# Indeterminate Equilibria in New Keynesian DSGE Model: An Application to the US Great Moderation

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(First Version)

Macro Theory Paper

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## Abstract

This paper tests "Bad Policy" Hypothesis which refers to the Great Moderation in the US. We examine this hypothesis by simulating model based impulse response functions for the both pre-Volcker period and post 1982 period. Deriving and simulating standard New Keynesian DSGE Model explicitly, we find that while post 1982 policy i.e. *active* policy, is consistent with the unique stable equilibrium characteristics; pre-Volcker or *passive* monetary policy generates equilibrium indeterminacy. Moreover, our simulated-impulse response functions show that the response of inflation and the output gap in post 82 period is weaker than the macroeconomic responses of the pre-Volcker period.

Keywords: The Great Moderation, Indeterminacy, Determinate Equilibrium, New Keynesian DSGE Model, Monetary Policy, Sunspot shocks.

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# 1 INTRODUCTION

The significant reduction in the volatility of both output growth and inflation in the US starting from the 1980s is known as *the Great Moderation*. The literature on the Great Moderation mostly focuses on trying to find an answer for what caused changes in both output growth and inflation in the US. In the Great Moderation context, some studies argue that pre-Volcker period –*as a passive policy period* – has been associated with a "Bad Policy" in terms of weak monetary policy response to inflation as well as output gap. Thus, according to these studies, pre-Volker monetary policy was responsible for high inflation rates experienced in the US. On the other hand, several studies conclude that after the start of Volcker's term as a Chairman of the Federal Reserve in 1980s, monetary policy became more active and hence named as a "Good Policy" in terms of reacting to inflation and output gaps strong. Therefore, these studies conclude that the volatility of both inflation and output has declined after 1980s in the US.

The literature on the US Great Moderation along with the tests of the"Bad Policy" Hypothesis is vast. Orphanides (2001) uses forward looking policy rules and finds a strong reaction to inflation forecasts during both pre-Volcker and post 1982. According to Orphanides, monetary policy was a source of instability during the Great Moderation. He also finds that prior to Volcker, monetary policy was too activist in reacting to perceived output gaps. Mcconnell and Quiros (2000) test and find a structural break in the volatility of the US output growth in 1984. According to Mcconnel and Quiros, the break in the durables is coincident with a break in the proportion of durables output accounted by inventories. Mcconnel and Quiros (2000) use Markov-Switching model and their data covers 1953:02-1999:02. Milani (2005) introduces learning to Great Moderation literature. He finds evidence of regime switch of US monetary policy from passive to active. Furthermore, Milani also shows the feedback coefficient to inflation was well above one in the 1960s and 1970s. According to Milani, monetary policy was not leading to macroeconomic instability. Gambetti, *et.al.*(2006) find that the contribution of real demand and supply shocks varies over time and hence changes the volatility of inflation and output. They also conclude that the monetary policy has a small role in terms of evolution of the persistence and volatility. Gambetti, *et.al.*(2006) use Markov-Chain Monte Carlo Methods as well as Time Varying Coefficients VAR over the period 1960:01 -2003:02.

Cecchetti, *et.al* (2007) examine international comparisons to understand US inflation dynamics. They find that Great Inflation and Inflation stabilization are linked due to the changes in the monetary policy regimes of several G-7 countries. Gali and Gambetti (2007) conclude that the Great Moderation can be explained by a sharp fall in the contribution of non – technology shocks to the variance of output. Moreover, according to Gali and Gambetti (2007), large changes in the patterns of co-movements among output, hours and labor productivity might cause the remarkable decline in macroeconomic volatility experienced by the US economy. Canova, *et.al*. (2007) examine the role of expectations in the Great Moderation Episode. They find that including or excluding expectations hardly changes the economic explanation of the Great Moderation. Furthermore, their results gives that contribution of sunspot shocks to output growth and inflation volatility do not line up well. Canova, *et.al*. (2007) use Bayesian Approach and also estimate New Keynesian DSGE Model. Their data is over the period 1960:01–1979:02 and 1982:04–1997:04.

Benati and Surico (2007) show that VAR analysis may misinterpret good policy for good luck. Moreover, they find that VAR evidence is inconsistent with the good policy explanation of the Great Moderation. According to Benati and Surico, VARs may fail to capture of the role of monetary policy played in recent macroeconomic stability. Furthermore, they argue that estimating DSGE models (with passive policy) has the potential to discriminate between the “good policy” and “good luck” explanations of the Great Moderation. Lubik and Schorfheide (2004) use Likelihood-based estimation of DSGE Model and Bayesian Approach. They estimate New Keynesian DSGE Model and their data covers 1960:01–1979:02, 1979:03–1997:04 and 1982:04–1997:04. Lubik and Schorfheide (2004) find that post 1982 US monetary policy is consistent with determinacy, whereas the pre-Volcker period is not. They estimate DSGE Model without restricting the parameters to the determinacy region. Likewise, this paper also tests whether or not pre-Volcker (or *passive*) period generates equilibrium indeterminacy. Moreover, we, then, examine if post-1982 *(*or *active)* period is consistent with the unique determinate equilibrium for the case of the US.

This paper is organized as follows: First, we set up New Keynesian Dynamic Stochastic General Equilibrium Model as a benchmark model. Then, we derive three equilibrium conditions of the model economy and solve the model dynamics explicitly. Section 3 provides calibration and simulation methodology of the study in detail. Section 4 discusses the results and finally Section 5 concludes.

## 2 THE MODEL

The model economy[2] is based on New Keynesian Dynamic Stochastic General Equilibrium Model setting. In this study, we, first, set up and solve maximization problems of the consumers and the firms explicitly. Second, we express equilibrium conditions for this New Keynesian economy by introducing and closing the model with monetary policy, i.e. simple interest rate rule. Third, we solve the model and examine equilibrium characteristics for both determinacy and indeterminacy. The simulation mechanics of the model is also introduced by using the similar approach for solving and simulating KPR Models.

