Okun's Law: Copula-based Evidence from G7 Countries

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Abstract

In this paper we present evidence on the association between unemployment and output in the G7 economies, which has direct implications for the validity of Okun's Law. Specifically, we investigate dependence and asymmetry between the residuals of the output and unemployment first difference equations using the copula methodology. We find that dependence between GDP and unemployment disturbances is strong only in USA and France followed by Canada, the UK and Germany. There is no dependence in Italy and Japan. This enhances the validity of Okun's Law in the former countries without invalidating it in Italy and Japan, since there is still a negative relationship given by the systematic part of the output-unemployment difference equations estimates. Also, there is asymmetry in the former five countries. Output disturbances are associated with unemployment ones only during recessions, while they are completely disentangled throughout contractions in the US, France, Canada, the UK and Germany. These findings imply that USA and France, and less so Canada, Great Britain and Germany provide the most favorable environment for counter-cyclical economic policies. In these economies, policy makers should react more than output-unemployment dynamic equations dictate in case of output slumps. However, during recoveries in these countries and in Italy as well as Japan during the whole business cycle, authorities ought to base stabilization policies solely on the systematic part of the relation between output and unemployment changes. Our results provide guidance to policy makers in addition to what is suggested by traditional empirical approaches, which focus on the estimation of the deterministic part of the output-unemployment relationship.

Keywords: Okun's Law; Dependence; Copula; Asymmetry

JEL: C14 , E10, E32

1. Introduction

The objective of this paper is to fully investigate for the first time the association between the residuals of output and unemployment first difference equations in terms of dependence and asymmetry in the G7 countries shedding light into cross-country comparisons between the world's largest economies. The nature of this association has direct and important implications for Okun's law and the conduct of economic policy.

The term Okun's law refers to the empirical regularity according to which a negative relationship exists between cyclical unemployment and cyclical output or unemployment changes and output changes. In particular, using quarterly US data, Okun stated "In the postwar period, on the average, each extra percentage point in the unemployment rate above four percent has been associated with about a three percent decrement in real GNP" (Okun, 1962). Except the theoretical importance of this regularity, since, combined with the Phillips curve, it gives the aggregate supply curve, there is renewed interest on Okun's law after the onset of the 2008 economic crisis. This is related to a central puzzle regarding this crisis, i.e. that ensuing Great Recession there has been sluggish employment growth during recovery, so called "jobless recovery" (Chinn et al., 2013; Jaimovich and Siu, 2012; Stock and Watson, 2012). Essentially, the basic issue has been whether structural unemployment has risen in the wake of the Great Recession and generally if the correlation between unemployment and output fluctuations varies over time and across countries. This is important for policy-makers in order to appraise the cost of lower output growth in terms of higher unemployment. Especially in monetary unions, like the EMU, this knowledge is very important both for central banks regarding monetary policy assessment and member countries for the formulation of other economic policies, e.g. related to their fiscal positions and labor markets.

The novelty of our work lies in that it is the first one to use the copula methodology, which offers important advantages relative to traditional empirical methods, to investigate the dependence as well as asymmetry between output and unemployment residuals in the G7 economies. A copula links two marginal distributions into a joint distribution (Joe, 2014; Nelsen, 2007). It captures dependence, because it contains all information on the joint distribution of two or more variables not utilized by traditional empirical techniques. Moreover, copulas are flexible in studying dependence separately from marginal distributions with no need for assumptions on the relation between those distributions (Dowd, 2008). So, we can fit different marginal distributions to different random variables, while commonly used approaches require fitting the same marginal distributions to all random variables. This implies much greater modelling flexibility compared to standard multivariate approaches, which is very useful, since different variables may be characterized by different marginal distributions. Additionally, using copulas we are able to examine all possible combinations of (upper and lower) tail dependence, which is not feasible with methods commonly applied in the literature. This is very important, because the type of dependence of variables like unemployment and output in Okun's law may be very different far away from their central masses than close to them. For example, output and unemployment may be strongly correlated close to average values but weakly correlated in situations of high unemployment and low output. Also, their dependence may differ between the two tails giving rise to asymmetry. Overall, copula methodology provides additional guidance to stabilization policy compared to the rest of the literature, because it informs policy makers on whether they should react more than the systematic part of the relationship between unemployment and output dictates in the wake of disturbances in order to reduce the severity of business cycles. Thus, our work contributes also to the literature on economic policy uncertainty (Çekin et al., 2019; Guo et al., 2018)

As far as dependence between output and unemployment is concerned, there is a very extensive literature, which supports Okun's law. However, there are important differences regarding the magnitude of the Okun coefficient. These are due to model specification (regarding the variables used and dynamic specification), model estimation methodology, estimation methodology of cyclical output and unemployment, sample period, data frequency, stage of development of the countries studied and the choice of regional versus national data (Perman et al., 2015). A paper related to ours is Moosa (1997), who examines the G7 economies and finds that the Okun coefficient is highest in North America and lowest in Japan and explains this evidence in terms of differences in labor market characteristics. He also estimates a rising absolute value of the coefficient over time due to labor market reforms. The other two works, which are relevant to ours, are Malley and Molana (2008) and Malley and Molana (2007), who conclude in favor of a persistent negative relationship between output and unemployment only in Germany using data for G7 countries. Ball et al. (2017) find that Okun's Law is a strong relationship in most countries, which is stable over time for the US since 1948 and 20 advanced countries since 1980. They also argue that the Okun coefficient varies substantially across countries reflecting special features of national labor markets. Dixon et al. (2017) reveal significant cross-country variations in the Okun coefficient but also symmetry, i.e positive and negative output gaps have the same effects on unemployment in 20 OECD countries over 1985-2013. Rahman and Mustafa (2017) show that Okun's Law is valid only for US and South Korea, while the evidence is weaker for Canada, Finland, France, Japan, Italy, Netherlands, New Zealand, Sweden, UK and Australia. However, it is invalid for Germany in 1971–2013. Grant (2018) find that a given unemployment gap has been associated with a smaller output gap since the Great Recession in the US. Nebot et al. (2019) conclude that Okun's coefficient for Germany, France and the Netherlands is similar and quite low, whereas it is much higher for Spain.

Regarding the issue of asymmetry, at least two theoretical studies (Gomme, 1999; Schettkat, 1996) suggest an asymmetric link between the relevant variables, i.e. that the relative change in cyclical unemployment depends on whether the output gap is positive (expansion) or negative (recession). At the empirical front, testing for asymmetry is very important, because if we ignore it when present, we are led to misspecified models, which produces poor forecasting and erroneous inference; thus it may lead to a rejection of the null hypothesis that there exists a long-run relationship between output and unemployment, when it actually exists, resulting in false policy implications, especially for unemployment policy. In light of this, many empirical papers have tested and confirmed the presence of asymmetry. For example, Virén (2001) finds that output growth has a strong effect on unemployment when unemployment is low and output is high, and vice versa for 20 OECD countries. Especially for the US, Cuaresma (2003) concludes that the contemporaneous effect of growth on unemployment is asymmetric and significantly higher in recessions than expansions. Also, unemployment shocks tend to be more persistent in the expansionary regime. This is in line with Silvapulle et al. (2004), who find that cyclical unemployment is more sensitive to negative compared to positive cyclical output using US data. Shin et al. (2014) find strong evidence of long-run asymmetry, i.e that unemployment is more sensitive to busts than booms, applying the NARDL framework to the unemployment-output relationship in the US, Canada and Japan. Moreover, particularly in Canada, they find dynamic asymmetries indicating that firms are quick to fire and slow to hire.

Recently, Valadkhani and Smyth (2015) conclude that the extent of within-regime asymmetry is stronger than across-regime asymmetry in the US. Moreover, Huang and Chang (2005) show that Okun coefficients differ remarkably across the business cycle, while confirming the validity of Okun's law for Canada. Wang and Huang (2017) find that Okun's coefficients are more negative in the low-growth regime, implying that the effect of output differences on unemployment differences is asymmetric, i.e more pronounced in recessions in the US over 1948:Q1-2016:Q4. Bournakis and Christopoulos in Bournakis et al. (2017) identify two regimes in Greece, i.e unemployment declines more in response to output increases under the high growth (above 1%) regime than in the low growth (below 1%) regime. Arabaci and Arabaci (2018) show that expansions and contractions have asymmetric effects on cyclical unemployment in Turkey. They identify an intermediate range of output gap, where cyclical unemployment does not decrease even though cyclical output is positive. Also, during recessions unemployment rises more than it falls during expansions. Finally, Nebot et al. (2019) estimate Okun's relationship for four European countries (France, Germany, the Netherlands and Spain) and confirm the existence of two regimes in each country but different thresholds across countries. Okun's relationship for Germany, France and the Netherlands is similar and different from Spain where it is much steeper.

In our analysis, first we investigate the stationarity and cointegration properties of the time series. Second, we employ VAR estimation of the unemployment and output differences for each country separately, extract the residuals and test them for serial correlation, ARCH effects and normality. Third, we standardize the residuals and transform them to copula data, which are then tested for independence and stability. Fourth, one copula model is selected for the residuals of each country's estimations out of a rich set of seventeen copula models considered.

Regarding the results, first we find that dependence of the residuals from the output and unemployment equations is relatively strong in the United States and France followed by Canada, the UK and Germany. In Italy and Japan there is independence. Second, we uncover asymmetry in all former countries. Specifically, negative output disturbances are associated with positive unemployment ones, while positive output disturbances are completely disentangled from negative unemployment ones. Thus, the relationship between the stochastic part of the unemployment-output equations differs across countries and within countries during the business cycle. As a consequence, countries are ranked in decreasing order in terms of effectiveness of stabilization policies in three groups: i) US, France; ii) Canada, the UK and Germany; iii) Italy and Japan. So, in the former two country groups, policy makers should follow more aggressive counter-cyclical policies than those implied by the deterministic part of the output-unemployment relations contrary to the latter two countries. Our findings are especially important for member states of a monetary union, like the EMU, Germany, France and Italy in our sample, which lack independent monetary policy instruments. The structure of the paper is as follows. In Section 2, we analyze the empirical methodology used for the study of dependence and asymmetry between output and unemployment residuals and in Section 3 we describe the data and empirical modelling. In Section 4, we present the results, while in Section 5 we discuss the main findings. Finally, we outline conclusions and policy implications in Section 6. The Appendix includes details on the findings.

2. Copulas, rank correlation and tail dependence

Copulas dates back to Sklar (1959), but only recently copula models have experienced widespread application in empirical models of joint probability distributions (see (Nelsen, 2007) for more details). These models use a copula function, which links two marginal probability functions that may or may not be related to one another.

A two–dimensional copula, $C(u_1, u_2)$, is a multivariate distribution function in the unit hypercube $[0,1]^2$ with uniform U(0,1) marginal distributions (Nelsen, 2007).[1] As long as the marginal distributions are continuous, a unique copula is associated with the joint distribution, H, and is described by equation (1). This function constitutes a form of the principal result of copula theory (Sklar's theorem). It is obtained as:

$$C(u_1, u_2) = H(H_1^{-1}(u_1), H_2^{-1}(u_2)) \tag{1}$$

Similarly, given a two-dimensional copula, $C(u_1, u_2)$, and two univariate distributions, $H_1(x)$ and $H_2(x)$, equation 1 is a two-variate distribution function with marginals $H_1(x)$ and $H_2(x)$, whose corresponding density function can be written as:

$$h(x,y) = c(H_1(x), H_2(y))h_1(x)h_2(y), \tag{2}$$

where the functions h_1 and h_2 are the densities of the distribution functions H_1 and H_2 respectively.

The density function of the copula, C, given its existence, can be derived using equation 1 and marginal density functions, h_i:

$$c(u_1, u_2) = \frac{h(H_1^{-1}(u_1), H_2^{-1}(u_2))}{h_1(H_1^{-1}(u_1))h_2(H_2^{-1}(u_2))} \tag{3}$$

[1]For simplicity we consider the bivariate case. The analysis, however, can be extended to a p-variate case with $p > 2$.

A rank based test of functional dependence is Kendall's τ. It provides information on co-movement across the entire joint distribution function, both at the center and at the tails of it. It is calculated from the number of concordant (P_N) and disconcordant (Q_N) pairs of observations in the following way:

$$\tau_N = \frac{P_N - Q_N}{\binom{N}{2}} = \frac{4P_N}{N(N-1)} - 1 \tag{4}$$

If a copula function (C) is known then τ can be calculated as:

$$\tau = 1 - 4\int\int_{[0,1]^2} \frac{\partial C}{\partial u_1}\frac{\partial C}{\partial u_2} \mathrm{d}u_1\, \mathrm{u}_2 \tag{5}$$

Often, though, information concerning dependence at the tails (at the lowest and the highest ranks) is extremely useful for economists, managers and policy makers. Tail (extreme) co-movement is measured by the upper, λ_U, and the lower, λ_L, dependence coefficients, such that λ_U, $\lambda_L \in [0,1]$, which are defined as:

$$\lambda_L = \lim_{u\downarrow 0}\mathbb{P}(U_1 < u | U_2 < u) = \lim_{u\downarrow 0}\frac{C(u,u)}{u} \tag{6a}$$

$$\lambda_U = \lim_{u\uparrow 1}\mathbb{P}(U_1 > u | U_2 > u) = \lim_{u\uparrow 1}\frac{1-2u+C(u,u)}{1-u} \tag{6b}$$

where, given the random vector (X,Y) with marginal distribution, U_1 for X and U_2 for Y, λ_U measures the probability that X is above a high quantile given that Y is also above that high quantile, while λ_L measures the probability that X is below a low quantile given that Y is also below that low quantile. In order to have upper or lower tail dependence, λ_U or λ_L need to be strictly positive respectively. Otherwise, there is upper or lower tail independence. Hence, the two measures of tail dependence provide information about the likelihood for the two random variables to boom and cllapse together. For example, in our work, positive upper and zero lower tail dependence estimates would provide evidence that large unexpected increases in output are matched by large unexpected unemployment declines, whereas extreme output slumps are not likely to be transmitted to unemployment.

