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Microeconomic foundation of the Phillips curve

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Abstract: We show the negative relation between the unemployment rate and the inflation rate, that is, the Phillips curve using an overlapping generations model under monopolistic competition. We consider the effects of exogeneous changes in labor productivity. An increase (decrease) in the labor productivity in a period induces a decrease (increase) in the employment, an increase (decrease) in the unemployment rate and a falling (rising) in the price of the goods in the same period. Then, given the price in the previous period the inflation rate falls (rises). This conclusion is based on the premise of utility maximization of consumers and profit maximization of firms. Therefore, we have presented a microeconomic foundation of the Phillips curve.

JEL classifications: E12, E24, E31

Keywords: Phillips Curve, Microeconomic foundation, Overlapping generations model, Monopolistic competition.

1. Introduction

Otaki and Tamai (2011, 2012) presented a microeconomic foundation of the negative relation between the unemployment rate and the inflation rate, that is, the Phillips Curve (Phillips (1958)) using an overlapping generations model (OLG model) under monopolistic competition. They have shown that, the lower the unemployment rate in a period (for example period $t - 1$), the higher the inflation rate from period t to period $t + 1$. Their logic is as follows. They assume that the low (or high) unemployment rate in period $t - 1$ raises (or lowers) the labor productivity in period t by learning effect. If the unemployment rate in period $t - 1$ increases, the labor productivity in period t falls. Then, by the behavior of firms in monopolistic competition the price of the goods in period t rises given nominal wage rate, and the inflation rate from period t to period $t + 1$ falls given the (expected) price of the goods in period $t + 1$. Alternatively, a decrease in the unemployment rate in period $t - 1$ raises the labor productivity in period t . Then, the price of the goods falls, and the inflation rate from period t to period $t + 1$ rises given the (expected) price of the goods in period $t + 1$. However, we do not find their conclusion that the low unemployment rate in period $t - 1$ explains the high inflation rate from period t to period $t + 1$ to be satisfactory. A fall in the price in period t means that the inflation rate from period $t - 1$ to period t falls, that is, the low unemployment rate in period $t - 1$ explains the low (not high) inflation rate from period $t - 1$ to period t .

Instead, in this paper we consider the effects of exogenous changes in labor productivity. It may be due to a change in the unemployment rate in the previous period as assumed by Otaki and Tamai (2011, 2012). We will show the negative relationship between the unemployment rate and the inflation rate in the same period. Our logic is as follows. If the labor productivity in a period, for example, period t increases, the employment decreases and the unemployment rate in period t increases. Then, by the behavior of firms in monopolistic competition the price of the goods falls given nominal wage rate, and the inflation rate from period $t - 1$ to period t decreases. Alternatively, if the labor productivity in period t decreases, the employment increases and the unemployment rate in period t decreases. Then, the price of the goods rises given nominal wage rate, and the inflation rate from period $t - 1$ to period t increases.

Some other references about Phillips curve are Lucas (1972), Calvo (1983), Mankiw and Reis (2002) and Woodford (1996). According to Otaki and Tamai (2011, 2012), every work on the Phillips curve presumes some market imperfection, and it implies that if there does not exist some price stickiness assumption or imperfect information, the negative correlation between inflation and unemployment will disappear. This paper will show that it isn't.

In Section 2 we analyze behaviors of consumers and firms. In Section 3 we consider the equilibrium of the economy with involuntary unemployment. In Section 4 we show the main results about the negative relation between the unemployment rate and the inflation rate.

2. Behaviors of consumers and firms

We consider a two-periods (young and old) OLG model under monopolistic competition according to Otaki (2007, 2009, 2011, 2015 and 2016). There is one factor of production, labor, and there is a continuum of goods indexed by $z \in [0,1]$. Each good is monopolistically produced by Firm z . Consumers are born at continuous density $[0,1] \times [0,1]$ in each period. They can supply only one unit of labor when they are young (period 1).

In the case of the two-period overlapping generations model, the fall in nominal wage rates and prices may increase the real value of the old generation's savings and increase consumption, but if we consider the three-period overlapping generations model that includes childhood period, we can show that the fall in nominal wage rates and prices may instead reduce consumption. See Tanaka (2020) for details. That point is irrelevant to the purpose of this paper.

