Rethinking the Ability-to-Pay and Equal Sacrifice Principles of Taxation: An Alternative Rationale for a Progressive Income Tax

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Rethinking the Ability-to-Pay and Equal Sacrifice Principles of Taxation: An Alternative Rationale for a Progressive Income Tax

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Abstract

Progressive income taxes have usually been justified on the basis of the ability-to-pay (ATP) and equal sacrifice principles, but how ATP and sacrifice should be measured remains unsettled. In this paper, I present an alternative rationale for progressive taxes on the basis of the concept of sustainable heterogeneity (SH). I conclude that income taxes have to be progressive for SH to be achieved, and therefore, progressive income taxes can be justified without relying on the ATP and equal sacrifice principles. In addition, for SH to be achieved, households should also be burdened with taxes to cover expenses for achieving policy objectives other than SH in proportion to their incomes, that is, roughly in relation to their consumption, such as the case with a value-added tax.

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Keywords: Ability-to-pay principle; Benefit principle; Equal sacrifice principle; Progressive tax; Social welfare; Sustainable heterogeneity

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1 INTRODUCTION

The fairness of the tax burden has been usually judged by two principles: the ability-to-pay (ATP) principle and the benefit principle. Progressive taxes have been usually justified by the ATP principle, particularly by the equal sacrifice principle that was presented by Mill (1848) on the basis of the ATP principle. In this case, if ATP is equal, the amount of tax imposed should be equal, and if ATP increases, the amount of tax imposed should increase. However, the ATP and equal sacrifice principles do not necessarily say that progressive taxes are necessary—it only says that they are allowable. Whether a progressive tax is appropriate differs depending on the measure used to evaluate ATP or sacrifice (Richter, 1983; Young, 1987) in addition to the shape of utility function. But how ATP and sacrifice should be measured remains unsettled. Three measures of sacrifice (i.e., equal absolute sacrifice, equal proportional sacrifice, and equal marginal sacrifice) have been particularly studied (Musgrave and Musgrave, 1973), but which measure most appropriately indicates the fairness of the tax burden is still an unresolved question.

The equal sacrifice principle has the problem that it potentially violates Pareto efficiency and probably usually does (Kaplow and Shavell, 2001; da Costa and Pereira, 2014). Of course, we can interpret this violation as a necessary evil and may have to accept it, but the problems with the ATP and equal sacrifice principles may indicate that the rationale for progressive taxes given on the basis of these principles is not necessarily sufficiently persuasive and flawless. The purpose of this paper is to explore an alternative rationale for progressive taxes, specifically, one that is based on the concept of sustainable heterogeneity (SH) that was presented by Harashima (2017a).[1]

SH is defined as the state at which all the optimality conditions of all heterogeneous households are indefinitely satisfied. Harashima (2017a) showed that even if households are heterogeneous in preferences, there is a unique balanced growth path (or steady state) on which SH is achieved. However, SH is politically vulnerable and is not necessarily achieved naturally. Interventions by government are necessary to achieve SH if households behave unilaterally in the sense that they behave without considering the optimality conditions of other households (Harashima, 2012). A government has to transfer money or other economic resources from relatively more-advantaged households to less-advantaged households to achieve SH. Progressive taxes are a means of this type of government transfer, which suggests that without relying on the ATP and equal sacrifice principles, progressive taxes might be able to be justified on the basis of the SH

[1] Harashima (2017a) is also available in English as Harashima (2010).

concept.

In this paper, I show that, income taxes should be progressive to achieve SH. This result is unchanged even if taxes to cover the expenses for achieving various policy objectives other than SH are also considered. Therefore, progressive income taxes can be justified without the use of the ATP and equal sacrifice principles. Households should, however, be burdened with other taxes to cover other policy expenses in proportion to their productivities, incomes, and approximate consumption, and these taxes should be similar in nature to a value-added tax (VAT).

2 SUSTAINABLE HETEROGENEITY

The nature of SH is briefly explained in this section following Harashima (2017a).

2.1 SH

Three heterogeneities—the rate of time preference (RTP), the degree of risk aversion (DRA), and productivity—are considered. Suppose that there are two economies (Economy 1 and Economy 2) that are identical except for RTP, DRA, and productivity. Each economy is interpreted as representing a group of identical households, and the population in each economy is constant and sufficiently large. The economies are fully open to each other, and goods and services and capital are freely transacted among them, but labor is immobilized in each economy. Households also provide laborers whose abilities are one of the factors that determine the productivity of each economy. Each economy can be interpreted as representing either a country or a group of identical households in a country. Usually, the concept of the balance of payments is used only for the international transactions, but in this paper, this concept and terminology are also used even if each economy represents a group of identical households in a country.

The production function of Economy i (= 1, 2) is

$$y_{i,t} = A_t^{\alpha} k_{i,t}^{1-\alpha},$$

where $y_{i,t}$ and $k_{i,t}$ are production and capital of Economy i in period t, respectively; A_t is technology in period t; and α ($0 < \alpha < 1$) is a constant and indicates the labor share. All variables are expressed in per capita terms. The current account balance in Economy 1 is τ_t and that in Economy 2 is $-\tau_t$. The accumulated current account balance

$$\int_0^t \tau_s ds$$

mirrors capital flows between the two economies. The economy with current account surpluses invests them in the other economy. Since $\frac{\partial y_{1,t}}{\partial k_{1,t}} \left(= \frac{\partial y_{2,t}}{\partial k_{2,t}}\right)$ are returns on investments,

$$\frac{\partial y_{1,t}}{\partial k_{1,t}} \int_0^t \tau_s ds \quad \text{and} \quad \frac{\partial y_{2,t}}{\partial k_{2,t}} \int_0^t \tau_s ds$$

represent income receipts or payments on the assets that an economy owns in the other economy. Hence,

$$\tau_t - \frac{\partial y_{2,t}}{\partial k_{2,t}} \int_0^t \tau_s ds$$

is the balance on goods and services of Economy 1, and

$$\frac{\partial y_{1,t}}{\partial k_{1,t}} \int_0^t \tau_s ds - \tau_t$$

is that of Economy 2. Because the current account balance mirrors capital flows between the economies, the balance is a function of capital in both economies such that

$$\tau_t = \kappa\left(k_{1,t}, k_{2,t}\right) .$$

This two-economy model can be easily extended to multi-economy models. Suppose that a country consists of *H* economies that are identical except for RTP, DRA, and productivity (Economy 1, Economy 2, …, Economy *H*). Households within each economy are identical. $c_{i,t}$, $k_{i,t}$, and $y_{i,t}$ are the per capita consumption, capital, and output of Economy *i* in period *t*, respectively; and θ_i, $\varepsilon_q = -\frac{c_{1,t}u_i''}{u_i'}$, ω_i, and u_i are RTP, DRA, productivity, and the utility function of a household in Economy *i*, respectively ($i = 1, 2, \ldots, H$). The production function of Economy *i* is

$$y_{i,t} = \omega_i A_t^{\alpha} k_{i,t}^{1-\alpha} . \tag{1}$$

In addition, $\tau_{i,j,t}$ is the current account balance of Economy *i* with Economy *j*, where *i*,

$j = 1, 2, \ldots, H$ and $i \neq j$.

