Fuzzy DEA models for sports data analysis: The evaluation of the relative performances of professional (virtual) football teams

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Abstract

The measurement of sports performances both of individual athletes and of an entire sports team, now highly widespread thanks to the enormous availability of sports data, is a crucial moment for professional sports clubs as the their survival is increasingly linked both to the results in the field obtained by its athletes and/or the team/s and to the achievement of many other sporting objectives. We here propose the use of the DEA methodology adapted to fuzzy logic to measure relative performances in the presence of uncertainty of a virtual sample of professional football teams along two dimensions: efficiency and effectiveness. The results obtained are especially interesting from the point of view of policy indications for the organization and management of the teams on the soccer pitch. The work then develops a second stage analysis structured in order to investigate on the one hand with the help of an econometric model the influence that a set of external factors can have on the performances and on the other, by calculating the gini coefficient, evaluates for various attitudes on the part of managers on uncertainty the degree of inequality in the distribution of sports performances of the groups that have participated in an ideal tournament. In conclusion, the work aims to develop, to our knowledge, an innovative and original way for the reference literature, a framework for analyzing sports data (and in particular for professional football clubs) in order to provide policy indications for improve their sports performances.

Keywords: relative performance, sports data, fuzzy logic, fuzzy DEA.

JEL classification: C44, C55,D81,L25

1.Introduction

Data Envelopment Analysis (DEA) (Cooper, Seiford, & Tone, 2007) is a benchmarking methodology (Bogetoft & Otto, 2019) that has developed within operational research and management sciences. Its application is vast and covers many areas (for a review of the literature on its application see for example (Liu, Lu, Lu, & Lin, 2013) (Emrouznejad, A., & Yang, 2017) (Emrouznejad, Parker, B., & Tavares, 2008)). With reference to sport (Liu, Lu, Lu, & Lin, 2013) in their survey they found that up to 2013 the number of papers that involved the application of the DEA in the sport sector were only 31 (0.99% of the total). (Bhat, H., Sultana, & Dar, 2019) found significant number of papers that have been published and that determine the athletic/economic/managerial efficiency by DEA in various sports; baseball, basketball, cricket, cycling, football, golf, handball, olympics, and tennis (e.g. (Lewis, Lock, & Sexton, 2009)). For football, the latter authors list 28 papers, none of which, however, considered fuzzy DEA models to evaluate the sporting performance of professional football clubs. To understand how important it is in general to measure performance in sports, it is enough to remember, for example, that professional football clubs all over the world try to grab the best talents, the best coaches and the best managers not only to make a show on the pitch (which is the elective purpose of a football match) but also and above all to win and consequently attract even more spectators and fans and achieve various corporate goals. If football clubs had unlimited resources, they could always buy the best talents and hire the best coaches and managers in order to always have the best and most competitive team in the different tournaments they participate in (and this is true for all sports clubs like those basketball, baseball etc. etc.). We know instead that due to the imposition of the budget constraint and the rules of fair play[1] for participation in the various European championships, professional football clubs cannot spend indefinitely but only a part of their revenues which also depend on sports performances (read notes 2 and 5). Better performances in the field increase the probability of victory and therefore the probability of qualifying for the different championships[2]or even the probability of winning

[1] (D'angelo, 2018) shows a study on this issue for Italian football teams.

[2] For example, the qualification for the Champions League for the 2018/2019 season the overall prize was worth € 1.95 billion with the following distribution criteria:

- 25% will be allocated to the starting fees (488 million euros);
- 30% will be allocated to the fixed amounts relating to services (585 million euros);
- 30% will be awarded on the basis of the ten-year performance coefficient rankings (585 million euros), the so-called historical ranking;
- 15% will be allocated to variable amounts (market pool) (292 million euros).

And performance bonuses for the Italian teams were: € 2.7 million for a win and € 900,000 for a draw. Clubs that qualify for the knockout stage will receive the following contributions:• qualification for the round of 16: € 9.5 million per club• qualification for the quarter-finals: € 10.5 million per club• qualification for the semi-finals: € 12 million per club• qualification for the final: € 15 million per club• The winner of the UEFA Champions League final will receive an additional € 4m. (source https://www.calcioefinanza.it/2018/08/22/premi-champions-league-2018-2019-italiane/)

them and achieving so many other goals. It is therefore evident the close connection between the team's sporting performances and the company's economic-financial performance. In particular, the management of technical/tactical performance of the soccer team on the pitch becomes a crucial moment for the management of the entire business of professional football clubs. Measuring them, however, requires the analysis of the sports data available. We remember that from an organisational point of view, footballers (the main resources for football club) are deployed on the pitch according to their physical, technical and psychological characteristics, their skills and talents[3] in order to achieve the best results on the pitch (for example, more goals and more shots on goal in attack and fewer fouls in the defensive phase etc.). A first dimension of technical/tactical performance referred to in the dedicated literature is operational efficiency while a second (i.e. sporty performance) is operational effectiveness (García-Sánchez, 2007). In order to measure these two dimensions for professional football teams (see note 10), the DEA methodology was used (as for example happens in (Barros & Garcia-del-Barrio, 2011) and (García-Sánchez, 2007) to name a few). In this paper, based on virtual sports data and taking a cue from the official statistics released by the leagues[4] football but in particular from the literature on the subject we will propose some "game" and "goal" models for football teams. The first model helps us to represent and describe in a schematic and simplified way the development of the football game in its essence (see Figure 1) by soccer teams the second the target objectives of a professional football club. The economic-theoretical support of those models that we will use is the production technology of results pursuit to the soccer team in the soccer pitch (see Appendix A) and a second of the objectives for the football clubs. Instead, to measure sports performance, we will solve the DEA fuzzy models for a sample of 20 virtual soccer teams using fuzzy logic (Zadeh L. , 1975) as a way to take into account the uncertainty both in sports data and in tactical / technical performance on the soccer pitch (a performance, for example, can be subject to execution errors and therefore can be uncertain) than in achieving sporting objectives. The paper describes the game of football during attack phase as a series of different types of passes (game model 1) and occupation of the field spaces (game model 2) both aimed at achieving the same field results (goals and shots on goal) (see right and left scheme in Figure 2). We will model these two aspect of the game of football as two separate economic technologies to represent producing results in the field. This separation, as we shall will see, allow us to separate policy indications for the football team from the point of view of its organization on the pitch. It should be noted that, in our opinion, those proposed here are just some of the possible production technologies for a professional football team. Work objective is therefore twofold: on the one hand, contributing to the literature on the measurement of the relative performance of professional football teams dealt with with the DEA methodology which in our opinion is still underdeveloped both from a geographical point of view (for Italy for example there is only one work and is of a comparative type (Bosca, Liem, Martinez, & Sala, 2009)) and which from a methodological point of view (for example only 2 papers use Data Envelopement Analysis with uncertainty (Barros & Garcia-del-Barrio, 2011) (Barros, Assaf, & Sá-Earp, 2010)) and on the other hand to offer an analysis framework of sports data useful for providing policy indications to improve sports performance for professional football teams and clubs (Barros & Garcia-del-Barrio, 2011). To our knowledge no work that applied the DEA to professional football clubs has considered DEA fuzzy models although some works have dealt with uncertainty as in (Barros, Assaf, & Sá-Earp, 2010), (Halkos & Tzeremes, 2011). Another purpose of the work is to show the actors interested in the performance measurements of professional football teams the application potential of our approach to different sports data. As we will see from the results of the fuzzy DEA models, it is possible to draw with a few of them a wide range of results to be interpreted and an equally wide range of policy indications. For example, a question that can be answered with the results of the fuzzy DEA models is: with the current level of operational efficiency of the collective in the pessimistic scenario (i.e. with the current organization and deployment of the team in the field) what was the degree of operational effectiveness of the club in achieving the ranking objectives defined in a blurred manner by the managers of the company? (as we will see this type of question is answered by the models of the "objective" type). As we have specified above, the improvement of game performances on the pitch (operational efficiency) is not pursued merely to obtain a more pleasant show, a dimension as fundamental as the others to attract more viewers and fans and increase revenues (an aspect relating to the economic dimension)), but it is an essential prerogative for the economic and financial survival of the club itself in the expected time that a better performance on the field increases the probability of victory and probably the probability of achieving the objectives and therefore of qualification for the respective championships to which the club participates (operational effectiveness). This is aimed at increasing revenues[5] (economic performance) and improve financial

[3] To give examples we can give three different examples of attackers: 1) the *creative shoter* is the creative attacker who combines dribbling with shots on goal, 2) the *penalty-box stricker* are the attackers from the penalty area, not very involved in the game and with little creativity, 3) the *target men* instead are the tips who prefer aerial play, and they too are not very creative and with a reduced contribution in terms of shots. We can associate specific indicators of individual performance to each of these types of attackers. Also for midfielders and defenders different types can be identified. (Gurpinar-Morgan, 2015).

[4] For example bundesliga (https://www.bundesliga.com/en/stats/bundesliga), Spanish league (https://www.laliga.com/estadisticas), the Italian Serie A (http://www.legaseriea.it/it/serie-a/statistiche-squadre).

[5] The revenues to which we refer are for example the prizes that the Leagues distribute in the event of qualifications, or to a possible increase in the value of the squad or of the individual player, deriving from a constant technical-sporting improvement, which can be

performance (eg debt reduction) as well as change the price of individual players, giving the company the possibility of greater capital gains from disposals and so on. In essence, the evaluation of sports performances is a crucial moment in the management of the business in the professional football sector (and in many other sports sectors) linked to other results and influenced by external factors. The paper is organized as follows: in section 2 we offer a review of the literature on the application of the DEA to professional football clubs, in section 3 we present the data (3.1), the DEA models of "game" and "target objectives" (3.2) and the structuring of a second stage analysis (3.3), in section 4 we present the results of the DEA (4.1) and second stage (4.2 and 4.3) models and finally in section 5 and 6 the discussion and conclusions, respectively .

2. Review of literature

DEA (Cooper, Seiford, & Tone, 2007) it has been applied in various ways to evaluate the performance of football clubs. (Barros & Garcia-del-Barrio, 2011) analyses efficiency drivers of a representative sample of Spanish football clubs by means of the two-stage data envelopment analysis (DEA) procedure proposed by (Simar & Wilson, 2007). In the first stage, the technical efficiency of football clubs is estimated using a bootstrapped DEA model in order to establish which of them are the most efficient; the ranking is based on total productivity in the period 1996–2004. In the second stage, the (Simar & Wilson, 2007) procedure is used to bootstrap the DEA scores with a truncated bootstrapped regression. Policy implications of the main findings are also considered. (Barros & Leach, 2006) uses data envelopment analysis (DEA) to evaluate the performance of English Premier League football clubs from 1998/99 to 2002/03 combining sport and financial variables. (Barros, Assaf, & Sá-Earp, 2010) introduces a two stage bootstrapped DEA model to analyze the technical efficiency of Brazilian first league football clubs over the period 2006-2007. In the first stage a bootstrapped DEA model is used to derive the efficiency scores and in the second stage, the determinants of technical efficiency are identified using a bootstrapped truncated regression. The outputs of their DEA model are: (i) number of attendance (ii) total receipts in thousand real and (iii) points in a league, while inputs are: (iv) operational cost (excluding labour costs) in thousand reals (v) total assets in thousand. and (vi) team payroll in thousand reals. In this way the authors combine sport and financial variables in estimating technical efficiency. In the second stage the author use (i) the number of victories in the league; (ii) the number of defeats in the league; (iii) the number of goals scored; (iv) the number of goals suffered. (Bosca, Liem, Martinez, & Sala, 2009) analyses technical efficiency of Italian and Spanish football during three seasons (2000/2001, 2001/2002, 2002/2003). The objective of their paper is to analyse whether the analysis of technical efficiency is of any help in shedding light on the sport but not the financial performance. According to the authors their approach can be of help to football coaches (and eventually to the more technically minded media people) in understanding how the technical efficiency of football clubs in different operations (say, defensive and offensive) and in different situations (home and away) is related to the sport performance of these clubs (basically, the number of points obtained along a season). At this end the offensive model consider in each game the following outputs and inputs: 1) goal scored (output), 2) shots on goals made by the team (input), 3) attacking play made by the team (input), 4) balls kicked into the opposing team's centre area (input) and 5) possession (input measured in min). For the defensive model the authors consider the following input and outputs variables: 1) goals conceded (output), 2) the inverse of shots-at-goal made by opposing team (input), 3) inverse of attacking plays made by opposing team in the goal area (input), 4) the inverse of passes to centre area made by opposing team, (input) and 5) the inverse of minutes of possession by opposing team (input). The authors use these variables as proxies to capture the attacking and defending capabilities of teams. (García-Sánchez, 2007) apply a three-stage-DEA model to the Spanish Professional Football League during the 2004/2005. In his paper the author separating the teams' economic behaviour into three components: operating efficiency—of the offence and defence—athletic or operating effectiveness, and social effectiveness. According to the author the advantage of this analysis is that it enables us to identify the strong and weak points of each team with respect to the tactical skills of its offence and defence, its effectiveness or precision of play, as well as its social relevance, which is strongly connected to the creation of present and future revenues. Thus, in the first stage the players' talent is used to produce two throughputs, one that is positive (goals scored) to be maximized and another that is negative (goals let in) to be minimized, which independently determine the efficiency of the offence and defence of a football team. Both efficiencies are introduced into the second stage as inputs to produce the final output, the points obtained by a team at the end of the season, evaluating the importance of each style of play — by the offence or defence — in the athletic or operating effectiveness. Finally, in the third stage, which is the result of the previous step, the athletic effectiveness is introduced as input, considering the number of spectators for the whole season as the outcome achieved in that season, enabling us to determine the social effectiveness.. (Halkos & Tzeremes, 2011) applies a probabilistic approach to investigate how the top European football clubs' current value and debt levels influence their performance. The composite output contains the sum of the number of European champions' cups (weighted by 5), UEFA cups/ Euroleague cups (weighted by 4), European cup winners' cups (weighted by 3), Intercontinental cups

transformed into a capital gain in in the event of sale, from the possible increase in revenues from TV rights or at the box office due to a greater number of audiences who decided to follow a more beautiful show and so on.

(weighted by 3) and FIFA Club World cups (weighted by 3). In addition the composite output contains also the sum of the number of domestic championships (weighted by 2) and domestic cups (weighted by 1). Both the number of the weighted domestic champions and domestic cups (includes all domestic cups, i.e. super cups, league cups, national cups, etc) are again weighted by FIFA world ranking score (FIFA, 2010). This extra weight has been added in order to reflect the different difficulty levels of obtaining a domestic cup and/ or championship among the different European leagues1. The authors assume that club revenues are used from the clubs in order to buy the best (in term of football quality) possible managers and players which can lead to team success (based on world, European and domestic championships and cups). (Espitia-Escuer & García-Cebrián, Comparison of efficiency measures for Spanish first division football teams using data envelopment and stochastic frontier analyses, 2014) analyze of the efficiency of Spanish First Division football teams during the seasons 98/99, 99/00, 00/01, 01/02, 02/03, 03/04, 04/05, 05/06, 06/07, 07/08, 08/09, 09/10 with DEA and Stochastic Frontier (S.F.). The variables of the resources used consists of the number of players used throughout the season, the attacking moves made, the minutes of possession and shots made, meanwhile the outputs generated consist in the number of points achieved during the season. (Haas D. , 2003) taking data for the year 2000 season estimate the technical efficiency of soccer teams in the Major League Soccer. The DEA inputs used by the teams are approached by the variables players' wage bill and wage of the head coach. Output is measured by points awarded, absolute number of spectators, and revenues. (Haas, Kocker, & Sutter, 2004) estimates CCR + BCC model using as inputs: 1) player's wage bill and 2) coach salary, while as outputs: 1) points awarded during the season, 2) total revenue and 3) average stadium utilization. (Espitia-Escuer & Garcıa-Cebrian, 2010) measure the efficiency of football teams in the Champions League from 2003 to 2007 and the method of calculating the efficiency will be both the traditional version of the DEA as well as the version proposed by Andersen and Petersen (1993), which allows discrimination among efficient units, using as inputs: 1) Attacking moves players used, 2) minutes of ball possession, 3) number of players and as outputs adopot: 1) games played and 2) goals attempt. (Espitia-Escuer & García-Cebrián, 2006) CCR model in professional soccer leage using as inputs: 1) Attacking moves players used, 2) minutes of ball possession and 3) shots at goal and points achieved throughout the season as output. (Espitia-Escuer & García-Cebrián, 2004) estimate for spanish first division team CCR + BCC model using as inputs: 1) Attacking moves players used, 2) minutes of ball possession, 3) shots and headers made. As outputs adopot: 1) points achieved and 2) goals scored (Zambom-Ferraresi, Lera-lópez, & Iráizoz, 2017) analyse the efficiency of English Premier League (EPL) clubs during three seasons (2012/13–2014/15). The methodologies employed are data envelopment analysis (DEA) and a bootstrapped DEA model using as input the ex ante measure of all squad market value, and as ouput: 1) Points achieved in the current season, 2) total revenue at the end of the season, 3) Percentage stadium capacity utilization during the entire season and 4) Index of social media impact of British clubs. (Villa & Lozano, 2016) applied network DEA model to 380 matches played in season 2013/14 by the 20 teams of the Spanish first division league to assess their scoring efficiency taking into account the offense and defense actions of the two teams. They used info on ball possession, shots at goal, corner kicks, penalty kicks, goalkeeper's saves, turnovers, steals executed and economic value of the team and total goals scored. The present paper brings you very close to that of (Bosca, Liem, Martinez, & Sala, 2009) for the choice of inputs and outputs of game models (models 1 and 2) and to the work of (García-Sánchez, 2007) for the dimensions of the performances considered.