This study considers a wide range of bivariate copula specifications. All of them are members of the elliptical copulas and Archimedean copulas, since they permit considerable flexibility in capturing dependence between output and unemployment. Elliptical and Archimedean copulas are two of the most commonly used copula families. The elliptical copulas we evaluate are the Gaussian (or Normal) and Student–t. Among the one parameter Archimedean copulas we consider, there are the Clayton, Gumbel, Frank, and Joe. Clayton-Gumbel, Joe-Gumbel, Joe-Clayton and Joe-Frank are among the two-parameter Archimedean copulas we examine.

Table 1 presents the copulas under consideration in our study, their respective dependence parameters, their relationship to Kendall's τ as well as to λ_U and λ_L (upper and lower dependence coefficients). Regarding the elliptical family, the Gaussian copula is symmetric and exhibits zero tail dependence. Thus, irrespective of the degree of the overall dependence, extreme changes in one random variable are not associated with extreme changes in the other random variable. The t-copula exhibits symmetric non-zero tail dependence (joint booms and slumps have the same probability of occurrence). Concerning the one parameter Archimedean copulas, the Clayton copula exhibits only left co-movement (lower tail dependence). The Gumbel and the Joe copulas exhibit only right co-movement (upper tail dependence). The Frank copula has zero tail dependence. As far as the two- parameter Archimedean copulas are concerned, the Gumbel-Clayton and the Joe-Clayton allow for potentially asymmetric upper and lower co-movement. The Joe-Gumbel exhibits only right co-movement, while the Joe-Frank exhibits zero tail dependence.

Table 1: Copula functions, parameters, Kendall's τ, tail dependence $^{(*)}$

Copulas	**Parameters**	**Kendall's τ**	**Tail dependence** (λ_L, λ_U)
1 Gaussian (N)	$\theta \in (-1,1)$	$\frac{2}{\pi}\arcsin(\theta)$	(0,0)
2 Student-t (t)	$\theta \in (-1,1)$, $\nu > 2$	$\frac{2}{\pi}\arcsin(\theta)$	$2t_{\nu+1}(-\sqrt{\nu+1}\sqrt{\frac{1-\theta}{1+\theta}})$, $2t_{\nu+1}(-\sqrt{\nu+1}\sqrt{\frac{1-\theta}{1+\theta}})$
3 Clayton (C)	$\theta > 0$	$\frac{\theta}{\theta+2}$	$(2^{\frac{-1}{\theta}}, 0)$
4 Gumbel (G)	$\theta \geq 1$	1- $\frac{1}{\theta}$	(0, 2 - $2^{\frac{1}{\theta}}$)
5 Frank (F)	$\theta \in R\backslash\{0\}$	$1-\frac{4}{\theta}+4\frac{D(\theta)}{\theta}$ with $D(\theta)=\int_0^\theta \frac{x/\theta}{\exp(x)-1}\mathrm{d}x$	(0,0)
6 Joe (J)	$\theta \geq 1$	1+$\frac{4}{\theta^2}\int_0^1 t\log(t)(1-t)^{2(1-\theta)/\theta}\,\mathrm{d}t$	(0, 2 - $2^{\frac{1}{\theta}}$)
7 Clayton-Gumbel (BB1)	$\theta_1 > 0$, $\theta_2 \geq 1$	$1-\frac{2}{\theta_2(\theta_1+2)}$	($2^{\frac{-1}{\theta_1\theta_2}}$, 2 - $2^{\frac{1}{\theta_2}}$)
8 Joe-Gumbel (BB6)	$\theta_1 \geq 1$, $\theta_2 \geq 1$	1+$\frac{4}{\theta_1\theta_2}\int_0^1(-\log(1-(1-t)^{\theta_1})$ $\times(1-t)(1-(1-t)^{-\theta_1}))\,\mathrm{d}t$	(0, 2 - $2^{\frac{1}{\theta_1\theta_2}}$)
9 Joe-Clayton (BB7)	$\theta_1 \geq 1$, $\theta_2 > 0$	1+$\frac{4}{\theta_1\theta_2}\int_0^1(-(1-(1-t)^{\theta_1})^{\theta_2+1}$ $\times\frac{(1-(1-t)^{\theta_1})^{-\theta_2}-1}{(1-t)^{\theta_2-1}})\mathrm{d}t$	($2^{\frac{-1}{\theta_2}}$, 2 - $2^{\frac{1}{\theta_1}}$)
10 Joe-Frank (BB8)	$\theta_1 \geq 1$, $\theta_2 \in (0,1]$	$1+\frac{4}{\theta_1\theta_2}\int_0^1(-\log(\frac{(1-t\theta_2)^{\theta_1}-1}{(1-\theta_2)^{\theta_1}-1})\times(1-t\theta_2)(1-(1-t\theta_2)^{-\theta_1}))\,\mathrm{d}t$	(0, 0)
13 Survival Clayton (SC)	$\theta > 0$	$\frac{\theta}{\theta+2}$	(0, $2^{\frac{-1}{\theta}}$)
14 Survival Gumbel (SG)	$\theta \geq 1$	1- $\frac{1}{\theta}$	$(2-2^{\frac{1}{\theta}}, 0)$
16 Survival Joe (SJ)	$\theta > 1$	1+$\frac{4}{\theta^2}\int_0^1 t\log(t)(1-t)^{2(1-\theta)/\theta}\,\mathrm{d}t$	(0, 2 - $2^{\frac{1}{\theta}}$)
17 Survival BB1 (SBB1)	$\theta > 0, \delta \geq 1$	$1-\frac{2}{\theta_2(\theta_1+2)}$	($2^{\frac{-1}{\theta_1\theta_2}}$, 2 - $2^{\frac{1}{\theta_2}}$)
18 Survival BB6 (SBB6)	$\theta > 1, \delta \geq 1$	1+$\frac{4}{\theta_1\theta_2}\int_0^1(-\log(1-(1-t)^{\theta_1})$ $\times(1-t)(1-(1-t)^{-\theta_1}))\,\mathrm{d}t$	(0, 2 - $2^{\frac{1}{\theta_1\theta_2}}$)
19 Survival BB7 (SBB7)	$\theta > 1, \delta > 0$	1+$\frac{4}{\theta_1\theta_2}\int_0^1(-(1-(1-t)^{\theta_1})^{\theta_2+1}$ $\times\frac{(1-(1-t)^{\theta_1})^{-\theta_2}-1}{(1-t)^{\theta_2-1}})\mathrm{d}t$	($2^{\frac{-1}{\theta_2}}$, 2 - $2^{\frac{1}{\theta_1}}$)
20 Survival BB8 (SBB8)	$\theta > 1, \delta > 0$	$1+\frac{4}{\theta_1\theta_2}\int_0^1(-\log(\frac{(1-t\theta_2)^{\theta_1}-1}{(1-\theta_2)^{\theta_1}-1})\times(1-t\theta_2)(1-(1-t\theta_2)^{-\theta_1}))\,\mathrm{d}t$	(0,0)

$^{(*)}$ Table adapted from (Joe, 2014) and (Schepsmeier et al., 2016).

3. Data and empirical models

3.1. Data

Data for unemployment and GDP have been downloaded from the OECD website.[2] The G7 countries have been considered, i.e. United States (USA), Canada (CAN), Japan (JPN), Great Britain (GBR), Germany (DEU), France (FRA) and Italy (ITA). The reasons for this choice are: (i) the application of copula methodology first to the biggest world economies, which exert the largest influence on economic conditions worldwide through trade and capital movements, therefore on smaller economies; (ii) better data availability (higher quality and quantity) in the G7 countries relative to other groups of economies, given that the use of copulas requires long time series. Regarding ΔGDP, quarterly seasonally adjusted data on percentage change relative to the previous quarter are used, which correspond to subject B1_GE and measure GPSA from the QNA table (doi: 10.1787/data-00017-en). As for unemployment U, the quarterly harmonized percentage of unemployment (total, all ages) has been used, which corresponds to subject LRHUTTTT and measure STSA of the LABOUR table (doi: 10.1787/mei-data-en). Data were downloaded for the period 1994:Q1 to 2018:Q2, since there are no comparable data before 1994. Sample code used to download the dataset can be found in Appendix E.

There are two broad ways of testing for Okun's law in the literature. The first uses cyclical components, while the second utilizes first differences of output and unemployment. Both have advantages and disadvantages. The former methodology requires use of filtering techniques, of which there are many alternatives, but there is no agreement in the literature as to which is the most appropriate. This is problematic, since the findings are very sensitive to the filtering method (Arčabić and Olson, 2019; Huang and Yeh, 2013; Lee, 2000; Moosa, 1997; Perman and Tavera, 2007; Silvapulle et al., 2004). However, the utilization of first differences requires assumptions on the stochastic processes that the data follow, i.e. that all variables correlated with output except unemployment (e.g. labor force, capital stock) are in equilibrium or change *pari passu* with the latter (Okun, 1970).

In light of these, we proceed with the first-difference version of Okun's law in line

[2]The OECD R package Persson (2016) has been used for database access and data download and manipulation. Data last accessed on January 04, 2019.

with Prachowny (1993), Lin et al. (2008), Zanin and Marra (2012), Sögner (2001). Consequently, we have constructed the following variable:

$$\Delta U_t = U_t - U_{t-1} \tag{7}$$

In order to obtain positively correlated data, which allows for the consideration of extended copula families, data were transformed as follows:

$$\Delta GDP_t = \frac{GDP_t - GDP_{t-1}}{GDP_{t-1}} \tag{8a}$$

$$-\Delta U_t = U_{t-1} - U_t \tag{8b}$$

3.2. Preliminary analysis

Figure 1 displays the time series of ΔU and ΔGDP for the G7 countries.

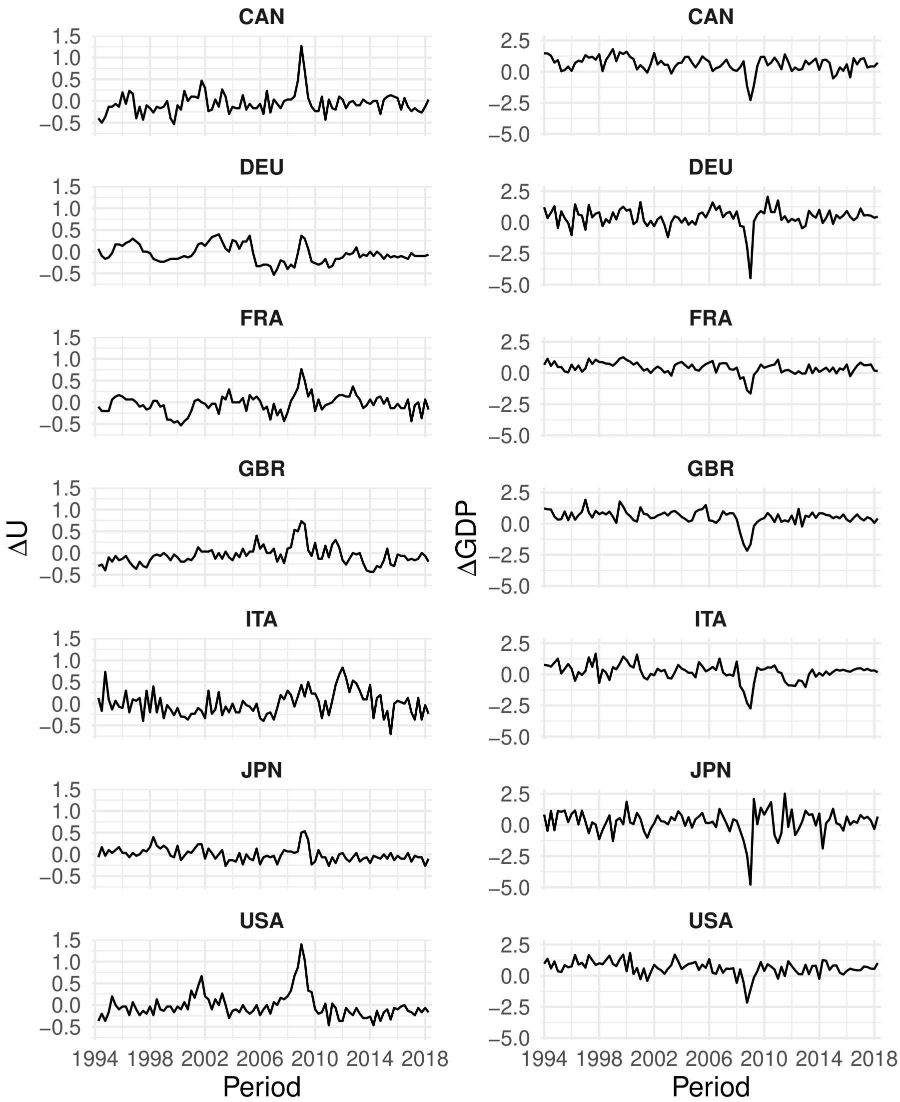

Figure 1: Time series plots of ΔU (left) and ΔGDP (right) data.

Table 2 displays descriptive statistics and basic tests of the $-\Delta U$ and ΔGDP variables, as defined in equations 8a and 8b respectively. Mean values of $-\Delta U$ were quite close to zero and substantially lower than the corresponding standard deviations, for example 0.0056 and 0.3083 for Canada. This is indicative of the absence of trend in $-\Delta U$ data. On the other hand, mean values of ΔGDP were found between 0.18 and 0.62 in all cases, which indicates a positive trend in ΔGDP data.

Negative skewness values (left skewed data) were observed in all countries for both

$-\Delta U$ and ΔGDP variables. Fat tails in the data are indicated by the high values of the kurtosis statistic in almost all countries, in both $-\Delta U$ and ΔGDP. As a matter of fact, normality was rejected by both Kolmogorov-Smirnov and Cramer von Mises tests, as in most cases p-values were found to be very small for both $-\Delta U$ and ΔGDP variables.