Harashima (2017a) showed that if and only if

$$\lim_{t\to\infty}\frac{\dot{c}_{i,t}}{c_{i,t}} = \left(\frac{\sum_{q=1}^{H}\varepsilon_q\omega_q}{\sum_{q=1}^{H}\omega_q}\right)^{-1}\left\{\left[\frac{\varpi\alpha\sum_{q=1}^{H}\omega_q}{Hmv(1-\alpha)}\right]^{\alpha} - \frac{\sum_{q=1}^{H}\theta_q\omega_q}{\sum_{q=1}^{H}\omega_q}\right\} \tag{2}$$

for any i (= 1, 2, … , H), all the optimality conditions of all heterogeneous economies are satisfied, where m, v, and ϖ are positive constants. Furthermore, if and only if equation (2) holds,

$$\lim_{t\to\infty}\frac{\dot{c}_{i,t}}{c_{i,t}} = \lim_{t\to\infty}\frac{\dot{k}_{i,t}}{k_{i,t}} = \lim_{t\to\infty}\frac{\dot{y}_{i,t}}{y_{i,t}} = \lim_{t\to\infty}\frac{\dot{A}_{t}}{A_{t}} = \lim_{t\to\infty}\frac{\dot{\tau}_{i,j,t}}{\tau_{i,j,t}} = \lim_{t\to\infty}\frac{\frac{d\int_0^t \tau_{i,j,s}ds}{dt}}{\int_0^t \tau_{i,j,s}ds}$$

is satisfied for any i and j ($i \neq j$). Because all the optimality conditions of all heterogeneous economies are satisfied, the state at which equation (2) holds is SH by definition.

2.2 SH with government intervention

SH is not necessarily naturally achieved if households behave unilaterally, but if the government properly transfers money or other types of economic resources from some economies to other economies, SH is achieved.

Let Economy 1+2+…+ (H – 1) be the combined economy consisting of Economies 1, 2, …, and (H – 1). The population of Economy 1+2+… + (H – 1) is therefore (H – 1) times that of Economy i (= 1, 2, 3, …, H). $k_{1+2+\cdots+(H-1),t}$ indicates the capital of a household in Economy 1+2+· · ·+ (H – 1) in period t. Let g_t be the amount of government transfers from a household in Economy 1+2+…+ (H – 1) to households in Economy H and $\bar{g}_t$ be the ratio of g_t to $k_{1+2+\cdots+(H-1),t}$ in period t to achieve SH. That is,

$$g_t = \bar{g}_t k_{1+2+\cdots,+(H-1),t} \cdot$$

$\bar{g}_t$ is solely determined by the government and therefore is an exogenous variable for households.

Harashima (2017a) showed that if

$$\lim_{t\to\infty}\bar{g}_t$$

$$= \left(\frac{\sum_{q=1}^{H} \varepsilon_q \omega_q}{\omega_H}\right)^{-1} \left\{\frac{\varepsilon_H \sum_{q=1}^{H} \omega_q - \sum_{q=1}^{H} \varepsilon_q \omega_q}{\sum_{q=1}^{H-1} \omega_q} \left[\frac{\varpi \alpha \sum_{q=1}^{H} \omega_q}{Hm\nu(1-\alpha)}\right]^{\alpha} - \frac{\varepsilon_H \sum_{q=1}^{H} \theta_q \omega_q - \theta_H \sum_{q=1}^{H} \varepsilon_q \omega_q}{\sum_{q=1}^{H-1} \omega_q}\right\}$$

is satisfied for any i (= 1, 2, · · ·, H) in the case that Economy H is replaced with Economy i, then equation (2) is satisfied (i.e., SH is achieved by government interventions even if households behave unilaterally). Because SH indicates a steady state, $\lim_{t\to\infty} \overline{g}_t$= constant.

Note that the amount of government transfers from households in Economy 1+2+ · · · + (H – 1) to a household in Economy H at SH is

$$(H-1)g_t = (H-1)\ k_{1+2+\cdots+(H-1),t} \lim_{t\to\infty} \overline{g}_t\ .$$

Note also that a negative value of g_t indicates that a positive amount of money or other type of economic resource is transferred from Economy H to Economy 1+2+ · · · + (H – 1) and vice versa.

3 POLICY EXPENSE TAX UNDER SH

Taxes are not only used as a tool of government transfers to achieve SH but also to cover the expenses for many policy objectives other than achieving SH (hereafter, this is called the "policy expense tax"). In this section, burdens of the policy expense tax under SH are examined in a two-economy model for simplicity. As shown in Section 2, a two-economy model can be easily extended to a multi-economy model.

3.1 Environment

Suppose that a country consists of two economies that are identical except for RTP and productivity, and households are identical within each economy and behave unilaterally. RTP and productivity are assumed to be negatively correlated because many empirical studies conclude that RTP is negatively correlated with incomes (e.g., Lawrance, 1991; Samwick, 1998; Ventura, 2003); this indicates that the economy with the higher productivity has a lower RTP than the economy with the lower productivity and vice versa. Let θ_i and ω_i be the respective RTP and productivity of a household in Economy i, and suppose that $\theta_1 < \theta_2$ and $\omega_1 > \omega_2$, which means that Economy 1 is more advantaged than Economy 2. Note that for simplicity, DRA is assumed to be identical in the two economies.

SH is achieved by the following government transfers from a household in Economy 1 to households in Economy 2 in period t

$$g_t = \overline{g}_t k_{1,t} \ ,$$

where I refer to g_t as "SH transfers." Note that government transfers from households in Economy 1 to a household in Economy 2 in period t are also $g_t = \overline{g}_t k_{1,t}$ because the number of households is identical between the two economies. Because

$$k_{1,t} = \frac{\omega_1}{\omega_2} k_{2,t} \tag{3}$$

as shown in Harashima (2017a), then

$$g_t = \overline{g}_t k_{1,t} = \overline{g}_t \frac{\omega_1}{\omega_2} k_{2,t} \ .$$

A positive value of g_t indicates that taxes are imposed on households in Economy 1, and households in Economy 2 receive economic resources from the government. A negative value means the reverse is true.

Let x_t be the policy expense tax in period t, and $x_{i,t}$ is x_t imposed on a household in Economy i (= 1, 2). It is assumed for simplicity that

$$x_{i,t} = \bar{x}_i \alpha y_{i,t} \ ,$$

where $\bar{x}_i$ is constant and exogenous for households, and $\alpha y_{i,t}$ indicates the labor income of a household in Economy i because α indicates the labor share. Here, Harashima (2017a) showed that, through arbitration in financial markets,

$$y_{i,t} = \left[\frac{(\omega_1 + \omega_2)\varpi\alpha}{2mv(1-\alpha)} \right]^{\alpha} k_{i,t}$$

and therefore

$$x_{i,t} = \bar{x}_i \alpha \left[\frac{(\omega_1 + \omega_2)\varpi\alpha}{2mv(1-\alpha)} \right]^{\alpha} k_{i,t} \ .$$

