Corporate Debt, Rentiers' Portfolio Dynamics, Instability and Growth: A neo-Kaleckian Perspective

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Abstract

Considering a stock-flow consistent neo-Kaleckian macro-model, along with firms' debt dynamics, in the long-run, we incorporate portfolio dynamics of rentiers and investigate the possibility of multiple equilibria and dynamic stability of the economy. Both the debt-led and the debt-burdened demand and growth regimes are possible. We find share buybacks, under certain conditions, not only may lead to the deterioration of the equilibrium rate of capital accumulation in the long-run but may potentially destabilize the entire economy. A strictly regulated financial market is desirable, as otherwise, the economy may lose its stability and produces the limit cycles.

Keywords: Capital Accumulation, Kaleckian Model, Stock-flow Consistency, Instability, Limit Cycle

JEL classification: C62, E12, E32, E44, O41.

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1 *Introduction*

The purpose of this paper is to present a new-Kaleckian stock-flow consistent macroeconomic model of the US economy to explain (i) how firms take their decisions related to financial and real sectors (ii) how the rentiers take their portfolio decisions and (iii) how the interaction between financial and real sectors leads to the occurrence of multiple equilibria and instability in the economy.

Although neo-Kaleckian theory of growth and distribution started with the contribution of Rowthorn (1981), Dutt (1984), Taylor (1985), Amadeo (1986), Blecker (1989), Bhaduri and Marglin (1990), Marglin and Bhaduri (1990), financial variables have been introduced much later in this tradition. While Hein (2006, 2007, 2008a, 2008b, 2012b), Lima and Meirelles (2007), Charles (2008a, 200b), and Parui (2020a) introduce firms' debt dynamics in the Kaleckian growth model and investigate the financial fragility and instability in the economy, Dutt (2006), Hein (2012a, 2012b), Kim (2012), Kim *et al.* (2014), Kapeller and Schütz (2015), Setterfield and Kim (2016), Setterfield *et al.* (2016), and Parui (2020b) are among others who capture the workers' debt dynamics in the Kaleckian tradition.

In a neo-Kaleckian model of growth and distribution with excess capacity, by endogenizing the retention rate and the level of debt, Charles(2008a) investigates the conditions for multiple equilibria and the possibility of instability. However, some problems with his model can be observed. First, Charles (2008a) assumes a positive relationship between the retention ratio and the debt level. According to him in case of a higher level of debt, "to preserve their financial autonomy and their ability to meet financial commitments" firms reduce dividend payment and increase retention ratio. However, in the era of financialization, precisely the opposite of that happens. A higher level of debt results in a higher level of risk and financial fragility. Consequently, shareholders would demand higher dividends for compensating the high level of risk. Dividend can also be perceived (by shareholders) as a signal for profitability, financial strength, and stability of firms (Baker and Wurgler (2012)). Therefore, when there is a higher level of risk caused by a higher level of debt, firms would prefer to provide a higher level of dividend as a signal of its financial stability to the shareholders. Dallery and Treeck (2011) also validate this. Second, Charles (2008a) assumes debt-capital ratio as the only determinant of targeted retention ratio. Along with the debt-capital ratio, however, expected future profit rate and expected growth rate are also key determinants of firms targeted dividend capital ratio. However, this is missing in his analysis.

Charles (2008b) develops a simple post-Keynesian macro-model and shows the instability in the economy because of a change in interest rate. Charles endogenizes the interest rate that fluctuates due to the change in debt-capital ratio or due to change in the exogenous interest rate of the riskless assets. According to him higher the debt-capital ratio, the higher are the borrower's risk and the rate of interest (of risky assets). While growing interest rate can destabilize the economy, on the other hand, an easy monetary policy can prevent the economy from collapsing.

Taylor and Rada (2003) construct a post-Keynesian macro-model related to cycles of business debt and equity. Over the past 50-100 years, according to them, six secular debt-equity cycles occur in data for the US, UK and Japan. A clockwise pattern is observable for the US and the UK while in case of Japan, a counterclockwise pattern is followed. The clockwise pattern arises for a "debt-led" capital accumulation rate and the "equity accelerated"[1] debt-capital ratio whereas, "debt-burdened" capital accumulation rate and "equity-decelerated" debt-capital ratio lead to the occurrence of counterclockwise cycles.

Ryoo (2010) in a stock-flow consistent macro model, shows the financial fragility that evolves endogenously through the interaction between real and financial sectors due to firms' and households' financial practices. Ryoo provides two distinct cycles: long waves and short cycles. Interaction between firms' and households' financial decisions generates long waves, whereas the interaction between effective demand and labor market dynamics produces short cycles. In his model, firms decide how much to accumulate and how to finance it; households take the consumption and portfolio decisions, and banks receive deposits and create loans. There are only two types of financial assets: equity and bank deposits. He assumes that there is a constant growth in the available labor force, and the long run growth is constrained by it.

The mechanism of long waves depends on two subsystems: (*i*) changes in firms' liability structure (i.e. the debt–capital ratio) and (*ii*) changes in households' portfolio composition (i.e. the equity–deposit ratio). In the long run a change in firms' debt–capital ratio depends on the ratio of the trend rate of profit to the interest payment obligation. If the level of profit is sufficiently high compared to interest payment obligations, firms are eager to take more debt while banks are willing to provide the required debt as higher profit level compared to an interest payment of firms is perceived by banks as having a lower probability of default. Banks play a passive role, and the availability of credit is independent of the financial position of banks. Households' equity to deposit ratio (or in other words households' portfolio decision) depends on households' optimism about stock markets which in turn depends on the difference between the rates of return on stocks and deposits. "Capital gains from holding stocks are not assumed away and enter the definition of the rate of return on equity". During good years, households tend to hold a more significant proportion of financial assets in the form of riskier assets i.e. on

[1] If the effect of a rise in equity-capital ratio on the change in debt-capital ratio is positive, Taylor and Rada (2003) call it as "equity-accelerated" debt-capital ratio. Otherwise, as they say, an "equity-decelerated" debt-capital ratio occurs.

equity. In this model, in the long run, these above mentioned two stable subsystems, when interacting with each other, can produce instability and cycles in the whole system.

For the short cycles, output growth is determined by the labor market condition and the profit signals in the goods market. Higher profitability instigates firms to expand output, whereas the tightened labor market provides negative incentives to the firms for expanding production. On the other hand, growth in the employment rate is determined by the difference between the output growth and the growth in the available labor force, which grows exponentially at a constant rate. Then Ryoo (2010) integrates long waves with short cycles and provides some simulation results.

However, Ryoo (2010) assumes equity finance as a pure residual of firms' financing constraint as it serves as a buffer to fill the gap between the funds needed for the investment plans and the funds available from retained earnings and bank loans. However, it is very unlikely that for a sufficiently long period funds needed for the investment plans of firms are lower than the available fund from retained earnings and borrowings so that they can adjust the excess funds by repurchasing stocks. We elaborately discuss it later in Section 3.

Few essential features of our model are: first, the model is stock-flow consistent. Second, although the economy is always in a wage-led demand regime, both the debt-led and the debt-burdened demand and growth regimes are possible. Third, the interaction between the dynamics of debt-capital ratio and the equity-debt ratio allows the existence of multiple equilibria and opens the possibility of instability in the economy. Fourth, share buybacks, under certain conditions, not only may lead to the deterioration of the equilibrium rate of capital accumulation in the long run, but may potentially destabilize the entire economy.

The outline of the rest of the paper is as follows. Section 2 sets up the model and talks about the short run analysis and the short run comparative statics. Section 3 discusses the long run analysis where we endogenize the debt-capital ratio and the equity-debt ratio. Section 4 discusses several possible cases which may arise because of the interaction between the debt-capital ratio and the equity-debt ratio dynamics. This is followed by the discussion of Hopf bifurcation where we analyse how the interaction between firms' and rentiers' financial practices can produce a limit cycle. Section 6 talks about the comparative statics. Section 7 offers some concluding remarks.

2 *The Model*

We assume a simple one-sector, closed economy, neo-Kaleckian growth model in which the economy consists of workers, rentiers and firms. Neither government intervention nor technical progress is there. Income is distributed between wages and profits as

$$Y = W + R \tag{2.1}$$

where, Y is nominal income, W is nominal wage income and R is nominal profit income. We assume excess supply of labor and under-utilization of capacity in the economy. For simplicity we assume away depreciation of capital. We assume two types of households-workers and rentiers. Workers do not hold any asset and consume whatever they earn i.e.,

$$C_W = W = [(1-\pi)u]\,K \tag{2.2}$$

where, C_W is consumption of workers, K is the existing capital stock, $u = \frac{Y}{K}$ is the degree of capacity utilization,[2] $\pi = \frac{R}{Y}$ is share of profit (which is fixed in the short run as well as in the long run), and $r = \frac{R}{K}$ is profit rate. So, $r = \pi u$.

Rentiers hold two types of assets (i) deposits with the banks, and (ii) equities that are issued by firms. Equities are considered to be a more risky asset compared to bank deposits. Banks play a passive role of allotting those deposits to the firms as credit.[3] Rentiers earn their income from two sources, interest income on the funds they lend to the firms and from a fraction of profit given to them as dividend by the firms.[4] Therefore, consumption of rentiers (C_R) can be represented as

$$C_R = c_r[(1-s_f)(R-iD)+iD] + c_q(P_eE + D) \tag{2.3}$$

$$\Rightarrow \frac{C_R}{K} = c_r[(1-s_f)(\pi u - id) + id] + (1+\lambda)c_q d \tag{2.4}$$

where c_r is the consumption propensity of rentiers out of income and c_q is the consumption propensity out of wealth,[5] i is the interest rate on both deposit and loan, D is total debt of firms to the rentiers, d is debt to capital ratio, $\lambda = \frac{P_eE}{D}$ is nominal market value of equity to debt ratio, s_f is retention rate of firms. Following neo-Kaleckian literature we assume a fraction of profit (or profit net of interest payment) is given as a dividend to the rentiers (see Charles (2008a, 2008b) for example). Following Charles (2008a, 2008b)

[2]For a fixed potential output-capital ratio, the actual output-capital ratio can be used as a proxy for the degree of capacity utilization.

[3]So effectively the rentiers lend money to the firms.

[4]By exercising the equities rentiers also can get capital gain or loss.

[5]$c_r \in (0,1)$, $c_q \in (0,1)$ and $c_r > c_q$.

Table 2.1: Balance sheet matrix

	Workers' households	Rentiers' households	Firms	$\sum$
Loans		$+D$	$-D$	0
Equities		$+P_eE$	$-P_eE$	0
Capital			K	K
$\sum$	0	$D + P_eE$	$K - (P_eE + D)$	K

Table 2.2: Transaction flow matrix

	Workers' households	Rentiers' households	Firms' current	Firms' capital	$\sum$
Consumption	$-C_W$	$-C_R$	$C_W + C_R$		0
Investment			I	$-I$	0
Wages	W		$-W$		0
Retained profits			$-s_fR$	s_fR	0
Distributed profits (dividends)		$(1 - s_f)R$	$-(1 - s_f)R$		0
(Value of) Change in equities		$-P_e\dot{E}$		$P_e\dot{E}$	0
Interest on loans		iD	$-iD$		0
Change in loans		$-\dot{D}$		$\dot{D}$	0
$\sum$	0	0	0	0	0

we assume investment function as

$$I = [\alpha_0 + \alpha_1 s_f(\pi u - id)]K \tag{2.5}$$

where, α_0 represents the state of animal spirits and α_1, the coefficient measuring the responsiveness of investment-capital ratio to a change in available internal funds. In order to make the model tractable we assume a very simple investment function (I) that depends only on the state of animal spirits (α_0), and on available internal funds. Our primary purpose in this paper is to see the short run impact of debt and the equity-debt ratio on aggregate demand, income distribution and economic growth and then to see the long run dynamics between debt-capital and equity-debt ratio. Therefore we concentrate on the role of internal funding on investment decisions and ignore the influence of capacity utilization. This kind of investment function can be found in Charles (2005, 2008a, 2008b), and the empirical evidence can be found in Ndikumana (1999).

The basic structure of the model is summarized by the balance sheet matrix in Table 2.1 and the transaction flow matrix in Table 2.2.

2.1 *Short run equilibrium*

In the short run, the goods market is cleared through changes in the level of output and capacity utilization. In equilibrium, nominal income must be equal to aggregate demand which in turn implies

$$\Rightarrow u^* = \frac{\alpha_0 + (1+\lambda)c_q d + (c_r - \alpha_1)s_f i d}{\pi\{1 - c_r + s_f(c_r - \alpha_1)\}} \tag{2.6}$$

where u^* is the equilibrium degree of capacity utilization. We assume Keynesian stability condition holds i.e. the induced increase in saving as u rises (i.e. $(1 - c_r + c_r s_f)\pi$) is greater than the induced increase in investment (i.e. $\alpha_1 s_f \pi$).

$$\text{In other words,} \{1 - c_r + s_f(c_r - \alpha_1)\} > 0 \tag{2.7}$$

For a meaningful solution of the equilibrium degree of capacity utilization, from equation (2.6) we assume

$$\alpha_0 > -(1+\lambda)c_q d + (\alpha_1 - c_r)s_f i d \tag{2.8}$$

If $(c_r - \alpha_1) > 0$, then $[\alpha_0 + (1+\lambda)c_q d + (c_r - \alpha_1)s_f i d]$ is unambiguously positive. If $(c_r - \alpha_1) < 0$ then $[\alpha_0 + (1+\lambda)c_q d > (\alpha_1 - c_r)s_f i d]$ is required for the meaningful solution. Putting the equilibrium value of degree of capacity utilization (u^*) into equation (2.5)[6] we get

$$g^* = \frac{\alpha_0(1 - c_r + s_f c_r) + \alpha_1 s_f\{(1+\lambda)c_q - (1 - c_r)i\}d}{[1 - c_r + s_f(c_r - \alpha_1)]} \tag{2.9}$$

where g^* is the equilibrium rate of capital accumulation. Given equation (2.7), for a positive equilibrium growth rate, from equation (2.9) we assume

$$\alpha_0(1 - c_r + s_f c_r) + \alpha_1 s_f\{(1+\lambda)c_q - (1 - c_r)i\}d > 0 \tag{2.10}$$

We also assume $s_f \in (0,1)$, and $\alpha_1 \in (0,1)$. Therefore, $\alpha_1 s_f \in (0,1)$.

The equilibrium rate of profit is

$$r^* = \pi u^* = \frac{\alpha_0 + (1+\lambda)c_q d + (c_r - \alpha_1)s_f i d}{\{1 - c_r + s_f(c_r - \alpha_1)\}}. \tag{2.11}$$

[6] Growth rate of the economy is expressed as $g = \frac{I}{K}$

2.2 *Comparative Statics*

Differentiating partially u^*, g^* and r^* with respect to α_0 we get,

$$\frac{\partial u^*}{\partial \alpha_0} = \frac{1}{\{S_F - s_f\alpha_1\}\pi} > 0; \quad \frac{\partial g^*}{\partial \alpha_0} = \frac{S_F}{\{S_F - s_f\alpha_1)\}} > 0; \quad \frac{\partial r^*}{\partial \alpha_0} = \frac{1}{\{S_F - s_f\alpha_1)\}} > 0 \tag{2.12}$$

where, $S_F = 1 - c_r + s_f c_r > 0$, and $\{S_F - s_f\alpha_1)\} = \{1 - c_r + s_f(c_r - \alpha_1)\}$. Due to a rise in animal spirits (α_0), equilibrium degree of capacity utilization, growth rate and rate of profit all increase.

Partial differentiation of u^*,and r^* with respect to d yields,

$$\frac{\partial u^*}{\partial d} = \frac{(1+\lambda)c_q + (c_r - \alpha_1)s_f i}{\{S_F - s_f\alpha_1)\}\pi}; \quad \frac{\partial r^*}{\partial d} = \frac{(1+\lambda)c_q + (c_r - \alpha_1)s_f i}{\{S_F - s_f\alpha_1)\}} \tag{2.13}$$

If $(c_r - \alpha_1) > 0$, the economy is always in a debt-led demand regime i.e. $\frac{\partial u^*}{\partial d} > 0$. But, if $(c_r - \alpha_1) < 0$ then if $(1+\lambda)c_q < (c_r - \alpha_1)s_f i$ the economy is in a debt-burdened demand regime i.e.$\frac{\partial u^*}{\partial d} < 0$. Otherwise it is in a debt-led demand regime. The movement in r^* is in the same direction as u^*. An increase in d, by reducing the available internal funds, reduces investment demand by $\alpha_1 s_f i$ unit (see equation (2.5)). But consumption demand of rentiers increases by $\{c_r s_f i + (1+\lambda)c_q\}$ unit (see equation (2.4)). If the latter is higher than the former, then for a given amount of capital the aggregate demand and hence the degree of capacity utilization rises.

Partial differentiation of g^* with respect to d yields,

$$\frac{\partial g^*}{\partial d} = \frac{\alpha_1 s_f\{(1+\lambda)c_q - (1-c_r)i\}}{\{S_F - s_f\alpha_1)\}} \tag{2.14}$$

The economy is in a debt-led growth regime if and only if $(1+\lambda)c_q > (1-c_r)i$. Otherwise, it is in a debt-burdened growth regime. An increase in d affects the equilibrium growth rate in two ways. First, by reducing the available internal fund it directly negatively affects the growth rate. On the other hand, through its effect on equilibrium degree of capacity utilization, it indirectly affects the equilibrium growth rate.[7]

Differentiating partially u^*, g^* and r^* with respect to i we get,

$$\frac{\partial u^*}{\partial i} = \frac{(c_r - \alpha_1)s_f d}{\{S_F - s_f\alpha_1)\}\pi} \gtreqless 0; \quad \frac{\partial g^*}{\partial i} = -\frac{\alpha_1 s_f(1-c_r)d}{\{S_F - s_f\alpha_1)\}} < 0; \quad \frac{\partial r^*}{\partial i} = \frac{(c_r - \alpha_1)s_f d}{\{S_F - s_f\alpha_1)\}} \gtreqless 0 \tag{2.15}$$

An increase in the interest rate, by reducing the available internal funds, reduces the investment rate (i.e. investment to capital ratio) by $\alpha_1 s_f d$ unit whereas the consumption

[7] The latter effect is ambiguous.

rate of rentiers (i.e. rentiers' consumption to capital ratio) increases by $c_r s_f d$ unit. If the latter is greater than the former then, for a given amount of capital, the aggregate demand and hence the degree of capacity utilization rises.

A rise in the interest rate leads to a fall in the equilibrium rate of capital accumulation. The reason is two fold. First, by reducing internal funds it directly negatively affects on the growth rate. On the other hand, through its effect on the equilibrium degree of capacity utilization, it indirectly affects the equilibrium growth rate. The latter effect[8] is however smaller than the former. Therefore, the impact of a rise in the interest rate on the equilibrium rate of capital accumulation is always negative.

