Human capital risk in life-cycle economies

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Abstract

I study the effect of market incompleteness on the aggregate economy in a model where agents face idiosyncratic, uninsurable human capital investment risk. The environment is a general equilibrium life-cycle model with a version of a Ben-Porath (1967) human capital accumulation technology, modified to incorporate risk. A CARA-normal specification keeps endogenous decisions independent of individual shock realizations. I study stationary equilibria of calibrated cases in which idiosyncratic uninsurable risk arises from *specialization risk* and *career risk*. Specialization risk is such that both mean and variance of the return from training are increasing in the endogenous decision to invest in human capital. In the case of *career risk*, however, only the mean return is increasing in the decision to invest in human capital. With career risk only, stationary equilibria resemble those studied by Aiyagari (1994), and one concludes that the impact of uninsurable idiosyncratic risk is relatively small. With a significant amount of specialization risk however, stationary equilibria are severely distorted relative to a complete markets benchmark. One aspect of this distortion is that human capital is only about 57 percent as large as its complete markets counterpart. This suggests that the two types of risk have very different and quantitatively significant general equilibrium implications. *Keywords*: Human capital risk, life-cycle, incomplete markets. *JEL codes*: E20, E21, E24.

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1 Introduction

1.1 Human capital risk

Dispersion of labor earnings increases over the life-cycle, a well documented feature of the US data. According to Deaton and Paxson (1994), within-cohort labor income inequality increases with age. In related work, Huggett Ventura and Yaron (2007) investigate the reasons behind this rise in earnings dispersion over the life-cycle which is primarily due to age effects in a partial equilibrium framework. Their study finds that about one-third of the variation in lifetime earnings is due to idiosyncratic human capital shocks. Other cross sectional studies also indicate that agents face a great deal of uncertainty when making their schooling decisions.[1] Taken together, it appears that investment in human capital is risky and part of the labor income uncertainty that agents face over their life-cycle is a manifestation of this idiosyncratic human capital risk. In addition, it is widely understood that human capital investment is uninsurable—there is a clear lack of complete markets with respect to this investment.

One main consequence of this type of labor income uncertainty is that it could deter investment in human capital, possibly leading to underaccumulation of human capital and overaccumulation of relatively less risky physical capital, in comparison to a case where agents can insure against this risk via complete markets. If a mechanism like this is at work in actual economies, the impact of market incompleteness on the aggregate economy could be large,[2] possibly calling for policy intervention to mitigate the

[1]Carneiro, Hansen and Heckman (2003) find that the substantial heterogeneity in the returns to schooling is unpredictable at the time when schooling decisions are made. In related work, Cunha, Heckman, and Navaro (2005) conclude that 40% of the variability in the returns to schooling is unforecastable at the time students decide to go to college, implying that this uncertainty is not due to observable factors like ability differences or differences in initial conditions but purely due to idiosyncratic shocks.

[2]Even more so if one takes the view that human capital is an engine of growth. With the exception of few, Levhari and Weiss (1974), Eaton and Rosen (1980), Krebs (2003), Benabou (2002), for example, this strand of the literature typically abstracts from the risky nature of human capital investment.

effect of this risk on household decisions to invest in training.[3]

I study the macroeconomic implications of labor income uncertainty arising from the risky nature of human capital investment. The specification here allows a direct analysis of the impact of risk on the process of human capital accumulation, isolate and quantify the effect of risk on individual decisions, and comment on divergent views in this literature on the role of market incompleteness on the aggregate economy.

1.2 Main ideas

In this paper, investment in human capital is studied as a time allocation problem in a life-cycle framework. Here agents allocate time away from the labor market when they are young to acquire education. Using a version of a Ben-Porath (1967) production function for human capital that allows for risky human capital, I study how uninsurable risk impacts an individual's decision to train in a general equilibrium life-cycle model.[4] If the returns to training are uncertain and uninsurable, agents may try to self-insure by holding larger precautionary savings and, more importantly, they may also endogenously alter their training decisions to mitigate the effect of human capital risk.

I consider two types of uninsurable idiosyncratic risks, namely, specialization risk and career risk. Higher risk is compensated by higher return, and higher return is often associated with higher levels of education. In the formulation of the returns from training, this aspect of education is captured by the specialization risk. The specialization risk is such that the endogenous decision to train increases both the expected returns from training and its variance, while the career risk only affects the mean return

[3]Krebs (2003) comes to this conclusion in an endogenous growth model where investment in risky human capital is modeled as a portfolio decision and physical capital is the risk-free asset.

[4]I abstract from the risks associated with physical capital investment. Underlying this assumption is the notion that for an individual human capital investment is more risky than investment in physical capital. Typically investment in human capital cannot be easily diversified or directly traded in the market since it is non-separable from the owner.

from training. The career risk is additive in the human capital accumulation technology and is the most common formulation in the literature which studies the impact of uninsurable idiosyncratic risk on the aggregate economy. Risks that look like the multiplicative specialization risk of this paper were first studied by Angeletos and Calvet (2006), but not in a human capital setting.

Market incompleteness arising because of these idiosyncratic uninsurable risks make the wealth distribution a relevant state for individual decisions, often making problems in this class intractable. Several papers, including Calvet (2001) and Angeletos and Calvet (2006), use constant absolute risk aversion (CARA) utility function and normally distributed shocks in order to ensure that an individual's risk-taking decision is independent of wealth. I employ the same technique along with certain other assumptions to ensure that an individual's decisions are independent of wealth. Due to the lack of wealth effects, within a generation all agents make identical decisions but they still differ in their labor quality, labor income, and consumption because of the different realizations of the two shocks. In the life-cycle model that I consider, therefore, there are both types of heterogeneity, within generation and across generation. Across generation heterogeneity is inherent in the life-cycle models.

I study calibrated versions of the model to assess the quantitative importance of incomplete markets. Risk related parameters—the variances of the two shock processes in the human capital accumulation technology—are chosen to match the portion of the variance in labor earnings over the life-cycle that is due to age effects. I study a baseline case which has a mixture of the two types of shocks, and also a more extreme case where there is only career risk. Such an analysis allows us to clearly see how each of these risks, the specialization and the career risk, influence the aggregate economy by altering an individual's decisions. In addition to studying the macroeconomic implications of market incompleteness due to risky human capital, in this paper I also explore the life-cycle features implied by the model.

1.3 Main findings

I first establish that the stationary equilibrium of this model with only career risk has properties similar to Aiyagari (1994). Aiyagari studied the macroeconomic impact of uninsurable idiosyncratic labor income risk arising due to shocks to labor endowments in a model where households live forever and where there is no human capital. The career-risk-only case of the present model has implications similar to Aiyagari (1994).[5] In particular, the precautionary savings induced by this risk has only a small quantitative impact on the macroeconomy.

I then study the baseline calibration where both shocks play a role. In the baseline calibration, almost all the variance in labor income early in life when agents are investing in training is due to specialization risk. Later in life, both risks play a role. I find that the effects of the specialization risk dominate and there is a very large impact on macroeconomic variables in the stationary equilibrium. In particular, there is a 43 percent underaccumulation of human capital relative to the complete markets case. Accordingly, since labor quality is dramatically lower, output, physical capital, consumption and other variables are also drastically affected by the idiosyncratic uninsurable uncertainty.

I conclude that uninsurable idiosyncratic specialization risk has a large impact on macroeconomic equilibrium, but that uninsurable idiosyncratic career risk does not.

Does human capital risk have a significant impact on actual economies? It may if the shocks resemble the baseline calibration. But if most of the risk in human capital investment is due to career risk, then the influence could diminish markedly.[6] In a way, the quantitative analysis nests both the views that are commonly seen in the literature, one following the tradition of Aiyagari (1994) that argues that the quantitative effects of incom-

[5]This is despite the fact that there are no borrowing constraints or wealth effects in the present model with human capital accumulation.

[6]This is in contrast to Krebs (2003) where market incompleteness due to uninsurable idiosyncratic human capital risk generates large quantitative effects for the macroeconomy.

plete markets are small, and a relatively recent view associated with Angeletos and Calvet (2006), Pijoan-Mas (2006) and Marcet, Obiols-Homs and Weil (2007), that suggests that these effects could be large.[7] One conclusion is that empirical studies based on micro data would need to identify the relative importance of these shocks in order to assess the implications of market incompleteness on the aggregate economy.

Apart from aiding our analysis in thinking about human capital investment and matching some of the salient features of the aggregate economy, the life-cycle model stays consistent with some of the features of the life-cycle that are often studied in partial equilibrium settings, for example the shape of mean earnings and the variance of labor earnings over the life-cycle.