With the exception of ΔGDP in the case of Japan, the Lung-Box test suggested the presence of serial correlation in the data. However, test results of ARCH effects were mixed; low p-values of the test were found for $-\Delta U$ in Canada, France, Great Britain and Italy and ΔGDP in the case of France, Great Britain and Italy. In all other cases there was no indication for the presence of ARCH effects in the data.

Table 2: Descriptive statistics

	CAN	DEU	FRA	GBR	ITA	JPN	USA
$-\Delta U$							
Mean	0.0056	0.0313	-0.0028	0.0446	0.0446	0.0189	-0.0040
Std.Dev.	0.3083	0.2512	0.1571	0.2213	0.2164	0.2401	0.2932
Min	-1.4000	-1.2667	-0.5333	-0.7333	-0.4000	-0.8333	-0.8333
Max	0.5000	0.5333	0.2667	0.5333	0.5333	0.5333	0.6000
Skewness	-2.0184	-1.9131	-0.9171	-1.1464	-0.2031	-0.2321	-0.4943
Kurtosis	8.7281	10.7927	4.5399	5.3915	2.2152	4.3287	2.9473
KS[a]	$<$1e-4	0.0002	0.0151	0.0009	0.0724	0.0032	0.0543
CvM[a]	$<$1e-4	0.0002	0.0270	0.0004	0.0325	0.0052	0.1586
Q(12)[b]	$<$1e-4	0.0150	0.0002	$<$1e-4	$<$1e-4	$<$1e-4	$<$1e-4
ARCH-LM[c]	0.0016	0.5648	0.0055	0.0027	0.0006	0.3351	0.1251
ΔGDP							
Mean	0.6134	0.3726	0.4167	0.5349	0.1835	0.2442	0.6192
Std.Dev.	0.6184	0.7960	0.4642	0.5926	0.6936	0.9631	0.5954
Min	-2.2842	-4.4861	-1.6453	-2.1715	-2.7375	-4.7942	-2.1638
Max	1.8114	2.0582	1.2609	1.9277	1.6659	2.5252	1.8312
Skewness	-1.3968	-2.4823	-1.3987	-1.7990	-1.2004	-1.6199	-1.2049
Kurtosis	7.3690	16.3263	7.4902	9.6054	6.6311	9.9347	7.3029
KS	0.0419	0.0025	0.1999	0.0000	0.0001	0.0004	0.0151
CvM	0.0240	0.0000	0.0462	0.0000	0.0000	0.0004	0.0036
Q(12)	0.0006	0.0004	0.0000	0.0000	0.0000	0.6844	0.0001
ARCH-LM	0.1847	0.9846	0.0442	0.0479	0.0477	0.4953	0.7647

a) p-values are displayed for the Kolmogorov-Smirnov (KS) and Cramer von Misses (CvM) test for normality.

b) Q(12) lists the p-values of the Ljung-Box test for time series independence taking into consideration 12 lags.

c) ARCH-LM lists p-values of the autoregressive conditional heteroskedasticity-Lagrange multiplier test, also using 12 lags.

Figure 2 displays the scatter plot of ΔU and ΔGDP data and the estimated equation:

$$\Delta GDP_t = \widehat{\alpha} + \widehat{\beta}\,\Delta U_t \tag{9}$$

OLS estimation results of a linear model above (eq. 9) are displayed in Table 3. At this point we do not analyze further the linear fit results, which nevertheless exhibit a negative relationship between unemployment and output changes as predicted by Okun's Law.

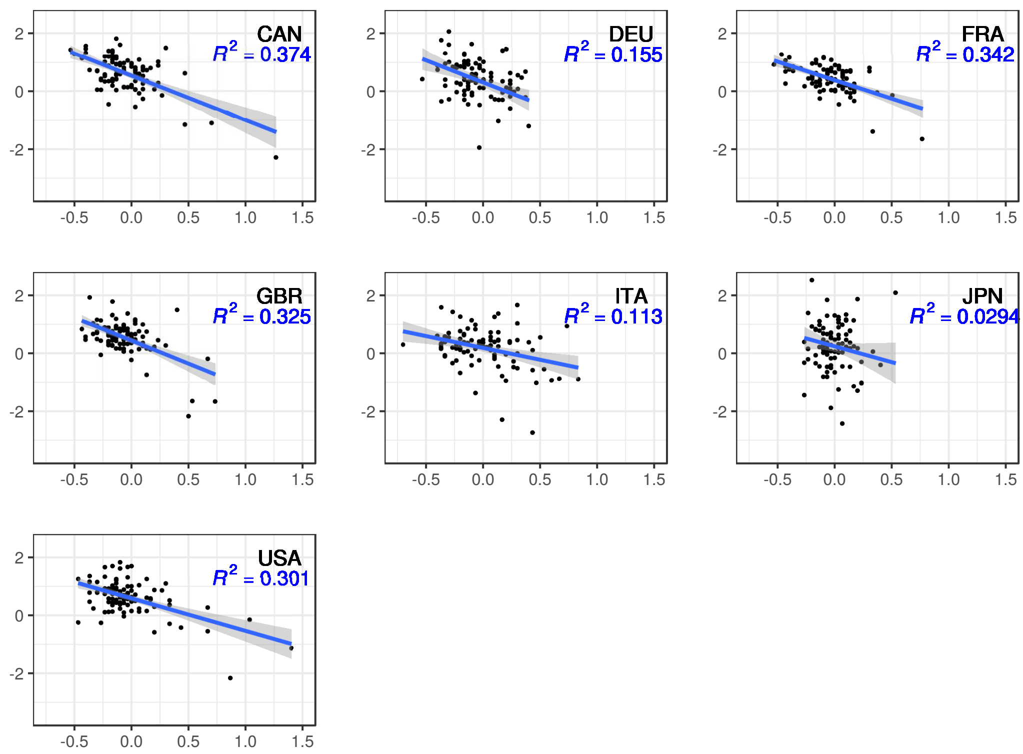

Figure 2: Okun law plot and linear regression of ΔGDP (vertical axis) on ΔU (horizontal axis).

Table 3: OLS estimation of the Okun Law parameter

Country	term	estimate	std.error	statistic	p.value
CAN	(Intercept)	0.5315	0.0511	10.4001	$< 1e^{-4}$
	ΔU	-1.5370	0.2040	-7.5345	0.0000
DEU	(Intercept)	0.2931	0.0771	3.8026	0.0003
	ΔU	-1.5313	0.3670	-4.1725	0.0001
FRA	(Intercept)	0.3710	0.0389	9.5331	$< 1e^{-4}$
	ΔU	-1.2801	0.1813	-7.0586	0.0000
GBR	(Intercept)	0.4391	0.0516	8.5178	$< 1e^{-4}$
	ΔU	-1.6013	0.2354	-6.8027	0.0000
ITA	(Intercept)	0.1857	0.0667	2.7837	0.0065
	ΔU	-0.8104	0.2340	-3.4625	0.0008
JPN	(Intercept)	0.2385	0.0969	2.4617	0.0156
	ΔU	-1.0893	0.6420	-1.6968	0.0930
USA	(Intercept)	0.5885	0.0510	11.5282	$< 1e^{-4}$
	ΔU	-1.1179	0.1750	-6.3897	0.0000

3.3. Testing for unit roots and cointegration

Stationarity or unit root presence in GDP time series is a controversial issue and has long been a puzzle in the literature (see for example Cushman (2016) and references cited therein). Similarly, the existence of unit root in unemployment data is questionable (see (Lee et al., 2013, 2009) and references cited therein).

We have performed various tests in order to test for the presence of unit root in the data (Hamilton, 1994; Phllips and Xiao, 1998), such as the Augmented Dickey–Fuller (ADF) test (Dickey and Fuller, 1981), the Kwiatkowski, Phillips, Schmidt & Shin (KPSS) test (Kwiatkowski et al., 1992), the Elliott, Rothenberg & Stock (ERS or DF-GLS) test (Elliott et al., 1996), the Zivot & Andrews (ZA) test (Zivot and Andrews, 1992) Finally, we have implemented the Carrion-i-Silvestre, Kim & Perron (CKP) test (Carrion-i Silvestre et al., 2009), which is the only one allowing for multiple structural breaks in both the level and slope of the trend function. Here, we should note that we have conducted the (Bai and

Perron, 2003) test before the (CKP) test in order to determine the appropriate number of breaks (up to five). Based on these findings, we have implemented the relevant version of the CKP test.

In order to check for cointegration (Hamilton, 1994) the Johansen (Johansen, 1991) and Gregory & Hansen (Gregory and Hansen, 1996) tests can be used.

To save space, all relevant results and details are available in the Appendix (Section A).

3.4. *Empirical modelling*

We have employed the VAR methodology, which allows all dependent variables to depend on their own lags and lags of all the other dependent variables, in order to extract the random component of the data series. VAR modelling is routinely used in applied macroeconomics research (Cover and Mallick, 2012; Stock and Watson, 2001).

A semi-parametric approach has been applied in the empirical part of this article (Chen and Fan, 2006; Fan and Patton, 2014; Huang et al., 2016; Mokni and Youssef, 2019; Patton, 2012):

1. ΔGDP and $-\Delta U$ variables were filtered via VAR modelling (details are given below).
2. Residuals of the resulting model were tested for autocorrelation and ARCH effects and transformed to copula data via an empirical cumulative density function and appropriately scaled by $n/(n+1)$.
3. Pairs of copula data for each country were used for copula selection and estimation.

As a first step, a VAR model of 4^{th} order was estimated, since quarterly data were used, for pairs of ΔGDP and $-\Delta U$ for each country (see equations 10).

$$-\Delta U_t = \mu_1 + \sum_{i=1}^{4} \Phi_{1,i}(-\Delta U_{t-i}) + \sum_{i=1}^{4} \Theta_{1,i}\Delta GDP_{t-i} + \alpha_1 t + u_t \tag{10a}$$

$$\Delta GDP_t = \mu_2 + \sum_{i=1}^{4} \Phi_{2,i}(-\Delta U_{t-i}) + \sum_{i=1}^{4} \Theta_{2,i}\Delta GDP_{t-i} + \alpha_2 t + v_t \tag{10b}$$

Each VAR model was estimated by OLS and the optimal value of the number of lags (n) was selected by applying the following information criteria: Akaike (AIC), Schwarz

(BIC), Hannan and Quinn (HQ) and Final prediction error (FPE). Results of the lag order selection procedure are shown in Table 4.

Table 4: Lag order selection according to four information criteria.

	AIC	HQ	SC	FPE	**USE**
CAN	1	1	1	1	1
DEU	1	1	1	1	1
FRA	2	2	1	2	2
GBR	1	1	1	1	1
ITA	2	1	1	2	2
JPN	4	1	1	4	1
USA	2	2	1	2	2

The last column (USE) indicates the lag order finally used.

As it can be seen from Table 4, a lag order of 2 was selected for France, Italy and USA, while a lag order of 1 was selected for Canada, Germany, Great Britain and Japan. In the second step, the VAR model selected in the first step was re-estimated and variables with $|t-\text{statistic}|<2$ were dropped from the model. Details about the final model selection are given in Table 5.

As an example, for Italy (ITA) the VAR model applied was:

$$-\Delta U_t^{\text{ITA}} = \mu_1 + \sum_{i=1}^{2} \Phi_{1,i}(-\Delta U_{t-i}^{\text{ITA}}) + \sum_{i=1}^{2} \Theta_{1,i}\Delta GDP_{t-i}^{\text{ITA}} + \alpha_1 t + u_t \tag{11a}$$

$$\Delta GDP_t^{\text{ITA}} = \mu_2 + \sum_{i=1}^{2} \Phi_{2,i}(-\Delta U_{t-i}^{\text{ITA}}) + \sum_{i=1}^{2} \Theta_{2,i}\Delta GDP_{t-i}^{\text{ITA}} + \alpha_2 t + v_t \tag{11b}$$

3.5. Copula selection and estimation

Residuals of each VAR model were converted to ranks in order to be used in the copula estimation process. As a first step we tested for independence, based on a procedure described by Genest and Fabre (2007). For those pairs that we found evidence for dependence we proceeded with copula estimation.

We estimated all copula families shown in Table 1. Copula estimation was performed using the VineCopula R package (Schepsmeier et al., 2016). Copula families were selected based on the Vuong (Vuong, 1989) and Clarke tests (Clarke, 2007), with Schwarz correction. Recently, the Clarke test has gained popularity due to its high power.

If one can choose between N copula families then each family is tested again all remaining (N-1) families. The copula family with the highest score is selected as the most appropriate one.

Vuong [3] and Clarke tests are nested tests that compare two models in order to find the best. If two models (model1 and model2 for example) are compared, then a score is assigned:

1. +1, if model 1 is better than model 2
2. -1, if model 2 is better than model 1
3. 0, if the test cannot discriminate between two models.

In case of ambiguity regarding the Vuong and Clarke test results, log-likelihood, AIC and BIC tests as well as goodness of fit *p*-values were also used as selection criteria (Manner and Reznikova, 2012). Cramer von Misses (CvM) goodness of fit has also been applied in order to properly discriminate the appropriate copula family (Berg, 2009; Genest et al., 2009; Kojadinovic et al., 2011) We have used 1,000 bootstrap repetitions to obtain *p*-values.

Copula invariance was tested with the Busetti & Harvey test (Busetti and Harvey, 2011). We have used a slightly modified code by Fousekis et al. (2017) to perform the computations in R environment.

[3] There is some evidence that in small sample data sets Vuong test might work better. Unpublished results are available upon request.

4. Results

4.1. Unit Root and Cointegration Tests

First, we perform unit root tests of the GDP and unemployment series in first differences. We emphasize the results of Elliott et al. (1996), which has significantly higher power than previous versions of the augmented Dickey-Fuller test. We also focus on the Zivot-Andrews test (Zivot and Andrews, 1992), which allows for one structural break in both the intercept and trend of a series. Finally, the CKP test, which allows for multiple breaks in both the level and slope of the trend function, rejects the null of unit root in most cases. The latter two tests are important, since we want to avoid the confusion of structural breaks in the series with nonstationarity. Overall, accounting for one or more structural breaks, changes in both unemployment and output are stationary in all countries. These findings are very similar according to the ERS test, so we can be confident about them (See Appendix, Section A.1 and Section A.2).