Differentiating partially u^*, g^* and r^* with respect to λ we get,

$$\frac{\partial u^*}{\partial \lambda} = \frac{c_q d}{\{S_F - s_f \alpha_1)\}\pi} > 0; \quad \frac{\partial g^*}{\partial \lambda} = \frac{\alpha_1 s_f c_q d}{\{S_F - s_f \alpha_1)\}} > 0; \quad \frac{\partial r^*}{\partial \lambda} = \frac{c_q d}{\{S_F - s_f \alpha_1)\}} > 0 \tag{2.16}$$

Unlike the debt to capital ratio, for a given level of debt, an increase in the equity to debt ratio unambiguously increases the equilibrium degree of capacity utilization, the equilibrium rate of profit and the equilibrium growth rate. For a given level of debt, due to a rise in the equity to debt ratio (i.e. a rise in the total value of equity), the consumption rate of rentiers increases by $c_q d$ unit while there is no initial change in investment demand. Hence the equilibrium degree of capacity utilization increases. Through its positive influence on u^*, the equity to debt ratio positively affects the equilibrium growth rate.

Differentiating partially u^*, g^* and r^* with respect to π we get,

$$\frac{du^*}{d\pi} = -\frac{u^*}{\pi} < 0; \quad \frac{dg^*}{d\pi} = \frac{dr^*}{d\pi} = 0 \tag{2.17}$$

Equation (2.17) explains that the economy is in a wage-led demand regime. This is because a rise in profit share (or a fall in wage share) distributes income from wage earners (who have a very high propensity to consume ($c_w = 1$)) to profit earners (who have a lower propensity to spend ($c_r < 1$)).

A rise in π, for a given value of u^*, raises the investment rate by $\alpha_1 s_f u^*$ unit whereas a rise in π, through its effect on u^*, reduces the investment rate by exactly the same unit (i.e. by $\alpha_1 s_f \pi \frac{\partial u^*}{\partial d} = \alpha_1 s_f u^*$ unit). Consequently, a rise in profit share has no impact on equilibrium growth rate.

The above discussed short run comparative static results are encapsulated in Table 2.3. In the next section, we proceed for the long run dynamics.

[8]The latter effect is ambiguous as well, and depends on the relative values of c_r and α_1.

Table 2.3: Impact of changes in various parameters on u^*, g^* and r^*

	u^*	g^*	r^*
α_0	positive	positive	positive
i	ambiguous	negative	ambiguous
d	ambiguous	ambiguous	ambiguous
λ	positive	positive	positive
π	negative	no effect	no effect

3 *Long Run*

We analyse the long run dynamics of the debt-capital ratio and equity-to-debt ratio in this section. In the long run we assume that the goods market always clears, i.e. equilibrium values of u, g and r are always attained. We assume the long-run equilibrium is attained when the debt-capital ratio (d) and the equity-debt ratio (λ) remain constant over time. Let's first focus on the dynamics of debt-capital ratio.

3.1 *Dynamics of the debt-capital ratio*

We know for business flows of funds, sources of funds must be equal to uses of funds, which in turn implies

$$\text{retained earnings} + \text{new borrowings} + \text{issuance of new equities} = \text{investment demand}$$

$$\Rightarrow s_f(\pi u - id)K + \dot{D} + P_e\dot{E} = I \tag{3.1}$$

According to Ryoo (2010), debt finance is endogenously determined through the relationship between firms' profitability and leverage ratio (i.e. the trend rate of corporate profitability to interest payment ratio) and the equity finance (x) is pure residual of firms' financing constraint[9] as it serves as a buffer to fill the gap between the funds needed for the investment plans and the funds available from retained earnings and bank loans. However, it is very unlikely that funds needed for the investment plans of firms are lower than the available fund from retained earnings and borrowings for a sufficiently long period and the excess funds are adjusted by repurchasing stocks. Instead, there are various alternative and plausible explanations for stock buybacks. Lazonick (2010) points out

[9]Rentiers' flow of funds (scaled by the value of capital) can be represented as,

$$\frac{\text{Rentiers' savings}}{\text{capital stock}} = \frac{\text{purchase of new equities} + \text{new bank deposits}}{\text{capital stock}}$$

$$\Longrightarrow (1-c_r)\left[(1-s_f)(\pi u - id) + id\right] - (1+\lambda)c_q d = xg + d\hat{D}$$

"the manifestation of the financialization of the U.S. economy is the obsession of corporate executives with distributing "value" to share holders, especially in the form of stock repurchases, even if they accomplish this goal at the expense of investment in innovation and the creation of U.S. employment opportunities". One of the central motive behind stock buybacks is investor exploitation. As long as market prices of stocks are below their 'intrinsic' values, firms try to buyback the stocks (see D'Mello and Shroff, 2000). Another motive is to remove low valuation stockholders for mitigating the possibility of takeovers by other firms. Firms sometimes choose to buyback shares for increasing the reported EPS (earning per share) (Baker et al., 1985; Brav et al., 2005). Share buybacks can also be used to signal the management's confidence about the future (Brav et al., 2005). As a result, it is more logical to assume that firms first decide how much of its investment is financed by the issuance of equities (or how much firms decide to spend for share buybacks) and then firms fill the gap between the funds needed and the funds available for the investment plans by debt financing. So instead of equity finance, it should be the debt finance as the accommodating variable.[10]

For simplicity we assume a fraction of investment (x) is always financed by issuance of equities[11] i.e instead of treating x as variable we treat it as a parameter. This assumption is found in Lavoie and Godley (2002), Dos Santos and Zezza (2008), Taylor and Rada (2007), and Taylor (2012) etc. So from equation (3.1) we get,

$$s_f(\pi u - id) + xg + \dot{d} + dg = g$$

$$\Rightarrow \dot{d} = (1 - d - x)g - s_f(\pi u - id)$$

$$\Rightarrow \dot{d} = \frac{(1-d-x)[\alpha_0 S_F + \alpha_1 s_f\{(1+\lambda)c_q - (1-c_r)i\}d] - s_f[\alpha_0 + \{(1+\lambda)c_q - (1-c_r)i\}d]}{[S_F - s_f\alpha_1]} \tag{3.2}$$

$$\implies \dot{d} = \frac{m - ld - hd^2}{[S_F - s_f\alpha_1]} \tag{3.3}$$

where $m = [(1-x)S_F - s_f]\alpha_0$, $l = [\alpha_0 S_F + s_f(1 - \alpha_1 + \alpha_1 x)\{(1+\lambda)c_q - (1-c_r)i\}]$, and $h = \alpha_1 s_f\{(1+\lambda)c_q - (1-c_r)i\}$. We assume $[(1-x)S_F - s_f] > 0$ which in turn ensures $m > 0$. We also assume $(1 - \alpha_1 + \alpha_1 x) > 0$. Now we focus on drawing the $\dot{d} = 0$ isocline in $d - \lambda$ plane.

[10] This argument is more conventional in Kaleckian literature and is found in Lavoie and Godley(2002), Dos Santos and Zezza (2008).

[11] $\triangle x < 0$ represents lower contribution of new equity issues for the financing of new investment, or share buybacks (see Hein and van Treeck; 2010). Thus a fall in x represents the phenomenon called "share buybacks" in our model.

In equilibrium, $\dot{d} = 0$

$$\Rightarrow \quad \lambda\big|_{\dot{d}=0} = \left\{ \frac{[(1-x)S_F - s_f]\alpha_0 - \alpha_0 S_F d + s_f(1-\alpha_1+\alpha_1 x)\{(1-c_r)i - c_q\}d}{(1-\alpha_1+\alpha_1 x)c_q s_f d + s_f c_q \alpha_1 d^2} \right\} + \left\{ \frac{\alpha_1 s_f\{(1-c_r)i - c_q\}d^2}{(1-\alpha_1+\alpha_1 x)c_q s_f d + s_f c_q \alpha_1 d^2} \right\} \tag{3.4}$$

To find out the slope of the $\dot{d} = 0$ isocline, we differentiate equation (3.4) with respect to d and get,

$$\frac{d\lambda}{dd}\Big|_{\dot{d}=0} = \alpha_0 \left\{ \frac{S_F\alpha_1 d^2 - 2\alpha_1\{(1-x)S_F - s_f\}d - (1-\alpha_1+\alpha_1 x)\{(1-x)S_F - s_f\}}{s_f c_q \Big[(1-\alpha_1+\alpha_1 x)d + \alpha_1 d^2\Big]^2} \right\} \tag{3.5}$$

We know, $S_F\alpha_1 = A > 0$; $-2\alpha_1\{(1-x)S_F - s_f\} = B \lesseqgtr 0$; $-(1-\alpha_1+\alpha_1 x)\{(1-x)S_F - s_f\} = C \lesseqgtr 0$.

Solving the quadratic term of the numerator of equation (3.5) we find existence of two inflexion points, $d^{'}, d^{''} = \frac{-B\pm\sqrt{B^2-4AC}}{2A}$. The discriminant is

$$\Delta = B^2 - 4AC = 4\alpha_1\{(1-x)S_F - s_f\}[1 - c_r + s_f(c_r - \alpha_1)] \tag{3.6}$$

From equation (2.7) we get, $[1 - c_r + s_f(c_r - \alpha_1)] > 0$. So, for $\Delta > 0$, $[(1-x)S_F - s_f] > 0$ must hold.[12] Let's assume $d^{'} = \frac{-B+\sqrt{\Delta}}{2A}$. We already have assumed $[(1-x)S_F - s_f] > 0$. Therefore, $B < 0$; $C < 0$; and $A > 0$. So, $d^{'} = \frac{-B+\sqrt{\Delta}}{2A} > 0$. $d^{''} = \frac{-(B+\sqrt{\Delta})}{2A}$ will be positive if $B + \sqrt{\Delta} < 0$. This is turn implies $d^{''} > 0$ if $AC > 0$. But $C < 0$ and $A > 0$, and hence $AC < 0$. Therefore we have a logical contradiction which proves $d^{''}$ to be negative. Hence, $d^{''} < 0 < d^{'}$. $\frac{d\lambda}{dd}\big|_{\dot{d}=0}$ has vertical asymptote at $d = 0$. Equation (3.5), therefore, can be expressed as

$$\frac{d\lambda}{dd}\Big|_{\dot{d}=0} = \frac{\alpha_0 A(d-d^{'})(d-d^{''})}{s_f c_q \Big[(1-\alpha_1+\alpha_1 x)d + \alpha_1 d^2\Big]^2}$$

$$\text{Therefore, } \forall d > d^{'}, \frac{d\lambda}{dd}\Big|_{\dot{d}=0} > 0 \text{ and } \forall d \in (0, d^{'}), \frac{d\lambda}{dd}\Big|_{\dot{d}=0} < 0. \tag{3.7}$$

Diagram of the $\dot{d} = 0$ isocline is shown in Figure 3.3a. Now we focus on the rentiers' portfolio dynamics.

[12] Note that $[(1-x)S_F - s_f] > 0$ implies $x < \frac{(1-c_r)(1-s_f)}{(1-c_r+c_r s_f)}$. But $(1-c_r)(1-s_f) = (1-c_r+s_f c_r) - s_f < (1-c_r+s_f c_r)$. Hence $[(1-x)S_F - s_f] > 0$ implies $x < \frac{(1-c_r)(1-s_f)}{(1-c_r+c_r s_f)} < 1$.

Source: OECD data. Author's calculation

Figure 3.1: Long term interest rate in USA (1984-2008).

3.2 *Dynamics of the equity-debt ratio*

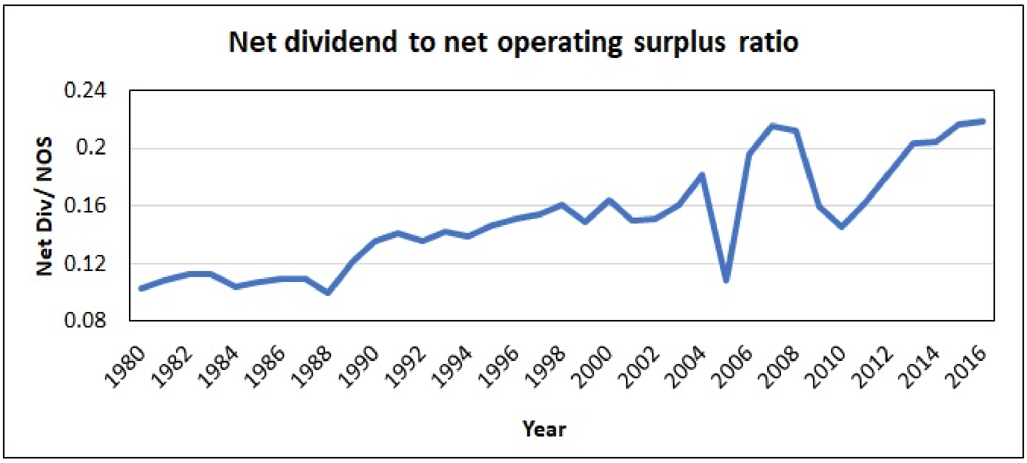
Let's assume

$$\dot{\lambda} = \theta[\lambda^d - \lambda]; \quad \theta > 0 \tag{3.8}$$

where, λ^d is desired equity to debt ratio of the rentiers. It depends on expectation regarding future capital gain.[13] . θ represents the speed of adjustment parameter. This speed of adjustment parameter can be affected by, *inter alia*, the regulation of the financial market. A strictly regulated financial market is associated with a smaller value of θ.

According to Abdullah and Hayworth (1993), "[T]he level of aggregate economic activity may influence stock prices through its impact on corporate profitability. An increase in output may increase cash flows and hence raise stock prices, while a recession would have the opposite effect." Fama (1990) shows that the stock returns are highly correlated with the future production growth rates for the period 1953-1987 for the US economy. In his analysis, production growth rates explain 43% of the stock return variance. Future production growth, as he argues, reflects information about future cash flows which in turn influence the stock prices. Schwert (1990) using 100 years of data (1889-1988) find a strong positive relationship between real stock returns and future production growth rates. In

[13] λ^d depends on relative rate of return on equity and debt. Return on equity consists of dividend and capital gain and rate of return on debt is the interest rate. For the USA for the last few decades the major difference between the returns comes from capital gain. Figure 3.1 shows the return on debt and Figure 3.2 shows the ratio of net dividend to net operating surplus. Note that starting from 1990, this ratio of net dividend to net operating surplus has not changed much till 2003. Similarly, starting from 1992, the long term interest rate did not change much.



Source: Bureau of Economic Analysis, December 21, 2018, Table 1.10; author's calculations.

Figure 3.2: Net dividend to net operating surplus in the USA (1980-2016).

his analysis, the future production growth rate is capable of explaining a large fraction of the variation in stock returns. Ratanapakorn and Sharma (2007) using quarterly data between 1975 and 1999, found that industrial production positively influences the stock prices. As they argue, a rise in industrial production increase the corporate earnings and hence enhances the present value of the firm. This leads to a rise in the investment in the stock market and therefore enhances the stock prices. Naik and Padhi (2012) find a bidirectional causality between industrial production and stock prices for India for the period 1994–2011. Therefore, we assume the expected capital gain depends on the growth rate.[14] In other words, the expectation of rentiers regarding future capital gain depends on economic growth. If the economy is performing well, they expect this to sustain for long, and they expect a sizeable future capital gain. So their desired equity to debt ratio (λ^d) increases.

Let us assume

$$\lambda^d = \varepsilon g; \qquad \varepsilon > 0 \tag{3.9}$$

ε measures the sensitivity of the desired equity to debt ratio to a change in growth rate. Equation (3.8) and (3.9) yields,

$$\dot{\lambda} = \theta[\varepsilon g - \lambda]$$

From equation (2.9) this implies that

$$\dot{\lambda} = \theta \left[\varepsilon \left\{ \frac{S_F \alpha_0 + \alpha_1 s_f c_q (1+\lambda) d - \alpha_1 s_f (1-c_r) i d}{1 - c_r + s_f (c_r - \alpha_1)} \right\} - \lambda \right] \tag{3.10}$$

[14] Note that capital gain per unit of equity (in value term) is equal to growth rate of equity prices as, $\frac{\dot{P}_e E}{P_e E} = \frac{\dot{P}_e}{P_e} = \hat{P}_e$

$$\Rightarrow \dot{\lambda} = \theta \left[\frac{S_F \alpha_0 \varepsilon + \alpha_1 s_f \varepsilon \{c_q - (1-c_r)i\} d - [1 - c_r + s_f(c_r - \alpha_1) - \alpha_1 s_f c_q \varepsilon d]\lambda}{1 - c_r + s_f(c_r - \alpha_1)} \right] \tag{3.11}$$

In equilibrium $\dot{\lambda} = 0$. Rearranging equation (3.11) we get

$$\Rightarrow \lambda\big|_{\dot{\lambda}=0} = \frac{\varepsilon S_F \alpha_0 + \varepsilon \alpha_1 s_f \{c_q - (1-c_r)i\} d}{1 - c_r + s_f(c_r - \alpha_1) - \alpha_1 s_f c_q \varepsilon d} \tag{3.12}$$

So putting $d = 0$ in equation (3.12) we get the vertical intercept of the $\dot{\lambda} = 0$ isocline as $\lambda\big|_{\dot{\lambda}=0}^{d=0} = \frac{\varepsilon S_F \alpha_0}{1-c_r+s_f(c_r-\alpha_1)} > 0$.