1.4 Recent related literature

Using an incomplete markets framework, several papers since Bewley (1977) have investigated the core implications of uninsurable idiosyncratic labor income risk on the aggregate economy. These papers typically abstract from aggregate uncertainty.[8] In Aiyagari (1994) individuals face uninsurable labor endowment risk which makes their labor income variable and uncertain. In addition, households face borrowing constraints in the credit market. In such a setup, households self-insure by increasing their precautionary savings when income risk rises. Due to market incompleteness and imperfect credit markets, the stationary equilibrium is characterized by higher capital stock, lower interest rates and higher output. However, the quantitative implications of the labor income risk are not very strik-

[7]Neither of these authors talked explicitly about human capital. Aiyagari (1994), Pijoan-Mas (2006) and Marcet, Obiols-Homs and Weil (2007) studied labor income risk whereas Angeletos and Calvet (2006) analyzed capital income risk. This paper is closer to Aiyagari (1994) and Marcet, Obiols-Homs and Weil (2007) but provides a different mechanism to explain the variance in labor income. It also nests both the views in the literature and argues that the nature of risk, specialization versus career risk, determines which view prevails in the quantitative analysis.

[8]Krusell and Smith (1998) among others incorporate both idiosyncratic and aggregate risk.

ing in Aiyagari (1994). In contrast recent papers by Marcet, Obiols-Homs and Weil (2007) and Pijoan-Mas (2006) find that market incompleteness could potentially have a large quantitative impact on the macroeconomy. In a model similar to Huggett (1997), another standard incomplete markets model with labor endowment shocks, Marcet, Obiols-Homs and Weil (2007) endogenize labor supply decisions. Their findings suggest that primarily due to wealth effects, market incompleteness can have a considerably large impact on the aggregate economy when labor elasticity is larger than consumption elasticity and leisure is a normal good.[9]

Another recent paper, Angeletos and Calvet (2006) that focuses on capital income uncertainty arising due to idiosyncratic uninsurable entrepreneurial risk, production as well as endowment risk, also finds large quantitative effects of market incompleteness. They argue that, in general, entrepreneurial risk reduces the demand for investment. At the same time, however, increase in uninsurable capital income risk raises precautionary savings which lowers the interest rate and therefore raises investment demand. As long as interest elasticity of savings is low, capital will be underaccumulated relative to the complete markets benchmark.

While Angeletos and Calvet (2006) study the aggregate implications of market incompleteness in a model with risky entrepreneurial income, a related paper, Krebs (2003), studies the impact of labor income risk in a model with risky human capital and risk-free physical capital. In Krebs (2003) this risky human capital is also the engine of growth. The decision to invest in human capital is modelled as a portfolio decision where an agent decides what fraction of savings to invest in risky human capital and how much to invest in the risk-free physical capital. He finds that risk lowers investment in human capital which in turn lowers growth and welfare.[10]

Based on Krebs (2003) it is not clear whether the quantitatively signifi-

[9] In the stationary equilibrium of some calibrated economies, for example, aggregate output is 50 percent lower relative to its complete markets benchmark. See Table 1 in Marcet, Obiols-Homs and Weil (2007).

[10] In the baseline calibration of Krebs (2003) complete elimination of labor income risk, which is entirely due to human capital risk, increases growth by 0.13 percent.

cant macroeconomic implications of risky human capital are due to market incompleteness or because human capital is the engine of growth. To isolate the role of uninsurable risky human capital, I abstract from growth in the steady state. Even though in this paper I ask a question similar to Krebs (2003), the framework used here is quite different and is more consistent with the traditional analyses of human capital investment. I consider a general equilibrium life-cycle model with a Ben-Porath production process for risky human capital investments.

A feature of this paper is that it can also produce some of the life-cycle features seen in the data in a general equilibrium framework. There is growing empirical-theoretic, primarily partial equilibrium literature studying the life-cycle features in the data, for example the rise in the variance of earnings and consumption and the hump-shaped mean earnings and consumption over the life-cycle. Deaton and Paxson (1994), Huggett, Ventura and Yaron (2007), Storesletten, Telmer and Yaron (2004), Heathcote, Storesletten and Violante (2005) among others study these issues. Deaton and Paxson (1994) use a complete markets framework to study the life-cycle features of their model and compare them to the U.S. cohort data.[11] They find that the age-profile of the dispersion of earnings and consumption increases over the life-cycle in the data and that non-separable preferences over consumption and leisure along with skill heterogeneity can explain these features. Storesletten, Telmer and Yaron (2004) argue that Deaton and Paxson's analysis has inconsistent implications for hours. As a result these authors attempt to explain the same features of the data using an incomplete markets framework where agents face idiosyncratic, uninsurable earnings risk, similar to Aiyagari (1994) and others but in a life-cycle context. In a related paper, Huggett, Ventura and Yaron (2007) use a partial equilibrium model with risky human capital to explain these same features in the data.

The paper is organized as follows. Section 2 presents the model while

[11] Deaton and Paxson (1994) also studied data from Great Britain and Taiwan which had features similar to the U.S..

section 3 discusses some intuition using a two-period overlapping generation model. In the final section I present quantitative results.

2 Model

The economy has an infinite sequence of overlapping generations of agents who live for T periods. Time is discrete and is indexed by $t = 0, 1, 2, ...$ In each period, a continuum of *ex ante* identical young agents with unit mass is born. Each agent is endowed with two units of time in each period until they retire. There is no population growth. Physical capital is risk-free and there are no credit market imperfections.[12] In terms of notation, subscripts indicate when the agent is born and the stage in an agent's life-cycle or the time in the model is in the parenthesis. In general, the aggregate variables do not have a subscript.

2.1 Incomplete markets

Each agent consumes a final good in every period such that the preferences of agent i born at date t are given by

$$U = E \sum_{j=0}^{T-1} \beta^j u(c_t^i(t+j)) \tag{1}$$

where $c_t^i(t+j)$ is the consumption of the final good of agent i who is born at date t at stage $t+j$ in the life-cycle, the discount factor is β, and E is the expectation operator. For tractability I assume that the utility function has the CARA form such that $u(c) = -\frac{1}{a}\exp[-ac]$ where a is the coefficient of absolute risk aversion.

The budget constraint of the agent for $0 \leq j < J_R$, where J_R is the exogenous date at which agents retire, is given by

$$c_t^i(t+j) = w(t+j)\{(1-\tau_t^i(t+j))\underline{x} + x_t^i(t+j)\} \\ + R(t+j-1)s_t^i(t+j-1) - s_t^i(t+j) \tag{2}$$

[12]Recent literature on incomplete markets, Magill and Quinzii (1996), Krebs (2003), Angeletos and Calvet (2006) among others, abstracts from credit market imperfections.

and

$$c_t^i(t+j) = R(t+j-1)s_t^i(t+j-1) - s_t^i(t+j) \tag{3}$$

for $J_R \leq j \leq T-1$. Time allocated to training by agent i is $0 \leq \tau_t^i(t+j) \leq 1$, $s_t^i(t+j)$ is the holdings of risk-free asset (capital) and $x_t^i(t+j)$ is the labor quality, measured in efficiency units, at date $t+j$. The wage rate per efficiency unit in period $t+j$ is $w(t+j)$ and $R(t+j)$ is the gross return on the risk-free asset holdings from period $t+j$ to $t+j+1$. Since there are no bequests, agents enter the first period without any assets and do not save in the last period of their life. Agents born at any date t however inherit the average aggregate labor quality $x(t)$ prevalent at that date and as a result $x_t(t) = x(t)$.

To ensure that all agents in the same stage of their life-cycle, agents within a generation, make the same training and asset holding decisions that are independent of the actual realization of the two shocks we assume that the labor quality of an agent $x_t^i(t+j)$, measured in efficiency units, enters additively such that it does not interact with the training decision in period $t+j$. The parameter $\underline{x} > 0$ converts units of time into efficiency units.[13] If $\tau_t^i(t+j) = 1$, agents receive $w(t+j)x_t^i(t+j)$ in labor income and when agents stop training, $\tau_t^i(t+j) = 0$, they allocate both units of their time to the labor market and their labor income becomes $w(t+j)\{\underline{x}+x_t^i(t+j)\}$. Capital income is the only source of income after retirement.