### 2. 1 The Household

The representative infinitely-lived utility maximizer household solves the following problem:

$$\max E_0 \sum_{t}^{\infty} \beta^t U(C_t, N_t) \tag{1}$$

s. t.

$$\int_0^1 P_t(i) C_t(i) di + Q_t B_t \leq B_{t-1} + W_t N_t - T_t \tag{2}$$

for t = 0, 1, 2, … plus solvency constraint.
The consumption index is defined as:

$$C_t \equiv \left[ \int_0^1 C_t(i)^{1-\frac{1}{e}} di \right]^{\frac{e}{e-1}} \tag{3}$$

and consider the following utility specification:

$$U(C_t, N_t) = \frac{C_t^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\varphi}}{1+\varphi} \tag{4}$$

Now, use Lagrangian and get the first order conditions to find the optimal demand:
Then, the optimal allocation of expenditures can be written as:

$$C_t(i) = \left( \frac{P_t(i)}{P_t} \right)^{-e} C_t \tag{5}$$

[2] See Gali's forthcoming textbook for this model economy derivation in detail. - Gali, J., (2007), "Lectures on Monetary Policy, Inflation and the Business Cycle", Lecture Notes, (February 2007), chapter-III.

which implies:

$$\int_0^1 P_t(i)C_t(i)di = P_t C_t \tag{6}$$

where

$$P_t \equiv \left[\int_0^1 P_t(i)^{1-e} di\right]^{\frac{1}{1-e}} \tag{7}$$

Other optimality conditions:

$$-\frac{U_{n,t}}{U_{c,t}} = \frac{W_t}{P_t} \quad \text{and} \quad Q_t = \beta E_t\left\{\frac{U_{c,t+1}}{U_{c,t}}\frac{P_t}{P_{t+1}}\right\} \qquad \text{(8) and (9)}$$

Then, by using the utility specification, log-linear optimal conditions become:

$$w_t - p_t = \sigma c_t + \varphi n_t \tag{10}$$

$$c_t = E_t\{c_{t+1}\} - \frac{1}{\sigma}(i - E_t\{\pi_{t+1}\} - \rho) \tag{11}$$

Equation (11) is consumer's Euler equation where $i_t \equiv -\log Q_t$ represents the "nominal interest rate" and $\rho \equiv -\log \beta$ is the "discount rate".

Finally, ad-hoc money demand can be written as:

$$m_t - p_t = y_t - \eta i_t \tag{12}$$

## 2. 2 The Firms

We assume that there is a continuum of firms that are indexed by $i \in [0,1]$. In this model, firms are monopolistically competitive and each produces a differentiated good in the economy. The firms are setting their prices based on Calvo – Price Setting (1983), i.e. prices are sticky and firms are able to reset their prices in any given period with the probability of $1-\theta$, which is independent across firms. Price stickiness is indexed by $\theta \in [0,1]$ and implied average price duration is $1/(1-\theta)$.

The production function is identical for all firms and that is:

$$Y_t(i) = A_t N_t(i)^{1-\alpha} \tag{13}$$

Aggregate price dynamics can be written as:

$$P_t = [\theta(P_{t-1})^{1-e} + (1-\theta)(P_t^*)^{1-e}]^{\frac{1}{1-e}} \tag{14}$$

dividing (14) by $P_{t-1}$ gives:

$$\prod_t^{1-e} = \theta + (1-\theta)\left(\frac{P_t^*}{P_{t-1}}\right)^{1-e} \tag{15}$$

and log-linearizing (15) around zero inflation steady state gives:

$$\pi_t = (1-\theta)(p_t^* - p_{t-1}) \tag{16}$$

Equation (16) can be also written as:

$$p_t = \theta p_{t-1} + (1-\theta)\, p_t^*) \tag{17}$$

Firm's optimal price setting problem then becomes:

$$\max_{P_t^*} \sum_{k=0}^{\infty} E_t \left\{ Q_{t,t+k} (P_t^* Y_{t+k|t} - \Psi_{t+k}(Y_{t+k|t})) \right\} \tag{18}$$

s.t.

$$Y_{t+k|t} = (P_t^* / P_{t+k})^{-e} C_{t+k}$$

where

$$Q_{t,t+k} \equiv \beta^k \left(\frac{C_{t+k}}{C_t}\right)^{-\sigma} \left(\frac{P_t}{P_{t+k}}\right) \tag{19}$$

for $k = 1, 2, \ldots$

Then, optimality condition is:

$$\sum_{k=0}^{\infty} \theta^k E_t \left\{ Q_{t,t+k} Y_{t+k|t} (P_t^* - \upsilon \varphi_{t+k|t}) \right\} = 0 \tag{20}$$

where

$$\varphi_{t+k|t} \equiv \Psi'_{t+k}(Y_{t+k|t}) \quad \text{and} \quad \upsilon \equiv \frac{e}{1-e} \qquad \text{(21) and (22)}$$

Equivalently,

$$\sum_{k=0}^{\infty} \theta^k E_t \left\{ Q_{t,t+k} Y_{t+k|t} \left( \frac{P_t^*}{P_{t-1}} - MC_{t+k|t} \Pi_{t-1,t+k} \right) \right\} = 0 \tag{23}$$

where

$$MC_{t+k|t} \equiv \frac{\varphi_{t+k|t}}{P_{t+k}} \quad \text{and} \quad \Pi_{t-1,t+k} \equiv \frac{P_{t+k}}{P_{t-1}} \qquad \text{(24) and (25)}$$

Now, we can write perfect foresight steady states as follows:

$$\frac{P_t^*}{P_{t-1}} = 1 \quad ; \quad \Pi_{t-1,t+k} = 1 \quad ; \quad Y_{t+k|t} = Y \quad ; \quad Q_{t,t+k} = \beta^k \quad ; \quad MC = \frac{1}{\upsilon}$$

and log-linearization around zero steady state can be written as:

$$p_t^* - p_{t-1} = (1-\beta\theta)\sum_{k=0}^{\infty}(\beta\theta)^k E_t\left\{(\hat{m}\hat{c}_{t+k|t} + p_{t+k} - p_{t-1}\right\} \tag{26}$$

or, we can also express equation (26) as:

$$p_t^* = \mu + (1-\beta\theta)\sum_{k=0}^{\infty}(\beta\theta)^k E_t\left\{mc_{t+k} + p_{t+k}\right\} \tag{27}$$

Then, flexible prices, in which $\theta = 0$, can be expressed as:

$$p_t^* = \mu + mc_t + p_t \tag{28}$$

$$\Rightarrow mc_t = -\mu \text{ (symmetric equilibrium)} \tag{29}$$

now, consider constant returns to scale[3], i.e. $\alpha = 0$;

$$\Rightarrow MC_{t+k|t} = MC_{t+k} \tag{30}$$

rewriting equation (26);

$$p_t^* - p_{t-1} = \sum_{k=0}^{\infty}(\beta\theta)^k E_t\left\{(1-\beta\theta)(\hat{m}\hat{c}_{t+k|t} + \pi_{t+k}\right\} \tag{31}$$

$$= \beta\theta E_t\left\{p_{t+1}^* - p_t\right\} + (1-\beta\theta)\hat{m}\hat{c}_t + \pi_t \tag{32}$$

where

$$\hat{m}\hat{c}_t \equiv mc_t - mc = mc_t + \mu \quad \text{and} \quad \mu \equiv \log\frac{e}{e-1} \qquad \text{(33) and (34)}$$

and, combining equation (16) and (32) gives the following expression:

$$\pi_t = \beta E_t\left\{\pi_{t+1}\right\} + \lambda\hat{m}\hat{c}_t \quad \text{where} \quad \lambda \equiv \frac{(1-\theta)(1-\beta\theta)}{\theta} \qquad \text{(35) and (36)}$$

## 2. 3 Equilibrium

*Goods market clears as follows:*

$$Y_t(i) = C_t(i) \tag{37}$$

[3] For the case of decreasing returns to scale, see Gali's textbook on Monetary Policy, Inflation and Business Cycles, February 2007, forthcoming.

for all $i \in [0,1]$ and all *t*.