3.3 *Jacobian elements of the system*

Partial differentiation of equation (3.11) w.r.t. d and λ respectively provides us

$$J_{21} = \frac{\partial \dot{\lambda}}{\partial d} = \frac{\theta \varepsilon \alpha_1 s_f \{(1+\lambda)c_q - (1-c_r)i\}}{1 - c_r + s_f(c_r - \alpha_1)} \tag{3.13}$$

$$J_{22} = \frac{\partial \dot{\lambda}}{\partial \lambda} = -\frac{\theta[1 - c_r + s_f(c_r - \alpha_1) - \alpha_1 s_f c_q \varepsilon d]}{1 - c_r + s_f(c_r - \alpha_1)} \tag{3.14}$$

Rearranging equation (3.2) we get,

$$\begin{aligned} \dot{d} = \; & \left\{ \tfrac{[(1-x)S_F - s_f]\alpha_0 - \alpha_0 S_F d - s_f(1-\alpha_1+\alpha_1 x)\{c_q-(1-c_r)i\}d - s_f(1-\alpha_1+\alpha_1 x)c_q \lambda d}{[1-c_r+s_f(c_r-\alpha_1)]} \right\} \\ & - \left\{ \tfrac{\alpha_1 s_f \{c_q-(1-c_r)i\}d^2 - \alpha_1 s_f c_q \lambda d^2}{[1-c_r+s_f(c_r-\alpha_1)]} \right\} \end{aligned} \tag{3.15}$$

Partial differentiation of equation (3.15) w.r.t. d and λ respectively provides us

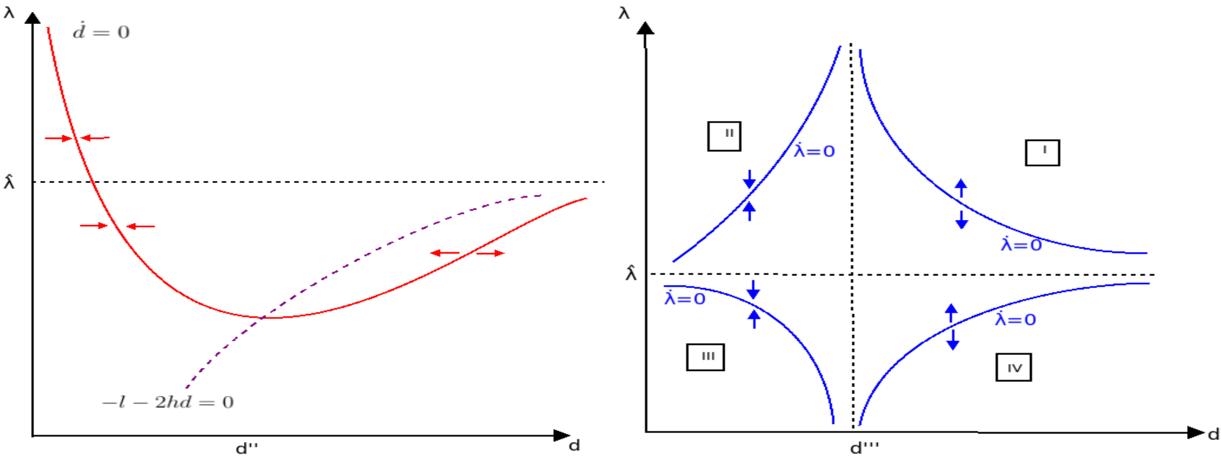
$$J_{11} = \frac{\partial \dot{d}}{\partial d} = \frac{-\alpha_0 S_F - s_f(1 - \alpha_1 + \alpha_1 x + 2\alpha_1 d)\{(1+\lambda)c_q - (1-c_r)i\}}{1 - c_r + s_f(c_r - \alpha_1)} = \frac{(-l - 2hd)}{1 - c_r + s_f(c_r - \alpha_1)} \tag{3.16}$$

$$J_{12} = \frac{\partial \dot{d}}{\partial \lambda} = -\frac{s_f c_q d(1 - \alpha_1 + \alpha_1 x + \alpha_1 d)}{1 - c_r + s_f(c_r - \alpha_1)} \tag{3.17}$$

where $M = -\alpha_0 S_F - s_f(1 - \alpha_1 + \alpha_1 x + 2\alpha_1 d)\{(1+\lambda)c_q - (1-c_r)i\} \gtreqless 0$; $N = s_f c_q d(1 - \alpha_1 + \alpha_1 x + \alpha_1 d) > 0$; $P = \varepsilon \alpha_1 s_f \{(1+\lambda)c_q - (1-c_r)i\} \gtreqless 0$; and $Q = [1 - c_r + s_f(c_r - \alpha_1) - \alpha_1 s_f c_q \varepsilon d] \gtreqless 0$.

Now the slope of the $\dot{d} = 0$ curve is

$$\begin{aligned} \frac{d\lambda}{dd}\bigg|_{\dot{d}=0} &= -\frac{J_{11}}{J_{12}} = \frac{-\alpha_0 S_F - s_f(1 - \alpha_1 + \alpha_1 x + 2\alpha_1 d)\{(1+\lambda)c_q - (1-c_r)i\}}{s_f c_q d(1 - \alpha_1 + \alpha_1 x + \alpha_1 d)} \\ &= \frac{-\alpha_0 S_F - s_f(1 - \alpha_1 + \alpha_1 x + 2\alpha_1 d)\{(1+\lambda)c_q - (1-c_r)i\}}{s_f c_q d(1 - \alpha_1 + \alpha_1 x + \alpha_1 d)} = \frac{M}{N} \end{aligned} \tag{3.18}$$



(a) diagram of the $\dot{d} = 0$ isocline (b) diagram of the $\dot{\lambda} = 0$ isocline

Figure 3.3

and the slope of the $\dot{\lambda} = 0$ curve is

$$\left.\frac{d\lambda}{dd}\right|_{\dot{\lambda}=0} = -\frac{J_{21}}{J_{22}} = \frac{\varepsilon\alpha_1 s_f\{(1+\lambda)c_q - (1-c_r)i\}}{[1 - c_r + s_f(c_r - \alpha_1) - \alpha_1 s_f c_q \varepsilon d]} = \frac{P}{Q} \tag{3.19}$$

Note that for $\{(1+\lambda)c_q - (1-c_r)i\} > 0$ i.e. when the economy is in a debt-led growth regime, J_{11} is unambiguously negative, and therefore, the $\dot{d} = 0$ isocline is negatively sloped (as J_{12} is always negative). However, for $\{(1+\lambda)c_q - (1-c_r)i\} < 0$ i.e. when the economy is in a debt-burdened growth regime, for smaller values of d (as long as $d < \frac{-l}{2h}$), $J_{11} < 0$ and therefore the $\dot{d} = 0$ isocline is negatively sloped. On the other hand, for a higher values of d ($\forall d > \frac{-l}{2h}$), $J_{11} > 0$ and therefore the slope of the $\dot{d} = 0$ isocline is positive. $\dot{\lambda} = 0$ isocline is vertically asymptotic at $d''' = \frac{1-c_r+s_f(c_r-\alpha_1)}{\alpha_1 s_f c_q \varepsilon} > 0$. Figure 3.3a represents the $\dot{d} = 0$ isocline. From the preceding discussion the following proposition can be inferred.

Proposition 1. *When the economy is in a debt-led growth regime, irrespective the the level of debt-capital ratio, J_{11} becomes negative. However, in case of a debt-burdened growth regime, $J_{11} \gtreqless 0$ according to whether $d \gtreqless -\frac{l}{2h}$.*

Now we explain equations (3.16), (3.17), (3.13), and (3.14) respectively. J_{11} shows the effect of an increase in the debt-capital ratio on a change in the debt-capital ratio itself. When the economy is in a debt-led growth regime, a rise in d has a negative effect on the change in the debt-capital ratio i.e. $J_{11} < 0$. The reason is as follows. For a given λ, a unit rise in d increases the investment rate by $\frac{\alpha_1 s_f\{(1+\lambda)c_q-(1-c_r)i\}}{\{1-c_r+s_f(c_r-\alpha_1)\}}$ unit (see equation (2.14)). For every unit rise in d, the retained earning rises by $\frac{s_f\{(1+\lambda)c_q-(1-c_r)i\}}{\{1-c_r+s_f(c_r-\alpha_1)\}}$ unit (as $\frac{\partial}{\partial d}(s_f(\pi u^* - id)) = \frac{s_f\{(1+\lambda)c_q-(1-c_r)i\}}{\{1-c_r+s_f(c_r-\alpha_1)\}}$). On the other hand, for a rise in d, finance through the issuance of new equities rises by $\frac{\alpha_1 x s_f\{(1+\lambda)c_q-(1-c_r)i\}}{\{1-c_r+s_f(c_r-\alpha_1)\}}$ unit. So, for a rise in d, the rise in finance through the retained earning and the issuance of new equities together are more than required for financing the new investment (as $\frac{(1+\alpha_1 x)s_f\{(1+\lambda)c_q-(1-c_r)i\}}{\{1-c_r+s_f(c_r-\alpha_1)\}} > \frac{\alpha_1 s_f\{(1+\lambda)c_q-(1-c_r)i\}}{\{1-c_r+s_f(c_r-\alpha_1)\}}$), and as a consequence, the debt level (normalized by the capital stock) decreases by $\frac{(1-\alpha_1+\alpha_1 x)s_f\{(1+\lambda)c_q-(1-c_r)i\}}{\{1-c_r+s_f(c_r-\alpha_1)\}}$ unit (as $\frac{\partial\left(\frac{\dot{D}}{K}\right)}{\partial d} = -\frac{(1-\alpha_1+\alpha_1 x)s_f\{(1+\lambda)c_q-(1-c_r)i\}}{\{1-c_r+s_f(c_r-\alpha_1)\}} < 0$). As we know $\dot{d} = \frac{\dot{D}}{K} - dg^*$, in the debt-led growth regime, a rise in d raises dg^*. Hence we get $J_{11} = \frac{\partial \dot{d}^*}{\partial d} < 0$ i.e. irrespective of the level of debt-capital ratio, the self-feedback effect of the debt-capital ratio is negative.

However, when the economy is in a debt-burdened growth regime, a rise in d has an ambiguous effect on the change in the debt-capital ratio and it depends on the level of debt-capital ratio. The reason is as follows. For a given λ, a rise in d now decreases the investment rate by $\frac{\alpha_1 s_f\{(1+\lambda)c_q-(1-c_r)i\}}{\{1-c_r+s_f(c_r-\alpha_1)\}}$ unit (from equation (2.14)). For every unit rise in d, finance through the issuance of new equities falls by $\frac{\alpha_1 x s_f\{(1+\lambda)c_q-(1-c_r)i\}}{\{1-c_r+s_f(c_r-\alpha_1)\}}$ unit. For a rise in d, the retained earnings decreases by $\frac{s_f\{(1+\lambda)c_q-(1-c_r)i\}}{\{1-c_r+s_f(c_r-\alpha_1)\}}$ unit. Consequently, for every unit rise in d, the finance through the retained earnings and the issuance of new equities together fall more than the fall in investment demand (as now $\frac{(1+\alpha_1 x)s_f\{(1+\lambda)c_q-(1-c_r)i\}}{\{1-c_r+s_f(c_r-\alpha_1)\}} < \frac{\alpha_1 s_f\{(1+\lambda)c_q-(1-c_r)i\}}{\{1-c_r+s_f(c_r-\alpha_1)\}} < 0$). Consequently, the debt level (normalized by the capital stock) increases by $\frac{(1-\alpha_1+\alpha_1 x)s_f\{(1+\lambda)c_q-(1-c_r)i\}}{\{1-c_r+s_f(c_r-\alpha_1)\}}$ unit (as $\frac{\partial\left(\frac{\dot{D}}{K}\right)}{\partial d} = -\frac{(1-\alpha_1+\alpha_1 x)s_f\{(1+\lambda)c_q-(1-c_r)i\}}{\{1-c_r+s_f(c_r-\alpha_1)\}} > 0$). On the other hand, in the debt-burdened growth regime, a rise in d reduces g^*. Therefore, the net effect of a rise in d on (dg^*) is ambiguous. Hence, the final result of a rise in d on $\dot{d}$ is ambiguous here and depends on the combination of d and λ.[15] If $d < -\frac{l}{2h}$, a rise in d negatively affects the change in the debt-capital ratio i.e. $J_{11} < 0$. On the other hand, a higher level of d ($d > -\frac{l}{2h}$) has a positive effect on the change in the debt-capital ratio and therefore $J_{11} > 0$. So much so, as $-\alpha_0 S_F - s_f(1-\alpha_1+\alpha_1 x + 2\alpha_1 d)\{(1+\lambda)c_q-(1-c_r)i\} = (-l-2hd)$, when the economy is in a debt-burdened growth regime, the magnitude of J_{11} also depends on the level of d in the sense that closer the value of d to $(-\frac{l}{2h})$, lower is the magnitude of J_{11}.

Remember that $-l-2hd=0$ implies $\lambda\big|_{-l-2hd=0} = \frac{(1-c_r)i-c_q}{c_q} - \frac{\alpha_0 S_F}{s_f c_q(1-\alpha_1+\alpha_1 x+2\alpha_1 d)} = \hat{\lambda} - \frac{\alpha_0 S_F}{s_f c_q(1-\alpha_1+\alpha_1 x+2\alpha_1 d)}$.[16] When $d=0$, $-l-2hd=0$ implies $\lambda\big|_{-l-2hd=0}^{d=0} = \hat{\lambda} - \frac{\alpha_0 S_F}{s_f c_q(1-\alpha_1+\alpha_1 x)} < \hat{\lambda}$. As d approaches to infinity, we get $\lim_{d\to\infty} \lambda\big|_{-l-2hd=0} = \hat{\lambda}$. Thus $\forall d \in [0,\infty)$, $-l-2hd=0$ implies $\lambda \leq \hat{\lambda}$. In other words, $-l-2hd=0$ is consistent only with the debt-burdened growth regime. For $-l-2hd=0$, we get $\frac{d\lambda}{dd}\big|_{-l-2hd=0} = \frac{2\alpha_0\alpha_1 S_F}{s_f c_q(1-\alpha_1+\alpha_1 x+2\alpha_1 d)^2} > 0$. Thus the slope is positive and depends on the value of d. Higher the value of d, lower is

[15] As l and h both are functions of λ.

[16] Note that $\{(1+\lambda)c_q-(1-c_r)i\}=0$ implies $\lambda = \frac{(1-c_r)i-c_q}{c_q}$. Let us assume $\frac{(1-c_r)i-c_q}{c_q} = \hat{\lambda}$. So, $\lambda > \hat{\lambda}$ implies the economy is in a debt-led growth regime while $\lambda < \hat{\lambda}$ is associated with a debt-burdened growth regime.

the value of $\frac{d\lambda}{dd}\Big|_{-l-2hd=0}$ i.e. as d increases, the slope decreases. The violet dotted curve in Figure 3.3a shows The diagram of $-l-2hd=0$.

J_{12} shows the effect of an increase in equity-debt ratio on the change in debt-capital ratio. For a given d, a rise in λ increases the investment rate by $\frac{\alpha_1 s_f c_q d}{\{1-c_r+s_f(c_r-\alpha_1)\}}$ unit (see equation (2.16)) and therefore finance through the issuance of new equities rises by $\frac{x\alpha_1 s_f c_q d}{\{1-c_r+s_f(c_r-\alpha_1)\}}$ unit. A rise in λ raises the retained earning rises by $\frac{s_f c_q d}{\{1-c_r+s_f(c_r-\alpha_1)\}}$ unit. So, for a rise in d, finance through the retained earning and the issuance of new equities together are more than required for financing the new investment (as $\frac{(1+\alpha_1 x)s_f c_q d}{\{1-c_r+s_f(c_r-\alpha_1)\}} > \frac{\alpha_1 s_f c_q d}{\{1-c_r+s_f(c_r-\alpha_1)\}}$). As a consequence, the debt level (normalized by the capital stock) decreases by $\frac{(1-\alpha_1+\alpha_1 x)s_f c_q d}{\{1-c_r+s_f(c_r-\alpha_1)\}}$ unit (as $\frac{\partial\left(\frac{\dot{D}}{K}\right)}{\partial d} = -\frac{(1-\alpha_1+\alpha_1 x)s_f c_q d}{\{1-c_r+s_f(c_r-\alpha_1)\}} < 0$).[17] Hence J_{12} is always negative. Note that the magnitude of J_{12}, however, depends positively on the level of debt-capital ratio.

J_{21} shows the effect of an increase in debt-capital ratio on the change in equity-debt ratio. When the economy is in a debt-led growth regime, a rise in d raises g^* which in turn raises the desired equity-debt ratio of the rentiers. Therefore, for a given λ, a change in d positively affects the change in the equity-debt ratio, i.e. J_{21} becomes positive. On the other hand, when the economy is in a debt-burdened growth regime, the opposite happens. Thus from equation (3.13) we get $\forall\lambda > \hat{\lambda}$, $\varepsilon\alpha_1 s_f\{(1+\lambda)c_q-(1-c_r)i\} > 0$ and $\forall\lambda < \hat{\lambda}$, $\varepsilon\alpha_1 s_f\{(1+\lambda)c_q-(1-c_r)i\} > 0 < 0$. Also note that higher the magnitude of $\{(1+\lambda)c_q-(1-c_r)i\}$, higher is the magnitude of J_{21}.

J_{22} shows the effect of an increase in equity-debt ratio on the change in equity-debt ratio. A unit rise in λ raises g^* which in turn raises the desired equity-debt ratio of the rentiers, and thereby, increases the change in the equity-debt ratio by $\theta\varepsilon\frac{\partial g^*}{\partial d} = \frac{\theta\varepsilon\alpha_1 s_f c_q d}{1-c_r+s_f(c_r-\alpha_1)}$ unit. On the other hand, holding λ^d constant, the equity-debt ratio leads to a fall in the change in the equity-debt ratio by θ unit. Thus, the net effect is ambiguous and depends on whether $\theta\varepsilon\frac{\partial g^*}{\partial d} > \theta$ or not. $\forall d < d''' = \frac{1-c_r+s_f(c_r-\alpha_1)}{\alpha_1 s_f c_q\varepsilon}$, $[1-c_r+s_f(c_r-\alpha_1)-\alpha_1 s_f c_q\varepsilon d] > 0$ and therefore, $J_{22} < 0$.[18] On the other hand, $\forall d > d'''$, $J_{22} > 0$. So much so, the magnitude of J_{22} also depends on the level of d in the sense that closer the value of d to d''', lower is the magnitude of J_{22}.

$d = d'''$ and $\lambda = \hat{\lambda}$ lines divide the diagram into four quadrants. As illustrated in Figure 3.3b, in the I quadrant, $J_{21} > 0$ and $J_{22} > 0$. Therefore, the slope of the $\dot{\lambda} = 0$ isocline is negative. In the II quadrant, as $J_{21} > 0$ and $J_{22} < 0$, $\dot{\lambda} = 0$ isocline is positively sloped. In the III quadrant, as both J_{21} and J_{22} are negative, and therefore, slope of the $\dot{\lambda} = 0$

[17] So, $\frac{\partial\dot{d}}{\partial\lambda} = -\frac{s_f c_q d(1-\alpha_1+\alpha_1 x+\alpha_1 d)}{1-c_r+s_f(c_r-\alpha_1)} < 0$.

[18] $[1-c_r+s_f(c_r-\alpha_1)-\alpha_1 s_f c_q\varepsilon d] > 0$ implies $\frac{\varepsilon\alpha_1 s_f c_q d}{1-c_r+s_f(c_r-\alpha_1)} < 1$ which in turn implies, $\theta\varepsilon\frac{\partial g^*}{\partial d} = \frac{\theta\varepsilon\alpha_1 s_f c_q d}{1-c_r+s_f(c_r-\alpha_1)} < \theta$. Consequently, $J_{22} < 0$.

isocline is negative. However, in the *IV* quadrant, $\dot{\lambda} = 0$ isocline is positively sloped (as here, $J_{21} < 0$ and $J_{22} > 0$).