3.2 Non-universal policy expense tax

3.2.1 The policy expense tax on the more-advantaged economy

I first examine the case that the government imposes the policy expense tax only on households in the more-advantaged Economy 1 (i.e., the policy expense tax consists only of $x_{1,t}$). Following Harashima (2017a), each household in Economy 1 maximizes its expected utility

$$E\int_0^\infty u_1(c_{1,t})\exp(-\theta_1 t)dt$$

subject to

$$\frac{dk_{1,t}}{dt}=\left[\frac{(\omega_1+\omega_2)\varpi\alpha}{2mv(1-\alpha)}\right]^\alpha k_{1,t}-c_{1,t}+\left[\frac{(\omega_1+\omega_2)\varpi\alpha}{2mv}\right]^\alpha(1-\alpha)^{1-\alpha}\int_0^t\tau_s\,ds$$
$$-\tau_t-\bar{g}_t k_{1,t}\ \ -x_{i,t}$$
$$=\left[\frac{(\omega_1+\omega_2)\varpi\alpha}{2mv(1-\alpha)}\right]^\alpha k_{1,t}-c_{1,t}+\left[\frac{(\omega_1+\omega_2)\varpi\alpha}{2mv}\right]^\alpha(1-\alpha)^{1-\alpha}\int_0^t\tau_s\,ds$$
$$-\tau_t-\bar{g}_t k_{1,t}\ \ -\bar{x}_1\alpha\left[\frac{(\omega_1+\omega_2)\varpi\alpha}{2mv(1-\alpha)}\right]^\alpha k_{1,t}$$

and each household in Economy 2 maximizes its expected utility

$$E\int_0^\infty u_2(c_{2,t})\exp(-\theta_2 t)dt$$

subject to

$$\frac{dk_{2,t}}{dt}=\left[\frac{(\omega_1+\omega_2)\varpi\alpha}{2mv(1-\alpha)}\right]^\alpha k_{2,t}-c_{2,t}-\left[\frac{(\omega_1+\omega_2)\varpi\alpha}{2mv}\right]^\alpha(1-\alpha)^{1-\alpha}\int_0^t\tau_s\,ds$$
$$+\tau_t+\bar{g}_t k_{2,t}$$

where E is the expectation operator.

Harashima (2017a) showed that if $\lim_{t\to\infty}\frac{\dot{c}_{1,t}}{c_{1,t}}=\lim_{t\to\infty}\frac{\dot{c}_{2,t}}{c_{2,t}}$ holds,

$$\lim_{t\to\infty}\bar{g}_t=\frac{\theta_2-\theta_1-\bar{x}_1\alpha\left[\frac{(\omega_1+\omega_2)\varpi\alpha}{2mv(1-\alpha)}\right]^\alpha}{\left(1+\frac{\omega_1}{\omega_2}\right)}=\text{constant},\qquad(4)$$

and

$$\lim_{t\to\infty}\frac{\dot{c}_{1,t}}{c_{1,t}}=\lim_{t\to\infty}\frac{\dot{c}_{2,t}}{c_{2,t}}=\varepsilon^{-1}\left[\left(\frac{\varpi\alpha}{mv}\right)^{\alpha}(1-\alpha)^{-\alpha}-\frac{\theta_1+-\theta_2-\bar{x}_1\alpha\left[\frac{(\omega_1+\omega_2)\varpi\alpha}{2mv(1-\alpha)}\right]^{\alpha}}{\left(1+\frac{\omega_1}{\omega_2}\right)}\right]$$

$$=\text{constant.}$$

Because

$$\omega_1^{\alpha}A_t^{\alpha}k_{1,t}^{-\alpha}=\frac{\varpi\alpha(\omega_1+\omega_2)}{2mv(1-\alpha)}$$

as Harashima (2017a) indicates, then by equation (4),

$$\lim_{t\to\infty}\bar{g}_t=\frac{\theta_2-\theta_1+\bar{x}_1\alpha\omega_1^{\alpha}A_t^{\alpha}k_{1,t}^{-\alpha}}{\left(1+\frac{\omega_1}{\omega_2}\right)}$$

$$=\frac{\theta_2-\theta_1-x_{1,t}k_{1,t}^{-1}}{\left(1+\frac{\omega_1}{\omega_2}\right)}$$

and therefore,

$$k_{1,t}\lim_{t\to\infty}\bar{g}_t=\frac{(\theta_2-\theta_1)k_{1,t}}{\left(1+\frac{\omega_1}{\omega_2}\right)}-\frac{x_{1,t}}{\left(1+\frac{\omega_1}{\omega_2}\right)}\ . \tag{5}$$

Equation (5) means that, although the policy expense tax ($x_{1,t}$) is nominally imposed only on households in Economy 1, households in both Economy 1 and Economy 2 bear the tax burden at SH.

The burden on a household in Economy 2 at SH is

$$\frac{x_{1,t}}{\left(1+\frac{\omega_1}{\omega_2}\right)}$$

and that in Economy 1 is

$$x_{1,t} - \frac{x_{1,t}}{\left(1 + \frac{\omega_1}{\omega_2}\right)} .$$

Because $\omega_1 > \omega_2$,

$$\left[x_{1,t} - \frac{x_{1,t}}{\left(1 + \frac{\omega_1}{\omega_2}\right)}\right] - \frac{x_{1,t}}{\left(1 + \frac{\omega_1}{\omega_2}\right)} = x_{1,t}\left(\frac{\omega_1 - \omega_2}{\omega_1 + \omega_2}\right) > 0 .$$

Hence, the burden on a household in Economy 1 is larger than that on a household in Economy 2 at SH.

The ratio of the burden on a household in Economy 2 to that on a household in Economy 1 (BD_{21}) is

$$BD_{21} = \frac{\frac{x_{1,t}}{\left(1 + \frac{\omega_1}{\omega_2}\right)}}{x_{1,t} - \frac{x_{1,t}}{\left(1 + \frac{\omega_1}{\omega_2}\right)}} = \frac{\omega_2}{\omega_1} .$$

The burden of the policy expense tax ($x_{1,t}$) is shared by households in both economies with the ratio ω_2 to ω_1 (i.e., the share of burden depends on their productivities). Notice that a household in Economy 1 shares more of the burden than a household in Economy 2 because $\omega_1 > \omega_2$, not because the policy expense tax is nominally imposed only on households in Economy 1.

3.2.2 The policy expense tax on the less-advantaged economy

Next, I examine the case that the government imposes the policy expense tax only on the less-advantaged Economy 2 (i.e., the policy expense tax consists only of $x_{2,t}$). By a similar procedure to that used in Section 3.2.1,

$$\lim_{t\to\infty} \bar{g}_t = \frac{\theta_2 - \theta_1 + x_{2,t} k_{2,t}^{-1}}{\left(1 + \frac{\omega_1}{\omega_2}\right)}$$

and

$$k_{1,t}\lim_{t\to\infty}\bar{g}_t = \frac{(\theta_2-\theta_1)k_{1,t}}{\left(1+\frac{\omega_1}{\omega_2}\right)} + \frac{x_{2,t}}{\left(1+\frac{\omega_2}{\omega_1}\right)} \ . \tag{6}$$

Equation (6) means that, although the policy expense tax ($x_{2,t}$) is nominally imposed only on households in Economy 2, households in both economies bear the tax burden at SH. Similar to the result shown in Section 3.2.1, the actual tax burdens at SH do not depend on where the policy expense tax is nominally imposed.

