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9 September 2020

Online at https://mpra.ub.uni-muenchen.de/102826/
MPRA Paper No. 102826, posted 15 Sep 2020 14:34 UTC

Generational Distribution of Fiscal Burdens: A Positive Analysis*

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Abstract

This study presents a political economy model with overlapping generations to analyze the effects of population aging on fiscal policy formation and the resulting distribution of fiscal burden across generations. The analysis shows that increased political power of the old, arising from population aging, leads to (i) an increase in the ratio of labor income tax revenue to GDP and the ratio of debt to GDP, and (ii) an increase in the ratio of capital income tax revenue to GDP in countries with high degrees of preferences for public goods, but an initial decrease followed by an increase in this ratio in countries with low degrees of preferences for public goods.

- Keywords: Generational burden, Overlapping generations, Political economy, Population aging, Public debt
- JEL Classification: D70, E24, E62, H60

*The authors gratefully acknowledge financial support from a Grant-in-Aid for Early-Career Scientists from the Japan Society for the Promotion of Science (No. 18K12802, Uchida) and a Grant-in-Aid for Scientific Research (C) from the Japan Society for the Promotion of Science (No. 18K01650, Ono).

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1 Introduction

How is the burden of fiscal policy distributed across generations? How do demographic changes affect the pattern of generational burdens? To answer these questions, several studies have explored the political determinants of fiscal policy in overlapping generations models. Examples are the works of Renström (1996), Beauchemin (1998), Boldrin and Rustichini (2000), Razin, Sadka, and Swagel (2004), and Razin and Sadka (2007), which are based on tractable models of the economy and voting process. Recently, Forni (2005), Bassetto (2008), Gonzalez-Eiras and Niepelt (2008, 2012), Mateos-Planas (2010), Ono and Uchida (2016), and Bishnu and Wang (2017) study taxation and public expenditure in a framework in which voting yields policies that are time consistent. All these works assume a balanced government budget in each period and thus, ignore the possibility of a shift of fiscal burdens onto future generations via public debt issuance.

The political economy of public debt has been addressed by several researchers, such as Cukierman and Meltzer (1989), Song, Storesletten, and Zilibotti (2012), Müller, Storesletten, and Zilibotti (2016), Röhrs (2016), Arawatari and Ono (2017), Arai, Naito, and Ono (2018), Ono and Uchida (2018), and Andersen (2019). In these studies, labor income tax on the working generation is the only tax instrument; capital income tax on the retired elderly, which is a possible additional instrument, is abstracted away from the analyses. An exception is Arcalean (2018), who considers dynamic fiscal competition over public spending financed by labor and capital taxes and public debt. His focus is on the effects of fiscal cross-border externalities on welfare and growth. In other words, these studies do not fully address the generational conflict over age-specific taxes and the resulting distribution of fiscal burden across generations. However, as indicated by Mateos-Planas (2010), demographic changes, such as increasing longevity and declining birth rates, affect voters' interests in taxing different factors and thus, drive the change in the mix of capital and labor income taxes over periods.

In our companion paper (Uchida and Ono, 2020), we address the generational conflict over taxes and public debt. The framework is an overlapping generations model in which labor supply is inelastic and public education expenditure, which benefits only young people, is financed by labor and capital taxes as well as public debt issue. Following Song, Storesletten, and Zilibotti (2012) and the subsequent literature, we assume probabilistic voting (e.g., Lindbeck and Weibull, 1987; Persson and Tabellini, 2000) in which fiscal policy in each period is determined to maximize the weighted sum of utility of the young and old. Within this setting, we investigate the effects of demographic changes on fiscal policy choice and its consequence for growth and welfare across generations. The present study generalizes our earlier work by assuming elastic labor supply and public goods that benefit both the young and old. Within this extended framework, we analyze the effects of population aging on the fiscal burden on current and future generations. In particular, we focus on the ratios of public debt, capital income tax revenue, and labor income tax revenue to GDP as measures of fiscal burden, and show that elastic labor supply as well as the degree of preferences for public goods are the keys to fit the results with empirical evidence in Organisation for Economic Co-operation and Development (OECD) countries.

We start our analysis by focusing on the inelastic labor supply case. The analysis shows that as population ages, the ratio of debt to GDP as well as the ratio of capital income tax revenue to GDP decrease irrespective of the degree of preferences for public goods. However, the result does not fit well with the empirical evidence. As depicted in Figure 1, a higher degree of aging is associated with a higher debt to GDP ratio. The evidence also shows that aging is non-linearly associated with the ratio of capital income tax revenue to GDP. For example, among countries sharing similar degrees of aging (Ireland, Korea, and the United States; and Israel, Iceland, and Slovakia), the first three countries show high ratios, whereas the last three show low ratios.

[Figure 1 is here.]

To bridge the gap between theory and evidence, we extend the analysis by assuming endogenous labor supply: individuals are endowed with one unit of labor in youth and supply it elastically in the labor market to balance the marginal costs and benefits of labor supply in terms of utility. We take a numerical approach to overcome the limitations of the analytical method. In particular, we calibrate the parameter that governs the degree of preferences for public goods to match the average of the ratio of government expenditure to GDP during 1995–2016 for each sample country (Figure 2). The calibration shows that a lower ratio of expenditure to GDP is associated with a lower degree of preferences for public goods.

[Figure 2 is here.]

Our numerical investigation shows that when the labor supply is elastic, the aging of the population results in higher labor income tax rates, which in turn reduces labor supply and GDP. Meanwhile, the decline in GDP leads to lower tax revenues, which leads to an increase in public debt issuance as an alternative source of revenue. Therefore, the aging of the population leads to an increase in the debt–GDP ratio. This result, which is opposite to that shown in the case of inelastic labor supply, fits well with the evidence observed in OECD countries (see Panel (c) in Figure 1). The numerical investigation also shows that in the case of elastic labor supply, the ratio of capital income tax revenue to GDP increases as population ages when the degree of preferences for public goods is high. However, when the degree is low, the ratio shows a U-shaped pattern: aging produces an initial decrease followed by an increase in the ratio of capital income tax revenue to GDP.

Such a U-shaped pattern is observed for 10 countries ranked lowest in sample countries in terms of the ratio of government expenditure to GDP (see Figure 2) and the associated degree of preferences for public goods. Our analysis, therefore, suggests that the degree of preferences for public goods is a key to account for the different patterns of the ratio observed in OECD countries.

The mechanism behind the different patterns is as follows. The ratio of capital income tax revenue to GDP is the product of the two factors: the ratio of capital income to GDP and the capital income tax rate. Population aging has the following effects on these two factors. First, as the population ages, the government raises the labor income tax rate and increases public debt issues to pass the fiscal burden onto the young. A higher labor income tax rate reduces saving, and a higher level of public debt strengthens the crowding-out effect in the capital market. These two effects in turn slow down capital accumulation, raise the interest rate, and thereby increase the ratio of capital income to GDP: this is the positive effect on the ratio. Second, as the population ages, the government lowers the capital income tax rate to reduce the fiscal burden on the old. However, as the population ages further, the government chooses to raise the capital income tax rate in response to public debt accumulation. Thus, aging produces an initial decrease followed by an increase in the capital income tax rate: this is the U-shaped effect on the ratio.

Which effect dominates the other depends on the degree of preferences for future public goods provision. The degree is associated with concern for the provision, and thus, with its marginal benefit through the choice of the capital income tax rate. This implies that the marginal benefit, showing the disciplining effect through the public goods provision (Song, Storesletten, and Zilibotti, 2012), increases as the degree increases. This property indicates that the possible negative effect via the capital income tax rate decreases as the degree becomes larger. This in turn implies that for high-degree countries, the positive effect via the capital income to GDP ratio outweighs the U-shaped effect via the capital income tax rate; the ratio of capital income tax revenue to GDP increases as the population ages. However, the opposite result holds for low-degree countries; aging produces an initial decrease followed by an increase in the ratio.

The present study is related to recent theoretical contributions on fiscal politics. Razin, Sadka, and Swagel (2004), Razin and Sadka (2007), Bassetto (2008), and Mateos-Planas (2010) focus on the association between population aging and capital income taxation. Mateos-Planas (2010) undertakes this analysis for the United States and shows that the tax rate initially decreases and then increases as population ages. The present study shows that such a U-shaped pattern holds for countries distinguished by low degrees of preferences for public goods. The role of preferences for public goods is also considered by Song, Storesletten, and Zilibotti (2012), who focus on public debt finance, but capital income taxation is abstracted away from their analysis. Thus, our study bridges the gap in the literature by comprehensively evaluating the effects of population aging on fiscal policy formation and the resulting fiscal burden on current and future generations.

The organization of the rest of this paper is as follows. Section 2 presents the model. Section 3 gives the characterization of political equilibrium, and then investigates policy response to increased political power of the old. Section 4 provides concluding remarks. The Appendix provides proofs of propositions and supplementary explanation for analytical and numerical methods.

2 Model

The discrete time economy starts in period 0 and consists of overlapping generations. Individuals are identical within a generation and live for two periods: youth and old age. Each young individual gives birth to $1+n$ children. The young population for period t is N_t and the population grows at a constant rate of $n(>-1)$: $N_{t+1}=(1+n)N_t$.

Individuals

Individuals display the following economic behavior over their life cycles. During youth, individuals work, receive market wages, and make tax payments. They use after-tax income for consumption and savings. In old age, they retire and receive and consume returns from savings.

Consider an individual born in period t. The individual is endowed with one unit of time. He/she supplies it elastically in the labor market and obtains labor income $w_t l_t$, where w_t is wage rate per unit of labor and $l_t \in (0,1)$ is the amount of labor supply. After paying tax $\tau_t w_t l_t$, where τ_t is the period t labor income tax rate, the individual distributes the after-tax income between consumption c_t and savings, s_t. Therefore, the period-t budget constraint for the youth becomes

$$c_t + s_t \leq (1-\tau_t) w_t l_t. \tag{1}$$

The period $t+1$ budget constraint in old age is

$$d_{t+1} \leq \left(1-\tau_{t+1}^K\right) R_{t+1} s_t, \tag{2}$$

where d_{t+1} is consumption, τ_{t+1}^K is the period-$t+1$ capital income tax rate, and R_{t+1} is the gross return from savings.

The preferences of an individual born in period t are specified by the following utility function:

$$\ln\left(c_t - \frac{(l_t)^{1+1/v}}{1+1/v}\right) + \theta \ln g_t + \beta\left(\ln d_{t+1} + \theta \ln g_{t+1}\right),$$

where $\beta \in (0,1)$ is a discount factor and $\theta(>0)$ is the degree of preferences for public goods. Following Greenwood, Hercowitz, and Huffman (1988) and Müller, Storesletten, and Zilibotti (2016), we assume that the disutility from labor effort is given by

$(l_t)^{1+1/v}/(1+1/v)$, where $v(>0)$ parameterizes the Frisch elasticity of labor supply. We substitute the budget constraints (1) and (2) into the utility function to form the unconstrained maximization problem:

$$\max_{\{s_t,l_t\}} \ln\left((1-\tau_t)w_t l_t - s - \frac{(l_t)^{1+1/v}}{1+1/v}\right) + \theta \ln g_t + \beta\left(\ln\left(1-\tau_{t+1}^K\right) R_{t+1}s_t + \theta \ln g_{t+1}\right).$$

By solving this problem, we obtain the following labor supply and savings functions:

$$l_t = [(1-\tau_t)w_t]^v, \tag{3}$$

$$s_t = \frac{\beta}{1+\beta}\cdot\frac{1/v}{1+1/v}[(1-\tau_t)w_t]^{1+v}. \tag{4}$$

The labor supply and savings decrease as the labor income tax rate, τ_t, increases, but they increase as the wage rate, w_t, increases.

Firms

There is a continuum of identical firms that are perfectly competitive profit maximizers and that produce the final output Y_t with a constant-returns-to-scale Cobb–Douglas production function, $Y_t = A(K_t)^\alpha (L_t)^{1-\alpha}$. Here, $A(>0)$ is total factor productivity, which is constant across periods, K_t is aggregate capital, L_t is aggregate labor, and $\alpha \in (0,1)$ is a constant parameter representing capital share in production.

In each period, a firm chooses capital and labor to maximize its profit, $A(K_t)^\alpha (L_t)^{1-\alpha} - \rho_t K_t - w_t L_t$, where ρ_t is the rental price of capital, and w_t is the wage rate. Profit maximization of a firm leads to

$$K_t : \rho_t = \alpha A(k_t)^{\alpha-1}(l_t)^{1-\alpha}, \tag{5}$$

$$L_t : w_t = (1-\alpha)A(k_t)^\alpha (l_t)^{-\alpha}, \tag{6}$$

where $k_t \equiv K_t/N_t$ is per capita capital. The arbitrage condition $\rho_t = R_t$ holds for all t, because the capital market is competitive and capital fully depreciates in a single period.

Government budget constraint

Government expenditure is financed by both taxes on capital and labor income and public debt issues. Let B_t denote aggregate inherited debt. The government budget constraint in period t is

$$\tau_t w_t l_t N_t + \tau_t^K R_t s_{t-1} N_{t-1} + B_{t+1} = R_t B_t + G_t,$$

where $\tau_t w_t l_t N_t$ is aggregate labor income tax revenue, $\tau_t^K R_t s_{t-1} N_{t-1}$ is aggregate capital income tax revenue, B_{t+1} is newly issued public debt, $R_t B_t$ is debt repayment, and G_t

is aggregate public expenditure. We assume a one-period debt structure to drive analytical solutions from the model. We also assume that the government in each period is committed to not repudiating the debt.

By dividing both sides of the constraint by N_t, we can obtain a per capita expression of the government budget constraint:

$$\tau_t w_t l_t + \frac{\tau_t^K R_t s_{t-1}}{1+n} + (1+n)b_{t+1} = R_t b_t + \frac{2+n}{1+n} g_t, \tag{7}$$

where $b_t \equiv B_t/N_t$ is per capita debt and $g_t \equiv G_t/(N_t + N_{t-1})$ is the per capita public expenditure.

Capital market-clearing condition

Public debt is traded in the domestic capital market. The market-clearing condition for capital is

$$K_{t+1} + B_{t+1} = N_t s_t,$$

which expresses the equality of total savings by young agents in period t, $N_t s_t$, to the sum of the stocks of aggregate physical capital and aggregate public debt at the beginning of period $t+1$. This condition is rewritten as

$$(1+n)(k_{t+1} + b_{t+1}) = s_t. \tag{8}$$

Economic Equilibrium

An economic equilibrium in the present framework is defined as follows.

- **Definition 1.** Given a sequence of policies, $\{\tau_t, \tau_t^K, g_t\}_{t=0}^{\infty}$, an *economic equilibrium* is a sequence of allocations, $\{c_t, d_t, s_t, l_t, k_{t+1}, b_{t+1}\}_{t=0}^{\infty}$, and prices, $\{w_t, \rho_t, R_t\}_{t=0}^{\infty}$, with the initial conditions $k_0 (> 0)$ and $b_0 (\geq 0)$ such that (i) given $\left(w_t, R_{t+1}, \tau_t, \tau_t^K, g_t, g_{t+1}\right)$, (c_t, d_{t+1}, s_t) solves the utility maximization problem; (ii) given (w_t, ρ_t), (K_t, L_t) solves a firm's profit maximization problem; (iii) given $(w_t, R_{t+1}, s_{t-1}, b_t)$, $(\tau_t, \tau_t^K, g_t, b_{t+1})$ satisfies the government budget constraint; (iv) an arbitrage condition, $\rho_t = R_t$, holds; (v) the labor market, $N_t l_t = L_t$, clears; and (vi) the capital market, $(1+n)(k_{t+1} + b_{t+1}) = s_t$, clears.

Definition 1 allows us to reduce the economic equilibrium conditions to a system of two difference equations, one representing the government budget constraint, and the other representing the capital market-clearing condition, for two state variables, physical capital k and public debt b. To show this, consider the labor supply in (3), the savings in (4), and factor prices in (5) and (6). These are written as functions of physical capital, k_t, and the labor income tax rate, τ_t as follows.

