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Coppola, Gianluigi

Dipartimento di Scienze Economiche e Statistiche, CELPE Centro di  
Economia del Lavoro e di Politica Economica, Università degli studi  
di Salerno

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## Health, Lifestyle and Growth:

*Gianluigi Coppola*  
*CELPE –Centro di Economia del lavoro e di Politica Economica*  
*University of Salerno (Italy)*  
84084 Fisciano, Via Ponte Don Melillo.  
E-mail [glcoppola@unisa.it](mailto:glcoppola@unisa.it)  
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*to Angelo*

### Abstract

In this paper we try to explain why lifestyle may have a positive impact on economic growth. First of all, we consider health affecting consumer's utility and we define also a Health Production Function where health is the output and the consumer's good are the inputs. In this approach we define lifestyle as the return to scale of the Health Production Function. A first result is that an increase of consumer's personal income may have a positive or a negative effect on health. According to this result, we modify the Solow Growth Model. We consider health as labour-augmenting. The result is a semi-endogenous model in which the population growth affects positively the income per capita growth, if lifestyle is positive.

Key words: Health, lifestyles, Growth

### **Introduction**

At the macro level the stylized facts show big differences in income per capita and in Health status among countries and/or regions. This may imply that low income per capita affects negatively health and vice versa.

It is useful to note that in the last 15 years the literature on economic growth focused primarily on the role of human capital accumulation while Health had a marginal role in the economic analysis. Secondly, if the differences among countries are strong (in income and in health), those ones among regions are stronger and also very important for economic growth.

This is essentially a theoretical paper in which the relationship between Health and Growth is built through the consumer lifestyle. Starting from Contoyannis and Jones's hypothesis (2004) a micro

model of consumer's choice is introduced in order to better define a measure of lifestyle and then to explain the effects of consumer's choices on his Health status. The first important result is that an increase of consumer's personal income may have a positive or a negative effect on his health if the same consumer has a good or a "bad" lifestyle.

At macro level For Weil (2005) one of the most important questions is: do the forces driving these differences come primarily from the side of health or from the side of income?

In this context we try to give an answer to the last question developing a simple modified Solow growth model in which health has a positive effects on labour productivity. For this reason this model includes the relationship among income, lifestyle and health status first obtained at micro level.

The main result of the model is that lifestyle may be crucial for the growth: a "good" lifestyle can generate a semi-endogenous growth, but a "bad" consumer's lifestyle may have also negative effects on the growth. The model also explains why improving in health have a positive effect on income while increasing in income may have a little effect on health (Weil, 2005)

## 1. A Micro Model

In this paragraph We develop a micro-funded model that explains the relationship between health and income, the effect of in income on health.

First of all, let's suppose an economy that produces 3 goods: 2 consumption good ( $x$  and  $z$ ), and Capital ( $K$ ). Saving rate ( $s$ ) is exogenous and constant

Starting from Grossman model (1972) the health capital and the demand for health have been widely modelled in economic literature. Among others, Contoyannis and Jones (2004) develop a static model of lifestyle and health production. In that model the assumptions are: 1) income is assumed to be endogenous, but there is no direct influence of lifestyle or health on wage; 2) health affects consumer's utility (unlike Grossman's dynamic model (1972) in which health is considered a stock that produces a flows of pecuniary and non pecuniary benefits as effect on investment on it). 3) health is a result of production function in which the inputs are *i)* a vector of goods, *ii)* a vector of

exogenous influences on health; *iii*) a vector of unobservable influences on health. 4) The money budget constraint and the time constraint close the model. The result is that maximizing the Consumer's utility with a Lagrangian function, they obtain the Marshallian demand for the goods, and for Health

We simplify and modify Contoyannis and Jones (2004) building-up a model of 2 equations: 1) the consumer's utility function; 2) the health production function.