The burden on a household in Economy 1 at SH is

$$\frac{x_{2,t}}{\left(1+\frac{\omega_2}{\omega_1}\right)}$$

and that in Economy 2 is

$$x_{2,t} - \frac{x_{2,t}}{\left(1+\frac{\omega_2}{\omega_1}\right)} \ .$$

Because $\omega_1 > \omega_2$,

$$\frac{x_{2,t}}{\left(1+\frac{\omega_2}{\omega_1}\right)} - \left[x_{2,t} - \frac{x_{2,t}}{\left(1+\frac{\omega_2}{\omega_1}\right)}\right] = -x_{2,t}\left(\frac{\omega_1-\omega_2}{\omega_1+\omega_2}\right) < 0 \ ;$$

therefore, the burden on a household in Economy 1 is larger than that on a household in Economy 2 at SH (see Section 3.2.1). The burden ratio, BD_{21}, is

$$BD_{21} = \frac{x_{2,t} - \frac{x_{2,t}}{\left(1+\frac{\omega_2}{\omega_1}\right)}}{\frac{x_{2,t}}{\left(1+\frac{\omega_2}{\omega_1}\right)}} = \frac{\omega_2}{\omega_1} \ .$$

The burden of $x_{2,t}$ is shared by households in both economies with the ratio ω_2 to ω_1, as was also the case in Section 3.2.1. An important point is that, even though the policy expense tax is nominally imposed only on households in Economy 2, a household in Economy 1 bears more of the burden than one in Economy 2 because $\omega_1 > \omega_2$.

3.3 Universal policy expense tax

I now examine the case where the government imposes the policy expense tax on households in both economies simultaneously; that is, the policy expense tax consists of both $x_{1,t}$ and $x_{2,t}$. By a similar procedure to those used in Section 3.2,

$$\lim_{t\to\infty}\bar{g}_t = \frac{\theta_2 - \theta_1 - x_{1,t}k_{1,t}^{-1} + x_{2,t}k_{2,t}^{-1}}{\left(1+\frac{\omega_1}{\omega_2}\right)}$$

and

$$k_{1,t}\lim_{t\to\infty}\bar{g}_t = \frac{(\theta_2 - \theta_1)k_{1,t}}{\left(1+\frac{\omega_1}{\omega_2}\right)} - \frac{x_{1,t} - x_{2,t}\frac{\omega_1}{\omega_2}}{\left(1+\frac{\omega_1}{\omega_2}\right)} .$$

Therefore, the burdens $x_{1,t}$ and $x_{2,t}$ are shared by households in both economies at SH.

The burden on a household in Economy 1 at SH is

$$x_{1,t} - \frac{x_{1,t} - x_{2,t}\frac{\omega_1}{\omega_2}}{\left(1+\frac{\omega_1}{\omega_2}\right)}$$

and that in Economy 2 is

$$x_{2,t} + \frac{x_{1,t} - x_{2,t}\frac{\omega_1}{\omega_2}}{\left(1+\frac{\omega_1}{\omega_2}\right)} .$$

Therefore,

$$BD_{21} = \frac{x_{2,t} + \frac{x_{1,t} - x_{2,t}\frac{\omega_1}{\omega_2}}{\left(1+\frac{\omega_1}{\omega_2}\right)}}{x_{1,t} - \frac{x_{1,t} - x_{2,t}\frac{\omega_1}{\omega_2}}{\left(1+\frac{\omega_1}{\omega_2}\right)}} = \frac{\omega_2}{\omega_1} . \tag{7}$$

As was the case for the analogous situations in Sections 3.2.1 and 3.2.2, the burdens of $x_{1,t}$ and $x_{2,t}$ are shared by households in both economies with the ratio ω_2 to ω_1, and the nominal policy expense tax imposed on households in each economy is irrelevant to the actual burdens at SH. In addition, equation (7) clearly indicates that, in any case, the actual burden on a household in Economy 1 is larger than that in Economy 2 at SH because $\omega_1 > \omega_2$.

3.4 Similarity to a VAT

Because household productivity (ω_i) is proportionate to incomes and furthermore roughly to the amounts of consumption, the policy expense tax $x_{i,t}$ can be interpreted to be approximately equivalent to a tax imposed based on the amount of consumption with a flat tax rate. Because the tax base of a VAT is the amount of consumption and the rate is flat, the nature of the policy expense tax appears to be very similar to that of a VAT.

Of course, the policy expense tax and VAT are not precisely equivalent. For example, because some income is transferred through g_t by the government at SH, consumption is not precisely in proportion to productivity so that consumption does not increase exactly at the same rate that ω_i increases under SH. Therefore, even if a large portion of $x_{i,t}$ can be collected with a VAT, a portion of $x_{i,t}$ will still need be additionally collected with other types of taxes to achieve SH.

4 PROGRESSIVE INCOME TAX

Section 2 implies that progressive taxes are necessary for SH, but Section 3 implies that under SH, the policy expense tax should be a proportional tax, such as a VAT. That is, these two different types of taxes need be integrated in a nation's tax system under SH, but how? In this section, I examine this question.

The environment for the examination is basically the same as that in Section 3, except that I use a multi-economy model with H economies. The H economies are identical except for RTP and productivity, and RTP and productivity are negatively correlated such that if $\theta_i > \theta_j$, then $\omega_i < \omega_j$ for $i, j = 1, 2, \ldots, H$ and $i \neq j$.

4.1 Income tax under SH

4.1.1 The total tax burden

Section 3 indicates that regardless of how the nominal policy expense tax is levied on households across economies, the burden is eventually borne by households in proportion to ω_i at SH. Taking this into consideration, it is assumed that the policy expense tax is imposed based on the amount of capital a household possesses (equivalently on a household's productivity as well as the amounts of production and income), with a proportional tax rate that is common to all economies such that

$$x_{i,t} = \tilde{x} k_{i,t} \quad , \tag{8}$$

where $x_{i,t}$ is the policy expense tax imposed on a household in Economy i in period t for i = 1, 2, …, H, and $\tilde{x}$ is the proportional tax rate, which is constant and common to all economies. Because $k_{i,t}$ is proportionate to ω_i as equation (3) indicates, a household in Economy i is burdened with the policy expense tax in proportion to ω_i from the beginning.

As Harashima (2012) shows, SH transfers to households in Economy i from a household in the other $H-1$ economies at SH in period t is

$$\begin{aligned} g_{i,t} &= \frac{\left(\sum_{q=1}^{H} k_{q,t} - k_{i,t}\right)}{H-1} \lim_{t\to\infty} \bar{g}_{i,t} \\ &= \frac{\left(\sum_{q=1}^{H} k_{q,t} - k_{i,t}\right)}{H-1} \left(\frac{\sum_{q=1}^{H} \varepsilon_q \omega_q}{\omega_i}\right)^{-1} \left\{ \frac{\left(\varepsilon_i \sum_{q=1}^{H} \omega_q - \sum_{q=1}^{H} \varepsilon_q \omega_q\right)}{\sum_{q=1}^{H} \omega_q - \omega_i} \left[\frac{\varpi \alpha \sum_{q=1}^{H} \omega_q}{Hmv(1-\alpha)}\right]^{\alpha} \right. \\ &\quad \left. - \frac{\varepsilon_i \sum_{q=1}^{H} \theta_q \omega_q - \theta_i \sum_{q=1}^{H} \varepsilon_q \omega_q}{\sum_{q=1}^{H} \omega_q - \omega_i} \right\} \quad , \end{aligned} \tag{9}$$

where $\varepsilon_q = -\frac{c_{q,t} u_q''}{u_q'}$ is the DRA of Economy q. Because DRA is assumed to be identical for any economy (i.e., ε_q is identical for any q (= 1, 2, …, H)), by equation (9),

$$g_{i,t} = \frac{\left(\sum_{q=1}^{H} k_{q,t} - k_{i,t}\right)}{H-1} \left(\frac{\sum_{q=1}^{H} \omega_q}{\omega_i}\right)^{-1} \left(\frac{\theta_i \sum_{q=1}^{H} \omega_q - \sum_{q=1}^{H} \theta_q \omega_q}{\sum_{q=1}^{H} \omega_q - \omega_i}\right) . \tag{10}$$