First, we substitute (6) into (3) to write the optimal labor supply as a function of τ_t and k_t :

$$l_t = l(\tau_t, k_t) \equiv \left[(1-\tau_t)(1-\alpha)A\left(k_t\right)^{\alpha}\right]^{v/(1+\alpha v)}. \tag{9}$$

Second, we reformulate the saving function in (4) by using (6) and (9) as follows:

$$s_t = s\left(\tau_t, k_t, l(\tau_t, k_t)\right) \equiv \frac{\beta}{1+\beta} \cdot \frac{1/v}{1+1/v}\left[(1-\tau_t)w\left(k_t, l(\tau_t, k_t)\right)\right]^{1+v}. \tag{10}$$

Third, we use firms' profit maximization with respect to L_t in (6) and the labor supply function in (9) to obtain the labor market-clearing wage rate:

$$w_t = w\left(k_t, l(\tau_t, k_t)\right) \equiv (1-\alpha)A\left(k_t\right)^{\alpha}\left[l(\tau_t, k_t)\right]^{-\alpha}. \tag{11}$$

Firms' profit maximization with respect to K_t in (5) and the labor supply function in (9) lead to

$$R_t = R\left(k_t, l(\tau_t, k_t)\right) \equiv \alpha A\left(k_t\right)^{\alpha-1}\left[l(\tau_t, k_t)\right]^{1-\alpha}. \tag{12}$$

With the use of the labor supply function in (9) and the factor prices in (11) and (12), we can reformulate the government budget constraint in Eq. (7) in terms of the state variables, k and b, and the government policy variables, τ, τ^K, and g as follows:

$$\begin{aligned} &\tau_t w\left(k_t, l(\tau_t, k_t)\right) l(\tau_t, k_t) + \tau_t^K R\left(k_t, l(\tau_t, k_t)\right)\left(k_t + b_t\right) + (1+n)b_{t+1} \\ &= R\left(k_t, l(\tau_t, k_t)\right) b_t + \frac{2+n}{1+n} g_t. \end{aligned} \tag{13}$$

We can also reformulate the capital market-clearing condition in (8) as follows:

$$\begin{aligned} (1+n)(k_{t+1}+b_{t+1}) &= s\left(\tau_t, k_t, l(\tau_t, k_t)\right) \\ &= \frac{\beta}{1+\beta} \cdot \frac{1/v}{1+1/v}\left[(1-\tau_t)(1-\alpha)A\left(k_t\right)^{\alpha}\right]^{\frac{1+v}{1+\alpha v}}. \end{aligned} \tag{14}$$

Thus, given the initial condition (k_0, b_0) and the sequence of policy variables, $\{\tau_t, \tau_t^K, g_t\}_{t=0}^{\infty}$, the sequence of physical capital and public debt in the economic equilibrium, $\{k_t, b_t\}_{t=0}^{\infty}$, is characterized by Eqs. (13) and (14).

In the economic equilibrium, the indirect utility of the young in period t, V_t^Y, and that of the old in period t, V_t^O, can be expressed as functions of policy variables, physical capital, and public debt. V_t^Y becomes:

$$\begin{aligned} V_t^Y = \ln &\left[c\left(\tau_t, k_t, l(\tau_t, k_t)\right) - \frac{\left(l(\tau_t, k_t)\right)^{1+1/v}}{1+1/v}\right] \\ &+ \theta \ln g_t + \beta\left[\ln\left(1-\tau_{t+1}^K\right) R\left(k_{t+1}, l(\tau_{t+1}, k_{t+1})\right) s\left(\tau_t, k_t, l(\tau_t, k_t)\right) + \theta \ln g_{t+1}\right], \end{aligned} \tag{15}$$

where $c\left(\tau_t, k_t, l(\tau_t, k_t)\right)$, representing consumption in youth, is defined by

$$c\left(\tau_t, k_t, l(\tau_t, k_t)\right) \equiv (1-\tau_t)w\left(k_t, l(\tau_t, k_t)\right) l(\tau_t, k_t) - s\left(\tau_t, k_t, l(\tau_t, k_t)\right).$$

The utility function of the old in period t, V_t^O, is

$$V_t^O = \ln d_t + \theta \ln g_t = \ln\left(1-\tau_t^K\right) R\left(k_t, l(\tau_t, k_t)\right)(1+n)(k_t+b_t) + \theta \ln g_t, \qquad (16)$$

where the second equality comes from (2), (8), and (12). The derivation of Eqs. (15) and (16) is provided in Appendix A.1.

3 Political Equilibrium

In this section, we consider voting on fiscal policy. We employ probabilistic voting à la Lindbeck and Weibull (1987). In this voting scheme, there is electoral competition between two office-seeking candidates. Each candidate announces a set of fiscal policies subject to the government budget constraint. As demonstrated by Persson and Tabellini (2000), the two candidates' platforms converge in the equilibrium to the same fiscal policy that maximizes the weighted average utility of voters.

In the present framework, both the young and old have an incentive to vote. Thus, the political objective is defined as the weighted sum of the utility of the young and old, given by $\tilde{\Omega}_t \equiv \omega V_t^O + (1+n)(1-\omega) V_t^Y$, where $\omega \in (0,1)$ and $1-\omega$ are the political weight placed on the old and young, respectively. A larger value of ω implies greater political power of the old. The weight on the young is adjusted by the gross population growth rate, $(1+n)$, to reflect their share of the population. To obtain the intuition behind this result, we divide $\tilde{\Omega}_t$ by $(1+n)(1-\omega)$ and redefine the objective function as follows:

$$\Omega_t = \frac{\omega}{(1+n)(1-\omega)} V_t^O + V_t^Y,$$

where the coefficient $\omega/(1+n)(1-\omega)$ of V_t^O represents the relative political weight on the old.

We substitute V_t^Y in (15) and V_t^O in (16) into Ω_t and obtain

$$\begin{aligned}
\Omega_t &= \frac{\omega}{(1+n)(1-\omega)} V^O\left(\tau_t, \tau_t^K, g_t; k_t, b_t\right) + V^Y\left(\tau_t, g_t, \tau_{t+1}, \tau_{t+1}^K, g_{t+1}, k_{t+1}; k_t\right) \\
&= \frac{\omega}{(1+n)(1-\omega)} \left[\ln\left(1-\tau_t^K\right) R\left(k_t, l(\tau_t, k_t)\right)(1+n)(k_t+b_t) + \theta \ln g_t\right] \\
&+ \ln\left[(1-\tau_t) w\left(k_t, l(\tau_t, k_t)\right) l(\tau_t, k_t) - s\left(\tau_t, k_t, l(\tau_t, k_t)\right) - \frac{\left(l(\tau_t, k_t)\right)^{1+1/v}}{1+1/v}\right] \\
&+ \theta \ln g_t + \beta \left[\ln\left(1-\tau_{t+1}^K\right) R\left(k_{t+1}, l(\tau_{t+1}, k_{t+1})\right) s\left(\tau_t, k_t, l(\tau_t, k_t)\right) + \theta \ln g_{t+1}\right]. \qquad (17)
\end{aligned}$$

The political objective function in (17) suggests that current policy choice affects decisions on future policy via physical capital accumulation. In particular, the period-t choice of τ_t^K, τ_t, and g_t affect the formation of physical capital in period $t+1$. This in turn influences decision making on period-$t+1$ fiscal policy. To demonstrate such an intertemporal effect, we employ the concept of Markov-perfect equilibrium under which fiscal policy today depends on the payoff-relevant state variables.

In the present framework, the payoff-relevant state variables are physical capital, k, and public debt, b. Thus, the expected provision of the public good and the rate of capital income tax for the next period, g_{t+1} and τ_{t+1}^K, respectively, are given by the functions of the period-$t+1$ state variables, $g_{t+1} = G(k_{t+1}, b_{t+1})$ and $\tau_{t+1}^K = T^K(k_{t+1}, b_{t+1})$. We denote by $-\underline{\tau}(<0)$ and $-\underline{\tau}^K(<0)$ the arbitrary lower limits of τ and τ^K, respectively. By using recursive notation with $z'(=k', b', g', \tau^{K\prime})$ denoting the next period $z(=k, b, g, \tau^K)$, we can define a Markov-perfect political equilibrium in the present framework as follows.

- **Definition 2.** A *Markov-perfect political equilibrium* is a set of functions, $\langle T, T^K, G, B \rangle$, where $T : \Re_+^2 \to (-\underline{\tau}, 1)$ is a labor income tax rate, $\tau = T(k, b)$, $T^K : \Re_+^2 \to \left(-\underline{\tau}^K, 1\right)$ is a capital income tax rate, $\tau^K = T^K(k, b)$, $G : \Re_+^2 \to \Re$ is a public goods provision rule, $g = G(k, b)$, and $B : \Re_+^2 \to \Re$ is a debt rule, $b' = B(k, b)$, so that given k and b, $\langle T(k, b), T^K(k, b), G(k, b), B(k, b) \rangle$ is a solution to the following problem:

$$\max \Omega = \frac{\omega}{(1+n)(1-\omega)} V^O \left(T(k, b), T^K(k, b), G(k, b); k, b\right)$$
$$+ V^Y \left(T(k, b), G(k, b), T(k', b'), T^K(k', b'), G(k', b'); k\right), \tag{18}$$
$$\text{s.t. } (1+n)(k' + B(k, b)) = s(T(k, b), l(T(k, b), k), k), \tag{19}$$
$$T(k, b) w(k, l(T(k, b), k), l) l(T(k, b), k) + T^K(k, b) R(k, l(T(k, b), k))(k + b) + (1+n) B(k, b)$$
$$= R(k, l(T(k, b), k)) b + \frac{2+n}{1+n} G(k, b), \tag{20}$$
given k and b,

where (19) comes from the capital market-clearing condition in (14), and (20) comes from the government budget constraint in (13).

3.1 Characterization of Political Equilibrium

To obtain the set of policy functions in Definition 2, we conjecture the following policy functions in the next period:

$$1 - \tau^{K\prime} = \frac{\bar{T}^K}{\alpha} \cdot \frac{k'}{k' + b'}, \tag{21}$$
$$1 - \tau' = \bar{T}, \tag{22}$$
$$g' = \bar{G} \cdot [A(k')^\alpha]^{\frac{1+v}{1+\alpha v}}, \tag{23}$$

where $\bar{T}^K$, $\bar{T}$, and $\bar{G}$ are positive constant parameters. To replace k' in (21) and (23) by k and b, respectively, we reformulate the capital market-clearing condition in (14) as

$(1+n)k' = s(\tau, k, l(\tau, k)) - (1+n)b'$, or

$$(1+n)k'\left(\tau, \tau^K, g, k, b\right) = s(\tau, k, l(\tau, k)) - R(k, l(\tau, k))b - \frac{2+n}{1+n}g + \tau w\left(k, l(\tau, k)\right) l(\tau, k) + \tau^K R(k, l(\tau, k))(k+b), \tag{24}$$

where the expression in (24) is derived by using the government budget constraint in (13).

Equation (24) indicates that the policy variables, τ^K, τ, and g, affect the formation of physical capital. To determine the effects, we partially differentiate k' with respect to the policy variables and obtain the following result:

$$\frac{\partial k'}{\partial \tau^K} > 0,\ \frac{\partial k'}{\partial g} < 0,\ \frac{\partial k'}{\partial \tau} \gtrless 0.$$

A rise in the capital income tax rate, τ^K, increases the tax revenue, which enables the government to reduce the public debt issue. This in turn weakens the crowding-out effect of the public debt, and thus, raises the capital stock: $\partial k'/\partial \tau^K > 0$. An increase in expenditure, g, has the opposite effect to that of a rise in τ^K: the government increases public debt issue to finance the increased expenditure. This strengthens the crowding-out effect and thus, lowers the capital stock: $\partial k'/\partial g < 0$.

A rise in the labor income tax rate, τ, produces several effects on capital stock, as observed in Eq. (24). First, it reduces the after-tax wage rate and labor supply of the young, and thus, lowers their savings. Second, the decreased labor supply lowers the interest rate and the debt repayment costs for the government, and thus, mitigates the crowding-out effect of debt on capital. In addition, the decreased interest rate reduces capital income tax revenue. Third, the rise in the tax rate increases labor income tax revenue; it also increases the marginal product of labor and the wage rate. However, at the same time, it lowers labor supply and works to reduce labor income tax revenue. The net effect, which is analyzed in detail below, depends on several factors, including the Frisch elasticity of labor supply, v, and the degree of preferences for public goods, θ.

With the use of (24), we can reformulate $\tau^{K\prime}$ in (21) and g' in (23) as follows:

$$1 - \tau^{K\prime} = \frac{\bar{T}^K}{\alpha} \cdot \frac{(1+n)k'}{(1+n)(k'+b')} = \frac{\bar{T}^K}{\alpha} \cdot \frac{(1+n)k'(\tau, \tau^K, g, k, b)}{s\left(\tau, k, l(\tau, k)\right)}, \tag{25}$$

$$g' = \bar{G} \cdot \left[A\left(k'(\tau, \tau^K, g, k, b)\right)^{\alpha}\right]^{\frac{1+v}{1+\alpha v}}. \tag{26}$$

Then, we can reformulate the political objective function in (17) as follows:

$$\begin{aligned}\Omega \simeq\ & \frac{\omega}{(1+n)(1-\omega)} \ln\left(1-\tau^K\right) R(k, l(\tau, k)) + \ln\left\{c(\tau, k, l(\tau, k)) - \frac{[l(\tau, k)]^{1+1/v}}{1+1/v}\right\} \\ & + \beta \ln \frac{\bar{T}^K}{\alpha} \cdot \frac{(1+n)k'(\tau, \tau^K, g, k, b)}{s\left(\tau, k, l(\tau, k)\right)} R\left(k'(\tau, \tau^K, g, k, b), l\left(\bar{T}, k'(\tau, \tau^K, g, k, b)\right)\right) \cdot s\left(\tau, k, l(\tau, k)\right) \\ & + \left(\frac{\omega}{(1+n)(1-\omega)} + 1\right) \theta \ln g + \beta\theta \ln \bar{G} \cdot \left[A\left(k'(\tau, \tau^K, g, k, b)\right)^{\alpha}\right]^{\frac{1+v}{1+\alpha v}},\end{aligned} \tag{27}$$

where the irrelevant terms are omitted from the expression.

In each period, the government, comprising elected politicians, choose τ^K, τ, and g to maximize its objective in (27). The associated debt issue, b', is determined to satisfy the government budget constraint. The first-order conditions with respect to τ^K, τ, and g are:

$$\tau^K : \underbrace{\underset{(-)}{\frac{(-1)\omega}{(1+n)(1-\omega)}} \frac{1}{1-\tau^K}}_{\text{(K.1)}} + \underbrace{\underset{(+)}{\frac{\beta}{1-\tau^{K\prime}}} \underset{(+)}{\frac{\partial\left(1-\tau^{K\prime}\right)}{\partial k'}} \underset{(+)}{\frac{\partial k'}{\partial \tau^K}}}_{\text{(K.2)}} + \frac{\beta}{R'}\left[\underbrace{\underset{(-)}{\frac{\partial R'}{\partial k'}} \underset{(+)}{\frac{\partial k'}{\partial \tau^K}}}_{\text{(K.3)}} + \underbrace{\underset{(+)}{\frac{\partial R'}{\partial l'}} \underset{(+)}{\frac{\partial l'}{\partial k'}} \underset{(+)}{\frac{\partial k'}{\partial \tau^K}}}_{\text{(K.4)}}\right] + \underbrace{\frac{\beta\theta}{g'} \underset{(+)}{\frac{\partial g'}{\partial k'}} \underset{(+)}{\frac{\partial k'}{\partial \tau^K}}}_{\text{(K.5)}} = 0. \tag{28}$$

$$\begin{aligned}
\tau : & \underbrace{\frac{\omega}{(1+n)(1-\omega)} \frac{1}{R} \underset{(+)}{\frac{\partial R}{\partial l}} \underset{(-)}{\frac{\partial l}{\partial \tau}}}_{\text{(T.1)}} + \frac{1}{c(\tau,k,l(\tau,k)) - [l(\tau,k)]^{1+1/v}/(1+1/v)} \\
& \left\{ \underbrace{-w\left(k,l(\tau,k)\right)l(\tau,k)}_{\text{(T.2)}} + (1-\tau)\left[\underbrace{\underset{(-)}{\frac{\partial w}{\partial l}} \underset{(-)}{\frac{\partial l}{\partial \tau}} l(\tau,k) + w\left(k,l(\tau,k)\right) \underset{(-)}{\frac{\partial l}{\partial \tau}}}_{\text{(T.3)}}\right] \right. \\
& \left. \underbrace{-\left(\underset{(-)}{\frac{\partial s}{\partial \tau}} + \underset{(+)}{\frac{\partial s}{\partial w}} \underset{(-)}{\frac{\partial w}{\partial l}} \underset{(-)}{\frac{\partial l}{\partial \tau}}\right)}_{\text{(T.4)}} - \underbrace{[l(\tau,k)]^{1/v} \underset{(-)}{\frac{\partial l}{\partial \tau}}}_{\text{(T.5)}} \right\} \\
& + \frac{\beta}{1-\tau^{K\prime}} \left\{ \underbrace{\underset{(+)}{\frac{\partial\left(1-\tau^{K\prime}\right)}{\partial k'}} \underset{(?)}{\frac{\partial k'}{\partial \tau}}}_{\text{(T.6)}} + \underbrace{\underset{(-)}{\frac{\partial\left(1-\tau^{K\prime}\right)}{\partial s}} \left(\underset{(-)}{\frac{\partial s}{\partial \tau}} + \underset{(+)}{\frac{\partial s}{\partial w}} \underset{(-)}{\frac{\partial w}{\partial l}} \underset{(-)}{\frac{\partial l}{\partial \tau}}\right)}_{\text{(T.7)}} \right\} + \frac{\beta}{R'} \left\{ \underbrace{\underset{(-)}{\frac{\partial R'}{\partial k'}} \underset{(?)}{\frac{\partial k'}{\partial \tau}}}_{\text{(T.8)}} + \underbrace{\underset{(+)}{\frac{\partial R'}{\partial l'}} \underset{(+)}{\frac{\partial l'}{\partial k'}} \underset{(?)}{\frac{\partial k'}{\partial \tau}}}_{\text{(T.9)}} \right\} \\
& + \underbrace{\frac{\beta}{s}\left(\underset{(-)}{\frac{\partial s}{\partial \tau}} + \underset{(+)}{\frac{\partial s}{\partial w}} \underset{(-)}{\frac{\partial w}{\partial l}} \underset{(-)}{\frac{\partial l}{\partial \tau}}\right)}_{\text{(T.10)}} + \underbrace{\frac{\beta\theta}{g'} \underset{(?)}{\frac{\partial g'}{\partial \tau}}}_{\text{(T.11)}} = 0
\end{aligned} \tag{29}$$