2.1 Consumers

We use the following notations.

$c^i(z)$: consumption of good z in period i , $i = 1, 2$.

$p^i(z)$: price of good z in period i , $i = 1, 2$.

X^i : consumption basket in period i , $i = 1, 2$.

$$X^i = \left\{ \int_0^1 c^i(z)^{1-\frac{1}{\eta}} dz \right\}^{\frac{1}{1-\frac{1}{\eta}}}, \quad i = 1, 2, \quad \eta > 1.$$

β : disutility of labor, $\beta > 0$.

W : nominal wage rate.

Π : profits of firms which are equally distributed to each consumer.

L : employment of each firm and the total employment.

L_f : population of labor or employment in the full-employment state.

$y(L)$: labor productivity. $y(L) \geq 1$.

δ is the definition function. If a consumer is employed, $\delta = 1$; if he is not employed, $\delta = 0$. The labor productivity is $y(L)$. We assume increasing or constant returns to scale technology. Thus, $y(L)$ is increasing or constant with respect to the employment of a firm L . We define the employment elasticity of the labor productivity as follows.

$$\zeta = \frac{y'}{y(L)} \cdot L$$

We assume $0 \leq \zeta < 1$. Increasing returns to scale means $\zeta > 0$. η is (the inverse of) the degree of differentiation of the goods. In the limit when $\eta \rightarrow +\infty$, the goods are homogeneous. We assume

$$\left(1 - \frac{1}{\eta}\right)(1 + \zeta) < 1$$

so that the profits of firms are positive.

The utility of consumers of one generation over two periods is

$$U(X^1, X^2, \delta, \beta) = u(X^1, X^2) - \delta\beta.$$

We assume that $u(X^1, X^2)$ is homogeneous of degree one (linearly homogeneous). The budget constraint is

$$\int_0^1 p^1(z)c^1(z)dz + \int_0^1 p^2(z)c^2(z)dz = \delta W + \Pi.$$

$p^2(z)$ is the expectation of the price of good z in period 2. The Lagrange function is

$$\mathcal{L} = u(X^1, X^2) - \delta\beta - \lambda \left(\int_0^1 p^1(z)c^1(z)dz + \int_0^1 p^2(z)c^2(z)dz - \delta W - \Pi \right).$$

λ is the Lagrange multiplier. The first order conditions are

$$\frac{\partial u}{\partial X^1} \left(\int_0^1 c^1(z)^{1-\frac{1}{\eta}} dz \right)^{\frac{1}{\eta}} c^1(z)^{-\frac{1}{\eta}} = \lambda p^1(z), \quad (1)$$

and

$$\frac{\partial u}{\partial X^2} \left(\int_0^1 c^2(z)^{1-\frac{1}{\eta}} dz \right)^{\frac{1}{\eta}} c^2(z)^{-\frac{1}{\eta}} = \lambda p^2(z). \quad (2)$$

They are rewritten as

$$\frac{\partial u}{\partial X^1} X^1 \left(\int_0^1 c^1(z)^{1-\frac{1}{\eta}} dz \right)^{-1} c^1(z)^{1-\frac{1}{\eta}} = \lambda p^1(z) c^1(z), \quad (3)$$

$$\frac{\partial u}{\partial X^2} X^2 \left(\int_0^1 c^2(z)^{1-\frac{1}{\eta}} dz \right)^{-1} c^2(z)^{1-\frac{1}{\eta}} = \lambda p^2(z) c^2(z). \quad (4)$$

Let

$$P^1 = \left(\int_0^1 p^1(z)^{1-\eta} dz \right)^{\frac{1}{1-\eta}}, \quad P^2 = \left(\int_0^1 p^2(z)^{1-\eta} dz \right)^{\frac{1}{1-\eta}}.$$