Note that SH transfers from households in the other $H-1$ economies to a household in Economy i at SH in period t is by equation (9),

$$(H-1) g_{i,t} = \left(\sum_{q=1}^{H} k_{q,t} - k_{i,t}\right) \lim_{t\to\infty} \bar{g}_{i,t} . \tag{11}$$

Let

$$k_{i,t} = \tilde{k}_t \omega_i \ . \tag{12}$$

Equation (12) indicates that $\tilde{k}_t$ is $k_{q,t}$ in the case that $\omega_q = 1$. Hence,

$$\frac{\sum_{q=1}^{H} k_{q,t} - k_{j,t}}{H-1} = \tilde{k}_t \frac{\sum_{q=1}^{H} \omega_q - \omega_j}{H-1} \ ; \tag{13}$$

therefore, by equations (10) and (13),

$$g_{i,t} = \frac{\tilde{k}_t \omega_i}{H-1}\left(\theta_i - \frac{\sum_{q=1}^{H} \theta_q \omega_q}{\sum_{q=1}^{H} \omega_q}\right) . \tag{14}$$

In addition, let

$$\tilde{\theta} = \frac{\sum_{q=1}^{H} \theta_q \omega_q}{\sum_{q=1}^{H} \omega_q} \ . \tag{15}$$

$\tilde{\theta}$ is clearly constant and indicates the average RTP of all households. Hence, by equations (14) and (15),

$$g_{i,t} = \frac{\tilde{k}_t \omega_i}{H-1}\left(\theta_i - \tilde{\theta}\right) . \tag{16}$$

The total amount of all tax burdens of a household in Economy i in period t at SH ($TB_{i,t}$) is the policy expense tax plus SH transfers from that household to households in the other $H-1$ economies. Because SH transfers from a household in Economy i to households in the other $H-1$ economies at SH in period t is, by equation (11),

$$-(H-1)g_{i,t} \ ,$$

by equations (8), (11), (12), and (16),

$$\begin{aligned} TB_{i,t} &= x_{i,t} - (H-1)g_{i,t} \\ &= \tilde{k}_t \omega_i\left(\tilde{x} + \tilde{\theta} - \theta_i\right) . \end{aligned} \tag{17}$$

Note that a negative value of $TB_{i,t}$ means that a household in Economy i bears no net tax burden but instead receives positive net transfers equivalent to $-TB_{i,t}$ from the government.

4.1.2 Progressiveness

By equation (1) (i.e., by the production function), the labor incomes of a household in Economy i in period t ($LI_{i,t}$) at SH are

$$\begin{aligned} LI_{i,t} &= \alpha y_{i,t} = \alpha \omega_i^{\alpha} A_t^{\alpha} k_{i,t}^{1-\alpha} \\ &= \alpha \omega_i A_t^{\alpha} \, \tilde{k}_t^{1-\alpha} \end{aligned} \tag{18}$$

because α indicates the labor share. On the other hand, because $\tilde{\theta}$ is the average RTP of all households and the real interest rate r_t is

$$\frac{\partial \tilde{y}_t}{\partial \tilde{k}_t} = (1-\alpha) A_t^{\alpha} \, \tilde{k}_t^{-\alpha} = r_t = \tilde{\theta} \tag{19}$$

at steady state for any economy through arbitration in financial markets (Harashima, 2017a), the capital incomes of a household in Economy i in period t ($CI_{i,t}$) at SH are

$$\begin{aligned} CI_{i,t} &= r_t k_{i,t} = \tilde{\theta} k_{i,t} \\ &= \tilde{\theta} \omega_i \tilde{k}_t \ . \end{aligned} \tag{20}$$

Let $\tilde{y}_t$ be $y_{q,t}$ in the case that $\omega_q = 1$. Hence, the ratio of the total tax burdens to the total incomes of a household in Economy i at SH (TBR_i) (i.e., the ratio of $TB_{i,t}$ to $LI_{i,t} + CI_{i,t}$ at SH) is, by equations (17), (18), and (20),

$$\begin{aligned} TBR_i &= \frac{TB_{i,t}}{LI_{i,t} + CI_{i,t}} = \frac{\tilde{k}_t \omega_i \left(\tilde{x} + \tilde{\theta} - \theta_i \right)}{\alpha \omega_i A_t^{\alpha} \, \tilde{k}_t^{1-\alpha} + \tilde{\theta} \omega_i \tilde{k}_t} \\ &= \frac{\tilde{x} + \tilde{\theta} - \theta_i}{\alpha \tilde{y}_t \tilde{k}_t^{-1} + \tilde{\theta}} \ . \end{aligned} \tag{21}$$

By equations (1) and (19),

$$\alpha \tilde{y}_t \tilde{k}_t^{-1} = \frac{\alpha}{1-\alpha} \tilde{\theta} \ . \tag{22}$$

Hence, by equations (21) and (22),

$$TBR_i = (1-\alpha)\frac{\tilde{x} + \tilde{\theta} - \theta_i}{\tilde{\theta}} \ . \tag{23}$$

Equation (23) clearly indicates that TBR_i is temporally constant. Equation (23) also indicates that TBR_i differs among households depending on the value of θ_i.

The right-hand side of equation (23) can be divided into two terms such that

$$TBR_i = (1-\alpha)\frac{\tilde{x}}{\tilde{\theta}} + (1-\alpha)\frac{\tilde{\theta} - \theta_i}{\tilde{\theta}} \ . \tag{24}$$

In equation (24), let

$$PBR_i = (1-\alpha)\frac{\tilde{x}}{\tilde{\theta}} \quad , \tag{25}$$

where PBR_i indicates the burden rate of the policy expense tax for a household in Economy *i* at SH; that is, it corresponds to the proportional tax rate of the policy expense tax as indicated by equation (8). Equation (25) clearly shows that PBR_i is constant and common to all economies.

Also in equation (24), let

$$\begin{aligned} IBR_i &= (1-\alpha)\frac{\tilde{\theta} - \theta_i}{\tilde{\theta}} \\ &= (1-\alpha)\left(1 - \frac{\theta_i}{\tilde{\theta}}\right) , \end{aligned} \tag{26}$$

where IBR_i indicates the burden rate of the income tax for a household in Economy *i* at SH. If IBR_i is positive (i.e., $\tilde{\theta} - \theta_i > 0$), a household in Economy *i* has income taxes imposed upon it, but if IBR_i is negative (i.e., $\tilde{\theta} - \theta_i < 0$), the household receives positive net transfers from the government. As equation (26) clearly indicates, IBR_i differs among economies depending on the value of θ_i. Note that because $0 < \alpha < 1$ and $\theta_i > 0$ for any *i*, then by equation (26), $IBR_i < 1$ for any *i*.