$$g : \underbrace{\left(\frac{\omega}{(1+n)(1-\omega)} + 1\right)\theta\frac{1}{g}}_{\text{(G.1)}} + \underbrace{\frac{\beta}{1-\tau^{K\prime}} \underset{(+)}{\frac{\partial\left(1-\tau^{K\prime}\right)}{\partial k'}} \underset{(-)}{\frac{\partial k'}{\partial g}}}_{\text{(G.2)}} + \frac{\beta}{R'}\left\{\underbrace{\underset{(-)}{\frac{\partial R'}{\partial k'}} \underset{(-)}{\frac{\partial k'}{\partial g}}}_{\text{(G.3)}} + \underbrace{\underset{(+)}{\frac{\partial R'}{\partial l'}} \underset{(+)}{\frac{\partial l'}{\partial k'}} \underset{(-)}{\frac{\partial k'}{\partial g}}}_{\text{(G.4)}}\right\} + \underbrace{\frac{\beta\theta}{g'} \underset{(+)}{\frac{\partial g'}{\partial k'}} \underset{(-)}{\frac{\partial k'}{\partial g}}}_{\text{(G.5)}} = 0, \tag{30}$$

where the symbols $(+)$, $(-)$, and $(?)$ below each term indicate that the sign of the term is positive, negative, and ambiguous, respectively. The derivation of Eqs. (28), (29), and (30) is provided in Appendix A.1.

The conditions in (28)–(30) suggest that the following three effects play roles in shaping policy: the general equilibrium effect of capital through the interest rate, R, represented by the terms (K.3) and (K.4) in (28), (T.8) and (T.9) in (29), and (G.3) and (G.4) in (30); the disciplining effect through the capital income tax rate in the next period, $\tau^{K\prime}$, represented by the terms (K.2) in (28), (T.6) and (T.7) in (29), and (G.2) in (30); and the disciplining effect through public goods provision in the next period, g', represented by the terms (K.5) in (28), (T.11) in (29), and (G.5) in (30).

To understand how these effects work, recall first the first-order condition with respect to τ^K in (28). The term (K.1) shows the marginal cost of taxation for the old; raising the tax rate increases their tax burden and thus, lowers their consumption. The terms (K.2)–(K.5) present the marginal cost or benefit for the young. The government can cut the labor income tax rate and thus, lower the tax burden on the young by raising the capital income tax rate. This creates a positive income effect on savings and capital formation. This in turn lowers the capital income tax burden in the next period, as presented by the term (K.2), and increases public good provision in the next period, as presented by the term (K.5). However, at the same time, it creates two opposing effects on the return from savings, R, as presented by the terms (K.3) and (K.4). Therefore, there are two marginal costs, presented by the terms (K.1) and (K.3), and three marginal benefits, presented by the terms (K.2), (K.4), and (K.5). The capital income tax rate is found to balance these costs and benefits.

Next, recall the first-order condition with respect to τ in (29) to consider the formation of the policy functions of the labor income tax rate. The term (T.1) shows the marginal cost of taxation for the old; raising the tax rate induces the young to reduce labor supply. This in turn lowers the return from savings, R, and thus, lowers the consumption of the old.

The terms (T.2)–(T.11) present marginal costs or benefits for the young. The terms (T.2)–(T.4) show the effect on the consumption in youth. A rise in the labor income tax rate causes disposable income to fall, as presented by the term (T.2). At the same time, the rise in the tax rate lowers the labor supply of the young, as demonstrated by the term (T.3). This in turn creates two opposing effects on consumption in youth: (a) increased marginal product of labor and the wage rate; and (b) decreased labor supply and thus, labor income for a given wage rate. The net effect is negative, as observed from Eqs. (9) and (11). In addition, the rise in the tax rate has two opposing effects on savings, as displayed by the term (T.4); the net effect is positive.

The term (T.5) shows that the rise in the tax rate reduces the disutility of labor. The terms (T.6) and (T.7) show the disciplining effect through the capital income tax rate in the next period. In particular, the term (T.6) includes various effects through capital, as argued above: the net effect depends on factors including the Frisch elasticity of labor supply as well as the degree of preferences for public goods. The term (T.7) indicates the effect through savings: the net effect on savings is negative and thus, the capital income tax burden in the next period decreases as the labor income tax rate increases.

The terms (T.8) and (T.9) display the general equilibrium effect of capital through the interest rate. The rise in the labor income tax rate raises (lowers) the return from savings if it lowers (raises) the capital level. The term (T.10) shows the effect on savings, which is negative, as observed in Eq. (10). Finally, the term (T.11) shows the disciplining effect through public goods provision in the next period. The rise in the labor income tax rate increases (reduces) the public goods provision if it raises (lowers) the capital level. In summary, the rise in the labor income tax rate produces various effects, including costs and benefits. The government chooses the tax rate to balance these costs and benefits.

Finally, recall the first-order condition with respect to g in (30). The term (G.1) shows the marginal benefit of public goods provision. The terms (G.2)–(G.5) present the marginal cost or benefit for the young. The effects presented by these terms are similar to those presented by the terms (K.2)–(K.5) in Eq. (28). However, they work in the opposite directions, because the government raises the labor income tax rate to finance increased expenditure for public goods provision, while it cuts the labor income tax rate to offset the increased capital income tax burden. Thus, there are three marginal costs, presented by the terms (G.2), (G.4), and (G.5), and the two marginal benefits, presented by the terms (G.1) and (G.3). The public good provision is set to balance these costs and benefits.

To simplify the presentation of the policy functions, we introduce the following notations:

$$
\begin{aligned}
\bar{T}^K &\equiv \frac{1-\left(\frac{\beta}{1+\beta}\frac{1/v}{1+1/v}-1\right)\frac{D_3}{D_1}}{\left[1+\theta+\frac{(1+n)(1-\omega)}{\omega}\left(\theta+\frac{\beta(1+\theta)\alpha(1+v)}{1+\alpha v}\right)\right]-\left(\frac{\beta}{1+\beta}\frac{1/v}{1+1/v}-1\right)\frac{D_2}{D_1}}, \\
\bar{T} &\equiv \frac{1}{1-\alpha}\cdot\frac{D_2\bar{T}^K-D_3}{D_1}, \\
\bar{G} &\equiv \frac{1+n}{2+n}\left(1+\frac{(1+n)(1-\omega)}{\omega}\right)\theta\left[(1-\alpha)\bar{T}\right]^{(1-\alpha)v/(1+\alpha v)}\bar{T}^K, \\
\bar{B} &\equiv \left[(1-\alpha)\bar{T}\right]^{(1-\alpha)v/(1+\alpha v)}\left[\bar{T}^K+(1-\alpha)\bar{T}-1\right]+\frac{2+n}{1+n}\bar{G},
\end{aligned}
$$

where D_1, D_2, and D_3 are defined by

$$D_1 \equiv \left[\tfrac{\omega(1-\alpha)v}{(1+n)(1-\omega)} + (1+v)\right]\left(\tfrac{\beta}{1+\beta}\tfrac{1/v}{1+1/v} - 1\right) - \beta\left(1+\theta\right)\alpha(1+v)\left[(-1)\tfrac{\beta}{1+\beta}\tfrac{1/v}{1+1/v}\tfrac{1+v}{1+\alpha v} + 1 + \tfrac{v(1-\alpha)}{1+\alpha v}\right],$$
$$D_2 \equiv \left[\tfrac{\omega(1-\alpha)v}{(1+n)(1-\omega)} + (1+v)\right]\left[1 + \left(1 + \tfrac{(1+n)(1-\omega)}{\omega}\right)\theta\right] + \tfrac{\beta(1+\theta)\alpha(1+v)(1-\alpha)v}{1+\alpha v},$$
$$D_3 \equiv \left[\tfrac{\omega(1-\alpha)v}{(1+n)(1-\omega)} + (1+v)\right] + \tfrac{\beta(1+\theta)\alpha(1+v)(1-\alpha)v}{1+\alpha v}.$$

The following proposition describes the optimal policy functions in the present framework.

- **Proposition 1** *There is a Markov-perfect political equilibrium characterized by the following policy functions:*

$$\begin{aligned} \tau^K &= 1 - \frac{\bar{T}^K}{\alpha} \cdot \frac{1}{1+b/k}, \\ \tau &= 1 - \bar{T}, \\ g &= \bar{G} \cdot \left[A\left(k\right)^{\alpha}\right]^{(1+v)/(1+\alpha v)}, \\ (1+n)b' &= \bar{B} \cdot \left[A\left(k\right)^{\alpha}\right]^{(1+v)/(1+\alpha v)}, \end{aligned}$$

Proof. See Appendix A.2.

Proposition 1 implies that the policy functions have the following features. First, the capital income tax rate is increasing in public debt but decreasing in physical capital. A higher level of public debt increases the burden of debt repayment. The government responds to the increased burden by raising the capital income tax rate. By contrast, a higher level of physical capital lowers the interest rate and thus, reduces the burden of debt repayment. This enables the government to lower the capital income tax rate. Second, the levels of public goods provision and public debt issues are linear functions of the output. This implies that the government finds it optimal to provide public goods and to issue public debt in proportion to the output. Third, the government borrows in the capital market as long as $\bar{B} > 0$; if this is the case, the government finds it optimal to shift a part of the burden onto future generations.

Having established the policy functions, we are now ready to demonstrate the accumulation of physical capital. We substitute the policy functions of b' and τ presented in Proposition 1 in the capital market-clearing condition in (14) and obtain

$$k' = \frac{1}{1+n}\left\{\tfrac{\beta}{1+\beta}\tfrac{1/v}{1+1/v}\left[\left(1-\alpha\right)\bar{T}\right]^{(1+v)/(1+\alpha v)} - \bar{B}\right\}\left[A\left(k\right)^{\alpha}\right]^{(1+v)/(1+\alpha v)}, \tag{31}$$

where k' denote the next-period capital stock. Given the initial condition k_0, the equilibrium sequence $\{k_t\}$ is determined by Eq. (31). A steady state is defined as an equilibrium sequence with $k = k'$. In other words, per capita capital is constant in a steady state. Eq. (31) indicates that there is a unique, stable steady-state equilibrium of k.

3.2 Policy Response to Increased Political Power of the Old

The result established in Proposition 1 indicates that the policy functions are affected by an increase in ω (i.e., increased political power of the old). Since ω increases as the population ages, the composition of government revenue (as a percentage of GDP) would change in response to population aging. To consider such aging effects, we focus on the ratios of government debt, capital income tax revenue, and labor income tax revenue to GDP.

3.2.1 Case of Inelastic Labor Supply

We first consider the case of inelastic labor supply, $v = 0$, and obtain the following result.

- **Proposition 2.** *Suppose that labor supply is inelastic:* $v = 0$. (i) *The ratio of government debt to GDP is positive (negative) and decreasing (increasing) in the political power of the old if* $\alpha (1 + \theta) < (>)1$*; and* (ii) *the ratio of capital income tax revenue to GDP is decreasing while the ratio of labor income tax revenue to GDP is increasing in the political power of the old:* $B_{t+1}/Y_t > (<)0$ *and* $\partial (B_{t+1}/Y_t) / \partial\omega < (>)0$ *if* $\alpha (1 + \theta) < (>)1$, $\partial \left(\tau_t^K R_r s_{t-1} N_{t-1}/Y_t\right) / \partial\omega < 0$, *and* $\partial (\tau_t w_t N_t/Y_t) / \partial\omega > 0$.

Proof. See Appendix A.3.

Proposition 2 shows that when labor supply is inelastic, the ratio of debt to GDP is positive as long as $\alpha(1 + \theta) < 1$; and under this condition, the ratio decreases as the political power of the old increases. This result appears to be inconsistent with the evidence observed in OECD countries. Proposition 2 also shows that the ratio of capital income tax revenue to GDP decreases as the political power of the old increases. This result seems to be intuitive, because greater political power of the old incentivizes the government to choose policies favoring the old who owe capital income tax burdens. However, the evidence in Figure 1 shows that the negative association does not hold for some countries. In particular, Ireland, Korea, and the United States show population aging rates below the average of sample OECD countries, while they show higher ratios of capital income tax revenue to GDP than other countries except for Sweden.

3.2.2 Case of Elastic Labor Supply

In the following analysis, we show that the inconsistency between theory and empirical findings could be resolved by assuming elastic labor supply. For the analysis, we take a numerical approach owing to the limitations of the analytical approach in the presence of elastic labor supply. Our strategy is to calibrate the model economy such that the steady-state equilibrium matches some key statistics of average OECD countries during 1995–2016. We then use the calibrated economy to run some quantitative experiments.

We fix the share of capital at $\alpha = 1/3$ following Song, Storesletten, and Zilibotti (2012) and Lancia and Russo (2016). Each period lasts 30 years; this assumption is standard in quantitative analyses of the two-period overlapping-generations models (e.g., Gonzalez-Eiras and Niepelt, 2008; Song, Storesletten, and Zilibotti, 2012; Lancia and Russo, 2016). Our selection of β is 0.99 per quarter, which is also standard in the literature (e.g., Kydland and Prescott, 1982; de la Croix and Doepke, 2002). Since agents in the present model plan over a generation that spans 30 years, we discount the future by $(0.99)^{120}$. Following Lancia and Russo (2016), we set the relative political weight on the old, $\omega/(1-\omega)$, to 0.8.

The net population growth rate, n, is taken from the average during 1995–2016 for each sample country. The preference weight on public good provision, θ, for each country is chosen so that the simulated version of the model matches the average of the ratio of government expenditure to GDP during 1995–2016. Table 1 summarizes the data on the average population growth rate and the associated estimation of θ for each sample country. Finally, in line with Trabandt and Uhlig (2011), we set $v = 3/2$ so that the top of the labor income tax Laffer curve is at 60% (see Appendix A.4 for the derivation).

[Table 1.]

We numerically investigate the effects of increased political power of the old (i.e., an increase in ω) on the ratios of government debt, capital income tax revenue, and labor income tax revenue to GDP for each sample country. To proceed with the simulation, we use the same parameter values of α, β, ω, and v for all countries, but apply the country-specific data of the net population growth rate, n, and the associated preference weight on public goods provision, θ, to each country. Figure 3 highlights six countries, including Iceland, Ireland, Israel, Korea, Slovakia, and the United States, selected from 31 sample countries. The results for the other countries are reported in Appendix A.5.

[Figure 3 is here.]

The numerical results show that the ratio of labor income tax revenue to GDP increases as the political power of the old, ω, increases. This finding is qualitatively similar to that demonstrated in the case of inelastic labor supply in Proposition 2. The results also show that the ratio of debt to GDP increases for all countries, and that the ratio of capital income tax revenue to GDP shows a U-shaped pattern for Ireland, Korea, and the United States, whereas it is monotonically increasing in ω for Iceland, Israel, and Slovakia. This finding differs substantially from that obtained in the case of inelastic labor supply.

