# Wage Discrimination: Direct vs. Reverse Regression Method

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# Wage Discrimination: Direct vs. Reverse Regression Method

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**ABSTRACT**

Two methods of estimating wage discrimination have been employed so far in the literature: the direct regression method and the reverse regression method. What distinguishes the two methods is their assumption about the wage determination process: The direct regression method assumes that employees are paid according to their qualifications; by contrast, the reverse regression method assumes that wages are determined by the nature of the job and it is the qualifications of employees within that job classification that may vary.

This paper develops a new measure of wage discrimination, referred to here as the combined method, which is based on combining the results of the two wage discrimination measures. Using randomly generated data, it can be demonstrated that the combined method will produce the correct estimate of wage discrimination, assuming that the two alternative wage determination processes outlined above are equally common in the labour market.

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## 1. Introduction

One of the topics that has attracted considerable attention in the literature is wage discrimination. In our discussion we use gender discrimination as an example, although the results apply equally to race discrimination or any other form of discrimination.

Two methods of estimating wage discrimination have been employed so far in the literature: the direct regression method and the reverse regression method. What distinguishes the two methods is their assumption about the wage determination process: The direct regression method assumes that employees are paid according to their qualifications; by contrast, the reverse regression method assumes that wages are determined by the nature of the job and it is the qualifications of employees within each job that may vary.

Consequently, according to the direct regression method, women are discriminated if they are paid less than men with similar job qualifications. By contrast, according to the reverse regression method, women are discriminated if they need more qualifications than men to be hired for a particular job.

In reality, both wage determination processes take place. And, since it is difficult to determine which of the two processes is more common, a reasonable compromise would be to assume that each of them represents half of the cases. Therefore, a more realistic way of measuring wage discrimination would be to develop a method for averaging the estimates of the two methods.

This paper develops a formula for combining the results of the two wage discrimination measures. Using randomly generate data, it can be shown that the new formula results will be correct if the two alternative wage determination process outlined above are equally common in the labour market.

In what follows, Section 2 summarizes the existing two methodologies. Section 3 develops a formula for combining the result of the two methods. Section 4 presents concluding remarks.

## 2. The Two Methods

### *2.1 The Direct Regression Method*

The direct regression method, originally developed by Oaxaca (1973) and Blinder (1973), assumes that in the absence of discrimination employers reward employees according to their qualifications. Consequently, the wage equation takes the following form:

$$(1)\quad Y = c + b_1 * X_1 + b_2 * X_2 + .... + u$$

where $Y$ is the wage rate, typically specified in natural log form; $X_1$, $X_2$, etc. are independent variables, typically in dummy form, representing such job qualifications as level of education and years of experience; and $u$ is an error term, assumed to satisfy the standard assumptions for a best, linear, unbiased estimator.

For the sake of simplicity, we assume that there is only one independent variable: years of schooling $E$. So the wage regression can be simplified as follows:

$$(2)\quad Y = c + b * E + u$$

The qualifications gap between two groups (e.g. male and female employees) can be estimated by using, for example, the male wage regression to estimate what the average female wage rate would have been if female qualifications were rewarded in the same way as male qualifications. The qualifications gap between the two genders will, therefore, be as follows:

$$(3)\quad \text{Qualifications gap} = b_m * (\overline{E}_m - \overline{E}_f)$$

where $b_m$ is the education regression coefficient from the male regression equation; $\overline{E}_m$ and $\overline{E}_f$ are the average years of schooling for male and female employees respectively. The remaining wage gap is attributed to wage discrimination:

$$(4)\quad \text{Wage discrimination} = (\overline{Y}_m - \overline{Y}_f) - b_m * (\overline{E}_m - \overline{E}_f)$$

where $\overline{Y}_m$ and $\overline{Y}_f$ are the average male and female wage rates respectively.