In this paper, I use a modified Ben-Porath's (1967) production function for human capital with time allocated to training as the input in the production technology.[14] Human capital investments require agents to give up labor income early in the life-cycle in order to generate higher future efficiency units. When allocating $\tau_t^i(t+j)$ units of time to training at date $t+j$, an agent is however uncertain about the number of efficiency units

[13]One aspect of this specification is that in a stationary equilibrium, the marginal cost of acquiring training is the same across all agents and is independent of their own individual labor quality.

[14]A one-input training technology is used here. In a richer multi-input specification, returns to training could depend on an agent's ability, the level of human and physical capital in the previous periods.

received in the subsequent period. Therefore, at the time when the training decision is made, the returns from training are uncertain making these investments risky. We assume the following functional form for the training technology

$$h_t^i(t+j+1) = \gamma(t+j)\tau_t^i(t+j)^{\phi}\epsilon_t^i\,(t+j+1) + \eta_t^i\,(t+j+1) \qquad (4)$$

for $0 \leq j < J_R - 1$. Here $0 < \phi < 1$ is the return elasticity of training and $\gamma(t+j)$ is the productivity of an agent who invested $\tau_t^i(t+j)$ time in training. The productivity is the same for all the agents in the same stage of the life-cycle but could vary over different stages of an agents's life-cycle. The two idiosyncratic shocks, the specialization shock $\epsilon_t^i\,(t+j+1)$, and the career shock $\eta_t^i\,(t+j+1)$, are normally distributed, independent and identical across agents within a generation and independent across generations. The risk associated with the returns from training could vary across generations implying that $\epsilon_t^i(t+j) \sim N(1, \sigma_\epsilon(t+j))$ and $\eta_t^i(t+j) \sim N(0, \sigma_\eta(t+j))$. The expected return from training in period $t+j+1$ is

$$Eh_t^i(t+j+1) = \gamma(t+j)\tau_t^i(t+j)^{\phi}$$

and the variance is

$$Vh_t^i(t+j+1) = \left(\gamma(t+j)\tau_t^i(t+j)^{\phi}\right)^2 \sigma_\epsilon^2(t+j+1) + \sigma_\eta^2(t+j+1).$$

In the case of complete markets, $\epsilon_t^i(t+j) = 0$ and $\eta_t^i(t+j) = 0$ for all the individuals and at all dates.

The *specialization risk* $\epsilon_t^i(t+j+1)$ is such that for the same level of risk, if individuals devote more time to training, they face a positive risk-return trade-off.[15] Allocating more time to training increases the expected future returns but at the same time it also increases the variance of the returns

[15] Numerous studies have estimated the returns to schooling. See Griliches (1977) and Card (2001) for a survey of this literature. In estimating the returns from education, this literature however has ignored the riskiness of these investments. For our analysis, the study by Palacios-Huerta (2003) is the most relevant. Using wage data from the March Current Population Survey (CPS) for 1964-1996, he reports the descriptive statistics of human capi-

from training. For instance, a college graduate who invests more years in training faces a higher return from training but also faces a greater risk relative to a high school graduate who devotes fewer years in training and hence faces lower risk-return trade-off. In Palacios-Huerta (2003), average real human capital return for white males with less than 5 years of experience with no high school, high school and college education are: 5.9 (6.2), 13.6 (9.7), 14.2 (11.3) respectively, with standard deviation of returns reported in the parenthesis. The *career risk* $\eta_t^i(t+j+1)$ can be interpreted as the general uncertainty associated with working in a job, whether the match is good, whether the agent is compatible with other workers, and other aspects of the labor market that are not incorporated in the model but are independent of the level of training. If there is only career risk, $\sigma_\epsilon(t) = 0$, then for the same level of risk, acquiring more training only increases the mean return from training but leaves the variance unaltered.

The uncertainty in the returns from training makes an agent's human capital, represented by the labor quality, random.[16] Random labor quality makes labor income uncertain in this paper. The labor quality of an agent i at date $t+j$ depends on the undepreciated level of inherited average aggregate labor quality and the human capital accumulated in the previous periods via training, the first and second terms respectively in the follow-

tal returns in Table 1 which suggests that there is evidence of a positive risk-return tradeoff in human capital investment for white males in the US. Other studies, for example a cross country study by Pereira and Martins (2002), and a study by Christainsen, Joensen and Nielsen (2006) using Danish data, also show that a positive risk-return trade-off exists in human capital investment.

[16]Uncertain labor quality is the source of labor income uncertainty in this paper. We abstract from labor income uncertainty arising because of labor market frictions. See, for example, Gomes, Greenwood and Rebelo (2001), Costain and Reiter (2005), Hall (2006), Rudanko (2007) among others.

ing equation

$$x_t^i(t+j) = (1-\delta_h)^j x(t) + \sum_{m=0}^{j-1} h_t^i(t+m+1)$$
$$= (1-\delta_h)^j x(t) + \sum_{m=0}^{j-1} \{\gamma(t+m)\tau_t^i(t+m)^{\phi}\epsilon_t^i(t+m+1) + \eta_t^i(t+m+1)\} \tag{5}$$

where the average aggregate labor quality inherited when young, $x(t)$, depreciates at rate δ_h. We also require that only inherited labor quality depreciates in order to abstract from wealth effects.

The labor quality of an agent has the following mean and variance

$$Ex_t^i(t+j) = (1-\delta_h)^j x(t) + \sum_{m=0}^{j-1} \gamma(t+m)\tau_t^i(t+m)^{\phi} \tag{6}$$

$$Vx_t^i(t+j) = \sum_{m=0}^{j-1} [\{\gamma(t+m)\tau_t^i(t+m)^{\phi}\}^2 \sigma_\epsilon^2(t+m+1) + \sigma_\eta^2(t+m+1)]. \tag{7}$$

The production technology of the final good is standard. There are two inputs, capital and labor and the production process exhibits constant returns to scale with the following specification

$$Y(t) = AK(t)^{\alpha} L(t)^{1-\alpha}, \tag{8}$$

where $K(t)$ is the aggregate capital stock and $L(t)$ is total labor supply measured in efficiency units at date t. The intensive form representation of the training technology is standard, $y(t) = Ak(t)^{\alpha}$, where $y(t)$ is the output per efficiency unit and $k(t)$ is the capital per efficiency unit. Total labor supply at date t, measured in efficiency units, is given by[17]

$$L(t) = \sum_{j=0}^{J_R-1} \{(1-\tau_{t-j}(t))\underline{x} + x_{t-j}(t)\}. \tag{9}$$

[17] I suppress superscript i since all the agents within a generation are identical with respect to their endogenous decisions.

Inputs are hired in competitive markets and are therefore paid their marginal products. Hence the wage rate per efficiency unit is $w(t) = (1-\alpha)Ak(t)^{\alpha}$ and the rental rate of capital is $r(t) = \alpha Ak(t)^{\alpha-1}$. Let δ_k be the rate at which capital depreciates, then $R(t) = r(t+1) + 1 - \delta_k$.

At the aggregate level there is no uncertainty. We can write the aggregate market clearing condition for physical capital as

$$L(t)k(t) = \sum_{j=1}^{T-1} s_{t-j}(t-1). \tag{10}$$

The law of motion of average aggregate labor quality is described by the following equation

$$x(t) = \frac{E\sum_{j=0}^{J_R-1} x_{t-j}(t)}{J_R}. \tag{11}$$

At date t, the average labor quality of any generation born in period $t-j$ is $Ex_{t-j}(t)$, for $j = 0, 1, ...J_R - 1$. Averaging over all the generations gives the average aggregate labor quality at date t which is inherited by the agents born at date t.

3 Some intuition

In this section I briefly describe some intuition coming from the simplest two-period case before considering the extended model described in the previous section. Let $T = 2$ so that the model collapses to a two-period overlapping generations model. For simplicity there is no retirement and since agents live for two periods, they hold assets and train only in the first period. We consider this case to illustrate the qualitative implications of our model. Agent specific superscript is dropped for clarity. CARA-normal specification allows us to get closed form solution for an individual's decisions. With closed form solutions we can clearly see the impact of risk on these decisions.