Ratio of Debt to GDP

To understand the mechanism behind the difference, consider first the ratio of debt to

GDP. The ratio when $v \geq 0$ (including both elastic and inelastic labor supply cases) is

$$\frac{B'}{Y} = \frac{\bar{B}}{\left[(1-\alpha)\,\bar{T}\right]^{v/(1+\alpha v)}}$$

$$= \frac{\overbrace{\left[(1-\alpha)\,\bar{T}\right]^{(1-\alpha)v/(1+\alpha v)}}^{(*1)} \left[\overbrace{\bar{T}^K}^{(*3)} + \overbrace{(1-\alpha)\bar{T}}^{(*4)} - 1\right] + \overbrace{\frac{2+n}{1+n}\bar{G}}^{(*5)}}{\underbrace{\left[(1-\alpha)\,\bar{T}\right]^{v/(1+\alpha v)}}_{(*2)}}.$$

The terms (*1) and (*2) correspond to the output, the term (*3) represents one minus the capital income tax rate, the term (*4) represents one minus the labor income tax rate, and the term (*5) corresponds to government expenditure. Figure 4 takes the case of Korea as an example and plots changes in $\bar{T}^K$, $\bar{T}$, $\bar{G}$, $\bar{B}$, and $\bar{T}^K + (1-\alpha)\bar{T} - 1$ against changes in ω to help understand the mechanism behind the difference between the elastic ($v > 0$) and inelastic ($v = 0$) cases.

[Figure 4 is here.]

Suppose first that labor supply is inelastic: $v = 0$. In this case, the terms (*1) and (*2) are reduced to one and thus, are independent of ω. The term (*3) increases while the term (*4) decreases as ω increases. This implies that the capital income tax revenue decreases while the labor income tax revenue increases as ω increases. The former works to increase government debt issue while the latter works in the opposite direction. The term (*5) increases as ω increases. This implies that the government chooses to increase debt issue to finance increased expenditure. Thus, the terms (*3) and (*5) show positive effects on the ratio of debt to GDP, while the term (*4) shows a negative one. Proposition 2 shows that the government borrows in the capital market (i.e., $B > 0$) and that the positive effects are outweighed by the negative one if and only if $\alpha\,(1+\theta) < 1$. Since the borrowing of the government is a natural assumption, we conclude that the ratio of debt to GDP is decreasing in ω when labor supply is inelastic.

Next, suppose that labor supply is elastic: $v > 0$. The terms (*1) and (*2) now depend on ω through labor supply. In particular, an increase in ω lowers $\bar{T}$ and thus, increases the labor income tax rate, τ, which has a negative effect on labor supply and thus, GDP. Given that $\bar{T}^K + (1-\alpha)\bar{T} - 1$ is negative in our numerical investigation, the negative effect on GDP via the term (*1) works to increase the ratio of debt to GDP. Given that the sign of the numerator is positive, the negative effect on GDP via the term (*2) also works to increase the ratio of debt to GDP. Thus, when labor supply is elastic, there are four positive effects via the terms (*1), (*2), (*3), and (*5), and one negative effect via the term (*4). The numerical results show that the former outweighs the latter; that is, increased political power of the old due to population aging leads to an increase in the ratio of debt to GDP. Therefore, by assuming elastic labor supply, we obtain a model prediction that is consistent with the evidence of the ratio of debt to GDP observed in OECD countries. In particular, the positive effect via the term (*2), which is not observed in the case of inelastic labor supply, is a key factor for the positive association between political power of the old and the ratio of debt to GDP.

Ratio of Capital Income Tax Revenue to GDP

Next, we consider the ratio of capital income tax revenue to GDP. The ratio, $\tau_t^K R_t s_{t-1} N_{t-1}/Y_t$, is rewritten as follows:

$$\frac{\tau_t^K R_t s_{t-1} N_{t-1}}{Y_t} = \tau_t^K \alpha \frac{k_t + b_t}{k_t}.$$

The expression shows that the ratio depends on two factors: the capital income tax rate, τ_t^K, and the ratio of capital income to GDP, $\left(\tau_t^K R_t s_{t-1} N_{t-1}\right)/Y_t = \alpha(k_t+b_t)/k_t$. Figure 5 takes the cases of Iceland and Korea as examples and plots changes in τ_t^K, $\alpha\left(k_t+b_t\right)/k_t$, and $\tau_t^K R_t s_{t-1} N_{t-1}/Y_t$ against changes in ω. We first assess the effect through the ratio of capital income to GDP, $\alpha\left(k_t+b_t\right)/k_t$, and then assess the effect through the capital income tax rate, τ_t^K.

[Figure 5 is here.]

To assess the effect through the ratio of capital income to GDP, $\alpha\left(k_t+b_t\right)/k_t$, recall the policy function of public debt presented in Proposition 1 and the capital market-clearing condition in (31). With the use of these conditions, the ratio of capital income to GDP, $\alpha\left(k_t+b_t\right)/k_t$, is given by

$$\alpha\frac{k+b}{k} = \alpha\frac{\frac{\beta}{1+\beta}\frac{1/v}{1+1/v}\left[(1-\alpha)\bar{T}\right]^{(1+v)/(1+\alpha v)}}{\frac{\beta}{1+\beta}\frac{1/v}{1+1/v}\left[(1-\alpha)\bar{T}\right]^{(1+v)/(1+\alpha v)} - \bar{B}}.$$

When the labor supply is inelastic, $v=0$, the ratio $\alpha\left(k+b\right)/k$ is reduced to $\alpha\left(k+b\right)/k = (1-\alpha\theta)/(1+\theta)$, showing that the ratio $\alpha\left(k+b\right)/k$ is independent of ω. Thus, the effect of ω through the ratio of capital income to GDP does not appear when $v=0$.

When labor supply is elastic, $v>0$, a change in ω affects the ratio $\alpha\left(k+b\right)/k$ through the two terms, $\bar{B}$ and $\bar{T}$. The term $\bar{B}$ increases, whereas the term $\bar{T}$ decreases as ω increases. This implies that the government, representing both the young and old, chooses a higher level of public debt and a higher labor income tax rate as the political power of the old, ω, increases. Changes in the labor income tax rate and public debt in turn have two opposing effects on the ratio of capital income to GDP, $\alpha\left(k+b\right)/k$. First, a higher labor income tax rate reduces savings, leading to a negative effect on the ratio, $\alpha\left(k+b\right)/k$. Second, a higher labor income tax rate and a higher level of public debt slow down capital accumulation and thereby increase the interest rate, leading to a positive effect on the ratio, $\alpha\left(k+b\right)/k$. In total, the latter positive effect dominates the former negative effect, implying that the ratio of capital income to GDP, $\alpha\,(k+b)\,/k$, increases as ω increases.

Next, consider the effect of ω through the capital income tax rate presented in Proposition 1: $\tau^K = 1 - \left(\bar{T}^K/\alpha\right)/\left(1/(1+b/k)\right)$. The expression shows that a change in ω has effects on the tax rate through the two terms, $\bar{T}^K$ and $\alpha(k+b)/k$. The term $\bar{T}^K$ increases and thus, τ^K decreases as ω increases irrespective of the status of labor supply, v. The negative effect on the capital income tax rate reflects the preferences of the old who want to reduce the fiscal burden through capital income taxation. The term $\alpha\,(k+b)\,/k$ is independent of ω when $v = 0$, while it is increasing in ω when $v > 0$, as argued in the last paragraph. The effects through the two terms suggest that when $v = 0$, the capital income tax rate decreases as ω increases. However, when $v > 0$, the negative effect through the term $\bar{T}^K$ outweighs the positive effect through the capital income to GDP ratio, $\alpha\,(k+b)\,/k$, for low values of ω; the opposite result holds for high values of ω. Thus, an increase in the political power of the old produces an initial decrease followed by an increase in the capital income tax rate.

Up to now, the results lead to the following implications for the ratio of capital income tax revenue to GDP. When the labor supply is inelastic, $v = 0$, the effect through the ratio of capital income to GDP does not appear; the effect through the capital income tax rate remains and works to lower the ratio of capital income tax revenue to GDP as ω increases. However, when the labor supply is elastic, $v > 0$, the positive effect through the ratio of capital income to GDP may outweigh the negative effect through the capital income tax rate for low values of ω. Which effect dominates depends on the degree of preferences for public goods, represented by θ.

To understand the role of preferences for public goods, recall the first-order condition with respect to τ^K in (28). The condition indicates that ω and θ affect the determination of the capital income tax rate via the terms (K.1) and (K.5), respectively. The term (K.1) shows the marginal cost of capital income taxation; the cost increases as ω increases. The term (K.5) shows the marginal benefit of capital income taxation, showing the disciplining effect through the public goods provision; the benefit reduces as θ decreases. These properties imply that the negative effect on the tax rate via the term (K.1) becomes stronger than the positive effect via the term (K.5) as θ decreases. This in turn implies that for countries with low values of θ, the negative effect through the capital income tax rate outweighs the positive effect through the debt-to-capital ratio. This is the source of the U-shaped pattern of the ratio of capital income tax revenue to GDP, which is observed for countries with low values of θ.

The numerical results show that among 31 sample countries, the following 10 countries show the U-shaped pattern: Australia, Chile, Estonia, Ireland, Japan, Korea, Latvia, Lithuania, Switzerland, and the United State (see Figures 3 and A1). As Figure 2 and

Table 1 show, these countries are ranked lowest among sample countries in terms of the ratio of government expenditure to GDP and the associated estimated level of the preferences for public goods, θ. The results predict that in these countries, the ratio of capital income tax revenue to GDP shows an initial decrease followed by an increase as population ages. This prediction fits well with the result shown by Mateos-Planas (2010), who undertake the analysis for the United States. In addition, Figure 1 shows that Ireland, Korea, and the United States show higher ratios of capital income tax revenue to GDP than do countries with a similar degree of population aging, such as Iceland, Israel, and Slovakia. Our numerical results suggest that the degree of preferences for public goods is a key factor accounting for these cross-country differences in the ratios.

4 Conclusion

This study analyzed the distribution of fiscal burden across generations in the political economy model of fiscal policy. The model includes (i) two tax instruments, that is, capital and labor income taxes, accompanied by debt finance; and (ii) household decisions on labor supply. The first element enables us to investigate the impact of population aging on the distribution of fiscal burden across generations; and the second element allows us to present the effects of aging on policy variables via households' labor decisions.

Given these features, we showed that aging, which implies increased political power of the old, leads to (i) an increase in the ratios of debt to GDP and labor income tax revenue to GDP; and (ii) an increase in the ratio of capital income tax revenue to GDP for countries with high degrees of preferences for public goods, but an initial decrease followed by an increase in this ratio for countries with low degrees of preferences for public goods. These model predictions are fit well with the evidence observed in OECD countries. In particular, the latter result suggests that the degree of preferences for public goods is a key factor to account for the different patterns of the ratio observed among OECD countries sharing similar demographic characteristics.

The results of this study provide important and useful information to predict the generational burden of fiscal policy in aging societies. In countries with low degrees of preferences for public goods, population aging first produces a shift of tax burden from older to younger generations. As the population ages further, the tax burden on both younger and older generations increases. This suggests a U-shaped pattern of fiscal burden on the old. This pattern was predicted by Mateos-Planas (2010), but his analysis was limited to the balanced government budget case. The present study instead allowed for government deficit, showing that the ratio of public debt to GDP increases as population ages. The result suggests that when debt finance is allowed, a shift of fiscal burden from older to younger generations could be strengthened at an early stage of population aging. However, further aging leads to increased fiscal burden on both younger and older generations. The increased fiscal burden is an inevitable consequence of population aging in the long run.

A Proofs and Supplementary Explanations

A.1 Reformulation of the First-order Conditions

A.1.1 Reformulation of V_t^Y in (15) and V_t^O in (16)

The utility function of the young in period t, V_t^Y, is

$$V_t^Y = \ln\left(c_t - \frac{(l_t)^{1+1/v}}{1+1/v}\right) + \theta \ln g_t + \beta\left(\ln d_{t+1} + \theta \ln g_{t+1}\right).$$

The term $c_t - (l_t)^{1+1/v}/(1+1/v)$ is rewritten as follows:

$$\begin{aligned} c_t - \frac{(l_t)^{1+1/v}}{1+1/v} &= (1-\tau_t) w l_t - s_t - \frac{(l_t)^{1+1/v}}{1+1/v} \\ &= (1-\tau_t) w\left(k_t, l(\tau_t, k_t)\right) l(\tau_t, k_t) - s\left(\tau_t, k_t, l(\tau_t, k_t)\right) - \frac{(l(\tau_t, k_t))^{1+1/v}}{1+1/v}, \quad (32) \end{aligned}$$

where the first line comes from the budget constraint in youth in (1), and the second line comes from the labor market-clearing wage rate in (11), the labor supply function in (9), and the saving function in (10). Rearranging the terms, we can reduce the expression in (32) as follows:

$$c_t - \frac{(l_t)^{1+1/v}}{1+1/v} = \frac{1}{1+\beta} \cdot \frac{1/v}{1+1/v}\left[(1-\tau_t)(1-\alpha) A (k_t)^{\alpha}\right]^{(1+v)/(1+\alpha v)}. \quad (33)$$

The term d_{t+1} is rewritten as follows:

$$\begin{aligned} d_{t+1} &= \left(1-\tau_{t+1}^K\right) R_{t+1} s_t \\ &= \left(1-\tau_{t+1}^K\right) R\left(k_{t+1}, l(\tau_{t+1}, k_{t+1})\right) s\left(\tau_t, k_t, l(\tau_t, k_t)\right), \quad (34) \end{aligned}$$

where the equality in the second line comes from (10) and (12).

With (9), (10), (11), and (12), we can reformulate the equation in (34) further as follows:

$$\begin{aligned} d_{t+1} = &\left(1-\tau_{t+1}^K\right) \cdot \alpha \left[(1-\tau_{t+1})(1-\alpha)\right]^{(1-\alpha)v/(1+\alpha v)} \left[A (k_{t+1})^{\alpha}\right]^{(1+v)/(1+\alpha v)} \frac{1}{k_{t+1}} \\ &\times \frac{\beta}{1+\beta} \cdot \frac{1/v}{1+1/v}\left[(1-\tau_t)(1-\alpha) A (k_t)^{\alpha}\right]^{\frac{1+v}{1+\alpha v}}. \quad (35) \end{aligned}$$

Thus, with (32) and (34), we can reformulate the expression in (15) as follows:

$$\begin{aligned} V_t^Y &= V^Y\left(\tau_t, g_t, \tau_{t+1}, \tau_{t+1}^K, g_{t+1}, k_{t+1}; k_t\right) \\ &\simeq \underbrace{(1+\beta)\frac{1+v}{1+\alpha v}\ln(1-\tau_t)}_{(\#1)} + \theta \ln g_t + \beta \ln\left(1-\tau_{t+1}^K\right) \\ &\quad + \underbrace{\beta\frac{(1-\alpha) v}{1+\alpha v}\ln(1-\tau_{t+1})}_{(\#2)} + \underbrace{(-1)\beta\frac{1-\alpha}{1+\alpha v}\ln k_{t+1}}_{(\#3)} + \beta\theta \ln g_{t+1}, \quad (36) \end{aligned}$$

where the irrelevant terms are omitted from the expression in (36). The term (#1) includes the effects of the period-t labor income tax rate on $c_t - (l_t)^{1+1/v} / (1+1/v)$ and s_t; the term (#2) includes the effect of the period-$t+1$ labor income tax rate on the interest rate R_{t+1} through the labor supply l_{t+1}; and the term (#3) includes the effect of physical capital on the interest rate R_{t+1}.

With the use of (9) and (12), the expression in (16) is further reformulated as follows:

$$V_t^O = V^O\left(\tau_t, \tau_t^K, g_t; k_t, b_t\right) \simeq \ln\left(1-\tau_t^K\right) + \frac{(1-\alpha)\,v}{1+\alpha v}\ln(1-\tau_t) + \theta \ln g_t, \tag{37}$$

where the irrelevant terms are omitted from the expression.

■

A.1.2 Derivation of First-order Conditions

Based on the specification of the utility and production functions, we can reformulate the first-order conditions in (28), (29), and (30) as follows:

$$\begin{aligned}
&\tau^K : \frac{(-1)\omega}{(1+n)(1-\omega)}\frac{1}{1-\tau^K} + \frac{\beta\,(1+\theta)\,\alpha(1+v)}{1+\alpha v} \\
&\times \frac{\alpha\left[(1-\tau)\,(1-\alpha)\right]^{(1-\alpha)v/(1+\alpha v)}\left[A\,(k)^{\alpha}\right]^{(1+v)/(1+\alpha v)}\left(1+\frac{b}{k}\right)}{(1+n)k'} = 0,
\end{aligned}\tag{38}$$

$$\begin{aligned}
&\tau : (-1)\left[\frac{\omega(1-\alpha)v}{(1+n)(1-\omega)} + (1+v)\right]\frac{1}{1+\alpha v}\frac{1}{1-\tau} \\
&+ \frac{\beta\,(1+\theta)\,\alpha(1+v)}{1+\alpha v}\cdot\frac{(1-\tau)^{(1-\alpha)v/(1+\alpha v)}\left[(1-\alpha)A\,(k)^{\alpha}\right]^{(1+v)/(1+\alpha v)}}{(1+n)k'} \\
&\times\left\{(-1)\frac{\beta}{1+\beta}\frac{1/v}{1+1/v}\frac{1+v}{1+\alpha v} + \frac{1}{1-\tau}\frac{v}{1+\alpha v}\left[\alpha\left(1-\tau^K\right)\left(1+\frac{b}{k}\right) - \alpha - (1-\alpha)\tau\right] + 1\right\} = 0,
\end{aligned}\tag{39}$$

$$g : \left(\frac{\omega}{(1+n)(1-\omega)} + 1\right)\frac{\theta}{g} - \beta\,(1+\theta)\,\frac{\alpha(1+v)}{1+\alpha v}\frac{\frac{2+n}{1+n}}{(1+n)k'} = 0. \tag{40}$$

The derivation of (38)–(40) is presented next.

