the benefit of the marginal unit of real money balances held from t to $t+1$, all expressed in time t consumption goods. Note that $R_m t \equiv p_t/p_t + 1$ is the real gross return on money held from t to $t+1$, that is, the inverse of the inflation rate, and $1+i_t \equiv R_t/R_m t$ is the gross nominal interest rate. The real return on money $R_m t$ must be less than or equal to the return on bonds R_t, because otherwise agents would be able to make arbitrarily large profits by choosing arbitrarily large money holdings financed by issuing bonds. In other words, the net nominal interest rate i_t cannot be negative, i.e., $i_t \geq 0$.

Firms. In each period, the representative firm takes (r_t, w_t) as given, rents capital and labor from households, and maximizes profits, $F\left(k_t, n_t\right) - r_t k_t - w_t n_t$. The first-order conditions for this problem are

$$F_k\left(k_t, n_t\right) = r_t, F_n\left(k_t, n_t\right) = w_t. \tag{18}$$

In words, inputs should be employed until the marginal product of the last unit is equal to its rental price. With constant return to scale, we get the standard result that pure profits are zero.

2.2 Primal approach to the Ramsey problem

We examine the second-best fiscal and monetary policy by utilizing the Primal approach developed by Atkinson and Stiglitz (1980) and Lucas and Stokey (1983). For this purpose we present the following useful definitions.

Definition: A *competive equilibrium* is an allocation $\{c_t, l_t, n_t, s_t, k_{t+1}, b_{t+1}, m_{t+1}\}_{t=0}^{\infty}$, a price system $\{p_t, w_t, r_t, R_t\}_{t=0}^{\infty}$, and a government policy $\left\{g_t, \tau_t^k, \tau_t^n, B_{t+1}, M_{t+1}\right\}_{t=0}^{\infty}$ such that (a) given the price system and the government poicy, the allocation solves both the firm's problem and the household's problem with $b_t = B_t$ and $m_t = M_t$ for all $t \geq 0$; (b) given the allocation and the price system, the government policy satisfies the sequence of government budget constraint (6) for all $t \geq 0$; (3) the time allocation constraint (2) and the resource constraint (5) are safisfied for all $t \geq 0$.

There are many competitive equilibria, indexed by different government policies. This multiplicity motivates the Ramsey problem.

Definition. Given k_0, b_0 and m_0, the *Ramsey problem* is to choose a competitive equilibrium that maximizes expression (1).

To construct the Ramsey problem, we firstly substitute repeatedly the flow budget constraint (7) to derive the household's present-value budget constraint

$$\sum_{t=0}^{\infty} q_t^0 \left(c_t + \frac{i_t}{1+i_t}\widehat{m}_{t+1}\right) = \sum_{t=0}^{\infty} q_t^0 \left(1-\tau_t^n\right) w_t n_t + \left[\left(1-\tau_0^k\right) r_0 + 1 - \delta\right] k_0 + b_0 + \frac{m_0}{p_0}, \tag{19}$$

where $q_t^0 = \prod_{i=0}^{t-1} R_i^{-1}$ is the Arrow-Debreu price, with the numeriare $q_0^0 = 1$, and we have imposed the transversality conditions, $\lim_{T\to\infty} q_T^0 \frac{b_{T+1}}{R_T} = 0$ and $\lim_{T\to\infty} q_T^0 \widehat{m}_{T+1} = 0$. Putting (16) in the definition of the Arrow-Debreu price leads to

$$q_t^0 = \beta^t \frac{u_c\left(c_t, l_t\right) - u_l\left(c_t, l_t\right) H_c\left(t\right)}{u_c\left(c_0, l_0\right) - u_l\left(c_0, l_0\right) H_c\left(0\right)}. \tag{20}$$

Substituting (13), (17), (20) and (4) into the present-value budget constraint (19) and rearranging it, we obtain the implementability condition

$$\sum_{t=0}^{\infty} \beta^t \left[u_c\left(c_t, l_t\right) c_t - u_l\left(c_t, l_t\right)\left(1 - l_t - \left(1-v\right) H\left(c_t, \widehat{m}_{t+1}\right)\right)\right] = A_1, \tag{21}$$

where A_1 is given by

$$A_1 = A\left(c_0, l_0, k_0, b_0, m_0, \tau_0^k\right) = \left[u_c\left(c_0, l_0\right) - u_l\left(c_0, l_0\right) H_c\left(0\right)\right]\left[\left(\left(1-\tau_0^k\right) r_0 + 1 - \delta\right) k_0 + b_0 + \frac{m_0}{p_0}\right].$$

The Ramsey problem is to maximize expression (1) subject to the implementability condition (21) and the feasibility constraint (5). Let ϕ be a Lagrange multiplier on equation (21) and define

$$U\left(c_t, l_t, \widehat{m}_{t+1}, \phi\right) = u\left(c_t, l_t\right) + \phi\left[u_c\left(c_t, l_t\right) c_t - u_l\left(c_t, l_t\right)\left(1 - l_t - \left(1-v\right) H\left(c_t, \widehat{m}_{t+1}\right)\right)\right].$$

Then we construct the Lagrangian

$$\mathcal{L} = \sum_{t=0}^{\infty} \beta^t \left\{U\left(c_t, l_t, \widehat{m}_{t+1}, \phi\right) + \theta_t\left[F\left(k_t, 1 - l_t - H\left(c_t, \widehat{m}_{t+1}\right)\right) + \left(1-\delta\right) k_t - c_t - g_t - k_{t+1}\right]\right\} - \phi A_1,$$

where $\{\theta_t\}_{t=0}^{\infty}$ is a sequence of Lagrange multipliers. After deriving the first-order conditions with respect to c_t, l_t, k_{t+1}, and $\widehat{m}_{t+1}$, for $t \geq 0$, we combine them and obtain the following optimality conditions:

$$\frac{U_l\left(c_t, l_t, \widehat{m}_{t+1}, \phi\right)}{U_c\left(c_t, l_t, \widehat{m}_{t+1}, \phi\right)} = \frac{F_n\left(k_t, n_t\right)}{F_n\left(k_t, n_t\right) H_c\left(c_t, \widehat{m}_{t+1}\right) + 1}, \tag{22}$$

$$\frac{U_c\left(c_t, l_t, \widehat{m}_{t+1}, \phi\right)}{\left[F_n\left(k_t, n_t\right) H_c\left(c_t, \widehat{m}_{t+1}\right) + 1\right]} = \frac{\beta U_c\left(c_{t+1}, l_{t+1}, \widehat{m}_{t+2}, \phi\right)}{\left[F_n\left(k_{t+1}, n_{t+1}\right) H_c\left(c_{t+1}, \widehat{m}_{t+2}\right) + 1\right]}\left[F_k\left(k_{t+1}, n_{t+1}\right) + 1 - \delta\right], \tag{23}$$

$$\left[\left(v\phi + 1\right) u_l\left(c_t, l_t\right) + \phi\left(u_{cl}\left(c_t, l_t\right) c_t - u_{ll}\left(c_t, l_t\right) n_t\right)\right] H_{\widehat{m}}\left(c_t, \widehat{m}_{t+1}\right) = 0, t \geq 0. \tag{24}$$

2.3 Model solution and intuitions

Proposition 1 *In a monetary model with capital accumulation and costless money, the optimal monetary policy is the Friedman rule. That is, the optimal inflation tax is zero, i.e., $I_t = 0$, which implies that the nominal interest rate is also zero, i.e., $i_t = 0$. In the long run, the optimal capital income tax is zero, i.e., $\tau^k = 0$.*

Proof The first-order condition for real balances (24) is satisfied when either $H_{\widehat{m}}(t) = 0$ or

$$(\upsilon\phi + 1) u_l(c_t, l_t) + \phi(u_{cl}(c_t, l_t) c_t - u_{ll}(c_t, l_t) n_t) = 0. \tag{25}$$

The Lagrange multiplier ϕ of the implementability condition, which measures the utility costs of distorting taxes, is nonnegative. It is easy to know that the left side of equation (25) is strictly positive, which displays that equation (25) cannot be hold and the solution has to be $H_{\widehat{m}}(t) = 0$. By equation (17), we know that the optimal inflation tax is zero, $I_t = 0$, which implies that the net nominal interest rate is zero, i.e., $i_t = 0$. In other words, the social planner follows the Friedman rule and satiates the economy with real money balances. To examine the limiting capital income tax, we consider the special case in which there is a $T \geq 0$ for which $g_t = g$ for all $t \geq T$. Assume that there exists a stationary solution to the Ramsey problem and that it converges to a time-invariant allocation, so that c, l, n, $\widehat{m}$ and k are constant after some time. Then because $U_c(t)$ and $[F_n(t) H_c(t) + 1]$ converge to constants, the stationary version of equation (23) implies

$$1 = \beta [F_k(k, n) + 1 - \delta].$$

Now because c, l and $\widehat{m}$ are constant in the limit, equations (14) and (15) imply that $R_t \left(= q_t^0 / q_{t+1}^0\right) \rightarrow \beta^{-1}$ and

$$1 = \beta \left[\left(1 - \tau^k\right) F_k(k, n) + 1 - \delta\right].$$

Combining the above two equalities implies that $\tau^k = 0$. $\square$

As is shown above, the baseline model can be looked as an extension of Correia and Teles (1996) by incorporating capital accumulation, or as an extension of Chamley (1986) by introducing money through a transaction technology. Proposition 2.1 shows that, in a combined monetary model with capital accumulation, we recover the optimality of the Friedman rule with zero norminal interest rate and the Chamley-Judd zero capital taxation theorem simultaneously.

