Trade and the equivalence between environmental tax and quota

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Abstract

In a two-sector general equilibrium model with pollution (arising from production) affecting the productivity, I examine in both autarky and trade equilibria the equivalence between tax and quota, that is, whether they can replace each other to achieve the same environmental goals. I show that (i) sometimes tax cannot achieve what quota can; (ii) the equivalence/non-equivalence between tax and quota may change due to trade liberalization; (iii) the choice of numeraire matters under tax regulation.

Keywords: Pollution tax; emission quota; production externalities; numeraire
JEL classification: F18; H23; Q58

1 Introduction

Tax and quota are among the most popular instruments to achieve environmental goals. Under tax regulation, the government determines the price of pollution discharge and lets the market determine the amount of pollution. In contrast, under quota regulation, the government determines the amount and lets the market determines the price of pollution discharge. The comparison between tax and quota often focuses on practical issues such as the ease of implementation, both technological and political. Many theoretical studies on this topic, pioneered by Weitzman (1974), focus on uncertainty or lake of information.[1]

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[1]See also Hoel and Karp (2001, 2002), Newell and Pizer (2003), Karp and Zhang (2005), and recently Grodecka and Kuralbayeva (2015).

In this paper, I try to compare tax and quota from another aspect. Assuming away implementation cost and uncertainty, I ask whether tax and quota can still substitute each other without any problem. Or more specifically, can an explicit pollution tax reproduce a pollution level that, if set alternatively as an emission quota, produces an implicit pollution tax equal to the explicit pollution tax and vice versa?

The comparison is conducted using a two-sector model with negative impacts of pollution on the production side. The labor is the only primary input and the pollution, which arises as joint products, is treated as an input by assuming underlying abatement activities. I examine the effects of quota and tax in autarky and in free trade. The main results are: (i) tax is less effective than quota in the sense that sometimes tax cannot achieve environmental goals that quota can; (ii) trade liberalization may change the equivalence/nonequivalence between tax and quota.

I also show that the choice of numeraire may hugely affect the effects of tax. In economic models, the numeraire is usually chosen arbitrarily or just for convenience. In this model, however, the choice of numeraire matters. This is because when adjusting the pollution tax, we actually change the relative price of pollution with respect to the numeraire good/factor. Therefore, given different numeraire, a change in tax may affect the economy through different channels. For example, let the wage be the numeraire, then the pollution tax takes effect mainly through the substitution between labor input and pollution discharge. However, if we choose a consumption good as the numeraire, the pollution tax works not only through the substitution channel but also through the negative impacts of pollution on production. In particular, in this model, choosing the wage as the numeraire makes the pollution tax more likely to be effective, compared to choosing consumption goods.

In a closely related work, Ishikawa and Kiyono (2006) compare different emission regulations in an open economy and derive the similar result that tax and quota may not be able to substitute each other. But the mechanisms driving the result are quit different: they assume the negative effects of emission on utility, whereas I assume the negative effects on production. The negative impacts of pollution on production are practically significant, especially in industries like forestry, agriculture, fishing, tourism, and alternative energy. For example, Reddy and Behera (2006) estimates a loss of $213.2 per household per annum on agriculture due to pollution in Kazipalle village, India. This paper also differs from Ishikawa and Kiyono (2006) in the type of pollution. They focus on the greenhouse gases (GHGs), which are transboundary emissions. In this paper, I focus on the non-transboundary pollution, which is also important in reality. As Sweeney (1993, pp.761) suggests, among a large number of types of pollution, many have relatively small damage range such as noise, radiation, NOx, SOx and particulates.

The rest of the paper is organized as follows. Section 2 explains the model. Section 3 considers the effects of quota and tax in autarky. Section 4 moves on to the analysis of a small open economy.

2 The Model

There are two consumption goods, manufacturing and agriculture goods, denoted by M and A. There is one primary input, labor, with the endowment L. Accompanying the production of consumption goods arises pollution as the joint product. Each firm can involve in abatement activities by allocating some labor to reduce the discharge pollution.

Firms Assume that, if firms conduct no abatement, the output of good j ($j = M, A$) is linearly proportionate to the labor input: $X_j = L_j / G^j(Z)$, where L_j is the labor input, Z is the total pollution discharge. The function $G^j(Z)$ measures the negative effects of pollution on the firms that produce good j, which satisfies $G^{j\prime}(Z) > 0$ and $G^j(0) > 0$.[2] Without any abatement, the pollution discharge arising from the production of good j, Z_j, is assumed proportionate to the scale of the production activity measured by the labor input: $Z_j = \gamma L_j$. Clearly, $Z = Z_M + Z_A$.

If firms conduct some abatement, given the same amount of labor in these firms, the pollution discharge declines: $Z_j < \gamma L_j$. Abatement actually introduces a substitution between labor input and pollution discharge, it is therefore convenient to treat pollution as an "input". Following Copeland and Taylor (1994), write the output as a linearly homogeneous function of Z_j and L_j: $X_j = F^j(L_j, Z_j) / G^j(Z)$, where $F^j(L_j, Z_j)$ is strictly increasing and strictly quasi-concave with respect to L_j and Z_j. For simplicity, further assume that both $F^i(L_j, Z_j)$ and $G^i(Z)$ are twice continuously differentiable.