A.1.3 Derivation of (38)

We reformulate the terms (K.2)–(K.5) in (28) as follows. First, from the conjecture of the policy function in (25), we have

$$\beta \ln\left(1-\tau^{K\prime}\right) \simeq \beta \ln k'(\tau, \tau^K, g, k, b),$$

where the terms that are irrelevant for τ^K are omitted from the expression. The differentiation of $\beta \ln\left(1-\tau^{K\prime}\right)$ with respect to τ^K leads to

$$\frac{\partial\left[\beta \ln\left(1-\tau^{K\prime}\right)\right]}{\partial \tau^K} = \frac{\beta}{k'}\frac{\partial k'}{\partial \tau^K}. \tag{41}$$

Next, consider the terms (K.3) plus (K.4). With the use of (9) and (12), we can rewrite $\beta \ln R'$ as follows:

$$\beta \ln R' = \beta \ln \alpha A \left(k'\right)^{\alpha-1} \left(l(\tau', k')\right)^{1-\alpha} \simeq \beta \ln \left(k'\right)^{\alpha(1+v)/(1+\alpha v)-1} = \beta \frac{\alpha-1}{1+\alpha v} \ln k'.$$

Thus, we obtain

$$\frac{\partial \left[\beta \ln R'\right]}{\partial \tau^K} = \beta \frac{\alpha-1}{1+\alpha v} \frac{1}{k'} \frac{\partial k'}{\partial \tau^K}. \tag{42}$$

Finally, consider the term (K.5). Based on the conjecture of the policy function in (26), we have

$$\beta\theta \ln g' \simeq \beta\theta\alpha \frac{1+v}{1+\alpha v} \ln k',$$

where irrelevant terms are omitted from the expression. Differentiation of $\beta\theta \ln g'$ with respect to τ^K leads to

$$\frac{\partial \left[\beta\theta \ln g'\right]}{\partial \tau^K} = \beta\theta\alpha \frac{1+v}{1+\alpha v} \frac{1}{k'} \frac{\partial k'}{\partial \tau^K}. \tag{43}$$

By using (41), (42), and (43), we have

$$\begin{aligned}
&\underbrace{\underset{(+)}{\frac{\beta}{1-\tau^{K\prime}}} \underset{(+)}{\frac{\partial \left(1-\tau^{K\prime}\right)}{\partial k'}} \underset{(+)}{\frac{\partial k'}{\partial \tau^K}}}_{\text{(K.2)}} + \frac{\beta}{R'} \left[\underbrace{\underset{(-)}{\frac{\partial R'}{\partial k'}} \underset{(+)}{\frac{\partial k'}{\partial \tau^K}}}_{\text{(K.3)}} + \underbrace{\underset{(+)}{\frac{\partial R'}{\partial l'}} \underset{(+)}{\frac{\partial l'}{\partial k'}} \underset{(+)}{\frac{\partial k'}{\partial \tau^K}}}_{\text{(K.4)}} \right] + \underbrace{\frac{\beta\theta}{g'} \underset{(+)}{\frac{\partial g'}{\partial k'}} \underset{(+)}{\frac{\partial k'}{\partial \tau^K}}}_{\text{(K.5)}} \\
&= \frac{\beta}{1+\alpha v} \left[1 + \alpha v + \alpha - 1 + \theta\alpha(1+v)\right] \frac{1}{k'} \frac{\partial k'}{\partial \tau^K} \\
&= \frac{\beta \left(1+\theta\right) \alpha(1+v)}{1+\alpha v} \frac{1}{(1+n)k'} R(k, l(\tau, k))(k+b) \\
&= \frac{\beta \left(1+\theta\right) \alpha(1+v)}{1+\alpha v} \frac{1}{(1+n)k'} \alpha \left[\left(1-\tau\right)\left(1-\alpha\right)\right]^{(1-\alpha)v/(1+\alpha v)} \left[A \left(k\right)^{\alpha}\right]^{(1+v)/(1+\alpha v)} \left(1 + \frac{b}{k}\right)
\end{aligned}$$

where the second and third equalities come from (24). Substituting this expression into (28) leads to (38).

A.1.4 Derivation of (39)

The terms (T.1)–(T.11) in (29) are reformulated as follows. First, consider the term (T.1), which expresses the first derivative of $\frac{\omega}{(1+n)(1-\omega)} \ln \left(1-\tau^K\right) R(k, l(\tau, k))$ with respect to τ. From (9) and (12), R is given by

$$R = \alpha \left[(1-\tau)(1-\alpha)\right]^{(1-\alpha)v/(1+\alpha v)} \left[A \left(k\right)^{\alpha}\right]^{(1+v)/(1+\alpha v)} \frac{1}{k}.$$

We substitute this into the term $\frac{\omega}{(1+n)(1-\omega)} \ln \left(1-\tau^K\right) R(k, l(\tau, k))$ and obtain

$$\frac{\omega}{(1+n)(1-\omega)} \ln \left(1-\tau^K\right) R(k, l(\tau, k)) \simeq \frac{\omega}{(1+n)(1-\omega)} \frac{(1-\alpha)v}{1+\alpha v} \ln(1-\tau),$$

where the irrelevant terms are omitted from the expression. Differentiation of $\frac{\omega}{(1+n)(1-\omega)} \ln\left(1-\tau^K\right) R(k, l(\tau, k))$ with respect to τ leads to

$$\frac{\partial\left[\frac{\omega}{(1+n)(1-\omega)} \ln\left(1-\tau^K\right) R(k, l(\tau, k))\right]}{\partial \tau} = \frac{\omega}{(1+n)(1-\omega)} \frac{(1-\alpha)v}{1+\alpha v} \frac{-1}{1-\tau}. \tag{44}$$

Next, consider the terms (T.2)–(T.5), expressing the first derivative of $\ln\left\{c(\tau, k, l(\tau, k)) - \frac{[l(\tau,k)]^{1+1/v}}{1+1/v}\right\}$ with respect to τ. With the use of (33), we have

$$\ln\left\{c(\tau, k, l(\tau, k)) - \frac{[l(\tau, k)]^{1+1/v}}{1+1/v}\right\} = \ln \frac{1}{1+\beta} \frac{1/v}{1+1/v} \left[(1-\tau)(1-\alpha)A(k)^{\alpha}\right]^{(1+v)/(1+\alpha v)}.$$

Differentiation of $\ln\left\{c(\tau, k, l(\tau, k)) - \frac{[l(\tau,k)]^{1+1/v}}{1+1/v}\right\}$ with respect to τ leads to

$$\frac{\partial\left\{c(\tau, k, l(\tau, k)) - \frac{[l(\tau,k)]^{1+1/v}}{1+1/v}\right\}}{\partial \tau} = \frac{1+v}{1+\alpha v} \frac{-1}{1-\tau}. \tag{45}$$

Third, consider the terms (T.6), (T.7), (T.8), (T.9), and (T.10), expressing the first derivative of $\beta \ln\left(1-\tau^{K\prime}\right) R's$ with respect to τ. With the use of (9) and (12), the term $\beta \ln\left(1-\tau^{K\prime}\right) R's$ is reformulated as follows:

$$\begin{aligned}
\beta \ln\left(1-\tau^{K\prime}\right) R's &= \beta \ln \frac{\bar{T}^K}{\alpha} \frac{(1+n)k'}{s} R's \\
&\simeq \beta \ln k'R' \\
&= \beta \ln k'\alpha\left[(1-\tau')(1-\alpha)\right]^{(1-\alpha)v/(1+\alpha v)} \left[A(k')^{\alpha}\right]^{(1+v)/(1+\alpha v)} \frac{1}{k'} \\
&\simeq \beta \ln (k')^{\alpha(1+v)/(1+\alpha v)},
\end{aligned}$$

where the irrelevant terms are omitted from the expression. Differentiation of $\beta \ln\left(1-\tau^{K\prime}\right) R's$ with respect to τ leads to

$$\frac{\partial\left[\beta \ln\left(1-\tau^{K\prime}\right) R's\right]}{\partial \tau} = \frac{\beta\alpha(1+v)}{1+\alpha v} \frac{1}{k'} \frac{\partial k'}{\partial \tau}. \tag{46}$$

Fourth, consider the term (T.11), expressing the first derivative of $\beta\theta \ln g'$ with respect to τ. Based on the conjecture of g' in (26), we have

$$\beta\theta \ln g' \simeq \beta\theta \frac{\alpha(1+v)}{1+\alpha v} \ln k'.$$

Thus, we have

$$\frac{\partial \beta\theta \ln g'}{\partial \tau} = \beta\theta \frac{\alpha(1+v)}{1+\alpha v} \frac{1}{k'} \frac{\partial k'}{\partial \tau}. \tag{47}$$

With the use of (38), (45), (46), and (47), we can rewrite the first-order condition in (29) as follows:

$$\begin{aligned}
&\frac{\omega}{(1+n)(1-\omega)} \frac{(1-\alpha)v}{1+\alpha v} \frac{-1}{1-\tau} + \frac{1+v}{1+\alpha v} \frac{-1}{1-\tau} + \beta(1+\theta) \frac{\alpha(1+v)}{1+\alpha v} \frac{1}{k'} \frac{\partial k'}{\partial \tau} = 0 \\
&\Rightarrow (-1)\left[\frac{\omega(1-\alpha)v}{(1+n)(1-\omega)} + (1+v)\right] \frac{1}{1+\alpha v} \frac{1}{1-\tau} + \beta(1+\theta) \frac{\alpha(1+v)}{1+\alpha v} \frac{1}{k'} \frac{\partial k'}{\partial \tau} = 0.
\end{aligned} \tag{48}$$

The remaining task is to compute $\partial k'/\partial\tau$. From (24), we have:

$$\begin{aligned}(1+n)k' &= s\left(\tau,k,l(\tau,k)\right)-R(k,l(\tau,k))b-\frac{2+n}{1+n}g+\tau w\left(k,l(\tau,k)\right)l(\tau,k)+\tau^{K}R(k,l(\tau,k))(k+b)\\ &=\frac{\beta}{1+\beta}\frac{1/v}{1+1/v}\left[(1-\tau)(1-\alpha)A\left(k\right)^{\alpha}\right]^{(1+v)/(1+\alpha v)}\\ &-\alpha\left[(1-\tau)(1-\alpha)\right]^{(1-\alpha)v/(1+\alpha v)}\left[A\left(k\right)^{\alpha}\right]^{(1+v)/(1+\alpha v)}\frac{b}{k}-\frac{2+n}{1+n}\\ &+\frac{\tau}{1-\tau}\left[(1-\tau)(1-\alpha)A\left(k\right)^{\alpha}\right]^{1/(1+\alpha v)}\cdot\left[(1-\tau)(1-\alpha)A\left(k\right)^{\alpha}\right]^{v/(1+\alpha v)}\\ &+\tau^{K}\alpha\left[\left(1-\tau\right)\left(1-\alpha\right)\right]^{(1-\alpha)v/(1+\alpha v)}\left[A\left(k\right)^{\alpha}\right]^{(1+v)/(1+\alpha v)}\left(1+\frac{b}{k}\right).\end{aligned}\tag{49}$$

By using (49), we can compute $\partial(1+n)k'/\partial\tau$ as follows:

$$\begin{aligned}\frac{\partial(1+n)k'}{\partial\tau} &= (1-\tau)^{(1-\alpha)v/(1+\alpha v)}\left[(1-\alpha)A\left(k\right)^{\alpha}\right]^{(1+v)/(1+\alpha v)}\\ &\times\left\{(-1)\frac{\beta}{1+\beta}\frac{1/v}{1+1/v}\frac{1+v}{1+\alpha v}+\underbrace{\frac{1}{1-\tau}\frac{v}{1+\alpha v}\left[\alpha\frac{b}{k}-\frac{1+v}{v}\tau-\tau^{K}\alpha\left(1+\frac{b}{k}\right)\right]+\frac{1}{1-\tau}}_{(*)}\right\},\end{aligned}$$

where the term (*) in the above expression is reformulated as follows:

$$(*)=\frac{1}{1-\tau}\frac{1}{1+\alpha v}\left[\alpha\left(1-\tau^{K}\right)\left(1+\frac{b}{k}\right)-\alpha-(1-\alpha)\tau\right]+1,$$

Therefore, we obtain

$$\begin{aligned}\frac{\partial(1+n)k'}{\partial\tau} &= (1-\tau)^{(1-\alpha)v/(1+\alpha v)}\left[(1-\alpha)A\left(k\right)^{\alpha}\right]^{(1+v)/(1+\alpha v)}\\ &\times\left\{(-1)\frac{\beta}{1+\beta}\frac{1/v}{1+1/v}\frac{1+v}{1+\alpha v}+\frac{1}{1-\tau}\frac{1}{1+\alpha v}\left[\alpha\left(1-\tau^{K}\right)\left(1+\frac{b}{k}\right)-\alpha-(1-\alpha)\tau\right]+1\right\}.\end{aligned}\tag{50}$$

Substitution of (50) into (48) leads to (39).

A.1.5 Derivation of (40)

First, consider the terms (G.2), (G.3), and (G.4), expressing the first derivative of $\beta\ln\left(1-\tau^{K\prime}\right)R's$ with respect to g. With the use of the conjecture in (25), we can rewrite $\beta\ln\left(1-\tau^{K\prime}\right)R's$ as follows:

$$\beta\ln\left(1-\tau^{K\prime}\right)R's=\beta\ln\frac{\bar{T}^{K}}{\alpha}\frac{(1+n)k'}{s}R's\simeq\beta\ln(k')^{\alpha(1+v)/(1+\alpha v)},$$

where the expression comes from (9) and (12). Differentiation of $\beta\ln\left(1-\tau^{K\prime}\right)R's$ with respect to g leads to

$$\frac{\partial\left[\beta\ln\left(1-\tau^{K\prime}\right)R's\right]}{\partial g}=\frac{\beta\alpha(1+v)}{1+\alpha v}\frac{1}{k'}\frac{\partial k'}{\partial g}=\frac{\beta\alpha(1+v)}{1+\alpha v}\frac{(-1)\frac{2+n}{1+n}}{(1+n)k'}.\tag{51}$$

Second, consider the term (G.5), expressing the first derivative of $\beta\theta \ln g'$ with respect to g. With the use of the conjecture in (26), we can rewrite $\beta\theta \ln g'$ as follows:

$$\beta\theta \ln g' \simeq \beta\theta \frac{\alpha(1+v)}{1+\alpha v} \ln k'.$$

Differentiation of $\beta\theta \ln g'$ with respect to g leads to:

$$\frac{\partial \beta\theta \ln g'}{\partial g} \simeq \beta\theta \frac{\alpha(1+v)}{1+\alpha v} \frac{1}{k'} \frac{\partial k'}{\partial g} = \beta\theta \frac{\alpha(1+v)}{1+\alpha v} \frac{(-1)\frac{2+n}{1+n}}{(1+n)k'}. \tag{52}$$

Substitution of (51) and (52) into (30) leads to (40).

■

A.2 Proof of Proposition 1

The procedure for finding optimal policy functions is as follows. First, substitute the first-order condition with respect to τ^K in (38) into the first-order condition with respect to g in (40) to write down g as a function of τ^K and τ : $g = g(\tau^K, \tau)$. Second, substitute $g = g(\tau^K, \tau)$ into the capital market-clearing condition in (24) to write down k' as a function of τ^K and τ : $k' = k'\left(\tau^K, \tau\right)$. Third, substitute $k' = k'\left(\tau^K, \tau\right)$ into the first-order condition with respect to τ^K in (38) and τ in (39) to obtain the two sorts of optimal relations between τ^K and τ, and solve them for τ^K and τ. Fourth, substitute the solutions for τ^K and τ into $g = g(\tau^K, \tau)$ to obtain the optimal policy function of g. Finally, substitute the optimal policy functions of τ^K, τ, and g into the government budget constraint in (20) to obtain the optimal policy function of b'.