They are prices of the consumption baskets in period 1 and period 2. By some calculations we obtain (please see Appendix)

$$u(X^1, X^2) = \lambda \left[\int_0^1 p^1(z) c^1(z) dz + \int_0^1 p^2(z) c^2(z) dz \right] = \lambda(\delta W + \Pi), \quad (5)$$

$$\frac{P^2}{P^1} = \frac{\frac{\partial u}{\partial X^2}}{\frac{\partial u}{\partial X^1}}, \quad (6)$$

$$P^1 X^1 + P^2 X^2 = \delta W + \Pi. \quad (7)$$

The indirect utility of consumers is written as follows

$$V = \frac{1}{\varphi(P^1, P^2)} (\delta W + \Pi) - \delta \beta. \quad (8)$$

$\varphi(P^1, P^2)$ is a function which is homogeneous of degree one. The reservation nominal wage rate W^R is a solution of the following equation.

$$\frac{1}{\varphi(P^1, P^2)} (W^R + \Pi) - \beta = \frac{1}{\varphi(P^1, P^2)} \Pi.$$

From this

$$W^R = \varphi(P^1, P^2) \beta.$$

The labor supply is indivisible. If $W > W^R$, the total labor supply is L_f . If $W < W^R$, it is zero. If $W = W^R$, employment and unemployment are indifferent for consumers, and there exists no involuntary unemployment even if $L < L_f$.

Indivisibility of labor supply may be due to the fact that there exists minimum standard of living even in the advanced economy (please see Otaki (2015)).

Let $\rho = \frac{P^2}{P^1}$. This is the expected inflation rate (plus one). Since $\varphi(P^1, P^2)$ is homogeneous of degree one, the reservation real wage rate is

$$\omega^R = \frac{W^R}{P^1} = \varphi(1, \rho) \beta.$$

If the value of ρ is given, ω^R is constant.

Otaki (2007) assumes that the wage rate is equal to the reservation wage rate in the equilibrium. However, there exists no mechanism to equalize them. We assume that β and ω^R are not so large.

2.2 Firms

Let

$$\alpha = \frac{P^1 X^1}{P^1 X^1 + P^2 X^2} = \frac{X^1}{X^1 + \rho X^2}, \quad 0 < \alpha < 1.$$

From (3) ~ (7),

$$\alpha(\delta W + \Pi) \left(\int_0^1 c^1(z)^{1-\frac{1}{\eta}} dz \right)^{-1} c^1(z)^{-\frac{1}{\eta}} = p^1(z).$$

Since

$$X^1 = \frac{\alpha(\delta W + \Pi)}{P^1},$$

we have

$$(X^1)^{\frac{1}{\eta}-1} = \left(\int_0^1 c^1(z)^{1-\frac{1}{\eta}} dz \right)^{-1} = \left(\frac{\alpha(\delta W + \Pi)}{P^1} \right)^{\frac{1}{\eta}-1}.$$

Therefore,

$$\alpha(\delta W + \Pi) \left(\frac{\alpha(\delta W + \Pi)}{P^1} \right)^{\frac{1}{\eta}-1} c^1(z)^{-\frac{1}{\eta}} = \left(\frac{\alpha(\delta W + \Pi)}{P^1} \right)^{\frac{1}{\eta}} P^1 c^1(z)^{-\frac{1}{\eta}} = p^1(z).$$

Thus,

$$c^1(z)^{\frac{1}{\eta}} = \left(\frac{\alpha(\delta W + \Pi)}{P^1} \right)^{\frac{1}{\eta}} P^1 (p^1(z))^{-1}.$$

Hence,

$$c^1(z) = \frac{\alpha(\delta W + \Pi)}{P^1} \left(\frac{p^1(z)}{P^1} \right)^{-\eta}.$$

This is demand for good z of an individual of younger generation. Similarly, his demand for good z in period 2 is

$$c^2(z) = \frac{(1 - \alpha)(\delta W + \Pi)}{P^2} \left(\frac{p^2(z)}{P^2} \right)^{-\eta}.$$