Mathematically, the optimality of the Friedman rule in this section is a generalization of other shopping time monetary models, such as Kimbrough (1986), Faig (1988), Guidotti and Vegh (1993), Chari, Christiano, and Kehoe (1996), Correia and Teles (1996), and Woodford (1990). However, the intuition for zero norminal interest rate is closely related to Correia and Teles

(1996). They suggest the simple argument of the Friedman rule, which states that a good that is costless to produce should be priced at zero. Since the marginal cost of the real money balances is zero, its marginal revenues should be zero. That is to say, the net norminal interest rate is zero, i.e., $i_t = 0$, which implies that the optimal inflation tax is also zero, i.e., $I_t = 0$. This argument will be verified in the following costly-money models. In another research line, Sidrauski (1967) and Chamley (1985) develop money-in-utility (MIU) models to establish the optimality of the Friedman rule.

Proposition 1 shows that the limiting capital income tax rate is still zero in shopping-time monetary economies. That is to say, introducing money through transaction technologies has no effect on savings behavior of the consumer and hence does not change zero capital income tax result of the standard RCK model. However, shopping-time models change the optimal allocation of the time endowment of the consumer and distorts the determination of the limiting labor income tax rate. As is shown in appendix A, the new term $u_l H_c$ in the formula of the limiting labor income tax rate makes the sign of the labor tax ambiguous, which implies that the limiting labor income tax may be positive, zero or negative.

3 The model with costly money (Model 2)

3.1 Setup

In this section we derive the optimal monetary policy and limiting capital tax results in the case in which money requires resources for its production. We assume that the government (the central bank)[10] employs labor (n_{2t}) and capital (k_{2t}) to produce real money balances with the constant-return-to-scale (CRS) production technology. The CRS property shows that the government earns no profits from producing real money balances. For the government, producing money provides another financing method for its expenditures[11]; for the individuals, holding money saves (time) resources for more leisure or labor supply. For analytical convenience, we assume the production function for real balances is Cobb-Douglas, namely,

$$\frac{m_{t+1}}{p_t} = k_{2t}^{\alpha_2} n_{2t}^{1-\alpha_2}, \alpha_2 \in (0, 1) . \tag{26}$$

For ease of exposition, we assume that the production technology of the consumption good is also Cobb-Douglas but with different factor income shares from the production function of money, namely, $F(k_{1t}, n_{1t}) = k_{1t}^{\alpha_1} n_{1t}^{1-\alpha_1}$, $\alpha_1 \in (0, 1)$, $\alpha_1 \neq \alpha_2$. We allow for differing tax rates on

[10] In most of the countries in the world, the central bank is the sole producer of the fait money.

[11] The government extracts factors taxes in the sector of money. The net benefits from producing money equal the total revenues $\left(m_{t+1}/p_t + \tau_{2t}^k r_{2t} k_{2t} + \tau_{2t}^n w_{2t} n_{2t}\right)$ minus the production cost $(r_{2t} k_{2t} + w_{2t} n_{2t})$. Due to the CRS property of the production function, the net value is $\left(\tau_{2t}^k r_{2t} k_{2t} + \tau_{2t}^n w_{2t} n_{2t}\right)$. Hence, producing money provides another financial method for government expenditures.

capital and labor used in the production of both the consumption good and money. n_{1t} and n_{2t}, labor used in the production of the consumption good and money, are taxed at rate τ_{1t}^n and τ_{2t}^n, respectively; k_{1t} and k_{2t}, capital used in the production of both sectors, are taxed at rate τ_{1t}^k and τ_{2t}^k, respectively. The transaction technology is also given by (3). The flow budget constraint and time allocation equation for the households are defined, for $t \geq 0$, by

$$c_t + k_{1t+1} + k_{2t+1} + \frac{b_{t+1}}{R_t} + \frac{m_{t+1}}{p_t} = \sum_{i=1,2} \left[\left(1 - \tau_{it}^k\right) r_{it} + (1 - \delta_i) \right] k_{it} + \sum_{i=1,2} (1 - \tau_{it}^n) w_{it} n_{it} + b_t + \frac{m_t}{p_t},$$

and

$$l_t + s_t + n_{1t} + n_{2t} = 1, \tag{27}$$

respectively. The resource constraint[12] is

$$c_t + k_{1t+1} + k_{2t+1} - (1 - \delta_1) k_{1t} - (1 - \delta_2) k_{2t} + g_t = F(k_{1t}, n_{1t}) = k_{1t}^{\alpha_1} n_{1t}^{1-\alpha_1}. \tag{28}$$

No arbitrage requires that the after-tax net rental rates of capital and the after-tax wage rates must be equalized across sectors:

$$\left(1 - \tau_{1t}^k\right) r_{1t} + (1 - \delta_1) = \left(1 - \tau_{2t}^k\right) r_{2t} + (1 - \delta_2), (1 - \tau_{1t}^n) w_{1t} = (1 - \tau_{2t}^n) w_{2t}. \tag{29}$$

Let $k_t = k_{1t} + k_{2t}$ and $n_t = n_{1t} + n_{2t}$. Then the household's flow budget constraint (FBC) can be rewritten as

$$c_t + k_{t+1} + \frac{b_{t+1}}{R_t} + \frac{m_{t+1}}{p_t} = \left(1 - \tau_{1t}^k\right) r_{1t} k_t + (1 - \tau_{1t}^n) w_{1t} n_t + (1 - \delta_1) k_t + b_t + \frac{m_t}{p_t}, \tag{30}$$

which is the same condition as (7) once $\left(\tau_t^k, \tau_t^n, r_t, w_t, \delta\right)$ are replaced by $\left(\tau_{1t}^k, \tau_{1t}^n, r_{1t}, w_{1t}, \delta_1\right)$. The restrictions of the private problem are the budget constraints (30) and the transaction technology (3), for $t \geq 0$. The first-order conditions of the private problem are identical to the ones of Model 1 in Section 2, but with those replacements listed above. We thus have

$$\frac{u_l(c_t, l_t)}{u_c(c_t, l_t) - u_l(c_t, l_t) H_c(t)} = (1 - \tau_{1t}^n) w_{1t}, \tag{31}$$

$$[u_c(t) - u_l(t) H_c(t)] = \beta [u_c(t+1) - u_l(t+1) H_c(t+1)] \left[\left(1 - \tau_{1t+1}^k\right) r_{1t+1} + 1 - \delta_1\right]. \tag{32}$$

$$R_t = \left(1 - \tau_{1t+1}^k\right) r_{1t+1} + 1 - \delta_1 = \frac{[u_c(c_t, l_t) - u_l(c_t, l_t) H_c(c_t, \widehat{m}_{t+1})]}{\beta [u_c(c_{t+1}, l_{t+1}) - u_l(c_{t+1}, l_{t+1}) H_c(c_{t+1}, \widehat{m}_{t+2})]}. \tag{33}$$

[12] Note that combining the household's budget constraint (30) and the government's budget constraint (35), we can recover the resource constraint of the economy (28).

$$\frac{R_t - R_{mt}}{R_t} = -\left(1-\tau_{1t}^n\right) w_{1t} H_{m/p}\left(c_t, \widehat{m}_{t+1}\right) = I_t. \tag{34}$$

The production cost of money is paid by the government, so the government's budget constraint (GBC) is changed as

$$g_t + r_{2t}k_{2t} + w_{2t}n_{2t} + B_t = \sum_{i=1,2}\left(\tau_{it}^k r_{it}k_{it} + \tau_{it}^n w_{it}n_{it}\right) + \frac{B_{t+1}}{R_t} - B_t + \frac{M_{t+1}-M_t}{p_t}. \tag{35}$$

The optimal production for both the consumption good and real balances gives rise to the marginal productivity conditions

$$r_{it} = \alpha_i k_{it}^{\alpha_i - 1} n_{it}^{1-\alpha_i}, w_{it} = \left(1-\alpha_i\right) k_{it}^{\alpha_i} n_{it}^{-\alpha_i}, i = 1, 2. \tag{36}$$

3.2 The Ramsey problem

The Ramsey problem is to choose $\{c_t, l_t, k_{t+1}, k_{2t}, m_{t+1}\}_{t=0}^{\infty}$ to maximize welfare, (1), subject to the implementability condition (21) with $\left(\tau_0^k, r_0, \delta\right)$ replaced by $\left(\tau_{10}^k, r_{10}, \delta_1\right)$, and the resource constraints[13], for $t \geq 0$,

$$c_t + k_{t+1} - \left(1-\delta_1\right)k_t - \left(\delta_1 - \delta_2\right)k_{2t} + g_t = \left(k_t - k_{2t}\right)^{\alpha_1}\left[1 - l_t - H\left(c_t, \widehat{m}_{t+1}\right) - \widehat{m}_{t+1}^{\frac{1}{1-\alpha_2}} k_{2t}^{-\frac{\alpha_2}{1-\alpha_2}}\right]^{1-\alpha_2}. \tag{37}$$