3.Method

In this section we will present: 1) data and variables (3.1), 2) two models for the analysis of sports performance of professional football clubs (3.2), 3) DEA fuzzy models (3.3) and 4) a second stage econometric model (3.4). A first type of model that we propose here to be used with sports data relating to professional football clubs in order to measure the sporting performance of their soccer team refers to the relative technical efficiency or otherwise said to the operational efficiency (García-Sánchez, 2007). In this model, a performance indicator refers to the relationship between the results on the soccer pitch (shots taken and goals scored) achieved by the soccer team and the level of technical-tactical performance of it represented by the number of different types of passes (model 1) and a another indicator that instead adopts the percentages of use of field spaces (model 2). The two models refer to two separate sports production technologies. The first technology has as production results the goals and shots on target for the opponent (model output) and as input the different types of passes (see Appendix A). The second technology, on the other hand, always results in goals and shots on goal in the opponent's goal but instead uses the percentages of exploitation of the right, left and central areas of the field as inputs. A second type of model proposed here instead focuses on measuring sports effectiveness (García-Sánchez, 2007) of the entire club (see note 10). The working hypothesis is that the sporting goals that the managers of the company pursue, such as the ranking goals, are obviously linked to the organization and strategies put in place for the soccer team in order to achieve a certain result (operational efficiency (García-Sánchez, 2007)). To measure operational effectiveness (García-Sánchez, 2007) instead we define a technology for producing goals to be attributed to the entire club (see note 10) rather than just the collective deployed on the soccer pitch. For us, the ranking goals of a soccer team is the result of the efforts not only of the footballplayers but also of the efforts of the athletic trainers, doctors, physiotherapists etc. etc. A crucial operational assumption concerns the time horizon in which performances are measured. In fact, assuming a short-term perspective, we believe that the dimensional operational scale of these two technologies can be considered as fixed and non-modifiable, therefore we will show the results of the DEA models under the assumption of constant returns to scale (our interpretation of the returns to scale can be founded in Appendix B). When, on the other hand, a longer-term horizon is assumed, with the obvious consequence of being able to consider all the variable resources, then the dimensional operational scale of a professional football team (club) can be modified to obtain economies (efficiency) of scale. In the latter case, then the most useful DEA models are those with variable scale assumptions. All DEA models we will solve them with simulated sports data. We will use fuzzy logic (Zadeh L. , 1975) instead to treat uncertainty in sports data so that we can treat the related variables as fuzzy numbers[6]. Finally, we will solve the fuzzy DEA models with two different approaches: 1) the one proposed in (Kao & Liu, 2000) and 2) the one proposed in (Léon, Liern, & Sirvent, 2003). The first will allow us to express the measurements in two different scenarios, pessimistic and optimistic, while the second will allow us to obtain possible measurements of the two types of performance (efficiency and operational effectiveness (García-Sánchez, 2007)).

3.1 Data and variables

The input and output variables of the DEA models have been designed taking inspiration from the official statistics on football teams that numerous specialized sites (see Note 4) offer to the general public but above all taking inspiration from the dedicated literature. In Table 1 we report the descriptive statistics of the variables to which exact (ideally sports) data generated with causal data generators have been assigned (RCoreTeam, 2016).

Table 1 Descriptive statistics of precise virtual (sports) input and output data (before fuzzy transformation).

Stat	Cross	Long Passes	Short Passes	Filtering	Right Side	Lato Sx	Central	Goals Scored	Shots Made	Ranking Points	Max Score
Min	12	36	147	1	32	35	11	20	10	17	114
Max	32	71	550	5	42	47	31	126	32	90	114
Range	20	35	403	4	10	12	20	106	22	73	0
Sum	404	1106	7062	61	724	833	443	1441	453	1033	2280
Median	19.5	55.5	356	3	36	43	23	68	24.5	43.5	114
Mean	20.2	55.3	353.1	3.05	36.2	41.65	22.15	72.05	22.65	51.65	114
Se.Mean	1.374	2.343	23.765	0.235	0.742	0.901	1.255	5.742	1.373	4.154	0
Ci.Mean	2.875	4.904	49.74	0.491	1.553	1.886	2.627	12.018	2.874	8.695	0
Var.	37.747	109.8	11295.36	1.103	11.011	16.239	31.503	659.418	37.713	345.187	0
Std.Var	6.144	10.479	106.28	1.05	3.318	4.03	5.613	25.679	6.141	18.579	0
Coef.Var	0.304	0.189	0.301	0.344	0.092	0.097	0.253	0.356	0.271	0.36	0

The data in Table 1 show that for our virtual sample of 20 professional football teams the average number of crosses per match was 20.2, with an average of 55.5 long passes, 356 short passes, 3 through balls and on average the left wing. of the field was used for 41.65%, the right one for 36.2 and the central one for 23%. The results resulting from this average level of play were 72.05 goals scored, 22.65 shots per game, while the average goal of the table was 51.65 points and the maximum of 114. The average range for cross matches was 20. with a minimum of crosses per match of 12 and a maximum of 32, while the number of cross matches that divided the champion in half was equal to 19.5. In other words, 10 teams made less than 19.5 crosses per match and the other ten more than 19.5 crosses per match. In total, all teams made 404 crosses per game. The reading of the data is similar for all the remaining variables of Table 1. Special attention deserves the variable called by us "max score" which has equal values for the minimum, maximum, average and median, zero for all other statistics while the sum does not make much sense. In the case of DEA fuzzy models the exact input and output (sports) data presented in Table 1 using fuzzy logic (Zadeh L. , 1975) were transformed into fuzzy numbers of symmetrical triangular type for most of them[7]. An example of a fuzzy number of a symmetrical triangular type with membership function not decreasing on the left and non-increasing on the right is the one shown below in Figure 1.

Insert Figure 1 Symmetrical triangular fuzzy number (the example of the Cross variable for team 1)

[6] By specialized sites we mean for example the one of the various Leagues where you can find statistics such as the average goals scored in a match in a football season etc. We were inspired in part by these statistics.

[7] Fuzzy numbers (Zadeh, 1965) can be mainly of three types: 1) trapezoidal, 2) symmetrical triangular and 3) asymmetrical triangular.

In the case of the variable in Figure 1 the vagueness (as a mode of expression of the uncertainty) on the variable "Cross" using fuzzy logic can be read as follows: the central value of the fuzzy variable "cross per game" for Team 1 in a hypothetical football season was 29 and with a symmetrical diffusion coefficient of 3 the lower minimum value is 26 while the upper maximum value is 32. To all other values in the left interval [26,29] and to all other values in the right interval [29,32] a certain degree of belonging to the "cross per game" class is attributed, decided with the membership function $\mu_{cross} \geq 0 \ \rightarrow [0,1]$ and which varies if a level is also adopted α-cut . So for example in the case of a cut $\alpha = 0$ the value 27 will belong to the fuzzy set "cross matches for team1" with a certain degree $\mu_{crosstteam1} > 0$, while with a cut $\alpha = 0.5$ the value 27 belongs to that same fuzzy set with degree $\mu_{crossteam1} = 0$, that is, it does not belong to that whole. The central value 29 always has a degree of belonging equal to 1 (Zadeh, 1975) (Zadeh, 1978).The fuzzy logic (Zadeh, 1975) for this variable leads to statements such as "on average in the football season considered the average number of crosses per match of Team 1 was more or less 29", or "on average in the football season considered the Average number of Team 1 crosses per match was around 29 ”. And using in particular the interpretation given to the membership function by (Zadeh, 1978), the degree of membership is interpreted as the ease with which, for example, team 1 has made a certain number of crosses on average. So, given the particular shape assigned to the membership function in Figure 1, it was very easy ($\mu_{crossteam1} = 1$) making an average of 29 crosses per game while it was less easy ($\mu_{crossteam2} < 1$) on average realize 26 or 31. Starting from this very brief clarification on fuzzy numbers and fuzzy logic as a way of accommodating uncertainty through vague linguistic categories (Zadeh, 1975) all the other fuzzy variables with their dispersion coefficients are reported in Table 2 below.

Tabella 2 Fuzzy data set

Teams	Cross	Long Passages	Short Passages	Filtering	Right Band	Left Band	Central Band	Goals Scored	Shots Made	Ranking Points	Max Score	Alfa1	Alfa2	Alfa3	Alfa4	Alfa5	Alfa6	Alfa7	Beta1	Beta2
1	29	65	490	3	35	35	30	86	25	90	114	3	1	8	1	6	3	7	0	3
2	12	63	254	3	34	35	31	75	25	79	114	3	1	7	1	7	4	6	0	3
3	15	71	305	4	37	39	24	76	20	73	114	5	1	5	2	8	2	5	0	2
4	16	36	179	4	38	35	27	63	13	69	114	2	2	9	1	6	3	6	0	3
5	13	71	292	3	35	42	23	50	23	68	114	5	2	9	2	7	4	5	0	4
6	13	56	291	4	32	43	25	65	10	66	114	3	1	8	1	5	3	6	0	3
7	22	64	506	4	33	42	25	64	28	63	114	5	1	9	2	6	4	5	0	2
8	17	62	413	4	33	45	22	66	28	59	114	3	2	8	2	7	8	7	0	3
9	19	41	352	4	32	43	25	20	27	53	114	4	3	9	2	6	6	6	0	2
10	29	50	455	4	39	44	17	93	22	44	114	3	1	8	1	7	4	6	0	3
11	20	43	308	3	41	45	14	126	18	43	114	3	2	7	2	8	3	7	0	2
12	32	63	339	2	41	47	12	55	24	43	114	5	3	8	1	7	4	8	0	4
13	29	55	385	2	42	47	11	94	26	42	114	3	3	9	2	5	5	9	0	5
14	20	53	550	2	34	46	20	82	25	41	114	3	3	8	1	8	4	8	0	6
15	27	49	429	1	37	43	20	101	15	41	114	7	2	7	2	7	3	7	0	5
16	20	47	399	2	32	45	23	57	32	41	114	3	2	8	1	6	6	8	0	4
17	14	67	147	2	33	39	28	110	12	38	114	3	2	9	1	9	5	6	0	5
18	16	40	379	2	38	36	26	27	29	38	114	3	1	10	2	8	6	6	0	6

5	6
0	0
4	5
3	4
7	6
1	1
9	8
1	1
3	5
114	114
25	17
28	23
61	70
23	17
39	43
38	40
3	5
360	229
52	58
18	23
19	20

And so, for example, the variable "long passes" for team 1 was assigned a central value of 65 and a symmetrical dispersion coefficient $alpha2$ =1 , the output variable "shots made" for team 1 was assigned a central tendency of 25 and a symmetrical dispersion coefficient of 3 (beta 2). The variables "goals scored" and "maximum score" were assigned exact values (except for the maximum scores in models 5 and 6). The values of the dispersion coefficients for these variables were not reported in Table 2[8]. The measurement of performances with DEA fuzzy models then requires that for the example of the cross variable for Team 1 the interpretation of the results, in a very general way, becomes for example: considering that Team 1 on average in each game in season X he performed 29 crosses more easily, that is a number of crosses that were positioned in the range between 26 and 32, what was his technical efficiency with the same result on the pitch relative to what the other teams did? And so for the output variables the reading of the measurements for Team 1 is in general the following: considering that on average in each game in season X Team 1 made more or less 25 shots on goal, [25-a, 25 + a], what was your technical efficiency given your level of play? And so on. These interpretations are then declined within the single fuzzy approach used to solve the DEA models.

3.2 The "game" models and "target objectives" models

On the soccer pitch there are two football teams, each of which mainly performs the function of attack and defense. In the attack phase, the game takes place through a series of different types of passes between the players of the same team (and dribbling to the opposing players or by stealing the ball from the opponent who is defending or attacking) who, occupying the different parts of the field (right, left and central), try to hold the ball gain ground and arrive without losing the ball to make as many shots and goals as possible (mainly head and foot) in the opponent's network (summarizing the basic principles of attack are represented by: 1) gaining the field in possession of the ball, 2) look for game solutions towards the opposing goal, 3) look for conditions to finish on the net), all with the opposition of the opposing team, in an alternation between attack and defense (the basic principles of defense are: 1) prevent game solutions towards goal, 2) to take away time and space from the opponent in order to recover the ball, 3) to prevent the conclusion at the net). The players must therefore learn the basic technique (i.e. the basic gestures such as kicking, receiving, guiding the ball, direct tackle, header) and the applied technique (i.e. the technical skills expressed in relation to the demands of the game). The technique applied for the defenders consists of taking a position, marking, interception, indirect tackling, defending the goal while for the attackers it consists in unmarking, passing, receiving, dribbling, shooting. This is the underlying spirit of a football match which can therefore be played at different levels of execution (for example a low level can mean a few of all the different types of passes in a rather large time frame between passes.). Following this basic essence of football played on the pitch the first model we propose here describes the game as a series of different types of passes and the level of play with: 1) the number of long passes in a match, 2) the number of short passes in a match, 3) the number of crosses in a match and 4) the number of throughers in a match. The results of this passing game are goals and shots on target by the opponent. The level of game activity is therefore described with 1) the number of shots on goal in a match and 2) and the number of goals scored in a match. We call this model of the game the “attacking pass level” model. The second game model, always valid during the attack phase, instead considers the use of the different areas of the field, right, left and central, as the development of the game, and always results in goals scored and shots on target by the opponent. We call this game model the "space occupation during attack" game model. These two models read together capture the underlying spirit of how the game takes place on the pitch in the attack phase: through a series of different types of passes, the players of a team using basic and applied techniques tactically occupy the different areas of the game. field in the attack phase in order to make the most shots on goal and the most goals (see Figure 2)

Insert Figure 2 Football

In Figure 2 on the left player A of the blue team (positioned in the left side of the field) sets an attack action by passing the ball to player B (positioned in the central part) with a short pass (4), player B with a long pass (3) reaches his teammate C (positioned in the left area of the field) continuing to develop the attack phase using precisely the left area of the field (he could have developed a different vision of the game and reached other his teammates positioned or in the center or in the right area). Player C in turn with a short pass (4) reaches his teammate D who is positioned in the central area of the field. In turn, player D with a through ball (2) between the players of the opposing team (K and L)

[8] If we had also treated these variables like the others, i.e. as fuzzy variables, Table 2 would have reported two other beta, beta 3 and beta 4.

continues to develop the attack phase by turning to a teammate positioned in the central area of the field (E). Player E, once player J of the opposing team has dribbled in the center of the pitch (unless he does not lose the ball, in an attempt to reach his teammate G with a long pass, because for example he was intercepted by a player of the opposing team J as shown in the figure on the right of Figure 1a), with another long pass (3) reaches player G located in the right area of the field. G can reach his team-mate F with a cross (1) that scores a goal, or reach H with a short pass that shoots the opposing team on goal without scoring. Schematically, our game models thus describe the development of the game on the pitch during the attack phase. Solved as DEA fuzzy models and following (García-Sánchez, 2007) these two "game" models lend themselves to measuring the technical efficiency of the team's tactical / technical performance in achieving the results of their own game activity in the attack phase (approximated with shots on goal and sawing goals). The scheme in Figure 2 is repeated several times in a macth and therefore with different combinations of players the crosses, throughers, long and short passes will be repeated several times but they will not always be able to reach the recipients (players of the same team) or because they are out aims, either because intercepted by the opponents or by mistake of the receiver. To be performed correctly and to reach teammates these two different technical/tactical performances (passes and game vision) require the players to use their tactical/technical skills and therefore the combined skills of the entire team to perform in repeated and correct steps and the right choice of the type of passage to be performed and an appropriate vision of the game[9]. These two schematic ways of actually representing interconnected performances lend themselves to measuring one of the two dimensions of sports performance on the pitch, that is, technical efficiency. Obviously they will depend on how the team is organized on the pitch in terms of players deployed, each of which has physical characteristics, tactical and strategic skills, talents of their own different from the other players (read note 3). The task of organizing the team on the pitch by drawing from the squad is the responsibility of the coaches. These first two models therefore belong to the type of model that we have defined as "game" and allow the measurement of operational efficiency (García-Sánchez, 2007). The effectiveness in reaching the sporting objective (such as for example a certain positioning in the standings, or qualification for the quarterfinals, semifinals or finals in the European international championships, etc.) is defined by (García-Sánchez, 2007) as the operational effectiveness and for us it is not intrinsically determined in the field (the intrinsic results in the field are goals scored, shots on goal etc.) but it is obviously related to it. Measuring then how effective the team has been in reaching a goal (generally this is set by the team manager) given a certain level of tactical operational efficiency in the field is instead the task assigned to the second type of models that we propose here and which we call “Strategic goals” and refers to the club (the football team in s + other resources). This second type of model proposed here was designed to measure operational effectiveness in achieving sporting objectives such as ranking positioning measured by points in the standings given a certain level of technical efficiency. The higher the team will be in the standings (the more points compared to the opponents) the more operational tactical efficiency has been translated into operational effectiveness. The "objective" models 3 and 4 that we propose here have the scores of efficiency of the previous game models (model 1 and model 2) and as output the positioning in the ranking approximated by the ranking points accumulated by the team. As we have said, however, the achievement of such a sporting goal is to be attributed to the effort of all the members of a professional football club of which the team is the essential element. Therefore we will refer this model to the effectiveness in achieving goals at the club level (see note 10). The inputs and outputs of the corresponding DEA models are shown in Table 3. The DEA models 1 and 2 in Table 3 are therefore the game models while the DEA models 3 and 4 are the objective models. When we move to a fuzzy logic (Zadeh, 1975) also to describe the outputs of models 3 and 4, then we build models 5 and 6 which become the reference models for evaluating operational effectiveness in the event of uncertainty. In these last two models, therefore, the objective is not set exactly by the managers and therefore is not described with exact expressions such as "reach x points in the standings", or "reach the maximum points in the standings" but is described in a blurry and vague with expressions such as: "reach at least x points in the standings", or "reach more or less x points in the standings", or "look for the best position in the standings" etc. Each of these blurry objectives has a specific meaning for each of the teams and we translate this with different fuzzy numbers.

Table 3 DEA models for evaluating the game performance of football teams

	Model 1 (Passage Model)	Model 2 (Position Model)	Model 3 (Efficacy Model)	Model 4 (Efficacy Model)
Inputs				
Cross	X			
Filtering	X			
Long Passes	X			
Short Passes	X			

[9] By game viewing we mean in which part of the field (right, left, central) the players can direct the different types of passes (crosses, throughs, long passes and cc.) In order to advance more effectively towards the goal of the opposing team. It goes without saying that this is a real technical skills of the team that the coach can best address from the sidelines.