Let M be the total savings of consumers of the older generation carried over from their period 1. It is written as

$$M = (1 - \alpha)(\bar{W}\bar{L} + L_f\bar{\Pi}).$$

\bar{W} , \bar{L} and $\bar{\Pi}$ are the nominal wage rate, the employment and the profit in the previous period. Then, their demand for good z is

$$\frac{M}{P^1} \left(\frac{p^1(z)}{P^1} \right)^{-\eta}.$$

The government expenditure constitutes the national income as well as consumptions of younger and older generations. The total demand for good z is written as

$$c(z) = \frac{Y}{P^1} \left(\frac{p^1(z)}{P^1} \right)^{-\eta}.$$

Y is the effective demand defined by

$$Y = \alpha(WL + L_f\Pi) + G + M.$$

G is the government expenditure (about this demand function please see Otaki (2007), (2009)). The total employment, the total profits and the total government expenditure are

$$\int_0^1 Ldz = L, \quad \int_0^1 \Pi dz = \Pi, \quad \int_0^1 Gdz = G.$$

We have

$$\frac{\partial c(z)}{\partial p^1(z)} = -\eta \frac{Y}{P^1} \frac{p^1(z)^{-1-\eta}}{(P^1)^{-\eta}} = -\eta \frac{c(z)}{p^1(z)}.$$

From $c(z) = Ly(L)$,

$$\frac{\partial L}{\partial p^1(z)} = \frac{1}{y(L) + Ly'} \frac{\partial c(z)}{\partial p^1(z)}.$$

The profit of Firm z is

$$\pi(z) = p^1(z)c(z) - \frac{W}{y(L)}c(z).$$

P^1 is given for Firm z . Note that the employment elasticity of the labor productivity is

$$\zeta = \frac{y'}{y(L)} \frac{L}{L}.$$

The condition for profit maximization with respect to $p^1(z)$ is

$$c(z) + \left[p^1(z) - \frac{y(L) - c(z)y' \frac{1}{y(L) + Ly'}}{y(L)^2} W \right] \frac{\partial c(z)}{\partial p^1(z)}$$

$$\begin{aligned}
&= c(z) + \left[p^1(z) - \frac{1 - Ly' \frac{1}{y(L) + Ly'}}{y(L)} W \right] \frac{\partial c(z)}{\partial p^1(z)} \\
&= c(z) + \left[p^1(z) - \frac{W}{y(L) + Ly'} \right] \frac{\partial c(z)}{\partial p^1(z)} = 0.
\end{aligned}$$

From this

$$p^1(z) = \frac{W}{y(L) + Ly'} - \frac{c(z)}{\frac{\partial c(z)}{\partial p^1(z)}} = \frac{W}{(1 + \zeta)y(L)} + \frac{1}{\eta} p^1(z).$$

Therefore, we obtain

$$p^1(z) = \frac{W}{\left(1 - \frac{1}{\eta}\right)(1 + \zeta)y(L)}.$$

With increasing returns to scale, since $\zeta > 0$, $p^1(z)$ is lower than that in a case without increasing returns to scale given the value of W .

3. The equilibrium with involuntary unemployment

Since the model is symmetric, the prices of all goods are equal. Then,

$$P^1 = p^1(z).$$

Hence

$$P^1 = \frac{w}{\left(1 - \frac{1}{\eta}\right)(1 + \zeta)y(L)}. \quad (9)$$

The real wage rate is

$$\omega = \frac{W}{P^1} = \left(1 - \frac{1}{\eta}\right)(1 + \zeta)y(L).$$

If ζ is constant, this is increasing with respect to L .

The aggregate supply of the goods is equal to

$$WL + L_f \Pi = P^1 Ly(L).$$

The aggregate demand is

$$\alpha(WL + L_f \Pi) + G + M = \alpha P^1 Ly(L) + G + M.$$

Since they are equal,

$$P^1 Ly(L) = \alpha P^1 Ly(L) + G + M, \quad (10)$$

or

$$P^1 Ly(L) = \frac{G+M}{1-\alpha}. \quad (11)$$

In real terms

$$Ly(L) = \frac{1}{1-\alpha}(g + m), \quad (12)$$

or

$$L = \frac{1}{(1-\alpha)y(L)}(g + m). \quad (13)$$

where

$$g = \frac{G}{P^1}, \quad m = \frac{M}{P^1}.$$

$\frac{1}{1-\alpha}$ is a multiplier. (12) and (13) mean that the employment L is determined by $g + m$. It can not be larger than L_f . However, it may be strictly smaller than L_f ($L < L_f$). Then, there exists involuntary unemployment. Since the real wage rate $\omega = \left(1 - \frac{1}{\eta}\right)(1 + \zeta)y(L)$ is increasing with respect to L , and the reservation real wage rate ω^R is constant, if $\omega > \omega^R$ there exists no mechanism to reduce the difference between them.