An interior solution of the Ramsey problem requires the following optimality conditions,

$$c_t : U_c\left(c_t, l_t, \widehat{m}_{t+1}, \phi\right) = \theta_t\left[\left(1-\alpha_1\right)\left(\frac{k_t - k_{2t}}{n_{1t}}\right)^{\alpha_1} H_c\left(t\right) + 1\right], t \geq 1 \tag{38}$$

$$l_t : U_l\left(c_t, l_t, \widehat{m}_{t+1}, \phi\right) = \theta_t\left(1-\alpha_1\right)\left(\frac{k_t - k_{2t}}{n_{1t}}\right)^{\alpha_1}, t \geq 1 \tag{39}$$

$$k_{t+1} : \theta_t = \beta\theta_{t+1}\left[\alpha_1\left(\frac{k_{t+1} - k_{2t+1}}{n_{1t+1}}\right)^{\alpha_1 - 1} + 1 - \delta_1\right], t \geq 0, \tag{40}$$

$$k_{2t} : \alpha_1\left(\frac{n_{1t}}{k_t - k_{2t}}\right)^{1-\alpha_1} = \frac{\left(1-\alpha_1\right)\alpha_2}{\left(1-\alpha_2\right)}\left(\frac{k_t - k_{2t}}{n_{1t}}\right)^{\alpha_1}\left(\frac{\widehat{m}_{t+1}}{k_{2t}}\right)^{\frac{1}{1-\alpha_2}} + \left(\delta_1 - \delta_2\right), t \geq 0, \tag{41}$$

$$\widehat{m}_{t+1} : \phi\left(1-v\right)u_l\left(c_t, l_t\right)H_{\widehat{m}}\left(t\right) = \theta_t\left(1-\alpha_1\right)\left(\frac{k_t - k_{2t}}{n_{1t}}\right)^{\alpha_1}\left[H_{\widehat{m}}\left(t\right) + \frac{1}{1-\alpha_2}\left(\frac{\widehat{m}_{t+1}}{k_{2t}}\right)^{\frac{\alpha_2}{1-\alpha_2}}\right], t \geq 0, \tag{42}$$

where ϕ and $\theta_t, t \geq 0$, are the multipliers associated with the implementability condition, (21), and the resource constraints, (37), respectively. Condition (41) is used to determine k_{2t}. Condition (42) differs from condition (24), for the problem without costs of producing money, in the extra term $\left(\widehat{m}_{t+1}/k_{2t}\right)^{\alpha_2/(1-\alpha_2)} / \left(1-\alpha_2\right)$.

[13] Notice that by substituting (27) and (26) into (28), we recover the resource constraint (37).

3.3 Optimal policy and intuitions

Proposition 2 *In a shopping-time monetary model with costly money, the optimal monetary policy follows the rules:*

$$I_t \begin{array}{c} > \\ = \\ < \end{array} \left(1-\tau_{2t}^n\right), \text{ if } v \begin{array}{c} < \\ = \\ > \end{array} 1. \tag{43}$$

In the steady state, the optimal tax rate on physical capital employed in the consumption sector is zero, i.e.,

$$\tau_1^k = 0;$$

and the optimal tax rule on physical capital employed in the money sector follows

$$\tau_2^k \begin{array}{c} > \\ = \\ < \end{array} 0, \text{ if } \left(r_2-\delta_2\right) \begin{array}{c} > \\ = \\ < \end{array} \left(r_1-\delta_1\right).$$

Proof The proof is put in Appendix B. □

Proposition 2 displays that if producing money uses resources of the market economy, then the Friedman rule does not hold generally. That is, the nomianl interest rate is not zero in general, which implies that the optimal inflation rate is not zero. The optimal inflation tax I_t (or the net nominal interest rate $i_t = I_t/\left(1-I_t\right)$) hinges not only on the optimal tax rate on the labor force employed in the money sector, τ_{2t}^n, but also on the degree of homogeneity of the transection technology, v. If $v < 1$, then the optimal inflation rate is larger thant the after-tax This case is similar to and also a generalization of the Correia and Teles (1996) model with capital.

It is shown that the limiting tax rate on capital employed in the consumption sector is also zero, i.e., $\tau_1^k = 0$, while the limiting tax on capital employed in the money sector depends. If the net (of depreciation) return rate of capital in the money sector is larger than the one in the concumption sector, then the government should tax the capital employed in the money sector to remove arbitrage opportunities; if the net return rate of capital employed in the money sector is less than the one in the consumption sector, then the government should subsidy the capital employed in the money sector because the optimal capital tax rate in the consumption sector is always zero. However, if the physical capital has the same net rate of return in both sectors, then the limiting tax rate on the capital employed in the money sector is also zero.

4 Models with capital-in-utility (CIU) (Models 3 and 4)

In this section we introduce physical capital (k_t) in the utility function of the household and investigate its model implications for optimal fiscal and monetary policy. Kurz (1968) pioneered this kind of capital-in-utility (CIU) model in the standard Ramsey-Cass-Koopman (RCK) framework, and examined its implications for growth performance. Later, a large literature on CIU explores its theoretical and empirical implications for savings and growth (Kurz, 1968; Cole, Mailath and Postlewaite, 1992; Zou, 1994, 1995), for business cycle (Boileau and Rebecca, 2007; Karnizova, 2010; Michallat and Saez, 2015), for asset pricing (Bakshi and Chen, 1995; Smith, 2002; Boileau and Rebecca, 2007), for wealth distribution (Luo and Young, 2009), for occupational choice in labor markets (Doepke and Zilibotti, 2008), and for rational bubbles (Zhou, 2016). In this section we will examine how CIU affects optimal fiscal and monetary policy in models with costless money and costely money that we have have discussed in Sections 2 and 3.

Keeping all of the other elements of Models 1 and 2, we introduce physical capital k_t in the utility function of the household in both models respectively. Then the objective function of the representative household is changed as follows:

$$\sum_{t=0}^{\infty} \beta^t u\left(c_t, l_t, k_t\right), \tag{44}$$

where $k_t \geq 0$ is the physical capital stock at time t, and the utility function satisfies $u_k > 0$, $u_{kk} < 0$, $u_{ik} \geq 0$, for $i \in \{c, l\}$. The dependence of the utility function on physical capital stock (or capital-in-utility) with $u_k > 0$ and $u_{kk} < 0$ captures Weber's idea: capital accumulation in a capitalist economy is motivated not only by the maximization of the long-run consumption, but also by the enjoyment (utility) from enhancing wealth itself.[14] Next we will examine the capital-in-utility models with costless money and costly money.

4.1 Costless-money model with capital-in-utility (Model 3)

In this subsection we re-examine the costless-money model presented in Section 2 but with the different objective function (44). The household's problem is maximizing (44), subject to the budget constraint (7), time allocation equation (2), and the shopping technology (3). The first-order necessary conditions with respect to c_t, l_t, b_{t+1} and m_{t+1} are the same as (8), (10), (11), and (12), in which the arguments (c_t, l_t) of the utility function are replaced by (c_t, l_t, k_t). However, the first-order necessary condition with respect to k_{t+1} is changed into

$$k_{t+1} : \lambda_t = \beta \left\{ u_k\left(c_{t+1}, l_{t+1}, k_{t+1}\right) + \lambda_{t+1} \left[\left(1 - \tau_{t+1}^k\right) r_{t+1} + 1 - \delta \right] \right\}, \tag{45}$$

in which the positive term $u_k\left(c_{t+1}, l_{t+1}, k_{t+1}\right) > 0$ stands for a new channel to savings CIU.[15] Combining these first-order conditions and compressing the arguments of (c_t, l_t, k_t) and $(c_t, \widehat{m}_{t+1})$

[14] Zou (1994) calls this kind of capital-in-utility the "the spirit of capitalism" approach, motivating many discussions in the literature. For more economic interpretations on the "spirit of capitalism" approach, please refer to Zou (1994).

[15] This new savings motive can be seen more clearly from the steady state version of equation (47) without taxes, $F_k = 1/\beta - 1 + \delta - u_k / \left(u_c - u_l H_c\right)$. The marginal product of capital F_k is lower than the one in the standard model without capital-in-utility, due to the new positive term $u_k / \left(u_c - u_l H_c\right) (> 0)$ here.

as (t), we obtain

$$\frac{u_l(t)}{u_c(t) - u_l(t) H_c(t)} = (1 - \tau_t^n) w_t, \tag{46}$$

$$[u_c(t) - u_l(t) H_c(t)] = \beta \left\{ u_k(t+1) + [u_c(t+1) - u_l(t+1) H_c(t+1)] \left[\left(1 - \tau_{t+1}^k \right) r_{t+1} + 1 - \delta \right] \right\}, \tag{47}$$

$$R_t = \frac{\left(1 - \tau_{t+1}^k\right) r_{t+1} + 1 - \delta}{\left[1 - \frac{\beta u_k(t+1)}{u_c(t) - u_l(t) H_c(t)}\right]} = \frac{[u_c(t) - u_l(t) H_c(t)]}{\beta [u_c(t+1) - u_l(t+1) H_c(t+1)]}, \tag{48}$$

$$\frac{R_t - R_{mt}}{R_t} = -(1 - \tau_t^n) w_t H_{m/p}(c_t, \widehat{m}_{t+1}) = I_t, \tag{49}$$

Equation (46) tells the marginal rate of substitution between consumption (net of its utility loss for the reduced leisure) and leisure equals their (after-tax) price ratios. In the consumption Euler equation (47), capital-in-utility ($u_k > 0$) generates a non-pecuniary return for physical capital $u_k(t+1) / [u_c(t+1) - u_l(t+1) H_c(t+1)]$, except for the pecuniary after-tax return $\left[\left(1 - \tau_{t+1}^k\right) r_{t+1} + 1 - \delta\right]$. The CIU brings about a positive savings effect opposite to the dissavings effect of capital taxation, which makes the signs of the limiting capital taxes ambiguous. We will examine this in the next subsection. The modified no-arbitrage condition for trades between capital and bond (48) also has a new positive term $\beta u_k(t+1) / [u_c(t) - u_l(t) H_c(t)]$. Equation (49) is exactly the one (17) in the costless-money model without CIU.