3.1 Individual decisions

For $T = 2$, equation (1) simplifies and the utility function of an agent born at date t is given by the following equation

$$U = -\frac{1}{a}\exp[-ac_t(t)] - \beta E\frac{1}{a}\exp[-ac_t(t+1)]. \tag{12}$$

The budget constraint in the first and second period of the life-cycle are $c_t(t) = w(t)\{(1-\tau_t(t))\underline{x} + x_t(t)\} - s_t(t)$ and $c_t(t+1) = w(t+1)\{\underline{x} + x_t(t+1)\} + R(t)s_t(t)$ respectively. Labor quality when old is given by

$$x_t(t+1) = (1-\delta_h)x(t) + \gamma(t)\tau_t(t)^{\phi}\epsilon_t(t+1) + \eta_t(t+1). \tag{13}$$

Since we use CARA preferences and assume that the shocks are normally distributed, we can rewrite the agent's problem as

$$U = -\frac{1}{a}\exp[-ac_t(t)] - \beta\frac{1}{a}\exp[-a\{Ec_t(t+1) - \frac{a}{2}Vc_t(t+1)\}], \tag{14}$$

where the expected value of date $t+1$ consumption is

$$Ec_t(t+1) = w(t+1)\{\underline{x} + (1-\delta_h)x(t) + \gamma(t)\tau_t(t)^{\phi}\} + R(t)s_t(t) \tag{15}$$

and the variance is

$$Vc_t(t+1) = w(t+1)^2\{(\gamma(t)\tau_t(t)^{\phi})^2\sigma_\epsilon^2 + \sigma_\eta^2\}. \tag{16}$$

Given initial inheritance of labor quality $x(t)$, wage rates $w(t)$, and $w(t+1)$ and interest rate $R(t)$, maximizing the agent's utility with respect to $c_t(t)$, $c_t(t+1)$, and $\tau_t(t)$ gives the following first order conditions

$$Ec_t(t+1) - c_t(t) = \frac{1}{a}\ln(\beta R(t)) + \frac{a}{2}Vc_t(t+1) \tag{17}$$

$$w(t)\underline{x} = \frac{\phi w(t+1)\gamma(t)\tau_t(t)^{\phi-1}}{R(t)} - \frac{\frac{a}{2}2\phi w(t+1)^2\gamma(t)^2\tau_t(t)^{2\phi-1}\sigma_\epsilon^2}{R(t)}. \tag{18}$$

These two equations collapse to the complete markets benchmark when $\sigma_\epsilon = 0$ and $\sigma_\eta = 0$. Relative to the complete markets case, the Euler equation (17) that determines the optimal level of asset holding has an extra term on the right hand side, $\frac{a}{2}Vc_t(t+1)$, the precautionary savings. Similarly, an extra term appears on the right hand side of equation (18) because markets are incomplete. When markets are complete, at the optimal level of training the marginal cost of acquiring training, the left hand side of equation (18) equals the present value of the future marginal benefit, the first term on the right hand side of this equation. When markets are incomplete, however, agents have to be compensated for the risk they face when they invest in training. Therefore, the second term on the right hand side is the risk premium on human capital investment. It is increasing in specialization risk σ_ϵ^2. Interestingly, from the two equations we see that while optimal asset holding is affected by both types of risk, the training decision, even when markets are incomplete, is not directly affected by the career risk. This suggests that individual decisions could differ depending on whether the agents face specialization risk, career risk or both, similar to the findings of Angeletos and Calvet (2006).

To explore this further, let the return elasticity of training ϕ equal $1/2$.[18] This allows us to get explicit solutions for the optimal level of asset holding and training of a young agent. Using these solutions we clearly see the role of risk and how it enters an individual's decisions in this simple two-period model with incomplete markets.

$$\begin{aligned} s_t(t) = {} & \frac{1}{1+R(t)} * \\ & \quad \frac{1}{a}\ln(\beta R(t)) + w(t)\{(1-\tau_t(t))\underline{x} + x(t)\} \\ & - w(t+1)\{\underline{x} + (1-\delta_h)x(t) + \gamma(t)\tau_t(t)^{0.5}\} + \frac{a}{2}V_t c_t(t+1)\} \end{aligned} \tag{19}$$

$$\tau_t(t) = \left(\frac{w(t+1)\gamma(t)}{2R(t)w(t)\underline{x} + aw(t+1)^2\gamma(t)^2\sigma_\epsilon^2}\right)^2. \tag{20}$$

Consider the case where $\sigma_\epsilon = 0$, and $\sigma_\eta > 0$. Based on the comparative static results for this case where changes in the time allocated to training

[18] In the human capital literature the estimated elasticity lies in the range $(0.5, 0.9)$. See Browning, Hansen and Heckman (1999) Table 2.3 and 2.4 for further details. Section 4.2.4. investigates the robustness of our results by varying ϕ.

only influence the mean return from training but not the variance, we see that the training decision is in fact unaffected by the career risk. Asset holding is however increasing in this risk because of the precautionary motive. Now let $\sigma_\epsilon > 0$, and $\sigma_\eta = 0$. In this case, since altering the decision to train impacts the variance of the return from training, agents train less when risk increases. As before, the direct effect of risk on real asset holding is positive for precautionary reasons. However, in this case risk also impacts real asset holding indirectly via training decisions. If the following inequality holds, then the saving decision is increasing in risk.

$$-1 \leq \frac{0.5w(t+1)\gamma(t)\tau_t(t)^{-0.5}}{w(t)\underline{x}} - \frac{\frac{a}{2}w(t+1)^2\gamma(t)^2\sigma_\epsilon^2}{w(t)\underline{x}} \tag{21}$$

From equation (18) we know that the right hand side of the above inequality equals the gross interest rate on the risk-free asset which is greater than 1 implying that this inequality holds.

4 Quantitative analysis

In the previous section we argued that the qualitative effect of risk, the specialization risk versus the career risk, on an agent's decision to hold risk-free assets and train could potentially be very different. This section explores the general equilibrium consequences of market incompleteness on decisions and the aggregate economy in various calibrated stationary equilibria of our model. It also investigates whether these qualitative results are in fact quantitatively significant.

Our calibration strategy, similar to Aiyagari (1994), is to first choose parameters such that they match some targets in the US data for the complete markets benchmark. To calibrate risk, we use the life-cycle features of our model. We consider three calibrated versions of our model that primarily differ along one dimension, the calibration of risk, which is chosen to endogenously generate variance of logarithm of labor income that matches data.[19] The three cases are the *Aiyagari-like* economy, the *career risk only*

[19]The only exception is the *Aiyagari-like* economy that differs in the calibration of other

economy, and the *baseline calibration* economy. In spite of the fact that all of these economies endogenously generate variance in labor income that matches the relevant data, the aggregate implications are strikingly different across these economies.

4.1 Calibration of complete market benchmark

In our calibration, the model period is eight years and an agent lives for 8 periods, or 64 years, from age 16 to 80. In the last two periods, at the age of 64, agents stop participating in the labor market and retire. An agent trains for 6 years between the ages $16-32$, the first two periods in this eight period model. We use standard values of α, β and δ_k. To match the share of physical capital in income we use $\alpha = 0.4$. The annual discount factor β is 0.975 and the annual depreciation rate of physical capital is 8 percent. We choose the depreciation rate of human capital as 4 percent such that initially when agents are accumulating human capital labor quality increases and after that it decreases gradually till they exit the labor market.[20] The other five parameters A, a, $\underline{x}$, $\gamma(t)$ and $\gamma(t+1)$ are chosen to match the following targets in the U.S. data. Our first goal is to make sure that the economy has a reasonable level of physical capital. We target capital-output ratio at 4.14 and consumption-output ratio at 0.748. The target for the gross annual interest rate on the risk free asset is 1.03 percent. Our second goal is to ensure that in equilibrium there is an appropriate level of education that matches the U.S. data. According to the Current Population Survey (CPS) on school enrollment,[21] 95.7 percent of the Americans between the age $15-17$, 72.5 percent from the age $18-19$, 39.4 percent from $20-24$ and 12.1 percent between the age $25-34$ are enrolled in school in 2006. The enrollment rate data allows us to calibrate time devoted to training within each period. In the first period, between the age $16-24$, we target 5.5 years of training and

parameters as well since we want to abstract from human capital accumulation in order to match key features of Aiyagari's (1994) model.

[20]In the robustness section, section 4.2.4., we vary δ_h.

[21]See Table S1401.

