An interim note on SPS allocation

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Abstract:

We show that the Strong Pareto Superior (SPS) allocation, due to (Marjit & Sarkar, 2017) does not hold in a special scenario.

Introduction:

(Marjit & Sarkar, 2017) have proved the existence of a unique inequality-preserving redistribution allocation in the contract curve. They call the allocation as Strong Pareto Superior allocation (SPS) and argue that the result is true irrespective of whether one considers an absolute measure of inequality or the leftist measure, or a relative measure of inequality or the rightist measure, even though they show that the results are different for both. The measures are called ASPS allocation and RSPS allocation respectively. In this paper, we cast doubt on their result of the existence of SPS allocation in a special scenario, which (Marjit & Sarkar, 2017) may not have specifically addressed.

Background to this paper:

(Venkatasubramanian et al., 2015) (VLS henceforth) had argued that some amount of income inequality is fair. By their way of fairness, they measure inequality by paying people according to their productivity and since people inherently differ in their productivity, they get paid differently. The resulting inequality is 'fair' according to them, which moves away from a leftist measure of being egalitarian in paying everyone equally, the result of which according to them will lead to bifurcations in pay-outs to the employees in the presence of perfectly competitive employees and firms.

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Even when people and firm's actions seem random from epistemic exterior stand point with their underlying behavior of selfishness being in contrast to the teleologically random movements exhibited by a set of molecules in a closed container; they argue that the thermodynamic measure of 'entropy' can be a measure of 'fair' inequality. We refer the reader to VLS paper for a convincing argument on why entropy can be a measure of a 'fairness' imbibed inequality.

(Marjit et al., 2019) gives an empirical illustration of their SPS allocation by using different measures of inequality, namely., absolute measure, relative measure using Gini and for generalized entropy with e = 0, 1, 2. However, the result does not seem to hold for the VLS measure of 'fairness' imbibed inequality. When we proceed along the lines of (Marjit et al., 2019), and introduce a transfer amount of 'T' from the person having high income post-growth to a person with low income post-growth, then for this entropy as a measure of inequality, there simply does not exist a transfer amount that preserves inequality post-distribution, casting doubt on the results of (Marjit & Sarkar, 2017), taking away the hope that we may have had the possibility of a distribution-neutral fiscal policy, as (Marjit et al., 2019) calls it. However, as (Marjit et al., 2019) show, the transfer amount do exist for the generalized entropy measure, which is an often cited measure of inequality in the literature.

Our claim:

The VLS fair measure of entropy is given by $S = \frac{1}{2} + \frac{\ln(2*\sigma^2*\Pi)}{2} + \mu;$

where, μ and σ are the mean and standard deviation of the income distribution.

In the following, we take 'a' and 'b' as two income levels and calculate $y_1 = \ln(a)$ and $y_2 = \ln(b)$; since for the VLS measure, the income distribution is a log-normal distribution.
Given: an income profile of (y_1, y_2), which becomes (g_1y_1, g_2y_2) after growth. (Without loss of generality, we assume $y_1 > y_2$, $g_1 > g_2$). Let 'T' be the transfer amount from person 1 to person 2, and after redistribution, the incomes profiles are $(g_1y_1 - T, g_2y_2 + T)$.
Since we want the inequality to remain same after transfer, we have $S_1 = S_2$

$$\frac{1}{2} + \frac{\ln(2*\sigma1^2*\Pi)}{2} + \mu1 = \frac{1}{2} + \frac{\ln(2*\sigma2^2*\Pi)}{2} + \mu2 \qquad \qquad \ldots\ldots\ldots\ldots\text{eq (1)}$$

$$\Rightarrow \quad \ln(\sigma1 / \sigma2) = \mu2 - \mu1 \qquad \qquad \ldots\ldots\ldots\ldots\text{eq (2)}$$

For the two given income profiles pre- and post- growth, for a transfer amount 'T' to exist, this boils down to the requirement of,

$$\frac{(y1 - y2)^2}{(g1y1 - g2y2)^2} > e^{\wedge}(g1y1 + g2y2 - y1 - y2)$$

Since the denominator of the LHS is a relatively big number, and since the RHS is raised to the power of an exponential, this inequality does not hold, at least for some incomes, thereby making it a proposition against the claim of the existence of a distribution-neutral transfer amount. A crude example is $y_1 = 5$, $y_2 = 2$, $g_1 = 3$, $g_2 = 2$.

This is equivalent to saying that either the (Marjit & Sarkar, 2017) result is not robust to the measure of inequality one takes; or that a 'fairness' imbibed income inequality measure will never require transfers. We assume the latter not to be the case because there's a change in inequality post- growth, which necessitates transfers.

We state it as a proposition, whose proof is obvious:
The existence of SPS allocation is untrue for a 'fair' measure of inequality, as measured by thermodynamic entropy.

Addressing our limitation:

The limitation of our argument is that, the VLS measure of entropy doesn't fall in the relative measure of inequality or in the absolute level of inequality or anywhere in between, but outside of it. For instance, let's consider an economy with two people having income (10, 20). Consider the growth process in this economy to have resulted in an income profile of (100, 200). Then by the relative measure, the inequality is preserved, since relative measure of inequality is scale invariant. Now, consider the growth process to have resulted in an income profile of (100,110). Then the absolute level of inequality is preserved since absolute measures are translation invariant. In literature, the absolute measure, which is dubbed the leftist measure happens to be a conservative estimate, however we will see now that the VLS measure of inequality as measured by entropy is far more conservative than the absolute measure itself, casting doubt on whether (Marjit & Sarkar, 2017) even addressed this issue. (We give further explanations in the following para.) Irrespective of whether they address this case or not, we do not have an inequality-preserving fiscal redistribution in terms of 'fairness', so to say.

For the given income profiles above, the VLS inequality measured by entropy goes up from 18.03 to 108.03, whereas the absolute measure has to remain intact at 18.03. In this sense, the VLS measure of entropy is more conservative than the absolute measure of inequality.

For clarification, we consider the in-between case of RSPS and ASPS, which (Marjit & Sarkar, 2017) had not explicitly stated. We need to see whether an income distribution which takes the form $y_1' = g_1y_1 + b$; and $y_2' = g_2y_2 + b$ post- growth will be robust to the result of existence of SPS allocation. This is an income transformation which after an inequality-preserving transfer amount 'T', makes it a relative-absolute SPS measure.

Following (Marjit & Sarkar, 2017), we proceed as follows:

$$\frac{y1-b}{y2-b} = \frac{g1y1-b-T}{g2y2-b+T}$$

$$\Rightarrow \quad T = \frac{[y1y2\,(g1-g2)+b(y1-y2)+b(g2y2-g1y1)]}{y1+y2-2b}$$

Implying, a transfer amount does exist for even a relative-absolute inequality measure.

To conclude, we wish to bring it to the attention that the VLS measure of inequality satisfy all the four axioms of (Marjit et al., 2019), namely, Transfer axiom, Symmetry axiom, Normalization axiom and Population Variance axiom. Hence, even though (Marjit & Sarkar, 2017; Marjit et al., 2019) are unpublished papers, our proposition of non-existence of distribution-neutral fiscal policy in terms of 'fairness' – is both valid from meeting the assumptions point of view – serves as an interim note on the SPS allocation; and is bad news for anyone concerned about justice and fairness, including us.

References:

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