### The consumer's utility function

We assume that the consumer's utility function is a Cobb Douglas where health ( $h$ ) is an input and for this reason it affects the consumer's utility function. The other 2 inputs are the goods  $x$  and  $z$ . In Formula the utility function is

$$U(h, x, z) = h^\alpha x^\beta z^\delta \quad [1.]$$

$\alpha$ ,  $\beta$  and  $\delta$  are respectively the elasticity of  $h$ ,  $x$  and  $z$ ;

$\alpha \geq 0$  may be considered the weight given to his own health by the consumer. If  $\alpha = 0$ , health is not important for the consumer. On the contrary if  $\alpha > 0$  then health is important

$\beta, \delta \leq 0$ . if  $\beta < 0$  (or  $\delta < 0$ )  $x$ , (or  $z$ ) isn't a good but a "bad" for the consumer (i.e. a medicinal).<sup>1</sup>

The individual consumes a good only if its elasticity is positive. We suppose that  $\beta > 0$  e  $\delta > 0$ -

. So we have  $\frac{dU(\cdot)}{dx} > 0$ ;  $\frac{dU(\cdot)}{dz} > 0$ . We also suppose that  $\frac{d^2U(\cdot)}{dx^2} < 0$ ;  $\frac{d^2U(\cdot)}{dz^2} < 0$

This is clearly a static equation. There is not dependence, but positive value of the elasticity means that the consumer knows the good's ophelimity.

### The health Production Function (HPF)

According Contoyannis and Jones (2004) consumption may affect consumer's health, and for this reason the consumer is a co-producer of his health. But the consumption of a good may better or worsen (or to be neutral) consumer's health status.

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<sup>1</sup> In the textbook a "bad" is an externality, something independent from the consumer's decision. Here a "bad" is a good that has a negativity impact on the utility of the consumer, and it can be used by the consumer according his own decision (i.e. a medicinal)

For sake of simplicity, We assume that a good can only better or worsen consumer's health status. In other words, there are no goods that can have a positive impact on health for small quantities and a negative for stronger doses. It assumes also that  $x$  improve health, while  $z$  worst health<sup>2</sup>.  $x$  can be defined as the virtuous good- in the sense of sustainable good - and  $s$  as the harmful good..

Health can also depends on the initial level of health status ( $h_0$ ), public health, ( $\Omega$ ) time ( $t$ ) and on a stochastic component  $\varepsilon$ . The Health Production Function (HPF) is

$$h(x, z, h_0, \psi, t, \varepsilon) = x^\rho z^{-\gamma} h_0 \psi e^{\theta t} e^\varepsilon \quad [2.]$$

The equation can be split into two parts:  $x^\rho z^{-\gamma}$  can be interpreted as the consumer's activity while the term  $h_0 \psi e^{\theta t} e^\varepsilon$  as other factors. For sake of simplicity We put  $h_0 \psi e^{\theta t} e^\varepsilon = \Omega$  and HPF becomes:

$$h(x, z, \Omega) = \Omega x^\rho z^{-\gamma} \quad [3.]$$

$(\rho - \gamma)$  is equal to the *elasticity of scale* and it can be positive, negative or null. Let  $\theta = \rho - \gamma$ .

Let's suppose that each input has a decreasing return to scale, as to say  $|\rho| < 1$  and  $|\gamma| < 1$ . So  $|\theta| \leq 1$ .

For Sassi and Hurst (2008) individual lifestyle are related to those individual behavioural that occupy a central position among health, because of their direct influences on individual health. Also Contoyannis and Jones (2004) define a lifestyle "as a set of behaviours which are considered to influence health"

For this reason We consider the parameter  $\theta$  as a proxy of the consumer lifestyle. If  $\theta > 0$  an increasing of the consumption has a positive effect on health, while for  $\theta < 0$  this effect is negative. With  $\theta = 0$  the consumer behaviour has no effect on health.

Substituting  $h(x, z, \Omega) = \Omega x^\rho z^{-\gamma}$  into  $U(h, x, z) = h^\alpha x^\beta z^\delta$ , it obtains

$$U(h, x, z) = \Omega x^{\alpha\rho} z^{-\alpha\gamma} x^\beta z^\delta \text{ or} \quad [4.]$$

$$U(h, x, z) = \Omega x^{\alpha\rho+\beta} z^{\delta-\alpha\gamma} \quad [5.]$$

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<sup>2</sup> The ancient Romans said "In Medius stat Virtus. That hypothesis doesn't matter in the model.

the  $x$ 's elasticity become  $\alpha\rho + \beta$  and the elasticity of  $z$   $\delta - a\gamma$ . The good  $z$  will be consumed only if  $\delta - a\gamma > 0$ . The choice of consuming  $z$  depends on 3 parameters: 1) the elasticity  $\delta$  of the good  $s$ , as to say the weight that the consumer confer to that good; 2)  $\alpha$ , the importance of his health, 3) and the measure of the damage of  $z$  on health ( $\gamma$ ).