The government's budget constraint and the resource constraint are the same as the ones in Section 2, (6) and (5), respectively. We derive the household's present-value budget constraint

$$\sum_{t=0}^{\infty} \left[\begin{array}{c} q_t^0 \left(c_t + \frac{i_t}{1+i_t} \widehat{m}_{t+1} - (1 - \tau_t^n) w_t n_t \right) \\ + q_{t+1}^0 \frac{u_k(t+1) k_{t+1}}{u_c(t+1) - u_l(t+1) H_c(t+1)} \end{array} \right] = \left[\left(1 - \tau_0^k\right) r_0 + 1 - \delta \right] k_0 + b_0 + \frac{m_0}{p_0}, \tag{50}$$

and the implementability condition

$$\sum_{t=0}^{\infty} \beta^t \left\{ u_c(c_t, l_t, k_t) c_t - u_l(c_t, l_t, k_t) [1 - l_t - (1 - v) H(c_t, \widehat{m}_{t+1})] + u_k(c_t, l_t, k_t) k_t \right\} = A_3, \tag{51}$$

where

$$A_3 = [u_c(0) - u_l(0) H_c(0)] \left\{ \left[\left(1 - \tau_0^k\right) r_0 + 1 - \delta \right] k_0 + b_0 + \frac{m_0}{p_0} \right\} + u_k(c_0, l_0, k_0) k_0.$$

The Ramsey problem is to maximize expression (44) subject to the implementability condition (51) and the feasibility constraint (5). Solving the Ramsey problem leads to the following optimality conditions:

$$\frac{U_l(t)}{U_c(t)} = \frac{F_n(t)}{F_n(t) H_c(t) + 1}, t \geq 1 \tag{52}$$

$$\frac{U_c(t)}{[F_n(t) H_c(t) + 1]} = \beta \left\{ U_k(t+1) + \frac{U_c(t+1)}{[F_n(t+1) H_c(t+1) + 1]} [F_k(t+1) + 1 - \delta] \right\}, t \geq 1 \tag{53}$$

$$\{(1+\upsilon\phi)\, u_l(t) + \phi\left[u_{cl}(t)\, c_t - u_{ll}(t)\, n_t + u_{kl}(t)\, k_t\right]\} H_{\widehat{m}}(t) = 0, t \geq 0 \tag{54}$$
$$U_c(0) - \phi A_{3c} = \beta U_c(1) \frac{[F_k(1) + 1 - \delta]}{[F_n(1)\, H_c(1) + 1]}, t = 0$$
$$U_l(0) - \phi A_{3l} = \beta U_c(1)\,[F_k(1) + 1 - \delta] \frac{F_n(0)}{F_n(1)}, t = 0.$$

Compared to (22)-(24) in Section 1, except for a new term $U_k(t+1)$ in equality (53), the arguments of the utility function are changed into (c, l, k). Then we have the following

Proposition 3 *In a costless monetary model with capital-in-utility, the optimal inflation tax is always zero, i.e.,*

$$I_t = 0,$$

which means that the (net) nominal interest rate is equal to zero, i.e., $i = 0$. *Suppose the economy converges to an interior steady state.*[16] *The optimal capital income tax rate in the steady state is*

$$\tau^k = \frac{1}{F_k(u_c - u_l H_c)} \frac{u_c F_n - u_l(F_n H_c + 1)}{u_c \eta_3 - u_l \eta_1} \left[u_k(\eta_1 - \eta_3 H_c) - \eta_2(u_c - u_l H_c)\right], \tag{55}$$

which shows that the optimal capital income tax *is positive, zero, or negative, if and only if* $[u_k(\eta_1 - \eta_3 H_c) - \eta_2(u_c - u_l H_c)]$ *is larger than, equal to, or less than zero. Namely,*

$$\tau^k \begin{array}{c} > \\ = \\ < \end{array} 0 \Leftrightarrow [u_k(\eta_1 - H_c\eta_3) - \eta_2(u_c - u_l H_c)] \begin{array}{c} > \\ = \\ < \end{array} 0.$$

Meanwhile, the formula for the optimal labor income tax rate in the steady state is

$$\tau^n = \frac{\phi}{1+\phi} \frac{1}{(u_c - u_l H_c) F_n} \left[(F_n H_c + 1)\, \eta_3 - F_n \eta_1\right], \tag{56}$$

which shows that the optimal labor income tax *is positive, zero, or negative, if and only if* $[(F_n H_c + 1)\, \eta_3 - F_n \eta_1]$ *is larger than, equal to, or less than zero. Namely,*

$$\tau^n \begin{array}{c} > \\ = \\ < \end{array} 0 \Leftrightarrow [(F_n H_c + 1)\, \eta_3 - F_n \eta_1] \begin{array}{c} > \\ = \\ < \end{array} 0.$$

[16] Different from the standard Ramsey model, we cannot prove the existence and uniqueness of the (non-degenerate) steady state. In this model, the steady-state version of the consumption Euler equation is $1/\beta = u_k/(u_c - u_l H_c) + \left[(1-\tau^k) F_k + 1 - \delta\right]$. The new term $u_k/(u_c - u_l H_c)$ prevents us from solving the steady state easily and brings about the possibility of multiple equilibria, as Kurz (1968) had already talked about this. For this reason, our paper assumes the existence of a steady state and focuses on the optimal taxation problem.

Note that

$$\eta_1 = u_{cc}c - u_{lc}n + u_l(1-v)H_c + u_{kc}k,$$
$$\eta_2 = u_{ck}c - u_{lk}n + u_{kk}k,$$
$$\eta_3 = u_{cl}c - u_{ll}n + u_{kl}k.$$

Proof The proof is placed in Appendix C. □

Proposition 3 tells that in the monetary growth model with capital in utility, the Friedman rule is still optimal, while the Chamley-Judd zero capital taxation theorem does not hold. The optimality of the Friedman rule in this case suggests that the optimal inflationt tax hinges on the production cost of real money balances, independent of capital accumulation and capital in utility. Once the production cost of money approaches zero, the net norminal interest rate will be zero. However, in this case, the limiting capital income tax is in general not zero, since the key term $u_k(\eta_1 - \eta_3 H_c) - \eta_2(u_c - u_l H_c)$ in equation (55) is not equal to zero generally. Thus, if the representative consumer cares about the utility from the physical capital stock, then the Chamley-Judd zero capital income taxation theorem will be overturned. Furthermore, the sign of the optimal capital tax rate relies only on the specification of the utility function and transaction technology rather than the production technology, as is shown by the term $u_k(\eta_1 - \eta_3 H_c) - \eta_2(u_c - u_l H_c)$ in equation (55). The sign of the limiting capital tax rate can be positive, negative or zero, which displays that capital should be taxed, subsidized or left alone in the long run. Similarly, the sign of the optimal labor income tax relies on the sign of the term $[(F_n H_c + 1)\eta_3 - F_n\eta_1]$.[17]

The ambiguous effects on optimal taxation of capital-in-utility come from the non-pecuniary return on capital driven by capital-in-utility, i.e., $u_k/(u_c - u_l H_c)$, in the following asset-pricing equation (i.e., rearranged consumption Euler equation (47)):

$$1 = \underbrace{\beta\frac{u_c(t+1) - u_l(t+1)H_c(t+1)}{u_c(t) - u_l(t)H_c(t)}}_{\text{SDF}}\left\{\underbrace{\frac{u_k(t+1)}{u_c(t+1) - u_l(t+1)H_c(t+1)}}_{\text{non-pecuniary return}} + \underbrace{\left[\left(1-\tau_{t+1}^k\right)r_{t+1} + 1 - \delta\right]}_{\text{pecuniary return}}\right\}. \tag{57}$$

Taxing capital discourages MPK-driven capital accumulation that - in the standard Ramsey settings - leads to lower steady-state capital. However, lower steady-state capital increases the numerator of the non-pecuniary comonent due to $u_{kk} < 0$, and thereby encouraging the capital-in-utility-driven capital accumulation. These two effects are opposite in direction and hard to determine which one dominates. Therefore, we cannot determine the sign of the limiting capital income tax in general. Actually, if the implied change in steady-state is relatively small, the