Left Side Connection		X		
Right Side Connection		X		
Attack From The Center		X		
Efficiency Score Mod 1			X	
Efficiency Score Mod 2				X
Outputs				
Shots (Including On Target)	X	X		
Goals Scored	X	X		
Points In The Standings			X	X

Model 5, therefore, is a review of model 3 of the objectives adapted to assess the operational effectiveness of football clubs [10] in case of uncertainty, however, using the scores of game models 1 as input. While model 6, which is the fuzzy version of model 4, unlike 5, uses the efficiency scores of game model 2 as input but with the same output. For these last two models, for example, the maximum score was 114, as happens in the Italian Serie A championship, a blurred goal of the type "good ranking in the standings of at least x points" can be translated with a fuzzy number of points in the standings in the range 70 and 90 below 114, for another team the same goal "good positioning ..." could be that associated with a range of points between 45 and 65 and so on. Another fuzzy objective that can be considered can be described as "at least the maximum score", or "close to the maximum score", or "no further than 30 points from the maximum score of 114" and so on[11]. It goes without saying then that model 5 evaluates the operational effectiveness of clubs in achieving their out-of-focus goal once the field team has been organized in a manner to be presented on a certain level of operational efficiency in passing, while model 6 evaluates the operational effectiveness of a club once the team is organized with a certain level of operational efficiency in the game vision. Game models 1 and 2 were solved by adopting two resolution approaches proposed in the literature for fuzzy DEA models[12]: 1) the possibilistic model proposed by (Léon, et al., 2003) and 2) and the DEA fuzzy model proposed by (Kao, et al., 2000) both implemented and optimized[13] in environment R with the deaR package (Coll-Serrano, et al., 2019). Models 5 and 6, on the other hand, were solved only as models (Kao, et al., 2000) and always with an output orientation ((Léon, et al., 2003) develop some fuzzy versions of the classical DEA models (in particular , the BCC model) by using the possibilistic approach (see footnote 19 and 22). (Kao, et al., 2000) instead adopt mathematical programming combined with the α-level crossing (Hatami-Marbini, et al. , 2011) The α cut approach to solving fuzzy patterns is to transform the fuzzy DEA model into a series of precise DEA models for each α cut level at the fuzzy variables[14]. (Kao & Liu, 2000) they add the construction of two scenarios for each cut α (read notes 25, 27 and 31)). The assumption we make with effectiveness models 3 and 4 is that all uncertainty dominates only the tactical organization of the team deployed on the field and that it has already been discounted in the previous measurement with the DEA fuzzy operational efficiency game models. (models 1 and 2) and that instead the objective to be achieved has been set exactly. At this point we evaluate to what extent the level of tactical operational efficiency has become the goal (operational effectiveness)[15]. To choose instead the measurement guidelines for the DEA fuzzy models, we note that in the game phase the coach / mister can push the players lined up on the field to save energy in order to improve performance (not wasting steps (model 1)) or to improve the vision of the game by better occupying the spaces on the pitch (model 2) that is not to overcrowd the spaces, or even invite his players to make more shots on goal etc. This suggests that the DEA 1 and 2 game models are solved both to minimize the inputs (in our case it means improving both in the number of different

[10] Here emerges the difference between the term "team" intended as the collective that plays on the field, that is, the players only and the term "club" which indicates the sports club with all its professional figures from the president to the technical director, and to continue the director sportsman, the general manager, the coach, the second-in-command coach, the athletic trainer, the football players, the nursery, the sports doctor, the physiotherapist, the observer and the storekeeper.

[11] Recall that all these objectives can be translated into fuzzy numbers such as 1) symmetrical triangular, 2) asymmetrical triangular and 3) trapezoidal

[12] In fuzzy models the expressions "equal to", "less than" and "maximize a function" of the precise DEA models become "at least equal to", "at least less than" and "maximize a fuzzy size". The inequalities between two fuzzy variables can be expressed by inserting a level of possibility defined by the decision maker (Guo & Tanaka, 2001).

[13] For these models, for example, maximizing is different from maximizing in standard DEA models. To give an example if the objective function is a symmetric triangular fuzzy variable of the type Z = (z, w), maximizing it means simultaneously maximizing z-(1-h) w and z + (1-h) w, being able to oscillate from a pessiministic opinion (worst case) to optimize it to an optimistic opinion (best case) to do so as described in (Guo & Tanaka, 2001).

[14] Per a categorization of fuzzy DEA models see for example (Emrouznejad, Tavana, & Hatami-Marbini, 2014), (Karsak, 2008), (Hatami-Marbini, Emrouznejad, & Tavana, 2011).

[15] This goal certainly has value for professional football teams that play in the highest category while for others or for amateurs the main goal should be to be promoted to the next championship and promotion to the next category is not just with victory. of the championship in their own category but the rules of engagement are different and for example in the Italian Serie B championship the first three qualify, also, therefore, with a different score from that maximum. Hence the usefulness of model 3 which considers the points earned in the ranking not the maximum score.

types of passes and in the occupation of the spaces in the field relative to what the other teams are doing) at the same level. of results on the pitch, both to maximize results on the pitch (increase shots on goal and goals scored) with the same level of play expressed and occupation of the spaces on the pitch. In other words, we are measuring the technical efficiency at the input and output of the two technologies underlying the game models 1 and 2. The rational search for the highest sporting effectiveness with a certain objective requires, on the other hand, to maximize the output of models 3 and 4. Model 3 measures how the tactical operational efficiency (do not waste the different types of steps in model 1) has turned into an exact target, and instead model 4 measures how a good capacity to occupy the spaces in the field well (model game 2) has become the same goal (in our case points in the standings) of course. However, the path to reach the ranking goal, like any other goal, is far from being simple and precise and very often its fixation requires considering a certain degree of uncertainty. Therefore, the DEA fuzzy efficacy models with a focus on the score in the ranking are those represented by models 5 and 6. In these last two models, the uncertainty was assumed only on the objectives (the outputs of the model) but not in the inputs. Therefore, the fuzzy output variable for these models takes the form of an asymmetrical triangular number on the left like the one shown in Figure 3 in order to represent the objective according to the blurred logic. The inputs of these models are instead acquired as exact values given by the solutions of game models 1 and 2.

Insert Figure 3 Variable "maximum score" triangular asymmetric

Figure 3 represents another possible way of setting a vague goal and can be read as follows: a score of 60 belongs to the fuzzy set "max score of 114" with some degree of about 0.6 if α = 0, while the score of 80 belongs to this fuzzy set, always using the same alpha cut equal to zeo, with a degree of about 0.8. Therefore only the score of 114 belongs with grade 1 to the fuzzy set "maximum score of 114". The variable "maximum score" was therefore inserted in models 5 and 6 no longer with an exact value but as a fuzzy number similar to that in Figure 2 with a central trend equal to the score in the ranking achieved by each team. In this way, we represented the uncertainty in achieving the full goal of ranking 114. The "maximum score" goal in this way became a blurred and vague goal as required by fuzzy logic (Zadeh, 1975). Models 5 and 6 therefore ultimately contain a certain input of efficiency scores that have already discounted the uncertainty but with a fuzzy interpretation in order to contain the uncertainty[16] and an uncertain output or the fuzzy variable "maximum score" as just described [17].

3.3 Fuzzy DEA models

3.3.1 Fuzzy DEA input model for professional football teams with constant return to scale

Let's assume that we have n = 20 professional football teams participating in a professional football tournament in a given football season. Let's assume that $\tilde{x}_{ij}$(with i inputs) e $\tilde{y}_{kj}$ (with k outputs) represent, respectively, the fuzzy inputs and the fuzzy outputs of the j-th professional football team. In particular, the inputs and outputs are those indicated in Table 3 for models 1 and 2. The fuzzy DEA model with constant returns to scale in the input-oriented version of the envelope will be as follows (Emrouznejad, Tavana, & Hatami-Marbini, 2014):

$$\min \theta_p$$
$$s.t. \sum_{j=1}^{n} \lambda_j \tilde{x}_{ij} \le \theta_p \tilde{x}_{ip}, \quad \forall i, \quad (1)$$
$$\sum_{j=1}^{n} \lambda_j \tilde{y}_{ij} \ge \tilde{y}_{ip}, \forall k$$
$$\lambda_j \ge 0, \forall j$$

Where θ_p^* will be the fuzzy efficiency score. The DEA fuzzy model (1) will then be solved both with the technology of model 1 (passage model) in Table 3 and with the technology of model 2 (position model) in Table 3. And as we have already said once you get the dual of this model we will solve them with the method proposed by (Kao, et al., 2000) and later with that proposed by (Léon, et al., 2003) (Emrouznejad, et al., 2014).

3.3.2 Fuzzy DEA output model for professional football clubs with constant return to scale

Let's assume that we have n = 20 professional football clubs, each of which has set a certain ranking among its sporting goals for its football team in a given football season. In this regard, it has taken care to involve athletic trainers, doctors, physiotherapists, first and second coaches for the players and goalkeepers in order to structure effective training programs for its athletes and also to guarantee a serene atmosphere through cooks and other workers, healthy and productive to get the best performance from their athletes. Once everything is organized, the club announces its first team, the one that will always play, made up of a specific squad of players who will take the field. We assume che $\tilde{x}_{ij}$(con i input) e $\tilde{y}_{kj}$ (con k output) represent, respectively, the fuzzy inputs and the fuzzy outputs of the j-th professional football team. In this case, the inputs will be the operational efficiency that the team has shown in the field

[16] The uncertainty in statistics takes the form of a random variable that has a probabilistic meaning. But following (Zadeh, 1978) assigning a probability to an uncertain event is different from saying that that event is possible.
[17] The curvature observed in the figure is given by a membership function equal to $x^{0.5}$

and the outputs of the model will be the objectives ranking (the inputs and outputs for models 3 and 4 of Table 3). The DEA fuzzy model with constant returns to scale in the output-oriented version of the envelope will be as follows:

$$\begin{aligned} &\max \delta_p \\ s.t. &\textstyle\sum_{j=1}^{n} \lambda_j \tilde{x}_{ij} \leq \tilde{x}_{ij},\ \forall i, \\ &\sum_{j=1} \lambda_j \tilde{y}_{ij} \geq \delta_p \tilde{y}_{ij}, \forall k \\ &\lambda_j \geq 0, \forall j \end{aligned} \qquad (2)$$

We built model (2) starting from standard DEA models with an output orientation (Cooper, et al., 2007) and replacing fuzzy inputs and outputs in it. The DEA fuzzy model in (2) is the one applied to the club with the inputs and outputs of models 3 and 4 in Table 3. It will be solved with two different types of inputs (once the efficiency scores of model (1) solved with the method of (Kao, et al., 2000) and once the efficiency scores of model 2.

3.4 Second stage analysis and models

In the analysis of efficiency with non-parametric methods such as the DEA, a fundamental part to complete the first stage analysis is the second stage through which we try to identify which are the factors external to the process that influence the performances. This last analysis is carried out with different econometric models (usually Tobit models given the censored nature of the efficiency scores) or following different techniques such as the one in (Simar, et al., 2007). An example of a second stage analysis present in this literature is in (Barros, et al., 2010). In general, the question to be answered with the second stage analysis is: what are the external factors that can influence the performances of a football team (i.e. its tactical efficiency on the pitch and/or its achievement of objectives?)? To identify some of them, let's think for example: 1) the weather conditions in which the match takes place, 2) the psychological support of the supporters, 3) the fact that one of the teams is playing the match away from home rather than at home, 4) if or less the manager ranks starters in most of the match or not, 5) if the team deployed on the pitch expresses a certain difference in experience between its organizational sectors (defense, attack, center of the field), 6) if the players' contract has expired or about to expire, 7) if the training took place regularly or took place in unfavorable conditions, 8) if there are players on the pitch who have just come out of an injury or are overburdened by a minor injury, 9) what is the weight of TV rights and the number of subscribers, 10) whether or not having a stadium owned influences the performances, 11) having realized capital gains from the sale of players, 12) the impact of the prestige and image of the club, 13) measure of the presence of foreign capital in the company's share capital, 14) the percentage of foreign players in the squad or on the field constantly, 15) the number of sponsors who support the team, 16) the level of indebtedness, 17) average player substitution time , 18) game module adopted (e.g. 3-4-3 or 4-3-2 and so on) and so on talking. Some of these factors can be considered to affect only the operational tactical efficiency in the field, others instead of affecting only the operational effectiveness in achieving the ranking objectives, and others instead that affect both dimensions. Even in the fuzzy DEA models used, the efficiency scores remain in the range [0,1] in the case of measurement with input orientation (and in any case censored on the left in output-oriented measurements) therefore in our opinion it can still be considered correct use a Tobit regression model, which in general can be indicated as follows:

$$y_i^* = \beta' x_i + \varepsilon_i \quad \varepsilon_i \sim N(0, \sigma^2) \qquad (3)$$

if:

1) $y_i^* > 0 \Rightarrow y_i = observed$
2) $y_i^* < 0 \Rightarrow y_i = 0$

The Tobit model (3) above is applied to estimate the effects of external factors with the two second-stage models reported in Table 3a.

Table 3a Second stage Tobit models.

Second Stage Models	Model 1	Model 2
Dependent Variables		
The Efficiency Scores	X	
The Effectiveness Scores		X
Independent Variables (Or Explanatory Factors)		
Number Of Holders In The Field	X	
Number Of Defeats In The Last 10 Days	X	
% Of Players With Regular Contracts	X	
% Of Foreign Players	X	
Average Duration Mister	X	
Average Salary	X	
Average Player Substitution Time	X	
Capital Gains From Disposals Of Players		X
Losses From Disposals Of Players		X

Net Gains From Player Disposals		X
Income From TV Rights		X
Football Squad Value		X
Positioning In The Ranking	X	X
Owned Stadium		X
% Of Players With Regular Contracts		X
Sponsor Weight		X
% Foreign-Owned Share Capital		X
Prestige Of The Club		X

Model Tobit in the (3) estimates the effects that external factors could have on operational efficiency while model 2 considers the external factors that are assumed to affect only operational effectiveness. The Tobit1 model in Table 3a above is estimated to evaluate the effect of the same external factors on both the tactical performance of the game level and the tactical performance of the positioning, therefore it uses as a dependent variable once the efficiency scores of the game model 1 and once those of model 2. For reasons of economy of the paper we decided to report only the results for the Tobit 1 model which uses as a dependent variable the efficiency scores obtained with models 1 and 2 with orientation to the input and assumptions of returns of constant scale solved the approach of (Kao, et al., 2000) both in the case of pessimistic and optimistic scenarios for three different levels of alpha (0,0.5,1) (Table 8a and 8b). Then we selected the efficiency scores of the game model 1 input crs alla (Kao, et al., 2000) in the pessimistic and optimistic scenarios for different levels of α and reported in Table 8a and for the Tobit 1 model with dependent variable the scores of the game model 2 we reported the results in Table 8b. The Tobit 2 model in Table 3a, on the other hand, was used to estimate the second stage effects of another set of external factors that are hypothesized to affect only operational effectiveness (results in Table 8c). Therefore the Tobit 2 model uses the operational effectiveness scores of the effectiveness model 5 as a dependent variable.

4.Results

4.1 Results of the DEA models

In this section we report only some of the numerous results obtained by the fuzzy DEA models. In particular, we will report the results of DEA fuzzy models 1 and 2 under the assumption that the operational dimensional scale of the teams is fixed and unchangeable and this corresponds to fuzzy DEA models with the assumption of constant returns to scale. In addition, we will report the results both in the event that the improvements in technical efficiency require adjustments in the game levels as in the number of different types of passes (game model 1) and in the exploitation of the field areas (game model 2) with the same results on the field (input orientation of the DEA fuzzy models), both in the case in which, at the same level of play, the adjustments affect the results of the field (shots on goal and goals scored) (output orientation of the models DEA fuzzy). In each of these circumstances, uncertainty will be considered in a fuzzy sense, treating it with the construction of scenarios (Kao, et al., 2000) and in a possible way (Léon, et al., 2003). For the evaluation of operational effectiveness (García-Sánchez, 2007) we will instead report only the results with the construction of scenarios (Kao, et al., 2000) with orientation to the output and in the absence of scale adjustments . Some of the results for game models (1 and 2) and some of the results for the objective model (5) are reported in Tables 4,5 and 6, respectively. The criterion that prompted us to select the results to be reported is to answer the following questions: 1) in the worst case scenario for the team under evaluation, i.e. when the team is playing with a level of play with the highest number of passages (in model 1) and / or overuse of space in the field (in model 2[18]) but with the lowest results (i.e. the lowest number of goals scored and hits on goal) while all the other teams are playing in the best situation for them (the opposite) what would be its operational efficiency given that no scale adjustments is possible?, and therefore how much could you improve your technical/tactical performance through an adjustment of the game levels in the worst situation for you with the same results and assuming that the operational dimensional scale of the team is fixed and unchangeable? The answer to this question can be provided with the results of game models 1 and 2 solved at (Kao, et al., 2000) with input orientation with worst-best resolution method reported in the worst columns in Table 5. Vice versa in the best situation for the team under evaluation and worst for the others (minimum input and

[18]For example, for a team the worst scenario would be an over-exploitation of the playing fields with the same game results on the field while all the others are under-using the bands with the same results, and vice versa the optimistic scenario would mean an under-use of the playing bands with the same results. of results for the team while all the others are overusing the pitches with the same results, so Team 1 of our sample in Table 2 is using the right, left and central bands of 35.35, and 30% respectively a dispersion coefficient of 6.3 and 7 respectively, this team in the pessimistic scenario would overuse the areas of the field by 41.38 and 37 respectively, while in the optimistic scenario it would be underutilizing the areas by 29.32 and 23, respectively, with the same of game results, while all the other teams are doing the opposite in each of the two situations.

maximum result) what would be its operational efficiency? and therefore how much is it possible to improve the technical/tactical performance in the situation most favorable to you, always assuming an unchangeable dimensional operational scale? We can always answer this question with game models 1 and 2 solved in (Kao, et al., 2000) always with orientation to input and assuming constant returns of scale but with the best-worst model solution method (best columns in Table 5). Another question we are looking for an answer to is: 2) admitting that the coach or/and manager of a team sets a certain level of tolerance in violating the constraints (often the individual performances of the players cannot be improved that much and therefore accept certain performances in the field) what is the h-possibilistic level of efficiency that we can at least obtain? On the other hand, the results of the fuzzy DEA model solved with a possibilistic approach can answer this question (Léon, Liern, & Sirvent, 2003)[19]. For the latter models, which start from the work of (Meada, et al., 1998) adopting the possible programming ((Léon, Liern, & Sirvent, 2003)pp 408)[20]. A level h = 1 defines the maximum level of possibility at which at least one level of efficiency h-possibilistic is obtained [21]. The results for models 1 and 2 solved in (Léon, et al., 2003) are reported in Table 4. Another question we try to answer is: 3) in the worst case scenario, i.e. when a team has achieved the performance lowest tactic while all the others have achieved the highest one, what was the level of operational effectiveness of the club in pursuing the goal (out of focus) "a satisfactory ranking in the standings" in the event that this goal is the one set at the highest level low among the lows (pessimistic scenario with $\alpha = 0$) and in the case in which instead it has been set at the highest level among the highest (optimistic scenario with $\alpha = 0$) always assuming that scale adjustments for the teasm cannot happen? To answer this question we solve and report instead the results of model 5 to (Kao, et al., 2000) with output orientation, with constant scaling and for different levels of α (results in Table 6).

Table 4 Efficiency models possibilistic approach (Léon, Liern, & Sirvent, 2003)[22]

Model 1 (Attack Model)					
Team	Input Crs				
	h=0	h=0.25	h=0.5	h=0.75	h=1
Team 1	**0.85229**	**0.82149**	**0.79055**	**0.76174**	**0.73219**
Team 2	1	1	1	1	1
Team 3	0.87462	0.83715	0.81499	0.79513	0.77726
Team 4	0.96848	0.96006	0.95169	0.94332	0.93491
Team 5	1	1	0.97797	0.91749	0.86656
Team 6	0.71538	0.7098	0.7044	0.69916	0.69409
Team 7	0.83977	0.8213	0.80409	0.78802	0.77299
Team 8	0.93501	0.92587	0.91677	0.90898	0.90219
Team 9	1	1	1	0.98156	0.96115
Team 10	0.86059	0.8558	0.85061	0.84339	0.83601
Team 11	**1**	**1**	**1**	**1**	**1**
Team 12	**0.88928**	**0.88558**	**0.88162**	**0.87647**	**0.87037**
Team 13	1	1	1	1	1
Team 14	1	1	1	1	1
Team 15	1	1	1	1	1
Team 16	1	1	1	1	1
Team 17	1	1	1	1	1
Team 18	1	1	1	1	1
Team 19	0.97827	0.96837	0.95789	0.94677	0.93495
Team 20	1	1	1	1	1
Mean	**0.945685**	**0.939271**	**0.932529**	**0.923102**	**0.914134**
Sd	0.080328	0.086679	0.091956	0.096567	0.102607
Range	[0.7153,1]	[0.7098,1]	[0.7044,1]	[0.6991,1]	[0.6940,1]
			Output Crs		

[19] In these models as in (Guo & Tanaka, 2001) the level of possibility is indicated with h ($0 \leq h \leq 1$) and is inserted in the constraints adopting the form: $z_1 - (1-h)w_1 \leq z_2 - (1-h)w_2$ where z1 and z2 are two fuzzy variables triangular fuzzy $Z_1 = (z_1, w_1)$ e $Z_2 = (z_2, w_2)$. When managers are available, these levels can be set differently and reduce the problem to a linear problem (Negi & Lee, 1993).