It is useful to note that consumer can decide to use  $z$  *even if* he knows that  $z$  is dangerous for its health. Following this approach, It does not depend only on the level of education. Even people well aware of the damage that produces the smoke may continue to smoke if prove a pleasure quite high in this activity. Including health in the consumer's utility function, it increases the consumption of those goods that benefit health and decreases that good which causes damage.

The Utility maximization problem: The optimal choice of  $x, z$  and  $h$

$L \underset{x,s}{Max} x^{\alpha\rho+\beta} z^{\delta-\alpha\gamma}$  et  $\Omega = 1$ .  $p_x x + p_z z = y$  is the consumer's budget constraint where  $p_x, p_z$  are

the prices of the good and  $y$  is the income

The consumer' maximizes his utility when

$$\underset{x,s}{Max} x^{\alpha\rho+\beta} z^{\delta-\alpha\gamma}$$

$$\underset{x,s}{max} L = U(x, z) - \lambda(p_x x + p_z z - y). \lambda \text{ is the Langrage Multiplier}$$

In optimal condition the quantity of goods consumed are

$$x = \frac{\alpha\rho + \beta}{\beta + \delta + \alpha(\rho - \gamma)} \frac{y}{p_x}$$

$$s = \frac{\delta - \alpha\gamma}{\beta + \delta + \alpha(\rho - \gamma)} \frac{y}{p_z}$$

The weight of health,  $\alpha$ , increases the consumption of "virtuous" good and reduce the consumption of harmful good

In optimal condition, the health level is

$$h = \left( \frac{\alpha\rho + \beta}{\beta + \delta + \alpha(\rho - \gamma)} \frac{y}{p_x} \right)^\rho \left( \frac{\delta - a\gamma}{\beta + \delta + \alpha(\rho - \gamma)} \frac{y}{p_z} \right)^{-\gamma} \quad [6.]$$

or

$$h = \left( \frac{\alpha\rho + \beta}{\beta + \delta + \alpha(\rho - \gamma)} \right)^\rho \left( \frac{\delta - \alpha\gamma}{\beta + \delta + \alpha(\rho - \gamma)} \right)^{-\gamma} \left( \frac{(p_z)^\gamma}{(p_x)^\rho} \right) y^{(\rho - \gamma)} \quad [7.]$$

where

$$\left( \frac{\alpha\rho + \beta}{\beta + \delta + \alpha(\rho - \gamma)} \right)^\rho \quad \text{and} \quad \left( \frac{\delta - \alpha\gamma}{\beta + \delta + \alpha(\rho - \gamma)} \right)^{-\gamma}$$

are respectively the share of good x and of good z weighted for their own elasticity with respect to health.

The level of health and the price of virtuous good are negatively correlated. If the price of good increases, it worsens the level of health while if it decrease then it improves health conditions. On the contrary  $h$  improves (worsens) if the price of z increases (decreases),

The lifestyle  $\theta = \rho - \gamma$  is the elasticity of health with respect to income. Unlike the other parameters that can have only one sign, the elasticity of health with respect to income may be positive or negative. If  $\rho - \gamma = 0$  income's growth do not affect the level of health. If  $\rho - \gamma < 0$ , income affects health negatively. If  $\rho - \gamma > 0$  affects it positively.

## 2. A Growth model with health

In literature there are many models that consider health as a factor of growth Lòpez-Casasnovas and others (2005). Rivera and Currais (1999a) use a conditional convergence regression where the growth of per capita income is a function of the determinants of the steady state and considering health as an important determinant of an enhanced labour force, they obtain the result that health affects income growth both positively and significantly. In an other paper (Rivera and Currais (1999b)) investment in health contributes in a significant way to explain variation in output through in human capital even in those countries which presumably have high level of health Heshmati (2001) build up a model that is an extension of the MRW model by incorporating health. The results show that Health Care Expenditure has positive effect on the economic growth and the speed of convergence

In this paper we want to consider the effect of individual lifestyle on economic growth

One of the result of the micro model In the previous paragraph is that

$$h = \left( \frac{\alpha\rho + \beta}{\beta + \delta + \alpha(\rho - \gamma)} \right)^\rho \left( \frac{\delta - \alpha\gamma}{\beta + \delta + \alpha(\rho - \gamma)} \right)^{-\gamma} \left( \frac{(p_z)^\gamma}{(p_x)^\rho} \right) y^{(\rho-\gamma)} \quad [8.]$$

or  $h = \nu y^\theta$  where  $\nu = \left( \frac{\alpha\rho + \beta}{\beta + \delta + \alpha(\rho - \gamma)} \right)^\rho \left( \frac{\delta - \alpha\gamma}{\beta + \delta + \alpha(\rho - \gamma)} \right)^{-\gamma} \left( \frac{(p_z)^\gamma}{(p_x)^\rho} \right)$  and  $-1 \leq \theta \leq 1$