TABLE 1. CALIBRATION OF RISK

	Specialization risk $\{\sigma_\epsilon(t+j)\}_{j=1}^{j=5}$	Career risk $\{\sigma_\eta(t+j)\}_{j=1}^{j=5}$
Aiyagari-like	$\{0\}_{j=1}^{j=5}$	{0.08, 0.03, 0.04, 0.04, 0.04}
Career risk only	$\{0\}_{j=1}^{j=5}$	{0.08, 0.03, 0.03, 0.06, 0.03}
Baseline calibration	{1.41, 1.84, 0, 0, 0}	{0, 0.01, 0.02, 0.02, 0.02}

Table 1: Calibrated values of the standard deviation of the two shock processes in different economies with incomplete markets.

about half year in the second period, age $24-32$.[22] As a consequence, in our model an individual must spend 6 years, two years in secondary and four years in post-secondary education, in school between the age $16-32$. Total number of years spent in school by an average American over this time period is consistent with our targets. The average school life expectancy in the U.S. is approximately 16 years, 6 years in primary education, starting at the age of 6, a little less than 6 years in secondary education starting at age 12 and 4 years in post-secondary and tertiary education.[23] The relative risk aversion implied by our model lies in the range $(1,3)$. Given that we have more targets relative to free parameters, our calibration is not precise but it is very close to the targets.

4.1.1 Two types of risks

In the second step of our calibration, we use the life-cycle features of our model to match the variance of labor earnings over the life-cycle which is primarily due to age effects to the U.S. data.[24] In our model since there is no

[22] A calculation based on CPS would suggest that agents train for approximately 5 years between the age 16-24 and 1 year between the age 24-32.

[23] These statistics are based on the estimates of the United Nation Educational, Scientific and Cultural Organization's (UNESCO) Global Education Digest 2004, see Figure 1.

[24] We follow Aiyagari (1994) in calibrating our model. Our targets for physical and human capital suggest that the data are generated under complete markets. However, when we calibrate risk, we assume that the data are generated under incomplete markets. One advantage of this procedure is that the complete markets case is identical across all the

growth and at age 16 all agents are *ex ante* identical, variance in labor earnings over the life-cycle is entirely due to age effects. But in the data there may be numerous other reasons why dispersion in labor earnings increases over the life-cycle, for example, time effects, cohort effects, differences in initial conditions, human capital risk, and other employment related risks. For our analysis, the Huggett, Ventura and Yaron (2007) paper provides a close approximation of the age effects in the variance of labor earnings.[25] In addition, in our model there are no other sources of within generation heterogeneity except for the heterogeneity due to the idiosyncratic human capital shocks. Therefore the variance of labor earnings over the life-cycle which is due to age effects must be entirely due to these shocks. Huggett, Ventura and Yaron (2007) investigate different sources of labor income dispersion and conclude that approximately 1/3 of the variance in lifetime earnings due to age effects is actually explained by idiosyncratic human capital risks.[26] As a result, we choose the variance of the two idiosyncratic human capital shock processes over the life-cycle such that the life-cycle pattern of labor earning variability generated by our model under market incompleteness is in fact 1/3 of the variance in labor earnings in the data.[27]

In our quantitative analysis, we consider three calibrated economies, the *Aiyagari-like* economy, the *career risk only* economy, and the *baseline cal-*

economies with endogenous training. Ideally, calibrated economies with market incompleteness must match the U.S. data. However, as will be apparent from Tables 2 and 3, the calibration strategy used here does not matter for two out of the three economies that we consider in this paper since market incompleteness has a very small and quantitatively insignificant impact.

[25]They use earnings data from the Panel Study of Income Dynamics (PSID) 1969-2004 family files. They assume that the earnings data is generated by three factors, cohort, time, and age effects. Cohort effects are the effects that impact all the agents born in a particular year, time effects impact all the generations alive at a particular date. Due to the linear relationship between time and age, the age effects cannot be isolated easily. As a result they isolate the age effects by either controlling for time or controlling for cohort. In our analysis, we use the regression results where they control for time. See Figure 2a in Huggett, Ventura and Yaron (2007).

[26]Identification of idiosyncratic human capital shocks in Huggett, Ventura and Yaron (2007) relies on the fact that closer to retirement agents do not invest in human capital. During this period when there is no investment in human capital, changes in the wage rate, product of the rental rate and human capital, are entirely due to changes in the rental rate and/or the human capital shocks. Using this identification strategy, they estimate the

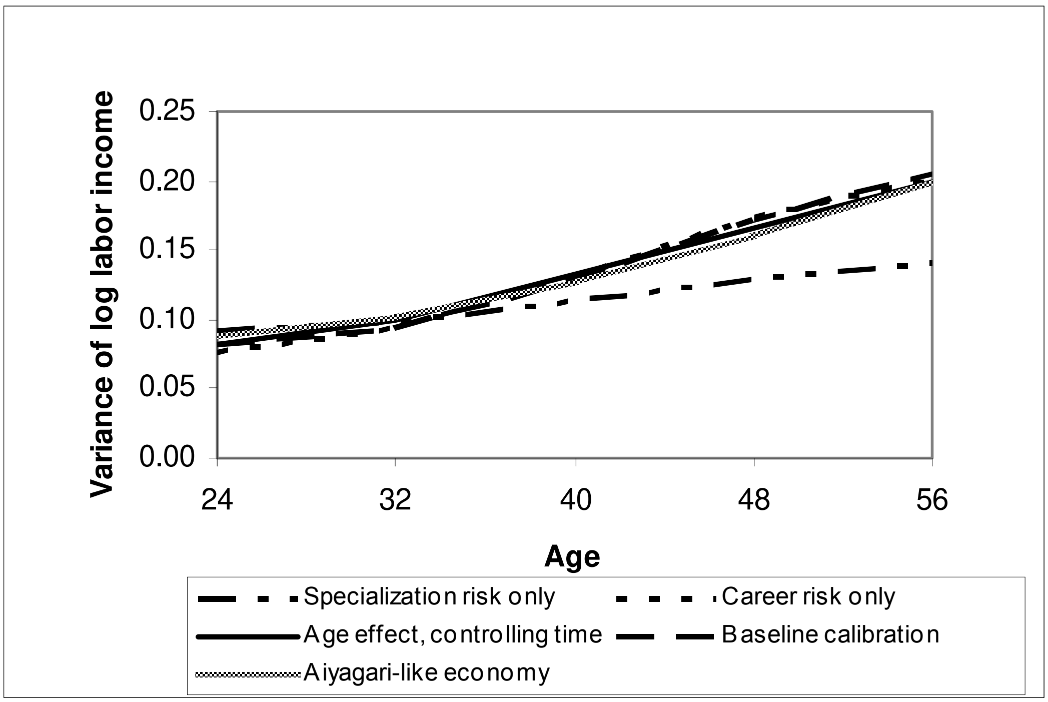

Figure 1: A comparison of the variance of logarithm of labor income in different calibrated cases of the incomplete markets version of our model with the relevant data from Huggett, Ventura and Yaron (2007).

ibration economy. Table 1 reports the underlying standard deviation of the sequence of shocks, $\{\sigma_\epsilon(t+j)\}_{j=1}^{j=5}$ and $\{\sigma_\eta(t+j)\}_{j=1}^{j=5}$, that are used in the calibration of the incomplete markets version of these economies. As noted earlier, the standard deviation of the two shocks varies across generations but remains constant for agents within a generation at all times. Notice that in this model there is no risk in the first period of life as agents have not yet invested in training and in the last two periods after they retire. Figure 1 illustrates how this calibration strategy allows us to match the earnings dispersion generated by our model with Huggett, Ventura and Yaron's (2007) measure of the variance of labor earnings which is due to idiosyncratic human capital shocks. All the economies, except the *specialization risk only* economy, can be calibrated to match the data. Specialization risk interacts with the time allocated to education, and agents invest in training only in the first two periods of their life-cycle. Therefore in the specialization risk only economy, the sequence of shocks cannot be calibrated to match the increase in the variance of labor earnings over the life-cycle.[28]

4.2 Results

Following the calibration procedure described above, we study stationary equilibria of calibrated economies to determine the quantitative macroeconomic consequences of market incompleteness in a general equilibrium life-cycle model.

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standard deviation of the human capital shocks.

[27] In our model, human capital investment refers to time the spent in education and hence agents train for 6 years after the age of 16. Human capital risk is both multiplicative and additive in our framework. In Huggett, Ventura and Yaron (2007) human capital shocks enter multiplicatively in the law of motion of human capital. For these shocks to play a role over most of the life-cycle, they assume that agents invest time in training until they are close to the retirement age. The time allocation problem, the trade-off between training and working exists for most part of the life-cycle, an unrealistic implication of their model. As a result, we consider that the risk identified in Huggett, Ventura and Yaron (2007) is in fact a proxy for both the risks that the agents face over their life-cycle, specialization and career risk.