[20] The same authors state that the difference lies in the fact that in order to obtain gains in computational and partly interpretative terms they adopt the envelopment rather than multiplier version of the DEA models as they do instead. (Guo & Tanaka, 2001)

[21] In general, for all fuzzy models of linear programming there are problems of interpretation of the violations of the constraints and not all the proposed interpretations coincide (Rommelfangem, 1996). And we believe that DEA fuzzy models are no exception, together with the specific problem of assigning rank to efficiency scores.

[22] To get a good idea about possibilistic programming see for example (Negi & Lee, 1993).

Team 1	**1.17331**	**1.2173**	**1.26493**	**1.31279**	**1.36576**
Team 2	1	1	1	1	1
Team 3	1.14336	1.19453	1.227	1.25765	1.28657
Team 4	1.03255	1.0416	1.05076	1.06009	1.06962
Team 5	1	1	1.02252	1.08993	1.15399
Team 6	1.39786	1.40884	1.41966	1.43029	1.44073
Team 7	1.1908	1.21758	1.24364	1.269	1.29368
Team 8	1.06951	1.08007	1.09079	1.10013	1.10841
Team 9	1	1	1	1.01879	1.04041
Team 10	1.16199	1.1685	1.17562	1.18569	1.19616
Team 11	1	1	1	1	1
Team 12	**1.12451**	**1.1292**	**1.13428**	**1.14093**	**1.14894**
Team 13	1	1	1	1	1
Team 14	1	1	1	1	1
Team 15	1	1	1	1	1
Team 16	1	1	1	1	1
Team 17	1	1	1	1	1
Team 18	1	1	1	1	1
Team 19	1.02221	1.03267	1.04396	1.05622	1.06957
Team 20	1	1	1	1	1
Mean	**1.065805**	**1.074515**	**1.083658**	**1.096076**	**1.108692**
Sd	0.104329	0.113044	0.121255	0.129107	0.139088
Range	[1,1.397]	[1,1.4088]	[1,1.4196]	[1,1.4303]	[1,1.4407]

Model 2 (Position Model)

Input Crs

Team 1	**1**	**1**	**1**	**1**	**1**
Team 2	1	1	1	0.99606	0.98921
Team 3	0.8808	0.86948	0.85318	0.83683	0.82162
Team 4	0.69374	0.69141	0.68901	0.68651	0.68138
Team 5	0.81139	0.80644	0.79841	0.7896	0.78029
Team 6	0.71558	0.70408	0.69151	0.67798	0.66249
Team 7	1	1	0.9868	0.96505	0.95157
Team 8	1	0.99501	0.98402	0.97482	0.96616
Team 9	0.93514	0.91752	0.9012	0.88603	0.87187
Team 10	0.95425	0.95002	0.94396	0.93778	0.93229
Team 11	1	1	1	1	1
Team 12	**0.97255**	**0.95781**	**0.94184**	**0.92508**	**0.90795**
Team 13	1	1	1	1	1
Team 14	1	1	1	1	1
Team 15	0.93902	0.92885	0.91706	0.90421	0.89019
Team 16	1	1	1	1	1
Team 17	1	1	1	1	1
Team 18	1	1	1	1	1
Team 19	1	1	1	1	1
Team 20	0.93854	0.91623	1	0.87989	0.8637
Mean	**0.942051**	**0.936843**	**0.930081**	**0.919464**	**0.915936**
Sd	0.095106	0.097523	0.100263	0.10385	0.107092
Range	[0.6937,1]	[0.6914,1]	[0.6890,1]	[0.6625,1]	[0.6625,1]

Output Crs

Team 1	**1**	**1**	**1**	**1**	**1**
Team 2	1	1	1	1.00396	1.01091
Team 3	1.13533	1.15012	1.17208	1.19499	1.21711
Team 4	1.44147	1.44632	1.45135	1.45664	1.46761
Team 5	1.23245	1.24002	1.25249	1.26647	1.28157
Team 6	1.39747	1.42029	1.44611	1.47496	1.50945
Team 7	1	1	1.01338	1.03621	1.05089
Team 8	1	1.00502	1.01623	1.02583	1.03503

Team 9	1.06936	1.0899	1.10964	1.12864	1.14695
Team 10	1.04794	1.05261	1.05937	1.06635	1.07263
Team 11	1	1	1	1	1
Team 12	**1.02823**	**1.04404**	**1.06175**	**1.08099**	**1.10138**
Team 13	1	1	1	1	1
Team 14	1	1	1	1	1
Team 15	1.06495	1.0766	1.09044	1.10594	1.12336
Team 16	1	1	1	1	1
Team 17	1	1	1	1	1
Team 18	1	1	1	1	1
Team 19	1	1	1	1	1
Team 20	1.06548	1.09143	1.1178	1.13651	1.15781
Mean	**1.074134**	**1.080818**	**1.089532**	**1.098875**	**1.108735**
Sd	0.132132	0.136189	0.14098	0.146355	0.153787
Range	[1,1.4415]	[1,1.4463]	[1,1.4513]	[1,1.4750]	[1,1.5094]

Let's start now with the interpretation of the results of "game" models 1 and 2 solved in (Léon, et al., 2003) (Table 4). Looking at the top of Table 4 we can interpret the results as follows: with $h = 1$ it means that the possibility of achieving at least a certain level of operational efficiency is equal to 1 when at the same time the possibility that the constraints are satisfied is equally equal to 1. So for example the possibility that team 1 has an operational efficiency level at least equal to 0.73219 (last column) is equal to 1. With the same possibility the operational efficiency level of team 11 is at least 1[23]. In both cases, given a certain level of violation of the constraints, the possible margins for improvement for team 11 are zero while for team 1 they are equal to 1-0.7321 = 0.2679 (i.e. around 26.8%). Therefore team 11 is with a possibility equal to 1 h-efficient while team 1 is h-inefficient. Similar interpretations can be given for each of the different levels of h. In our opinion, the path from level $h = 0$ to level $h = 1$ is interesting, (by reading Table 4 from left to right by column) to analyze how the h-possibility level of (in)efficiency of the teams changes as the possibility of obtaining it increases. In this passage, Team 1 in model 1(with input orientation andcrs assumption) clearly worsens its possible operational efficiency in the game level (passing from 0.85229 to 0.73219) while maintaining its position operational efficiency unchanged in model 2. This team according to the results of model 2 is maintaining at every level of possibility the possibility of achieving its field results without over-utilization and without under-utilization of field spaces. Reading the results of the two possible game models for this team, we could say that team 1 with the same goals scored to the opponent and shots on the opponent's goal is organized in the field in order to excel in the tactics of occupying the spaces in the field in attack phase in any scenario of uncertainty more than it knows how to do in the passages. In another perspective we suggest to interpret these results in the sense that, with the same results achieved (goals scored and shots on goal), the search for a greater possibility for Team 1 to obtain at least a certain h-possible level of tactical efficiency in the game levels of the passages (passing da $h = 0 \rightarrow h = 1$) requires a lower possible level of operational efficiency in the passages (model 1) and therefore a greater adjustment in the quantites of the passages (passes from 0.85229 to 0.73219) than its tactical positioning efficiency (model 2) requires for the skills / tactical qualities for the exploitation of the areas of the field [24]. For Team 12, on the other hand, we record that the search for a greater possibility (passage from $h = 0$ to $h = 1$) of obtaining at least a certain h-possibility level of operational efficiency is always associated with a lower h-possibility level of operational efficiency in both models 1 and 2. In each of the different scenarios of possible uncertainty, this team needs significant adjustments in the two different levels of play (this could be achieved in different ways, for example by motivating the players more, improving and enhancing the qualities and individual talents during training, making substitutions etc. etc.). In addition, these adjustments are required for this team whether you are looking for an improvement in the organization of resources in the field to save energy with the same results in the field (orientation to the input of the DEA fuzzy models) or if you are looking for an improvement in results in the field (orientation to the output of the fuzzy DEA models) at the same levels of play. So,on average in our virtual sample of professional football teams, this transition of possibility levels is always associated with worsening in the h-possibility level of operational efficiency for models 1 and 2 (see Mean lines of Table 4 from left to right) . In other words, we can say that managers who accept a reduction in the level of possibility of reaching at least a certain level of operational efficiency in the levels of play and in the exploitation of the field areas can on average hope that the teams they manage can earn a higher operational tactical efficiency level with the same results in the field. In other words, organizing a team in order to obtain a higher level of operational tactical efficiency in the game levels and in the exploitation of the spaces on the pitch is possible if you accept a lower

[23] It is possible to note that the efficiency levels with h = 1 in the possible approach coincide with the worst best of the model (Kao, et al., 2000) with α = 1.
[24] To realize this interpretation, one can read the example offered in general for possibilistic programming by (Negi, et al., 1993).

level of possibility that you will reach it. The advantage of having a greater possibility of reaching a certain degree of possible h-efficiency is therefore associated with a greater request for adjustment in the technical / tactical organization in the field of the team from the point of view of the skills required to execute the passes and use the same spaces of the field (therefore greater commitment on the part of players and coaches, to avoid waste of passages and over or under utilization of field spaces). In fact, on average, the efficiency adjustment required for a score as possible would go on average from 1-0.9456 = 0.0544 with h = 0 to an adjustment of 1-0.9141 = 0.0859 for the 1 crs-input model with h = 1 and under the same conditions an adjustment that would go from 1-0.9420 = 0.058 to 1-0.9159 = 0.0841 in the 2 input crs model. This translated in terms of the variables of model 1 with input orientation means that in the most possible case, a proportional reduction of all types of passage with the same result is necessary, higher than that which would be necessary if an efficiency result h were accepted - as little as possible but higher. The sporting manager should therefore ask the team (policy indication) not to waste passes during the attack phase of the game if more results are not being obtained which may require changing the mix of resources (of players) on the field. Then looking at Team 1 in its efficiency scores for model 2 with orientation to the output with assumption of crs, since with increasing h the score is always 1, then the manager of this team can hope for an efficiency level h-possibility always relatively high. Therefore this team is relatively always h possibly efficient and may not look for more goals on goal and shots made. On average, however, in our sample also for the 2 crs model there is the same pejorative dynamic both in the orientation to input and output. A more compact reading can derive from the graphs in Figures 4 and 5 where the h-possibilistic efficiency levels (x axis) are reported for each value of h. According our opinion the policy indications will be of strategic nature (during the entire tournament) than tactical (during the match).

Insert Figure 4 Trend of h-possible efficiency level for each level of h. Model 1 input and output crs.

Insert Figure 5 Trend of h-possible efficiency level for each level of h. Model 2 input and output crs.

Hence, interesting cases in our opinion from the point of view of strategic policy (understood as a way of dealing with the other teams) are the intersections of the lines in Figures 4 and 5 that are observed for the different levels of increasing h ($h = 0 \rightarrow h = 1$). For example Team 5 in Figure 4 on the level h=0 has a h-possibilistic level of efficiency greater than that of teams 19 (s) 4 (d), 8 (h) and 12 (i), but at level $h = 0.75$ (i.e. at a higher level of possibility of reaching a certain level of efficiency) its h-possiblistic level of operational efficiency in the game level is lower than that of Teams 4, 8 and 19 (letters d, h and l). The first intuitive interpretation that emerges from reading the crossings for team 5 is that for this team looking for a greater chance of achieving certain levels of efficiency is not the best strategy when it meets teams 4,8 and 19 as in the transition $h = 0 \rightarrow h = 1$ because his performance worsens more than the levels of play of these other 3 teams. A possible strategic policy indication is that the coach of Team 5 could give an indication not to waste passage (for example by maintaining a higher psychological concentration during these meetings during the tournament) when his team meets teams 19, 4 and 8 in order to achieve a level of technical efficiency greater than that of teams 19.4 and 8.. For team 7, on the other hand, compared to team 1, being less efficient from a technical point of view with a high possibility of being so is instead the best strategy when it meets team 1 which always achieves a performance with the same high level of possibility as that of team 7 inferior already starting from h = 0.25. As you can guess, all this does not translate *sict et simpliciter* into a victory (hence the second technology of the objective model). Turning now to interpreting the results of performance measurements relating to game level and positioning in the pessimistic and optimistic scenarios of Table 5 we show the results of DEA fuzzy models 1 and 2 solved with the approach proposed in (Kao, et al., 2000). With these models, there are two operational efficiency results, one in the worst scenario and the other in the best one for each level of α (read note 21, 25 and 27). The ratio of these models is different and does not concern a level of satisfaction or the possibility of obtaining a certain level of efficiency or tolerance of acceptance of major or minor violations of constraints, but through the α cut on the inputs and fuzzy ouput these models with cut α[25] construct different pessimistic and optimistic scenarios (see note 18 and 27) and evaluate the fuzzy efficiency in these intervals - For example in the pessimistic scenario for each level α the model considers the minimum output and input values of the DMUs under evaluation and of the other DMUs , respectively, and the maximum value of the inputs and outputs of the DMUs under evaluation and of the others respectively (see note 27). In this way, different ranges of fuzzy numbers are considered in each scenario (pessimistic and optimistic). And so the best of the worst input values in the pessimistic case (where the highest input values are taken) and the best of the best input values in the optimistic case (where the lowest input valuesare taken) are realized at α the higher the former and the lower the latter. The same but opposite is true for the output values[26].

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[25] As in (Meada, et al., 1998) these authors, to solve the DEA fuzzy models, combine the approach of parametric programming (Zimmerman, 1978) with the idea of the α level of fuzzy sets (Zadeh, 1978), but they call their approach "of the α cut" .. At each level of α there are two DEA programs in the pessimistic and optimistic case (see note 19 below)

[26] An exercise to do to clarify the ideas is to refer to Figure 1 in the text and image to make cuts at different levels of α (vertical axis) and project the points found on the figure below on the horizontal axis. It will be noted that for each different level of α, different ranges of values are obtained on the horizontal axis.

Table 5 Efficiency models α-level approach (Kao & Liu, 2000)[27]

Model 1 (Attack Model)						
Team	Input Crs					
	α=0		α=0.5		α=1	
	Worst	Best	Worst	Best	Worst	Best
Team 1	0.5601	1	0.62456	1	0.73219	0.73219
Team 2	0.7897	1	0.99995	1	1	1
Team 3	0.58146	1	0.66281	1	0.77726	0.77726
Team 4	0.68221	1	0.79496	1	0.93491	0.93491
Team 5	0.53313	1	0.66341	1	0.86656	0.86656
Team 6	0.49163	1	0.56847	0.8846	0.69409	0.69409
Team 7	0.57459	1	0.64268	1	0.77299	0.77299
Team 8	0.64218	1	0.74772	1	0.90219	0.90219
Team 9	0.7015	1	0.82612	1	0.96115	0.96115
Team 10	0.67493	1	0.75016	0.93752	0.83601	0.83601
Team 11	1	1	1	1	1	1
Team 12	0.55333	1	0.67131	1	0.87037	0.87037
Team 13	0.67263	1	0.78251	1	1	1
Team 14	0.58135	1	0.7142	1	1	1
Team 15	0.64441	1	0.75581	1	1	1
Team 16	0.77864	1	0.90859	1	1	1
Team 17	1	1	1	1	1	1
Team 18	0.70122	1	0.86855	1	1	1
Team 19	0.6788	1	0.79854	1	0.93495	0.93495
Team 20	0.70348	1	0.85943	1	1	1
Mean	0.6772645	1	0.781989	0.991106	0.9141335	0.9141335
Sd	0.135469952	0	0.128193144	0.02868967	0.102607037	0.102607037
Range	[0.4916,1]	[1,1]	[0.5685,1]	[0.8846,1]	[0.6941,1]	[0.6941,1]
Model 2 (Position Model)						
Team 1	0.80872	1	0.94457	1	1	1
Team 2	0.69272	1	0.79959	1	0.98921	0.98921
Team 3	0.62914	1	0.70709	1	0.82162	0.82162
Team 4	0.515	1	0.60138	0.7815	0.68138	0.68138
Team 5	0.46096	1	0.59814	1	0.78029	0.78029
Team 6	0.43676	1	0.49458	0.8436	0.66249	0.66249
Team 7	0.62174	1	0.76347	1	0.95157	0.95157
Team 8	0.56761	1	0.73624	1	0.96616	0.96616
Team 9	0.50887	1	0.67072	1	0.87187	0.87187
Team 10	0.65195	1	0.75312	1	0.93229	0.93229
Team 11	0.90935	1	1	1	1	1
Team 12	0.46845	1	0.61455	1	0.90795	0.90795
Team 13	0.63304	1	0.82839	1	1	1
Team 14	0.56323	1	0.74102	1	1	1
Team 15	0.68489	1	0.76361	1	0.89019	0.89019
Team 16	0.62946	1	0.85936	1	1	1
Team 17	0.83333	1	0.91509	1	1	1
Team 18	0.59325	1	0.81967	1	1	1
Team 19	0.60727	1	0.77663	1	1	1
Team 20	0.52496	1	0.6504	1	0.8637	0.8637
Mean	0.617035	1	0.751881	0.981255	0.915936	0.915936
Sd	0.124948	0	0.125886	0.058569	0.107092	0.107092
Range	[0.4367,0.6217]	[1,1]	[0.49445,1]	[0.7815,1]	[0.6625,1]	[0.6625,1]

[27] In the model of (Kao, et al., 2000), for each level of α, the worst level of efficiency in the pessimistic scenario (minimum level of efficiency) is obtained by placing the outputs of the unit under evaluation and the inputs of the others at the lowest level and the input level of the units underestimated high and the outputs of the others at the highest level, while the best level of efficiency (maximum level of efficiency) is obtained by doing the opposite both for the unit under evaluation and for the rest.