In light of this evidence, there is no scope for conducting cointegration testing in any country, since both series examined are I(0). The findings as a whole imply that VAR modeling is sufficient to capture the short-run relationship between unemployment and output in first differences, since there is no long-run relationship.

4.2. VAR modelling

Table 5 shows the estimation results of VAR modelling according to Equation 10 and Table 4. As it can be seen from Table 5, GDP_1 (first lag) is present in almost all equations with two exceptions concerning the GDP_1 equations of Japan and USA, while the first lag of $-\Delta U$ is kept as explanatory variable only in USA's GDP_1 equation and $-\Delta U$ equations for Germany, UK, Japan and the US. Second order lags and trends are only sporadically observed, while most equations retain the constant in the right-hand side.

It has to be noted, as mentioned in Section 3.4, that all coefficients have $|t - \text{statistic}| > 2$. At this point, we use VAR modeling as a filtering method in order to get rid of autocorrelation in the data.

Table 5: VAR estimation results

country	variable	ΔGDP_1	$-\Delta U_1$	ΔGDP_2	$-\Delta U_2$	const	trend
CAN	$-\Delta U$	0.2194				-0.0847	
	ΔGDP	0.4980				0.2996	
DEU	$-\Delta U$	0.0462	0.7375				
	ΔGDP	0.3089				0.2580	
FRA	$-\Delta U$	0.2205			0.3655	-0.0704	
	ΔGDP	0.5922				0.1637	
GBR	$-\Delta U$	0.0685	0.5339				
	ΔGDP	0.5998				0.2068	
ITA	$-\Delta U$	0.1262			0.3526		
	ΔGDP	0.5884					
JPN	$-\Delta U$	0.0565	0.2512			-0.0893	0.0016
	ΔGDP					0.2515	
USA	$-\Delta U$	0.1842	0.3402	0.1237		-0.3100	0.0027
	ΔGDP		0.7506			0.9150	-0.0064

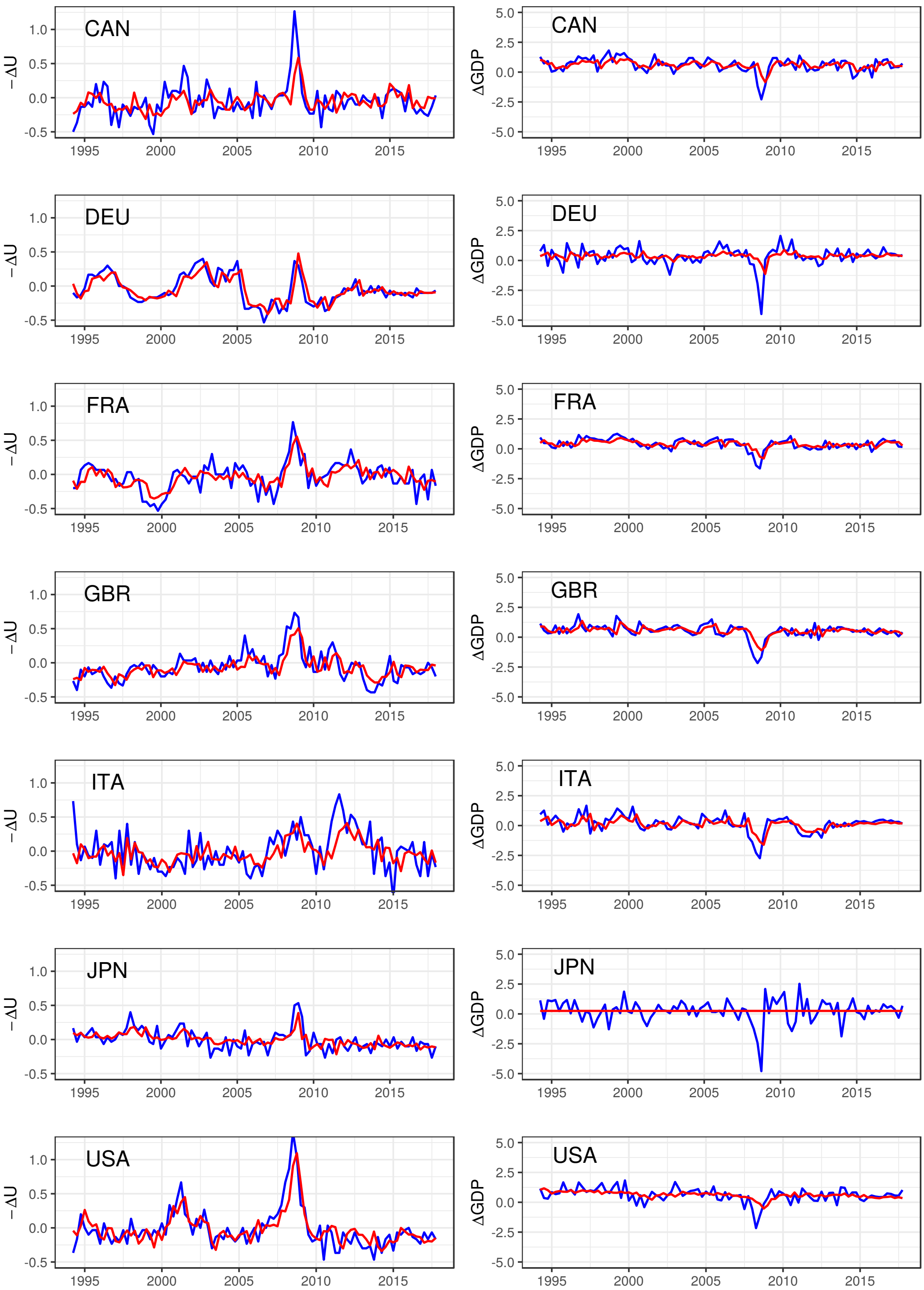

Figure 3: Plot of observed (blue) and fitted (red) values of Unemployment changes (left panel) and GDP changes (right panel) as estimated by VAR, described in Equation 10 and Table 4.

Figure 3 presents a realization of the fitted VAR model versus the observed values. In general, we observe a good fit.

Table 6 presents the test statistics for serial correlation of the residuals obtained after

VAR filtering. So, there is no significant serial correlation observed in the residuals in most cases. In Canada, France, UK and Italy the ARCH-LM test indicates the presence of ARCH effects in the residuals with 12 lags and 24 lags only in the UK of the ΔGDP_1 equation. Also in Japan and the US VAR equation systems, the Portmanteau serial correlation test gave p-value < 0.05 in the case of 12 lags. We consider these test results minor problems and proceed with our analysis. The vast majority of the remaining test results strongly indicate the absence of ARCH-LM effects and serial correlation in the residuals of equation 10.

Table 6: ARCH-LM and Serial correlation test results

Country	L	arch $-\Delta U$	arch ΔGDP	arch-mul	Portmanteau	BG
CAN	12	0.8198	0.0478	0.8293	0.3305	0.4813
CAN	24	0.9926	0.4197	0.4872	0.5608	0.7451
DEU	12	0.4294	0.7725	0.1724	0.1523	0.4554
DEU	24	0.6669	0.9986	0.4872	0.3259	0.5899
FRA	12	0.3053	0.0291	0.0361	0.0747	0.1275
FRA	24	0.7213	0.5775	0.5449	0.1360	0.1151
GBR	12	0.1127	0.0175	0.1833	0.4446	0.5655
GBR	24	0.6549	0.0107	0.4872	0.4210	0.3929
ITA	12	0.1349	0.0070	0.0221	0.4928	0.0884
ITA	24	0.0945	0.1926	0.5449	0.8599	0.4819
JPN	12	0.1381	0.5468	0.0032	0.0206	0.0976
JPN	24	0.5857	0.9658	0.4872	0.2485	0.3084
USA	12	0.3533	0.9503	0.0050	0.0490	0.0723
USA	24	0.5569	0.9452	0.5449	0.1034	0.3015

p-values of the test statistics are presented. They have been computed with the R package vars (Pfaff, 2008, 2015).
L refers to the number of lags used in the corresponding test.
arch $-\Delta U$ and arch ΔGDP refer to the ARCH-LM test for $-\Delta U$ and ΔGDP variables of equation 10, while the arch-mul refers to the corresponding multivariate ARCH-LM test for both variables.
The multivariate Portmanteau and Breusch-Godfrey (BG) tests (serial correlation) are given in the last two columns.

4.3. *Copula Independence*

We conduct the test of independence of the copula data derived from the output and unemployment equations in each of the examined economies (Genest and Fabre, 2007). According to the evidence presented in Table 7, there is dependence in Canada, Germany, France, Great Britain and USA and independence in Italy and Japan. So, we exclude these two countries from the analysis, which follows.

Table 7: Copula Independence test

	$t-$statistic	$p-$value
CAN	3.0450	0.0023
DEU	2.4372	0.0148
FRA	3.9642	$< 1e-4$
GBR	2.7791	0.0055
ITA	1.0063	0.3143
JPN	0.1836	0.8543
USA	3.6877	0.0002

4.4. *Copula invariance*

Concordance between two variables and the corresponding Kendall's τ can vary over time. It is thus necessary to extract information about the stability and invariance of Kendall's τ before proceeding with the estimation of a static copula model. In general, copula stability is to be expected, due to the relatively short sample used in this study (1994:Q1–2018:Q2). However, computing and plotting the Kendall's τ for sub-samples of the data set ensures that this hypothesis is justified.

Figure 4 displays how Kendall's dependence measure evolves over time. It shows rolling-window estimates (van den Goorbergh et al., 2005) of Kendall's τ using window sizes of 60 data points, that is, Kendall's τ in period t is computed using the 60 previous observations until period t.

The stability of Kendall's τ is well demonstrated in the cases of Canada, Germany, France, UK and US.

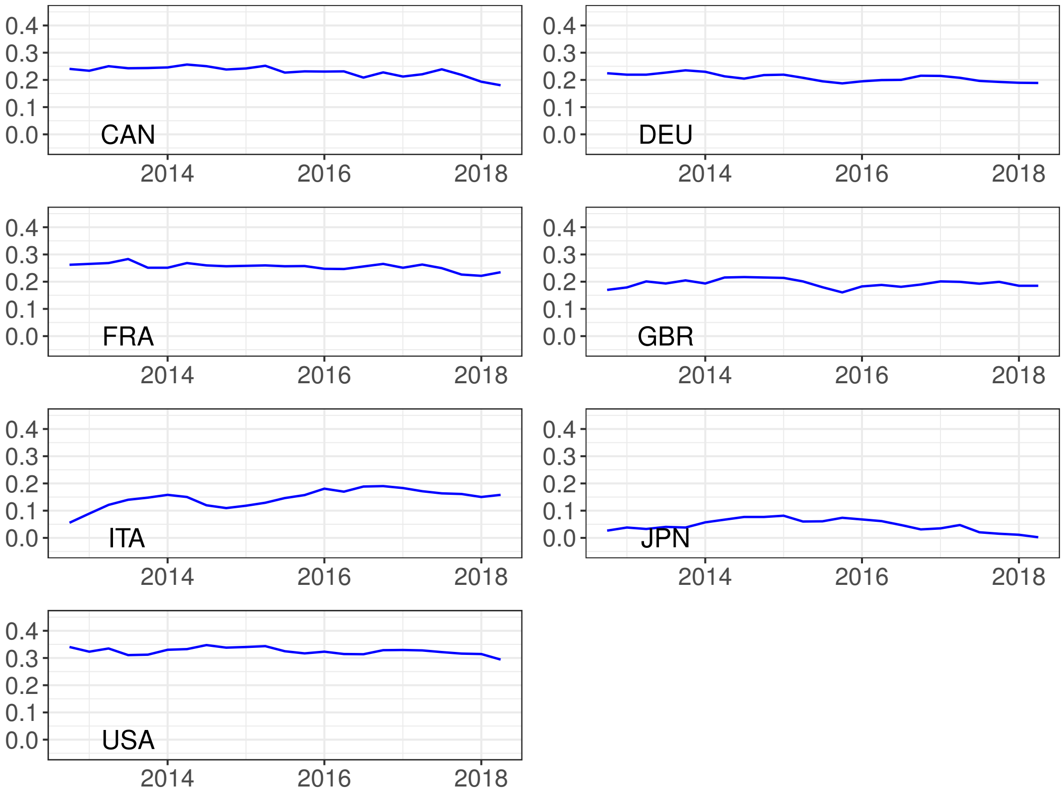

Figure 4: Dynamic Kendall's τ (vertical axis) vs Time.

Furthermore, the Busetti-Harvey test (Busetti and Harvey, 2011) of copula invariance has been also used in order to assess copula invariance. Table 8 lists the results of this test, including the values of the corresponding statistic. The critical values corresponding to significance levels of 5% and 10% are 0.461 and 0.743 respectively. The test statistics have been computed for three quantiles, i.e at $\tau = 0.25, 0.50, 0.75$.

In all cases the test statistic takes values lower than the corresponding critical values at the 5% level of significance. For example, for the US values of 0.267, 0.448 and 0.098 have been obtained, all of which are lower than 0.461.

So, we can be confident about the stability of the copula estimations.

Table 8: Busetti-Harvey test of copula invariance

	$\tau = 0.25$	$\tau = 0.50$	$\tau = 0.75$
CAN	0.027	0.039	0.225
DEU	0.240	0.293	0.204
FRA	0.157	0.277	0.075
GBR	0.177	0.074	0.085
USA	0.267	0.448	0.098

4.5. *Copula selection and estimation*

We proceed with copula selection and estimation only for the five countries, for which we have established dependence (Canada, Germany, France, UK and USA). Table 10 lists the results of copula estimation. Copula family selection was based on both Vuong and Clarke tests (Table 9) and the AIC, BIC and log-likelihood information criteria. We have estimated the latter for all copula families given in Table 1.

Table 9: Vuong (Vg) and Clarke (Ck) test results of copula selection procedure.