So, let $Y_t \equiv \left[ \int_0^1 Y_t(i)^{1-\frac{1}{e}} di \right]^{\frac{e}{e-1}}$. Thus, $Y_t = C_t$ for all *t*.

Now, we can rewrite consumer's Euler equation (11) and one can get:

$$y_t = E_t\{y_{t+1}\} - \frac{1}{\sigma}(i - E_t\{\pi_{t+1}\} - \rho) \tag{38}$$

*Labor Market clears as follows:*

$$N_t = \int_0^1 N_t(i) di \tag{39}$$

$$= \int_0^1 \left( \frac{Y_t(i)}{A_t} \right)^{\frac{1}{1-\alpha}} di = \left( \frac{Y_t}{A_t} \right)^{\frac{1}{1-\alpha}} \int_0^1 \left( \frac{P_t(i)}{P_t} \right)^{-\frac{e}{1-\alpha}} di \tag{40}$$

taking logs of the above expression is then:

$$(1-\alpha)n_t = y_t - a_t + d_t \tag{41}$$

where

$$d_t \equiv (1-\alpha) \log \int_0^1 \left( \frac{P_t(i)}{P_t} \right)^{-\frac{e}{1-\alpha}} di \text{ (second order condition)} \tag{42}$$

and first order approximation is:

$$y_t = a_t + (1-\alpha)n_t \tag{43}$$

Marginal Cost and Output can be written by using the following expressions:

$$mc_t = (w_t - p_t) - mpn_t$$

$$= \sigma y_t + \varphi n_t) - (y_t - n_t) - \log(1-\alpha)$$

$$= \left( \sigma + \frac{\varphi + \alpha}{1-\alpha} \right) y_t - \frac{1+\varphi}{1-\alpha} a_t - \log(1-\alpha) \tag{44}$$

under flexible prices equation (44) becomes:

$$mc_t = \left( \sigma + \frac{\varphi + \alpha}{1-\alpha} \right) y_t^n - \frac{1+\varphi}{1-\alpha} a_t - \log(1-\alpha) \tag{45}$$

which implies

$$y_t^n = -\delta_y + \ell_{ya} a_t \tag{46}$$

where

$$\delta_y \equiv \frac{(\mu - \log(1-\alpha))(1-\alpha)}{\sigma + \varphi + \alpha(1-\alpha)} > 0, \quad \ell_{ya} \equiv \frac{1+\varphi}{\sigma + \varphi + \alpha)(1-\alpha)} \qquad \text{(47) and (48)}$$

which implies:

$$\hat{mc}_t = \left(\sigma + \frac{\varphi + \alpha}{1-\alpha}\right)(y_t - y_t^n) \tag{49}$$

where

$$\bar{y} \equiv y_t - y_t^n \text{ is the output gap.} \tag{50}$$

Now, *New Keynesian Phillips Curve* can be written as:

$$\pi_t = \beta E_t\{\pi_{t+1}\} + \lambda\left(\sigma + \frac{\varphi + \alpha}{1-\alpha}\right)\bar{y}_t \tag{51}$$

and *Dynamic IS Equation* is:

$$\bar{y}_t = E_t\{\bar{y}_{t+1}\} - \frac{1}{\sigma}(i - E_t\{\pi_{t+1}\} - r_t^n) \tag{52}$$

where $r_t^n$ is the "natural rate of interest, given by;

$$\begin{aligned} r_t^n &\equiv \rho + \sigma E_t\{\Delta y_{t+1}^n\} \\ &= \rho + \sigma \ell_{ya} E_t\{\Delta a_{t+1}\} \end{aligned} \tag{53}$$

Monetary policy - by determining $i_t$ closes the model.

## 2. 4 Monetary Policy

Suppose monetary policy follows the following simple interest rate rule:

$$i_t = \chi + \phi_\pi \pi_t + \phi_y y_t + v_t \tag{54}$$

where $v_t$ is exogenous shock with zero mean, and $y_t$ is the output gap realized at time t. Monetary policy reacts the economy with $\phi_\pi$ and $\phi_y$ policy response parameters.

Hence, combining (51), (52) and (54) gives;

$$\begin{bmatrix} y_t \\ \pi_t \end{bmatrix} = \frac{1}{\sigma + \phi_y + \kappa\phi_\pi}\begin{bmatrix} \sigma & 1-\beta\phi_\pi \\ \sigma\kappa & \kappa + \beta(\sigma + \phi_y \end{bmatrix} + \frac{1}{\sigma + \phi_y + \kappa\phi_\pi}\begin{bmatrix} 1 \\ \kappa \end{bmatrix}(\hat{r}_t^n - v_t) \tag{55}$$

and according to Bullard and Mitra (2002);

*Uniqueness* $\Leftrightarrow$ $\frac{1}{\sigma + \phi_y + \kappa\phi_\pi}\begin{bmatrix} \sigma & 1-\beta\phi_\pi \\ \sigma\kappa & \kappa + \beta(\sigma + \phi_y \end{bmatrix}$ has both eigenvalues within the unit circle,

given $\phi_\pi \geq 0$ and $\phi_y \geq 0$,

$\kappa(\phi_\pi - 1) + (1-\beta)\phi_y > 0$ is necessary and sufficient.

## 2. 5 Solving the Model

The economy is characterized by three New Keynesian Equations (51), (52) and (54):

$$\bar{y}_t = E_t\{\bar{y}_{t+1}\} - \frac{1}{\sigma}(i - E_t\{\pi_{t+1}\} - r_t^n)$$

$$\pi_t = \beta E_t\{\pi_{t+1}\} + \lambda\left(\sigma + \frac{\varphi + \alpha}{1-\alpha}\right)\bar{y}_t$$

$$r_t^n = \rho r_{t-1}^n + v_t \tag{56}$$

where $r_t^n$ is the natural rate and $v$ is a shock. Rewrite these equations in a matrix form as a linear system of equations:

$$\begin{bmatrix} E_t y_{t+1} \\ E_t \pi_{t+1} \\ E_t r_{t+1} \end{bmatrix} = W \begin{bmatrix} y_t \\ \pi_t \\ r_t \end{bmatrix} + R\varepsilon_{t+1} \tag{57}$$

use Jordan Decomposition and express the system as follows:

$$\begin{bmatrix} E_t y_{t+1} \\ E_t \pi_{t+1} \\ E_t r_{t+1} \end{bmatrix} = P \begin{bmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{bmatrix} P^{-1} \begin{bmatrix} y_t \\ \pi_t \\ r_t \end{bmatrix} + R\varepsilon_{t+1} \tag{58}$$

The left-hand-side of this system is not a differential equations type of vector. Hence, rewrite this system as a differential equation system in order to solve the model.