4 *Possible Cases*

In this section, we explain different possible cases which may arise due to the interaction between the debt and the portfolio dynamics. Depending on whether the economy is in a debt-led or a debt-burdened growth regime, we can have several possible cases. We now analyze these cases step by step.

4.1 *Case 1*

As illustrated in Figure 4.1, in *case 1,* the $\dot{d} = 0$ isocline is passing through quadrants *II*, *III* and *IV*. However, slope of the $\dot{d} = 0$ isocline changes its sign in the *III* quadrant. In *case 1,* therefore, $d' < d'''$. In the *II* quadrant we get only one steady state named E_1. There may be two equilibria named E_2 and E_3 in the *III* quadrant while two equilibria named E_4 and E_5 are possible in the *IV* quadrant. So, maximum five different equilibria are possible in *case 1.*

Consider point E_1: Here the economy is in a debt-led growth regime (as $\lambda > \hat{\lambda}$). $\lambda > \hat{\lambda}$ and equation (3.13) implies $J_{21} > 0$, whereas $\lambda > \hat{\lambda}$ and equation (3.16) implies $J_{11} < 0$. As $d < d'''$, equation (3.14) yields that $J_{22} < 0$. J_{12} is always negative. Thus at E_1, the determinant of the Jacobian matrix is positive (as $\text{Det(J)} = (\overbrace{J_{11}}^{-}\overbrace{J_{22}}^{-} - \overbrace{J_{12}}^{-}\overbrace{J_{21}}^{+}) > 0$), and the trace of the Jacobian matrix is negative ($\text{tr(J)} = (\overbrace{J_{11}}^{-} + \overbrace{J_{22}}^{-}) < 0$). Hence, point E_1 is a stable steady state.

Let us explain the stability of the steady state E_1 intuitively. Because of some exogenous shock, let us assume that the debt-capital ratio deviates from its steady state value. Suppose that the debt-capital ratio is greater than its steady state value. First, in the debt-led growth regime, if d is greater than the steady state value d^*, it must fall due to $\frac{\partial \dot{d}}{\partial d} = J_{11} < 0$. This is the direct effect. Second, as the debt-capital ratio is greater than its steady state value, the equity-debt ratio increases due to $J_{21} > 0$. This leads to a fall in the debt-capital ratio due to $J_{12} < 0$. This is the indirect effect. In the debt-led growth regime, both the direct and indirect effects are stable. As a result, in this case, if the debt-capital ratio rises from the steady state value, it again comes back to its steady state. Hence, the steady state is stable.

Consider point E_2: At the steady state E_2, the economy is in a debt-burdened growth regime (i.e. $\lambda < \hat{\lambda}$ here). $\lambda < \hat{\lambda}$ and equation (3.13) implies $J_{21} < 0$. Slope of the $\dot{d} = 0$ isocline is negative here, i.e. $\frac{d\lambda}{dd}\Big|_{\dot{d}=0} = -\frac{J_{11}}{J_{12}} < 0$. As $J_{12} < 0$, J_{11} must be negative then.[19] At E_2, the debt-capital ratio is so small that $d < d'''$. Equation (3.14) and $d < d'''$ together implies $J_{22} < 0$. At point E_2, slope of the $\dot{d} = 0$ isocline is less than the slope of the $\dot{\lambda} = 0$ isocline i.e.

$$\frac{d\lambda}{dd}\Big|_{\dot{d}=0} = -\frac{J_{11}}{J_{12}} < \frac{d\lambda}{dd}\Big|_{\dot{\lambda}=0} = -\frac{J_{21}}{J_{22}} < 0$$

$$\Rightarrow (J_{11}J_{22} - J_{12}J_{21}) > 0$$

Therefore, the determinant is positive, and the trace is negative ($\mathrm{tr(J)} = J_{11} + J_{22} < 0$). So point E_2 is a stable steady state.

Debt-capital ratio, suppose due to some reason, deviates from the steady state and is now lower than its steady state value. There exist two opposite effects near E_2. First, as the debt-capital ratio is lower than its steady state value, it must rise due to $\frac{\partial \dot{d}}{\partial d} = J_{11} < 0$. This is the direct stable effect. Second, the fall in debt-capital ratio leads to a rise in the equity-debt ratio due to $J_{21} < 0$. As $J_{12} < 0$, this rise in equity-debt ratio leads to a fall in the debt-capital ratio. This second effect is an indirect unstable effect. However, as λ is very close to $\hat{\lambda}$ near E_2, magnitude of $\{(1+\lambda)c_q - (1-c_r)i\}$ is very small in size. Therefore the negative effect of J_{21} is very weak (see Figure 4.1 and equation (3.13)). As a result, the fall in debt-capital ratio leads to a small amount of rise in the equity-debt ratio which in turn through equation (3.17) leads to a negligible amount of fall in the debt-capital ratio (as d is low near E_2, the negative effect of J_{12} is very weak too). Therefore, the direct stable effect dominates the indirect unstable effect and results the steady state to be stable.

Consider point E_3: As at E_3 $\lambda < \hat{\lambda}$, the economy is in a debt-burdened growth regime and $J_{21} < 0$. As $d > \frac{-l}{2h}$, $J_{11} > 0$. At E_3, the debt-capital ratio is so small that $d < d'''$. Equation (3.14) and $d < d'''$ together implies $J_{22} < 0$. At point E_3, slope of the $\dot{d} = 0$ isocline is greater than the slope of the $\dot{\lambda} = 0$ isocline i.e.

$$\frac{d\lambda}{dd}\Big|_{\dot{d}=0} = -\frac{J_{11}}{J_{12}} > 0 > \frac{d\lambda}{dd}\Big|_{\dot{\lambda}=0} = -\frac{J_{21}}{J_{22}}$$

$$\Rightarrow (J_{11}J_{22} - J_{12}J_{21}) < 0$$

Hence, the determinant is negative. Consequently, E_3 emerges as a saddle point.

[19] The other way of finding why $J_{11} < 0$ is that as d is sufficiently low ($d < \frac{-l}{2h}$), J_{11} must be negative here.

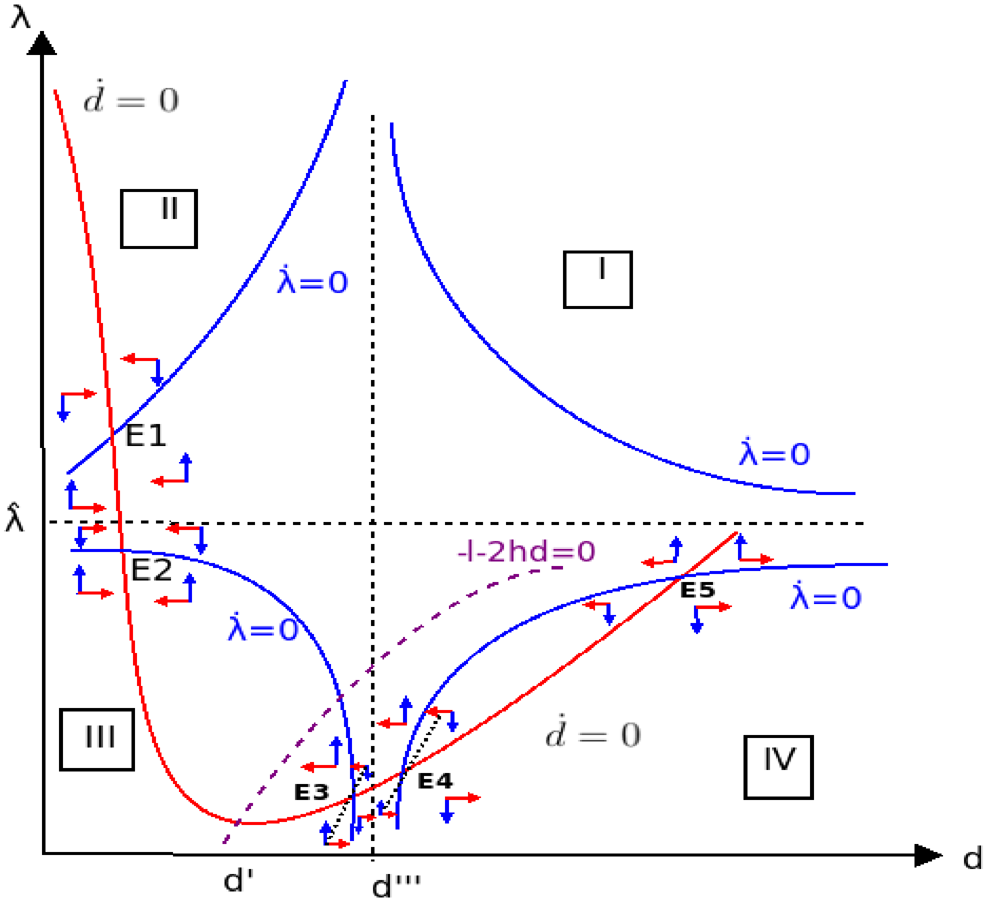


Figure 4.1: *case 1*

Let us discuss it intuitively. Equity-debt ratio, suppose due to some reason, deviates from the steady state, and is now lower than its steady state value. There exist two opposite effects near the steady state E_3. First, as the equity-debt ratio is lower than its steady state value, it must rise due to $J_{22} < 0$. This is the direct stable effect. However, as d is close to d''', the negative effect of J_{22} is very weak (see Figure 4.1 and equation (3.14)). As a result, the initial fall in equity-debt ratio leads to a small amount rise in the equity-debt ratio. Second, the fall in equity-debt ratio leads to a rise in the debt-capital ratio due to $J_{12} < 0$. As $J_{21} < 0$, this rise in debt-capital ratio leads to a fall in the equity-debt ratio. This second effect is an indirect unstable effect. However, as the gap between λ and $\hat{\lambda}$ is high near E_3, magnitude of $\{(1+\lambda)c_q - (1-c_r)i\}$ is large in size. Therefore the negative effect of J_{21} is strong (see Figure 4.1 and equation (3.13)). As a result, the rise in debt-capital ratio leads to a large amount of fall in the equity-debt ratio. Consequently, the indirect unstable effect dominates the direct stable effect and results the steady state to be unstable. There is only one stable arm that reaches to the equilibrium point E_3. Hence E_3 emerges as a saddle point.

Consider point E_4: At the steady state E_4 the economy is in a debt-burdened growth regime, and here $J_{21} < 0$. $d > \frac{-l}{2h}$ implies $J_{11} > 0$. $d > d'''$ implies (from equation (3.14)) $J_{22} > 0$. At point E_4, slope of the $\dot{\lambda} = 0$ isocline is greater than the slope of the $\dot{d} = 0$ isocline i.e.

$$\left.\frac{d\lambda}{dd}\right|_{\dot{\lambda}=0} = -\frac{J_{21}}{J_{22}} > \left.\frac{d\lambda}{dd}\right|_{\dot{d}=0} = -\frac{J_{11}}{J_{12}} > 0$$
$$\Rightarrow (J_{11}J_{22} - J_{12}J_{21}) < 0$$

As the determinant is negative, E_4 is a saddle point.

Suppose due to some reason, equity-debt ratio deviates from the steady state, and is now higher than its steady state value. First, as the equity-debt ratio is higher than its steady state value, it must rise due to $J_{22} > 0$. Second, the rise in equity-debt ratio leads to a fall in the debt-capital ratio due to $J_{12} < 0$. As $J_{21} < 0$, this fall in debt-capital ratio leads to a rise in the equity-debt ratio. So, both the effects are unstable. Consequently, the steady state is unstable. There is only one stable arm that reaches to the equilibrium point E_4. Hence E_4 emerges as a saddle point.

Consider point E_5: Here the economy is in a debt-burdened growth regime. Consequently, $J_{21} < 0$. On the other hand, $d > d'''$, and therefore, $J_{22} > 0$. As $d > \frac{-l}{2h}$, $J_{11} > 0$. At point E_5, slope of the $\dot{d} = 0$ isocline is greater than the slope of the $\dot{\lambda} = 0$ isocline i.e.

$$\left.\frac{d\lambda}{dd}\right|_{\dot{d}=0} = -\frac{J_{11}}{J_{12}} > \left.\frac{d\lambda}{dd}\right|_{\dot{\lambda}=0} = -\frac{J_{21}}{J_{22}} > 0$$

$$\Rightarrow (J_{11}J_{22} - J_{12}J_{21}) > 0$$

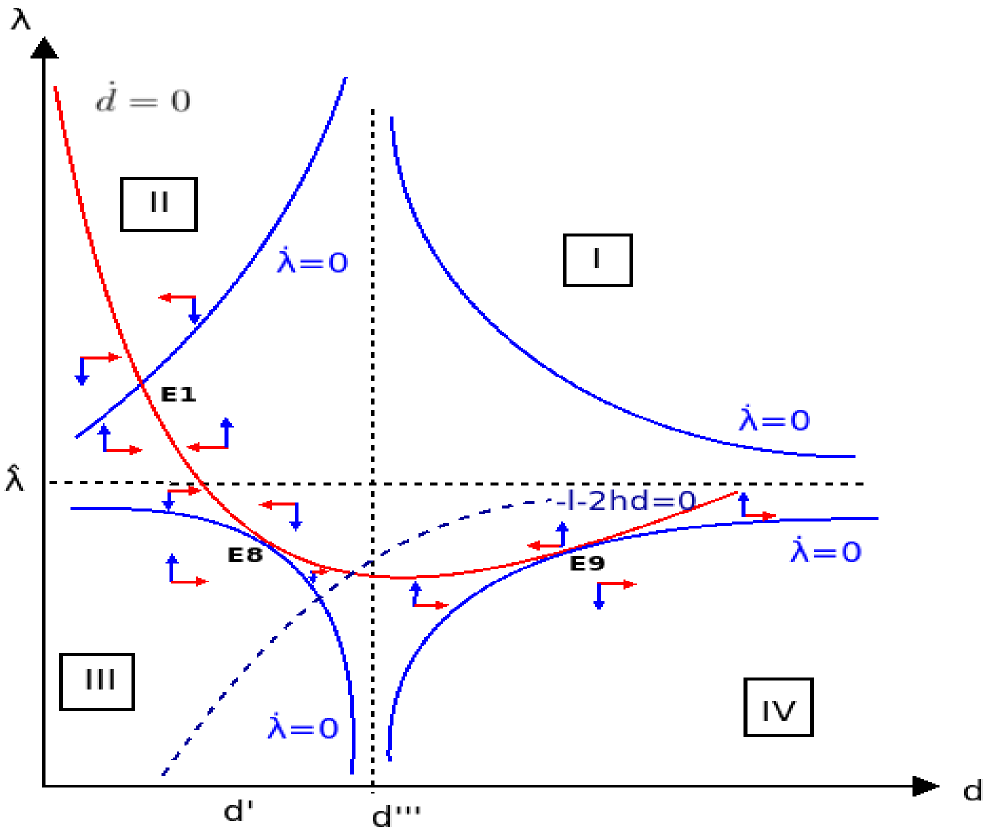


Figure 4.2: *case 1:* when $\dot{d} = 0$ and $\dot{\lambda} = 0$ isoclines are tangent at *III* & *IV* quadrants

The determinant and the trace ($\text{tr(J)} = J_{11} + J_{22} > 0$) both are positive here. So point E_5 is an unstable steady state.

Let us discuss it intuitively. Equity-debt ratio, suppose due to some reason, deviates from the steady state and is now lower than its steady state value. There exist two opposite effects near the steady state E_5. First, as the equity-debt ratio is lower than its steady state value, it must fall due to $J_{22} > 0$. This is the direct unstable effect. Second, the fall in equity-debt ratio leads to a rise in the debt-capital ratio due to $J_{12} < 0$. As $J_{21} < 0$, this rise in debt-capital ratio leads to a fall in the equity-debt ratio. This second effect is also an indirect unstable effect. Consequently, E_5 is an unstable steady state.

In *case 1,* instead of E_2 and E_3, a new equilibrium E_8 and instead of E_4 and E_5 a new equilibrium E_9 are also possible. As illustrated in Figure 4.2, E_8 and E_9 are both saddle point unstable steady states.

4.2 *Case 2*

Case 2 is represented in Figure 4.3. Here the $\dot{d} = 0$ isocline is passing through quadrants *II*, *III* and *IV*. However, slope of the $\dot{d} = 0$ isocline changes its sign at the $d = d'''$ line.

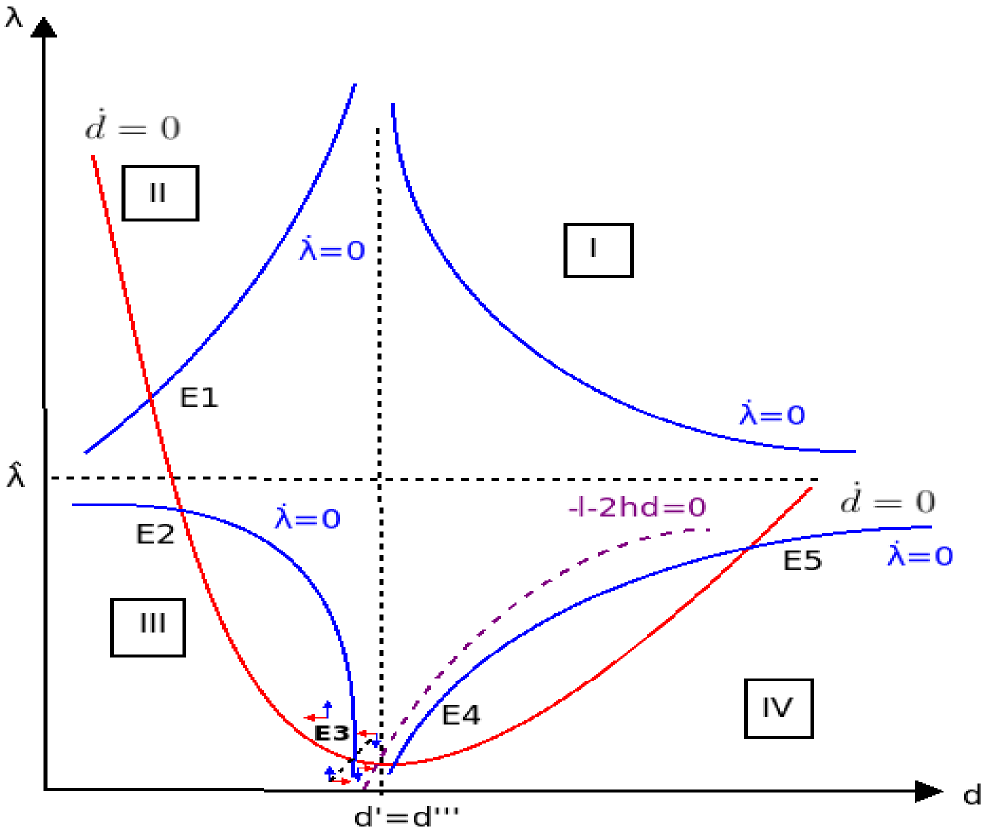


Figure 4.3: *case 2*

In the II quadrant we get only one steady state named E_1. Two equilibria named E_2 and E_3 are possible in the III quadrant while E_4 and E_5 these two equilibria are possible in the IV quadrant. So, in total, maximum five different equilibria are possible in *case* 2. Note that, in *case 2*, $d''' = d'$. For equilibria points E_1, E_2, E_4 and E_5 the analysis is the same as the corresponding equilibria points in *case 1*. Therefore, we focus only on equilibrium point E_3 here.