[17] Li, Wang and Zou (2020) derived very similar results about the indeterminacy of the limiting factor income taxation in a non-monetary model with capital-in-utility.

whole non-pecuniary term can increase, thereby encouraging "capital–in-utility-driven" capital accumulation. As a result, taxing capital might be relatively less or more distortionary in the CIU specification than in the standard neoclassical model, capital tax has an ambiguous effect on steady-state capital accumulation in a model with capital-in-utility and hence the limiting capital tax rate may have any sign. In particular, if capital is not in utility (i.e., $u_k = 0$, which implies that $u_k(\eta_1 - \eta_3 H_c) - \eta_2(u_c - u_l H_c) = 0$), then the limiting capital income tax is zero (i.e., $\tau^k = 0$). Meanwhile, the formula for the limiting labor income tax is degenerated to the one in Model 1. The degenerate case without CIU is essentially Model 1 that examined in Section 2. In the case without CIU, the asset-pricing equation is degenerated as the standard one:

$$1 = \underbrace{\beta \frac{u_c(t+1) - u_l(t+1) H_c(t+1)}{u_c(t) - u_l(t) H_c(t)}}_{\text{SDF}} \underbrace{\left[\left(1 - \tau_{t+1}^k\right) r_{t+1} + 1 - \delta\right]}_{\text{pecuniary return}},$$

which shows that taxing capital leads to lower levels of physical capital and does harm to long run economic growth. Hence, physical capital should be untaxed. These results correspond to the Chamley-Judd zero capital income taxation theorem in a neoclassical growth model without or with money.

Comparing our model to the one without capital-in-utility, we know that zero capital tax results do not hold in any case. As is argued in Jones, Manuelli and Rossi (1997), there is nothing special for physical capital as a stock variable. Meanwhile, the limiting tax on labor income (as a flow variable) is also ambiguous and its sign depends on specifications on both the utility function and the production technology.

To develop more intuitions for optimal capital taxation, we assume that there is no money in the economy (i.e., $s_t = H(c_t, \widehat{m}_{t+1}) = 0$) the instantaneous utility function of the representative consumer is additively separable with respect to its three arguments, namely,

$$u(c, l, k) = \gamma_c u(c) + \gamma_l v(l) + \gamma_k w(k), \gamma_i > 0, i \in \{c, l, k\}. \tag{58}$$

Hence we know that $u' > 0$, $u'' < 0$, $v' > 0$, $v'' < 0$, $w' > 0$, and $w'' < 0$, due to the assumed properties of $u(c, l, k)$. Then we have the following

Corollary 1 *Assume that there is no money and the utility function takes the form in (58). The limiting capital income tax is positive, zero, or negative, if and only if the capital elasticity of marginal utility of capital is less than, equal to, or larger than the consumption elasticity of marginal utility of consumption. That is,*

$$\tau^k \begin{array}{c} > \\ = \\ < \end{array} 0 \Longleftrightarrow \frac{w''(k) k}{w'(k)} \begin{array}{c} < \\ = \\ > \end{array} \frac{u''(c) c}{u'(c)}.$$

Meanwhile, the optimal labor income tax is nonnegative, namely,

$$\tau^n = \frac{1}{u_c F_n} \frac{\Phi}{1+\Phi} \left(-u_{ll} n - u_{cc} c F_n\right) \geq 0.$$

Furthermore, if the utility function is constant-relative-risk-aversion (CRRA), i.e., $u(c,l,k) = \gamma_c \left(c^{1-1/\theta_c} - 1\right) / (1-1/\theta_c) + \gamma_l \left(l^{1-1/\theta_l} - 1\right) / (1-1/\theta_l) + \gamma_k \left(k^{1-1/\theta_k} - 1\right) / (1-1/\theta_k)$, *where* $\theta_i, i \in \{c,l,k\}$ *are the constant elasticities of intertemporal substitution (EIS) for three types of utility goods, then we know that*

$$\tau^k \begin{array}{c} > \\ = \\ < \end{array} 0 \Longleftrightarrow \theta_k \begin{array}{c} < \\ = \\ > \end{array} \theta_c.$$

Proof We easily prove Corollary 1 by substitution. □

Corollary 1 explores a special case with additively separable utility functions by assuming away money and shopping technologies. It is shown that optimal capital taxes depend on the relative values of the marginal utility elasticities for different utility goods (consumption goods and capital goods). If the marginal utility of capital responses more sensitively to one percent change of capital stock, compared to the response of the marginal utility of consumption to one percent change of consumption, then the optimal capital tax will be positive; if not, the optimal capital tax will be negative. If they (consumption and capital goods) have the same sensitivity, then the optimal capital tax will be zero. Simple calculations gives rise to $\epsilon_c = -u'(c)/u''(c)c$, $\epsilon_n = -v'(1-n)/v''(1-n)n$, and $\epsilon_k = -w'(k)/w''(k)k$. In particular, if the additively separable utility functions are constant-relative-risk-aversion (CRRA), namely,

$$u(c,l,k) = \gamma_c \frac{\left(c^{1-1/\theta_c} - 1\right)}{(1-1/\theta_c)} + \gamma_l \frac{\left(l^{1-1/\theta_l} - 1\right)}{(1-1/\theta_l)} + \gamma_k \frac{\left(k^{1-1/\theta_k} - 1\right)}{(1-1/\theta_k)},$$

where $\theta_i, i \in \{c,l,k\}$ are the constant elasticities of intertemporal substitution (EIS) for three types of utility goods, then we have that

$$\tau^k \begin{array}{c} > \\ = \\ < \end{array} 0 \Longleftrightarrow \theta_k \begin{array}{c} < \\ = \\ > \end{array} \theta_c.$$

That is, if the EIS of the consumption goods, θ_c, is larger than (equal to, or less than) that of the capital goods, θ_k, then the limiting capital income tax is positive (zero, or negative).

4.2 The costly-money model with capital-in-utility (Model 4)

In this section we examine a costly-money model with capital-in-utility. We formulate this case by either introducing physical capital in the utility function in Model 2 or incorporating the production technology of real money balances in Model 3.

The household's optimization problem is to maximize the objective function, (44), subject to the budget constraint, (30), the time allocation equation, (27), and the shopping technology, (3). The first-order necessary conditions are

$$\frac{u_l(c_t, l_t, k_t)}{u_c(c_t, l_t, k_t) - u_l(c_t, l_t, k_t) H_c(c_t, \widehat{m}_{t+1})} = (1 - \tau_{1t}^n) w_{1t}, \tag{59}$$

$$u_c(t) - u_l(t) H_c(t) = \beta \left\{ u_k(t+1) + \left[\left(1 - \tau_{1t+1}^k\right) r_{1t+1} + 1 - \delta_1 \right] [u_c(t+1) - u_l(t+1) H_c(t+1)] \right\}, \tag{60}$$

$$R_t = \frac{\left(1 - \tau_{1t+1}^k\right) r_{1t+1} + 1 - \delta}{\left[1 - \frac{\beta u_k(t+1)}{u_c(t) - u_l(t_t) H_c(c_t, \widehat{m}_{t+1})}\right]} = \frac{u_c(c_t, l_t, k_t) - u_l(c_t, l_t, k_t) H_c(c_t, \widehat{m}_{t+1})}{\beta [u_c(c_{t+1}, l_{t+1}, k_{t+1}) - u_l(c_{t+1}, l_{t+1}, k_{t+1}) H_c(c_{t+1}, \widehat{m}_{t+2})]}, \tag{61}$$

$$\frac{R_t - R_{mt}}{R_t} = -(1 - \tau_{1t}^n) w_{1t} H_{m/p}(c_t, \widehat{m}_{t+1}) = I_t. \tag{62}$$

Compared to the first-order conditions (31)-(34) in Model 2, here there is a new term about u_k in equalities (60) and (61), and the arguments in the utility function are (c, l, k).

The household's present-value budget constraint and the implementability condition are (50) and (51), respectively, with $\left(\tau_0^k, r_0, \delta, k_0\right)$ replaced by $\left(\tau_{10}^k, r_{10}, \delta_1, k_{10}\right)$. The resource constraint is the same as the one in Model 2, (37).