So, Adding and subtracting $\begin{bmatrix} y_{t+1} \\ \pi_{t+1} \\ r_{t+1} \end{bmatrix}$ to previous system gives:

$$\begin{bmatrix} E_t y_{t+1} \\ E_t \pi_{t+1} \\ E_t r_{t+1} \end{bmatrix} = P \begin{bmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{bmatrix} P^{-1} \begin{bmatrix} y_t \\ \pi_t \\ r_t \end{bmatrix} + R\varepsilon_{t+1} + \begin{bmatrix} y_{t+1} \\ \pi_{t+1} \\ r_{t+1} \end{bmatrix} - \begin{bmatrix} y_{t+1} \\ \pi_{t+1} \\ r_{t+1} \end{bmatrix} \tag{59}$$

rearranging;

$$\begin{bmatrix} y_{t+1} \\ \pi_{t+1} \\ r_{t+1} \end{bmatrix} = P \begin{bmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{bmatrix} P^{-1} \begin{bmatrix} y_t \\ \pi_t \\ r_t \end{bmatrix} + R\varepsilon_{t+1} + \begin{bmatrix} y_{t+1} \\ \pi_{t+1} \\ r_{t+1} \end{bmatrix} - \begin{bmatrix} E_t y_{t+1} \\ E_t \pi_{t+1} \\ E_t r_{t+1} \end{bmatrix} \tag{60}$$

The last two vector difference, $\begin{bmatrix} y_{t+1} \\ \pi_{t+1} \\ r_{t+1} \end{bmatrix} - \begin{bmatrix} E_t y_{t+1} \\ E_t \pi_{t+1} \\ E_t r_{t+1} \end{bmatrix}$ is the “forecast error” vector of the system.

Rewrite the previous system as:

$$\begin{bmatrix} y_{t+1} \\ \pi_{t+1} \\ r_{t+1} \end{bmatrix} = P \begin{bmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{bmatrix} P^{-1} \begin{bmatrix} y_t \\ \pi_t \\ r_t \end{bmatrix} + R\varepsilon_{t+1} + \begin{bmatrix} y_{t+1} - E_t y_{t+1} \\ \pi_{t+1} - E_t \pi_{t+1} \\ r_{t+1} - E_t r_{t+1} \end{bmatrix} \tag{61}$$

now, multiplying both sides by matrix $P^{-1}$:

$$P^{-1} \begin{bmatrix} y_{t+1} \\ \pi_{t+1} \\ r_{t+1} \end{bmatrix} = P^{-1} P \begin{bmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{bmatrix} P^{-1} \begin{bmatrix} y_t \\ \pi_t \\ r_t \end{bmatrix} + P^{-1} R\varepsilon_{t+1} + P^{-1} \begin{bmatrix} y_{t+1} - E_t y_{t+1} \\ \pi_{t+1} - E_t \pi_{t+1} \\ r_{t+1} - E_t r_{t+1} \end{bmatrix} \tag{62}$$

note that $r_{t+1} = \rho r_t + v_{t+1}$ and hence $E_t r_{t+1} = \rho r_t$. Hence, ($r_{t+1} - E_t r_{t+1}$) $= v_{t+1}$.

Furthermore, call ($y_{t+1} - E_t y_{t+1}$) $= \eta_y$ and ($\pi_{t+1} - E_t \pi_{t+1}$) $= \eta_\pi$.

Hence, the system becomes:

$$P^{-1} \begin{bmatrix} y_{t+1} \\ \pi_{t+1} \\ r_{t+1} \end{bmatrix} = \begin{bmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{bmatrix} P^{-1} \begin{bmatrix} y_t \\ \pi_t \\ r_t \end{bmatrix} + P^{-1} R\varepsilon_{t+1} + P^{-1} \begin{bmatrix} \eta_y \\ \eta_\pi \\ v_{t+1} \end{bmatrix} \tag{63}$$

Now, define the following expressions:

$$P^{-1} \begin{bmatrix} y_{t+1} \\ \pi_{t+1} \\ r_{t+1} \end{bmatrix} = Q_{t+1} \quad \text{and} \quad P^{-1} \begin{bmatrix} y_t \\ \pi_t \\ r_t \end{bmatrix} = Q_t$$

Hence,

$$Q_{t+1} = \begin{bmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{bmatrix} Q_t + P^{-1} R\varepsilon_{t+1} + P^{-1} \begin{bmatrix} \eta_y \\ \eta_\pi \\ v_{t+1} \end{bmatrix} \tag{64}$$

or, equivalently,

$$\begin{bmatrix} q_{1t+1} \\ q_{2t+1} \\ q_{3t+1} \end{bmatrix} = \begin{bmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{bmatrix} \begin{bmatrix} q_{1t} \\ q_{2t} \\ q_{3t} \end{bmatrix} + P^{-1} R\varepsilon_{t+1} + P^{-1} \begin{bmatrix} \eta_y \\ \eta_\pi \\ v_{t+1} \end{bmatrix} \tag{65}$$

The above expression is a linear system of difference equations. Note that this system has two control variables (output gap and inflation), one predetermined variable (natural rate of interest as a state) and one state variable which is natural rate of interest, $r_t$. Moreover, the system has three lambdas: $\lambda_1, \lambda_2$ and $\lambda_3$. Suppose that $|\lambda_1| > 1$, $|\lambda_2| > 1$ and $|\lambda_3| < 1$.

In this case,

# of predetermined variables = # of stable eigenvalues (i.e. $\lambda_3$). Therefore, the only way to have a stable solution is hence defining $q_{1t} = q_{2t} = 0$ for all t. In other words, from the previous representation of the system it can be written as:

$$q_{1t+1} = \lambda_1 q_{1t} + P^{-1} R \varepsilon_{t+1} + P^{-1} \begin{bmatrix} \eta_y \\ \eta_\pi \\ v_{t+1} \end{bmatrix} \tag{66}$$

and,

$$q_{2t+1} = \lambda_2 q_{2t} + P^{-1} R \varepsilon_{t+1} + P^{-1} \begin{bmatrix} \eta_y \\ \eta_\pi \\ v_{t+1} \end{bmatrix} \tag{67}$$

It is now obvious that since $q_{1t} = q_{2t} = 0$ for all t, $\eta_y$ along with $\eta_\pi$ has no impact on the variables of the system. Thus, even though agents re-form their expectations, there is "only one path" for the control variables and this path leads a *unique stable determinate Rational Expectations Equilibrium.*

To show this in another way recall $P^{-1} \begin{bmatrix} y_t \\ \pi_t \\ r_t \end{bmatrix} = Q_t$ and rewrite this expression as follows:

$$\begin{bmatrix} q_{1t} \\ q_{2t} \\ q_{3t} \end{bmatrix} = \begin{bmatrix} p_{11} & p_{12} & p_{13} \\ p_{21} & p_{22} & p_{23} \\ . & . & . \end{bmatrix} \begin{bmatrix} y_t \\ \pi_t \\ r_t \end{bmatrix} \tag{68}$$

since $q_{1t} = q_{2t} = 0$ for all t,

$$p_{11} y_t + p_{12} \pi_t + p_{13} r_t = 0 \tag{69}$$

$$p_{21} y_t + p_{22} \pi_t + p_{23} r_t = 0 \tag{70}$$

and from these equations, each control variable (output gap and inflation) can be pinned down as a function of the state variable, $r_t$.