Consider point E_3: At the steady state E_3 the economy is in a debt-burdened growth regime, and $J_{21} < 0$. As $d < \frac{-l}{2h}$, $J_{11} < 0$. At E_3, as $d < d'''$, $J_{22} < 0$. At point E_3, slope of the $\dot{d} = 0$ isocline is greater than the slope of the $\dot{\lambda} = 0$ isocline i.e.

$$0 > \left.\frac{d\lambda}{dd}\right|_{\dot{d}=0} = -\frac{J_{11}}{J_{12}} > \left.\frac{d\lambda}{dd}\right|_{\dot{\lambda}=0} = -\frac{J_{21}}{J_{22}}$$

$$\Rightarrow (J_{11}J_{22} - J_{12}J_{21}) < 0$$

Hence, the determinant is negative. Consequently, E_3 emerges as a saddle point.

Similar to *case 1,* instead of E_2 and E_3, a new equilibrium E_8 and instead of E_4 and E_5 a new equilibrium E_9 are possible. E_8 and E_9 are both saddle point unstable steady states (see Figure 4.4).

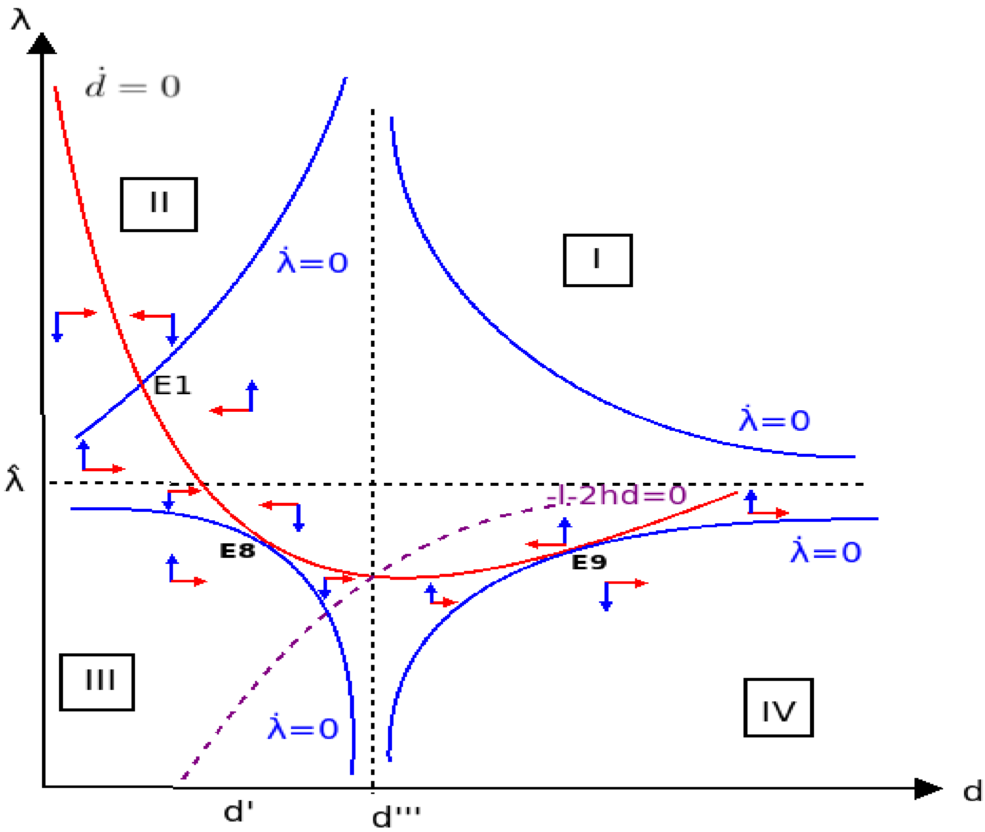


Figure 4.4: *case 2:* when $\dot{d} = 0$ and $\dot{\lambda} = 0$ isoclines are tangent at *III* & *IV* quadrants

4.3 *Case 3*

As illustrated in Figure 4.5, here the $\dot{d} = 0$ isocline is passing through quadrants II, III and IV. But, slope of the $\dot{d} = 0$ isocline changes its sign in the IV quadrant. Therefore, in *case 3,* $d' > d'''$. We get a unique steady state (E_1) in II quadrant. Two equilibria (E_2 and E_3) are possible in the III quadrant, whereas in the IV quadrant, we get two equilibria (E_4 and E_5). Therefore, at most, five different equilibria are possible in *case* 3. For equilibria E_1, E_2, and E_5, the analysis is the same as the corresponding equilibria points in *case 1* while for point E_3, the analysis is the same as in E_3 of *case 2.* So, we focus only on equilibrium point E_4 here.

Consider point E_4**:** At the steady state E_4 the economy is in a debt-burdened growth regime. $J_{21} < 0$ here. As $d < \frac{-l}{2h}$, $J_{11} < 0$. At E_4, as $d > d'''$, $J_{22} > 0$. Here, the slope of the $\dot{\lambda} = 0$ isocline is greater than the slope of the $\dot{d} = 0$ isocline i.e.

$$\left.\frac{d\lambda}{dd}\right|_{\dot{\lambda}=0} = -\frac{J_{21}}{J_{22}} > 0 > \left.\frac{d\lambda}{dd}\right|_{\dot{d}=0} = -\frac{J_{11}}{J_{12}}$$

$$\Rightarrow (J_{11}J_{22} - J_{12}J_{21}) < 0$$

As the determinant is negative, E_4 is a saddle point.

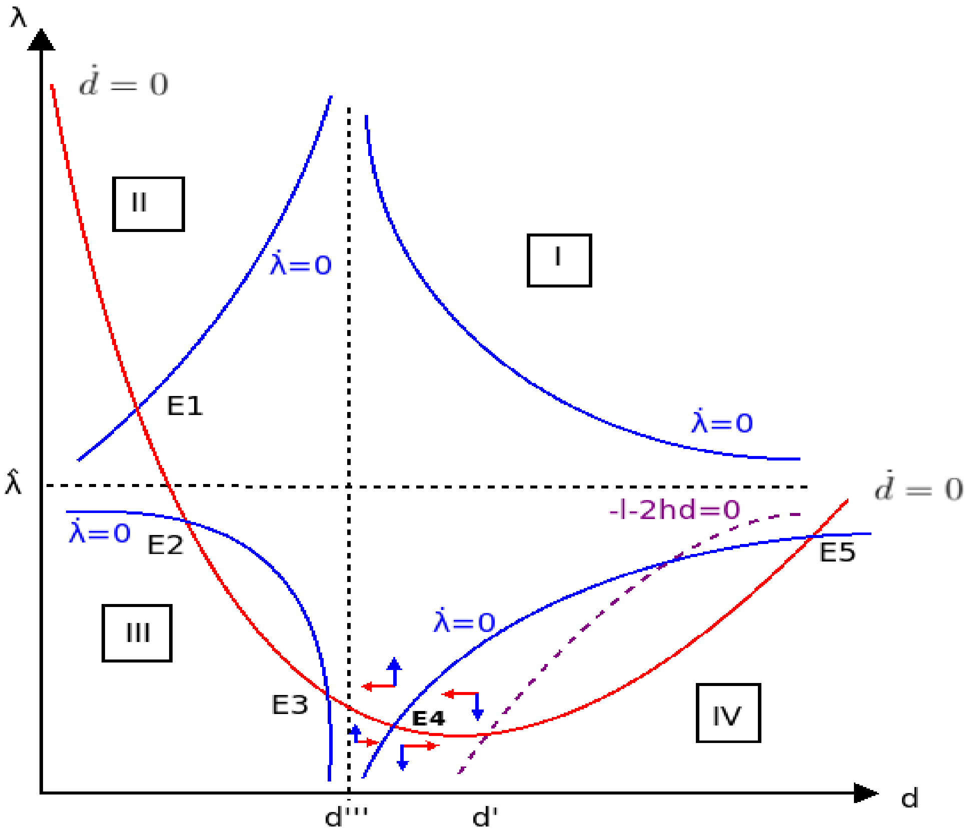


Figure 4.5: *case 3*

Similar to *case 1,* instead of E_2 and E_3, a new equilibrium E_8 and instead of E_4 and E_5 a new equilibrium E_9 are possible. E_8 and E_9 are both saddle point unstable steady states (see Figure 4.6).

4.4 *Case 4*

Case 4 is represented in Figure 4.7. Here the $\dot{d} = 0$ isocline is passing through quadrants II, I and IV, and slope of the $\dot{d} = 0$ isocline changes its sign on the IV quadrant. As a result, $d''' < d'$ here. In the II quadrant, we get a unique steady state (E_1). There are two equilibria (E_6 and E_7) in the I quadrant while E_4 and E_5- these two equilibria are in the IV quadrant. So, at most, five different equilibria are possible in *case 4*. For equilibria E_1 and E_5, the analysis is the same as the corresponding equilibria in *case 1,* while for point E_4, the analysis is the same as the corresponding point in *case 3.* Therefore, we now focus only on equilibria E_6 and E_7.

Consider point E_6: At the steady state E_6, the economy is in a debt-led growth regime i.e. here $\lambda > \hat{\lambda}$. Here, $J_{21} > 0$, and $J_{11} < 0$. As $d > d'''$, $J_{22} > 0$. At point E_6, slope of

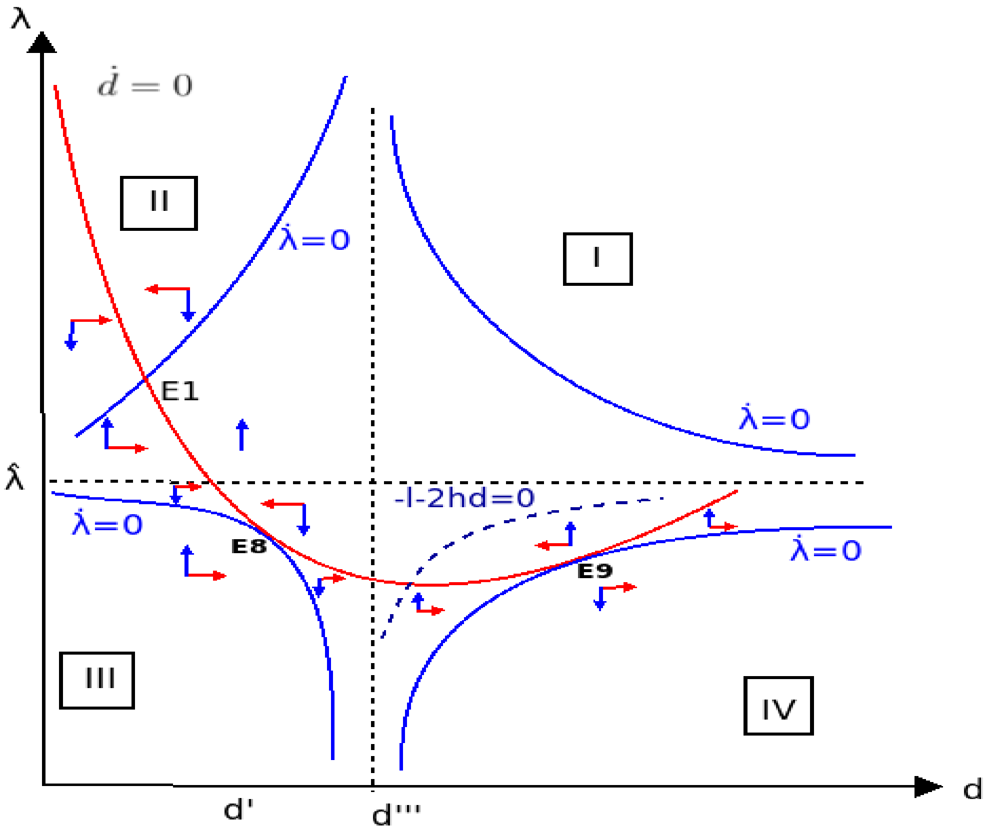


Figure 4.6: *case 3:* when $\dot{d} = 0$ and $\dot{\lambda} = 0$ isoclines are tangent at III & IV quadrants

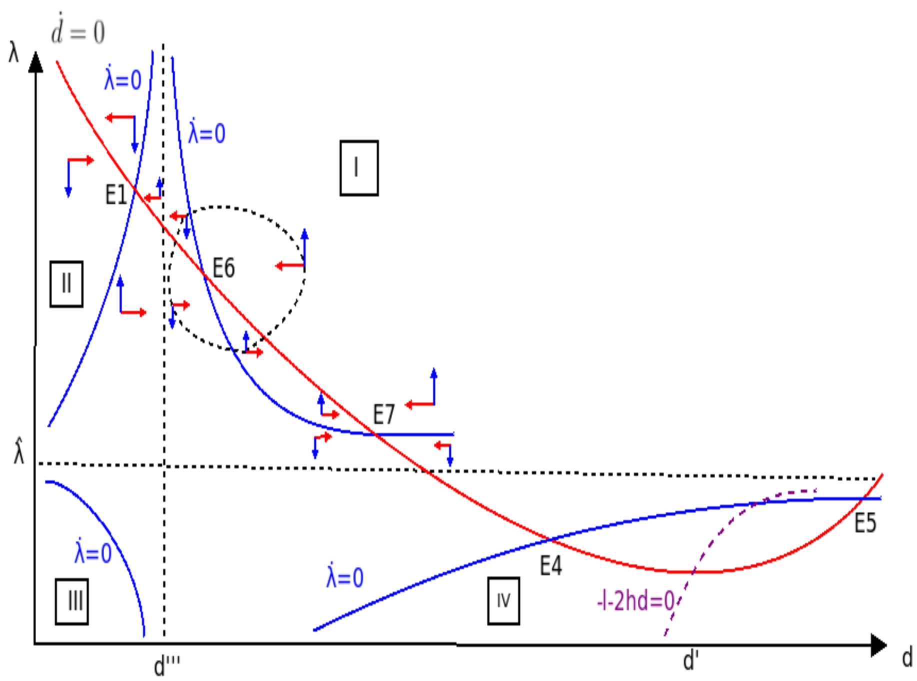


Figure 4.7: *case 4*

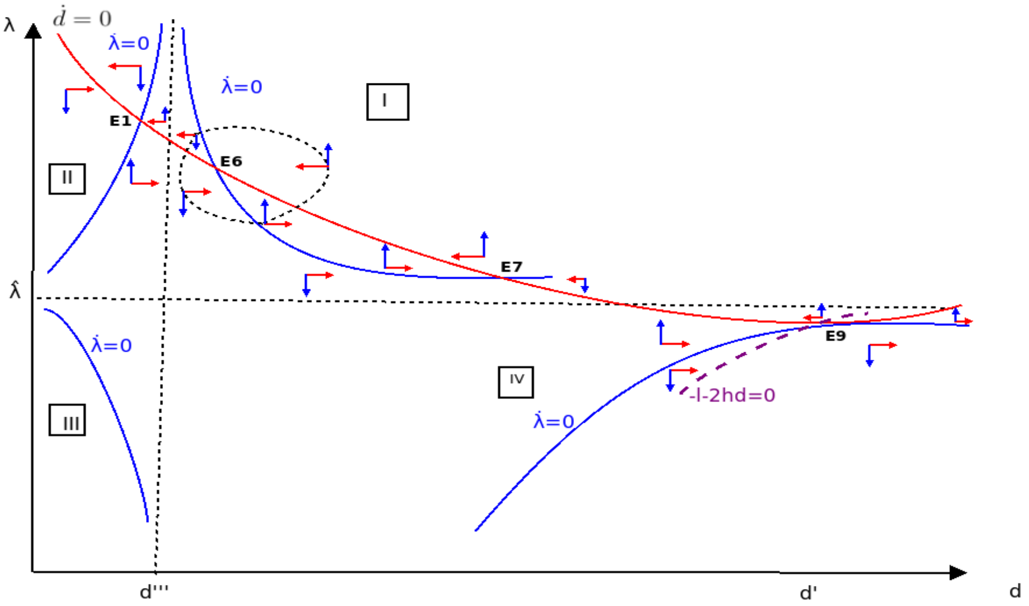


Figure 4.8: *case 4:* when $\dot{d} = 0$ and $\dot{\lambda} = 0$ isoclines are tangent at *IV* quadrant

the $\dot{d} = 0$ isocline is greater than the slope of the $\dot{\lambda} = 0$ isocline i.e.

$$0 > \left.\frac{d\lambda}{dd}\right|_{\dot{d}=0} = -\frac{J_{11}}{J_{12}} > \left.\frac{d\lambda}{dd}\right|_{\dot{\lambda}=0} = -\frac{J_{21}}{J_{22}}$$

$$\Rightarrow (J_{11}J_{22} - J_{12}J_{21}) > 0$$

The determinant is positive. However, sign of the the trace is ambiguous (as tr(J) $= J_{11} + J_{22} = \frac{(-l-2hd)}{1-c_r+s_f(c_r-\alpha_1)} + \frac{-\theta[1-c_r+s_f(c_r-\alpha_1)-\alpha_1 s_f c_q \varepsilon d]}{1-c_r+s_f(c_r-\alpha_1)} \gtreqless 0$). Therefore, E_6 is a stable steady state if $\theta < \hat{\theta} = \frac{-\alpha_0 S_F - s_f(1-\alpha_1+\alpha_1 x+2\alpha_1 d)\{(1+\lambda)c_q-(1-c_r)i\}}{[1-c_r+s_f(c_r-\alpha_1)-\alpha_1 s_f c_q \varepsilon d]}$, and unstable if $\theta > \hat{\theta}$. However, starting with a low value of θ if it increases to $\hat{\theta}$, the economy loses its stability and gives birth to a limit cycle. More discussion regarding Hopf-bifurcation is provided in Section 5.