In the optimistic case with an α = 1 → 0 (reading Table 5 from right to left), i.e. going from selecting the central input and output value with α = 1 to selecting the best (lowest) input values and of outputs (the highest) in the case of α = 0 on average the performances of our sample improve going from an average of 0.9141 to an average of 1 for the model 1 and from an average of 0.9159 to an average of 1 for the model 2. The teams that in model 1 always remain the best performing are teams 2,11,13,14,15,16 17 and 19, while for model 2 the ever green teams are teams 1,11,13, 14, 16,17,18 and 19. (read all the best columns of Table 5 for model 1 and 2). To clarify the context in which we move, if in the optimistic case with α = 0 we take team 1 this means that we are evaluating it with the variable cross (one of its inputs in model 1) with a value of 26 (see Table 2 where this fuzzy variable is represented as a symmetric triangular with a central tendency of 29 and a symmetrical dispersion coefficient, alpha 1, equal to 3) which is the lower end of the left range (26-29) (29-3 = cross-alpha1 = 26) of this fuzzy variable (see Figure 1). The same logic applies to all his other input variables while for the other teams, always remaining in this optimistic scenario for team 1 with cut alpha = 0, we are taking the extreme values of the right range of their input values (for example of team 3 for the same variable cross we are using the value 20 (15 + 5 = cross + alpha1)). The value instead of the fuzzy output variable "shots taken" always for team 1 we are using is the value 28 (shots made + beta 1 = 25 + 3) and for the remaining teams we are using the most extreme values of the left ranges of their output variables (the lowest of the lowest values). In the pessimistic version of the model to evaluate team 1 we would have instead considered for its input values the highest ones (which are located at the upper end of the right range) and its lowest output values (which are located at the lower end of the range). their left range) while we would have done the opposite for the other teams. Returning to the efficiency score reading, the performances of this team, in the pessimistic scenario, like those of all the others, improve if we read Table 5 from left to right ($\alpha = 0 \rightarrow 1$[28]) that is, for increasing levels of α cuts. These two ways of accommodating the uncertainty in the fuzzy DEA models used here appear to us very adequate for evaluating the performance of football teams [29] as they can grasp actually real aspects for football teams. For this type of DEA fuzzy model we report the results for models 1 and 2 input crs for all teams and for the three alpha levels in Figures 5 and 6, as we did for that of (Léon, et al., 2003) . Moving on to models 2 (the position ones) and the reading of the results with respect to the levels h for the fuzzy DEA models a la (Léon, et al., 2003) and that for the different α values for those (Kao, et al., 2000) are identical but refer to different tactical performances (occupations of spaces instead of number of steps) in situations of different out of focus context. Moving on to the policy indications, from the first model (model 1) "policy" indications are obtained for the level of play intended with the number of passes, crosses and filtering on the field, while from the second (model 2) indications of "policy" are obtained ”For the game intended as the occupation of the spaces of the field always in the attack phase. As we have shown above, the interpretations of the results must take into account the specificity required by the fuzzy models used and the fuzzy logic used to define variables and objectives (Zadeh, 1975). Let's now turn to what (García-Sánchez, 2007) calls the operational effectiveness of football teams. To this end, it is necessary to refer to the results of models 3,4, 5 and 6. Once again we make a choice of economics of the paper and we report only some results, i.e. those that allow us to answer the following questions: 1) assuming that for the teams professionals who play in the highest degree of football championships (eg Serie A in Italy) the goal is unique for all, that is to aim for the maximum score to win the championship, which team was most effective in transforming its operational tactical efficiency (which in the short term can be a difficult constraint to overcome and for this reason the crs assumption is used in DEA fuzzy models) in a blurry lens such as “at least a good positioning? In this regard, the Ouput-oriented model 5 is the one that can best give the answer to this question and some of the many [30] results are reported in Table 6. In particular, we report the results of model 5 with output orientation under the crs assumption using as its inputs the operational tactical efficiency scores of model 1 solved at (Kao, et al., 2000) with input orientation with crs assumption. The results of model 5 therefore refer to the athletic or operational effectiveness of the teams in reaching their out-of-focus goal of maximum score when the achievement of this goal is influenced by uncertainty and when the team is operating at an optimal operational scale level of model 5) once the organizational

[28] This behavior of the efficiency scores in relation to the α value had already been noted in (Kao, et al., 2000).

[29] Consider for example one of the league football seasons in which team X is believed to have always played on average in an optimistic scenario for her when she met a team Y that is believed to have always played worse.

[30] The number of results are equal to the number of types of effectiveness models 5 that can be solved (with orientation to input or output in combination with the assumptions crs and vrs) multiplied by the number of inputs that can be used and in each of those models. These inputs are 24 for each game model, 1 and 2 (total 48), solved at (Kao, et al., 2000) and 20 for each game model solved at (Léon, et al., 2003). Therefore, the interpretation of the results for the effectiveness models, including model 5 of which we report in the paper, requires a lot of attention and interpretative scrupulousness. To facilitate the task, we have chosen to use only the efficiency scores of the game models 1 and 2 solved in (Kao, et al., 2000) as the input of the effectiveness model 5. The interpretative combinations are: 1) with worst-case game model efficiency scores 1 and 2 for each α (0, 0.5, and 1) solved with input and output orientations in combination of the cre and vrs assumptions for each of the effectiveness models a la (Kao, et al., 2000) solved in turn with input and output orientation in combination with the assumptions crs and vrs and 2) with efficiency scores of game models 1 and 2 in the best case for each α solved with input and output orientations in combination of the cre and vrs assumptions for each of the effectiveness models a la (Kao & Liu, 2000) solved in turn with input and output orientation in combination with the crs and vrs assumptions.

efficiency has already discounted the uncertainty in the operational tactical efficiency in model 1 (level of the game) in the worst case and in the best case and when in turn the size scale of the team in these two models it is set at optimal level (crs assumption of game model 1 using in efficiency scores of model 1 in the case of input-crs for each level of α). We have assumed for this model exact values for the inputs and fuzzy for its outputs, which we remember being the efficiency scores the first and the ranking score the second (see Figure 2).

Table 6 Operational effectiveness (model 5 DEA fuzzy (Kao, et al., 2000) output crs)

Model 5 Worst And Best Case With Orientation To Output And CRS Assumption						
	Case With Efficiency Scores In The Worst Case Of Model 1 With Orientation To The Input And Assumption Crs As Input Of Model 5 A La (Kao, Et Al., 2000) With A = 0 Worst Case					
Team	α=0		α=0.5		α=1	
	Worst	Best	Worst	Best	Worst	Best
1	1.60342	1	1.19923	1	1	1
2	2.54329	1.00179	1.91478	1.24921	1.60625	1.60625
3	2.07431	1	1.54268	1	1.27989	1.27989
4	2.51098	1	1.89225	1.13814	1.58871	1.58871
5	2.02661	1	1.51313	1	1.2598	1.2598
6	1.91138	1	1.47985	1	1.19694	1.19694
7	2.37923	1	1.76785	1	1.46553	1.46553
8	2.9198	1	2.13746	1.13329	1.74897	1.74897
9	3.96744	1	2.73234	1.28245	2.12681	2.12681
10	4.89075	1	3.25147	1.30416	2.46481	2.46481
11	7.72939	1.26857	5.01547	1.9446	3.73687	3.73687
12	3.88809	1	2.66566	1.07601	2.06772	2.06772
13	4.72638	1	3.28359	1.31638	2.57338	2.57338
14	4.21264	1	2.91574	1.14508	2.2784	2.2784
15	4.39491	1	3.14583	1.26929	2.52555	2.52555
16	5.47128	1	3.85247	1.53368	3.05162	3.05162
17	7.2463	1.26857	5.23041	2.00857	4.22857	4.22857
18	6.02223	1	3.9498	1.40845	2.96516	2.96516
19	9.83758	1	6.06167	1.49093	4.36294	4.36294
20	40.78103	1	12.26497	1.6395	6.64936	6.64936
Mean	6.056852	1.026947	3.390833	1.296987	2.508864	2.508864
Sd	8.461637	0.082634	2.489291	0.30068	1.38308	1.38308
Range	[1.603,40.78]	[1,1.2685]	[1.199,12.264]	[1,2.008]	[1,6.649]	[1,6.649]
	Case With Worst-Case Efficiency Scores Of Model 1 With Orientation To The Input And Assumption Crs As Input Of Model 5 A La (Kao, Et Al., 2000) With A = 0 Best Case					
1	1.40741	1	1.12865	1	1	1
2	1.58333	1	1.35099	1	1.13924	1.13924
3	1.75385	1	1.47826	1	1.23288	1.23288
4	1.80952	1	1.54545	1	1.30435	1.30435
5	1.86885	1	1.5814	1	1.32353	1.32353
6	2.07273	1	1.68595	1	1.36364	1.36364
7	2.03571	1	1.71429	1	1.42857	1.42857
8	2.23529	1	1.85455	1	1.52542	1.52542
9	2.78049	1	2.17021	1.02395	1.69811	1.69811
10	3.5625	1	2.68421	1.08228	2.04545	2.04545
11	3.8	1	2.79452	1.08917	2.09302	2.09302
12	3.45455	1	2.68421	1.08917	2.09302	2.09302
13	3.45455	1	2.72	1.09615	2.14286	2.14286
14	3.5625	1	2.79452	1.10323	2.19512	2.19512
15	3.35294	1	2.72	1.10323	2.19512	2.19512
16	3.45455	1	2.75676	1.10323	2.19512	2.19512
17	3.5625	1	2.91429	1.125	2.36842	2.36842
18	4.22222	1	3.13846	1.125	2.36842	2.36842
19	7.125	1	4.97561	1.23022	3.6	3.6
20	28.5	1	9.71429	1.30534	5.29412	5.29412

Mean	4.279925	1	2.720331	1.073799	2.030321	2.030321
Sd	5.846927	0	1.863672	0.083672	0.977028	0.977028
Range	[1.40,28.5]	[1,1]	[1.128,9.714]	[1,1.3053]	[1,5.2942]	[1,5.2942]

An example of how we have inserted uncertainty on the maximum target of 114 points in model 5 for team 1 using fuzzy logic (Zadeh, 1975) is represented by Figure 5 below

Insert Figure 6 Fuzzy target of maximum target for team 1

As you can see, the central trend in Figure 6 is equal to 90 (the most possible value), that is, we have adopted the score actually achieved by team 1 as the central trend of the fuzzy number of asymmetric triangular type "max target" for team 1, " and assumed a lower uncertainty about its achievement on low scores and greater for higher ones (setting a left dispersion coefficient lower than the right one). Having fixed this, the α value is shown on the y axis, which determines the α cut used in the models of (Kao, et al., 2000). In the pessimistic case[31] the value of the output variable in Figure 5 which for example is identified at the level α = 0.5 in approximately 87 (left range), and instead 102 is the one identified in the optimistic case (right range). Therefore with this value of α for each level of operational efficiency (measured with models 1 and 2) the operational effectiveness of team 1 measured, in the pessimistic version of model 5 it should have reached level 87 which is positioned much lower than at the maximum target (114) and at a distance of 3 from the score it actually reached in the standings (90), when the opposite happened for all the other teams., While the level of operational or athletic effectiveness of the teams 1 in the optimistic case of model 5 is evaluated with respect to the value 102 placed relatively higher than that achieved in the ranking is closer to the maximum score, as the opposite is true for all the other teams. And therefore with orientation to the ouput of model 5 with hiring crs, it is evaluated in the pessimistic and optimistic scenario of how much a team could have achieved the "max score" objective. Having clarified the interpretative context of model 5, we move on to interpret the athletic effectiveness scores of the teams, in the worst and best scenarios, reported in Table 6 distinguishing when, again in the out crs orientation of model 5, we used the scores as its input. of operational efficiency deriving from model 1 to (Kao, et al., 2000) with crs input orientation both in the pessimistic situation, at the top of the table, and when instead we used the efficiency scores as input of model 5 (always out crs) always operational of model 1 (always inputr crs) but in the optimistic case. Using in model 5 the results of the input models crs a la (Kao, et al., 2000) for model 1 means that we are considering the degree of effectiveness in achieving the objectives when the level of efficiency in the play levels (passing, cross and filtering) with the same game results (goals scored and goals on goal) are measured first in the worst situation and then in the best one. In the worst scenario (with α = 0) or the first column of the upper part of Table 5 we find that team 20 (last in the standings with 17 points, see Table 2) was the least effective according to model 5. In the case of transformation of the tactical efficiency deriving from model 1 solved as a DEA fuzzy model a la (Kao, et al., 2000) with alpha = 0 in the pessimistic scenario (upper part of Table 6), the operational effectiveness in the pessimistic scenario (worst columns in the upper part of Table 6) was on average better at the alapha cut = 1 (2.508), i.e. when the effectiveness of the teams in the pessimistic scenario was evaluated with the central trends (i.e. their ranking scores achieved) and the team that best transformed its tactical efficiency in the level of play in its worst case scenario with an alpha = 0 cut of model 1 is team 1. While the team that has been most effective in transforming its tactical efficiency in the level of play in the best scenario of model 1 with alpha = 0 in the worst scenario of model 5 (worst columns of the lower part of Table 6) has always been team 1 (1). Which is also confirmed as the most effective in the case of optimistic scenarios of model 5 (best columns of Table 6) together with teams from 2 to 8 in the case of alpha = 0.5 for the effectiveness model 5 (third last column of the part low of Table 6). In the best scenario of model 5 with alpha = 0.5 (third to last column best in the upper part of Table 6) Team 20 (1.6395) was relatively more effective than Team 17 (2.00857).

Insert Figure 7 Example of a fuzzy goal for the soccer teams of the Italian Serie A championship

Insert Figure 8 Model results 5 out crs with input the efficiency scores of model 1 inp crs with alpha = 0

The policy indications with model 5 concern all those indications that, given the constraint of operational efficiency in the field, allow to achieve higher results in the ranking. And that is all those indications that can increase the probability of victory and the number of victories or in any case lower the probability of defeats or the number of defeats to reach the best ranking. One possible way is to create greater psychological awe on the pitch, or create a better reputation. Finally, the peer group analyzes for all models can be found in the additional files peers_group-analysis.xlsx.

4.2 Second stage results

[31] For clarity and completeness we note that the best-worst (optimistic view) and worst-best (pessimistic view) methods adopted in the work of (Kao, et al., 2000) and described in the text are accompanied by two other methods that are adopted in level of the α-level approach to solving fuzzy DEA models and are: best-best method and worst-worst method. In these last two cases, all DMUs are evaluated from an optimistic point of view in the first case and all DMUs are evaluated from a pessimistic point of view in the second. Furthermore, in addition to the α level approach, another widespread approach to solve fuzzy DA models is that of defuzzification. For a more general overview of the methods of solving fuzzy DEA models see (Hatami-Marbini, et al., 2011).

To conduct the second stage analysis, we also generated virtual data in R (RCoreTeam, 2016) for 18 external factors, some of which are believed to affect only the operational efficiency while for others only the operational effectiveness (Table 7). For some of them it can be assumed that they influence both dimensions of performances, while for others it seems more logical to assume that they influence only one of the two.

Table 7 Statistics describing external factors

Coef.	0.12	0.757	0.449	0.029	0.477	0.367	-0.08	0.246	0.41	0.451	2.442	0.568
Std.De	1.146	2.007	0.786	2.882	19.663	13308.56	1412.383	13291.793	9288.186	192561.63	0.366	17.875
Var	1.313	4.029	0.618	8.305	386.618	177117762	1994826.5	176671758	86270392	3.708e+10	0.134	319.524
Ci.Me	0.536	0.939	0.368	1.349	9.202	6228.59	661.01	6220.75	4347	90121.62	0.171	8.366
Se.Me	0.256	0.449	0.176	0.644	4.397	2975.884	315.819	2972.135	2076.901	43058.09	0.082	3.997
Mean	9.55	2.65	1.75	98.1	41.25	36264.898	-17672.165	53937.063	22643.887	426940.23	0.15	31.45
Media	10	2	2	100	42.5	34062.145	-17757.99	50117.17	22932.528	446101.12	0	29.5
Sum	191	53	35	1962	825	725297.97	-353443.3	1078741.3	452877.73	8538804.6	3	629
Range	4	7	2	10	70	42070.377	4714.432	42093.773	35206.581	774866.57	1	60
Max	11	7	3	100	80	60705.082	-15071.007	79462.825	39851.997	862628.69	1	70
Min	7	0	1	90	10	18634.7	-19785.44	37369.05	4645.416	87762.12	0	10
Vars	1) Titular In The Field	2)Sconf_Ultime10gg	3) Ranking Ranking	4) Players With A Regular Contract	5) Percentage Of Foreign Footballers Holders	7)Capital Gain	8) Capital Loss	9)Net Capital Gain	10) Revenue From Tv Rights	11) Football Squad Value	12) Stage Property	13) For Foreign Share Capital

14) Prestige Of The Club	15) Team Sponsors	16) Average Coach Duration	17) Average Engagement	18) Average Replacement Time
0.449	0.471	0.401	0.487	0.278
0.786	0.801	12.473	7853.598	20.108
0.618	0.642	155.568	61679008	404.344
0.368	0.375	5.837	3675.6	9.411
0.176	0.179	2.789	1756.118	4.496
1.75	1.7	31.1	16114.949	72.319
2	1.5	30	14837.417	78.247
35	34	622	322298.99	1446.38
2	2	36	32506.035	72.399
3	3	48	32540.047	102.51
1	1	12	34.012	30.111

The average team of our sample, reading Table 7 from left to right, fielded 9.55 starters, suffered 2.65 defeats in the last 10 days, has a medium-low ranking 1.75 (we have included this in the model as a variable category with 1 = low ranking, 2 medium 3 high), 98.1 percent of its players have a regular contract, employed 42.5% of foreign players, realized on average capital gains of 36.264 million euros, losses of 17.672 million euros and a net capital gain of 53,937 million diueri, has received an average of 22.643 million euros from TV rights, has an average pink value of 426 940 million euros, on average its share capital is of foreign ownership for 31.45% , has a medium-low prestige 1.75 (this variable is also categorical on three levels) has an average of 1.75 sponsors and finally his coach has been coaching the team for 31.1 months, the average salary of his players is about € 16.114 ml per season and on average the first replacement took place at 72'[32]. In this section we try to answer the following questions: 1) in what direction and which factors affect the operational effectiveness of a team in achieving its goal in the worst case scenario for you, i.e. when you are engaging your organizational tactical efficiency in field to pursue a lower goal among the lows while all the other teams are engaging their level of efficiency to pursue the same goal but at a higher level? and vice versa what are the factors that influence operational effectiveness in the opposite scenario (optimistic for you)? To answer this question, we estimate model 2 in Table 3 as a Tobit (Kleiber, et al., 2008), using the operative efficacy score obtained with the DEA fuzzy 5 model solved at (Kao & Liu, 2000) as the dependent variable. both in the case of pessimistic and optimistic scenarios obtained in turn, with the efficiency scores deriving from fuzzy DEA game models. Another question we try to answer is instead 2) what factors influence the operational tactical efficiency of the game levels (game model 1) and positioning (game model 2) in pessimistic and optimistic scenarios and in what direction? We try to answer this question instead with model 1 of Table 2 also estimated as a Tobit. Turning to the interpretation of the results shown in Table 8a, b and c we can disaggregate the interpretation of the results as follows:

1) Influence of external factors on the operational tactical efficiency of worst-case and best-scenario levels of play with different values of α (Table 8a)
2) Influence of external factors on operational tactical efficiency in the worst and best scenario game vision with different values of α (Table 8 b)
3) Influence of external factors on operational effectiveness in worst and best scenarios with different values of α (Table 8 c).