So far, we have seen the determinacy case. However, if the number of stable eigenvalues are more than the number of predetermined variables (natural rate of interest), then $q_{1t} \neq q_{2t} \neq 0$ for all t.

Recall system (66) and (67):

$$q_{1t+1} = \lambda_1 q_{1t} + P^{-1} R \varepsilon_{t+1} + P^{-1} \begin{bmatrix} \eta_y \\ \eta_\pi \\ v_{t+1} \end{bmatrix} \quad ; \quad q_{2t+1} = \lambda_2 q_{2t} + P^{-1} R \varepsilon_{t+1} + P^{-1} \begin{bmatrix} \eta_y \\ \eta_\pi \\ v_{t+1} \end{bmatrix}$$

But, now since $q_{1t} \neq q_{2t} \neq 0$ for all t, *forecast errors* do have impact on the system variables. This leads equilibrium indeterminacy instead of unique equilibrium. In this case, forecast errors; $\eta_y$ and $\eta_\pi$ drive business cycles. These errors are also called *sunspot shocks*.

## 2. 6 The Mechanics

The mechanics of the standard New Keynesian Model can be grasped by using a general approach to solving KPR models[4]. Hence, now consider the RBC model's mechanics and see how one can calibrate and simulate New Keynesian DSGE Model through this process[5].

So now, instead of New Keynesian Model variables, we consider RBC Model variables; $c_t, k_t, a_t$ and examine the model mechanics as follows:

Recall;

$$\begin{bmatrix} q_{1t} \\ q_{2t} \\ q_{3t} \end{bmatrix} = \begin{bmatrix} 0 \\ b \\ d \end{bmatrix} \quad \text{or, equivalently,} \quad P^{-1} \begin{bmatrix} c_t \\ k_t \\ a_t \end{bmatrix} = \begin{bmatrix} 0 \\ b \\ d \end{bmatrix} \quad \Rightarrow \begin{bmatrix} c_t \\ k_t \\ a_t \end{bmatrix} = P \begin{bmatrix} 0 \\ b \\ d \end{bmatrix} \qquad (71)$$

excluding zero value from the system;

[4] See Cochrane, John H. (2001). "Solving Real Business Cycles Models by Solving Systems of First Order Conditions", *Lecture Notes, Graduate School of Business, University of Chicago,* page 14.

[5] In the RBC Model, instead, there are one control variable, Consumption, one state variable, Capital and one exogenous variable, Technology. So, we only assume $q_{1t} = 0$, since then, the number of predetermined variables will be equal to the stable eigenvalues. This in fact leads a unique determinate equilibrium.

$$\begin{bmatrix} c_t \\ k_t \\ a_t \end{bmatrix} = P[\cdot,2:3]\begin{bmatrix} b \\ d \end{bmatrix} \tag{72}$$

$$\Rightarrow \begin{bmatrix} b \\ d \end{bmatrix} = P[\cdot,2:3]^{-1}\begin{bmatrix} c_t \\ k_t \\ a_t \end{bmatrix} \tag{73}$$

and in order to solve the system above, rewrite as:

$$\begin{bmatrix} b \\ d \end{bmatrix} = P[2:3,2:3]^{-1}\begin{bmatrix} k_t \\ a_t \end{bmatrix} \tag{74}$$

substituting system (74) into (72) gives the following expression:

$$\begin{bmatrix} c_t \\ k_t \\ a_t \end{bmatrix} = P[\cdot,2:3]P[2:3,2:3]^{-1}\begin{bmatrix} k_t \\ a_t \end{bmatrix} \tag{75}$$

for $c_t$, the decision rule can be hence written as:

$$c_t = P[1,2:3]P[2:3,2:3]^{-1}\begin{bmatrix} k_t \\ a_t \end{bmatrix} \text{ or, } c_t = \Phi_{c,ka}\begin{bmatrix} k_t \\ a_t \end{bmatrix} \tag{76}$$

Therefore, now, we can simulate the above system using the decision rule of consumption. Furthermore, based on the decision rule, we can also simulate;

$$\begin{bmatrix} k_{t+1} \\ a_{t+1} \end{bmatrix} = \left(W[2:3,1]\Phi_{c,ka} + W[2:3,2:3]\right)\begin{bmatrix} k_t \\ a_t \end{bmatrix} + R[2:3,1]\varepsilon_{t+1} \tag{77}$$

and finally we generate shocks by;

$$a_{t+1} = \rho a_{t+1} + \varepsilon_{t+1} \tag{78}$$

## 3 CALIBRATION and SIMULATION

In this section, we calibrate New Keynesian Model and then simulate by using the mechanism discussed in the previous section[6]. For the calibration exercise, we have three periods[7]: (i) whole sample or mixed policy period (1954-2008), (ii) pre-Volcker period (1954-1979), (iii) post-82 period (1980-2008). For the mixed policy period, we take the following calibration values[8]:

$\beta = 0.99$, $\sigma = \varphi = 1$, $\kappa = 0.0024$, $\rho = 0.5$, $\phi_\pi = 1.5, \phi_y = 0.5 . \theta = 2/3, \eta = 4$

for the pre-Volcker passive period calibration, we choose;

$\beta = 0.99$, $\sigma = 1$, $\kappa = 0.77$, $\rho = 0.5$, $\phi_\pi = 0.77, \phi_y = 0.17$,

and, for the post-82 active period calibration, we choose;

$\beta = 0.99$, $\sigma = 1$, $\kappa = 0.77$, $\rho = 0.5$, $\phi_\pi = 2.19, \phi_y = 0.30$.

Having used the calibration values above, we solve the model for both pre-Volcker period (Bad policy) and post 82 period (Good policy), then analyze the equilibrium characteristics. In this paper, we only consider monetary policy shocks and use the following shock process in order to get the simulated-impulse response functions of the economy:

Set $\hat{r}_t^n = 0$. (no real shocks) and let $v_t$ follow an AR (1) process that is:

$$v_t = \rho_v v_{t-1} - \varepsilon_t^v \tag{79}$$

Now, by using calibrated values and generating shocks, we are now able to plot the impulse-response functions of the output, inflation and the nominal interest rates to one unit of v shock.

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6 All MATLAB codes for the simulation, impulse responses and graphs can be provided upon request.

7 The whole sample data period (1954:03-2008:01) is selected and extended based on Gali and Gambetti (2007); the pre-Volcker period data (1954:03, 1979:02) is based on Milani (2005); and post 82 period data sample (1979:03 – 2008: 01) is selected based on Milani (2005), Lubik and Schorfheide (2004).

8 Whole sample calibration is taken from Walsh’s Textbook and Gali’s forthcoming Textbook on Monetary Policy, Inflation and Business Cycles, February 2007. Passive policy and active policy calibrations are taken from Canova and Gambetti (2008) as well as Gali’s Textbook.