Consider point E_7**:** At the steady state E_7 the economy is in a debt-led growth regime i.e. here $\lambda > \hat{\lambda}$. Here, $J_{21} > 0$, and $J_{11} < 0$. As $d > d'''$, $J_{22} > 0$. Here, the slope of the $\dot{\lambda} = 0$ isocline is greater than the slope of the $\dot{d} = 0$ isocline i.e.

$$0 > \left.\frac{d\lambda}{dd}\right|_{\dot{\lambda}=0} = -\frac{J_{21}}{J_{22}} > \left.\frac{d\lambda}{dd}\right|_{\dot{d}=0} = -\frac{J_{11}}{J_{12}}$$

$$\Rightarrow (J_{11}J_{22} - J_{12}J_{21}) < 0$$

As the determinant is negative, E_7 emerges as a saddle point.

Let us discuss it intuitively. Equity-debt ratio, suppose due to some reason, deviates from the steady state and is now higher than its steady state value. There exist two opposite effects near the steady state E_7. First, as the equity-debt ratio is higher than its steady state value, it must rise due to $J_{22} > 0$. This is the direct unstable effect. However, as d is much higher than d''', the positive effect of J_{22} is strong (see Figure 4.7 and equation (3.14)). As a result, the initial rise in equity-debt ratio causes a further and large amount of rise in the equity-debt ratio. Second, the rise in equity-debt ratio leads to a fall in the debt-capital ratio due to $J_{12} < 0$. As $J_{21} > 0$, this fall in debt-capital ratio leads to a fall in the equity-debt ratio. This second effect is an indirect stable effect. However, as the gap between λ and $\hat{\lambda}$ is low near E_7, the negative effect of J_{21} is weak (see Figure 4.7 and equation (3.13)). As a result, the fall in debt-capital ratio leads to a small amount of fall in the equity-debt ratio. Consequently, the direct unstable effect dominates the stable indirect effect and results in the steady state to be unstable. There is only one stable arm that reaches to the equilibrium point E_7. Hence E_7 emerges as a saddle point.

In *case 4*, instead of E_4 and E_5, a new saddle point unstable steady state E_9 is also possible. Figure 4.8 illustrates this.

Table 4.1 summarizes the results of the stability related to various steady states.

Table 4.1: *Summary of stability of the steady states*

Case	Quadrant	Steady state	Sign of the elements of J	Nature of the steady state
1	*II*	E_1	$J_{11} < 0,\ J_{12} < 0,\ J_{21} > 0,\ J_{22} < 0$	stable
	III	E_2	$J_{11} < 0,\ J_{12} < 0,\ J_{21} < 0,\ J_{22} < 0$	stable
		E_3	$J_{11} > 0,\ J_{12} < 0,\ J_{21} < 0,\ J_{22} < 0$	saddle point unstable
		E_8	$J_{11} < 0,\ J_{12} < 0,\ J_{21} < 0,\ J_{22} < 0$	saddle-node
	IV	E_4	$J_{11} > 0,\ J_{12} < 0,\ J_{21} < 0,\ J_{22} > 0$	saddle point unstable
		E_5	$J_{11} > 0,\ J_{12} < 0,\ J_{21} < 0,\ J_{22} > 0$	unstable
		E_9	$J_{11} > 0,\ J_{12} < 0,\ J_{21} < 0,\ J_{22} > 0$	saddle-node
2	*III*	E_3	$J_{11} < 0,\ J_{12} < 0,\ J_{21} < 0,\ J_{22} < 0$	saddle point unstable
	Results of other steady states are similar to *case 1*			
3	*IV*	E_4	$J_{11} < 0,\ J_{12} < 0,\ J_{21} < 0,\ J_{22} > 0$	saddle point unstable
	Results of other steady states are similar to *case 1*			
4	*II*	E_1	$J_{11} < 0,\ J_{12} < 0,\ J_{21} > 0,\ J_{22} < 0$	stable
	I	E_6	$J_{11} < 0,\ J_{12} < 0,\ J_{21} > 0,\ J_{22} > 0$	stable/unstable/limit cycle
		E_7	$J_{11} < 0,\ J_{12} < 0,\ J_{21} > 0,\ J_{22} > 0$	saddle point unstable
	IV	E_4	$J_{11} < 0,\ J_{12} < 0,\ J_{21} < 0,\ J_{22} > 0$	saddle point unstable
		E_5	$J_{11} > 0,\ J_{12} < 0,\ J_{21} < 0,\ J_{22} > 0$	unstable
		E_9	$J_{11} > 0,\ J_{12} < 0,\ J_{21} < 0,\ J_{22} > 0$	saddle-node

From the analysis of Section 4, two points are worth remembering.

Remark 1. *If the economy is in a debt-led growth regime, a lower level of debt-capital ratio (i.e. $d < d'''$) is a sufficient condition for achieving the unique stable steady state. However, it is not a necessary condition for achieving stability (for example see E_6 of case 4, Figure 4.7).*

Remark 2. *When the economy is in a debt-burdened growth regime, a lower level of* *debt-capital ratio (i.e.* $d < -\frac{l}{2h}$*) is a necessary (although not sufficient) condition for achieving the stable steady state (See* E_2 *of case 1, Figure 4.1).*[20]

5 *Hopf Bifurcation*

In this section, we discuss the possibilities of emergence of cycle as a solution to the dynamical systems represented by equation (3.15) and (3.11). Consider the steady state E_6 of *case 4*. We get the following proposition.

Proposition 2. *For an appropriate value of the speed of adjustment parameter* θ*, the characteristic equation to (3.15) & (3.11) evaluated at the steady state* E_6 *has purely imaginary roots and for the same dynamical system,* $\theta = \hat{\theta} = \frac{-\alpha_0 S_F - s_f(1-\alpha_1+\alpha_1 x+2\alpha_1 d)\{(1+\lambda)c_q-(1-c_r)i\}}{[1-c_r+s_f(c_r-\alpha_1)-\alpha_1 s_f c_q \varepsilon d]}$ *provides a point of Hopf bifurcation.*

Proof. See Appendix A.1. □

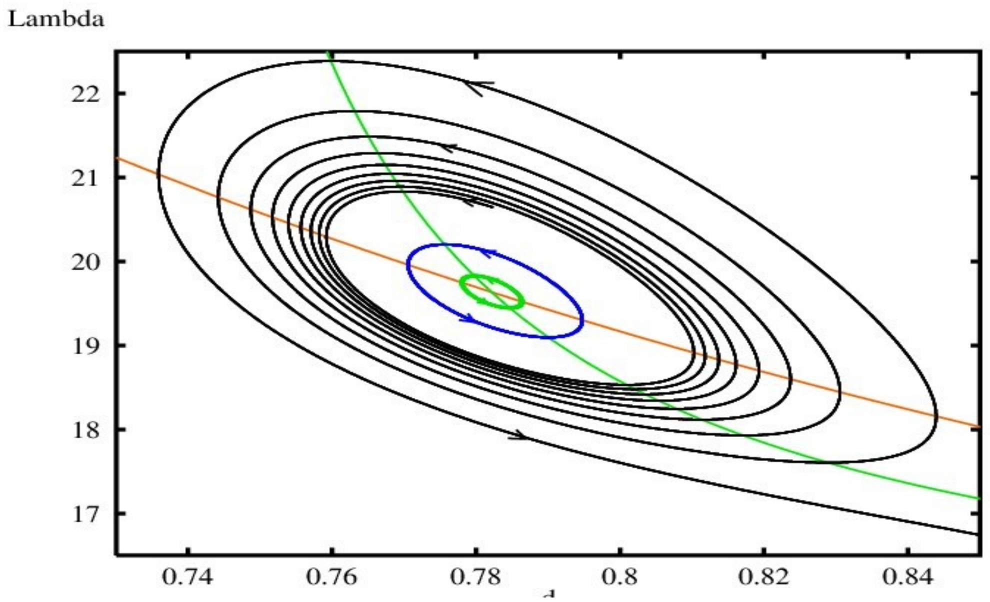
Note that for $\theta < \hat{\theta}$, the trace become negative and hence we have a stable equilibrium. However when $\theta > \hat{\theta}$, the equilibrium is unstable. When θ rises to $\hat{\theta}$, the system with a stable equilibrium point loses its stability and gives birth to a limit cycle.[21] Therefore, for ensuring stability in the economy, a strictly regulated financial market is desirable.

Using XPPAUT software we find that the Hopf bifurcation is sub-critical in nature i.e. an unstable limit cycle exists (shown by blue curve in Figure 5.1a). We draw the solution path from $t = 0$ to $t = 200$, and we find that the solution path is not a perfect closed orbit. For example, for an initial condition close to the long-run equilibrium, the solution path converges to the equilibrium (shown through green curve in Figure 5.1a), whereas for the initial condition further away from the long-run equilibrium, the solution path diverges from the equilibrium (as shown by orange curve in Figure 5.1a). Consequently, we conclude that in this numerical example, the sub-critical Hopf bifurcation occurs and the periodic solution is unstable. Instead of calibrating a real economy, the primary purpose of this numerical study is to confirm whether the model produces the limit cycle and to observe its basic properties. Therefore, we introduce the values so that we obtain economically meaningful outcomes. For the simulation we set $\alpha_0 = 0.1$, $\alpha_1 = 0.6$, $c_q = 0.03$, $s_f = 0.4$, $i = 0.8$, $c_r = 0.4$, $\varepsilon = 100$, $x = -0.6$, $\hat{\theta} = 3.008$. We get the equilibrium values $d^* = 0.78215$ and $\lambda^* = 19.635$ for the steady state E_6 of *case 4*.[22]

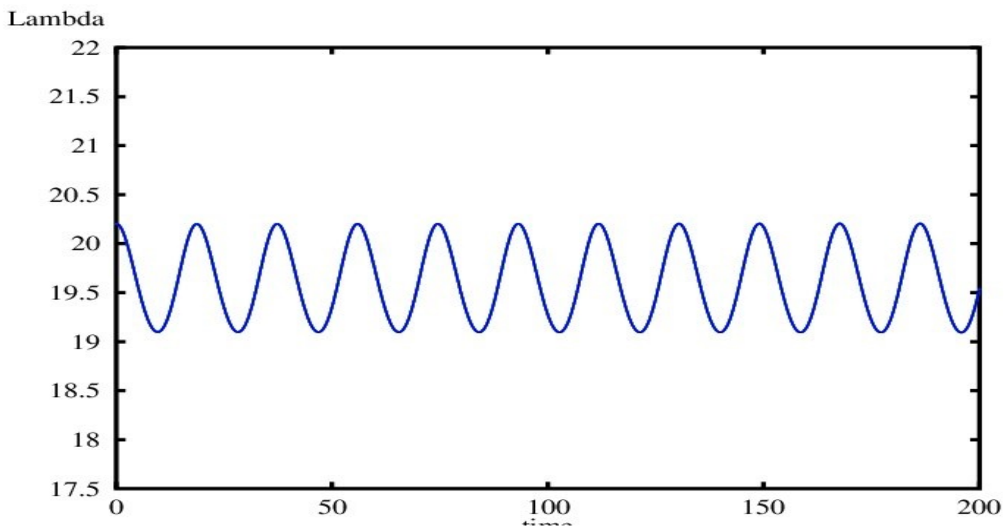
[20]To see that it is not a sufficient condition, see Figure 4.2 where at E_8, $d < -\frac{l}{2h}$, nonetheless the steady state is not a stable one.

[21]Note that the limit cycle can arise only when the economy is in a debt-led growth regime.

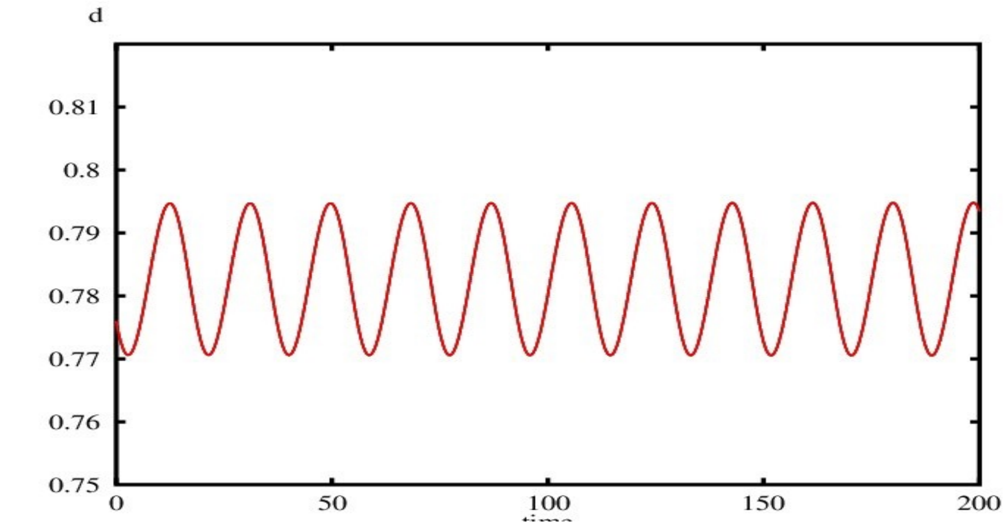
[22]Note that here equations (2.7), (2.8), (2.10), $[(1-x)S_F - s_f] > 0$, $d > d'''$, $\lambda > \hat{\lambda}$ - all are satisfied.



(a) *Solution paths in* (d, λ) *plane: an unstable limit cycle due to sub-critical Hopf bifurcation*



(b) *Transitional dynamics of equity-debt ratio*



(c) *Transitional dynamics of debt-capital ratio*

Figure 5.1: Limit cycle

Figure 5.1a shows cyclical patterns in the (d, λ)-plane. In the (d, λ)-plane anti-clockwise cycle emerges. Figure 5.1b shows the transitional dynamics of the equity-debt ratio and Figure 5.1c shows the transitional dynamics of the debt-capital ratio.

In what follows, we explain the reason behind the occurrence of a limit cycle under the debt-led growth regime. First, when the economy is in a debt-led growth regime, the self-feedback effect of the debt–capital ratio is negative, i.e. $J_{11} = \frac{\partial \dot{d}}{\partial d} < 0$. Besides, here the self-feedback effect of the equity-debt ratio is positive, i.e. $J_{22} = \frac{\partial \dot{\lambda}}{\partial \lambda} > 0$. When the speed of adjustment parameter θ is small, the self-feedback effect of the equity-debt ratio is dominated by the self-feedback effect of the debt–capital ratio and so the economy achieves stability (As the trace becomes negative here). On the contrary, when the opposite happens, the economy becomes unstable. Thus, limit cycle occurs in the boundary between the unstable and the stable feedback effect i.e. when θ reaches its critical value $\hat{\theta}$.

6 *Comparative Statics*

In this section we investigate how various parameters influence the equilibrium values of the debt-capital ratio and the equity-debt ratio. Table 6.1 summarizes the results of the comparative statics.

The total differentiation of equations (3.15) & (3.11) shows the effects of parametric changes in the economy which imply

$$\begin{bmatrix} J_{11} & J_{12} \\ J_{21} & J_{22} \end{bmatrix} \begin{bmatrix} dd \\ d\lambda \end{bmatrix} = \begin{bmatrix} 0 \\ -\theta g \end{bmatrix} d\varepsilon + \begin{bmatrix} g \\ 0 \end{bmatrix} dx + \begin{bmatrix} -\frac{s_f(1-c_r)d(1-\alpha_1+\alpha_1 x+\alpha_1 d)}{1-c_r+s_f(c_r-\alpha_1)} \\ \frac{\theta\varepsilon\alpha_1 s_f(1-c_r)d}{1-c_r+s_f(c_r-\alpha_1)} \end{bmatrix} di \quad (6.1)$$

Frome equation (6.1) we get, $\frac{dd^*}{d\varepsilon} = \frac{\theta g^* J_{12}}{(J_{11}J_{22}-J_{12}J_{21})}$, $\frac{d\lambda^*}{d\varepsilon} = \frac{-\theta g^* J_{11}}{(J_{11}J_{22}-J_{12}J_{21})}$; $\frac{dd^*}{dx} = \frac{g^* J_{22}}{(J_{11}J_{22}-J_{12}J_{21})}$, $\frac{d\lambda^*}{dx} = \frac{-g^* J_{21}}{(J_{11}J_{22}-J_{12}J_{21})}$; $\frac{dd^*}{di} = \frac{-\frac{s_f(1-c_r)d(1-\alpha_1+\alpha_1 x+\alpha_1 d)}{1-c_r+s_f(c_r-\alpha_1)}J_{22} - \frac{\theta\varepsilon\alpha_1 s_f(1-c_r)d}{1-c_r+s_f(c_r-\alpha_1)}J_{12}}{(J_{11}J_{22}-J_{12}J_{21})}$,

and $\frac{d\lambda^*}{di} = \frac{\frac{\theta\varepsilon\alpha_1 s_f(1-c_r)d}{1-c_r+s_f(c_r-\alpha_1)}J_{11} + \frac{s_f(1-c_r)d(1-\alpha_1+\alpha_1 x+\alpha_1 d)}{1-c_r+s_f(c_r-\alpha_1)}J_{21}}{(J_{11}J_{22}-J_{12}J_{21})}$. Note that for comparative static, the analysis of *case 2 & 3* are the same as in *case 1*. Therefore, we focus on two cases only: *case 1* and *case 4*.

6.1 *Effect of a change in ε*

Case 1: Let us consider *case 1* first. Here only two equilibria are stable: E_1 and E_2.

Table 6.1: *Summary of various comparative statics results*

(a) *Summary of comparative statics results for a change in* ε

Case	Steady states	Effect on d^*	Effect on λ^*	Effect on g^*
1	E_1	negative	positive	ambiguous
	E_2	negative	positive	positive
2	Same as in *case 1*			
3	Same as in *case 1*			
4	E_1	negative	positive	ambiguous
	E_6	negative	positive	ambiguous

(b) *Summary of comparative statics results for a change in* i

Case	Steady states	Effect on d^*	Effect on λ^*	Effect on g^*
1	E_1	positive	ambiguous	ambiguous
	E_2	positive	negative	negative
2	Same as in *case 1*			
3	Same as in *case 1*			
4	E_1	positive	ambiguous	ambiguous
	E_6	ambiguous	ambiguous	ambiguous

(c) *Summary of comparative statics results for a change in* x

Case	Steady states	Effect on d^*	Effect on λ^*	Effect on g^*
1	E_1	negative	negative	negative
	E_2	negative	positive	positive
2	Same as in *case 1*			
3	Same as in *case 1*			
4	E_1	negative	negative	negative
	E_6	positive	negative	ambiguous

Consider point E_1: As illustrated in Figure 6.1, for a rise in ε, the vertical intercept of the $\dot{\lambda} = 0$ isocline rises, and the slope of the $\dot{\lambda} = 0$ isocline becomes steeper. Besides, the vertical asymptote decreases.[23] However, there is no change in the $\dot{d} = 0$ isocline. Consequently, d^* decreases and λ^* increases.