Recall the first-order condition with respect to g in (40), which is rewritten as

$$\frac{2+n}{1+n} g = \left(\frac{\omega}{(1+n)(1-\omega)} + 1\right) \theta \frac{1+\alpha v}{\beta\,(1+\theta)\,\alpha(1+v)} (1+n)k'.$$

We substitute the first-order condition with respect to τ^K in (38) into the above expression to obtain $g = g(\tau^K, \tau)$, or

$$\frac{2+n}{1+n} g = \left(\frac{\omega}{(1+n)(1-\omega)} + 1\right) \theta \frac{\alpha\left[(1-\tau)\,(1-\alpha)\right]^{(1-\alpha)v/(1+\alpha v)} \left[A\,(k)^{\alpha}\right]^{(1+v)/(1+\alpha v)} \left(1+\frac{b}{k}\right)}{\frac{\omega}{(1+n)(1-\omega)} \frac{1}{1-\tau^K}}. \tag{53}$$

Next, we substitute (53) into the capital market-clearing condition in (24) to obtain

$$
\begin{aligned}
(1+n)k' = & \frac{\beta}{1+\beta}\frac{1/v}{1+1/v}\left[(1-\tau)(1-\alpha)A\left(k\right)^{\alpha}\right]^{(1+v)/(1+\alpha v)} \\
& - \alpha\left[(1-\tau)(1-\alpha)\right]^{(1-\alpha)v/(1+\alpha v)}\left[A\left(k\right)^{\alpha}\right]^{(1+v)/(1+\alpha v)}\frac{b}{k} \\
& - \left(\frac{\omega}{(1+n)(1-\omega)}+1\right)\theta\frac{\alpha\left[(1-\tau)\left(1-\alpha\right)\right]^{(1-\alpha)v/(1+\alpha v)}\left[A\left(k\right)^{\alpha}\right]^{(1+v)/(1+\alpha v)}\left(1+\frac{b}{k}\right)}{\frac{\omega}{(1+n)(1-\omega)}\frac{1}{1-\tau^{K}}} \\
& + \frac{\tau}{1-\tau}\left[(1-\tau)(1-\alpha)A\left(k\right)^{\alpha}\right]^{1/(1+\alpha v)}\cdot\left[(1-\tau)(1-\alpha)A\left(k\right)^{\alpha}\right]^{v/(1+\alpha v)} \\
& + \tau^{K}\alpha\left[(1-\tau)\left(1-\alpha\right)\right]^{(1-\alpha)v/(1+\alpha v)}\left[A\left(k\right)^{\alpha}\right]^{(1+v)/(1+\alpha v)}\left(1+\frac{b}{k}\right).
\end{aligned}
$$

By rearranging the terms, we have

$$
\begin{aligned}
(1+n)k' = & \left[(1-\tau)(1-\alpha)\right]^{(1-\alpha)v/(1+\alpha v)}\left[A\left(k\right)^{\alpha}\right]^{(1+v)/(1+\alpha v)} \\
& \times\left\{\left(\frac{\beta}{1+\beta}\frac{1/v}{1+1/v}-1\right)(1-\tau)(1-\alpha)\right. \\
& \left.\times-\left[1+\left(1+\frac{(1+n)(1-\omega)}{\omega}\right)\theta\right]\alpha\left(1-\tau^{K}\right)\left(1+\frac{b}{k}\right)+1\right\}. \qquad (54)
\end{aligned}
$$

Eq. (54) shows that $(1+n)k'$ is expressed as a function of τ^K and τ.

Third, we substitute (54) into the first-order condition with respect to τ^K in (38) and obtain

$$
\begin{aligned}
& \frac{\omega}{(1+n)(1-\omega)}\frac{1}{1-\tau^{K}} \\
& = \frac{\beta\left(1+\theta\right)\alpha(1+v)}{1+\alpha v}\times\frac{\alpha\left[(1-\tau)\left(1-\alpha\right)\right]^{(1-\alpha)v/(1+\alpha v)}\left[A\left(k\right)^{\alpha}\right]^{(1+v)/(1+\alpha v)}\left(1+\frac{b}{k}\right)}{X} \\
& = \frac{\beta\left(1+\theta\right)\alpha(1+v)}{1+\alpha v} \\
& \times\frac{\alpha\left(1+\frac{b}{k}\right)}{\left(\frac{\beta}{1+\beta}\frac{1/v}{1+1/v}-1\right)(1-\tau)(1-\alpha)-\left[1+\left(1+\frac{(1+n)(1-\omega)}{\omega}\right)\theta\right]\alpha\left(1-\tau^{K}\right)\left(1+\frac{b}{k}\right)+1},
\end{aligned}
$$

where X is defined by

$$
\begin{aligned}
X \equiv & \left[(1-\tau)(1-\alpha)\right]^{(1-\alpha)v/(1+\alpha v)}\left[A\left(k\right)^{\alpha}\right]^{(1+v)/(1+\alpha v)} \\
& \times\left\{\left(\frac{\beta}{1+\beta}\frac{1/v}{1+1/v}-1\right)(1-\tau)(1-\alpha)-\left[1+\left(1+\frac{(1+n)(1-\omega)}{\omega}\right)\theta\right]\alpha\left(1-\tau^{K}\right)\left(1+\frac{b}{k}\right)+1\right\}
\end{aligned}
$$

By rearranging the terms, we have

$$
\begin{aligned}
& \left(\frac{\beta}{1+\beta}\frac{1/v}{1+1/v}-1\right)(1-\tau)(1-\alpha)+1 \\
& = \left[1+\theta+\frac{(1+n)(1-\omega)}{\omega}\left(\theta+\frac{\beta\left(1+\theta\right)\alpha(1+v)}{1+\alpha v}\right)\right]\alpha\left(1-\tau^{K}\right)\left(1+\frac{b}{k}\right). \qquad (55)
\end{aligned}
$$

This describes the optimal relationship between τ^K and τ.

Third, we substitute (54) in the first-order condition with respect to τ into (39) to obtain

$$\begin{aligned}
&\left[\tfrac{\omega(1-\alpha)v}{(1+n)(1-\omega)}+(1+v)\right]\left\{\left(\tfrac{\beta}{1+\beta}\tfrac{1/v}{1+1/v}-1\right)(1-\tau)(1-\alpha)\right.\\
&\left.-\left[1+\left(1+\tfrac{(1+n)(1-\omega)}{\omega}\right)\theta\right]\alpha\left(1-\tau^K\right)\left(1+\frac{b}{k}\right)+1\right\}\\
&=\beta\left(1+\theta\right)\alpha(1+v)\left(1-\tau\right)\left(1-\alpha\right)\\
&\times\left\{(-1)\tfrac{\beta}{1+\beta}\tfrac{1/v}{1+1/v}\tfrac{1+v}{1+\alpha v}+\tfrac{v}{1+\alpha v}\tfrac{1}{1-\tau}\left[\alpha\left(1-\tau^K\right)\left(1+\frac{b}{k}\right)-\alpha-(1-\alpha)\tau\right]+1\right\}\\
&=\beta\left(1+\theta\right)\alpha(1+v)\left(1-\alpha\right)\\
&\times\left\{\left[(-1)\tfrac{\beta}{1+\beta}\tfrac{1/v}{1+1/v}\tfrac{1+v}{1+\alpha v}+1+\tfrac{v(1-\alpha)}{1+\alpha v}\right](1-\tau)+\tfrac{v}{1+\alpha v}\alpha\left(1-\tau^K\right)\left(1+\frac{b}{k}\right)-\tfrac{v}{1+\alpha v}\right\}.
\end{aligned}$$

By rearranging the terms, we have

$$\begin{aligned}
&\underbrace{\left\{\left[\tfrac{\omega(1-\alpha)v}{(1+n)(1-\omega)}+(1+v)\right]\left(\tfrac{\beta}{1+\beta}\tfrac{1/v}{1+1/v}-1\right)-\beta\left(1+\theta\right)\alpha(1+v)\left[(-1)\tfrac{\beta}{1+\beta}\tfrac{1/v}{1+1/v}\tfrac{1+v}{1+\alpha v}+1+\tfrac{v(1-\alpha)}{1+\alpha v}\right]\right\}}_{D_1}\\
&\times(1-\tau)(1-\alpha)\\
&=\underbrace{\left\{\left[\tfrac{\omega(1-\alpha)v}{(1+n)(1-\omega)}+(1+v)\right]\left[1+\left(1+\tfrac{(1+n)(1-\omega)}{\omega}\right)\theta\right]+\tfrac{\beta(1+\theta)\alpha(1+v)(1-\alpha)v}{1+\alpha v}\right\}}_{D_2}\alpha\left(1-\tau^K\right)\left(1+\frac{b}{k}\right)\\
&-\underbrace{\left\{\left[\tfrac{\omega(1-\alpha)v}{(1+n)(1-\omega)}+(1+v)\right]+\tfrac{\beta(1+\theta)\alpha(1+v)(1-\alpha)v}{1+\alpha v}\right\}}_{D_3},
\end{aligned}\tag{56}$$

where D_1, D_2, and D_3 are defined by

$$\begin{aligned}
D_1&\equiv\left[\tfrac{\omega(1-\alpha)v}{(1+n)(1-\omega)}+(1+v)\right]\left(\tfrac{\beta}{1+\beta}\tfrac{1/v}{1+1/v}-1\right)-\beta\left(1+\theta\right)\alpha(1+v)\left[(-1)\tfrac{\beta}{1+\beta}\tfrac{1/v}{1+1/v}\tfrac{1+v}{1+\alpha v}+1+\tfrac{v(1-\alpha)}{1+\alpha v}\right],\\
D_2&\equiv\left[\tfrac{\omega(1-\alpha)v}{(1+n)(1-\omega)}+(1+v)\right]\left[1+\left(1+\tfrac{(1+n)(1-\omega)}{\omega}\right)\theta\right]+\tfrac{\beta(1+\theta)\alpha(1+v)(1-\alpha)v}{1+\alpha v},\\
D_3&\equiv\left[\tfrac{\omega(1-\alpha)v}{(1+n)(1-\omega)}+(1+v)\right]+\tfrac{\beta(1+\theta)\alpha(1+v)(1-\alpha)v}{1+\alpha v}.
\end{aligned}$$

Eqs. (55) and (56) characterize the atonal τ and τ^K. Substitution of (56) into (55) leads to

$$1-\tau^K=\underbrace{\frac{1-\left(\tfrac{\beta}{1+\beta}\tfrac{1/v}{1+1/v}-1\right)\tfrac{D_3}{D_1}}{\left[1+\theta+\tfrac{(1+n)(1-\omega)}{\omega}\left(\theta+\tfrac{\beta(1+\theta)\alpha(1+v)}{1+\alpha v}\right)\right]-\left(\tfrac{\beta}{1+\beta}\tfrac{1/v}{1+1/v}-1\right)\tfrac{D_2}{D_1}}}_{=\bar{T}^K}\cdot\frac{1}{\alpha}\cdot\frac{1}{1+b/k},\tag{57}$$

showing that the conjecture of τ^K in (21) is verified. In addition, we substitute (57) into (56) to obtain

$$1-\tau=\frac{1}{1-\alpha}\cdot\frac{D_2\bar{T}^K-D_3}{D_1}\equiv\bar{T},\tag{58}$$

showing that the conjecture of τ in (22) is verified.

Fourth, we substitute (57) and (58) into (53) to derive the policy function of g :

$$g = \underbrace{\frac{1+n}{2+n}\left(1+\tfrac{(1+n)(1-\omega)}{\omega}\right)\theta\left[(1-\alpha)\bar{T}\right]^{(1-\alpha)v/(1+\alpha v)}\bar{T}^{K}}_{=\bar{G}}\left[A(k)^{\alpha}\right]^{(1+v)/(1+\alpha v)}. \tag{59}$$

Finally, substitution of τ^K, τ, and g into the government budget constraint in (20) leads to the following policy function of b':

$$(1+n)b' = \bar{B}\left[A(k)^{\alpha}\right]^{(1+v)/(1+\alpha v)}, \tag{60}$$

where $\bar{B}$ is defined by

$$\bar{B} \equiv \left[(1-\alpha)\bar{T}\right]^{(1-\alpha)v/(1+\alpha v)}\left[\bar{T}^{K}+(1-\alpha)\bar{T}-1\right]+\frac{2+n}{1+n}\bar{G}.$$

■

A.3 Proof of Proposition 2.

Suppose that $v=0$ holds. The policy functions of b_{t+1}, τ_t^K, and τ_t presented in Proposition 1 are reduced to

$$\begin{aligned}
b_{t+1} &= \frac{1}{1+n}\cdot\frac{\beta(1-\alpha(1+\theta))\frac{(1+n)(1-\omega)}{\omega}}{(1+\theta)\left(1+\frac{(1+n)(1-\omega)}{\omega}(1+\alpha\beta)\right)}A(k)^{\alpha},\\
\tau_t^K &= 1-\frac{1}{(1+\theta)\left(1+\frac{(1+n)(1-\omega)}{\omega}(1+\alpha\beta)\right)}\cdot\frac{1}{\alpha(1+b_t/k_t)},\\
\tau_t &= 1-\frac{\frac{1+\beta}{1-\alpha}\cdot\frac{(1+n)(1-\omega)}{\omega}}{(1+\theta)\left(1+\frac{(1+n)(1-\omega)}{\omega}(1+\alpha\beta)\right)}.
\end{aligned}$$

The ratio of B_{t+1} to Y_t becomes

$$\frac{B_{t+1}}{Y_t} = \frac{(1+n)b_{t+1}N_t}{A(k)^{\alpha}N_t} = \frac{\beta(1-\alpha(1+\theta))}{(1+\theta)\left(\frac{\omega}{(1+n)(1-\omega)}+(1+\alpha\beta)\right)}.$$

The expression indicates that B_{t+1}/Y_t is positive and decreasing in ω if and only if $\alpha(1+\theta)<1$.

The ratio of $\tau_t^K R_r s_{t-1}N_{t-1}$ to Y_t becomes

$$\begin{aligned}
\frac{\tau_t^K R_r s_{t-1}N_{t-1}}{Y_t} &= \frac{\tau_t^K\alpha A(k)^{\alpha-1}(k_t+b_t)(1+n)N_{t-1}}{A(k)^{\alpha}N_t}\\
&= \alpha\left(1+\frac{b_t}{k_t}\right)-\frac{1}{(1+\theta)\left(1+\frac{(1+n)(1-\omega)}{\omega}(1+\alpha\beta)\right)}.
\end{aligned}$$

In period 0, the ratio becomes

$$\frac{\tau_0^K R_0 s_{-1} N_{-1}}{Y_0} = \alpha\left(1+\frac{b_0}{k_{t0}}\right) - \frac{1}{(1+\theta)\left(1+\frac{(1+n)(1-\omega)}{\omega}(1+\alpha\beta)\right)}.$$

Given the initial conditions of k_0 and b_0, the equation indicates that the ratio $\tau_0^K R_0 s_{-1} N_{-1}/Y_0$ is decreasing in ω. In period $t \geq 1$, we have

$$1+\frac{b_t}{k_t} = \frac{1-\alpha\theta}{\alpha(1+\theta)}.$$

Thus, the ratio becomes

$$\frac{\tau_t^K R_r s_{t-1} N_{t-1}}{Y_t} = \frac{1-\alpha\theta}{(1+\theta)} - \frac{1}{(1+\theta)\left(1+\frac{(1+n)(1-\omega)}{\omega}(1+\alpha\beta)\right)},$$

showing that $\tau_t^K R_r s_{t-1} N_{t-1}/Y_t$ is decreasing in ω.

The ratio of $\tau_t w_t N_t$ to Y_t becomes

$$\frac{\tau_t w_t N_t}{Y_t} = \frac{\tau_t (1-\alpha) A (k)^\alpha N_t}{A(k)^\alpha N_t} = (1-\alpha) - \frac{1+\beta}{(1+\theta)\left(\frac{\omega}{(1+n)(1-\omega)} + (1+\alpha\beta)\right)}.$$

The equation indicates that $\tau_t w_t N_t / Y_t$ is increasing in ω.