Let's now consider a Solow Growth Model with constant saving rate ( $s$ ), diminishing return of capital and labour, Labour augmenting technology, constant return to scale. The production function is

$$Y = K^a (AL)^{1-a} \quad [9.]$$

where  $K, A, L$  are respectively the capital, the technical progress and the labour.  $\alpha$  is capital elasticity

Considering that health is labour augmenting  $h = \nu Y^\theta$  than the production function becomes

$$Y = K^a (AhL)^{1-a} \quad [10.]$$

or

$$Y = K^a (A \nu Y^\theta L)^{1-a} \quad [11.]$$

$$Y = K^{\frac{a}{1-\theta(1-a)}} (A \nu L)^{\frac{1-a}{1-\theta(1-a)}} \quad [12.]$$

The parameter  $\theta$  becomes crucial in order to determine return to scale. For  $\theta > 0$  there are increasing returns to scale because

$$\frac{1}{1-\theta(1-a)} > 1$$

In steady state the Income growth rate is (see appendix for demonstration)

$$\dot{Y} = \left( \frac{1}{1-\theta} \right) (\dot{A} + \dot{L}) \quad [13.]$$

and Income per capita growth rate is



$$\frac{Y}{L} = \left( \frac{1}{1-\theta} \right) (\dot{A}) + \left( \frac{\theta}{1-\theta} \right) (\dot{L}) \quad [14.]$$

$\dot{Y}$ ,  $\dot{A}$ ,  $\dot{L}$  are respectively the growth rate of income, technical progress, and population growth

This is the most important results is that for  $\theta > 0$  economic growth is positive correlated with demographic growth and also that growth rate is higher than the Solow model. In other words if lifestyle is positive the model is a “semi-endogenous” growth model in which the population growth has a positive impact on economic growth. The result is similar to Arrow (1962) learning by doing model. In that model the technical progress depends on income through learning by doing process while in the model just developed the income impacts on labour productivity through lifestyle.

We can also quantify the impact on the income of a “health shock” and the impact on health of an “income shock” in terms of elasticity. The first is  $\frac{1-\alpha}{1-\theta(1-\alpha)}$  and the second is  $\frac{\theta(1-\alpha)}{1-\theta(1-\alpha)}$ . The

level of positive shock depends on the labour elasticity and on the lifestyle. If  $|\theta| \leq 1$  the effect of a health shock on income is greater that a shock of income on health

## Conclusions

In this paper a growth model with Health has been built. The crucial hypothesis are (i) that individuals are co-producers of their health and (ii) health affects positively labour productivity.

First I develop a consumer’s micro model with health and two goods. Both of them are positively correlate with the Consumer’s Utility. Health is the output of a ”consumer’s production function” with the two goods are inputs. The first good has a positive impact on health while the second good has a negative impact. The result is that the elasticity of consumer’s income on health depends on a parameter, named lifestyle, that is equal to the algebraically sum of the goods’ elasticity with respect to health. It may be positive, negative or neutral.

Secondly, this micro-behaviour rule is introduced in Solow growth model with constant return to scale. The result is that if lifestyle is positive (and less than 1) the growth of income per capita is higher than the technical progress and it depends positively on the population growth rate.



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## Appendix

In the Solow model:

$$Y = K^m (AL)^n$$

$$Y = K^m A^n L^n$$

$$\dot{A} = \lambda$$

$$\dot{L} = n$$

$$\dot{K} = \frac{dK}{dt} \frac{1}{K}$$

$$\dot{K} = s \frac{Y}{K}$$

$$\dot{K} = s \frac{K^m A^n L^n}{K} = s K^{m-1} A^n L^n$$

In steady state

$\dot{K}, \dot{A}, \dot{L}, \dot{Y}$  are constant

$$\dot{A} = \lambda$$

$$\dot{L} = n$$

$$\frac{d}{dt} K^{m-1} A^n L^n = 0$$

$$\dot{Y} = m\dot{K} + n\dot{A} + n\dot{L}$$

In steady state

$$\frac{d}{dt} K^{m-1} A^n L^n = 0$$

$$A^n L^n (m-1) K^{m-2} \frac{dK}{dt} + K^{m-1} L^n n A^{(n-1)} \frac{dA}{dt} + K^{m-1} A^n n L^{(n-1)} \frac{dL}{dt} = 0$$

$$(m-1) \frac{dK}{dt} \frac{1}{K} + n \frac{dA}{dt} \frac{1}{A} + n \frac{dL}{dt} \frac{1}{L} = 0$$