Table 8a Model 1 Tobit Estimates for Effects of External Factors on Operating Efficiency in Game Levels

	Model 1 (Worst α=0)	Model 1 (Worst α=0.5)	Model 1 (Worst α=1)	Model 1 (Best α=0.5)	Model 1 (Best α=1)

[32] From a business point of view, for example, having your own stadium has a significant impact on the income deriving from tickets to the stadium (on average in a season for a Serie A Italian football team it can be equal to 20% of turnover). impacts on commercial and sponsorship agreements to sell the corresponding brand (on average also by 20%), TV rights vary greatly from team to team and from country to country as they depend on agreements with individual broadcasters and closely connected with European championships. This makes football a business that is not very cost-efficient. While for a commercial or industrial company according to the classical economy the main objective is to maximize profits for a football club the fundamental objective is to maximize victories.

(Intercept)	0.20666	1.09131	6.60945*	1.54830	5.71998*
	(1.38333)	(1.52324)	(2.85841)	(3.41547)	(2.89379)
Owners In The Field	0.03340	0.02159	0.00547	0.07491	0.01932
	(0.03006)	(0.03327)	(0.04504)	(0.04180)	(0.04645)
Defeats Last 10 Days	-0.03010*	-0.03127*	-0.01690	-0.02862	-0.03186
	(0.01354)	(0.01492)	(0.02384)	(0.03231)	(0.02786)
Positioning In The Ranking	-0.03437	-0.03412	0.01032	-0.04831	0.02594
	(0.04618)	(0.05120)	(0.07031)	(0.21680)	(0.07647)
Regular Contract Players	0.00011	-0.00610	-0.05532*	-0.02184	-0.05299
	(0.01185)	(0.01303)	(0.02588)	(0.03410)	(0.02848)
Percentage Of Foreign Holders	-0.00182	-0.00057	0.00422	0.00160	0.00225
	(0.00178)	(0.00196)	(0.00314)	(0.00591)	(0.00258)
Coach Average Duration	0.00140	0.00038	-0.00522	-0.00463	-0.00050
	(0.00335)	(0.00371)	(0.00549)	(0.00732)	(0.00600)
Average Engagement	0.00001**	0.00001*		0.00000	0.00001
	(0.00000)	(0.00000)		(0.00002)	(0.00001)
Average Time For Replacements	0.00143	0.00129	-0.00135	0.00381	0.00170
	(0.00186)	(0.00205)	(0.00310)	(0.00290)	(0.00330)
Log(Scale)	-2.29623***	-2.20223***	-1.99854***	-3.45547***	-2.00234***
	(0.17162)	(0.17079)	(0.23540)	(0.00000)	(0.23410)
Log(Average Engagement)			-0.01564		
			(0.03150)		
AIC	-7.65665	-4.76993	17.93993	17.23281	17.08456
BIC	2.30067	5.18739	27.89725	27.19013	27.04188
Log Likelihood	13.82833	12.38497	1.03003	1.38360	1.45772
Deviance	21.92569	21.42853	21.69124	5.88379	20.91933
Total	20	20	20	20	20
Left-Censored	0	0	0	0	0
Uncensored	18	18	11	2	11
Right-Censored	2	2	9	18	9
Wald Test	19.55845	10.60869	5.50698	11.23246	5.27698

***P < 0.001, **P < 0.01, *P < 0.05

Table 8b Tobit model1 estimates for the effects of external factors on operational efficiency in game viewing

	Model 1 (Worst α=0)	Model 1 (Worst α=0.5)	Model 1 (Worst α=1)	Model 1 (Best α=0.5)	Model 1 (Best α=1)
(Intercept)	0.14884	0.36450	0.71043	0.68159	0.87103
	(1.27304)	(1.39769)	(1.64407)	(0.58113)	(1.00943)
Owners In The Field	0.00476	0.00743	0.01564	0.00085	0.00082
	(0.02688)	(0.02963)	(0.03419)	(0.01227)	(0.02131)
Sconfitte Ultime10gg	-0.00451	-0.00421	-0.00701	0.00290	-0.00382
	(0.01223)	(0.01343)	(0.01606)	(0.00558)	(0.00970)
Positioning In The Ranking	-0.01454	-0.01527	0.01615	-0.00373	-0.00194
	(0.03991)	(0.04538)	(0.05101)	(0.01822)	(0.03164)
Players With Regular Contracts	0.00167	-0.00095	-0.00602	-0.00790	-0.00370
	(0.01069)	(0.01173)	(0.01409)	(0.00488)	(0.00848)
Percentage Of Foreign Players	-0.00210	-0.00237	-0.00032	0.00092	0.00045
	(0.00183)	(0.00202)	(0.00230)	(0.00083)	(0.00145)
Coach Average Duration	-0.00410	-0.00368	-0.00085	-0.00255*	-0.00214
	(0.00273)	(0.00307)	(0.00348)	(0.00125)	(0.00217)
Log(Average Engagement)	0.04573*	0.04598*	0.03549	-0.00061	0.02233
	(0.01877)	(0.02078)	(0.02362)	(0.00857)	(0.01489)
Average Time In Substitutions	0.00116	0.00324	0.00527*	0.00152*	0.00350**
	(0.00166)	(0.00183)	(0.00221)	(0.00076)	(0.00132)

Log(Scale)	-2.38529***	-2.29281***	-2.24060***	-3.16947***	-2.61730***
	(0.15811)	(0.16446)	(0.22137)	(0.15811)	(0.15811)
AIC	-18.65395	-11.17989	11.73127	-50.85463	-27.93452
BIC	-8.69663	-1.22257	21.68859	-40.89731	-17.97720
Log Likelihood	19.32698	15.58995	4.13437	35.42732	23.96726
Deviance	20.00000	21.02735	23.45124	20.00000	20.00000
Total	20	20	20	20	20
Left-Censored	0	0	0	0	0
Uncensored	20	19	12	20	20
Right-Censored	0	1	8	0	0
Wald Test	14.99837	12.19567	13.29694	25.87899	20.89117

***P < 0.001, **P < 0.01, *P< 0.05

Table 8c Model 2 Tobit Estimates for Effects of External Factors on Operational Effectiveness

	Model 2 (Worst α=0)	Model 2 (Worst α=0.5)	Model 52 (Worst α=1)	Model 2 (Best α=0)	Model 2 (Best α=0.5)	Model 2 (Best α=1)
(Intercept)	-19.1735**	-12.5048**	-19.3680**	-5.87266	2.34380	0.75656
	(6.92090)	(3.82854)	(7.25734)	(3.90070)	(26.35830)	(5.53668)
Log(Capital Gain)	1.53950***	0.99578***	1.42307**			
	(0.46659)	(0.25352)	(0.47838)			
Log(Revenues From TV Rights)	-0.57732	-0.30915	-0.38685			0.08714
	(0.36717)	(0.19238)	(0.37207)			(0.23192)
Log(Pink Value)	-0.22078	-0.29873	-0.19199		-0.74203	-0.28586
	(0.29826)	(0.16072)	(0.29901)		(1.54478)	(0.27361)
Positioning In The Ranking	0.42573	0.17420	0.49296*	0.59890	10.21392	0.79598**
	(0.23292)	(0.13204)	(0.23447)	(0.62791)	(6.25614)	(0.25892)
Owned Stadium	-2.3977***	-1.7224***	-2.2585***	-1.22971**	-2.79645	-0.87073
	(0.46615)	(0.28699)	(0.48517)	(0.39936)	(3.16084)	(0.53274)
Regular Contract Players	0.13376*	0.09527**	0.12505*	0.06992	0.18091	0.04546
	(0.06128)	(0.03195)	(0.05867)	(0.03996)	(0.28432)	(0.05223)
Team Sponsor Number	0.38575*	0.32960***	0.42636*	0.11031	0.84896	0.13769
	(0.16695)	(0.09758)	(0.17800)	(0.14619)	(1.11800)	(0.17430)
Percentage Of Foreign Share Capital	0.02569*	0.01652**	0.02021	0.00885	0.02520	0.00842
	(0.01065)	(0.00585)	(0.01111)	(0.00645)	(0.05653)	(0.00952)
Prestige Club	-0.93020**	-0.6047***	-0.95694**	-0.81281	-10.13584	-0.87324**
	(0.28419)	(0.15680)	(0.29148)	(0.63160)	(6.29229)	(0.30718)
Log(Scale)	-0.8375***	-1.3982***	-0.7801***	-1.4722***	0.62755**	-0.7133***
	(0.17819)	(0.19083)	(0.17958)	(0.20300)	(0.21160)	(0.15811)
Capital Gain				0.00002*		
				(0.00001)		
Revenue From TV Rights				-0.00001	-0.00000	
				(0.00001)	(0.00008)	
Pink Value				-0.00000		
				(0.00000)		
Loss					-0.00048	-0.00009
					(0.00063)	(0.00009)
AIC	108.07904	40.60558	109.20215	90.64179	130.85404	95.57867
BIC	119.03210	51.55863	120.15520	101.59484	141.80710	106.53172
Log Likelihood	-43.03952	-9.30279	-43.60107	-34.32089	-54.42702	-36.78933
Deviance	22.26432	24.72826	22.40464	26.72406	28.30024	20.00000
Total	20	20	20	20	20	20
Left-Censored	0	0	0	0	0	0
Uncensored	20	20	20	20	20	20
Right-Censored	0	0	0	0	0	0
Wald Test	67.55520	59.16615	57.83662	15.82157	4.79745	14.37441

$^{***}P < 0.001$, $^{**}P < 0.01$, $^{*}P < 0.05$

The Tobit models of Tables 8a and 8b have as dependent variable (intercept) the efficiency scores of game model 1 (game levels) and positioning model 2 (game vision) with input orientation and crs assumption both solved with the approach (Kao, et al., 2000). In Table 8c the estimated Tobit models have as their dependent variable the efficacy scores deriving from the 5 DEA fuzzy model with output orientation and crs assumption resolved with the approach (Kao, et al., 2000). Let us now begin the interpretations on the influence of external factors on the levels of tactical efficiency in the level of play of Table 8a and note that: 1) the number of losses recorded in the last 10 days has a negative effect, that is, a unit increase in the number of " losses in the last 10 days ”reduces the level of tactical efficiency of the level of play in both the worst and best scenarios, even if the significant statistical evidence only affects the worst scenario with alpha = 0 and 0.5.A plausible explanation may be that if in the past (last 10 matches) the team lost many matches this could already be attributed to a low level of tactical efficiency of the game and that a further defeat in that period would have further reduced efficiency. operational tactics of the game level. In our opinion this could be due to an emotional effect of demoralization and therefore the team performs worse even after the period considered just passed. We also observe that the average player engagement, when the team is playing worse than its opponents (pessimistic scenario), has a small but positive effect on operational efficiency in the levels of play. An explanation may lie in the fact that incentivizing the players more from an economic point of view when the team performs poorly could increase the average level of operational efficiency in the field due to a greater commitment linked to an economic motivational aspect.So there can be both a 'euphoria' effect and a greater commitment to the best-paid players. Finally, another external variable that appears to have an influence on the operational efficiency of the game levels in the pessimistic scenario is the number of players on the pitch with regular contracts. Also in this case, a possible explanation is that a greater number of players on the pitch with a regular contract do their job better, dragging the team to higher levels of operational efficiency. Turning to Table 8b, to interpret the effects of external variables on the operational tactical efficiency in the game vision, we see that also in this case the average engagement has a statistically significant and positive effect in the pessimistic scenario. This time, however, the interpretation is percentage, given that the variable is inserted with its logarithmic transformation. And therefore a unitary percentage increase in the average engagement increases operational efficiency in the game vision. Therefore, a first general conclusion is that increasing the average salary of the players improves sporting performances both in the execution of passes and in the view of the game when the team was playing worse (pessimistic scenario for her).The policy indication could be to "increase the average player engagement when the team performs at low levels of operational efficiency in order to increase the possibility of having a higher level of operational efficiency for both levels of play. The variable "average time in replacements" or the average time expressed in minutes in which a replacement takes place in the field, in particular for the first replacement, increases the operational efficiency of the position model in both the pessimistic and optimistic scenarios. A possible explanation for this is that leaving the same formation on the pitch for longer benefits the team itself in terms of exploiting the areas of the pitch. And this effect is greater in the pessimistic scenario for the team under evaluation. And as if giving the team time to fuel by avoiding replacing players during a match is a choice with greater benefits than that instead of making substitutions when the team is playing worse (well-paid players like to play).A policy indication is "both when the team performs well and when it performs poorly, wait for substitutions (at least up to 74 minutes for our sample)" is on average the best policy to implement. Finally, let's explore which external factors influence the team's operational effectiveness. In this regard, in Table 8c we note that: 1) a greater capital gain from the sale of players always has a positive effect on the operational effectiveness in achieving the objectives. Intuitively, a greater capital gain means having greater financial resources for the club that can be reinvested to pursue the objectives more effectively through the acquisition of other more promising players to replace others. Having a stadium owned both in the worst scenario (having achieved lower goals) and in the best one (i.e. having achieved higher goals) helps the club to reach the ranking objectives (remember that in this model Tobit the intercept is the efficacy score obtained from model 5 with output orientation). Ultimately we note that to favor greater effectiveness, especially in the pessimistic scenario are a) the number of players with regular contracts on the field, b) a greater number of sponsors that a team manages to attract, c) a greater percentage of foreign capital in the social capital of the football club, and finally d) the prestige of the club.

4.3 The effects of managers' attitudes on operational efficiency

As we have seen in the model of (Léon, et al., 2003) the parameter h is a degree of possibility of reaching a certain level of possible efficiency fixed a priori in the model. If this value of h can be in some way decided or in any case influenced by managers or whoever works for it, then it captures the different attitude towards uncertainty from these actors. A manager who wants to obtain a greater possibility of achieving a certain result of tactical efficiency is accepting in terms of the DEA fuzzy model a lower uncertainty if not certainty since with $h = 1$ the DEA fuzzy model corresponds to the DEA deteministic model. Tactically organizing the team so as to achieve at least a certain level of operational efficiency on the pitch in the two game levels with the same field results is reflected in the value of h in the model of

(Léon, et al., 2003). An interesting question to answer may then be to what extent does the different attitude of football mangers towards uncertainty (according to our more lenient interpretation as h decreases) generates differences (inequalities) between performances on the pitch? The way we propose here to study them is to calculate the Gini coefficient at the operational tactical efficiency scores of models 1 and 2 solved with models (Léon, et al., 2003) for the different levels of h to measure the degree of inequality. . In Table 9 we report these values while in Figures 8 and 9 we report the Lorenz curves.

Table 9 Operational efficiency inequalities in play and positioning levels.

Models	Gini Coefficient				
	H-Level				
	0	0.25	0.5	0.75	1
Model 1 (Game Levels)	0.04021703	0.04465133	0.04879757 0	0.05334839	0.058220
	Range=[0.04021,0.0582];	Diff=0.0180			
Model 2 (Positioning)	0.04521358	0.04813325	0.05154476	0.05498856	0.058711
	Range=[0.04521,0.05871],	Diff=0.1349			

In particular, Table 9 reports the Gini index for the levels of operational efficiency in the case in which both model 1 (created to evaluate efficiency in game levels) and model 2 (created to evaluate efficiency in occupy the field spaces) are solved with orientation to the input and with the assumption of crs. We remind you that the input orientation of the operational tactical efficiency models proposed here give indications of how to improve the organization of the team from the point of view of the characteristics of the players, their skills and their talents in order to reduce the number of passes, crosses and throughers with the same game results (goals scored and shots on goal) (game model 1) and with game model 2 improve the organization of resources in the field in order to make the most of the areas of the field avoiding for example, overcrowding always with the same game results. In both models it has been assumed that the operational dimensional scale of the team cannot be changed and that they are operating at constant returns to scale.[33]. It is from these points of view that the Gini indices in Table 9 must be read. For the effectiveness models the point of view is instead that of transforming the levels of operational efficiency into objectives (blurred). Usually the setting of the objectives is the task of the managers while the coaches have the one to better manage the resources on the pitch to obtain "field" results. Having clarified this we see that the inequality in relative game performance increases with the level of h in both models. An interpretation of this result is that with managers of all teams less risk averse (i.e. who prefer lower performances but more possible given a certain degree of tolerance of violation of the constraints, $h = 0$) greater inequalities in the performance of the teams are generated. In principle, this can be interpreted as an increase in the competitiveness between the teams as in contrast to the managerial preferences just clarified there would be the preferences of the coaches who instead seek a higher level of operational efficiency as their prestige is linked to this. their image, their reputation etc. pushing the teams both to improve the game levels and the vision of the pitch spaces. This interpretation, in our opinion, would bring out a trade-off between managerial preferences and coaches' preferences on how to organize a team on the pitch and on the objectives to be pursued.