Indeed, we can combine no abatement and abatement cases into one production function as follows.

$$X_j = \frac{F^j(L_j, Z_j)}{G^j(Z)}, \quad Z_j \in (0, \gamma L_j], \tag{1}$$

where $F^j(L_j, \gamma L_j) \equiv L_j$.

Firms under perfect competition maximize the profit by taking the total pollution discharge (Z) as given. The linear homogeneity of $F^j(L_j, Z_j)$ implies that firms' decision can

[2] Mayeres and Proost (2001) term this type of negative effect "feedback effect". Given certain function forms, an externality on production and that on utility can be equivalent, since an externality on production eventually enters utility function through consumption.

be described by the cost minimization problem:

$$\min_{a_i,e_j} wa_j + re_j, \quad \text{s.t.} F^j(a_j, e_j) = 1, \tag{2}$$

where w and r is the wage rate and the price of pollution discharge. Recall that $Z_j \in (0, \gamma L_j]$, which implies $e_j/a_j \in (0, \gamma]$. Clearly, (2) has interior solutions only if r/w is not too low; otherwise, firms may have no incentive to conduct abatement and the corner solution arises. Precisely, we have

$$a_j^* = \begin{cases} 1 & \frac{r}{w} \in (0, \eta], \\ a_j(r, w) & \frac{r}{w} \in (\eta, \infty), \end{cases}$$
$$e_j^* = \begin{cases} \gamma & \frac{r}{w} \in (0, \eta], \\ e_j(r, w) & \frac{r}{w} \in (\eta, \infty), \end{cases}$$

where

$$\eta \equiv \frac{\partial F^j(1, \gamma)/\partial e_j}{\partial F^j(1, \gamma)/\partial a_j}.$$

Let $c^j(r, w) \equiv wa_j(r, w) + re_j(r, w)$, which is linearly homogenous, then the unit cost of good j is $G^j(Z)c^j(r, w)$. Perfect competition together with profit maximization gives the following Kuhn-Tucker condition:

$$G^j(Z)c^j(r, w) \geq p_j, \quad X_j \geq 0, \quad \left(G^j(Z)c^j(r, w) - p_j\right)X_j = 0. \tag{3}$$

As long as good j is produced, we have $G^j(Z)c^j(r, w) = p_j$. For convenience, define the sensitivity as

$$\varepsilon_j \equiv -\frac{\partial \ln X_j}{\partial \ln Z} = \frac{d \ln G^j(Z)}{d \ln Z} > 0, \tag{4}$$

which captures how fast the output of good j declines with the total pollution discharge.

Households Assume the representative household has the utility function of Cobb-Douglas type:

$$u = C_A^b C_M^{1-b}, \tag{5}$$

where C_j is the consumption on good j and b is the spending share on agriculture good. The government transfers the tax revenue to households in a lump-sum fashion, so the income $Y = wL + rZ$. The household maximizes the utility subject to the budget constraint $Y = p_A C_A + p_M C_M$, yielding the domestic demands: $C_A = bY/p_A$ and $C_M = (1-b)Y/p_M$.

Quota and tax In this paper, I focus on two instruments of environmental regulation: emission quota and pollution tax. Under quota regulation, the government issues emission permits to control directly the amount of total pollution discharge: $Z = Q$. The permits can be traded freely, so the same price of permits prevails in the economy. Obviously, with a quota $Q < \gamma L$, the model becomes a two-factor two-sector Heckscher-Ohlin model with labor endowment L and "environment endowment" Q, with a difference that the level of Q affects the productivity as well.

On the other hand, under tax regulation, the government imposes tax on pollution discharge to control directly the price of pollution discharge. Note that the pollution tax should be imposed in a relative sense, say, the government determines the level of r/w or r/p_j rather than just the level of r. This is because we have not specified the numeraire. If the government changes just r, all other prices can change by the same ratio and the real economy (outputs, pollution discharge, and utility level) will remain unchanged. Note that the effects of changing r/w or r/p_j is the same as the effects of changing r in a model with the numeraire $w = 1$ or $p_j = 1$. To show that in terms of what the pollution tax is imposed, or equivalently the choice of numeraire, may affect the effects of pollution tax, it is convenient to let the numeraire not specified throughout the model.