# 4 RESULTS

We first find that New Keynesian Model generates a unique stable Rational Expectations Equilibrium for the mixed policy (1954-2008) by using the suggested parameters discussed in the previous section. Moreover, Figure 6[9] gives the simulated-impulse response functions of the output gap, inflation and the nominal interest rates for the mixed period –whole sample case. From this figure, it can be seen that while inflation has a small response to shock with persistency, both output gap and interest rates response to shock immediately. Obviuously, after the second period, the effect of one unit of shock starts to die away from the system. However, it takes about 8 periods for the variables to reach their steady state values.

Second, we examined New Keynesian Model for the pre-Volcker period. We find that simulation of New Keynesian DSGE Model by using the suggested calibration for the pre-Volcker period generates indeterminate equilibria[10]. Furthermore, Figure 7 represents the responses of the output gap, inflation and the nominal interest rates to one unit of “v” shock as we consider pre-Volcker “bad policy” period. In this case, inflation, output as well as interest rates response to one unit of shock. In this figure, note that the response of the inflation to shock is higher than the response of the output. Also, the effect of the shock on both inflation and output gap is high i.e inflation and output rise a lot in the second period, then the impact starts to decrease as time passes.

Third, we simulate and examine equilibrium characteristics of New Keynesian Model for the post-82 period. We find that simulation of New Keynesian DSGE Model by using the suggested calibration for the post-82 period generates determinate unique equilibrium. Figure 8 represents the responses of the output gap, inflation and the nominal interest rates to one unit of “v” shock as we consider post 82 (good policy) period. Now, comparing impulse response functions of pre-Volcker period (Figure 7) and post 82 period (Figure 8) is crucial. From the Figure 8, it is obvious that the response of inflation and the output gap in post 82 period is weaker than the responses of the pre-Volcker period. In other words, if we give a same one unit of “v” shock to each period, this shock will raise the inflation rate and output gap higher in pre-Volcker period than it does in post 82 period. This means that the effect of the shock is much stronger in the pre-Volcker period. By this result, one can also clarify that pre-Volcker period was a sort of “bad” policy in terms of monetary policy reaction to both inflation and the output gap.

[9] See appendix for all figures.
[10] MATLAB codes regarding the indeterminacy solution is avaliable upon request.

## 5 CONCLUSION

This paper tests "Bad Policy" Hypothesis which refers to the Great Moderation in the US. We examine this hypothesis by simulating model based impulse response functions for the both pre-Volcker period and post 1982 period. First, the model economy –New Keynesian Dynamic Stochastic General Equilibrium Model has been set up with consumers, firms and monetary policy. Second, we derived the all optimality conditions in this economy and obtained the equilibrium conditions based on three NK equations. Then, having solved the model in matrix form, we analyzed equilibrium characteristics: determinacy and indeterminacy. Fourth, we delve deeper into the model dynamics and showed how the model mechanics work for a simulation exercise. The next step was based on calibration and simulation of the model in order to examine equilibrium characteristics in both pre-Volcker period and Post-82 period. After doing this, we gave a monetary policy "v" shock to the system and obtained impulse response functions of inflation, output gap and nominal interest rate. Our main finding is that while post 1982 policy (i.e. *active* policy) is consistent with the unique stable equilibrium characteristics; pre-Volcker or *passive* monetary policy generates equilibrium indeterminacy. This result is consistent with the literature of the indeterminacy tests of the Great Moderation[11]. Furthermore, our simulated-impulse response functions show that the response of inflation and the output gap in post 82 period is weaker than the responses of the pre-Volcker period. In other words, if we give a same one unit of "v" shock to each period, this shock will raise the inflation rate and output gap higher in pre-Volcker period than it does in post 82 period. This means that the effect of the shock is much stronger in the pre-Volcker period. A future study may then use our simulated –impulse response functions and compare them with real – data impulse response functions for the both pre-Volcker and post 82 periods. This future goal is one of the main motivations to present this paper as well.

[11] See Lubik, T., Schorfheide, F. (2003). "Testing for Indeterminacy: An Application to US Monetary Policy", *American Economic Reviw*, (June 2003), pp. 1-51.