Insert Figure 9 Lorenz curve for the efficiency scores on the play levels of the model
Insert Figure 10 Lorenz curve for the efficiency scores on the positioning model obtained from the model

5.Discussion

The paper addressed the issue of how to improve the sports performances of a sport team or an athlete by proposing, in our opinion, an innovative and original sports data analysis framework based on the DEA modeling in the presence of uncertainty. To this end, based on the numerous statistics offered for professional football matches, we have built a virtual sample of 20 professional football teams and applied two fuzzy DEA models. In general, this approach is closely linked to the theory of production according to which each decision-making unit (for us the football team and the club) can be attributed a production technology. For example (García-Sánchez, 2007) in his work he indicates 4 different approaches that have been used to define the production function in sport. The first looks at the match (football or baseball) as the unit of reference and efficiency concerns the level of play using as inputs such as ball possession, corner kicks and so on[34], and the match results as output. The second approach focuses on the efficiency of managers, since the

[33] In the output models, on the other hand, the indication refers to improving the organization of the team to maximize the results on the field (goals scored and shots on goal) with the same levels of play and without changing the vision of the game understood as the ability to exploit the areas of the field.
[34] The players are the inputs of a production function, often inputs that are not perfectly replaceable, think of the top player and the rest of the players

variation in results is attributed to the responsibility of the coaches. The third approach looks at the level of X-inefficiency of athletes. And finally the fourth focuses on the overall efficiency of the team in each season. Here we have adopted as a reference unit the football teams that take the field to define a first technology (Appendix A) and the club to define a second one. The first technology looked at the production of results in the playing field (shots on goal and goals scored) with the use of passes and occupation of the field areas, the other looked at the production of (quantitative) sporting objectives of the clubs using as input the performance of the soccer team on the pitch. We then measured the relative technical efficiency of these technologies by incorporating uncertainty into them through fuzzy logic (Zadeh, 1975). As happens in (García-Sánchez, 2007) we looked at the levels of play, however separating the technical/tactical performance of the players during the execution of passes (model 1 of play) on the one hand and the technical/tactical performance for the development of a game view (game model 2) from the other. Basically we looked at what in reality is a unitary and complex technology that uses passes and areas of the pitch in an interrelated way as two separate technologies that produce the same categories of outputs (goals scored and shots on goal) but use different sets of inputs. (number of different types of passages and percentage of space occupation). So, for each soccer team we observed different production plan, for example team 1 produced a certain number y_{11} and y_{12} of output (shots on goal and goals scored) with a certain number $x_{11}, x_{12}, x_{13}, x_{14}$ inputs (crosses, filtering, long passes and short passes), while team 2 produced a certain number y_{21} and y_{22} output with a certain number $x_{21}, x_{22}, x_{23}, x_{24}$ inputs and so on for remainig teams. In other words, we processed sports data produced by sports statistics for professional football teams using the DEA methodology in order to measure the relative performances (in particular the technical efficiency) of professional football teams assuming uncertainty on those observed production plans. In fact, a possible source of uncertainty can be attributed to errors on the pitch committed for example by a player who with a long passages tries to intercept his teammate but fails on the first passage, thus making his individual performance uncertain and therefore performance uncertain technical / tactics of the whole team who will use an uncertain number of passes during the whole match. But also the data can be the cause of inaccurate measurements due to measurement errors.The approach used here is, in our opinion, of the hybrid type compared to the 4 mentioned (García-Sánchez, 2007). In fact, in part, with game models 1 and 2, it fits into the first approach described by (García-Sánchez, 2007) adopting the football team as a reference unit and attributing to it a production plan (for a given technology production) inferable from game models 1 and 2 in which the technical/tactical performance of the players are its inputs through which two types of products are produced: shots on target for the opponent and goals scored (therefore the work does not fall into the analysis of X-inefficiency (Leibenstein, 1966) (Ariyaratne, et al., 2000)). In part, however, it also falls into the fourth approach with models 3,4, and 5 and 6. With these latest models we have concentrated in fact on achieving certain categories of sporting objectives that a professional football club can pursue. With models 3,4,5 and 6 we therefore evaluated the operational effectiveness, while with models 1 and 2 the operational efficiency in the game levels (García-Sánchez, 2007). In particular, with models 5 and 6 we assessed the operational effectiveness in terms of the ranking objective by adopting the fuzzy logic (Zadeh, 1975) in order to define this objective in an uncertain way as a manager of a professional soccer team. With this model, the production technology has as input a certain level of operational efficiency of the team with a given organizational structure and as output the classification objective treated as uncertain. To decide which variables to use in our DEA models, we then referred to the numerous statistics that are produced for football teams and in particular we referred to the literature on the subject. Differently from the reference literature we used fuzzy DEA models to measure soccer team performance assuming uncertainty about the execution in the technical/tactical performance of the players and in the setting of goals. Thus alluding to the existence of some of the reasons that in fact could lead us not to resort to the theory of neoclassical production and its analyzes (Leibenstein, 1966) such as when footballers (workers) are not encouraged to give the maximum effort. As for the operational efficiency in the game levels we have thought of two valid models both for the attack phase of the teams in which the first considers as input the tactical/technical skills used in the attack phase to kick the ball (making passes, assists, etc.) and develop the game on the pitch in order to achieve the greatest number of goals scored and shots on goal as a result. The second game model we developed, on the other hand, considered the "game vision" intended as a tactic for occupying the spaces on the pitch during the attack phase and therefore used the percentages of use of the three areas of the field, left, as input variables. right and center. Also for this latter model, the outputs considered always concerned field results, goals scored and shots on goal. Models 3, 4, 5 and 6 which instead refer to operational effectiveness (García-Sánchez, 2007) differ from each other due to the fact that the first (model 3) considered the points actually achieved as targets. from each single team, and the second (model 4) has set as a single exact objective for all teams (target) the maximum achievable score in a championship. Models 5 and 6, assuming uncertainty about the same objectives, described them in a blurred way by inserting them in the DEA fuzzy model as asymmetrical triangular fuzzy numbers. The results of the operational efficiency models are of two types: one of a possible type (Léon, et al., 2003) and the other defined pessimistic and optimistic scenarios for different values of the alpha cut (Kao, et al., 2000). In (García-Sánchez, 2007) among the reasons that can lead to differences in operational efficiency between teams can be attributed: 1) to the different ability of coaches and managers to organize the resources at their disposal, 2) the inadequacy of the operational dimension to which the team was organized [35]. This last point refers to the scale dimension in which increasing and descending as well as constant returns to scale occur. Here we have always reported the results for the DEA models with constant scale back assumptions, thus assuming a short-term view (production technology with fixed inputs) as we have assumed that team scale adjustments cannot occur instantly. .A scaling adjustment in the presence of increasing returns to scale (see Appendix B) would mean that an increase in all the types of passages that the team performs on average corresponds to a more than proportional increase in the results.. It is also evident that the differences in the productivity of the teams can also manifest themselves in the presence of the same managerial ability to organize the team. It is not uncommon in fact that for example the same coach with the same skills reaches different performances in two different teams. And this effect is in our opinion largely attributable to uncertainty as well as obviously to the presence of external factors. Differently from the work of (García-Sánchez, 2007 [36]), here we have considered only the attacking phase of a team and not the defensive one. From a policy point of view, our work is close to that of (Bosca, et al., 2009) in the sense that our models are useful, in our opinion, to coaches and managers to understand how the technical efficiency of teams in attacking movements and in different situations (for example in pessimistic and optimistic game scenarios) it is linked to the more general performance in achieving objectives (points in the standings and results on the pitch). The combined reading of the results of game models 1 and 2 with the effectiveness models 5 and 6 respond to this unified need. Furthermore, our game models (1 and 2) are very close from the point of view of the inputs and outputs considered to those of (Bosca, et al., 2009). Keeping in mind that both operational efficiency and operational effectiveness can be influenced by external factors and that different attitudes on the part of managers on the possibility of obtaining certain results influence the distribution of efficiency and effectiveness levels, generating a greater or lesser degree of competitiveness in tournaments (and an example of this is the analysis of the gini coefficients on inequalities conducted in section 4.3) the paper also considered a second analysis structured in two parts. The first adopts an econometric model to estimate the effect of external factors on operational tactical efficiency and operational effectiveness while the second part of the second stage analysis calculated the gini coefficient to measure the inequality in the distribution of relative performances with respect to different attitudes that managers may have about uncertainty. The results of the fuzzy DEA game and goal models therefore show a great variety of results and therefore a great variety of interpretations and policy indications to improve the performances of football teams. All DEA game models estimated with input orientation give, for example, indications on how to improve game levels in the different types of passes (eg number of passes[37]) for each level of field results, or give indications on how to optimally use (and relative) the use of the field areas (game models 2) when in different scenarios overuse and underuse of the parts of the field can be achieved. With reference to the models solved in (Kao, et al., 2000), the results must be interpreted in two different scenarios: the pessimistic and the optimistic. On the other hand, when we refer to the models a la (Léon, et al., 2003) the efficiency results must be interpreted in a possible sense. However, the policy indications deriving directly from the fuzzy DEA models cannot be considered without taking into account the possible influence of external factors on both operational efficiency and operational effectiveness. For example, we have seen that for some team that were inefficient in the pessimistic scenario, the policy indication deriving from the fuzzy DEA model was to improve the organization on the field from the point of view of the game vision (and this can happen for example through the replacement of players with better vision). At the same time, however, we have seen in the second stage analysis that the level of efficiency can be positively influenced by the average replacement time (Table 8b). Therefore, a policy indication may be to wait (ie increase the average replacement time) in replacements. It should be noted that with the DEA models the policy indications concern the single team while with the second stage the indications are valid on average. Finally, it may happen that the team plays according to the more or less implicit preferences of managers and coaches on the possible

[35] The results in the case of returns of varying scale were not presented. It should be noted that these latter models estimate boundaries which in addition to having traits with constant returns to scale (i.e. in these traits it is assumed that an increase in all the inputs leads to a proportional increase in all the outputs of the model) increasing scale (an increase of all inputs leads to a more proportional increase of all outputs of the model)) and decreasing scale (an increase of all inputs leads to a less than proportional increase of all outputs of the model)). In football sports teams, for example, increasing returns to scale can be found, for example in the presence of champion degrees when it is ascertained that the purchase of a large sample increases the game results more than proportionally, while returns of increasing scale can be linked to coordination problems on the pitch, finally there can be constant returns of scale when increasing the level of play does not correspond to an increase or decrease in productivity on the field, both the average and marginal productivity of the factors remain constant.

[36] (García-Sánchez, 2007) adds a third dimension which is social effectiveness understood as the link between the results on the pitch, the rankings that attract a greater number of fans.

[37] In reality, the coaches can give directions to each player on 1) increase or decrease the minimum number of passes to be made before being able to score in one of the areas indicated 2) limit the number of touches available (for example, let a maximum of three play touches), 3) give constraints to the joker player (play first, only right, only left, etc.). For the coaches, other issues are also important such as the quality of passing and movement with and without the ball, indicating to the attackers that they must spread out to cover as much space as possible, play wide, giving an indication to the defenders that they must cover the spaces trying to narrow the playing area as much as possible and by cutting as many passing lines as possible, keep the game intensity high https://www.youcoach.it/en/article/pass-and-movement-attacking-techniques

levels of efficiency and this aspect has been attempted to grasp it with the DEA fuzzy models (Léon, et al., 2003). In other words, preferences for higher constraint violations could lead to higher but less possible levels of efficiency. This choice (regarding h) could theoretically be made by many or all managers of teams participating in the same tournament. And the first reflection that emerges is that this can change the degree of competitiveness of the tournament as there would be more aggressive teams and concentrate on not violating the constraints and more accommodating teams. This could generate different distributions of team performances within a tournament. Assuming that this reasoning is valid, we used Gini's measure of inequality to measure inequalities in performances in two scenarios: the pessimistic and the optimistic one for the results of models 1 and 2 at (Kao, et al., 2000) (Table 9). The information obtained is that greater inequality towards low scores could be the result of less competitive teams and therefore a less competitive and dynamic tournament. A focus on high efficiency scores with low inequality in efficiency levels could instead be the result of highly competitive teams and a tournament with a high degree of dynamism. In conclusion, the work was presented as a proposal for an analysis framework for the evaluation of the relative performances of football teams based on the DEA fuzzy methodology in order to take into account blurred objectives. In fact, the use of fuzzy logic (Zadeh, 1975) in the DEA methodology allows us to treat inputs, outputs and objectives in a blurry way to accommodate uncertainty in a different way from what can be done with probabilistic logic. We have thus found a wealth of interpretative results and policy indications. All analyzes were performed with the statistical software R (RCoreTeam, 2016), the deaR package for DEA fuzzy models (Coll-Serrano, et al., 2019), and the ineq package (Zeileis, 2014) for inequalities of Gini.

6.Conclusions

The work considered professional football clubs as an example and after having amply illustrated the importance of measuring sports performance for them, it proposed a framework for analyzing sports data referring to football team matches in order to measure operational efficiency of the collective in the field but also to measure the sporting effectiveness of the club in a context of uncertainty. We have in fact underlined that the measurement of sports performances both of individual athletes (within a team or individually considered) and of an entire team is a crucial moment for professional sports clubs as their survival over time is closely linked to both results on the playing field (with the intention of winning the match) obtained by individuals and/or by the team and both to the achievement of other objectives of a sporting nature such as positioning in the standings, or the qualification or/and the shift to international tournaments and so on[38]. The basic question addressed here is the following: Was a professional football team efficient in achieving its field results? That is, compared with that of the other teams, the ratio between goals scored and shots on target by the opponent and the number of passes and occupation of the pitches during a championship season was relatively higher or lower than the same ratio for the other teams that participated in the same sporting tournament during a football season (was it more or less efficient than them?) And, again in relation to the other teams, his level of operational efficiency and degree of effectiveness allowed him to achieve respect algi ranking goals programmed by the manager? To answer these questions: 1) we have described the game of football in its essence with "game models" (see Figure 2), 2) we have attributed to each soccer professional club a production technology for its soccer team (see appendix A) and a production technology to the whole club, and finally 3) we have built and solved several fuzzy DEA models to evaluate, using sports data, such as game statistics (see note 4), both the operational efficiency of the team and the degree of operational effectiveness of the club (with the second technology). For measurement needs and above all to separate the policy indications useful for the team we have solved two DEA fuzzy models. A first game model that we called of "passes" and a second that we called of "position" (DEA fuzzy models 1 and 2). These two DEA models have been proposed to measure the operational efficiency for the two levels of play of football teams (García-Sánchez, 2007) intended as an expression of technical ability in performing different types of passes (game models 1) and as an expression of the capacity to occupy the spaces on the pitch (game model 2). Therefore, the efficiency improvements for these two game models require adjustments in the number of steps to be performed and in the percentage of use of the field spaces. The second type of model that we have called instead of strategic "objectives", and to which the models belong DEA, 3,4,5 and 6, of which 5 and 6 resolved as DEA fuzzy models, was designed to measure the operational effectiveness of the club (García-Sánchez, 2007). In other words, to what extent the club, given a football team with a fixed organizational asset that guarantees a given level of operational efficiency in the field, has been able to achieve the (blurred) ranking objectives set by the managers. However, the main feature of this work was that of having dealt with the uncertainty (which can also be determined by measurement errors in the statistical data) in DEA models (section 3.3) using fuzzy logic (Zadeh, 1975) to analyse sport data. According to

[38] Examples of international tournaments for individual athletes are the Australian Open, Wimbledon, Roland Garros and US Open tennis tournaments organized by the ATP (Association of Tennis Professionals) and its ranking or those organized by the Women's Tennis Association (WTA) for tennis players. professional, or examples of national tournaments for teams is the men's basketball tournament organized by the Italian Serie A Basketball League and the women's one (Serie A1), or the Italian Serie A women's football championship.

this logic (whose theory and application is present in (Zimmermann, 2011)) uncertainty can be described with linguistic categories of a blurred type (Zadeh, 1975) which, for example, lend themselves to using linguistic expressions such as "all ' approximately "," close to "," more or less to "and so on. It goes without saying then that each of these expressions has a different value and meaning from team to team. So for example with the game model 1 the reading of its results is: with approximately: x_1 cross, x_2 long passes, x_3 short passes and x_4 throughs the i-th team has scored y_1 goals and approximately y_2 shots in leads to the opposing team thus reaching a relative fuzzy efficiency level of θ. We have therefore translated these vague expressions within our DEA fuzzy models into fuzzy numbers for both inputs and outputs using both symmetrical and asymmetric triangular fuzzy numbers. We have solved the DEA fuzzy models using both the approach proposed by (Kao, et al., 2000) and the possibilistic approach proposed by (Léon, et al., 2003). We have solved with both approaches both models 1 and 2 (game) to measure operational efficiency (García-Sánchez, 2007) and models 5 and 6 to evaluate operational effectiveness (García-Sánchez, 2007). The fuzzy DEA models solved with the approach (Kao, et al., 2000) allowed us to build two different scenarios, one pessimistic and one optimistic, in each of which by adopting the alpha cut technique we obtained the relative performance measurements of football teams in many different pessimistic and optimistic scenarios. The approach instead of (Léon, et al., 2003) allowed us to obtain possible measurements of relative performances. In our opinion, all the relative performance measurements and second stage analyzes are useful to give indications of tactical / organizational improvement of the team and the pursuit of objectives for the club, answering questions such as: considering that the team.

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Appendix A

A production technology for a professional football team and its representation

The production technology for a generic decision-making unit can be mathematically represented through the technological set T (Bogetoft, et al., 2019):

$$T = \{(x, y) \in R_+^m \times R_+^n | \; x \; può \; produrre \; y \; \} \quad (1)$$

Where x and y indicate respectively the vector of inputs and that of outputs with m inputs used and n outputs produced. In the case of a professional football team we here define the vector x of the inputs as consisting of: 1) cross, 2) filtering, 3) long passes, 4) short passes, 5) percentage of the right field space used, 6) percentage of left field space used and 7) percentage of center field space used. While the vector of the outputs produced by a football team consists of: 1) goals scored against the opponent and 2) shots on goal against the opponent. For us, this will be the homogenous manufacturing technology for each team. In this way we are representing the production technology of a football team through the technological sets (T) without specifying the precise relationship that exists between the inputs and outputs of a football team (production function). On this technology we introduce the standard assumptions that are usually made on a production technology: 1) free availability of inputs and outputs, 2) convexity and 3)) returns to scale. The assumption of free availability we do for both (inputs) and (outputs) implies that if a team produces a certain amount of goals and shots with a certain amount of crosses, throughs, long passes, short passes and a certain amount of amount of percentage of occupation of the spaces of the right, left and central fields then it can produce the same amount of goals and shots with more crosses, with more filtered, with more long and short passes and with more percentage of occupation of the spaces of the field. This is the assumption of free availability of inputs. If, on the other hand, a football team with a given amount of input (crosses, throughs, long passes, short passes, percentage of right field space, percentage of left field space and percentage of center field space) produces a certain amount of output (goals scored and shots on target) then the team can use the same amount of those inputs to produce less output (i.e. it can have the surplus of goals scored and shots on target). This is the assumption of free availability of the outputs. We can represent this last assumption as follows: if (x_1(cross)=10,$y_1(shots\ on\ goal)$=5)∈T and if $x_1' \geq x_1$ e $y_1' \leq y_1$ then $x_1', y_1' \in T$. In this work, however, we assumed uncertainty about the production technology of the teams using fuzzy logic (Zadeh, 1975) and therefore we referred to fuzzy sets (Zimmermann, 2011). Therefore indicating with $\tilde{x}, \tilde{y}$ the fuzzy sets of inputs and outputs of a team with membership function $\mu_x(x)\ e\ \mu_y(y)$ we can write the fuzzy technological ensemble in a general way $\tilde{T}$ as follows:

$$\tilde{T} = \{(\tilde{x}, \tilde{y}) with\ support\ in\ R_+^m \times R_+^n\} \quad (2)$$