■

A.4 Calibration of v

The labor income tax revenue is

$$\begin{aligned}\tau_t w_t l_t N_t &= \tau_t (1-\alpha) A (k_t)^\alpha (l_t)^{-\alpha} l_t N_t \\ &= \left[(1-\alpha) A (k_t)^\alpha\right]^{(1-\alpha)v/(1+\alpha v)} N_t \tau_t (1-\tau_t)^{(1-\alpha)v/(1+\alpha v)},\end{aligned}$$

where the equality in the second line comes from substituting the labor supply function in Eq. (9) into the expression in the first line. The revenue-maximizing tax rate, denoted by $\tau_{\max}$, satisfies the following first-order condition:

$$(1-\tau_{\max})^{(1-\alpha)v/(1+\alpha v)} - \tau_{\max}\frac{(1-\alpha)v}{1+\alpha v}(1-\tau_{\max})^{(1-\alpha)v/(1+\alpha v)-1} = 0,$$

which leads to

$$\tau_{\max} = \frac{1+\alpha v}{1+v}.$$

Following Trabandt and Uhlig (2011), we set v so that the top of the labor income tax Laffer curve is at 60%. Setting $\tau_{\max} = 0.6$ and $\alpha = 1/3$, we obtain $v = 3/2$.

■

A.5 Numerical Results

Figure A.1 reports the numerical results for 25 sample countries, including Australia, Austria, Chile, Czech Republic, Denmark, Estonia, Finland, France, Germany, Greece, Hungary, Italy, Japan, Lithuania, Luxemburg, the Netherlands, Norway, Poland, Portugal, Republic of Latvia, Slovenia, Spain, Sweden, Switzerland, and the United Kingdom. The ratio of capital income tax revenue to GDP shows a U-shaped pattern for Australia, Chile, Estonia, Japan, Latvia, Lithuania, and Switzerland whereas it is monotonically increasing in ω for other countries.

[Figure A.1.]

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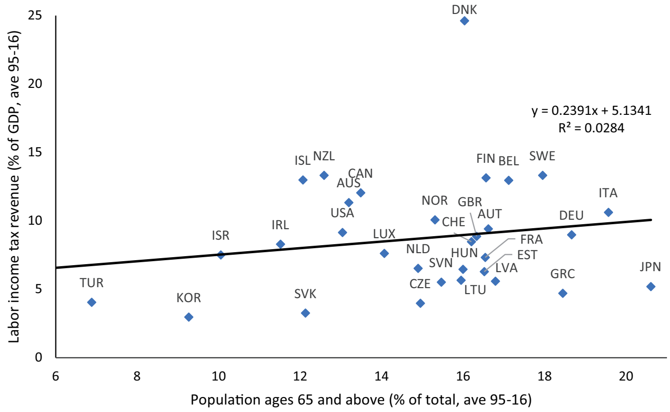
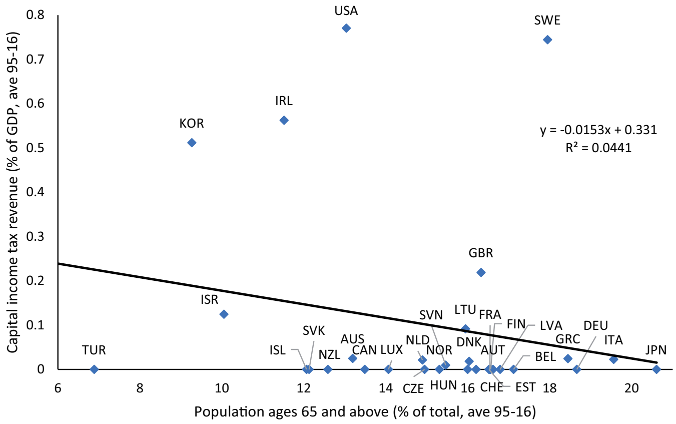
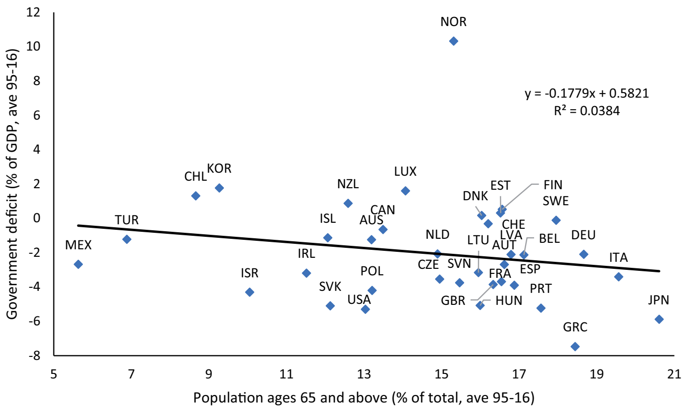


(a)



(b)



(c)

Figure 1: Each panel plots the data for OECD countries during 1995–2016. The vertical axis takes the average share of population aged 65 years and over. The horizontal axis takes the average ratio of labor income tax revenue to GDP (Panel (a)), the average ratio of capital income tax revenue to GDP (Panel (b)), and average ratio of deficit to GDP (Panel (c)). In Panel (c), the budget deficit is taken as an approximate variable for the public debt.
Source: OECD.Stat (https://stats.oecd.org/) (accessed on September 25, 2019).

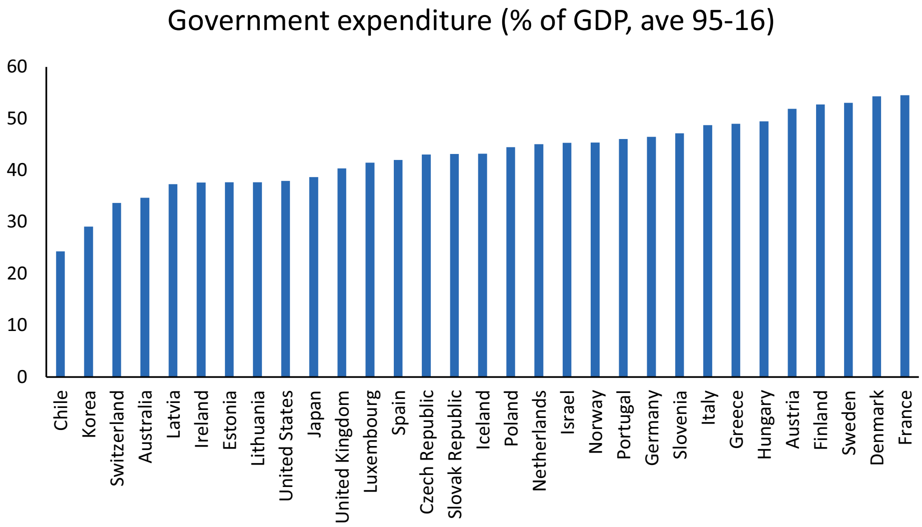


Figure 2: Average ratios of government expenditure to GDP for OECD countries during 1995–2016. Belgium, Canada, Mexico, Turkey, and New Zealand are not included in the figure because of missing data. The same applies to Table 1.
Source: OECD.Stat (https://stats.oecd.org/) (accessed on September 25, 2019).

	AUS	AUT	CZE	DNK	FIN	FRA	DEU	GRC	HUN	ISL	IRL
n	0.511	0.147	0.0326	0.139	0.11	0.181	0.0174	0.0251	-0.07	0.375	0.491
θ	1.78	18.8	3.64	61.5	26.7	77.1	5.5	8.36	8.84	3.89	2.28
	ITA	JPN	KOR	LUX	NLD	NOR	POL	PRT	SVK	ESP	SWE
n	0.0957	0.0178	0.2	0.661	0.147	0.298	-0.0239	0.0428	0.0178	0.25	0.179
θ	8.1	2.38	1.14	3.33	4.68	5.01	4.24	5.22	3.67	3.37	32.8
	CHE	GBR	USA	CHL	EST	ISR	SVN	LVA	LTU		
n	0.281	0.193	0.318	0.38	-0.117	0.855	0.0543	-0.287	-0.284		
θ	1.62	2.85	2.31	0.812	2.15	5.38	6.17	2.04	2.1		

Table 1: The average population growth rate for OECD countries during 1995–2016 and the associated estimation of θ.
Source: OECD.Stat (https://stats.oecd.org/) (accessed on September 25, 2019).

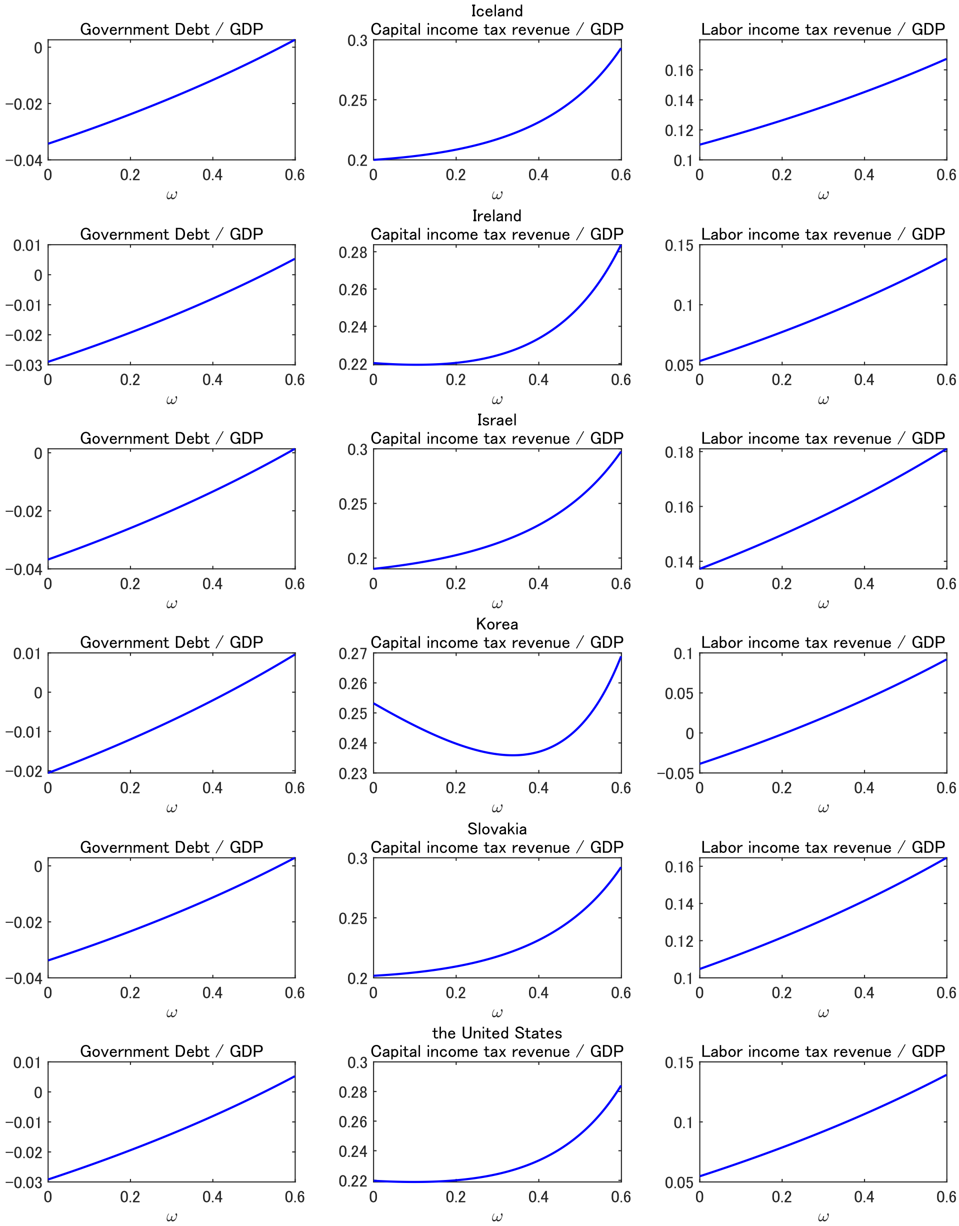


Figure 3: Predicted ratios of government debt, capital income tax revenue, and labor income tax revenue to GDP for Iceland, Ireland, Israel, Korea, Slovakia, and the United States.

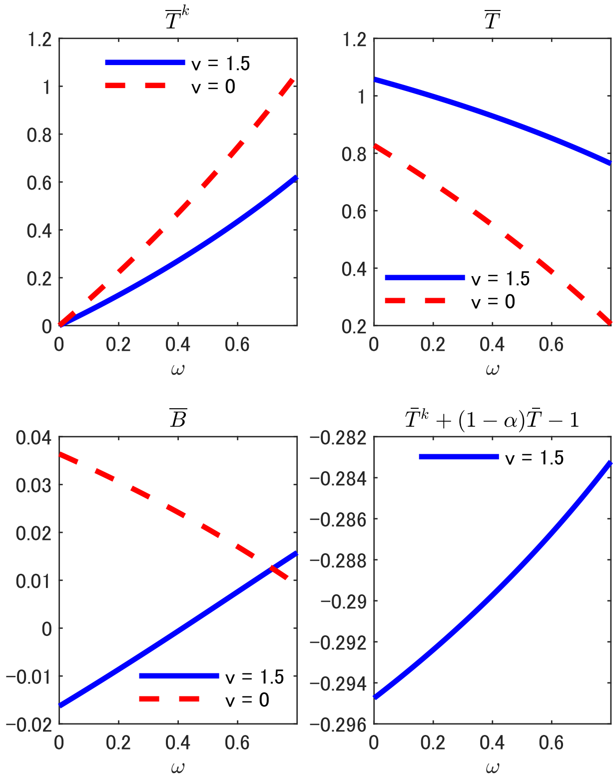


Figure 4: Numerical illustration of the effects of ω on $\bar{T}^K$, $\bar{T}$, $\bar{G}$, $\bar{B}$, and $\bar{T}^K + (1-\alpha)\bar{T} - 1$ for the two cases of Korea: $v = 0$ and $v = 1.5$.

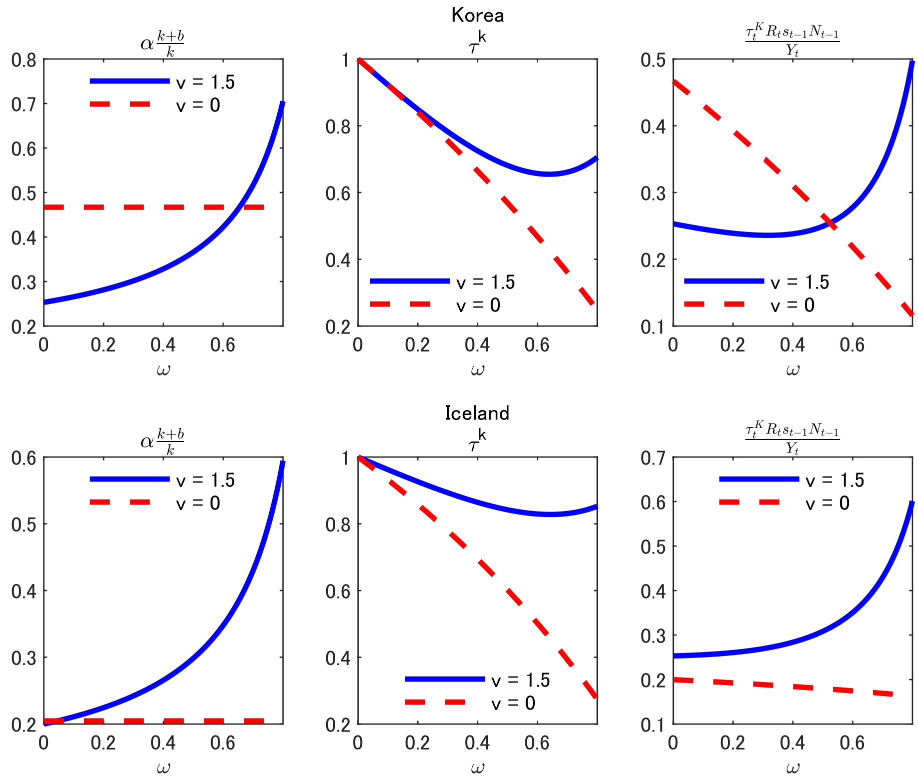


Figure 5: Numerical illustration of the effects of ω on b_t/k_t, τ_t^K, and $\tau_t^K R_t s_{t-1} N_{t-1}/Y_t$ for the cases of Korea (upper panels) and Iceland (lower panels). The results for the cases of $v = 0$ and 1.5 are plotted by dotted and solid curves, respectively.