	CAN		DEU		FRA		GBR		USA	
	Vg	Ck	Vg	Ck	Vg	Ck	Vg	Ck	Vg	Ck
1 N	3	5	3	5	**7**	8	4	2	3	-3
2 t	0	3	-3	-7	-1	-1	-3	1	-5	-1
3 C	0	0	7	8	4	4	5	2	**14**	**14**
4 G	3	7	2	9	1	2	1	10	-2	-4
5 F	2	1	2	2	2	**13**	4	11	0	0
6 J	0	0	0	0	-1	-13	0	-1	-12	-11
7 BB1	-3	-2	-3	-6	-3	1	-3	-5	2	1
8 BB6	-2	-6	-5	-8	-3	-12	-5	-7	-13	-13
9 BB7	-2	0	-3	-4	-2	-5	-3	-11	2	1
10 BB8	-4	-5	-4	-5	-2	-4	-2	-3	-11	-13
13 SC	1	2	1	1	1	-13	1	-1	-4	-12
14 SG	**6**	**10**	**9**	**10**	6	**13**	**8**	**13**	9	**14**
16 SJ	0	-1	6	9	0	3	5	2	13	**14**
17 SBB1	0	-2	-3	-5	-2	3	-3	-3	1	4
18 SBB6	-1	-5	-3	-4	-2	-2	-3	-5	1	3
19 SBB7	1	0	-3	-2	-2	0	-3	-1	1	3
20 SBB8	-4	-7	-3	-3	-3	3	-3	-4	1	3

Bold face numbers indicate selected copula family based on highest score.

Canada

Regarding Canada (CAN), the survival Gumbel copula is chosen by both Vuong and Clarke tests (see Tables 9 and 10). Survival Gumbel copula scored 6 using the Vuong test, while the second choices had a score of only 3. The Clarke test gave 10 for the survival Gumbel, being only 7 for the second best choice (Gumbel copula). The AIC and BIC information criteria (Table C.1) also corroborated the survival Gumbel copula, whilst only the LogLikehood criterion suggested the survival BB7 copula. So, we select the survival Gumbel family. The value of $Kendall's\ \tau = 0.229$ implies moderate dependence between the residuals of the output and unemployment first difference equations. The values of $\lambda_L = 0.294$ and $\lambda_U = 0$ indicate asymmetry between output and unemployment residuals, specifically that their dependence holds only during recessions.

Germany

Vuong test corroborated for the survival Gumbel copula for Germany (DEU) with a score of 9 and Clarke gave it a score of 10 (Table 9). The AIC, BIC and log-likelihood criteria give results in favor of the Clayton copula family. As a consequence, we choose the Clayton copula supported by all information criteria. However, τ, λ_L and λ_U estimated values were found very similar in both cases (Clayton and survival Gumbel copulas), making the interpretation of the results (see following Discussion section) relatively independent of the selected copula.[4] The size of Kendall's $\tau = 0.187$ shows weak dependence concerning output and unemployment residuals, i.e. lower than Canada, France, UK and the US (Table 10). Also, the evidence is in favor of asymmetry; output and unemployment residuals evolve in opposite directions only during recessions in line with those of the above mentioned countries (Cuaresma, 2003; Holmes and Silverstone, 2006; Silvapulle et al., 2004).

France

Concerning France (FRA), the Normal copula family is chosen by Vuong test, while the survival Gumbel and F copulas have been chosen by the Clarke test (Table 9). The AIC, BIC give also results in favor of Survival Gumbel, while the LogLikelihood crite-

[4] Survival Gumbel, or rotated 180^o Gumbel copula, and Clayton are very similar to each other. Basically there are no major differences between these two candidates.

rion corroborates towards the SBB1 copula family (Table C.3). So, we end up with the Survival Gumbel copula, which is chosen by two out of three information criteria. However, for all four copula families the estimated Kendall's τ lies between 0.267 and 0.289 implying relatively strong dependence concerning the residuals of output and unemployment difference equations. Also, all four copulas show no upper tail dependence. They only differ with respect to lower tail dependence; survival Gumbel and SBB1 are in favor of strong dependence, while Normal and F imply moderate independence (Table 10). In a nutshell, no relationship between unemployment and output disturbances is found in expansions, but we tend to be in favor of strong dependence during recessions.

United Kingdom

For the United Kingdom (GBR) both Vuong and Clarke tests have produced results in favor of the survival Gumbel copula with scores 8 and 13 respectively (Table 9). Two of the three information criteria (AIC, BIC) were in favor of the survival Gumbel copula (Table C.4), while the LogLikelihood criterion favors SBB1. Consequently, we prefer the former copula family. However, in both cases, the estimated Kendall's τ is almost identical being equal to 0.199, 0.201 respectively (Table 10). This makes the interpretation of results independent of the copula selection. The findings imply moderate dependence between unemployment and output residuals, as in the cases of Canada and Germany. The moderately high value of λ_L (0.258) and the zero value of λ_U show moderately strong dependence between output and unemployment residuals during recessions and no dependence throughout recoveries. So, the unexplained parts of output and unemployment first differences move in opposite directions during recessions, but are not associated at all, i.e. higher unexpected output growth is not accompanied by lower unexpected unemployment change, during expansions. In other words, UK is similar to Germany in terms of lower tail dependence and Canada, Germany, France and the US regarding upper tail dependence.

United States

For the US, the Clayton copula is selected by the Vuong test with a score of 14, but Clarke test provided mixed results giving equal score (14) to three copula families (Clayton, survival Gumbel and survival Joe) (see Table 9). The selection between Clayton, survival Gumbel and Survival Joe copulas was based on information criteria, where all LogLikelihood, AIC and BIC values were in favor of Clayton copula (see Appendix, Table C.5). It must be noted that τ, λ_L and λ_U estimated values were found very similar for all three copula families, making the interpretation of the results (see following Discussion section) relatively independent of the selected copula. Figure D5 displays the corresponding copula plot. The size of $\tau = 0.297$ (Table 10) indicates relatively strong dependence between the residuals of the equations of output and unemployment differences. The values of $\lambda_L = 0.441$ and $\lambda_U = 0$ (Table 10) show strong asymmetry between unemployment and output disturbances in the recessionary vs recovery phases of the business cycle. In particular, the unexplained parts of ΔGDP and $-\Delta U$ move together downwards, but not upwards. Effectively, output and unemployment unexplained components evolve in opposite directions during recessions, but are completely disentangled, i.e. higher output residuals is not accompanied by lower unemployment ones, during expansions. These findings are in line with those of (Cuaresma, 2003; Holmes and Silverstone, 2006; Silvapulle et al., 2004).

Final results after copula selection are summarized in Table 10.

Table 10: Copula estimation results

	family	θ	τ	λ_L	λ_U	logL	AIC	BIC
CAN	14 SG	1.298	0.229	0.294	0.000	7.146	−12.292	−9.727
DEU	3 C	0.459	0.187	0.221	0.000	5.116	−8.232	−5.667
FRA	14 SG	1.365	0.267	0.338	0.000	8.866	−15.732	−13.178
GBR	14 SG	1.249	0.199	0.258	0.000	5.098	−8.196	−5.632
USA	3 C	0.846	0.297	0.441	0.000	13.238	−24.475	−21.921

Refer to Table 1 for copula families and parameters.
Only θ parameter is given as $\delta = 0$ in all cases.
logL stands for log-likelihood criterion.

5. Discussion

Generally, in terms of dependence we distinguish four groups of countries, namely France, US (strong), Canada, Great Britain (medium), Germany (weak), while Japan and Italy exhibit no dependence in line with Moosa (1997). Regarding (a)symmetry, all five countries which exhibit dependence (France, US, Canada, Great Britain and Germany) are also characterized by moderate to strong asymmetry, i.e output and unemployment first difference residuals are negatively associated during recessions, but completely disentangled throughout recoveries. Overall, we can distinguish four groups of countries. The first group comprises US and France, the only economies characterized by moderately strong dependence and strong asymmetry, where unemployment and output disturbances are linked exclusively during recessions. The second team is composed by Canada and the UK, both of which exhibit medium dependence and strong asymmetry, i.e output and unemployment disturbances are associated throughout contractions. Germany is unique in that it is characterized by weak dependence and moderate asymmetry. Finally, Italy and Japan constitute the fourth group, where we find no evidence of dependence between unemployment and output equation residuals throughout the whole business cycle. Thus, unemployment and output differences are related exclusively according to the deterministic part of the respective equations in the latter countries.

In order to explain the above findings, we could argue that the flexible US and Canadian labor markets would be expected to imply a stronger response of unemployment to output during expansions compared to the response implied by our analysis given the lack of job security provisions and restrictions on layoffs, which inhibit employers from reducing workforce during recessions and increasing it during expansions. A possible explanation is that the response of the economy to unexpected output increases takes mostly the form of an increase in labor force participation, productivity (Lim et al., 2019; Lin et al., 2008) and hours worked (Okun, 1962; Prachowny, 1993). An explanation for the different reactions of the economy in expansions compared to contractions is that employers are more pessimistic during the downturns than optimistic in the upturns due to risk aversion (Silvapulle et al., 2004). Regarding Germany, the policies followed in the early 2000's, which led to the liberalization of the labor market along with the continued presence of decentralized collective bargaining agreements, which cover large sectors of the German economy and promote wage flexibility in the wake of output disturbances, partially explain the unemployment-output residuals association in Germany only during recessions in our analysis. We can only explain the lack of dependence between output and unemployment unexpected equation components during expansions by the same mechanisms outlined above, i.e. higher labor force participation, productivity and working hours as well as risk-aversion of the employers.

As far as Italy is concerned, the liberalization of labor market institutions initiated in the late 1990's, through e.g. the introduction of part-time regulation, along with the broad collective agreement coverage should imply a higher responsiveness of unemployment to output changes compared to the past. However, the residuals of the respective equations seem to be completely disentangled during the whole business cycle partially in contrast with earlier findings (Zanin and Marra, 2012). Again, there is the possibility of a response in the wake of non-expected output variations in the form of changes in labor force participation, productivity and especially hours worked during expansions (Zwick et al., 2016). Moreover, the labor market reforms introduced in France in the late 1990's, through e.g facilitating part-time work concurrent with the introduction of reduced weekly working hours, have made unemployment responsive to output disturbances during recessions, but not throughout recoveries partially in line with Zanin and Marra (2012). On the other hand, the mostly informal rigidities still characterizing the Japanese industrial relations may explain why output and unemployment residual dynamics are completely disentangled. At the same time, the UK labor market would be expected to imply a stronger dependence between unemployment and output dynamics than we actually see above, given that this country's labor market is the least regulated in Europe regarding the terms and conditions of employment and working time, minimum wages and trade union power (Freeman, 2001; Moosa, 1997). This is so, even if we account for the VAR estimates (see Table 5).

From a policy point of view, no country provides a very favorable environment for counter-cyclical economic policies, since in all countries there is no evidence for unemployment-output residual dependence during expansions. In other words, unemployment responds to output according to the deterministic part of the VAR estimations, but the disturbances of the output and unemployment equations do not respond to each other during recoveries. Having that in mind, USA and France are characterized by strong dependence and asymmetry where unemployment responds to output disturbances only during recessions.

In Canada and the UK moderate dependence is combined with asymmetry, where unemployment responds moderately when output falls unexpectedly, but not at all when it rises above what is predicted by VAR estimates. Germany exhibits weak dependence along with asymmetry, where unemployment rises moderately during unexpected slumps. Finally, disturbances in the first differences of output and unemployment seem to be completely disentangled throughout the whole business cycle in both Italy and Japan. As a consequence, these variables are related only according to the deterministic part of the VAR estimates. Thus, stabilization policies in the US, France, Canada, Germany and the UK are predicted to be relatively effective in avoiding unemployment hikes in the wake of recessions, but less effective in achieving lower unemployment during recoveries. This holds especially for the US and France and less so for the remaining three countries. Obviously such policies will be less effective in Italy and Japan, where policy makers can smooth business cycles based on the output-unemployment relationship shown by the coefficient estimates of the VAR equations.

6. Conclusions

In this paper we have examined the association between the residuals of the output and unemployment first difference equations in the G7 economies using copulas for 1994:Q1–2018:Q2, which has implications for the validity of Okun's Law. The copula methodology provides flexibility to fit dependence using a joint distribution separately from marginal distributions along with flexibility over the choice of the type of dependence. After examining our series in terms of stationarity, we extract the copula data by employing VAR methodology and investigate dependence along with asymmetry between disturbances in output and unemployment variations. We conclude that dependence between output and unemployment unexpected variations is relatively strong only in USA and France, followed by Canada, UK and Germany. Italy and Japan exhibit no dependence. Also, we find asymmetry in all the former five countries. Specifically, output disturbances are associated with unemployment disturbances only during recessions, while they are completely disentangled from each other throughout expansions in these economies.

These findings imply that USA, France, and less so Canada, UK and Germany provide the most favorable conditions for effective counter-cyclical economic policies due to their dependence and symmetry characteristics regarding the output-unemployment relationship. In other words, policy makers in these countries should react more in the wake of output slumps than VAR estimates imply to avoid deepening of recessions. Moreover, in Italy and Japan, stabilization policy should be focused on smoothing business cycles based solely on the deterministic part of the output-unemployment first difference relationship. Our findings should prove useful to policy makers in addition to what is suggested by traditional empirical approaches, which do not analyze dependence and asymmetry of disturbances in the output-unemployment relation. In future research, we aim to investigate thoroughly using copula techniques the role of labor force participation, working hours, capital stock and technical progress in the output-unemployment relationship in the context of Okun's Law.

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Appendix A Unit Root Tests

In this section we present unit root test results for the ΔGDP and $-\Delta U$ time series. For the sake of completeness and to facilitate readability of the tables we also present critical values of the tests at 0.01, 0.05, 0.10 levels of significance. Related references can be found in the main text of the manuscript.