*2.2 The Reverse Regression Method*

The reverse regression method, originally proposed by Kapsalis (1979, 1980), Roberts (1979, 1980), and Dempsters (1979), assumes that in the absence of discrimination wages are determined by the nature of the job and it is the qualifications of employees within that job classification that may vary.[1] Consequently, the reverse regression method replaces the direct wage equation (2) with the following reverse regression equation:

(5) $E = d + k * Y + v$

The interpretation of $k$ is that the higher the wage rate is, the higher the expected qualifications will be. The reverse regression method uses the inverse of the $k$ coefficient as a more accurate estimate of the $b$ coefficient in the initial wage equation (2). Consequently, the qualifications gap in equation (3) earlier is replaced by the following equation:

(6) $Qualifications\ gap = 1/k_m * (\overline{E}_m - \overline{E}_f)$

The direct and reverse regression coefficients are related by the following relationship (Malinvaud, 1970, p. 7):

(7) $b_m * k_m = R^2 \quad or \quad 1/k_m = b_m / R^2$

Consequently, equation (6) can be written as follows:

(8) $Qualifications\ gap = b_m / R^2 * (\overline{E}_m - \overline{E}_f)$

And the extent of wage discrimination can be written as:

(9) $Wage\ discrimination = (\overline{Y}m - \overline{Y}_f) - b_m / R^2 * (\overline{E}_m - \overline{E}_f)$

In reality, qualifications include a number of independent variables. To be able to regress qualifications as a function of the wage rate it would be necessary to replace the qualification variables with a single variable. This has been done in the literature by regressing first wages as a function of qualifications and then using the estimated wage values as an independent variable in the reverse regression. However, this step is redundant given the results of equation (9).

[1] A complementary justification for the reverse regression method is that, while wage rates are measured fairly accurately, qualifications are represented by proxy variables. For example, skills are approximated by the level of education and experience by age. As a result, the measurement of qualifications is subject to a measurement error, which biases downwards the regression coefficients of qualifications and, as a result, biases downwards the estimate of the qualifications gap and upwards the estimate of the wage discrimination.

*Proposition 1: The reverse regression estimate of wage discrimination can be calculated by simply dividing the direct regression estimate of the qualifications gap by $R^2$.*

From the above result, it is obvious that the two methods will produce identical results in the case of $R^2=1$. However, when $R^2$ is not zero the reverse regression method will produce a larger qualifications gap and a smaller wage discrimination estimate than the direct method. Moreover, the smaller the $R^2$ the larger would be the difference between the two methods.

## 3. The Combined Method

In reality, both wage determination processes take place. And, since it is difficult to determine which of the two processes is more common, a reasonable approach would be to assume that each of them represents half of the hiring cases. Therefore, a more realistic way of measuring wage discrimination would be to develop a method of averaging the estimates of the two previous methods.

The method proposed here is that of a geometric mean of the two estimates of the qualifications gap – i.e. the square root of the product of the two measures of qualifications gap:

$$\textit{Qualifications gap} = \sqrt{(b_m * (\overline{E}_m - \overline{E}_f)) * (b_m / R^2 * (\overline{E}_m - \overline{E}_f))} \quad \textit{or}$$

$$\text{(10)} \quad \textit{Qualifications gap} = b_m / R * (\overline{E}_m - \overline{E}_f)$$

Consequently, according to the combined method, the extent of wage discrimination would be given by the following formula:

$$\text{(11)} \quad \textit{Wage discrimination} = (\overline{Y}_m - (\overline{Y}_f) - b_m / R * (\overline{E}_m - \overline{E}_f)$$

*Proposition 2: The combined method estimate of wage discrimination can be calculated by simply dividing the direct regression estimate of the qualifications gap by R. When the two competing hypothesis of wage determination are equally common, the proposed combined method will produce the correct estimate of wage discrimination.*[2]

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[2] For an example of randomly generated data to test the three alternative measures of wage discrimination please visit the following address: https://drive.google.com/file/d/1E8W4dODke7qHPhiP8DZkn9_awJTxNe8Q/view?usp=sharing

## 4. Conclusion

The direct regression and reverse regression methods of measuring wage discrimination are based on competing assumptions about the wage determination method. In the absence of information as to which of the two competing hypothesis is more prevalent, this paper proposes using an averaging method. The proposed combined method of wage discrimination simply requires dividing the qualifications gap by the R regression coefficient. If the two competing wage determination processes are equally prevalent, then the combined method will produce the correct estimate of wage determination.

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