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## Appendix

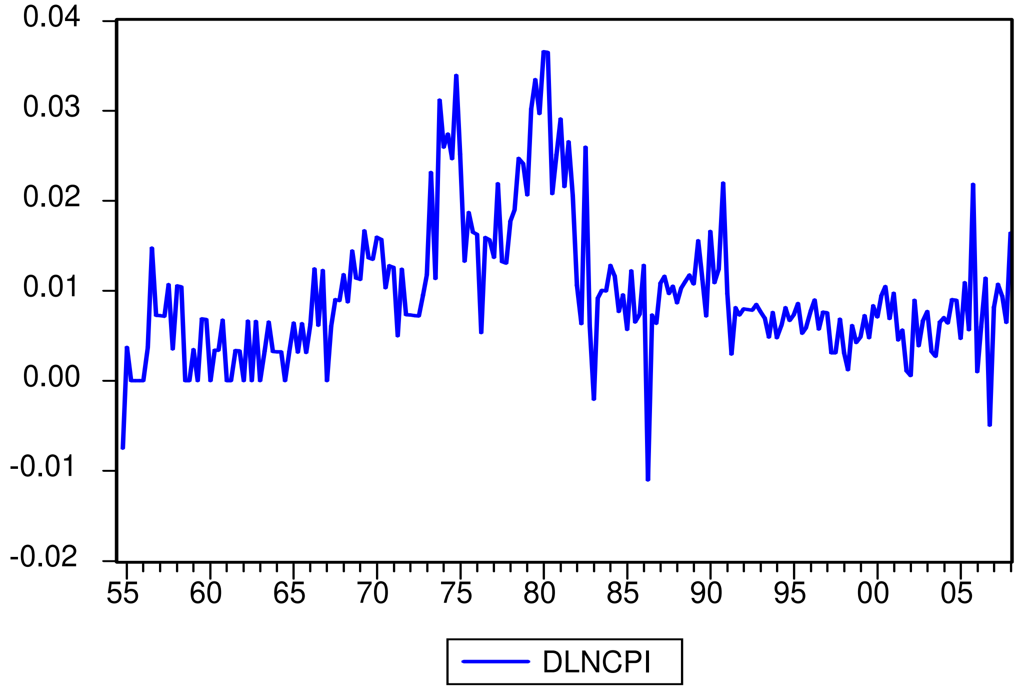



**Figure 1: Change in Consumer Prices (1954 – 2008)**

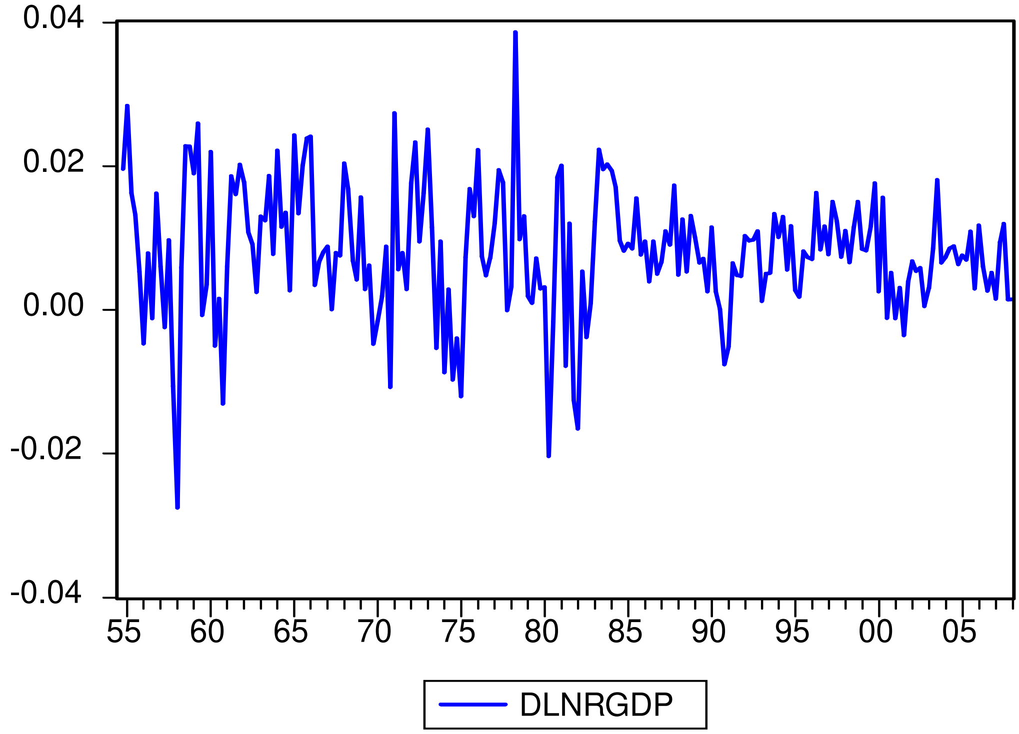



**Figure 2: US Output Growth (1954 – 2008)**

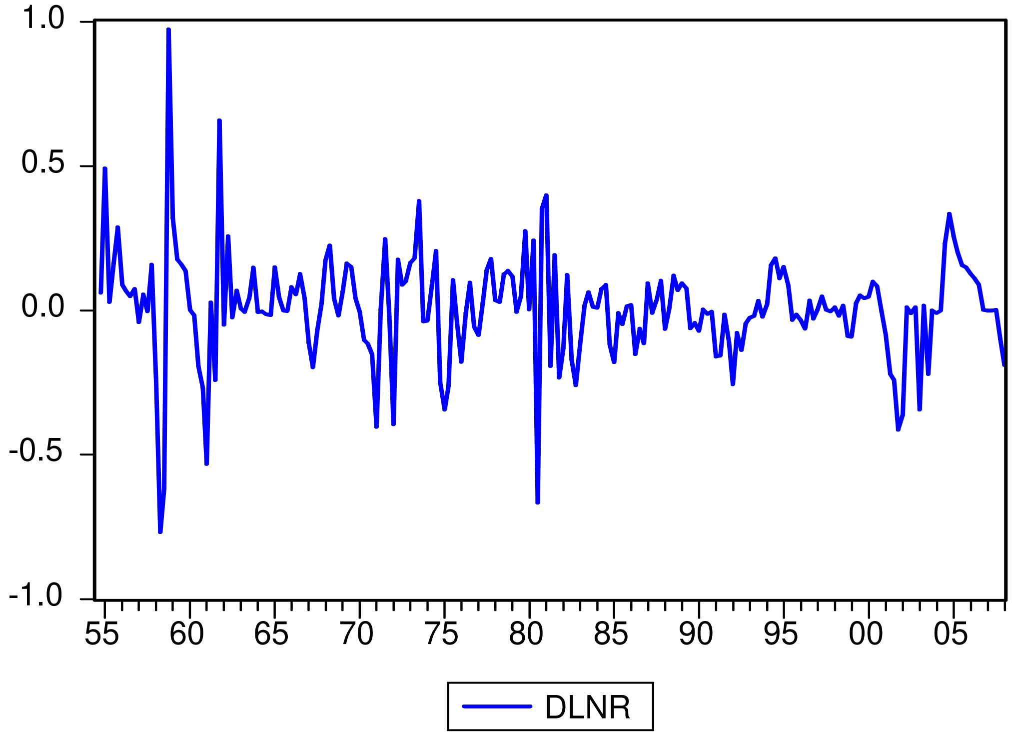



**Figure 3: Change in the Effective Fed Funds rates (1954 – 2008)**

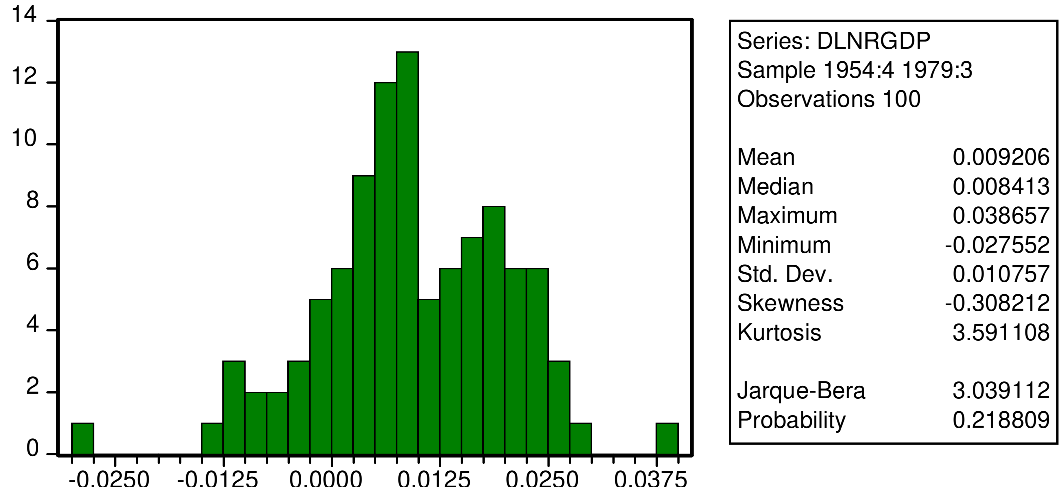
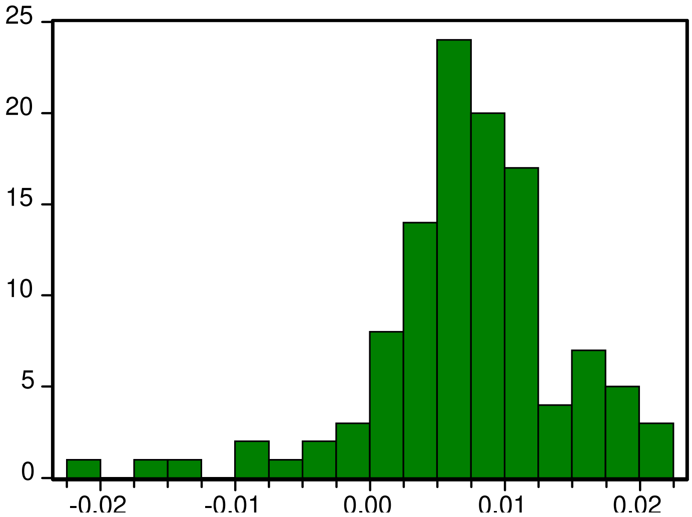