The Ramsey problem is to maximize the objective function, (44), subject to the implementability condition, (51), and the resource constraint, (37). The associated optimality conditions are

$$c_t : U_c(t) = \theta_t \left[(k_t - k_{2t})^{\alpha_1} (1 - \alpha_1) n_{1t}^{-\alpha_1} H_c(c_t, \widehat{m}_{t+1}) + 1 \right], t \geq 1$$

$$l_t : U_l(t) = \theta_t (k_t - k_{2t})^{\alpha_1} (1 - \alpha_1) n_{1t}^{-\alpha_1}, t \geq 1$$

$$k_{t+1} : \theta_t = \beta \left\{ U_k(t+1) + \theta_{t+1} \left[\alpha_1 (k_{t+1} - k_{2t+1})^{\alpha_1 - 1} n_{1t+1}^{1-\alpha_1} + 1 - \delta_1 \right] \right\}, t \geq 0$$

$$k_{2t} : \alpha_1 \left(\frac{k_t - k_{2t}}{n_{1t}} \right)^{\alpha_1 - 1} = \frac{(1 - \alpha_1) \alpha_2}{(1 - \alpha_2)} \left(\frac{k_t - k_{2t}}{n_{1t}} \right)^{\alpha_1} \left(\frac{\widehat{m}_{t+1}}{k_{2t}} \right)^{\frac{1}{1-\alpha_2}} + (\delta_1 - \delta_2)$$

$$\widehat{m}_{t+1} : \phi (1 - v) u_l(c_t, l_t, k_t) H_{\widehat{m}}(c_t, \widehat{m}_{t+1}) = \theta_t (1 - \alpha_1) \left(\frac{k_t - k_{2t}}{n_{1t}} \right)^{\alpha_1} \left[H_{\widehat{m}}(c_t, \widehat{m}_{t+1}) + \frac{1}{1 - \alpha_2} \left(\frac{\widehat{m}_{t+1}}{k_{2t}} \right)^{\frac{\alpha_2}{1-\alpha_2}} \right], t \geq$$

where

$$U(t) = u(c_t, l_t, k_t) + \phi [u_c(c_t, l_t, k_t) c_t - u_l(c_t, l_t, k_t) [1 - l_t - (1 - v) H(c_t, \widehat{m}_{t+1})] + u_k(c_t, l_t, k_t) k_t],$$

$$U_c(t) = u_c(t) + \phi [u_{cc}(t) c_t + u_c(t) - u_{lc}(t) n_t + u_l(t)(1 - v) H_c(t) + u_{kc}(t) k_t],$$

$$U_l(t) = u_l(t) + \phi [u_{cl}(t) c_t - u_{ll}(t) n_t + u_l(t) + u_{kl}(t) k_t],$$

$$U_k(t+1) = u_k(t+1) + \phi [u_{ck}(t+1) c_{t+1} - u_{lk}(t+1) n_{t+1} + u_{kk}(t+1) k_{t+1} + u_k(t+1)].$$

Then we have the following

Proposition 4 *In a costly-money model with capital-in-utility, the optimal monetary policy follows the following rules:*

$$I_t \;\substack{>\\=\\<}\; \left(1-\tau^n_{2t}\right), \text{ if } v \;\substack{<\\=\\>}\; 1. \tag{63}$$

Suppose the economy converges to an interior steady state. In the steady state, the formula of the limiting tax on capital employed in the consumption sector is

$$\tau^k_1 = \frac{1}{F_{k_1}\left(u_c - u_l H_c\right)} \frac{u_c F_{n_1} - u_l\left(F_{n_1} H_c + 1\right)}{u_c \eta_3 - u_l \eta_1} \left[u_k\left(\eta_1 - \eta_3 H_c\right) - \eta_2\left(u_c - u_l H_c\right)\right].$$

It is positive, zero, or negative, if and only if $\left[u_k\left(\eta_1 - \eta_3 H_c\right) - \eta_2\left(u_c - u_l H_c\right)\right]$ *is larger than, equal to, or less than zero, i.e.,*

$$\tau^k_1 \;\substack{>\\=\\<}\; 0 \Leftrightarrow \left[u_k\left(\eta_1 - H_c\eta_3\right) - \eta_2\left(u_c - u_l H_c\right)\right] \;\substack{>\\=\\<}\; 0.$$

The formula of the limiting tax on capital employed in the money sector is

$$\tau^k_2 = \frac{\left(r_2 - \delta_2\right) - \left(r_1 - \delta_1\right)}{r_2} + \frac{r_1}{r_2}\tau^k_1. \tag{64}$$

Then we know that

$$\tau^k_2 \;\substack{>\\=\\<}\; \frac{\left(r_2 - \delta_2\right) - \left(r_1 - \delta_1\right)}{r_2}, \text{ if } \tau^k_1 \;\substack{>\\=\\<}\; 0.$$

Proof The proof of the optimal monetary policy rules here is very similar to the case with costly money in Model 2, and the derivations of τ^k_1 and τ^n are very similar to the case with costless money in Model 3. Hence we omit them here. The results on τ^k_2 are due to the no-arbitrage condition of factor mobility, i.e. (29). □

Proposition 4 displays that in the model with costly money and CIU, the Friedman rule is not optimal in genenal and the optimal inflation tax depends on the optimal tax on the labor employed in the money sector, τ^n_{2t}, and the degree of homogeneity of the transaction function, v. Note that the optimal tax rates τ^n_{2t} in the expressions of (43) and (63) are different, since they are endogenously determined in the analytical framework of Ramsey taxation.

In this case, the limiting taxes on the capital income are more complicated. The sign of the limiting tax on capital employed in the consumption good is determined by the sign of the expression, $\left[u_k\left(\eta_1 - H_c\eta_3\right) - \eta_2\left(u_c - u_l H_c\right)\right]$, which is indeterminate. The reason for this determinacy is very similar to Model 3, which is omitted here. The limiting tax rate on the capital employed in the money sector relies on two factors: the relative values of net real returns of capital employed in the two sectors, $[(r_2-\delta_2)-(r_1-\delta_1)]/r_2$ and r_1/r_2, and the limiting tax rate on the capital employed in the consumption sector, τ_1^k. If the limiting tax τ_1^k is zero (i.e., $\tau_1^k=0$), then the limiting tax is equal to the difference between net real returns of capital in both sectors (i.e., $\tau_2^k=[(r_2-\delta_2)-(r_1-\delta_1)]/r_2$). If the limiting tax τ_1^k is positive (i.e., $\tau_1^k>0$), then the limiting tax is larger than the difference of net real returns of capital in both sectors (i.e., $\tau_2^k>[(r_2-\delta_2)-(r_1-\delta_1)]/r_2$); and vice versa.

5 Conclusion

In the paper we reexamine the optimal fiscal and monetary policy in a combined shopping-time monetary model with capital accumulation. With different combinations of two important channels (i.e., the production cost of money and capital in utility), we examine four models in which we derive many interesting results. In the costless-money model without CIU, we recover the classical results in dynamic taxation theory and optimal monetary theory: both the Friedman rule and the Chamley-Judd zero capital income taxation theorem hold. When producing money is costly, the Friedman rule is not optimal and the optimal inflation rate relies on the optimal tax rate on the labor force employed in the money sector and the homogeneity of the transaction technology. Meanwhile, the tax structure for capital income is changed accordingly. When the consumer cares about the utility from the physical capital stock, the Chamley-Judd theorem will not hold and the limiting taxes on the physical capital deviate from zero due to the tradeoffs between the non-pecuniary return and pecuniary return of capital accumulation. In the more complicated Model 4, neither the Friedman rule nor the Chamley-Judd theorem holds. The production cost of money and CIU interact in determining the optimal fiscal and monetary policy.

6 Mathematical appendix

6.1 Appendix A

Proof of Proposition 1. Firstly, we derive the implementability condition. Iterating the household's flow budget constraint from period zero, we have

$$b_0=q_T^0\frac{b_{T+1}}{R_T}+q_T^0\frac{m_{T+1}}{p_T}+\sum_{t=0}^{T}q_t^0c_t+\sum_{t=0}^{T-1}q_t^0\frac{i_t}{1+i_t}\widehat{m}_{t+1}-\sum_{t=0}^{T}q_t^0\left(1-\tau_t^n\right)w_tn_t+$$
$$\sum_{t=0}^{T}q_t^0k_{t+1}-\sum_{t=0}^{T-1}q_{t+1}^0\left[\left(1-\tau_{t+1}^k\right)r_{t+1}+1-\delta\right]k_{t+1}-\left[\left(1-\tau_0^k\right)r_0+1-\delta\right]k_0-\frac{m_0}{p_0}.$$

Using the no-arbitrage condition (15), taking the limits on the both sides with $T\to+\infty$, and imposing the transversality conditions $\lim_{T\to+\infty}q_T^0\frac{b_{T+1}}{R_T}=0$ and $\lim_{T\to+\infty}q_T^0\frac{m_{T+1}}{p_T}=0$, we obtain the present-value budget constraint:

$$\sum_{t=0}^{+\infty} q_t^0 \left[c_t - (1-\tau_t^n) w_t n_t + \frac{i_t}{1+i_t} \widehat{m}_{t+1} \right] = \left[\left(1-\tau_0^k\right) r_0 + 1 - \delta \right] k_0 + b_0 + \frac{m_0}{p_0}.$$

Substituting (13), (17), (20) and (4) into the present-value budget constraint and rearranging it, we obtain the implementability condition, (21):

$$\sum_{t=0}^{\infty} \beta^t \left[u_c(c_t, l_t) c_t - u_l(c_t, l_t) (1 - l_t - (1-v) H(c_t, \widehat{m}_{t+1})) \right] = A,$$

where A is given by

$$A = A\left(c_0, l_0, k_0, b_0, m_0, \tau_0^k\right) = \left[u_c(c_0, l_0) - u_l(c_0, l_0) H_c(0)\right] \left[\left(\left(1-\tau_0^k\right) r_0 + 1 - \delta \right) k_0 + b_0 + \frac{m_0}{p_0} \right].$$

Secondly, we solve the Ramsey problem using the Primal approach. The Ramsey problem is to maximize expression (1) subject to the implementability condition (21) and the feasibility constraint (5). Let ϕ be a Lagrange multiplier on equation (21) and define

$$U(c_t, l_t, \widehat{m}_{t+1}, \phi) = u(c_t, l_t) + \phi \left[u_c(c_t, l_t) c_t - u_l(c_t, l_t) (1 - l_t - (1-v) H(c_t, \widehat{m}_{t+1})) \right].$$