4. Phillips Curve

4.1 Exogenous changes in labor productivity

We consider exogenous changes in labor productivity given nominal wage rate. It may be due to a change in the unemployment rate in the previous period as assumed by Otaki and Tamai (2011, 2012). Suppose that the labor productivity $y(L)$ in a period, for example, period t increases to $\theta y(L)$ with a constant $\theta > 1$ given L . From (13) if g and m are constant, employment L decreases, that is, the unemployment rate in period t increases. (9) means that the price of the goods in period t given W falls because η and ζ are constant. Let P_t and P_{t-1} be the price of the goods (price of the consumption basket) in period t and that in period $t - 1$. Then, the inflation rate from period $t - 1$ to t , $\frac{P_t}{P_{t-1}} - 1$, falls given P_{t-1} .

Alternatively, a decrease in the labor productivity ($\theta < 1$) increases employment, decreases the unemployment rate, and raises the price of the goods and the inflation rate from period $t - 1$ to t .

Therefore, we obtain the negative relationship between the unemployment rate and the inflation rate in the same period.

Figure 1 depicts an example the Phillips Curve. U_t denotes the unemployment rate in period t .

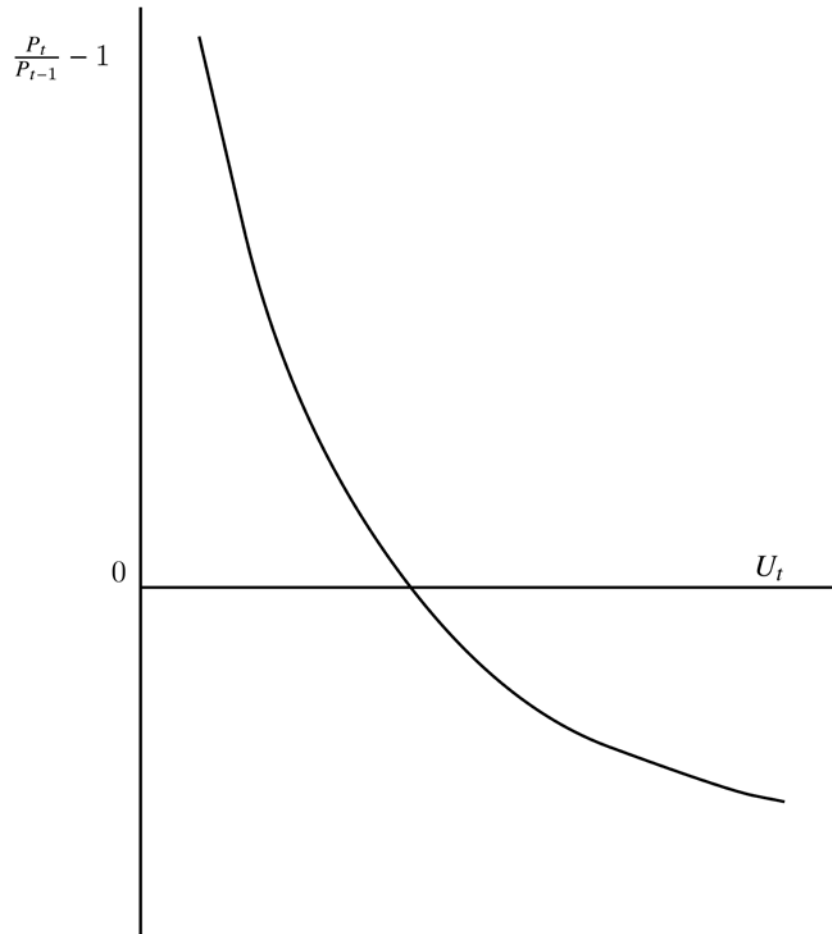


Figure 1: Phillips Curve

4.2 Analysis by Otaki and Tamai (2011, 2012)