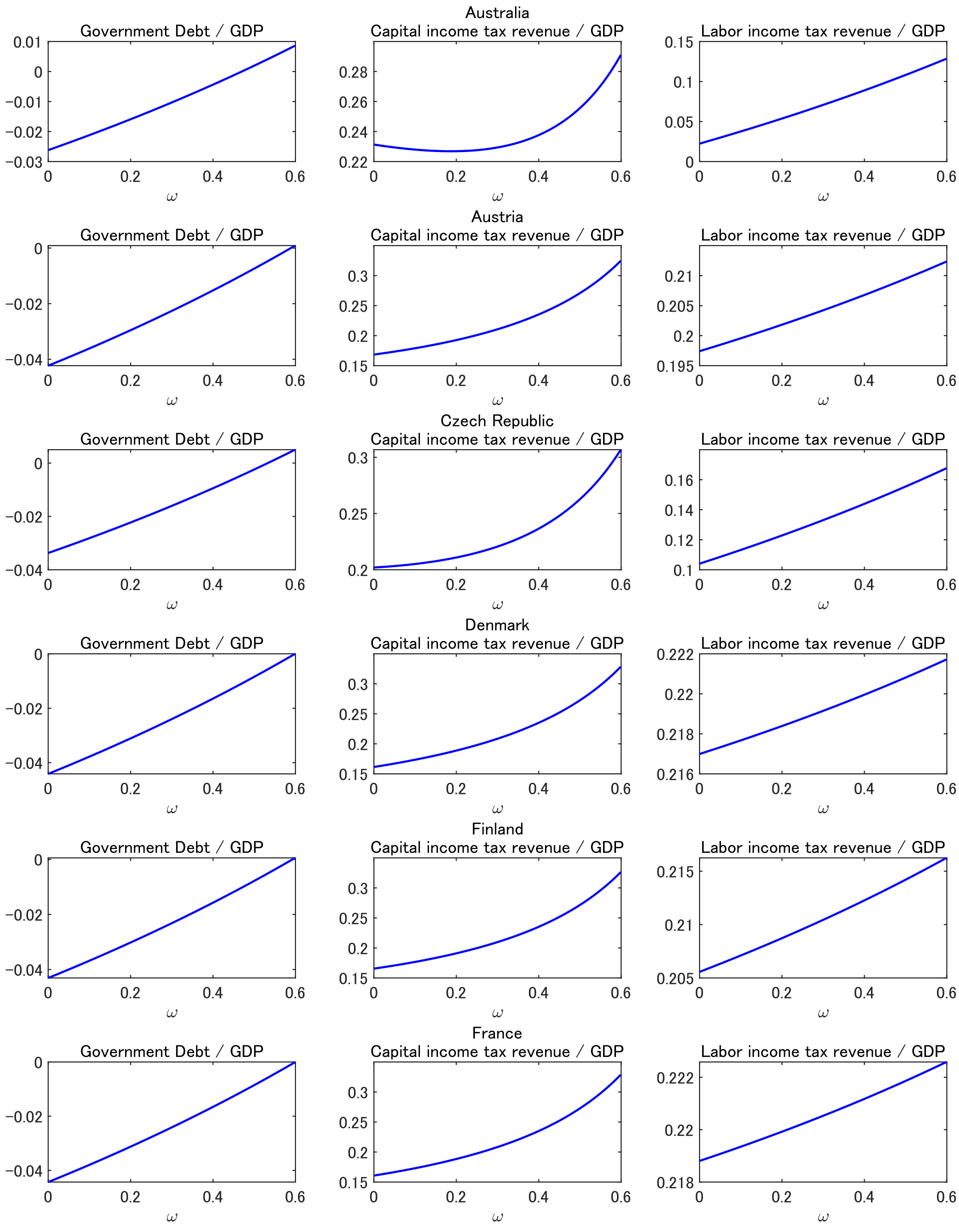
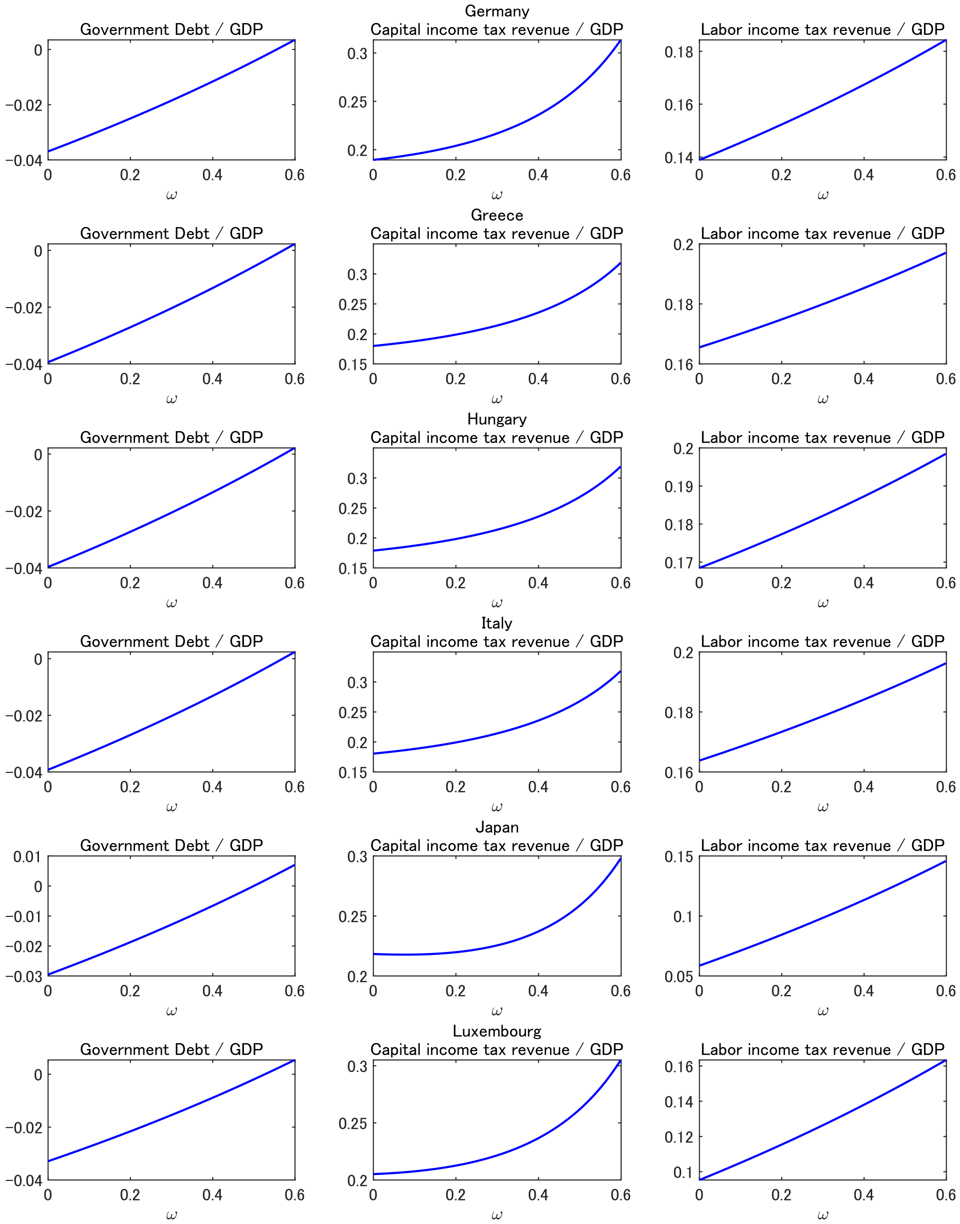
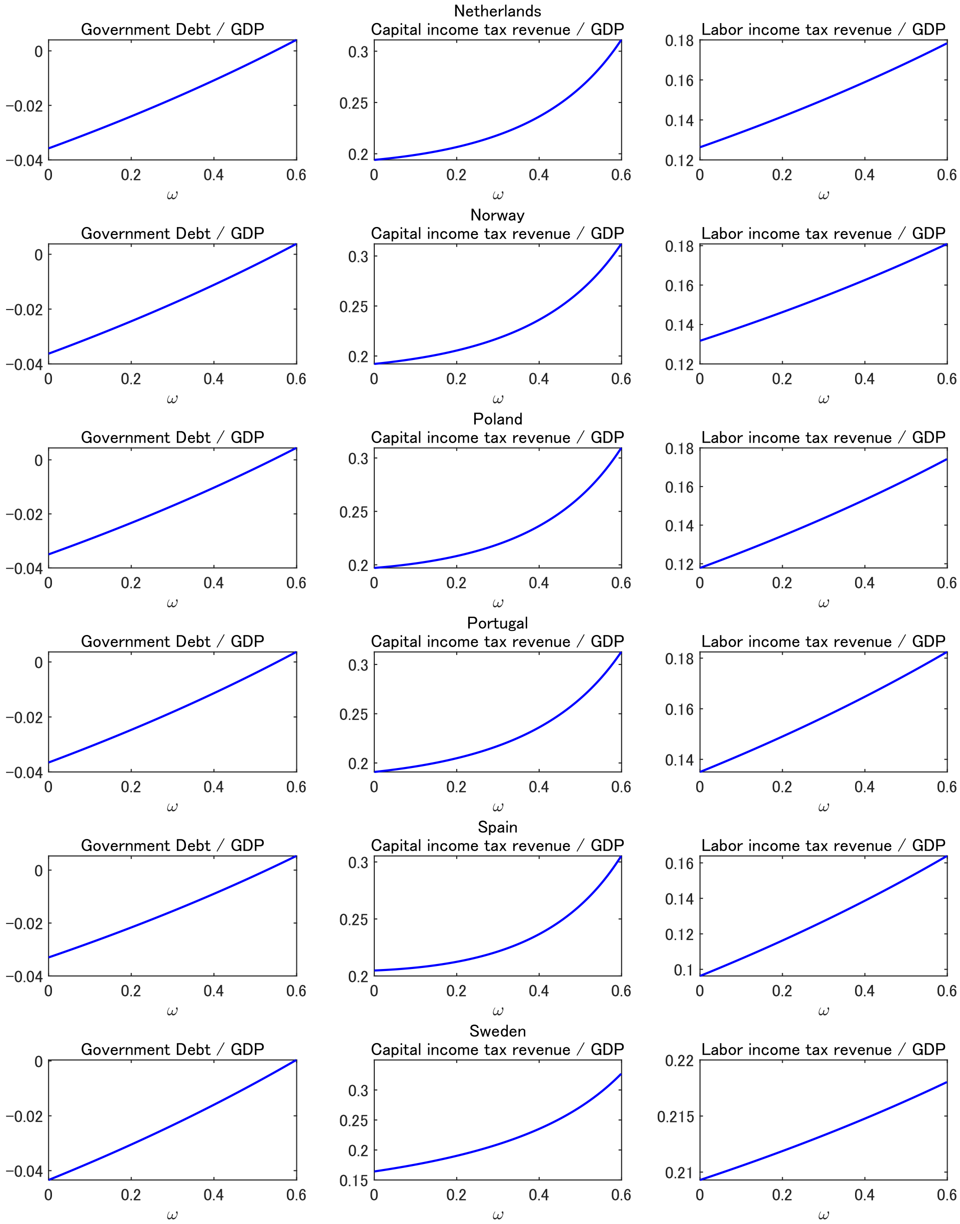
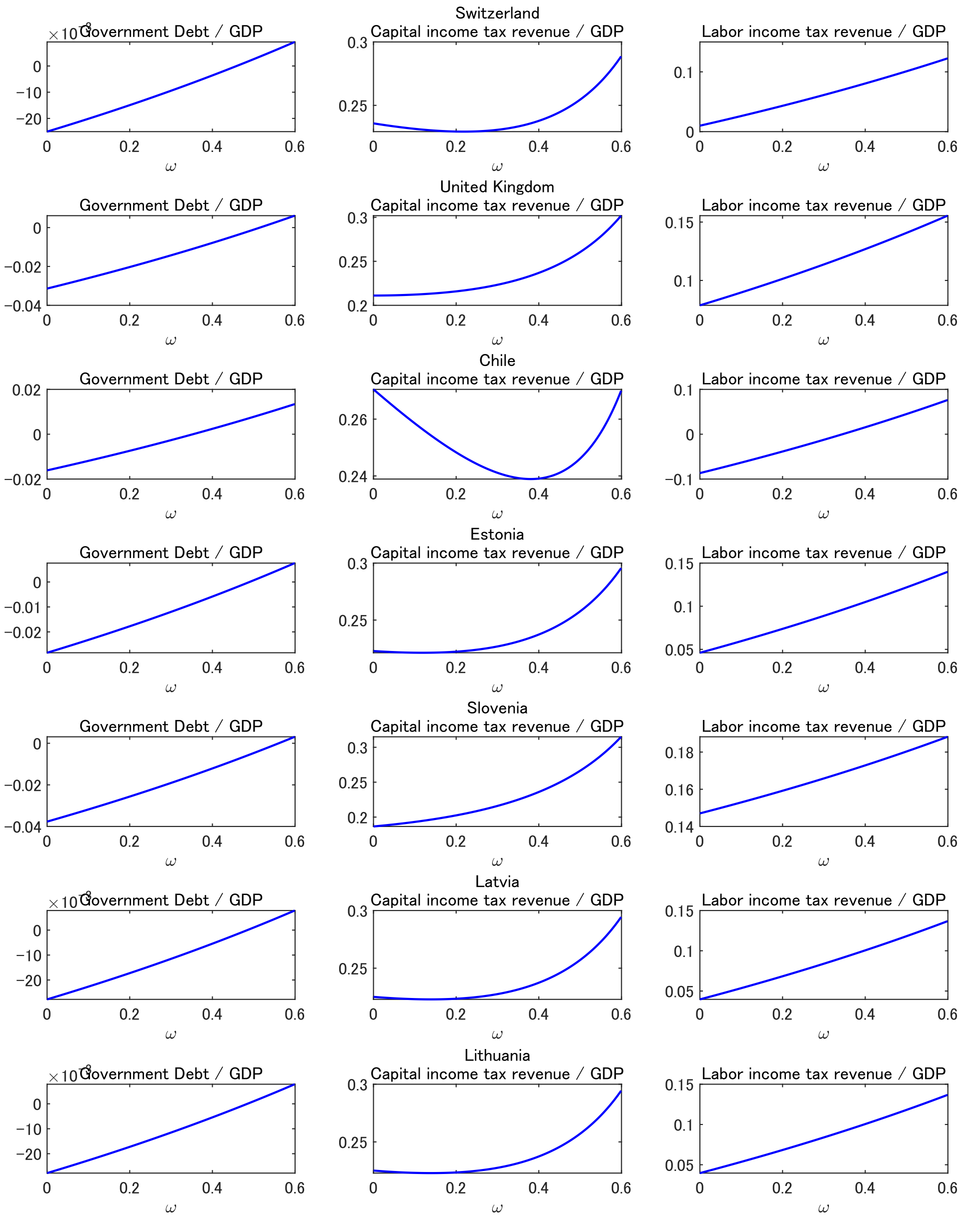


Figure A.1: Predicted ratios of government debt, capital income tax revenue, and labor income tax revenue to GDP for 25 sample countries.


Germany
Greece
Hungary
Italy
Japan
Luxembourg
Government Debt / GDP
Capital income tax revenue / GDP
Labor income tax revenue / GDP
ω


Netherlands
Government Debt / GDP
Capital income tax revenue / GDP
Labor income tax revenue / GDP
Norway
Government Debt / GDP
Capital income tax revenue / GDP
Labor income tax revenue / GDP
Poland
Government Debt / GDP
Capital income tax revenue / GDP
Labor income tax revenue / GDP
Portugal
Government Debt / GDP
Capital income tax revenue / GDP
Labor income tax revenue / GDP
Spain
Government Debt / GDP
Capital income tax revenue / GDP
Labor income tax revenue / GDP
Sweden
Government Debt / GDP
Capital income tax revenue / GDP
Labor income tax revenue / GDP
ω


Switzerland
$\times10^{-3}$
Government Debt / GDP
Capital income tax revenue / GDP
Labor income tax revenue / GDP
0
−10
−20
0.3
0.25
0.1
0
0.2
0.4
0.6
ω
United Kingdom
Government Debt / GDP
Capital income tax revenue / GDP
Labor income tax revenue / GDP
0
−0.02
−0.04
0.3
0.25
0.2
0.15
0.1
Chile
Government Debt / GDP
Capital income tax revenue / GDP
Labor income tax revenue / GDP
0.02
0
−0.02
0.26
0.24
0.1
0
−0.1
Estonia
Government Debt / GDP
Capital income tax revenue / GDP
Labor income tax revenue / GDP
0
−0.01
−0.02
0.3
0.25
0.15
0.1
0.05
Slovenia
Government Debt / GDP
Capital income tax revenue / GDP
Labor income tax revenue / GDP
0
−0.02
−0.04
0.3
0.25
0.2
0.18
0.16
0.14
Latvia
$\times10^{-3}$
Government Debt / GDP
Capital income tax revenue / GDP
Labor income tax revenue / GDP
0
−10
−20
0.3
0.25
0.15
0.1
0.05
Lithuania
$\times10^{-3}$
Government Debt / GDP
Capital income tax revenue / GDP
Labor income tax revenue / GDP
0
−10
−20
0.3
0.25
0.15
0.1
0.05