Then we construct the Lagrangian

$$\mathcal{L} = \sum_{t=0}^{\infty} \beta^t \left\{ U(c_t, l_t, \widehat{m}_{t+1}, \phi) + \theta_t \left[F(k_t, 1 - l_t - H(c_t, \widehat{m}_{t+1})) + (1-\delta) k_t - c_t - g_t - k_{t+1} \right] \right\} - \phi A,$$

where $\{\theta_t\}_{t=0}^{\infty}$ is a sequence of Lagrange multipliers. First-order conditions for this problem are

$$c_t : U_c(c_t, l_t, \widehat{m}_{t+1}, \phi) = \theta_t \left[F_n(k_t, n_t) H_c(c_t, \widehat{m}_{t+1}) + 1 \right], t \geq 1$$

$$l_t : U_l(c_t, l_t, \widehat{m}_{t+1}, \phi) = \theta_t F_n(k_t, n_t), t \geq 1$$

$$k_{t+1} : \theta_t = \beta \theta_{t+1} \left[F_k(k_{t+1}, n_{t+1}) + 1 - \delta \right], t \geq 0$$

$$\widehat{m}_{t+1} : \left[\phi (1-v) u_l(c_t, l_t) - \theta_t F_n(k_t, n_t) \right] H_{\widehat{m}}(c_t, \widehat{m}_{t+1}) = 0, t \geq 0$$

$$c_0 : U_c(0) = \theta_0 \left[F_n(k_0, n_0) H_c(c_0, \widehat{m}_1) + 1 \right] + \phi A_c, t = 0$$

$$l_0 : U_l(0) = \theta_0 F_n(0) + \phi A_l, t = 0$$

where

$$U_c(0) = u_c(0) + \phi \begin{bmatrix} u_{cc}(0) c_0 + u_c(0) + u_l(0)(1-v) H_c(0) \\ -u_{lc}(0)(1 - l_0 - (1-v) H(0)) \end{bmatrix},$$

$$U_l(c_0, l_0) = u_l(0) + \phi \left[u_{cl}(0) c_0 - u_{ll}(0)(1 - l_0 - (1-v) H(0)) + u_l(0) \right],$$

$$A_c = \frac{\left[u_{cc}(0) - u_{lc}(0) H_c(0) - u_l(0) H_{cc}(0)\right]}{\left[u_c(0) - u_l(0) H_c(0)\right]} A - \left[u_c(0) - u_l(0) H_c(0)\right] \left(1 - \tau_0^k\right) F_{kn}(0) H_c(0) k_0,$$

$$A_l = \frac{[u_{cl}(0) - u_{ll}(0) H_c(0)]}{[u_c(0) - u_l(0) H_c(0)]} A - [u_c(0) - u_l(0) H_c(0)] \left(1 - \tau_0^k\right) F_{kn}(0) k_0.$$

Combining the above first-order conditions, we have the following optimality conditions:

$$\frac{U_l(c_t, l_t, \widehat{m}_{t+1}, \phi)}{U_c(c_t, l_t, \widehat{m}_{t+1}, \phi)} = \frac{F_n(k_t, n_t)}{F_n(k_t, n_t) H_c(c_t, \widehat{m}_{t+1}) + 1}, t \geq 1,$$

$$\frac{U_c(c_t, l_t, \widehat{m}_{t+1}, \phi)}{[F_n(k_t, n_t) H_c(c_t, \widehat{m}_{t+1}) + 1]} = \frac{\beta U_c(c_{t+1}, l_{t+1}, \widehat{m}_{t+2}, \phi)}{[F_n(k_{t+1}, n_{t+1}) H_c(c_{t+1}, \widehat{m}_{t+2}) + 1]} [F_k(k_{t+1}, n_{t+1}) + 1 - \delta], t \geq 1,$$

$$[(\upsilon\phi + 1) u_l(c_t, l_t) + \phi(u_{cl}(c_t, l_t) c_t - u_{ll}(c_t, l_t) n_t)] H_{\widehat{m}}(c_t, \widehat{m}_{t+1}) = 0, t \geq 0,$$

$$U_c(0) - \phi A_{1c} = \beta U_c(1) \frac{[F_k(1) + 1 - \delta]}{[F_n(1) H_c(1) + 1]}, t = 0,$$

$$U_l(0) - \phi A_{1l} = \beta U_c(1) [F_k(1) + 1 - \delta] \frac{F_n(0)}{F_n(1)}, t = 0.$$

Thirdly, the optimality of the Friedman rule and zero capital income taxation is verified in the main tex in Section 2.1. Finally, from the first order conditions with respect to c_t and l_t, in the steady state, we have

$$u_c F_n - u_l(F_n H_c + 1) = \frac{\phi}{1+\phi} [(F_n H_c + 1)(u_{cl} c - u_{ll} n) - F_n(u_{cc} c - u_{lc} n + u_l(1-v) H_c)].$$

Solving from (13) and (18) gives rise to

$$u_c F_n - u_l(F_n H_c + 1) = (u_c - u_l H_c) F_n \tau^n.$$

Combining the above equalities leads to the formula for the limiting labor income tax:

$$\tau^n = \frac{\phi}{1+\phi} \frac{(F_n H_c + 1)(u_{cl} c - u_{ll} n) - F_n [u_{cc} c - u_{lc} n + u_l(1-v) H_c]}{(u_c - u_l H_c) F_n},$$

which may be positive, negative or zero. □

6.2 Appendix B

Proof of Proposition 2. From (42), the solution is

$$H_{\widehat{m}}(t) = \frac{\frac{\theta_t w_{1t}}{1-\alpha_2} \left(\frac{\widehat{m}_{t+1}}{k_t - k_{1t}}\right)^{\frac{\alpha_2}{1-\alpha_2}}}{\phi(1-v) u_l(c_t, l_t) - \theta_t w_{1t}}.$$

Notice that, as we saw in Section 2.1, $\phi(1-v) u_l(c_t, l_t) - \theta_t w_{1t} \neq 0$. Combining the above equation with the necessary condition (34) of the private problem gives us the equality

$$\frac{\frac{\theta_t w_{1t}}{1-\alpha_2} \left(\frac{\widehat{m}_{t+1}}{k_t - k_{1t}}\right)^{\frac{\alpha_2}{1-\alpha_2}}}{\theta_t w_{1t} - \phi(1-v) u_l(c_t, l_t)} = \frac{I_t}{(1 - \tau_{1t}^n) w_{1t}}.$$

If $v=1$, then we have $\frac{I_t}{(1-\tau_{1t}^n)w_{1t}} = \frac{1}{1-\alpha_2}\left(\frac{\widehat{m}_{t+1}}{k_t-k_{1t}}\right)^{\frac{\alpha_2}{1-\alpha_2}}$. Using the no-arbitrage condition for labor mobility and the production function of money (26), we have that $1-\tau_{2t}^n = I_t$. If $v>1$, then $\frac{I_t}{(1-\tau_{1t}^n)w_{1t}} < \frac{1}{1-\alpha_2}\left(\frac{\widehat{m}_{t+1}}{k_t-k_{1t}}\right)^{\frac{\alpha_2}{1-\alpha_2}}$. By the similar procedure, we obtain $1-\tau_{2t}^n > I_t$. Conversely, if $v>1$, then the similar argument gives us $1-\tau_{2t}^n < I_t$.

Substituting (38) into (40), we have

$$\frac{U_c(t)}{1+F_{n_1}(t)H_c(t)} = \beta\frac{U_c(t+1)}{1+F_{n_1}(t+1)H_c(t+1)}\left[F_{k_1}(t+1)+1-\delta_1\right].$$

In the steady state, it turns out to

$$1=\beta\left(r_1+1-\delta_1\right).$$

Meanwhile, equality (32) turns out to

$$1=\beta\left[\left(1-\tau_1^k\right)r_1+1-\delta_1\right].$$

Combining them gives rise to $\tau_1^k=0$. In the steady state, plugging $\tau_1^k=0$ into (29) leads to

$$\tau_2^k=\frac{(r_2-\delta_2)-(r_1-\delta_1)}{r_2},$$

which establishes the results presented in Proposition 3.1. □

6.3 Appendix C: Proof of Proposition 3

Proof of Proposition 3. The present-value budget constraint is derived as

$$\left(\begin{array}{c}\sum_{t=0}^{+\infty} q_t^0\left[c_t-(1-\tau_t^n)w_tn_t+\frac{i_t}{1+i_t}\widehat{m}_{t+1}\right]\\ +\sum_{t=0}^{+\infty}\left\{q_t^0-q_{t+1}^0\left[\left(1-\tau_{t+1}^k\right)r_{t+1}+1-\delta\right]\right\}k_{t+1}\end{array}\right)=\left[\left(1-\tau_0^k\right)r_0+1-\delta\right]k_0+b_0+\frac{m_0}{p_0}. \tag{65}$$

Combining the no-arbitrage condition (15) and the first-order condition w.r.t c, l and b, we obtain

$$R_t-\left[\left(1-\tau_{t+1}^k\right)r_{t+1}+1-\delta\right]=\frac{u_k(t+1)}{u_c(t+1)-u_l(t+1)H_c(t+1)}.$$

Multiplying the both sides of the above equation with q_{t+1}^0 gives rise to

$$q_t^0-q_{t+1}^0\left[\left(1-\tau_{t+1}^k\right)r_{t+1}+1-\delta\right]=q_{t+1}^0\frac{u_k(t+1)}{u_c(t+1)-u_l(t+1)H_c(t+1)}. \tag{66}$$

Substituting (66) into (65) leads to the present-value budget constraint, (50). Putting (20) with the arguments (c,l,k) in the utility function, (46) and (49) into (50) leads to the implementability condition, (51).