Because $0 < \alpha < 1$, by equation (26),

$$\frac{dIBR}{d\theta_i} = -\frac{1-\alpha}{\tilde{\theta}} < 0 \ . \tag{27}$$

Equation (27) clearly indicates that $\frac{dIBR}{d\theta_i}$ = constant; that is, IBR_i decreases linearly as θ_i increases. In addition, because ω_i is negatively correlated with θ_i, that is,

$$\frac{d\theta_i}{d\omega_i} < 0 \ , \tag{28}$$

then by inequality (27),

$$\frac{dIBR_i}{d\omega_i} = \frac{dIBR_i}{d\theta_i}\frac{d\theta_i}{d\omega_i} > 0 \ . \tag{29}$$

Furthermore, because the amount of incomes $(LI_{i,t} + CI_{i,t})$ is an increasing function of ω_i as equations (18) and (20) indicate, that is, because

$$\frac{d\omega_i}{d(LI_{i,t} + CI_{i,t})} > 0 \ , \tag{30}$$

by inequality (29),

$$\frac{dIBR_i}{d(LI_{i,t} + CI_{i,t})} = \frac{dIBR_i}{d\omega_i}\frac{d\omega_i}{d(LI_{i,t} + CI_{i,t})} > 0 \ . \tag{31}$$

Inequality (31) indicates that the burden rate of the income tax (IBR_i) should increase as incomes increase. That is, the income tax under SH should be progressive.

4.1.3 Progressiveness in actual income taxes

Equation (26) indicates that income taxes should not be imposed on households in Economy *i* if θ_i is higher than $\tilde{\theta}$. This means that income taxes should not be imposed on roughly half of the households in a country because $\tilde{\theta}$ indicates the average RTP. However, in actuality, a majority of households will pay at least some income taxes in many countries because the policy expense tax can be levied in various forms, including an income tax.

Suppose that a part of $x_{i,t}$ (i.e., $\beta x_{i,t}$) is levied in the form of an income tax for any *i*, where β ($0 < \beta < 1$) is a constant. In this case, IBR_i becomes

$$IBR_i = (1-\alpha)\frac{\beta\tilde{x}}{\tilde{\theta}} + (1-\alpha)\frac{\tilde{\theta} - \theta_i}{\tilde{\theta}} \tag{32}$$

by equation (24). Equation (32) indicates that even if θ_i is higher than $\tilde{\theta}$, households have to pay income taxes if

$$\beta\tilde{x} + \tilde{\theta} - \theta_i > 0 \ .$$

An important point is that equation (32) clearly indicates that inequalities (29) and (31) still hold even in this case; therefore, income taxes still have the property of being progressive.

4.2 Rent incomes and progressive tax

4.2.1 Rent incomes

Harashima (2019b) showed that some households can persistently obtain incomes from economic rents (hereafter, "rent incomes") thanks to monopoly rents derived from ranking value and preference, a concept introduced by Harashima (2018b).[2] Because ranking value and preference generate monopoly powers, some people can obtain incomes that are much higher than those of ordinary people (e.g., superstars in professional sports). Harashima (2017b) showed that companies can obtain monopoly rents due to ranking preference and value by differentiating their products. Because the strategy of product differentiation is one of the most important strategies for companies (Porter, 1980, 1985) and is actually pursued by many companies, monopoly rents will exist widely, ubiquitously, and massively across an economy at the present time and in the future.

An important point, as Harashima (2018a, 2018b, 2019a) indicates, is that these monopoly rents are probably distributed very unevenly, particularly to a few people involved with the companies (e.g., owners or executives). Accessibility to the monopoly rents is highly likely to be heterogeneous among people and family lines, and as a result, many of these monopoly rents will probably be enjoyed by only a small number of people and family lines. How should we deal with these monopoly rents with regard to SH and progressive taxes? I examine this problem in this section.

4.2.2 Rent incomes and the total tax burden

The environment for the examination is the same as that in Section 4.1 except that some households can obtain rent incomes. Suppose that each household in Economy *j* equally obtains rent incomes $z_{j,t}$ in period *t*, and conversely, the income of each household in the

[2] Harashima (2018b) is also available in English as Harashima (2016).

other $H-1$ economies is reduced by $\frac{z_{j,t}}{H-1}$ in period t. Households in the other $H-1$ economies do not obtain any rent income. It is assumed that $z_{j,t}$ is in proportion to $k_{j,t}$ such that

$$z_{j,t} = \left(\bar{\mu}_j + \epsilon_t\right)\bar{z}k_{j,t} \quad , \tag{33}$$

where $\bar{z}$ (> 0) and $\bar{\mu}_j$ (> 0) are constants and ϵ_t is i.i.d. with zero mean.

Harashima (2019b) indicates that SH transfers to households in Economy j from a household in the other $H-1$ economies at SH in period t is

$$\begin{aligned} E\left(g_{j,t}\right) &= \frac{\sum_{q=1}^{H} k_{q,t} - k_{j,t}}{H-1} \lim_{t\to\infty} E\left(\bar{g}_{j,t}\right) \\ &= \frac{\sum_{q=1}^{H} k_{q,t} - k_{j,t}}{H-1}\left(\frac{\sum_{q=1}^{H}\varepsilon_q\omega_q}{\omega_j}\right)^{-1}\left\{\frac{\left(\varepsilon_j\sum_{q=1}^{H}\omega_q - \sum_{q=1}^{H}\varepsilon_q\omega_q\right)}{\sum_{q=2}^{H}\omega_q - \omega_j}\left[\frac{\varpi\alpha\sum_{q=1}^{H}\omega_q}{Hmv(1-\alpha)}\right]^{\alpha}\right. \\ &\qquad \left. - \frac{\varepsilon_j\sum_{q=1}^{H}\theta_q\omega_q - \theta_j\sum_{q=1}^{H}\varepsilon_q\omega_q}{\sum_{q=1}^{H}\omega_q - \omega_j}\right\} - k_{j,t}\frac{\bar{\mu}_j\bar{z}}{H-1} \quad . \end{aligned} \tag{34}$$

Because temporal rent incomes are indifferent to SH (Harashima, 2019b), ϵ_t is ignored for simplicity; that is, $E\left(g_{j,t}\right) = g_{j,t}$. Because $\varepsilon_q = \varepsilon$ for any q, then by equation (34),

$$g_{j,t} = \frac{\sum_{q=1}^{H} k_{q,t} - k_{j,t}}{H-1}\left[\left(\frac{\sum_{q=1}^{H}\omega_q}{\omega_j}\right)^{-1}\left(\frac{\theta_j\sum_{q=1}^{H}\omega_q - \sum_{q=1}^{H}\theta_q\omega_q}{\sum_{q=1}^{H}\omega_q - \omega_j}\right)\right] - k_{j,t}\frac{\bar{\mu}_j\bar{z}}{H-1} \ . \tag{35}$$

By equation (12),

$$\frac{\sum_{q=1}^{H} k_{q,t} - k_{j,t}}{H-1} = \tilde{k}_t\frac{\sum_{q=1}^{H}\omega_q - \omega_j}{H-1} \ ,$$

and therefore, by equation (35),

$$g_{j,t} = \frac{\tilde{k}_t\omega_j}{H-1}\left(\theta_j - \tilde{\theta} - \bar{\mu}_j\bar{z}\right) \ . \tag{36}$$