The economic intuition behind the fall in d^* and the rise in λ^* is as follows. A rise in ε, *ceteris paribus*, raises the desired equity-debt ratio of rentiers and thereby pushes the $\dot{\lambda} = 0$ isocline upwards. For a given λ, at the old steady state E_1, the debt-capital ratio is higher than required for $\dot{\lambda} = 0$ to be satisfied. As the economy is in a debt-led growth regime, this higher level of d puts upward pressure on equity-debt ratio through equation (3.13). As a result, equity-debt ratio (λ) starts rising. As soon as λ rises, debt market deviates from its equilibrium position. Given the level of d, λ is now higher than required for $\dot{d} = 0$ to be satisfied. As $\frac{\partial \dot{d}}{\partial \lambda} = J_{12} < 0$, debt-capital ratio must fall. Combination of higher equity-debt ratio and lower debt-capital ratio ultimately ensure to achieve the new equilibrium point E_1' either monotonically or spiraling around E_1.

As demonstrated in equation (6.2), at E_1, ε has an ambiguous effect on the long run equilibrium rate of capital accumulation.

$$\frac{dg^*}{d\varepsilon} = \left\{ \overbrace{\frac{\partial g^*}{\partial d}}^{+} \overbrace{\frac{dd^*}{d\varepsilon}}^{-} + \overbrace{\frac{\partial g^*}{\partial \lambda}}^{+} \overbrace{\frac{d\lambda^*}{d\varepsilon}}^{+} \right\} \gtreqless 0 \tag{6.2}$$

At E_1, as ε increases, λ^* increases which in turn enhances the equilibrium rate of capital accumulation. On the other hand a rise in ε decreases d^*, and so g^* declines. Hence, the final result of a rise in ε on g^* is ambiguous.

Consider point E_2: For a rise in ε, vertical intercept of the $\dot{\lambda} = 0$ isocline rises, while the $\dot{\lambda} = 0$ isocline become steeper.[24] At E_2, a rise in ε leads to a rise in λ^* and a fall in d^*. At E_2, ε has an overall positive effect on the long run equilibrium rate of capital accumulation (see equation (6.3)).

$$\frac{dg^*}{d\varepsilon} = \left\{ \overbrace{\frac{\partial g^*}{\partial d}}^{-} \overbrace{\frac{dd^*}{d\varepsilon}}^{-} + \overbrace{\frac{\partial g^*}{\partial \lambda}}^{+} \overbrace{\frac{d\lambda^*}{d\varepsilon}}^{+} \right\} > 0 \tag{6.3}$$

[23] As $\frac{\partial}{\partial \varepsilon}\left(\lambda\Big|_{\dot{\lambda}=0}^{d=0}\right) = \frac{S_F \alpha_0}{1-c_r+s_f(c_r-\alpha_1)} > 0$, and $\frac{\partial}{\partial \varepsilon}\left(\frac{d\lambda}{dd}\Big|_{\dot{\lambda}=0}\right) = \frac{\alpha_1 s_f\{(1+\lambda)c_q-(1-c_r)i\}[1-c_r+s_f(c_r-\alpha_1)]}{[1-c_r+s_f(c_r-\alpha_1)-\alpha_1 s_f c_q \varepsilon d]^2} > 0$. Also note that, $\frac{\partial d'''}{\partial \varepsilon} = \frac{\partial}{\partial \varepsilon}\left(\frac{1-c_r+s_f(c_r-\alpha_1)}{\varepsilon \alpha_1 s_f c_q}\right) < 0$.

From here onward in this paper, the $\dot{\lambda} = 0$ isocline is represented through blue solid line and after the change (or shift), the new position of the $\dot{\lambda} = 0$ isocline is represented through the blue dotted line. Similarly, $\dot{d} = 0$ isocline is represented through red solid line and after the change (or shift), the new position of the $\dot{d} = 0$ isocline is represented through the red dotted line.

[24] As $\frac{\partial}{\partial \varepsilon}\left(\lambda\Big|_{\dot{\lambda}=0}^{d=0}\right) = \frac{S_F \alpha_0}{1-c_r+s_f(c_r-\alpha_1)} > 0$, and $\frac{\partial}{\partial \varepsilon}\left(\frac{d\lambda}{dd}\Big|_{\dot{\lambda}=0}\right) = \frac{\alpha_1 s_f\{(1+\lambda)c_q-(1-c_r)i\}[1-c_r+s_f(c_r-\alpha_1)]}{[1-c_r+s_f(c_r-\alpha_1)-\alpha_1 s_f c_q \varepsilon d]^2} < 0$.

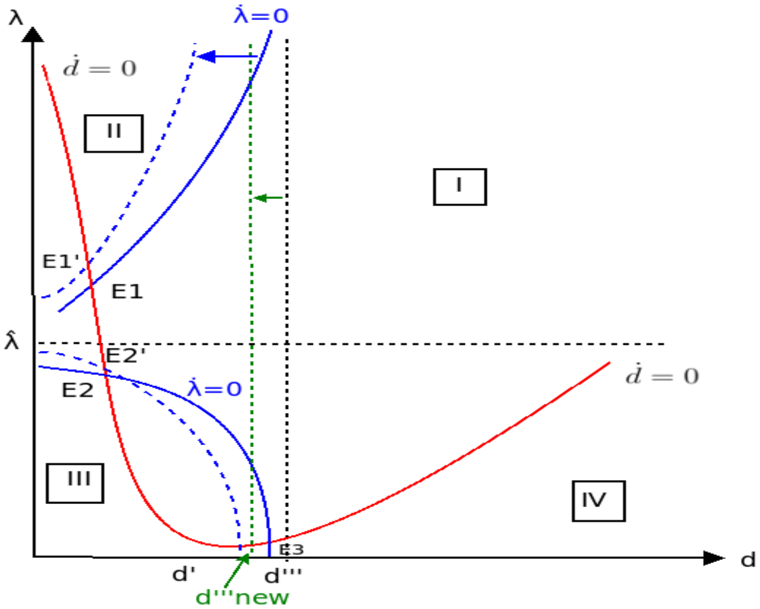


Figure 6.1: *Effect of a rise in* ε*: case 1*

At E_2, as ε increases, λ^* increases which in turn enhances the equilibrium rate of capital accumulation. In addition, a rise in ε decreases d^*. As the economy is in a debt-burdened growth regime, a fall in d^* in turn raises g^*. Hence, ε has an overall expansionary effect on the long run equilibrium rate of capital accumulation. Thus when the economy is in a relatively weak debt-burdened growth regime (as λ and $\hat{\lambda}$ are close to each other at E_2), and the debt-capital ratio is relatively low (as $d < \frac{-l}{2h}$ and $d < d'''$ at E_2), a rise in ε, the extent to which the rate of capital accumulation stimulates desired equity-deposit ratio of rentiers, raises the equilibrium rate of capital accumulation in the long run.

Case 4: Here point E_1 is a stable equilibrium. The analysis for E_1 is the same as in *case 1.* However for point E_6, its stability depends on the value of the parameter θ. Suppose $\theta < \hat{\theta}$ so that E_6 is a stable steady state. At E_6, as $\frac{\partial}{\partial \varepsilon}\left(\frac{d\lambda}{dd}\big|_{\dot{\lambda}=0}\right) > 0$, the slope of $\dot{\lambda} = 0$ curve becomes flatter (Figure 6.2). Consequently, for a rise in ε, d^* decreases and λ^* increases.

A rise in ε raises λ^* in E_4. This (λ^*) in turn enhances g^*. On the other hand, a rise in ε decreases d^*. As the economy is in a debt-led growth regime, a fall in d^* in turn decreases g^*. Hence, ε has an overall ambivalent effect on the long run equilibrium rate of capital accumulation (see equation (6.4)).

$$\frac{dg^*}{d\varepsilon} = \left\{ \overbrace{\frac{\partial g^*}{\partial d}}^{+} \overbrace{\frac{dd^*}{d\varepsilon}}^{-} + \overbrace{\frac{\partial g^*}{\partial \lambda}}^{+} \overbrace{\frac{d\lambda^*}{d\varepsilon}}^{+} \right\} \gtreqless 0 \tag{6.4}$$

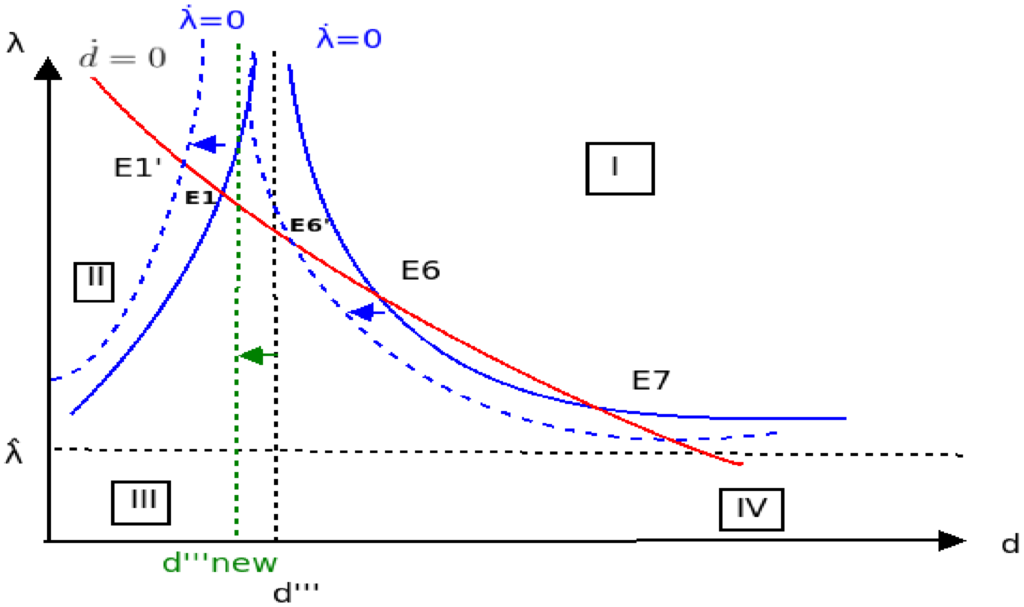


Figure 6.2: *Effect of a rise in ε: case 4*

6.2 *Effect of a change in the rate of interest, i*

Case 1: There are two stable steady states in *case 1*- E_1 and E_2.

Consider point E_1**:** As illustrated in Figure 6.3, for a rise in the interest rate, the $\dot{d}=0$ curve shifts upward whereas the $\dot{\lambda}=0$ isocline becomes flatter.[25] Therefore, we get $\frac{dd^*}{di}>0$ and $\frac{d\lambda^*}{di}\gtreqless 0$. Interest rate has only a direct negative effect on g^* in the short run. However, as demonstrated in equation (6.5), at E_1, i has an ambiguous effect on the long run equilibrium rate of capital accumulation.

$$\frac{dg^*}{di}=\left\{\overbrace{\frac{\partial g^*}{\partial d}}^{+}\overbrace{\frac{dd^*}{di}}^{+}+\overbrace{\frac{\partial g^*}{\partial \lambda}}^{+}\overbrace{\frac{d\lambda^*}{di}}^{+/-}+\overbrace{\frac{\partial g^*}{\partial i}}^{-}\right\}\gtreqless 0 \tag{6.5}$$

At E_1, as i increases, d increases which in turn enhances the equilibrium rate of capital accumulation (because here the economy is in a debt-led growth regime). On the other hand, interest rate has an ambiguous effect on λ^* while λ^* has a positive effect on g^*. Finally, interest rate has a direct negative effect on the equilibrium rate of capital accumulation. Hence, the final result of a rise in i on g^* is ambiguous.

[25]Partially differentiating $\lambda|_{\dot{d}=0}$ (from equation (3.4)) with respect to i we get, $\frac{\partial}{\partial i}\left(\lambda|_{\dot{d}=0}\right)=\frac{(1-c_r)}{c_q}>0$. Therefore, given the value of d, for a rise in the interest rate, a higher value of λ is required so that $\dot{d}=0$ is satisfied. Therefore the $\dot{d}=0$ curve shifts upward. Partially differentiating $\lambda|_{\dot{\lambda}=0}$ (from equation (3.12)) with respect to i we get, $\frac{\partial}{\partial i}\left(\lambda|_{\dot{\lambda}=0}\right)=\frac{-\varepsilon\alpha_1 s_f(1-c_r)d}{[1-c_r+s_f(c_r-\alpha_1)-\alpha_1 s_f c_q \varepsilon d]}<0$. Hence, for a given value of d, to satisfy $\dot{\lambda}=0$, a lower value of λ is required. Therefore the $\dot{\lambda}=0$ isocline becomes flatter here.

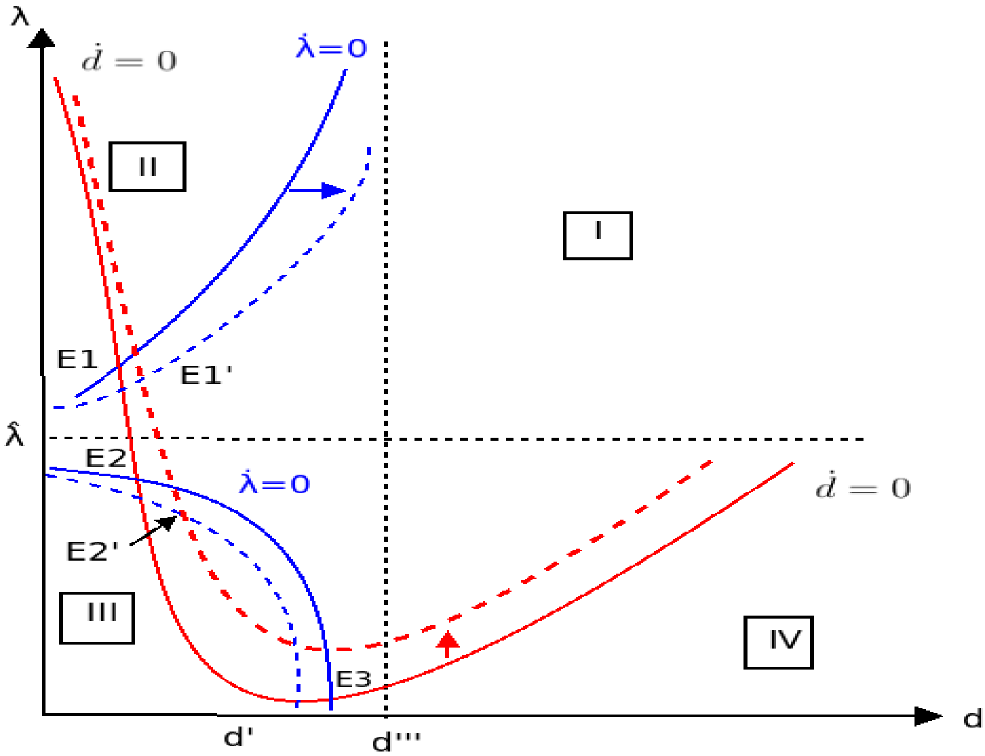


Figure 6.3: *Effect of a rise in i: case 1*

Consider point E_2: Because of a rise in the interest rate, the $\dot{d}=0$ curve shifts upward. However, the $\dot{\lambda}=0$ isocline becomes steeper.[26] As a result, as shown in Figure 6.3, a rise in the interest rate leads to a rise in d^* and a fall in λ^*. At E_2, i has a vividly negative effect on the long run equilibrium rate of capital accumulation. At E_2, as i increases, d increases which in turn reduces the equilibrium rate of capital accumulation (because the economy is in a debt-burdened growth regime). On the other hand, interest rate has a negative effect on λ^* while λ^* has a positive effect on g^*. So, a rise in i, through its negative effect on λ, decreases g^* indirectly. Finally, interest rate has a direct negative effect on the equilibrium rate of capital accumulation. Hence, the final result of a rise in i on g^* is unequivocally negative (see (6.6)).

$$\frac{dg^*}{di} = \left\{ \overbrace{\frac{\partial g^*}{\partial d}}^{-} \overbrace{\frac{dd^*}{di}}^{+} + \overbrace{\frac{\partial g^*}{\partial \lambda}}^{+} \overbrace{\frac{d\lambda^*}{di}}^{-} + \overbrace{\frac{\partial g^*}{\partial i}}^{-} \right\} < 0 \tag{6.6}$$

Case 4: For a rise in the interest rate, while the $\dot{d}=0$ isocline shifts upward, the $\dot{\lambda}=0$ curve becomes flatter here (see Figure 6.4).[27] The interest rate here has an ambiguous

[26] As $\frac{\partial}{\partial i}\left(\lambda|_{\dot{\lambda}=0}\right) = \frac{-\varepsilon\alpha_1 s_f(1-c_r)d}{[1-c_r+s_f(c_r-\alpha_1)-\alpha_1 s_f c_q \varepsilon d]} < 0$, for a given value of d, to satisfy $\dot{\lambda}=0$, a lower value of λ is required. Therefore the $\dot{\lambda}=0$ isocline becomes steeper here.