[28] The specification of risk in the specilization risk only economy is the following $\{\sigma_\epsilon(t+j)\}_{j=1}^{j=5} = \{1.732\}_{j=1}^{j=5}$ and $\{\sigma_\eta(t+j)\}_{j=1}^{j=5} = \{0\}_{j=1}^{j=5}$.

4.2.1 Aiyagari-like economy

We construct a version of Aiyagari (1994) in a life-cycle context, *Aiyagari-like* economy. Such a construction ensures that the predictions based on an appropriately calibrated model, the Aiyagari-like economy, are consistent with the quantitative findings of the existing literature that studies the effect of uninsurable idiosyncratic labor income risk on the aggregate economy. In addition this construction rules out the likelihood that some of the features of our model–life-cycle model with CARA preferences with no wealth effects and no borrowing constraints–are shaping the main findings of the paper.

Aiyagari (1994) studied uninsurable labor endowment risk in a model with no human capital where agents live forever. This is a life-cycle model, and unlike Aiyagari there are no borrowing constraints and no wealth effects. To collapse our model with human capital accumulation to an Aiyagari-like economy, we set $\gamma(t)$ and $\gamma(t+1)$ equal to zero such that agents do not invest in training. Due to lack of education, specialization risk does not play any role in this calibration. We endow the agents with an average steady state labor quality $\hat{x}$ from our complete markets benchmark, column 1, panel *B*, Table 2. We set $\delta_h = 0$. The rest of the parameters are calibrated to meet the targets mentioned in the previous section.

Panel *A* of Table 2 reports the aggregate macroeconomic variables in the stationary equilibrium of the Aiyagari-like economy. Both qualitative and quantitative results in the Aiyagari-like economy are similar to Aiyagari (1994). When markets are incomplete, the savings rate (which equals $\delta_k k/y$) increases relative to the complete markets benchmark but the increase is not quantitatively significant. In Aiyagari (1994), the increase in the savings rate is in the range $\{0.06, 7.33\}$, expressed in percentage points.[29] In

[29] In the quantitative analysis of Aiyagari (1994) logarithm of labor endowment shock follows an AR(1) process with different values of the autoregressive coefficient and the coefficient of variation based on various studies. He reports his results based on different combinations of relative risk aversion, coefficient of variation and serial correlation. See Table II in Aiyagari (1994). In reporting the range here, we do not consider the case where the net return on capital is negative, the last entry in Table II.

TABLE 2. ECONOMIES WITH CAREER RISK ALONE

	Aiyagari-like economy Panel A		Career risk only economy Panel B	
	C.M.	I.M.	C.M.	I.M.
$\hat{x}$	0.154	0.154	0.1540	0.1542
$\hat{k}$	1.344	1.354	1.275	1.276
$\hat{K}/\hat{Y}$	4.15	4.17	4.02	4.03
$\hat{C}/\hat{Y}$	0.747	0.746	0.755	0.755
$\hat{L}/\hat{Y}$	0.386	0.385	0.395	0.394
$\hat{R}$	1.032	1.031	1.034	1.034
$\hat{w}$	1.553	1.558	1.521	1.521
$\hat{Y}$	4.644	4.658	4.272	4.276

Table 2: Comparing the quantitative macroeconomic effects of incomplete markets (IM) with the complete markets (CM) in the stationary equilibria of the economies with career risk alone – the Aiyagari-like economy and the career risk only economy.

the Aiyagari-like economy, the increase in savings rate is 0.11 percentage points. Higher savings rate raises output in both cases relative to their respective complete markets benchmark. Another consequence of increased savings is that the annual net interest rate (expressed in percentage points) declines by $\{0.02, 2.88\}$ in Aiyagari (1994) and 0.035 in the Aiyagari-like economy.[30] The Aiyagari-like economy, where there is no human capital accumulation, therefore corresponds to the less extreme cases in Aiyagari (1994).

4.2.2 Career risk only economy

Can market incompleteness in a model where investment in human capital is risky also produce small and quantitatively insignificant results? Consider the *career risk only* economy that differs from the Aiyagari-like econ-

[30]This is computed based on the annual interest rate reported in Table 2. For convenience, all the tables report the annual interest rate instead of the period interest rate.

omy along two dimensions, agents endogenously decide how much to train and human capital depreciates to maintain a constant level of labor quality in the stationary equilibrium.

Table 2, panel *B* presents results for the career risk only economy. Relative to the complete markets benchmark, incomplete markets have small and quantitatively insignificant effects on the aggregate economy, even smaller than the Aiyagari-like economy. Like the Aiyagari-like economy, the savings rate increases and output is also higher when markets are incomplete. A striking result in the career risk only economy is that when markets are incomplete there is *overaccumulation* of human capital, though quantitatively insignificant but in compliance with Angeletos and Calvet (2006).[31] These results demonstrate that allowing agents to optimally decide how much time to allocate to training when returns from training are uncertain and uninsurable is not enough to generate large quantitative effects. Moreover, when individuals face career risk, the risk where a change in training only affects the mean return but not the variance of the return from training, the effect of market incompleteness are marginal.

Our next specification of the incomplete markets incorporates specialization risk as well.

4.2.3 Baseline calibration economy

In the incomplete markets case of the *baseline calibration* economy, we assume that agents face both risks. Since the empirical literature provides no guidance on how to assign weights to these shocks, we assign these weights in a way that attributes almost all the variability in labor earnings to specialization risk early in the life-cycle and later, the additional risk that agents face is entirely due to the career shocks. The reason behind such an extreme calibration is that in this economy we want to explore the role of specialization risk while still matching the data on life-cycle labor income variability. From Figure 1 it is clear that specialization risk alone cannot

[31] Labor quality-output ratio also increases, albeit very marginally.

	Complete markets	Incomplete markets
$\hat{x}$	0.154	0.088
$\hat{k}$	1.275	1.381
$\hat{K}/\hat{Y}$	4.02	4.22
$\hat{C}/\hat{Y}$	0.755	0.742
$\hat{L}/\hat{Y}$	0.395	0.382
$\hat{R}$	1.034	1.030
$\hat{w}$	1.521	1.575
$\hat{Y}$	4.272	3.366

TABLE 3. BASELINE CALIBRATION

Table 3: Aggregate effects of market incompleteness in the stationary equilibrium of the baseline calibration economy.

match the data and hence we combine it with career risk.

As mentioned before, the two economies with endogenous training decision, the career risk only economy and the baseline calibration economy, differ only in the way the risks are calibrated. As a result, the complete markets case is identical for the two economies, column 1 in panel *B* of Table 2 and column 2 in Table 3. However, note that the stationary equilibria with uninsurable idiosyncratic human capital risk are strikingly different. The stationary equilibrium of the baseline calibration economy clearly illustrates that the specialization risk plays a very crucial role in determining the aggregate implications of market incompleteness. Market incompleteness has a significant impact on human capital accumulation causing the average labor quality to decline by 43 percent. This is in stark contrast with the findings of Aiyagari (1994) where the quantitative effect of incomplete markets was marginal. Agents self-insure in this economy by allocating less time to training. Due to such a sharp decrease in average labor quality, capital and the labor supply reduce dramatically by 12 and 19 percent respectively. Unlike the other economies studied here, output *falls* and the decrease is substantial–17 percent relative to the complete markets benchmark. Like Aiyagari (1994) and other studies in this literature, agents in this economy also insure themselves against the uninsurable risk by holding more risk-free assets. As a result, capital-output ratio increases by 4.9 percent, even though in the aggregate both capital and output fall relative to the complete markets case. The annual net interest rate decreases by 0.37 percentage points.

In the baseline calibration economy, the underaccumulation of human capital is large relative to the findings of Krebs (2003). In Krebs (2003) investment in human capital is 4.2 percentage points (of GDP) lower relative to the complete markets case. The comparable number for this economy is 19.8 percentage points.[32] Therefore, not accounting for the time allocation problem in an economy where specialization risk is the main source of uninsurable human capital risk can lead to a substantial misstatement of the impact of risk on human capital investment. Investment in physical capital in Krebs (2003) is 3.95 percentage points (of GDP) higher relative to complete markets benchmark. In our baseline calibration economy, investment in physical capital relative to output, the savings rate, increases by 1.20 percentage points, an increase which is 1/3 of the increase in Krebs. Therefore it appears that in this economy agents self insure but more by altering their decision to train and less via the traditional precautionary savings channel.