Growth of K in steady state

$$\frac{dK}{dt} \frac{1}{K} = \frac{n}{(1-m)} \frac{dA}{dt} \frac{1}{A} + \frac{n}{(1-m)} \frac{dL}{dt} \frac{1}{L}$$

Growth of Y in steady state

$$\frac{dY}{dt} \frac{1}{Y} = m \frac{n}{(1-m)} \frac{dA}{dt} \frac{1}{A} + m \frac{n}{(1-m)} \frac{dL}{dt} \frac{1}{L} + n \frac{dA}{dt} \frac{1}{A} + n \frac{dL}{dt} \frac{1}{L}$$

$$m \frac{n}{1-m} + n = \frac{n}{1-m}$$

$$\frac{dY}{dt} \frac{1}{Y} = \frac{n}{(1-m)} \frac{dA}{dt} \frac{1}{A} + \frac{n}{(1-m)} \frac{dL}{dt} \frac{1}{L}$$

$$\frac{dY}{dt} \frac{1}{Y} = \frac{n}{(1-m)} \frac{dA}{dt} \frac{1}{A} + \frac{(m+n-1)}{(1-n)} \frac{dL}{dt} \frac{1}{L}$$

In our case the production function is

$$Y = K^{\frac{\alpha}{1-\theta(1-\alpha)}} (A \nu L)^{\frac{1-\alpha}{1-\theta(1-\alpha)}}$$

if

$$m = \frac{\alpha}{1-\theta(1-\alpha)}$$

$$n = \frac{1-\alpha}{1-\theta(1-\alpha)}$$

$$1-m = \frac{1-\theta(1-\alpha)-\alpha}{1-\theta(1-\alpha)}$$

$$1-m = \frac{(1-\alpha)-\theta(1-\alpha)}{1-\theta(1-\alpha)}$$

then in steady state

$$\dot{K} = \frac{dK}{dt} \frac{1}{K} = \frac{n}{(1-m)} \frac{dA}{dt} \frac{1}{A} + \frac{n}{(1-m)} \frac{dL}{dt} \frac{1}{L}$$

$$\frac{dK}{dt} \frac{1}{K} = \frac{(1-\alpha)}{(1-\alpha)-\theta(1-\alpha)} \frac{dA}{dt} \frac{1}{A} + \frac{(1-\alpha)}{(1-\alpha)-\theta(1-\alpha)} \frac{dL}{dt} \frac{1}{L}$$

-

$$\frac{dK}{dt} \frac{1}{K} = \frac{1}{1-\theta} \frac{dA}{dt} \frac{1}{A} + \frac{1}{1-\theta} \frac{dL}{dt} \frac{1}{L}$$

The growth rate of Y is

$$\dot{Y} = \frac{\alpha}{1-\theta(1-\alpha)} \dot{K} + \frac{1-\alpha}{1-\theta(1-\alpha)} (\dot{A} + \dot{L})$$

$$\dot{Y} = \frac{\alpha}{1-\theta(1-\alpha)} \left( \frac{1}{1-\theta} \right) (\dot{A} + \dot{L}) + \frac{1-\alpha}{1-\theta(1-\alpha)} (\dot{A} + \dot{L})$$

$$\dot{Y} = \frac{\alpha + (1-\theta)(1-\alpha)}{1-\theta(1-\alpha)} \left( \frac{1}{1-\theta} \right) (\dot{A} + \dot{L})$$

$$\dot{Y} = \left( \frac{1}{1-\theta} \right) (\dot{A} + \dot{L})$$

$$\frac{Y}{L} = \left( \frac{1}{1-\theta} \right) (\dot{A}) + \left( \frac{\theta}{1-\theta} \right) (\dot{L})$$

The Growth rate of h is

$$\dot{h} = \left( \frac{\theta}{1-\theta} \right) (\dot{A} + \dot{L})$$