Otaki and Tamai (2011, 2012) suppose that the low (or high) unemployment rate in a period, for example, period $t - 1$ raises (or lowers) the labor productivity in period t by learning effect. If the unemployment rate in period $t - 1$ increases, the labor productivity in period t falls. Then, from (9) the price of the goods rises, and the inflation rate from period t to period $t + 1$ falls given the (expected) price of the goods in period $t + 1$. Alternatively, a decrease in the unemployment rate in period $t - 1$ raises the labor productivity in period t . Then, the price of the goods falls, and the inflation rate from period t to period $t + 1$ rises given the (expected) price of the goods in period $t + 1$. Thus, they have shown the negative relation between the unemployment rate in period $t - 1$ and the inflation rate from period t to period $t + 1$, $\frac{P_{t+1}}{P_t} - 1$. On the other hand, a fall in the price in period t means that the inflation rate from period $t - 1$ to period t falls, that is, the low unemployment rate in period $t - 1$ explains the low (not high) inflation rate from period $t - 1$ to period t , $\frac{P_t}{P_{t-1}} - 1$.

Their Phillips curve is depicted in Figure 2. U_{t-1} denotes the unemployment rate in period $t - 1$.

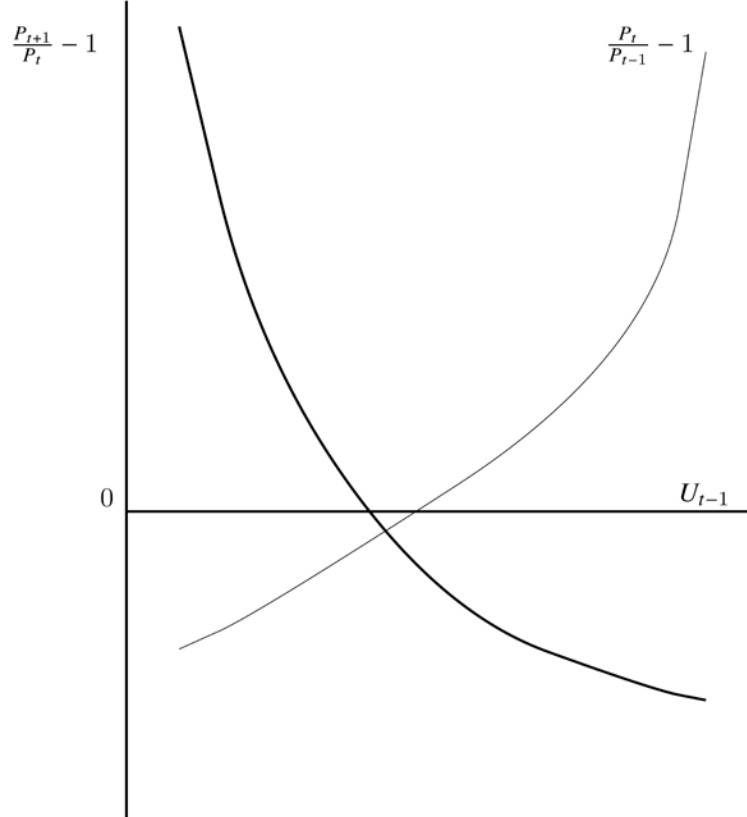


Figure 2: Phillips curve by Otaki and Tamai (2011, 2012)

5. Conclusion

We have shown that in an overlapping generations model under monopolistic competition changes in labor productivity bring about the negative relation between the unemployment rate and the inflation rate in the same period. This conclusion is based on the premise of utility maximization of consumers and profit maximization of firms. Therefore, we have presented a microeconomic foundation of the Phillips curve.