Because SH transfers from a household in Economy j to households in the other $H-1$ economies at SH in period t is, by equation (11),

$$-(H-1)g_{j,t}$$

then by equations (8), (12), and (36), the total amount of tax burden of a household in Economy *j* at SH ($TB_{Rent,j,t}$) is

$$\begin{aligned} TB_{Rent,j,t} &= x_{j,t} - (H-1)g_{j,t} \\ &= \tilde{k}_t \omega_j \left(\tilde{x} + \tilde{\theta} - \theta_j + \bar{\mu}_j \bar{z}\right) . \end{aligned} \tag{37}$$

4.2.3 Rent incomes and progressiveness

Because the labor incomes of a household in Economy *j* in period *t* are the same as those indicated by equation (18), the total incomes (labor incomes plus rent incomes) of a household in Economy *j* ($LI_{Rent,j,t}$) at SH are, by equations (12), (18), and (33),

$$\begin{aligned} LI_{Rent,j,t} &= \alpha y_{j,t} + z_{j,t} = \alpha y_{j,t} + k_{j,t} \bar{\mu}_j \bar{z} \\ &= \alpha \omega_j A_t^{\alpha} \, \tilde{k}_t^{1-\alpha} + \bar{\mu}_j \bar{z} \tilde{k}_t \omega_j \ . \end{aligned} \tag{38}$$

Capital incomes ($CI_{j,t}$) are not affected by rent incomes at an SH that is achieved by appropriate government interventions. That is, equation (20) holds at SH even if rent incomes exist. Let $TBR_{Rent,j}$ be $TBRj$ in the case that households in Economy *j* have rent incomes. By equations (20), (22), (37), and (38),

$$\begin{aligned} TBR_{Rent,j} &= \frac{TB_{Rent,j,t}}{LI_{Rent,j,t} + CI_{j,t}} = \frac{\tilde{k}_t \omega_j \left(\tilde{x} + \tilde{\theta} - \theta_j + \bar{\mu}_j \bar{z}\right)}{\alpha \omega_j A_t^{\alpha} \, \tilde{k}_t^{1-\alpha} + \bar{\mu}_j \bar{z} \tilde{k}_t \omega_j + \tilde{\theta} \omega_i \tilde{k}_t} \\ &= \frac{\tilde{x}}{\frac{\tilde{\theta}}{1-\alpha} + \bar{\mu}_j \bar{z}} + \frac{\tilde{\theta} - \theta_j + \bar{\mu}_j \bar{z}}{\frac{\tilde{\theta}}{1-\alpha} + \bar{\mu}_j \bar{z}} \ . \end{aligned} \tag{39}$$

Equation (39) indicates that as with TBR_j, $TBR_{Rent,j}$ is temporally constant.

Let $IBR_{Rent,j}$ be IBR_j in the case that households in Economy *j* have rent incomes, and therefore, by equation (39),

$$IBR_{Rent,j} = \frac{\tilde{\theta} - \theta_j + \bar{\mu}_j \bar{z}}{\frac{\tilde{\theta}}{1-\alpha} + \bar{\mu}_j \bar{z}} \ . \tag{40}$$

By equation (40),

$$\frac{dIBR_{Rent,j}}{d\theta_j} = -\left(\frac{\tilde{\theta}}{1-\alpha} + \bar{\mu}_j \bar{z}\right)^{-1} < 0 \ . \tag{41}$$

Hence, $IBR_{Rent,j}$ decreases linearly as θ_j increases. In addition, by inequalities (28) and (41),

$$\frac{dIBR_{Rent,j}}{d\omega_j} > 0 \ . \tag{42}$$

Furthermore, by inequalities (30) and (42),

$$\frac{dIBR_{Rent,j}}{d(LI_{j,t} + CI_{j,t})} > 0 \ . \tag{43}$$

On the other hand, because $0 < \alpha < 1$, for any $\theta_j (> 0)$,

$$\frac{\tilde{\theta}}{1-\alpha} > \tilde{\theta} - \theta_j \ . \tag{44}$$

Hence, by equations (26) and (40) and inequality (44), for $\tilde{\theta} - \theta_j > 0$,

$$IBR_{Rent,j} = \frac{\tilde{\theta} - \theta_j + \bar{\mu}_j \bar{z}}{\frac{\tilde{\theta}}{1-\alpha} + \bar{\mu}_j \bar{z}} > \frac{\tilde{\theta} - \theta_j}{\frac{\tilde{\theta}}{1-\alpha}} = (1-\alpha)\left(\frac{\tilde{\theta} - \theta_j}{\tilde{\theta}}\right) = IBR_j \tag{45}$$

because $\bar{\mu}_j \bar{z} > 0$. That is, for $\tilde{\theta} - \theta_j > 0$, $IBR_{Rent,j}$ is higher than IBR_j. By inequalities (43) and (45), therefore, income taxes should be still progressive even when rent incomes exist.

Note that if $\tilde{\theta} - \theta_j + \bar{\mu}_j \bar{z} \leq 0$, no income tax is imposed on Economy *j*, and if $\tilde{\theta} - \theta_j + \bar{\mu}_j \bar{z} < 0$, Economy *j* instead receives positive net transfers from the government.

4.2.4 Significantly high rent incomes and progressiveness

Because rent incomes of households in Economy *j* are extracted from the incomes of households in the other $H - 1$ economies, the rent incomes of a household in Economy *j* ($k_{j,t}\bar{\mu}_i \bar{z} = z_{j,t}$) can be as large as almost equal to

$$\sum_{q=1}^{H} y_{q,t} - y_{j,t} \ .$$

As a result, the value of $\bar{\mu}_j \bar{z}$ can be far larger than the value of $\frac{\tilde{\theta}}{1-\alpha}$ in equation (39), which means that the rent incomes of a household in Economy j can be far larger than its labor and capital incomes ($LI_{j,t}+CI_{j,t}$).

In addition, by inequality (44), the value of $\bar{\mu}_j \bar{z}$ can be also far larger than the value of $\tilde{\theta} - \theta_j (> 0)$ as well as the value of $\frac{\tilde{\theta}}{1-\alpha}$. If it really is far larger than these values, by equation (40),

$$IBR_{Rent,j} \cong 1 \ .$$

That is, the income tax rate for a household that obtains extremely high rent incomes should be nearly 100% to achieve SH.

4.3 Similarity to negative income tax

Because θ_i and ω_i are negatively correlated, θ_i and labor and capital incomes are also negatively correlated, as inequalities (28) and (30) indicate. This negative correlation can be most simply described by

$$LI_i + CI_i = \frac{\chi}{\theta_i} \quad , \tag{46}$$

where χ is a positive constant. Suppose for simplicity that there is no rent income. Therefore, by equation (26), the amount of the income tax burden for a household in Economy i is

$$\begin{aligned} IBR_i \times (LI_i + CI_i) &= (1-\alpha)\left(1 - \frac{\theta_i}{\tilde{\theta}}\right)(LI_i + CI_i) \\ &= (1-\alpha)\left(LI_i + CI_i - \frac{\chi}{\tilde{\theta}}\right) \ . \end{aligned} \tag{47}$$

Equation (47) clearly indicates that this progressive income tax is equivalent to a negative income tax with a nominal income tax rate of $1-\alpha$ and a tax exemption of $\frac{\chi}{\tilde{\theta}}$. This type of negative income tax was endorsed by Friedman (1962).