Critical values of unit root tests

Table A.2: Critical values of the Augmented Dickey–Fuller (ADF) test

		Level of significance		
Type	Statistic	0.01	0.05	0.10
none	tau1	−2.60	−1.95	−1.61
drift	tau2	−3.51	−2.89	−2.58
	phi1	6.70	4.71	3.86
trend	tau3	−4.04	−3.45	−3.15
	phi2	6.50	4.88	4.16
	phi3	8.73	6.49	5.47

H_0 : There is a unit root

Reject the *null* if test-value $<$ crit-value

Table A.3: Critical values of the Kwiatkowski, Phillips, Schmidt & Shin (KPSS) test

	Level of significance		
Type	0.01	0.05	0.10
mu	0.739	0.463	0.347
tau	0.216	0.146	0.119

H_0 : There is no unit root

Reject the *null* if test-value $>$ crit-value

Table A.4: Critical values of the Elliott, Rothenberg & Stock (ERS or DF-GLS) test

	Level of significance		
Model	0.01	0.05	0.10
constant	−2.59	−1.94	−1.62
trend	−3.58	−3.03	−2.74

H_0 : There is a unit root

Reject the *null* if test-value $<$ crit-value

Table A.5: Critical values of the Zivot & Andrews (ZA) test

	Level of significance		
Model	0.01	0.05	0.10
intercept	−5.34	−4.80	−4.58
trend	−4.93	−4.42	−4.11
both	−5.57	−5.08	−4.82

H_0 : there is a unit root

Reject the *null* if test-value $<$ crit-value

A.1 Unit root tests for Unemployment time series

In the table shown below, L denotes the lag order.

Table A.6: Unit root test results for Canada (CAN) ΔU data.

Test	Model	L=0	L=1	L=2
ADF	none	-6.016***	-4.980***	-4.674***
	drift	-6.124***	-5.056***	-4.761***
	trend	-6.129***	-5.026***	-4.727***
KPSS	mu	0.366**	0.254***	0.210***
	tau	0.185*	0.129**	0.107***
ERS	constant	-4.190***	-3.126***	-2.722***
	trend	-5.549***	-4.317***	-3.916***
ZA	intercept	-6.708***	-5.496***	-5.117**
	trend	-6.391***	-5.240***	-4.971***
	both	-6.665***	-5.460**	-5.230**

Table A.7: Unit root test results for Germany (DEU) ΔU data

Test	Model	L=0	L=1	L=2
ADF	none	-3.027***	-3.593***	-3.056***
	drift	-3.148**	-3.703***	-3.151**
	trend	-3.232*	-3.925**	-3.397*
KPSS	mu	1.097	0.606*	0.444**
	tau	0.235	0.131**	0.097***
ERS	constant	-2.924***	-3.475***	-2.950***
	trend	-3.266**	-3.970***	-3.437**
ZA	intercept	-4.727*	-5.491***	-4.935**
	trend	-3.371	-4.027	-3.523
	both	-4.700	-5.440**	-4.875*

Table A.8: Unit root test results for France (FRA) ΔU data

Test	Model	L=0	L=1	L=2
ADF	none	-4.873***	-3.260***	-3.614***
	drift	-4.935***	-3.289**	-3.642***
	trend	-4.954***	-3.272*	-3.630**
KPSS	mu	0.575*	0.363**	0.270***
	tau	0.252	0.160*	0.119**
ERS	constant	-4.826***	-3.183***	-3.490***
	trend	-4.997***	-3.306**	-3.659***
ZA	intercept	-5.569***	-3.933	-4.336
	trend	-5.273***	-3.496	-3.859
	both	-5.979***	-4.278	-4.639

Table A.9: Unit root test results for United Kingdom (GBR) ΔU data

Test	Model	L=0	L=1	L=2
ADF	none	-4.277***	-3.428***	-3.471***
	drift	-4.425***	-3.546***	-3.570***
	trend	-4.383***	-3.495**	-3.498**
KPSS	mu	0.912	0.550*	0.412**
	tau	0.792	0.479	0.359
ERS	constant	-3.412***	-2.635***	-2.503**
	trend	-4.135***	-3.247**	-3.141**
ZA	intercept	-6.117***	-5.189**	-5.278**
	trend	-5.555***	-4.599**	-4.682**
	both	-6.054***	-5.077*	-5.151**

Table A.10: Unit root test results for Italy (ITA) ΔU data

Test	Model	L=0	L=1	L=2
ADF	none	-6.903***	-3.659***	-3.109***
	drift	-6.866***	-3.639***	-3.082**
	trend	-6.897***	-3.629**	-3.171*
KPSS	mu	0.556*	0.419**	0.319***
	tau	0.341	0.259	0.198*
ERS	constant	-6.579***	-3.470***	-2.814***
	trend	-6.665***	-3.551**	-2.903*
ZA	intercept	-8.539***	-4.963**	-4.307
	trend	-7.560***	-4.089	-3.839
	both	-8.828***	-5.162**	-4.822*

Table A.11: Unit root test results for Japan (JPN) ΔU data

Test	Model	L=0	L=1	L=2
ADF	none	-6.264***	-4.547***	-3.594***
	drift	-6.235***	-4.531***	-3.580***
	trend	-6.926***	-5.089***	-4.205***
KPSS	mu	1.284	0.910	0.744
	tau	0.097***	0.073***	0.063***
ERS	constant	-5.944***	-4.340***	-3.400***
	trend	-6.365***	-4.668***	-3.711***
ZA	intercept	-7.552***	-5.687***	-4.888**
	trend	-6.927***	-5.121***	-4.231*
	both	-7.524***	-5.667***	-4.862*

Table A.12: Unit root test results for United States (USA) ΔU data

Test	Model	L=0	L=1	L=2
ADF	none	-4.040***	-3.098***	-3.151***
	drift	-4.027***	-3.089**	-3.134**
	trend	-4.053***	-3.122	-3.190*
KPSS	mu	0.525*	0.307***	0.224***
	tau	0.466	0.273	0.200*
ERS	constant	-3.086***	-2.325**	-2.256**
	trend	-3.613***	-2.748*	-2.705
ZA	intercept	-5.247**	-4.282	-4.232
	trend	-4.434**	-3.455	-3.501
	both	-5.167**	-4.144	-4.074

A.2 Unit root tests for ΔGDP time series

Table A.13: Unit root test results for Canada (CAN) ΔGDP data

Test	Model	L=0	L=1	L=2
ADF	none	-3.822***	-3.389***	-2.951***
	drift	-5.643***	-5.311***	-4.906***
	trend	-5.819***	-5.507***	-5.131***
KPSS	mu	0.955	0.634*	0.528*
	tau	0.136**	0.092***	0.079***
ERS	constant	-3.921***	-3.394***	-2.893***
	trend	-5.491***	-5.067***	-4.604***
ZA	intercept	-6.324***	-5.893***	-5.478***
	trend	-5.955***	-5.665***	-5.318***
	both	-6.269***	-6.067***	-5.853***

Table A.14: Unit root test results for Germany (DEU) ΔGDP data

Test	Model	L=0	L=1	L=2
ADF	none	-6.248***	-4.527***	-3.741***
	drift	-7.157***	-5.395***	-4.564***
	trend	-7.119***	-5.366***	-4.541***
KPSS	mu	0.076***	0.058***	0.051***
	tau	0.076***	0.058***	0.051***
ERS	constant	-5.526***	-3.953***	-3.153***
	trend	-6.579***	-4.889***	-4.022***
ZA	intercept	-7.497***	-5.735***	-4.877**
	trend	-7.174***	-5.442***	-4.618**
	both	-7.452***	-5.684***	-4.884*

Table A.15: Unit root test results for France (FRA) ΔGDP data

Test	Model	L=0	L=1	L=2
ADF	none	-3.573***	-2.839***	-2.714***
	drift	-4.952***	-3.855***	-3.928***
	trend	-5.249***	-4.024**	-4.223***
KPSS	mu	1.268	0.799	0.611*
	tau	0.194*	0.125**	0.097***
ERS	constant	-4.685***	-3.546***	-3.605***
	trend	-5.304***	-4.067***	-4.270***
ZA	intercept	-5.659***	-4.503	-4.703*
	trend	-5.464***	-4.169*	-4.440**
	both	-5.985***	-4.730	-5.109**

Table A.16: Unit root test results for United Kingdom (GBR) ΔGDP data

Test	Model	L=0	L=1	L=2
ADF	none	-3.560***	-2.948***	-3.096***
	drift	-4.892***	-4.132***	-4.601***
	trend	-5.067***	-4.287***	-4.827***
KPSS	mu	1.137	0.709*	0.549*
	tau	0.243	0.154*	0.121**
ERS	constant	-3.772***	-3.028***	-3.225***
	trend	-4.943***	-4.143***	-4.621***
ZA	intercept	-5.645***	-4.896**	-5.574***
	trend	-5.332***	-4.553**	-5.193***
	both	-6.100***	-5.391**	-6.332***

Table A.17: Unit root test results for Italy (ITA) Δ*GDP* data

Test	Model	L=0	L=1	L=2
ADF	none	-4.976***	-4.088***	-4.045***
	drift	-5.146***	-4.239***	-4.225***
	trend	-5.322***	-4.393***	-4.425***
KPSS	mu	1.062	0.677*	0.531*
	tau	0.240	0.156*	0.124**
ERS	constant	-4.421***	-3.531***	-3.420***
	trend	-5.316***	-4.376***	-4.400***
ZA	intercept	-5.863***	-4.943**	-5.050**
	trend	-5.749***	-4.834**	-4.979***
	both	-6.502***	-5.652***	-5.978***

Table A.18: Unit root test results for Japan (JPN) Δ*GDP* data

Test	Model	L=0	L=1	L=2
ADF	none	-8.327***	-5.581***	-5.506***
	drift	-8.754***	-6.015***	-6.047***
	trend	-8.712***	-5.998***	-6.020***
KPSS	mu	0.072***	0.065***	0.061***
	tau	0.056***	0.051***	0.047***
ERS	constant	-7.712***	-5.127***	-4.830***
	trend	-8.544***	-5.874***	-5.765***
ZA	intercept	-9.223***	-6.372***	-6.394***
	trend	-8.800***	-6.166***	-6.174***
	both	-9.135***	-6.344***	-6.428***

Table A.19: Unit root test results for United States (USA) ΔGDP data

Test	Model	L=0	L=1	L=2
ADF	none	-4.080***	-2.655***	-2.114**
	drift	-6.536***	-4.347***	-3.688***
	trend	-6.849***	-4.502***	-3.890**
KPSS	mu	1.098	0.797	0.636*
	tau	0.249	0.188*	0.154*
ERS	constant	-5.920***	-3.810***	-3.207***
	trend	-6.863***	-4.515***	-3.901***
ZA	intercept	-7.491***	-5.068**	-4.492
	trend	-7.442***	-4.986***	-4.467**
	both	-7.927***	-5.461**	-4.937*

Breakpoints

Bai and Perron (2003) test to indetify the number of breakpoints in time series.

Table A.20: Breakpoints according to Bai-Perron test

Series	Country	Breaks	Dates
ΔGDP	CAN	1	2000:Q1
	DEU	0	
	FRA	1	2007:Q2
	GBR	2	2007:Q4, 2011:Q2
	ITA	2	2008:Q1, 2013:Q1
	JPN	0	
	USA	2	2006:Q1, 2009:Q3
ΔU	CAN	0	
	DEU	4	1997:Q3, 2001:Q1, 2005:Q1, 2008:Q3
	FRA	2	2007:Q4 2013:Q1
	GBR	3	1997:Q4 2007:Q3 2011:Q3
	ITA	3	1998!Q3, 2007:Q1, 2013:Q3
	JPN	3	2003:Q1 2007:Q1 2010:Q3
	USA	2	2006:Q1 2009:Q3

Carrion-i-Silvestre test under possible multiple breaks

Carrion-i-Silvestre et. al.(2009) developed five tests in order to test for unit roots in time series with possible multiple breaks. The test has H_0 : there is a unit root under structural breaks.

We present below the results concerning the five tests, where: P_T represents feasible point optimal statistic, MP_T is the feasible point optimal statistic, MZ_α, MSB, and MZ_t represent the M-class test statistics. The asymptotic critical values are computed via bootstrap. Rejection of the null hypothesis in the GLS unit root tests implies the absence of a unit root.

In the tables below with evaluate the test-statistic with 1% (3 asterisk), 5% (2 asterisk) or 10% (1 asterisk) significance level. If the test did not pass the 10% significance level there is no asterisk and the value in parentheses represent the critical value at the 10% significance level.

We obtained the number of the structural breaks from the Table A.20. If there is no structural break we did not perform the test.

There are three possible model specifications for the test:

- Model = 0 : for the constant case, without structural breaks
- Model = 1 : for the linear time trend case, without structural breaks
- Model = 2 : for the linear time trend that is affected by multiple structural breaks; the structural break affects both the level and the slope of the time trend

Obviously the last case (Model = 2) is of interest here. However we list results of all possible models for the sake of completeness.