Appendix: Derivations of (5), (6), (7) and (8)

From (3) and (4)

$$\frac{\partial u}{\partial X^1} X^1 \left(\int_0^1 c^1(z)^{1-\frac{1}{\eta}} dz \right)^{-1} \int_0^1 c^1(z)^{1-\frac{1}{\eta}} dz = \frac{\partial u}{\partial X^1} X^1 = \lambda \int_0^1 p^1(z) c^1(z) dz,$$

$$\frac{\partial u}{\partial X^2} X^2 \left(\int_0^1 c^2(z)^{1-\frac{1}{\eta}} dz \right)^{-1} \int_0^1 c^2(z)^{1-\frac{1}{\eta}} dz = \frac{\partial u}{\partial X^2} X^2 = \lambda \int_0^1 p^2(z) c^2(z) dz.$$

Since $u(X^1, X^2)$ is homogeneous of degree one,

$$u(X^1, X^2) = \frac{\partial u}{\partial X^1} X^1 + \frac{\partial u}{\partial X^2} X^2.$$

Thus, we obtain

$$\frac{\int_0^1 p^1(z) c^1(z) dz}{\int_0^1 p^2(z) c^2(z) dz} = \frac{\frac{\partial u}{\partial X^1} X^1}{\frac{\partial u}{\partial X^2} X^2},$$

and

$$u(X^1, X^2) = \lambda \left[\int_0^1 p^1(z) c^1(z) dz + \int_0^1 p^2(z) c^2(z) dz \right] = \lambda(\delta W + \Pi).$$

From (1) and (2), we have

$$\left(\frac{\partial u}{\partial X^1} \right)^{1-\eta} \left(\int_0^1 c^1(z)^{1-\frac{1}{\eta}} dz \right)^{-1} c^1(z)^{1-\frac{1}{\eta}} = \lambda^{1-\eta} p^1(z)^{1-\eta},$$

and

$$\left(\frac{\partial u}{\partial X^2} \right)^{1-\eta} \left(\int_0^1 c^2(z)^{1-\frac{1}{\eta}} dz \right)^{-1} c^2(z)^{1-\frac{1}{\eta}} = \lambda^{1-\eta} p^2(z)^{1-\eta}.$$

They mean

$$\left(\frac{\partial u}{\partial X^1} \right)^{1-\eta} \left(\int_0^1 c^1(z)^{1-\frac{1}{\eta}} dz \right)^{-1} \int_0^1 c^1(z)^{1-\frac{1}{\eta}} dz = \lambda^{1-\eta} \int_0^1 p^1(z)^{1-\eta} dz,$$

and

$$\left(\frac{\partial u}{\partial X^2} \right)^{1-\eta} \left(\int_0^1 c^2(z)^{1-\frac{1}{\eta}} dz \right)^{-1} \int_0^1 c^2(z)^{1-\frac{1}{\eta}} dz = \lambda^{1-\eta} \int_0^1 p^2(z)^{1-\eta} dz.$$

Then, we obtain

$$\frac{\partial u}{\partial X^1} = \lambda \left(\int_0^1 p^1(z)^{1-\eta} dz \right)^{\frac{1}{1-\eta}} = \lambda P^1,$$

and

$$\frac{\partial u}{\partial X^2} = \lambda \left(\int_0^1 p^2(z)^{1-\eta} dz \right)^{\frac{1}{1-\eta}} = \lambda P^2.$$

From them we get

$$u(X^1, X^2) = \lambda(P^1 X^1 + P^2 X^2),$$

$$\frac{P^2}{P^1} = \frac{\frac{\partial u}{\partial X^2}}{\frac{\partial u}{\partial X^1}},$$

and

$$P^1 X^1 + P^2 X^2 = \delta W + \Pi.$$

Since $u(X^1, X^2)$ is homogeneous of degree one, λ is a function of P^1 and P^2 , and $\frac{1}{\lambda}$ is homogeneous of degree one because proportional increases in P^1 and P^2 reduce X^1 and X^2 at the same rate given $\delta W + \Pi$. We obtain the following indirect utility function.

$$V = \frac{1}{\varphi(P^1, P^2)} (\delta W + \Pi) - \delta \beta.$$

$\varphi(P^1, P^2)$ is a function which is homogenous of degree one.

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