The Ramsey problem is to maximize expression (44) subject to the implementability condition (51) and the feasibility constraint (5). The first-order conditions for this problem are

$$c_t : U_c(t) = \theta_t \left[F_n(t) H_c(c_t, \widehat{m}_{t+1}) + 1 \right], t \geq 1 \tag{67}$$

$$l_t : U_l(t) = \theta_t F_n(t), t \geq 1 \tag{68}$$

$$k_{t+1} : \theta_t = \beta \left\{ U_k(t+1) + \theta_{t+1} \left[F_k(t+1) + 1 - \delta \right] \right\}, t \geq 0 \tag{69}$$

$$\widehat{m}_{t+1} : \left[\phi(1-v) u_l(c_t, l_t, k_t) - \theta_t F_n(t) \right] H_{\widehat{m}}(c_t, \widehat{m}_{t+1}) = 0, t \geq 0 \tag{70}$$

$$c_0 : U_c(0) = \theta_0 \left[F_n(0) H_c(0) + 1 \right] + \phi A_c,$$

$$l_0 : U_l(0) = \theta_0 F_n(0) + \phi A_l,$$

$$k_0 : U_k(0) = \phi A_k - \theta_0 \left[F(0) + (1-\delta) \right],$$

where

$$U(t) = u(c_t, l_t, k_t) + \phi \left[u_c(c_t, l_t, k_t) c_t - u_l(c_t, l_t, k_t) \left[1 - l_t - (1-v) H(c_t, \widehat{m}_{t+1}) \right] + u_k(c_t, l_t, k_t) k_t \right],$$

$$U_c(t) = u_c(t) + \phi \left[u_{cc}(t) c_t + u_c(t) - u_{lc}(t) n_t + u_l(t)(1-v) H_c(t) + u_{kc}(t) k_t \right],$$

$$U_l(t) = u_l(t) + \phi \left[u_{cl}(t) c_t - u_{ll}(t) n_t + u_l(t) + u_{kl}(t) k_t \right],$$

$$U_k(t+1) = u_k(t+1) + \phi \left[u_{ck}(t+1) c_{t+1} - u_{lk}(t+1) n_{t+1} + u_{kk}(t+1) k_{t+1} + u_k(t+1) \right],$$

$$U_c(0) = u_c(0) + \phi \left[\begin{array}{c} u_{cc}(0) c_0 + u_c(0) + u_l(0)(1-v) H_c(0) \\ -u_{lc}(0)(1 - l_0 - (1-v) H(0)) \end{array} \right],$$

$$U_l(0) = u_l(0) + \phi \left[u_{cl}(0) c_0 - u_{ll}(0)(1 - l_0 - (1-v) H(0)) + u_l(0) \right],$$

$$A_{3c} = \frac{\left[u_{cc}(0) - u_{lc}(0) H_c(0) - u_l(0) H_{cc}(0) \right]}{\left[u_c(0) - u_l(0) H_c(0) \right]} A_3 - \left[u_c(0) - u_l(0) H_c(0) \right] \left(1 - \tau_0^k \right) F_{kn}(0) H_c(0) k_0,$$

$$A_{3l} = \frac{\left[u_{cl}(0) - u_{ll}(0) H_c(0) \right]}{\left[u_c(0) - u_l(0) H_c(0) \right]} A_3 - \left[u_c(0) - u_l(0) H_c(0) \right] \left(1 - \tau_0^k \right) F_{kn}(0) k_0.$$

Combining these conditions, we have

$$\frac{U_l(t)}{U_c(t)} = \frac{F_n(t)}{F_n(t) H_c(t) + 1}, t \geq 1$$

$$\frac{U_c(t)}{\left[F_n(t) H_c(t) + 1 \right]} = \beta \frac{U_c(t+1)}{\left[F_n(t+1) H_c(t+1) + 1 \right]} \left[F_k(t+1) + 1 - \delta \right], t \geq 1$$

$$\left\{ (1 + v\phi) u_l(t) + \phi \left[u_{cl}(t) c_t - u_{ll}(t) n_t + u_{kl}(t) k_t \right] \right\} H_{\widehat{m}}(t) = 0, t \geq 0 \tag{71}$$

$$U_c(0) - \phi A_{3c} = \beta U_c(1) \frac{\left[F_k(1) + 1 - \delta \right]}{\left[F_n(1) H_c(1) + 1 \right]}, t = 0$$

$$U_l(0) - \phi A_{3l} = \beta U_c(1)[F_k(1) + 1 - \delta]\frac{F_n(0)}{F_n(1)}, t = 0.$$

From equalities (70) and (71), by the similar procedure to that in the proof of Proposition 1, we conclude that the Friedman rule is optimal, namely, $I_t = i_t = 0$.

To examine the optimal tax rates, we consider the special case in which there is a $T \geq 0$ for which $g_t = g$ for all $t \geq T$. Assume that there exists a stationary solution to the Ramsey problem and that it converges to a time-invariant allocation, so that c, l, $\widehat{m}$ and k are constant after some time. The steady state of the economy can be found by solving the steady state version of equations (67)-(69):

$$u_c + \phi[u_{cc}c + u_c - u_{lc}n + u_l(1-v)H_c + u_{kc}k] = \theta(F_nH_c + 1), \tag{72}$$

$$\theta = \beta[u_k + \phi(u_{ck}c - u_{lk}n + u_{kk}k + u_k) + \theta(F_k + 1 - \delta)], \tag{73}$$

$$u_l + \phi(u_{cl}c - u_{ll}n + u_l + u_{kl}k) = \theta F_n, \tag{74}$$

Equations (72)-(74) are rewritten as

$$F_nH_c + 1 = \frac{1+\phi}{\theta}u_c + \frac{\phi}{\theta}\left[\underbrace{u_{cc}c - u_{lc}n + u_l(1-v)H_c + u_{kc}k}_{\equiv\eta_1}\right], \tag{75}$$

$$1 - \beta(F_k + 1 - \delta) = \beta u_k\frac{1+\phi}{\theta} + \beta\frac{\phi}{\theta}\left(\underbrace{u_{ck}c - u_{lk}n + u_{kk}k}_{\equiv\eta_2}\right), \tag{76}$$

$$F_n = \frac{1+\phi}{\theta}u_l + \frac{\phi}{\theta}\left(\underbrace{u_{cl}c - u_{ll}n + u_{kl}k}_{\equiv\eta_3}\right). \tag{77}$$

We solve equations (75) and (77) for $(1+\phi)/\theta$ and ϕ/θ as follows:

$$\frac{1+\phi}{\theta} = \frac{(F_nH_c + 1)\eta_3 - F_n\eta_1}{u_c\eta_3 - u_l\eta_1}, \tag{78}$$

$$\frac{\phi}{\theta} = \frac{u_cF_n - u_l(F_nH_c + 1)}{u_c\eta_3 - u_l\eta_1}. \tag{79}$$

The steady-state version of consumption Euler equation (47) is changed as

$$[1 - \beta(F_k + 1 - \delta)](u_c - u_lH_c) = \beta u_k - \beta(u_c - u_lH_c)\tau^kF_k. \tag{80}$$

Substituting (78)-(80) into (76) yields us the formula for the capital income tax rate (55), namely,

$$\tau^k = \frac{1}{F_k(u_c - u_lH_c)}\frac{u_cF_n - u_l(F_nH_c + 1)}{u_c\eta_3 - u_l\eta_1}[u_k(\eta_1 - \eta_3H_c) - \eta_2(u_c - u_lH_c)].$$

From equation (79), the term $\frac{u_c F_n - u_l(F_n H_c+1)}{u_c\eta_3 - u_l\eta_1} = \frac{\phi}{\theta}$ is nonnegative, because the Lagrange multiplier ϕ is nonnegative, while the insatiable utility function implies that θ is strictly positive. Note that F_k and $(u_c - u_l H_c)$ are both nonnegative. Hence the sign of the limiting capital income tax is determined completely by the sign of the term $[u_k(\eta_1 - H_c\eta_3) - \eta_2(u_c - u_l H_c)]$.

From equalities (75) and (77), we have

$$u_c F_n - u_l(F_n H_c + 1) = \frac{\phi}{1+\phi}\left[(F_n H_c + 1)\eta_3 - F_n\eta_1\right]. \tag{81}$$

Equation (46) yields us

$$F_n(u_c - u_l H_c)\tau^n = u_c F_n - u_l(F_n H_c + 1). \tag{82}$$

Combining equations (81) and (82), we derive the formula for the optimal labor income tax, i.e., (56), whose sign is also indeterminate. □

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