Of course, there is no guarantee that equation (46) exactly holds in the real world. Therefore, a progressive income tax is not always and precisely equivalent to a negative income tax. However, because it is highly likely that θ_i and ω_i are negatively correlated, the similarity between the progressive income tax discussed in Section 4.1 and a negative income tax will actually exist to some extent. Hence, the idea of a negative income tax seems to be basically consistent with the concept of SH.

5 RATIONALE FOR A PROGRESSIVE INCOME TAX

5.1 Equality and efficiency

As discussed in the Introduction, the equal sacrifice principle potentially violates Pareto efficiency and probably usually does (Kaplow and Shavell, 2001; da Costa and Pereira, 2014). SH with an appropriate progressive income tax is Pareto efficient because all of the optimality conditions of all heterogeneous households are indefinitely satisfied at SH, and SH is the only such state in a heterogeneous population. In addition, SH transfers and a policy expense tax at SH are consistent with Pareto efficiency, and the policy expense tax x_t is identical to a lump-sum tax if ω_i is identical across households. From the point of view of economic efficiency, therefore, the rationale for a progressive income tax based on the concept of SH is different from the rationales for the ATP and equal sacrifice principles.

The reason for this difference is that the ATP and equal sacrifice principles have to meet both economic efficiency and equal sacrifice (more broadly, economic equality) criteria, whereas SH requires only the criterion of economic efficiency. Because the economic equality and efficiency criteria are basically independent from each other, there is no guarantee that both can be satisfied simultaneously. Economic equality and efficiency may, in fact, be incompatible and contradictory. If that is true, when the equal sacrifice criterion is satisfied, Pareto efficiency will be always violated.

On the other hand, SH says nothing *a priori* about economic equality and only requires efficiency. Hence, Pareto efficiency can be always satisfied. Nevertheless, because all heterogeneous households are equally "happy" indefinitely at SH, we may interpret that if SH is achieved, economic equality is also achieved. Of course, whether this interpretation is justifiable depends on the definition of economic equality.

In any case, it will be true that SH expresses some kind of normative standard because SH is the only state at which all heterogeneous households are equally "happy" indefinitely. It seems highly likely that many people would agree that such a state is socially desirable. On the other hand, whether economic equality is socially favorable depends on how economic equality is defined. For example, if economic equality is defined as indicating the state at which all households obtain completely identical incomes regardless of any other circumstance, many people might consider this kind of economic equality to be socially unfavorable. In this sense, SH may express a clearer and more widely agreeable normative standard than economic equality.

5.2 Alternative rationale for progressive tax

In most industrialized countries, income taxes are actually progressive, which means that a majority of people appear to agree that a progressive income tax is socially beneficial. However, as discussed in the Introduction, the ATP principle does not necessarily clearly provide a reason for why progressiveness is beneficial for society. The benefits for society are generally explained by the *ad hoc* introduction of the equal sacrifice criterion.

On the other hand, SH with appropriate government interventions inevitably necessitates a progressive income tax, as shown in Section 4. If all households behave unilaterally, SH can be achieved only through the appropriate use of progressive taxes. Achieving and maintaining SH therefore clearly provide a strong rationale for the necessity of progressive taxes. Hence, without relying on the ATP principle, we can justify a progressive tax on the basis of the concept of SH.

5.3 The benefit principle of taxation

According to the benefit principle of taxation, taxes should be paid by people who benefit from government activities, and the more they benefit, the more taxes they should pay. SH transfers are clearly irrelevant to the benefit principle, but the policy expense tax may be relevant to it because the policy expense tax is levied on people to cover costs for implementing measures for various policy objectives, and these costs can be seen collectively as corresponding to the benefits people receive.

However, the policy expense tax is in essence irrelevant to the benefit principle because, as shown in Section 3, a household is burdened with the policy expense tax in proportion to its productivity regardless to the amount of benefits it receives. Collectively or as an aggregated value, the total amount of policy expense tax may correspond to the combined benefits that all people receive, but there is no one-to-one correspondence between the amount of policy expense tax a person pays and the benefits that person receives. Even if a household receives few benefits, it has to pay a larger amount of policy expense tax if its productivity is higher. Therefore, the results of Section 3 indicate that the benefit principle is meaningful only if the policy expense tax is considered collectively or as an aggregated value.

The basic or ultimate principle underlying tax burdens is therefore neither the ATP nor the benefit principle. SH solely determines how households are burdened with taxes except for some local or objective taxes. A household has to pay taxes, and furthermore, a richer household has to pay more taxes progressively, to achieve SH because SH is the economically most efficient state in a heterogeneous population. In addition, it is the only state at which all heterogeneous households are equally "happy" indefinitely. Therefore, achieving SH is highly likely to be regarded as significantly important and favorable for a society with a heterogeneous population.

5.4 The base of a nation's tax system

As shown in Section 3, no matter how unevenly the policy expense tax x_t is nominally imposed on households, its burdens are eventually shared by all households in proportion to their productivities (ω_i) at SH. Therefore, the burdens of the policy expense tax x_t are similar to those of the VAT discussed in Section 4. On the other hand, SH transfers inevitably necessitate the property of progressiveness, particularly a progressive income tax, as shown in Section 4. Because government expenditures eventually can be covered almost exclusively by the policy expense tax and SH transfers even though a government can rely on borrowings for a while, the combination of a progressive income tax and VAT can be seen as the base of a nation's tax system.

Of course, many other types of taxes can be introduced. Some taxes complement the VAT and progressive income tax (e.g., corporate taxes and inheritance taxes), whereas others are used as tools to achieve specific policy objectives (e.g., environmental taxes and a tobacco tax).

6 CONCLUDING REMARKS

Progressive income taxes have usually been justified on the basis of the ATP and equal sacrifice principles, but how ATP and sacrifice should be measured is still unsettled. In addition, the equal sacrifice principle potentially violates Pareto efficiency and probably usually does.

In this paper, I present an alternative rationale for progressive taxes on the basis of the concept of SH. A government has to transfer money or other economic resources from relatively more-advantaged households to less-advantaged households to achieve SH. Taxes are used as a means to implement these government transfers, and I showed that income taxes should be progressive to achieve SH. This result is unchanged even if a separate policy expense tax is also considered. Progressive taxes therefore can be justified without the ATP and equal sacrifice principles.

In addition, the results in Section 3 indicate that the benefit principle is relevant only collectively (in the aggregate) to the policy expense tax, and therefore it cannot provide any useful information about how much policy expense tax burden each household should bear. Because both the ATP and benefit principles do not seem to provide sufficiently useful information about the allocation of tax burdens among households, it seems highly likely that SH determines each household's tax burden and that the basic or ultimate principle for taxation is neither the ATP nor benefit principle but the principle of achieving SH.

Progressive income taxes to achieve SH have a similar nature to a negative income tax because it is highly likely that RTPs and productivities are negatively correlated. On the other hand, the burdens of the policy expense tax are similar in nature to the VAT because these burdens should be shared among households in proportion to their productivities, incomes, and approximate amounts of consumption. In this sense, the combination of a progressive income tax and VAT can be seen as the base of a nation's tax system.

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