Table A.21: Carrion-i-Silvestre test Canada

Series	Model	Breaks	P_T	MP_T	MZ_α	MSB	MZ_t
ΔGDP	0	1	1.371*	1.100**	-23.058**	0.147***	-3.384***
			(1.956)	(1.211)	(-14.830)	(0.166)	(-3.163)
	1	1	1.372*	1.100**	-23.058**	0.147***	-3.385***
			(1.956)	(1.211)	(-14.831)	(0.166)	(-3.163)
	2	1	4.673***	3.974***	-37.007***	0.116***	-4.301***
			(4.741)	(4.741)	(-30.314)	(0.127)	(-3.877)

Table A.22: Carrion-i-Silvestre test Germany

Series	Model	Breaks	P_T	MP_T	MZ_α	MSB	MZ_t
ΔU	0	4	1.812*	1.711*	-14.568*	0.185**	-2.691**
			(1.956)	(1.956)	(-11.090)	(0.212)	(-2.335)
	1	4	5.149**	5.250**	-17.478**	0.169*	-2.950**
			(5.544)	(5.544)	(-17.326)	(0.186)	(-2.896)
	2	4	15.096	13.972	-26.953	0.136	-3.669
			(10.898)	(10.898)	(-35.170)	(0.119)	(-4.136)

Table A.23: Carrion-i-Silvestre test France

Series	Model	Breaks	P_T	MP_T	MZ_α	MSB	MZ_t
ΔGDP	0	1	1.362*	1.288*	-20.512**	0.155***	-3.176***
			(1.956)	(1.956)	(-14.831)	(0.166)	(-3.163)
	1	1	3.471***	3.543***	-25.722***	0.139***	-3.586***
			(3.833)	(3.833)	(-24.485)	(0.142)	(-3.471)
	2	1	8.098*	7.326*	-28.056*	0.133*	-3.732*
			(8.356)	(8.356)	(-25.023)	(0.142)	(-3.511)
ΔU	0	2	1.480*	1.490*	-16.719**	0.173**	-2.885**
			(1.956)	(1.956)	(-14.831)	(0.212)	(-2.335)
	1	2	5.155**	5.256**	-17.799**	0.166**	-2.961**
			(5.544)	(5.544)	(-17.326)	(0.168)	(-2.896)
	2	2	10.909	9.931	-19.811	0.159	-3.147
			(8.462)	(8.462)	(-24.045)	(0.143)	(-3.447)

Table A.24: Carrion-i-Silvestre test United Kingdom

Series	Model	Breaks	P_T	MP_T	MZ_α	MSB	MZ_t
ΔGDP	0	2	1.840*	1.528*	-17.022**	0.170**	-2.894**
			(1.956)	(1.956)	(-14.831)	(0.212)	(-2.335)
	1	2	3.254***	3.228***	-28.515***	0.132***	-3.770***
			(3.833)	(3.833)	(-24.485)	(0.142)	(-3.471)
	2	2	8.914*	6.823**	-35.097**	0.119**	-4.171**
			(9.587)	(8.241)	(-29.001)	(0.130)	(-3.791)
ΔU	0	3	2.229	1.904*	-12.870*	0.197**	-2.537**
			(1.956)	(1.956)	(-11.090)	(0.212)	(-2.335)
	1	3	5.234**	5.051**	-18.800**	0.161**	-3.033**
			(5.544)	(5.544)	(-17.326)	(0.168)	(-2.896)
	2	3	6.828**	6.252**	-39.700**	0.112**	-4.442**
			(7.327)	(7.327)	(-33.284)	(0.122)	(-4.071)

Table A.25: Carrion-i-Silvestre test Italy

Series	Model	Breaks	P_T	MP_T	MZ_α	MSB	MZ_t
ΔGDP	0	2	1.305*	1.165**	-21.622**	0.152***	-3.279***
			(1.956)	(1.211)	(-14.831)	(0.166)	(-3.163)
	1	2	2.947***	2.976***	-31.021***	0.127***	-3.932***
			(3.833)	(3.833)	(-24.485)	(0.142)	(-3.471)
	2	2	6.817**	6.364**	-36.363**	0.117**	-4.258**
			(7.755)	(7.755)	(-29.868)	(0.129)	(-3.845)
ΔU	0	3	2.447	2.332	-11.588*	0.203**	-2.352**
			(1.956)	(1.956)	(-11.090)	(0.212)	(-2.335)
	1	3	7.668	7.695	-12.019	0.202	-2.431
			(6.780)	(6.780)	(-14.000)	(0.186)	(-2.607)
	2	3	12.834	11.836	-22.379	0.149	-3.341
			(8.622)	(8.622)	(-31.082)	(0.127)	(-3.933)

Table A.26: Carrion-i-Silvestre test Japan

Series	Model	Breaks	P_T	MP_T	MZ_α	MSB	MZ_t
ΔU	0	3	2.313	2.298	-10.773	0.215*	-2.315*
			(1.956)	(1.956)	(-11.090)	(0.235)	(-2.105)
	1	3	6.973	6.690*	-13.669	0.191	-2.610*
			(6.780)	(6.780)	(-14.000)	(0.186)	(-2.607)
	2	3	8.736	7.457*	-35.081**	0.119*	-4.184**
			(8.625)	(8.625)	(-34.995)	(0.127)	(-4.166)

Table A.27: Carrion-i-Silvestre testUSA

Series	Model	Breaks	P_T	MP_T	MZ_α	MSB	MZ_t
ΔGDP	0	2	1.443*	1.390*	-17.760**	0.168**	-2.977**
			(1.956)	(1.956)	(-14.831)	(0.212)	(-2.335)
	1	2	3.634***	3.695***	-26.116***	0.137***	-3.579***
			(3.833)	(3.833)	(-24.485)	(0.142)	(-3.471)
	2	2	7.977*	7.214**	-32.964**	0.123**	-4.059**
			(9.214)	(7.935)	(-30.146)	(0.128)	(-3.861)
ΔU	0	2	2.835	2.389	-10.297	0.220*	-2.267*
			(1.956)	(1.956)	(-11.090)	(0.235)	(-2.105)
	1	2	7.082	6.624*	-13.952	0.188	-2.625*
			(6.780)	(6.780)	(-14.000)	(0.186)	(-2.607)
	2	2	14.925	12.095	-19.674	0.159	-3.132
			(10.114)	(10.114)	(-23.644)	(0.143)	(-3.420)

Appendix B VAR related formulas

Information criteria used for lag selection in VAR modelling.

Each VAR model was estimated by OLS and the optimal value of number of lags (n) was selected by applying the following information criteria:

$$AIC(n) = \ln\left[\det\left(T^{-1}\sum_{t=1}^{T}\widehat{\mathbf{u}}_t\widehat{\mathbf{u}}_t'\right)\right] + \frac{2}{T}nK^2 \qquad \text{[Akaike]} \quad (B.1)$$

$$HQ(n) = \ln\left[\det\left(T^{-1}\sum_{t=1}^{T}\widehat{\mathbf{u}}_t\widehat{\mathbf{u}}_t'\right)\right] + \frac{2\ln(\ln(T))}{T}nK^2 \qquad \text{[Hannan–Quinn]} \quad (B.2)$$

$$SC(n) = \ln\left[\det\left(T^{-1}\sum_{t=1}^{T}\widehat{\mathbf{u}}_t\widehat{\mathbf{u}}_t'\right)\right] + \frac{\ln(T)}{T}nK^2 \qquad \text{[Schwarz]} \quad (B.3)$$

$$FPE(n) = \det\left(T^{-1}\sum_{t=1}^{T}\widehat{\mathbf{u}}_t\widehat{\mathbf{u}}_t'\right)\left(\frac{T+n^\star}{T-n^\star}\right)^K \qquad \text{[Akaike FPE]} \quad (B.4)$$

where $\widetilde{\sum}_u(n) = T^{-1}\sum_{t=1}^{T}\widehat{\mathbf{u}}_t\widehat{\mathbf{u}}_t'$, n is the lag order and $n^\star$ is the total number of parameters in each equation.

Appendix C Copula estimation results

Note: in some cases estimation of p-value goodness of fit was not possible.

Table C.1: Canada (CAN) copula estimation (empirical $\tau = 0.310$)

family	θ	δ	τ	λ_L	λ_U	p-value	logL	AIC	BIC
1 N	0.367	0.000	0.239	0.000	0.000	0.727	6.038	−10.077	−7.512
2 t	0.342	4.943	0.222	0.139	0.139	0.527	7.001	−10.002	−4.873
3 C	0.536	0.000	0.211	0.274	0.000	0.468	5.924	−9.847	−7.283
4 G	1.278	0.000	0.218	0.000	0.280	0.593	5.463	−8.926	−6.361
5 F	2.032	0.000	0.217	0.000	0.000	0.475	4.927	−7.854	−5.289
6 J	1.360	0.000	0.169	0.000	0.335	0.739	4.151	−6.301	−3.737
7 BB1	0.313	1.143	0.243	0.144	0.166	0.714	6.676	−9.352	−4.223
8 BB6	1.001	1.277	0.218	0.000	0.280	0.615	5.460	−6.920	−1.792
9 BB7	1.211	0.411	0.244	0.185	0.227	0.780	6.770	−9.539	−4.411
10 BB8	2.234	0.755	0.211	0.000	0.000	0.285	5.198	−6.396	−1.267
13 SC	0.462	0.000	0.188	0.000	0.223	0.907	4.831	−7.661	−5.097
14 SG	1.298	0.000	0.229	0.294	0.000	0.283	7.146	−12.292∗	−9.727∗
16 SJ	1.406	0.000	0.186	0.363	0.000	0.774	6.216	−10.433	−7.868
17 SBB1	0.171	1.216	0.243	0.232	0.036	0.709	7.530	−11.060	−5.931
18 SBB6	1.001	1.297	0.229	0.294	0.000	0.257	7.145	−10.290	−5.161
19 SBB7	1.289	0.321	0.243	0.288	0.115	0.772	7.875∗	−11.750	−6.621
20 SBB8	6.000	0.313	0.212	0.000	0.000	0.294	4.808	−5.615	−0.487

Table C.2: Germany (DEU) copula estimation (empirical $\tau = 0.310$)

family	θ	δ	τ	λ_L	λ_U	p-value	logL	AIC	BIC
1 N	0.318	0.000	0.206	0.000	0.000	0.476	4.402	−6.804	−4.240
2 t	0.309	30.000	0.200	0.000	0.000	0.431	4.291	−4.583	0.546
3 C	0.459	0.000	0.187	0.221	0.000	0.849	5.116∗	−8.232∗	−5.667∗
4 G	1.194	0.000	0.163	0.000	0.213	0.189	2.779	−3.558	−0.994
5 F	1.591	0.000	0.172	0.000	0.000	0.205	3.174	−4.349	−1.784
6 J	1.217	0.000	0.110	0.000	0.232	0.791	1.504	−1.008	1.556
7 BB1	0.451	1.005	0.188	0.217	0.007	0.837	5.117	−6.234	−1.105
8 BB6	1.001	1.194	0.163	0.000	0.214	0.190	2.775	−1.549	3.579
9 BB7	1.011	0.454	0.189	0.217	0.015	0.823	5.118	−6.236	−1.107
10 BB8	6.000	0.255	0.168	0.000	0.000	0.203	3.092	−2.185	2.944
13 SC	0.303	0.000	0.131	0.000	0.101	0.591	2.145	−2.291	0.274
14 SG	1.227	0.000	0.185	0.241	0.000	0.516	5.033	−8.066	−5.502
16 SJ	1.331	0.000	0.158	0.316	0.000	0.589	4.860	−7.720	−5.156
17 SBB1	0.001	1.227	0.185	0.241	0.000	0.361	5.033	−6.066	−0.937
18 SBB6	1.086	1.163	0.181	0.268	0.000	0.373	5.061	−6.121	−0.992
19 SBB7	1.282	0.135	0.185	0.282	0.006	0.552	5.153	−6.306	−1.177
20 SBB8	1.331	1.000	0.158	0.316	0.000	0.128	4.860	−5.720	−0.592

Table C.3: France (FRA) copula estimation (empirical $\tau = 0.310$)

family	θ	δ	τ	λ_L	λ_U	p-value	logL	AIC	BIC
1 N	0.439	0.000	0.289	0.000	0.000	0.889	8.934	−15.867	−13.313
2 t	0.434	30.000	0.285	0.001	0.001	0.831	8.756	−13.512	−8.404
3 C	0.653	0.000	0.246	0.346	0.000	0.680	8.469	−14.938	−12.384
4 G	1.317	0.000	0.240	0.000	0.307	0.093	6.513	−11.025	−8.471
5 F	2.551	0.000	0.267	0.000	0.000	0.508	7.744	−13.487	−10.934
6 J	1.370	0.000	0.173	0.000	0.342	0.940	4.155	−6.309	−3.755
7 BB1	0.486	1.103	0.271	0.275	0.126	0.860	8.900	−13.799	−8.691
8 BB6	1.001	1.316	0.240	0.000	0.307	0.088	6.508	−9.015	−3.908
9 BB7	1.130	0.586	0.267	0.307	0.154	0.800	8.869	−13.737	−8.629
10 BB8	6.000	0.366	0.255	0.000	0.000	0.258	7.369	−10.738	−5.630
13 SC	0.506	0.000	0.202	0.000	0.254	0.278	5.532	−9.065	−6.511
14 SG	1.365	0.000	0.267	0.338	0.000	0.765	8.866	−15.732∗	−13.178∗
16 SJ	1.518	0.000	0.225	0.421	0.000	0.862	7.821	−13.643	−11.089
17 SBB1	0.084	1.320	0.273	0.309	0.002	0.920	8.953∗	−13.906	−8.798
18 SBB6	1.001	1.364	0.267	0.338	0.000	0.826	8.865	−13.730	−8.623
19 SBB7	1.402	0.280	0.267	0.360	0.084	0.849	8.870	−13.740	−8.632
20 SBB8	2.323	0.820	0.264	0.000	0.000	0.459	8.239	−12.479	−7.371

Table C.4: United Kingdom (GBR) copula estimation (empirical $\tau = 0.310$)

family	θ	δ	τ	λ_L	λ_U	p-value	logL	AIC	BIC
1 N	0.314	0.000	0.203	0.000	0.000	0.404	4.296	−6.593	−4.029
2 t	0.316	10.922	0.205	0.029	0.029	0.456	4.578	−5.155	−0.026
3 C	0.460	0.000	0.187	0.221	0.000	0.327	4.552	−7.104	−4.539
4 G	1.216	0.000	0.178	0.000	0.232	0.267	3.109	−4.218	−1.654
5 F	1.866	0.000	0.201	0.000	0.000	0.192	4.211	−6.421	−3.857
6 J	1.255	0.000	0.127	0.000	0.262	0.735	1.764	−1.528	1.036
7 BB1	0.400	1.038	0.197	0.188	0.050	0.370	4.588	−5.176	−0.047
8 BB6	1.001	1.215	0.177	0.000	0.232	0.286	3.105	−2.210	2.919
9 BB7	1.001	0.459	0.187	0.221	0.001	0.325	4.552	−5.104	0.025
10 BB8	6.000	0.295	0.198	0.000	0.000	0.239	4.199	−4.399	0.730
13 SC	0.346	0.000	0.147	0.000	0.135	0.238	2.699	−3.398	−0.834
14 SG	1.249	0.000	0.199	0.258	0.000	0.188	5.098	−8.196∗	−5.632∗
16 SJ	1.348	0.000	0.165	0.328	0.000	0.751	4.589	−7.178	−4.614
17 SBB1	0.024	1.237	0.201	0.249	0.000	0.110	5.104∗	−6.208	−1.079
18 SBB6	1.001	1.248	0.199	0.258	0.000	0.184	5.098	−6.195	−1.067
19 SBB7	1.279	0.182	0.199	0.281	0.022	0.134	5.061	−6.121	−0.992
20 SBB8	6.000	0.297	0.200	0.000	0.000	0.274	4.199	−4.398	0.731