[27] As $\frac{\partial}{\partial i}\left(\lambda|_{\dot{\lambda}=0}\right) = \frac{-\varepsilon\alpha_1 s_f(1-c_r)d}{[1-c_r+s_f(c_r-\alpha_1)-\alpha_1 s_f c_q \varepsilon d]} > 0$, for a given value of d, to satisfy $\dot{\lambda}=0$, a higher value

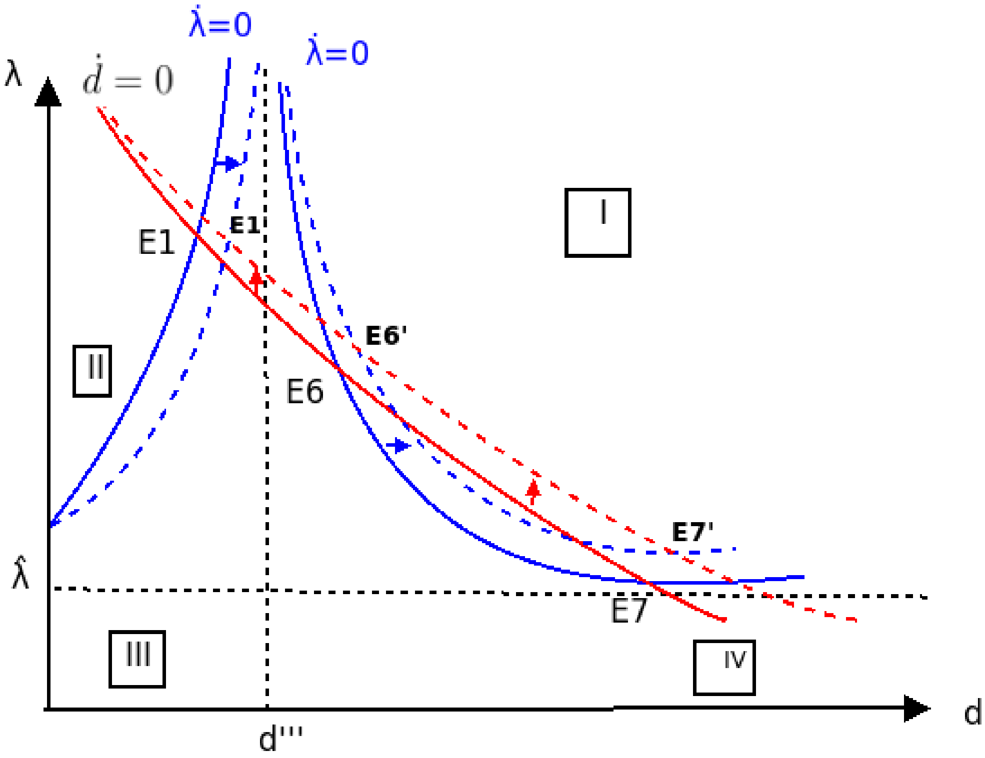


Figure 6.4: *Effect of a rise in i: case 4*

effect on both d^* and λ^*. Interest rate has an ambiguous effect on g^* too which is shown mathematically by equation (6.7).

$$\frac{dg^*}{di} = \left\{ \overbrace{\frac{\partial g^*}{\partial d}}^{+} \overbrace{\frac{dd^*}{di}}^{+/-} + \overbrace{\frac{\partial g^*}{\partial \lambda}}^{+} \overbrace{\frac{d\lambda^*}{di}}^{+/-} + \overbrace{\frac{\partial g^*}{\partial i}}^{-} \right\} \gtreqless 0 \tag{6.7}$$

6.3 *Effect of a change in the fraction of investment which is financed by issuance of new equities, x*

Case 1: Two equilibria, E_1 and E_2, are stable in *case 1.*

Consider point E_1:

As depicted in Figure 6.5, for a fall in x, i.e. when "share buybacks" happens, $\dot{d} = 0$ isocline shifts upward. However, x has no impact on the $\dot{\lambda} = 0$ isocline.[28] Therefore, as x increases, d^* and λ^* both decrease whereas a fall in x leads to a rise in both λ^* and d^*.

of λ is required. Therefore the $\dot{\lambda} = 0$ isocline becomes flatter here.

[28]Differentiating equation (3.4) partially w.r.t. x we get $\frac{\partial}{\partial x}\left(\lambda|_{\dot{d}=0}\right) = \frac{-[1-c_r+s_f(c_r-\alpha_1)]\alpha_0 s_f c_q d}{[(1-\alpha_1+\alpha_1 x)c_q s_f d + s_f c_q \alpha_1 d^2]^2} < 0$. So as x rises, *ceteris paribus*, λ decreases. This is possible if the $\dot{d} = 0$ curve shifts downward. On the other hand, if x decreases, $\dot{d} = 0$ isocline shifts upward.

The economic intuition is as follows. If x decreases i.e. when the phenomenon called "share buybacks" happens, for the same level of investment, *ceteris paribus,* now firms get less funds through the issuance of new equities and as a result the $\dot{d} = 0$ isocline shifts upwards. For a given λ, at the old steady state E_1, the debt-capital ratio is lower than required for new $\dot{d} = 0$ to be satisfied. As $\frac{\partial \dot{d}}{\partial d} = J_{11} < 0$, debt-capital ratio starts rising. As soon as d rises, $\dot{\lambda} = 0$ is no more satisfied. Given the level of λ, d is now higher than required for $\dot{\lambda} = 0$ to be satisfied. As $\frac{\partial \dot{\lambda}}{\partial d} = J_{21} > 0$, equity-debt ratio also starts rising. Combination of higher equity-debt ratio and higher debt-capital ratio ultimately ensure to achieve the new equilibrium point E_1' either monotonically or spiraling around E_1.

As demonstrated in equation (6.8), at E_1, x has an unequivocally negative effect on the long run equilibrium rate of capital accumulation.

$$\frac{dg^*}{dx} = \left\{ \overbrace{\frac{\partial g^*}{\partial d}}^{+} \overbrace{\frac{dd^*}{dx}}^{-} + \overbrace{\frac{\partial g^*}{\partial \lambda}}^{+} \overbrace{\frac{d\lambda^*}{dx}}^{-} \right\} < 0 \tag{6.8}$$

At E_1, as x increases, λ^* decreases and as a result g^* falls. On the other hand, a rise in x decreases d^*. As the economy is in a debt-led growth regime, this fall in d^* reduces g^*. Hence, the final result of a rise in x on g^* is vividly negative. Thus when the economy is in a debt-led growth regime and the debt-capital ratio is significantly low (as $d < \frac{-l}{2h}$ and $d < d'''$ at E_1), repurchases of equities by firms (i.e. a fall in x) stimulates the equilibrium rate of capital accumulation in the long run.

Consider point E_2: Here $\frac{dd^*}{dx} < 0$ and $\frac{d\lambda^*}{dx} > 0$. Thus as x increases, d^* decreases and λ^* increases, and when x decreases, λ^* decreases while d^* increases (see Figure 6.5). The reason is as follows. If x increases, for the same level of investment, *ceteris paribus,* now firms get more funds through the issuance of new equities, and as a result d falls (as now $\dot{d} < 0$). As here the economy is in a debt-burdened growth regime, for a fall in d, g^* rises which in turn raises the desired level of equity-debt ratio. Hence the equilibrium equity-debt ratio rises too. The opposite happens for a fall in x (i.e. when "share buybacks" happens).

A rise in x, has an unambiguously positive effect on the long run equilibrium rate of capital accumulation at E_2. As x increases, λ^* rises which in turn boosts g^*. On the other hand, a rise in x reduces d^*. As the economy is in a debt-burdened growth regime, this fall in d^* in turn enhances g^*. Hence, the final result of a rise in x on g^* is vividly positive.

$$\frac{dg^*}{dx} = \left\{ \overbrace{\frac{\partial g^*}{\partial d}}^{-} \overbrace{\frac{dd^*}{dx}}^{-} + \overbrace{\frac{\partial g^*}{\partial \lambda}}^{+} \overbrace{\frac{d\lambda^*}{dx}}^{+} \right\} > 0 \tag{6.9}$$

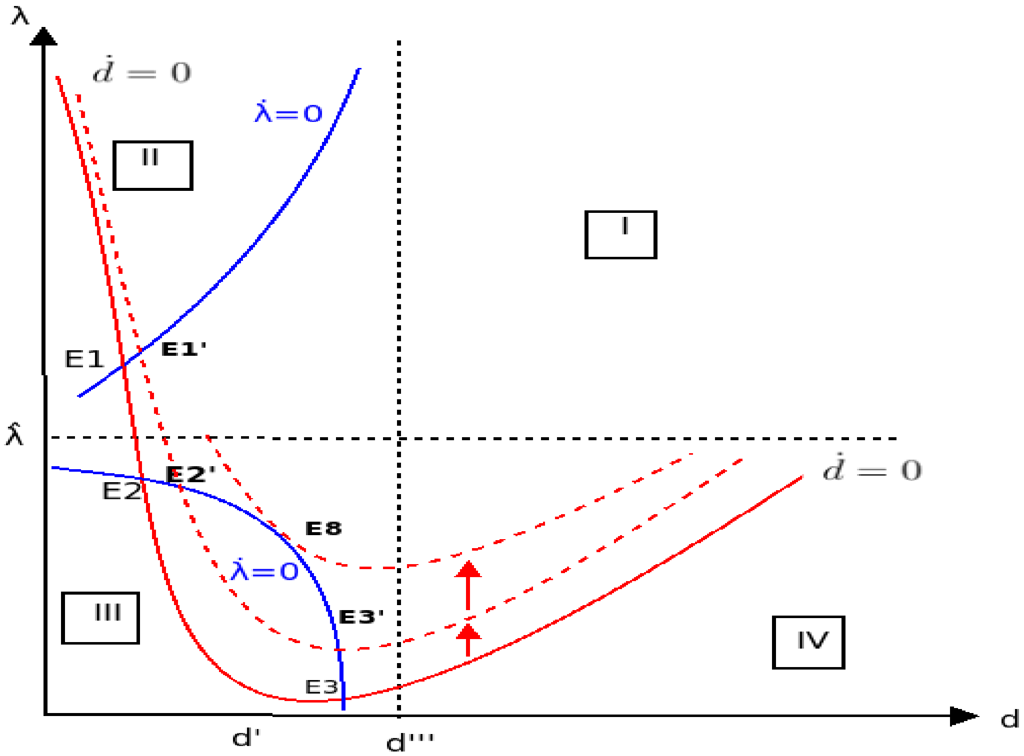


Figure 6.5: *Effect of a fall in x: case 1*

Note that for a decrease in x, the two equilibria E_2 and E_3 come closer and for a sufficient fall in x, both the equilibria converge to a unique saddle-point unstable steady state E_8. Thus, in the era of financialization, as more of a share buy-back is happening (and less proportion of investment is financed through issuance of new equities), when the economy is in a debt-burdened growth regime and the debt-capital ratio is low (i.e. when $d < \frac{-l}{2h}$ and $d < d'''$), not only the equilibrium rate of capital accumulation declines, the stable equilibrium E_2 may lose its stability as well in the sense that at all, no stable steady state exists in the economy.

Case 4: The analysis for E_1 is the same as in *case 1*. Now consider the steady state E_6. Here $\frac{dd^*}{dx} > 0$ and $\frac{d\lambda^*}{dx} < 0$. Thus as x increases, d^* increases and λ^* decreases, whereas for a fall in x, the opposite happens (see Figure 6.6).

A rise in x decreases λ^* in E_4 and so g^* falls. On the other hand, a rise in x increases d^*. As the economy is in a debt-led growth regime, a rise in d^* in turn raises g^*. Hence, x has an overall ambivalent effect on the long run equilibrium rate of capital accumulation (see equation (6.10)).

$$\frac{dg^*}{dx} = \left\{ \overbrace{\frac{\partial g^*}{\partial d}}^{+} \overbrace{\frac{dd^*}{dx}}^{+} + \overbrace{\frac{\partial g^*}{\partial \lambda}}^{+} \overbrace{\frac{d\lambda^*}{dx}}^{-} \right\} \gtreqless 0 \tag{6.10}$$

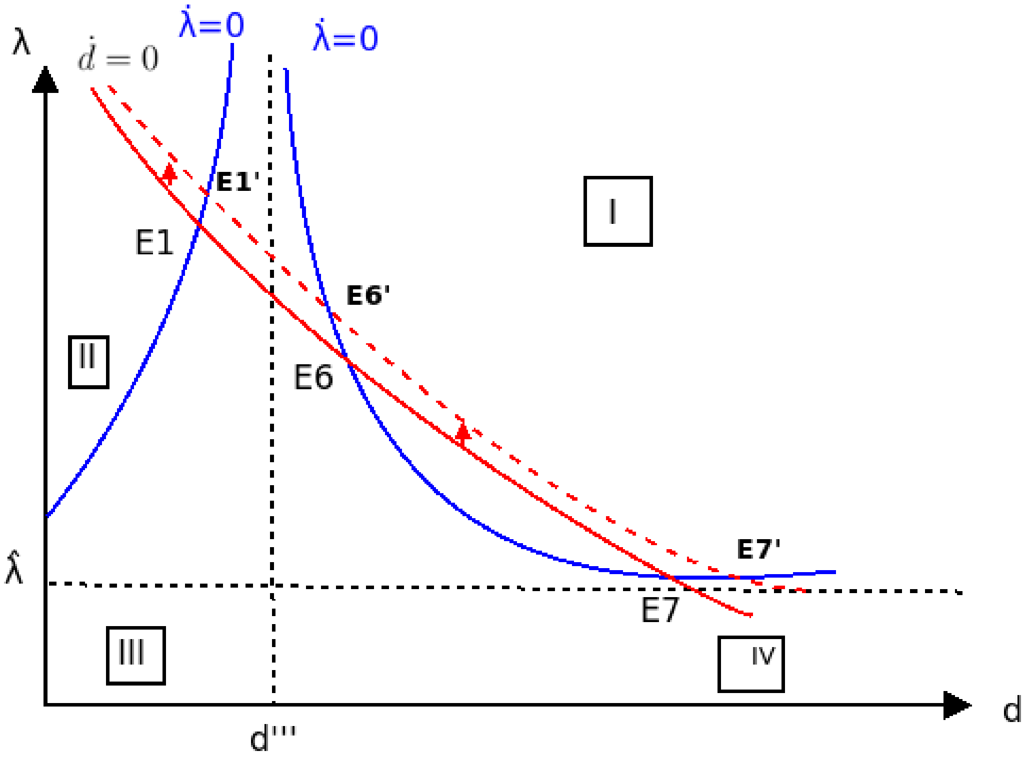


Figure 6.6: *Effect of a fall in x: case 4*

7 *Conclusion*

In this paper, we dealt with a neo-Kaleckian growth model in which the debt-capital ratio and the equity-debt ratio are endogenous variables. First, we examined the short-run stability and comparative statics. We concluded that although the economy is in a wage-led demand regime, debt-led or debt-burdened demand and growth regimes are possible. In the long-run, the equity-debt ratio and the debt–capital ratio are endogenous. The study of the dynamics shows that several cases and multiple equilibria are possible. We find a few interesting results.

1. If the economy is in a debt-led growth regime, a lower level of debt-capital ratio (i.e. $d < d'''$) is sufficient for achieving a unique stable steady state. On the other hand, when the economy is in a debt-burdened growth regime, a lower level of debt-capital ratio (i.e. $d < -\frac{l}{2h}$ and $d < d'''$) is a necessary (although not sufficient) condition for achieving a stable steady state.

2. Irrespective of whether the economy is in a debt-led or a debt-burdened growth regime, a rise in ε changes the rentiers' asset-portfolio in favour of equities (i.e. the equity-debt ratio λ rises) and reduces the debt level of firms (i.e. the debt-capital ratio d falls). Besides, if the economy is in a debt-burdened growth regime with a

low level of debt-capital ratio (i.e. $d < -\frac{l}{2h}$ and $d < d'''$), a rise in ε can enhance the equilibrium rate of capital accumulation too.

3. When the economy is in a strong debt-led growth regime (so that λ is much higher than $\hat{\lambda}$) and debt-capital ratio is significantly high ($d > d'''$), in other words when the economy is at the steady state E_6 of *case 4*, whenever the speed of adjustment parameter θ rises to $\hat{\theta}$ the economy loses its stability and produces an unstable limit cycle. This suggests that more regulated financial markets are desirable for ensuring stability in the economy.

4. In a weak debt-burdened growth regime (where $\lambda < \hat{\lambda}$ and λ is much closer to $\hat{\lambda}$) where the debt-capital ratio is sufficiently low (i.e. $d < -\frac{l}{2h}$ and $d < d'''$), in other words when the economy is at the stable steady stateE_2, a rise in the interest rate or a share buyback (i.e. a fall in x) not only reduces the equilibrium growth rate, but has a potential destabilizing effect on the economy as well. In the era of financialization as more shares are repurchased, as x falls significantly, the stable equilibrium E_2 may lose its stability altogether and may produce a saddle point unstable steady state E_8.

5. A rise in the interest rate always causes a fall in the equilibrium rate of capital accumulation in the short run. Nevertheless, in the long run, opposite of that may happen.

Few limitations of this paper can be found. First, in our model, banks have played a passive role. Active participation of banks may make the model more realistic. Second, our model is based on a closed economy where there is no role of the government. Third, the interest rate is exogenous in our model. Introduction of an endogenous interest rate (something like Taylor (1983) type) and an active role of government may make it more interesting. We have assumed away the technological change in our model. Focusing on how technological change occurs through time, especially in the era of financialization in the context of the US economy would be an interesting exercise. These issues are, however, left for future research.

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A Appendix

A.1 Proof of Proposition 2

Proof. The characteristic equation to (3.15) & (3.11) is

$$\mu^2 + (-\mathrm{tr}(\mathrm{J}))\mu + \mathrm{Det}(\mathrm{J}) = 0.$$

A necessary condition of the Hopf bifurcation for complex roots is $\mathrm{Det}(\mathrm{J}) > 0$, which is satisfied at E_6 of *case 4*. The trace of the Jacobian matrix can be made either positive or negative by appropriately selecting the value of θ while leaving the other parameters constant. To see this, notice that tr(J) $= J_{11}+J_{22} = \frac{(-l-2hd)}{1-c_r+s_f(c_r-\alpha_1)} + \frac{-\theta[1-c_r+s_f(c_r-\alpha_1)-\alpha_1 s_f c_q \varepsilon d]}{1-c_r+s_f(c_r-\alpha_1)}$. Hence when $\theta = \hat{\theta} = \frac{-\alpha_0 S_F - s_f(1-\alpha_1+\alpha_1 x+2\alpha_1 d)\{(1+\lambda)c_q-(1-c_r)i\}}{[1-c_r+s_f(c_r-\alpha_1)-\alpha_1 s_f c_q \varepsilon d]} > 0$ ($\because J_{22} > 0, J_{11} < 0$), the following equation holds exactly:

$$\mathrm{tr(J)} = 2 * \mathrm{Re}\mu = \left[\frac{(-l-2hd) - \theta[1-c_r+s_f(c_r-\alpha_1)-\alpha_1 s_f c_q \varepsilon d]}{1-c_r+s_f(c_r-\alpha_1)}\right] = 0$$

where tr(J) is the trace of J and Reμ is the real part of its characteristic roots. As the determinant of the Jacobian matrix is positive, the product of the roots is positive in a neighborhood of the equilibrium, assuring Im$\mu \neq 0$. Now differentiating the trace of the Jacobian matrix with respect to θ and then evaluating it at $\theta = \hat{\theta}$ we get

$$\frac{\partial(\frac{tr(J)}{2})}{\partial\theta}\bigg|_{\theta=\hat{\theta}} = \frac{-[1-c_r+s_f(c_r-\alpha_1)-\alpha_1 s_f c_q \varepsilon d]}{1-c_r+s_f(c_r-\alpha_1)} > 0$$

So the trace is smooth, differentiable and monotonically increasing in the speed of adjustment parameter, θ. The trace disappears at $\theta = \hat{\theta}$. Also note that $tr(J) \gtreqless 0 \iff \theta \gtreqless \hat{\theta}$. From the preceding discussion, all conditions for Hopf bifurcation are satisfied at $\theta = \hat{\theta}$.[29] $\square$

[29]The method of the proof is based on Gandolfo (1997).