**Figure 4: Descriptive Statistics of the Pre-Volcker Output Growth**




| Series: DLNRGDP | |
|---|---|
| Sample 1980:1 2008:1 | |
| Observations 113 | |
| Mean | 0.007163 |
| Median | 0.007347 |
| Maximum | 0.022304 |
| Minimum | -0.020393 |
| Std. Dev. | 0.007050 |
| Skewness | -0.891178 |
| Kurtosis | 5.374034 |
| Jarque-Bera | 41.49375 |
| Probability | 0.000000 |

**Figure 5: Descriptive Statistics of the Post-1982 Output Growth**

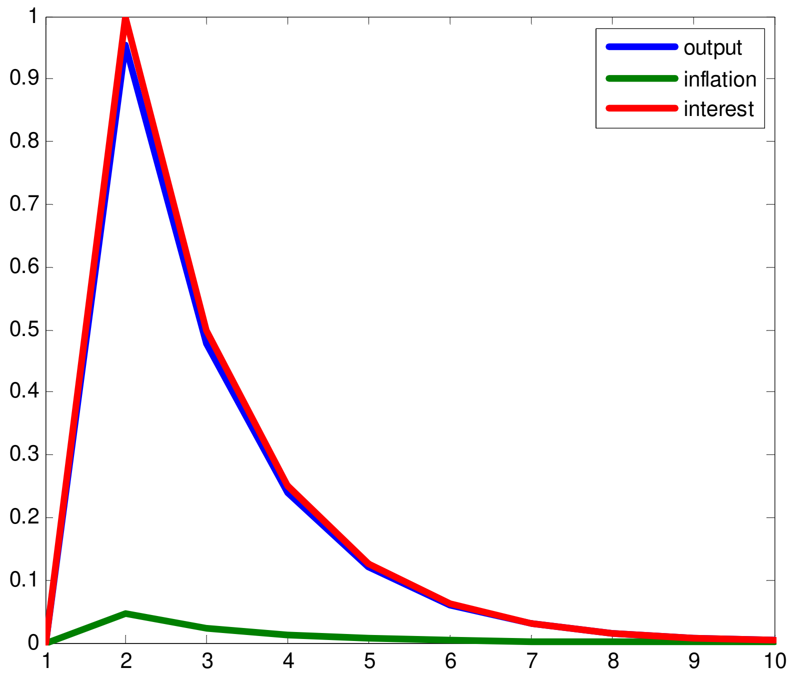



**Figure 6: Impulse Response Function of NK Model with Determinacy (Mixed Policy – Whole Sample)**

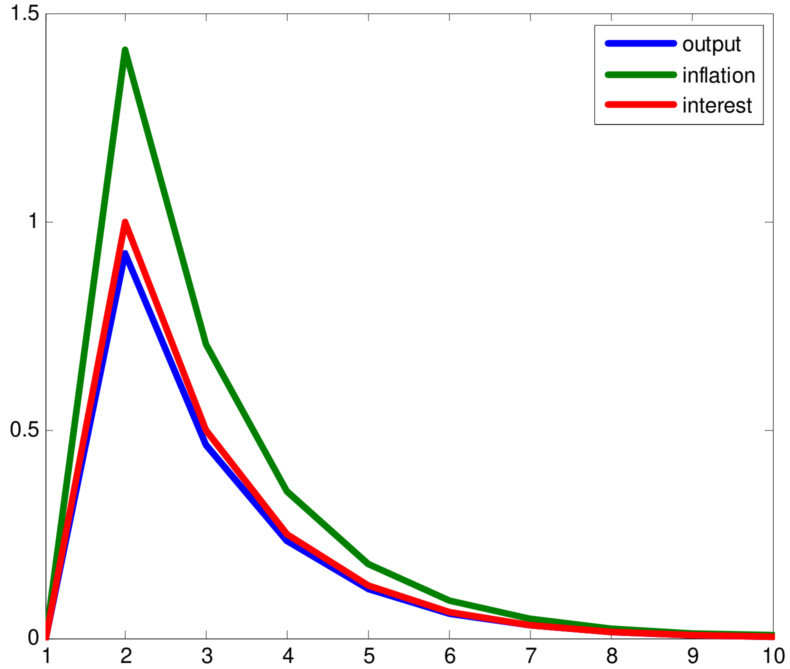



**Figure 7: Impulse Response Function of NK Model with Indeterminacy (Passive Policy - Pre-Volcker Period)**

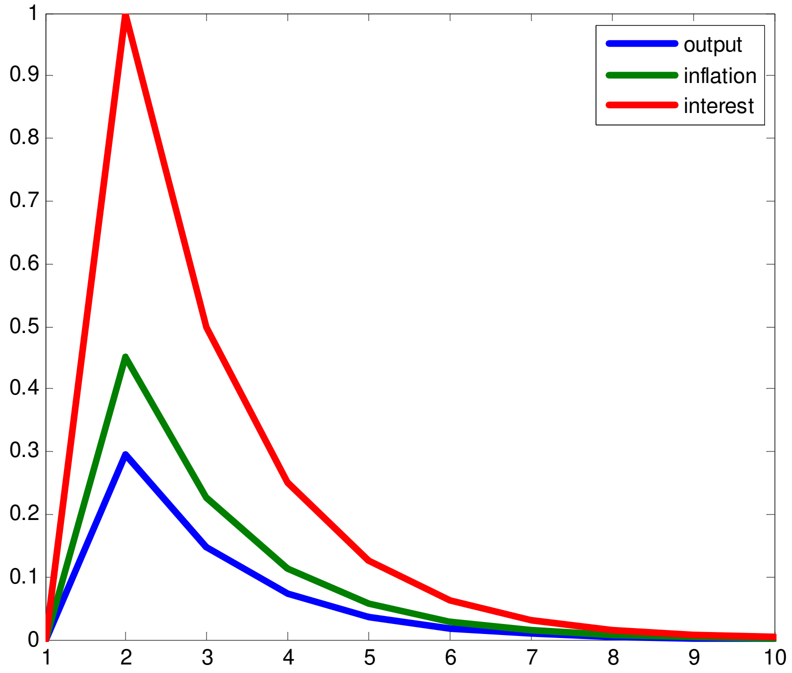



**Figure 8: Impulse Response Function of NK Model with Determinacy (Active Policy – Post 82 Period)**