Table C.5: United States (USA) copula estimation (empirical $\tau = 0.310$)

family	θ	δ	τ	λ_L	λ_U	p-value	logL	AIC	BIC
1 N	0.439	0.000	0.290	0.000	0.000	0.040	8.958	−15.916	−13.362
2 t	0.429	8.313	0.282	0.085	0.085	0.077	9.435	−14.871	−9.763
3 C	0.846	0.000	0.297	0.441	0.000	0.428	13.238*	−24.475*	−21.921*
4 G	1.303	0.000	0.233	0.000	0.298	0.000	5.609	−9.218	−6.664
5 F	2.513	0.000	0.263	0.000	0.000	0.003	7.356	−12.712	−10.158
6 J	1.292	0.000	0.142	0.000	0.290	0.886	2.403	−2.805	−0.252
7 BB1	0.845	1.001	0.298	0.441	0.001	0.404	13.232	−22.465	−17.357
8 BB6	1.001	1.303	0.233	0.000	0.298	0.000	5.601	−7.202	−2.094
9 BB7	1.001	0.846	0.298	0.441	0.001	0.397	13.235	−22.469	−17.361
10 BB8	6.000	0.346	0.239	0.000	0.000	0.000	6.569	−9.139	−4.031
13 SC	0.412	0.000	0.171	0.000	0.186	0.281	3.416	−4.831	−2.277
14 SG	1.422	0.000	0.297	0.372	0.000	0.674	12.061	−22.121	−19.567
16 SJ	1.682	0.000	0.275	0.490	0.000	0.315	13.179	−24.358	−21.804
17 SBB1	0.001	1.421	0.297	0.371	0.000	0.574	12.054	−20.108	−15.000
18 SBB6	1.680	1.001	0.275	0.490	0.000	0.434	13.178	−22.356	−17.249
19 SBB7	1.682	0.001	0.276	0.490	0.000	0.443	13.178	−22.357	−17.249
20 SBB8	1.682	1.000	0.275	0.490	0.000	0.509	13.179	−22.358	−17.250

Appendix D Copula plots

Copula data have been plotted in two ways for five out of seven countries under study, since no dependence between the residuals of output and unemployment first difference equations was found for Japan and Italy. In the left panel the contour plot of the normalized copula data is displayed. In the right panel the χ and λ statistics plot is displayed. Blue points correspond to ”lower” mode and red points correspond to ”upper” mode.

$$\chi_i = \frac{\hat{F}_{U_1,U_2}(u_{i,1},u_{i,2}) - \hat{F}_{U_1}(u_{i,1})\hat{F}_{U_2}(u_{i,2})}{\hat{F}_{U_1}(u_{i,1})(1-\hat{F}_{U_1}(u_{i,1}))\hat{F}_{U_2}(u_{i,2})(1-\hat{F}_{U_2}(u_{i,2}))}$$
$$\lambda_i = 4\text{sgn}(\tilde{F}_{U_1}(u_{i,1}),\tilde{F}_{U_2}(u_{i,2})) \cdot max(\tilde{F}_{U_1}(u_{i,1})^2,\tilde{F}_{U_2}(u_{i,2})^2)$$

- $\hat{F}$, empirical distribution function.
- λ_i is a measure of the distance between a data point $(u_{i,1},u_{i,2})$ and the center of the bivariate data set.
- χ corresponds to correlation coefficient between U_1 and U_2.
- Under independence:

$$\chi_i \sim \mathcal{N}(0,1/N)$$
$$\lambda_i \sim \mathcal{U}[-1,1]$$

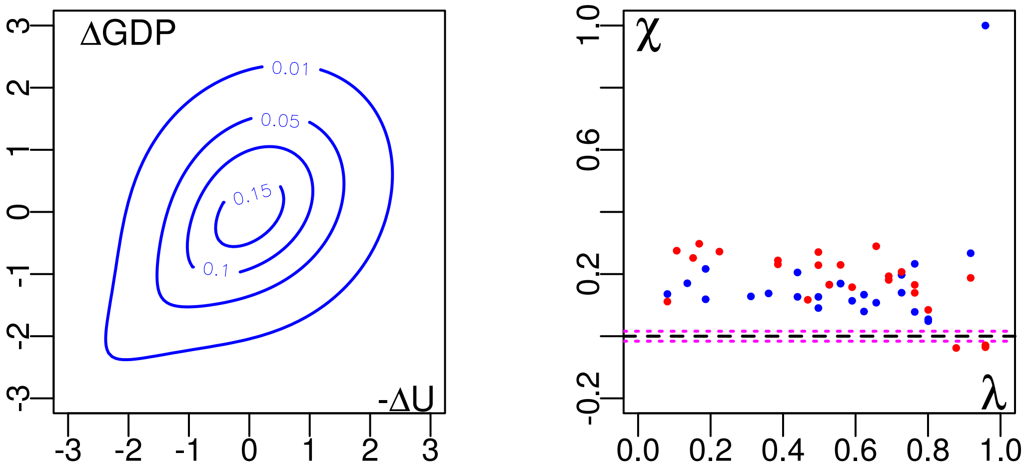

Figure D1: Contour plot of normalized copula data and χ plot of copula data for Canada

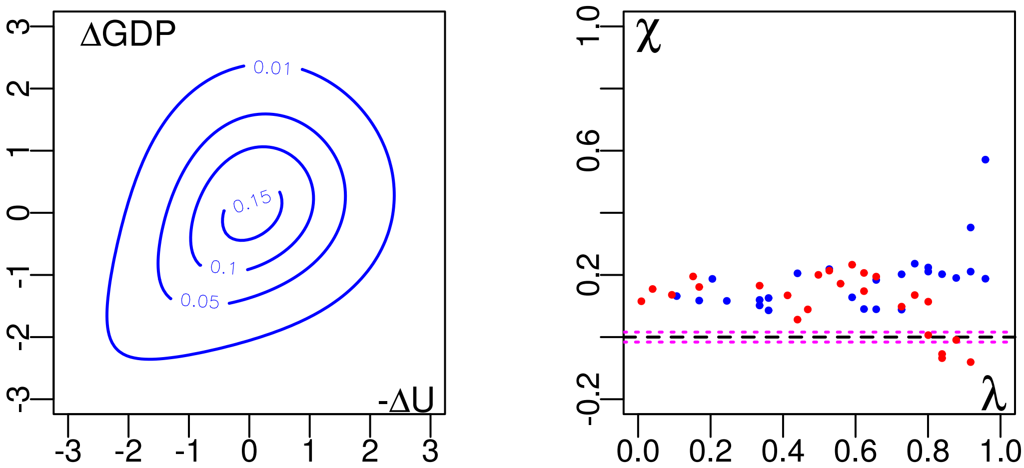

Figure D2: Contour plot of normalized copula data and χ plot of copula data for Germany

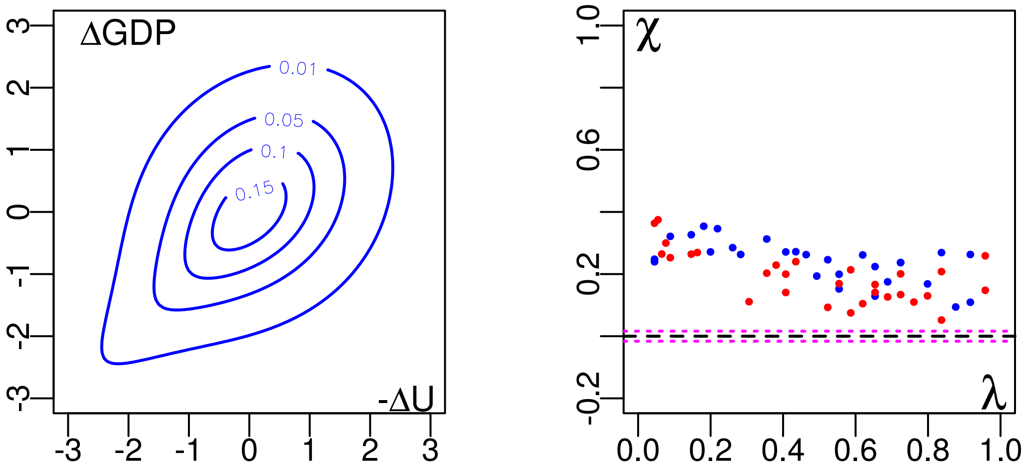

Figure D3: Contour plot of normalized copula data and χ plot of copula data for France

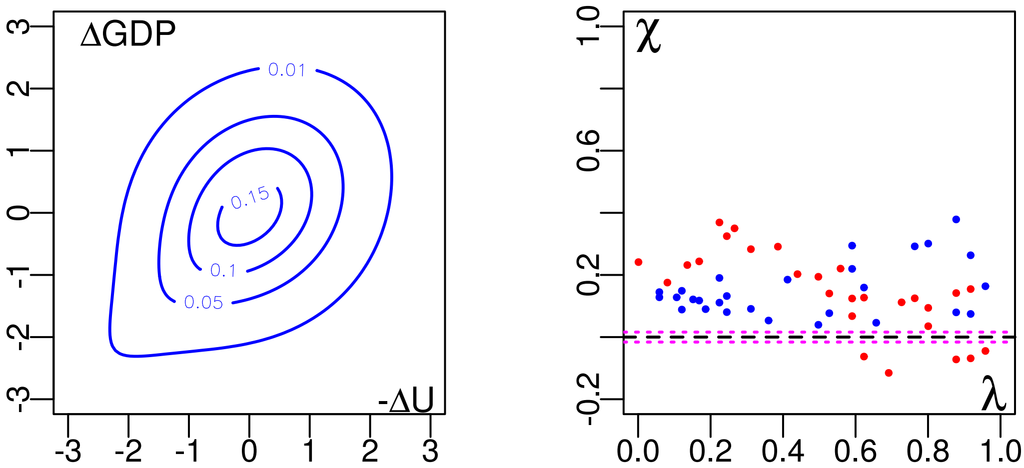

Figure D4: Contour plot of normalized copula data and χ plot of copula data for Great Britain

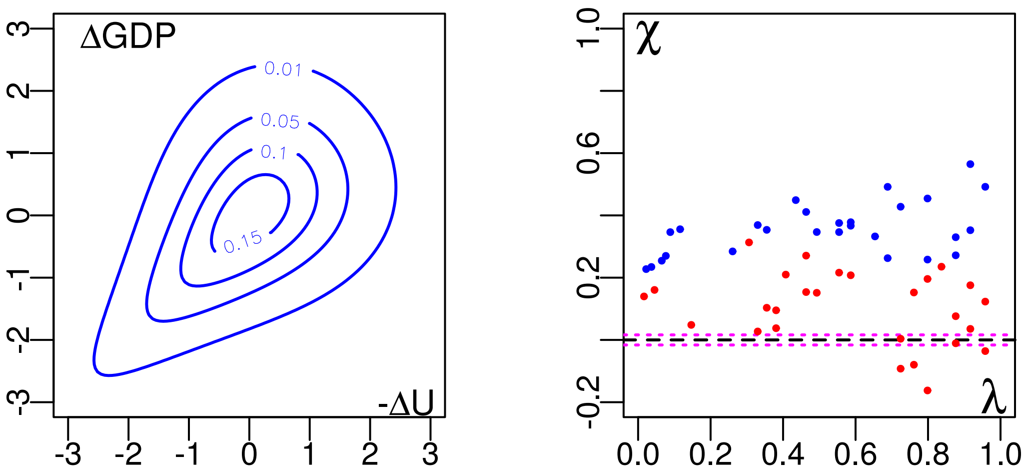

Figure D5: Contour plot of normalized copula data and χ plot of copula data for the USA

Appendix E R code

Listing 1: Code to download data for this article (R OECD version)

```
library(OECD)
Start_Period <- "1994-Q1"
End_Period   <- "2018-Q2"
une <- get_dataset(
  "STLABOUR",
  filter = "CAN+DEU+FRA+GBR+ITA+JPN+USA.LRHUTTTT.STSA.Q",
  start_time = Start_Period,
  end_time = End_Period,
  pre_formatted = TRUE
)

gdp <- get_dataset(
  "QNA",
  filter = "CAN+DEU+FRA+GBR+ITA+JPN+USA.B1_GE.GPSA.Q",
  start_time = Start_Period,
  end_time = End_Period,
  pre_formatted = TRUE
)
```

Listing 2: Code to download data for this article (R SDMX version)

```
library(rsdmx)
library(tibble)
bas_url <- "https://stats.oecd.org/restsdmx/sdmx.ashx/GetData/"
ser_une <- "STLABOUR/CAN+DEU+FRA+GBR+ITA+JPN+USA.LRHUTTTT.STSA.Q/"
tim_qry <- "all?startTime=1994-Q1&endTime=2018-Q2"
une_url     <- paste0(bas_url, ser_une, tim_qry)
sdmx     <- readSDMX(une_url)
une     <- as_tibble(sdmx)
ser_gdp <- "QNA/CAN+DEU+FRA+GBR+ITA+JPN+USA.B1_GE.LNBQRSA.Q/"
```

```
gdp_url      <- paste0(bas_url, ser_gdp, tim_qry)
sdmx      <- readSDMX(gdp_url)
gdp       <- as_tibble(sdmx)
```