With the introduction of fuzzy sets, the assumption of free availability therefore involves the inequality relations for fuzzy isies to which various answers have been given in the literature (Rommelfangem, 1996). In general then and without going into details we can transform that assumption as follows: if $(\tilde{x}_1, \tilde{y}_1) \in \tilde{T}$ and if $\tilde{x}_1' \geq \tilde{x}_1$ e $\tilde{y}_1' \leq \tilde{y}_1$ then $(\tilde{x}_1', \tilde{y}_1') \in \tilde{T}$. When we have two possible production plans, a further assumption that we can introduce is that of convexity. According to this assumption, if for a football team there are two possible production plans that belong to the technological whole T, $(x^0, y^0) \in T\ e (x^1, y^1) \in T$, then also a weighted average of them belongs to T. More formally $\forall \lambda [0,1]\ we\ have\ \ \lambda(x^0, y^0) + (1-\lambda)(x^1, y^1) \in T$. Other assumptions that can be introduced instead concern the returns to scale. And so a technology exhibits constant returns to scale if with $\lambda \geq 0$ for each $(x, y) \in T\ we\ have\ that\ \lambda(x, y) \in T$. With $0 \leq \lambda \leq 1$ on the other hand, there are non-increasing returns to scale and with $\lambda \geq 1$ non-descending returns to scale. In other words, if a football team increases by the same level the number of its crosses, through filters, long and short passes and the percentage of occupancy of the field spaces achieves a proportional increase in all its outputs such as goals and shots in leads to the adversary then its production technology presents constant returns to scale, while if it presents a more than proportional increase in its outputs it presents increasing returns to scale and vice versa if it presents less than proportional increases in all its outputs then there are returns of scale describing. With due consideration, all these assumptions can be introduced in the case of fuzzy technologies. In this case, all the variables that define the technology must be considered fuzzy variables and all relations and operations must be traced back to the theory of fuzzy sets (Zimmermann, 2011) and adapted to DEA models (Emrouznejad, et al., 2014) (Guo , et al., 2001) (Kao, et al., 2000) (Meada, et al., 1998) (Hatami-Marbini, et al., 2011). And therefore a whole $\tilde{X} = \{x, \mu_x(x)\}$ fuzzy is convex if for each $x_1, x_2 \in \tilde{X}$ and each $\lambda[0,1] \rightarrow \mu_x(\lambda x_1 + (1-\lambda)x_2) \geq \min\{\mu_X(x_1), \mu_X(x_2)\}$ (Zadeh, 1965). To give an example limited to a one-dimensional set reported in the text if the Cross set is fiuzzy set like this [(26,0.3), (27,0.6), (28,0.85), (29,1), (30, 0.5), (31,0.3), (32,0.1)] with its membership function $\mu_{CROSS}(x)$ then given two values that belong to it $x_1(=27), x_2(=32) \in Cross$ this set is covesso se for each $\lambda[0,1] \rightarrow \mu_{CROSS}(\lambda \cdot 27 + (1-\lambda)32) \geq \min(\mu_{CROSS}(27), \mu_{CROSS}(32))\ that\ is \geq \min(0.7, 0.3)$ or ≥ 0.3 and therefore if $\lambda = 0.5$ we have that $\mu_{CROSS}(O.5 \cdot 17 + 0.5 \cdot 32) \geq 0.3\ or\ \mu_{CROSS}(29.5) \geq 0.3$. Putting together the two assumptions of convexity and free availability, the new deterministic T technology can be represented as follows:

$$T = \{(x, y) |\ x \geq \lambda^1 x^1 + \lambda^2 x^2 + \cdots + \lambda^j x^j\ and\ y \leq \lambda^1 y^1 + \lambda^2 y^2 + .. + \lambda^j y^j\ con\ \lambda^1 + \lambda^2 + .. + \lambda^j = 1\ e\ (\lambda^1, \lambda^2, \ldots, \lambda^j) > 0\} \quad (3)$$

Of course we can also get only "weakly" more inputs and outputs by replacing $\geq, \leq$ with $>, <$. Applying the fuzzy theory (Zimmermann, 2011) the technological ensemble in 3 can be expressed as a fuzzy set as follows:

$$\tilde{T} = \{(\tilde{x}, \tilde{y}) |\ \tilde{x} \geq \lambda^1 \tilde{x}^1 + \lambda^2 \tilde{x}^2 + \cdots + \lambda^j \tilde{x}^j\ and\ \tilde{y} \leq \lambda^1 \tilde{y}^1 + \lambda^2 \tilde{y}^2 + .. + \lambda^j \tilde{y}^j\ con\ \lambda^1 + \lambda^2 + .. + \lambda^j = 1\ e\ (\lambda^1, \lambda^2, \ldots, \lambda^j) > 0\} \quad (4)$$

That is, in the case of a football team if we consider a simple technology with an input (cross) and an output (shots made) we have the following fuzzy technology with assumption of free availability of inputs and outputs and convex technology:

$$\tilde{T}_{soccer\_team} = \{(\widetilde{cross}, \widetilde{tiri}) |\ \widetilde{cross} \geq \lambda^1 \widetilde{cross}^1 + \lambda^2 \widetilde{cross}^2 + \cdots + \lambda^j \widetilde{cross}^j \ and\ \tilde{y} \leq \lambda^1 \widetilde{tiri}^1 + \lambda^2 \widetilde{tiri}^2 +..+ \lambda^j \widetilde{tiri}^j \ con\ \lambda^1 + \lambda^2 +..+ \lambda^j = 1\ e\ (\lambda^1, \lambda^2, \ldots, \lambda^j) > 0\} \quad (5)$$

In which $\widetilde{cross}, \widetilde{tiri}$ are two fuzzy sets of crosses and shots for a football team whose technology incorporates uncertainty according to the fuzzy theory (Zimmermann, 2011). As we can see these sets we have defined them in our application as symmetrical triangular fuzzy numbers (see Table 2). In the case of a more complex technology such as the one we have proposed here, the fuzzy sets of inputs ($\tilde{x}$) and outputs ($\tilde{y}$) can be represented as follows:

$$\tilde{x} = [\widetilde{cross}, \widetilde{filtering}, long\ \widetilde{passages}, short\ \widetilde{passages}, field\ \widetilde{rigth}\ side, field\ \widetilde{left}\ side, \widetilde{central}\ side, \widetilde{dribbling}, \widetilde{lost\ ball}], \quad (6a)$$

$$\tilde{y} = [goals, \widetilde{shots},\ \widetilde{tanking\ points}]\ (6b)$$

In which ~ indicates that they are all fuzzy sets, except for the "goals scored" variable which enters the trecnologica as an exact value. How well is a football team that uses a similar technology performing relative to another team that uses the same technology? This depends on how it does it, i.e. on the values of all the variables. A simple example is given in Table 1a below.

Table 1a Relative technical efficiency of football teams

Soccer team	cross	Shots made	Really Efficient?
1	10	35	No
2	10	45	No
3	10	35	No
4	10	32	No
5	10	50	Si

Finally, a further assumption that can be made concerns the assumption of additivity According to this assumption, when there are two possible production plans, one can look at their sum and assimilate that also the sum or their sum belongs to the technological set T and therefore possible, that is : self $(x^1, y^1) \in T\ e\ (x^2, y^2) \in T$then $(x^1 + x^2), (y^1 + y^2) \in T$. This assumption implies for example that: $3(x, y) = 2(x, y) + (x, y) \in T$. Additivity is an interesting hypothesis as you can think of the added plans as if you were managing two autonomous production lines. In essence, this assumption excludes the positive or negative externalities between two flat productions. These four assumptions: 1) free availability, 2) returns to scale, 3) convexity and 4) additivity can be taken individually or jointly and have interrelations (Bogetoft, et al., 2019). Regardless of what is being said here, in our opinion, the introduction of additivity assumption in the production technologies of football teams requires attention. For example, according to this assumption, if a football team that is making 10 crosses with 3 shots on goal in a first phase of the game / match and 3 crosses with 2 shots on goal in another phase of the game / match, it goes without saying that the team he can also make 10 + 3 crosses = 13 and 3 + 2 shots on goal = 5. This can hardly be said for the goals scored against the opponent. Ultimately, the minimum assumption that can be made on our production technology in our opinion is that of free availability. If, on the other hand, we made some assumptions about the specific relationship between inputs and outputs, we would have a production function available. For example, in the case of a Cobb-Douglas production function with two inputs and one output, we could specify the technological set as follows:

$$T = \{(x_1, x_2, y) \in R_+^3 | y \leq x_1^{\alpha 1} x_2^{\alpha 2}\}\ (7)$$

Therefore a Cobb-Douglas production function for a football team can be:

$$y_{tiri\_in\_porta} = cross^{\alpha 1} filtering^{\alpha 2}\ (8)$$

The only assumption of free availability requires that the production function be weakly increasing or that if $x' > x \rightarrow f(x') > f(x)$. And so in general we can assume for the production functions that correspond to the assumptions made for T. So we could describe the production technology for a (professional) football team using both the technological set T and the production functions f. The choice depends on the research needs and the application. In general, when 1) the exact relationship between inputs and outputs is not known and 2) the technology provides two or more outputs, the most useful representation is that with technological sets (T) as we have done here for soccer teams professionals. To measure performance we can define a distance function on T (which is also a further alternative to describe the production technology for a football team). The best known and most used are Farrell's distance function (Farrell, 1957) and Shepard's (Shephard, 1970) and both can refer to the input side and the output side. Farrell's, at the input θ and to the ouput δ can be indicated as follows:

$$\theta = \theta(x, y) = min\{\theta > 0 | (\theta x, y) \in T\} \text{ (9)}$$

$$\delta = \delta(x, y) = max\{\delta > 0 | (x, \delta y) \in T\} \text{ (10)}$$

Where is it $\theta \; and \; \delta$ represent respectively the maximum contraction and proportional expansion of the inputs and outputs that allow to produce y, the former, and that it is possible given the inputs the latter. Finally in the case of convex multiouput technologies we can use the dual representation with the cost, revenue and profit functions. In multi-input and multi-output situations it is often easy to represent and describe the technology using correspondences, that is to describe the technology from the input side (L) or from the output side (P). In the case of the production technology for professional football teams proposed here we have:

$$P_{soccer\ team}(x) = \{(goala\ , shots) | \ ((cross, filtering, long\ passages, short\ passages, right\ side, left\ side, central, dribbling, lost\ ball), (goals, shots))) \in T_{soccer\ team}\} \text{ (11)}$$

$$L_{soccer\ team}(y) = \{(cross, filteringi, long\ passage, short\ passages, side\ right, side\ left, centr, dribbling, lost\ ball) | \ ((x), (goals, shots))) \in T_{soccer\ team}\} \text{ (12)}$$

The production correspondences P are often called the set of production possibilities, while the consumption correspondences L are often called the set of required inputs. Therefore, if we know P for all the values of the inputs and L for all the values of the outputs, then reconstruct the technological set T. Otherwise we certainly know the technology through the correspondences but not the technological set. Often in applications we know the matches on the basis of the input and output data. Respectively, the lower and upper extremes of these two sets are the frontiers or the isoquants for L and the isoquants for P. A second technology does not look at the results (goals and shots) (output) directly produced by those inputs (passes and game vision) but to the final consequences that those outputs can produce or to the final outcomes / objectives. And therefore this technology does not look at victories, which can be the consequences of many goals or rather more goals than those that an opponent could inflict in a match and which can be considered intermediate outcomes, but rather at points in the standings as objectives / final outcomes of those outputs. Therefore, we will describe this technology as follows:

$$T_{soccerteamputcomes} = \{(\theta, p) \in R \times R | \ \theta \ have \ p\} \text{ (13)}$$

Where is it θ is the level of technical efficiency achieved by a team and p are the points earned in the ranking (goal). Even this technology since it is a quantitative technology lends itself to measurements of technical efficiency. In particular, with reference to the output, it indicates how much the points in the standings achieved by a team could have been expanded given the level of technical efficiency of a team relative to the other teams. To be clearer, if, for example, with an efficiency level of 0.8 one team reached 80 points in the standings and another with the same level of technical efficiency reached 70 points in the standings, this second team, albeit similarly efficient, was less effective than the former and could have reached 80 points or increased his points in the standings by 80- 70 = 10 if he had placed himself on the frontier of production possibilities, see Figure 11.

Insert Figure 11 Production possibility frontier for a football team

As can be seen in Figure A1, team 1 and team 12 recorded a very similar level of technical efficiency but while team 1 achieved a ranking score (goal) close to 90, team 12 achieved an overall ranking goal. 'roughly half (40 pt). Or note team 2 which compared to team 1 recorded a level of efficiency much higher than that of team 1 but achieved a much lower score than team 1. It can be said that both teams 1 and 2 were effective with respect to their ranking objectives but team 2 was more efficient although the ability to transform the level of efficiency into effectiveness was superior for team 1 which transformed, compared to team 2, a more technical level of efficiency. low in a higher ranking goal than team 2. The least effective and relatively less efficient teams were teams 19 and 20.

Appendix B

Return to scale in a football soccer production technology

In this appendix we will show a possible way of interpreting the assumptions of variable return to scales for the technology presented in Appendix A but using a reduced version of it with only short passes, long passes, crosses and shots on goal. So let's assume that we have 3 teams and that each of them in the attack phase has performed the same sequence of steps indicated in Figure 12 for a certain number of times to reach the shooting area and perform a certain number of shots on goal against the opponent. The sequence is specifically: 3 short passes (A$\rightarrow B, B \rightarrow C, C \rightarrow D$), 1 long pass (D$\rightarrow E$) and a cross (E$\rightarrow F$) as in Figure 1 below. At the end of this sequence Player F may or may not successfully complete a shot on target's target. So let's suppose that for the 3 teams we have the situation shown in Table 1 B below.

Enter Figure 12 A possible attack sequence

Table 1 Returns to scale

Team	sequence	Shoot in the mirror	Cumulative shooting frequency
1	yes	0	0
1	yes	1	1
1	yes	1	2
2	yes	0	0
2	yes	0	0
2	yes	1	1
3	yes	1	1
3	yes	1	2
3	yes	1	3

Team 1, like all the others, has completed this sequence 3 times and in these three repetitions of the same sequence has successfully made 2 shots on goal or starting from the second has made a proportional number of shots on goal compared to the first time he ran the sequence. In other words, at each unit increase of the same sequence after the first one, he has made a shot on goal for each of them, accumulating 2 shots on goal. Team 2 after repeating the same sequence also three times has accumulated only 1 goal shot while team 3 has accumulated 3. If repeated this pattern indefinitely then we can say that team 1, 2 and 3 according to our interpretation Team 1 operates in correspondence of constant returns to scale, descending team 2 and increasing team 3. As by proportionally increasing all their inputs I pass from one repetition to another of that same sequence, team 2 after two repetitions starting from first made 2 shots on goal, one for each repetition, team 1 starting from the first neither made 1 while team 3 neither made 3. Therefore, extending the reasoning to the technology in Appendix A we can say that on it they can be assumed both constant and increasing and decreasing returns to scale. Therefore the DEA -VRS models with rhytons of variable scale can be assumed.

FIGURES

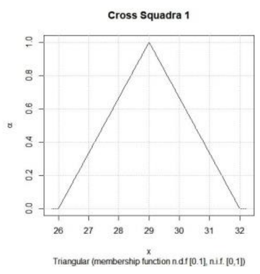

Figure 1 *Symmetrical triangular fuzzy number (the example of the Cross variable for team 1)*

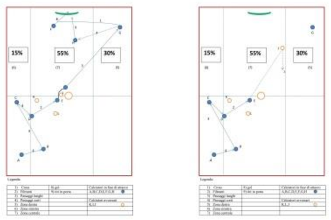

Figure 2 *Football*

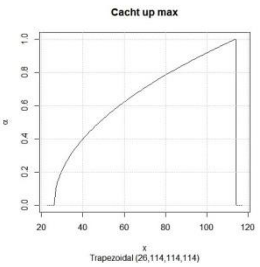

Figure 3 *Variable "maximum score" triangular asymmetric*

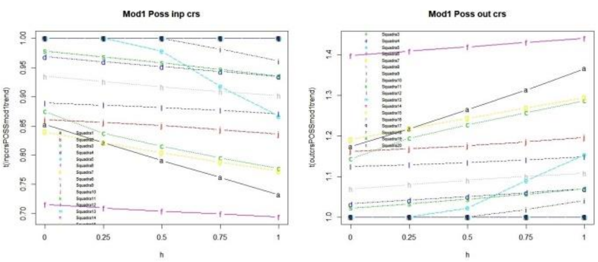

Figure 4 *Trend of h-possible efficiency level for each level of h. Model 1 input and output crs.*

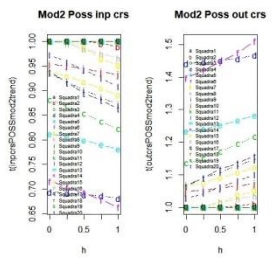

Figure 5 *Trend of h-possible efficiency level for each level of h. Model 2 input and output crs.*

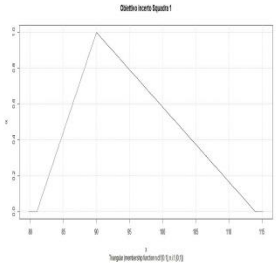

Figure 6 *Fuzzy target of maximum target for team 1*

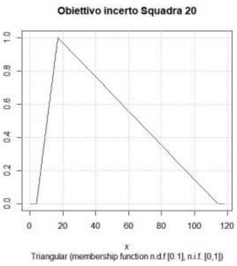

Figure 7 *Example of a fuzzy goal for the soccer teams of the Italian Serie A championship*

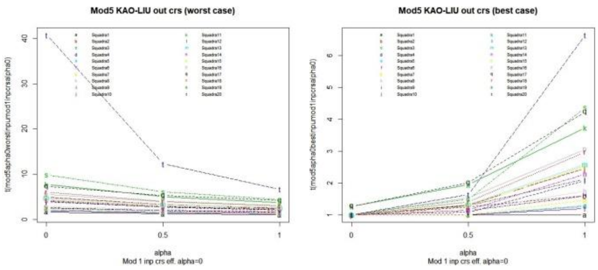

Figure 8 *Model results 5 out crs with input the efficiency scores of model 1 inp crs with alpha = 0*

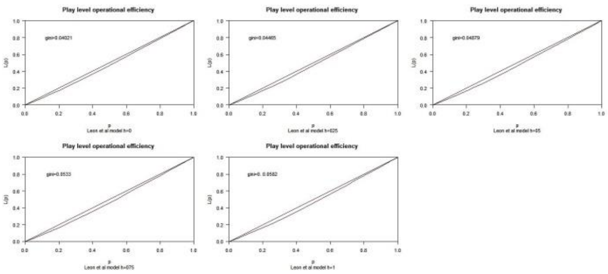

Figure 9 *Lorenz curve for the efficiency scores on the play levels of the model*

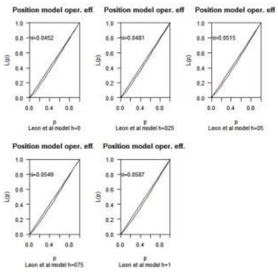

Figure 10 *Lorenz curve for the efficiency scores on the positioning model obtained from the model*

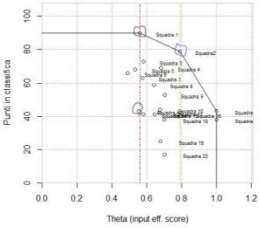

Figure 11 *Production possibility frontier for a football team*

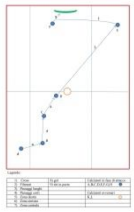

Figure 12 *A possible attack sequence*