In the quantitative analysis we compared three different economies, the Aiyagari-like economy, the career risk only economy and the baseline calibration economy. Such a comparison is convincing in that all three economies endogenously generate the same life-cycle pattern of labor income variance even when they have extremely different aggregate macroeconomic implications. In the Aiyagari-like economy and the career risk only economy, incomplete markets have very small quantitative effects. On the contrary, the baseline calibration economy with higher weight on specialization risk, market incompleteness decreases labor quality dramati-

[32]Investment in human capital in our model is captured by the time allocated to training. In the first period, time devoted to training relative to output $(\hat{\tau}_0/\hat{y})$ decreases by 19.2 percentage points. The decrease is lower in the second period, 0.6 percentage points. Thus the overall decrease in investment in human capital relative to output is 19.8 percentage points.

cally and all other aggregate variables are also impacted considerably. Thus depending on the weights of the two shocks, the quantitative implications of incomplete markets could be as large as our baseline case or as small as the case with only career shocks. Unless empirical studies isolate the relative weights of these shocks in the data, it is hard to take a stand on the exact role of market incompleteness.

4.2.4 Robustness analysis

Here we evaluate whether our quantitative findings are robust to alternative calibrations of the elasticity parameter ϕ and the depreciation rate of human capital δ_h. Table 4, panels *A* and *B*, presents results where we vary each parameter one at a time while holding the other parameters fixed at values discussed in the previous section.

By increasing the return elasticity of training, ϕ equals 0.9, the quantitative effects of market incompleteness become more pronounced for the baseline calibration economy. For example, relative to the complete markets benchmark, average labor quality falls by 60 percent, capital and labor fall by 23 and 30 percent respectively and aggregate output declines by 27 percent. As before, the quantitative effects remain insignificant in the stationary equilibrium of the career risk only economy. Lower annual depreciation rate of human capital, 2 percent, does not change our quantitative results appreciably, see Table 4, panel *B*. Relative to the complete markets benchmark, market incompleteness due to career risk continues to have small effects whereas these effects stay large for the baseline calibration economy. Average labor quality declines by 42 percent relative to the complete markets benchmark and as a result output, capital and labor are lower in the baseline calibration with a lower rate of human capital depreciation. However, not surprisingly, lower depreciation rate of human capital in fact raises the average labor quality in the stationary equilibria of all calibrated economies, row 1, panel *B*, Table 4.

TABLE 4. ROBUSTNESS ANALYSIS

	Complete markets	Incomplete markets	
		Career risk only	Baseline calibration
		$\phi = 0.9$	
		Panel A	
$\hat{x}$	0.171	0.171	0.067
$\hat{k}$	1.223	1.226	1.359
$\hat{K}/\hat{Y}$	3.925	3.931	4.181
$\hat{C}/\hat{Y}$	0.761	0.761	0.746
$\hat{L}/\hat{Y}$	0.401	0.401	0.385
$\hat{R}$	1.036	1.031	1.036
$\hat{w}$	1.496	1.497	1.560
$\hat{Y}$	4.372	4.377	3.178
		$\delta_h = 0.02$	
		Panel B	
$\hat{x}$	0.240	0.241	0.139
$\hat{k}$	1.253	1.255	1.364
$\hat{K}/\hat{Y}$	3.983	3.987	4.191
$\hat{C}/\hat{Y}$	0.758	0.757	0.745
$\hat{L}/\hat{Y}$	0.397	0.397	0.380
$\hat{R}$	1.035	1.035	1.031
$\hat{w}$	1.510	1.511	1.563
$\hat{Y}$	5.557	5.565	4.347

Table 4: Robustness analysis with respect to the return elasticity of training and the depreciation rate of human capital are reported in panels A and B respectively. All other parameters are held fixed at their initial calibrated values.

TABLE 5. TIME IN TRAINING

	Complete markets	Incomplete markets	
		Career risk only	Baseline calibration
		Panel A	
$\hat{\tau}_0$	0.683	0.684	0.202
$\hat{\tau}_1$	0.066	0.067	0.054
		Panel B	
$\hat{\tau}_0$		0.683	0.193
$\hat{\tau}_1$		0.066	0.049

Table 5: Fraction of one unit of time devoted to training in the first two periods of the life-cycle. No time is allocated to training thereafter. Panels A and B report general equilibrium and partial equilibrium results respectively.

4.3 Life-cycle features of the model

In this section we take a closer look at the underlying training and asset holding decisions in different calibrated economies with human capital accumulation – career risk only and baseline calibration economy. In these economies, the training and asset holding decisions are identical for all the agents within a generation. However, within generation heterogeneity due to different realization of the shocks generates a distribution of earnings and consumption for each cohort. As a result, we also explore the cross sectional distribution of earnings and consumption implied by our model.

4.3.1 Individual decisions

Time allocated to training as a fraction of one unit of time in the first two periods of the life-cycle, where each period corresponds to eight years, is reported in Table 5. Panel *A* reports the overall effects whereas panel *B* isolates the partial equilibrium effects of idiosyncratic uninsurable human capital risk by holding wage and interest rate fixed at the complete markets level.

In the case when there is no uncertainty, agents spend 6 years in training, 5.5 years between the age $16 - 24$ and half a year between $24 - 32$.[33] When labor income uncertainty is due to career risk, the time allocated to training does not change much relative to the complete markets benchmark, see column 3, panel A. However, in the baseline calibration where specialization risk plays a central role, agents train for 2 years, 1.6 and 0.4 years in the first and second periods respectively. Relative to the complete markets benchmark, training is merely $1/3$ which is why labor quality is dramatically lower in the stationary equilibrium of the baseline calibration economy. As noted earlier this is a striking result.

Panel B evaluates the direct impact of risk on an individual's training decision when we hold wage and interest rate fixed at the complete markets level. From the two period case discussed in section 3 we know that career risk does not directly influence the training decision. This is validated here in column 3, panel B as the level of training in the career risk only economy is the same as in the complete markets benchmark, column 2, panel A. However, in the baseline calibration economy, time devoted to training is even lower relative to the comparable case in panel A. Reducing the time allocated to training lowers efficiency units but at the same time it increases the time supplied to the labor market by an individual. Thus the overall impact on the equilibrium wage rate is ambiguous in a general equilibrium setting. However, we know that the wage rate increases in the baseline calibration economy relative to the complete markets benchmark, see Table 3. Not allowing the wage rate per efficiency unit to increase in panel B partly explains why training is lower in this panel in relation to the corresponding column in panel A.

The decision to hold risk-free assets for precautionary purposes when markets are incomplete has been studied extensively. Figure 2 shows that the asset holding as a fraction of the total income increases over the working life of an agent, irrespective of whether markets are complete or not. One reason is that agents accumulate human capital when young and there-

[33] Time devoted to training in the first period is $0.683 * 8 = 5.5$ years. Similarly training in the second period is $0.066 * 8 = 0.5$ years.

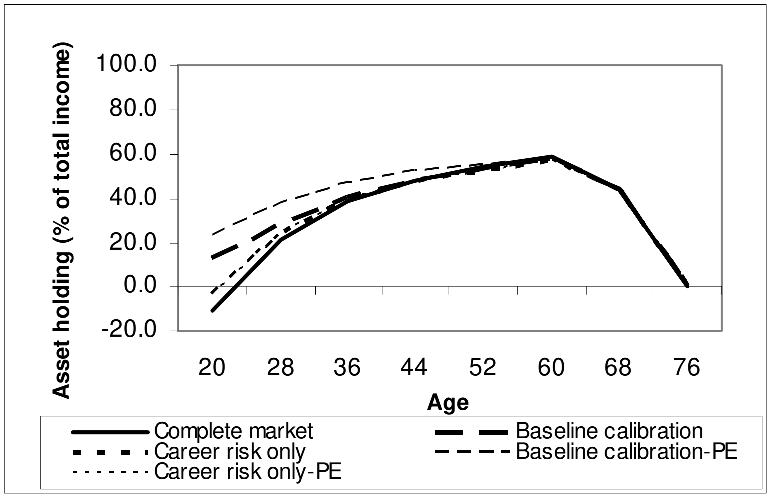
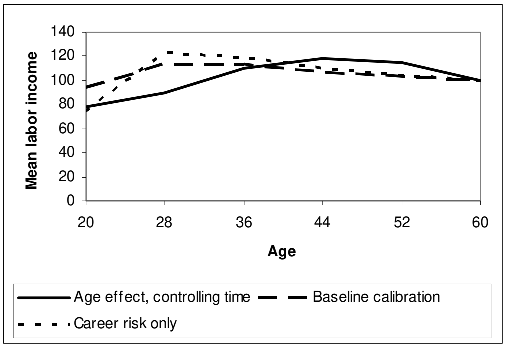


Figure 2: Life-cycle pattern of asset holding as a fraction of total income in the different economies for the general equilibrium and the partial equilibrium (PE) case.

fore their asset holdings are relatively low in the earlier part of their life-cycle. Figure 2 also shows that the asset holding decision gets distorted by risk, a common result in this literature. Agents tend to save more when markets are incomplete for precautionary purposes. Consequently, in both economies, the career risk only economy and the baseline calibration economy, real assets are higher relative to the case when markets are complete. In a partial equilibrium setting, in both economies asset holding as a fraction of total income is higher relative to their corresponding general equilibrium economies with incomplete markets, implied by equation (19), albeit marginally in the career risk only economy.[34]

Therefore, *both* decisions, the decision to train less and the decision to save more allow agents to self-insure against uninsurable idiosyncratic human capital risk. In addition, the large quantitative effects for the baseline calibration economy are primarily due to the partial equilibrium effects of

[34]For the partial equilibrium case, total income is computed using prices, wage and interest rate, from the stationary equilibrium of the complete markets economy.



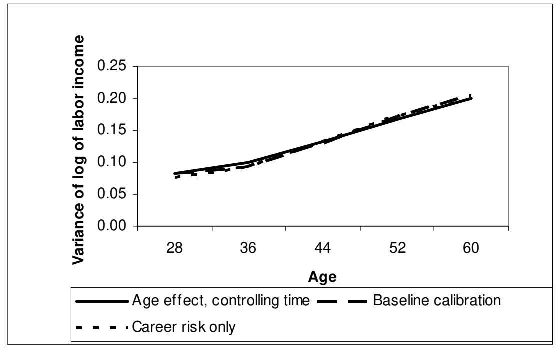


Figure 3: The left graph plots mean labor income and the right plots the variance of logarithm of the labor income over the life-cycle in the different economies.

idiosyncratic uninsurable human capital risk on these endogenous decisions.

4.3.2 Life-cycle pattern of income and consumption

Figure 3 compares the distribution of life-cycle earnings generated by our model with the data. The solid line in the left graph of Figure 3 is the age effects in mean earnings according to the estimates of Huggett, Ventura and Yaron (2007). Following Huggett, Ventura and Yaron (2007), we scale the mean labor income such that at the end of an agent's work life mean labor income is 100. The average labor income generated by our general equilibrium model has the familiar hump shape which is seen in the data. However, our model generates the hump in the earlier periods of the life-cycle, close to age 30, but in the data labor earnings peak when agents are in their mid 40s. One reason for this mismatch could be that in our model we do not allow labor quality to increase due to reasons other than schooling, like on-the-job training, learning by doing and other mechanisms that improve labor quality over the entire working life of an agent. Not surprisingly, the variance of the logarithm of labor earnings, the right graph in Figure 3, matches data in all the different economies that we consider since

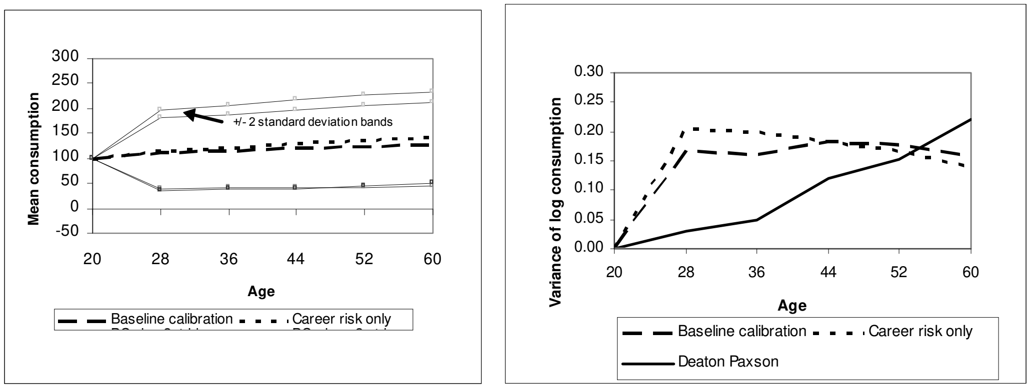


Figure 4: The left graph plots mean consumption in the two economies along with $+/-2$ standard deviation bands which suggests that negative consumption is a rare event in these economies. The right graph plots the variance of the logarithm of consumption.

we calibrated the risk to match this aspect of the data.

Our model does not match the moments of life-cycle consumption data very well. In our specification the two shocks have a normal distribution and consumption is residual. This suggests that in our quantitative analysis there can be some instances for which consumption is negative. The bands in the left graph of Figure 4 for mean consumption contain 95% of the values suggesting that the shocks in our model are sufficiently small to preclude high incidences of negative consumption. Mean age-consumption in our model has an upward slope, a common implication of life-cycle models with additively separable utility function defined over consumption alone. Bullard and Feigenbaum (2007) argue that inclusion of both consumption and leisure in the utility function would produce a hump-shaped consumption along with a U-shaped leisure profile, given a hump-shaped life-cycle productivity pattern. Comparing the consumption inequality over the life-cycle generated by our model to Deaton and Paxson's (1994) measure of consumption inequality in the data, we find that our model generates higher consumption volatility relative to data even when we account for just 1/3 of the variance in labor earnings.[35] Storesletten, Telmer and Yaron (2004) suggest that without a social security system, variance in consumption is roughly 20% higher relative to data. In this model there is no form of social insurance and that could partly explain high consumption variance. Another reason that our model generates high consumption variance is that an agent's endogenous decisions are unaffected by the actual realization of the two shocks. This causes consumption to be residual and therefore more volatile. Apart from being more volatile, the variance of consumption generated by the model is concave whereas in the data it is somewhat linear. In the model when agents are young, they accumulate human capital which is risky and their holdings of risk-free assets are relatively low. As a result shocks to training have a larger impact on consumption in the early periods of the life-cycle. Over time as holdings of risk-free asset increases, human capital shocks tend to have a lower impact. Overall, in both the economies, the life-cycle consumption implications of our model do not match data very well. By incorporating leisure, allowing for wealth effects and introducing some form of social insurance, we could potentially correct the implications of our model for consumption.

Even though the aggregate implications of these two economies with incomplete markets, the career risk only economy and the baseline calibration economy, are undoubtedly very different, the life-cycle features are somewhat similar. The two economies endogenously generate the same life-cycle pattern of labor income variability. Other life-cycle features, such as mean labor income, mean consumption and the variance of consumption over the life-cycle are also not very different across these two economies. One reason could be that these divergent decisions occur only in the early part of the life-cycle and their influence gets smoothed over many periods thus understating the striking differences in the endogenous decisions in the two economies with incomplete markets.

[35] Deaton and Paxson (1994) use consumption data from the Consumption Expenditure Survey (CEX), 1980-1990. They remove the cohort effects from the data when measuring the age-profile of consumption inequality over the life-cycle. See Deaton and Paxson (1994) for further details.

5 Conclusion

This paper studies the impact of uninsurable idiosyncratic human capital risk, using a Ben-Porath version of production function for human capital, in a general equilibrium life-cycle model. Our results indicate that uninsurable idiosyncratic human capital risk alone is not enough to generate large quantitative macroeconomic effects, contrary to the earlier findings in this literature. The nature of risk, career versus specialization, is crucial in determining the quantitative implications of market incompleteness for the aggregate economy. Calibrated stationary equilibrium with only career risk has properties similar to Aiyagari (1994) reconfirming the long held belief that the quantitative implications of uninsurable idiosyncratic risk are inconsequential. However, our baseline calibration with high weight on the specialization risk clearly illustrates that market incompleteness could have large, quantitatively significant, macroeconomic implications. Stationary equilibrium in this case is severely distorted relative to the complete markets benchmark.

Based on our quantitative analysis, we conclude that in order to comment on the macroeconomic effect of uninsurable idiosyncratic human capital risk, empirical studies based on micro data would need to determine the relative weights of these shocks in data. Even though the life-cycle features generated by our model for different calibrations of risk are not very different, the decision to train is particularly dissimilar in these economies. This dissimilarity in the level of education is primarily due to specialization risk. Empirical cross country studies estimating risks associated with different types of education that require varying degrees of specialization could however be indicative of the level of specialization risk that exists in these countries. Such studies could thereby alert us about the likely extent of distortion in these countries due to uninsurable idiosyncratic human capital risk.

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