3 Autarky

Consider the effects of quota and tax in autarky. For the purpose, first characterize the equilibrium in autarky. The clearing in good markets requires $bY/p_A = X_A$ and $(1-b)\,Y/p_M = X_M$. Given the Cobb-Douglas utility function, both goods are essential and thus produced in autarky equilibrium, which gives, by (3), $G^j(Z)\,c^j(r,w) = p_j$. The goods market clearing condition can be rewritten into

$$X_A G^A(Z)\,c^A(r,w) = bY, \tag{6}$$

$$X_M G^M(Z)\,c^M(r,w) = (1-b)\,Y. \tag{7}$$

On the other hand, the production of good j demands labor $X_j G^j(Z)\,a_j(r,w)$, so the labor clearing requires

$$\sum_j X_j G^j(Z)\,a_j(r,w) = L. \tag{8}$$

At the same time, the production of good j yields pollution $X_j G^i(Z) e_j(r,w)$, so the total pollution is

$$\sum_j X_j G^i(Z) e_j(r,w) = Z. \tag{9}$$

There are six variables (X_A, X_M, Y, r, w, and Z) to solve from the four equations above. Since we have not specified the numeraire, only the relative prices r/w and Y/w matter.[3] Under quota regulation, Z is exogenous and we can solve the four equations for X_A, X_M, Y/w, and r/w. Under tax regulation, if the government determines r/w, we can solve for X_A, X_M, Y/w, and Z. If the government determines the pollution tax in terms of other prices, say r/p_A or r/p_M, we have slightly different equations for equilibrium. However, it turns out that the effects of a change in r/p_j can be readily obtained once we have derived the effects of a change in r/w. Therefore, for the purpose of comparative statics, it is sufficient to focus on (6) to (9).

Taking the logarithmic differential of (6) to (9) and using $dL = 0$, $\partial c^j(r,w)/\partial r = e_j^*$, and $\partial c^j(r,w)/\partial w = a_j^*$, we can obtain

$$\hat{X}_A + \varepsilon_A \hat{Z} + \theta_{ZA}(\hat{r} - \hat{w}) = \hat{Y} - \hat{w}, \tag{10}$$

$$\hat{X}_M + \varepsilon_M \hat{Z} + \theta_{ZM}(\hat{r} - \hat{w}) = \hat{Y} - \hat{w}, \tag{11}$$

$$\sum_j \lambda_{Lj}\left[\hat{X}_j + \varepsilon_j \hat{Z} + \hat{a}_j^*\right] = 0, \tag{12}$$

$$\sum_j \lambda_{Zj}\left[\hat{X}_j + \varepsilon_j \hat{Z} + \hat{e}_j^*\right] = \hat{Z}, \tag{13}$$

where θ_{ij} denotes the income share of factor i in good j (e.g., $\theta_{ZM} \equiv rZ_M/(p_M X_M)$), whereas λ_{ij} denotes the allocation share of factor i in good j (e.g., $\lambda_{ZM} \equiv Z_M/Z$). As usually, the hat over a variable denotes the proportionate change, e.g., $\hat{Z} \equiv dZ/Z$.

To further simplify (12) and (13), note that the cost minimization implies $\theta_{Lj}\hat{a}_j^* + \theta_{Zj}\hat{e}_j^* = 0$, so we have

$$\hat{a}_j^* = \theta_{Zj}\sigma_j(\hat{r} - \hat{w}), \quad \hat{e}_j^* = -\theta_{Lj}\sigma_j(\hat{r} - \hat{w}), \tag{14}$$

where $\sigma_j \equiv -d\ln\left(e_j^*/a_j^*\right)/d\ln(r/w) = -\left(\hat{e}_j^* - \hat{a}_j^*\right)/(\hat{r} - \hat{w}) > 0$ measures the elasticity of substitution between labor input and pollution discharge in producing good j. By (14), we can obtain

$$\sum_j \lambda_{Lj}\hat{a}_j^* = \delta_L(\hat{r} - \hat{w}), \quad \sum_j \lambda_{Zj}\hat{e}_j^* = -\delta_Z(\hat{r} - \hat{w}), \tag{15}$$

where $\delta_L \equiv \lambda_{LM}\theta_{ZM}\sigma_M + \lambda_{LA}\theta_{ZA}\sigma_A$ and $\delta_Z \equiv \lambda_{ZM}\theta_{LM}\sigma_M + \lambda_{ZA}\theta_{LA}\sigma_A$ have their economic

[3] The four equations (6) to (9) have implied that the income satisfies $Y = wL + rZ$, i.e., the Warlas's law.

meanings: δ_L (δ_Z) measures the aggregate proportionate saving in labor input (pollution discharge) given a proportionate rise in r/w while keeping all outputs unchanged.[4] Substituting (15) into (12) and (13), we can rewrite (10) to (13) as

$$\hat{X}_A - \left(\hat{Y} - \hat{w}\right) + \theta_{ZA}\left(\hat{r} - \hat{w}\right) + \varepsilon_A \hat{Z} = 0, \tag{16}$$

$$\hat{X}_M - \left(\hat{Y} - \hat{w}\right) + \theta_{ZM}\left(\hat{r} - \hat{w}\right) + \varepsilon_M \hat{Z} = 0, \tag{17}$$

$$\sum_j \lambda_{Lj}\hat{X}_j + \delta_L\left(\hat{r} - \hat{w}\right) + \sum_j \lambda_{Lj}\varepsilon_j \hat{Z} = 0, \tag{18}$$

$$\sum_j \lambda_{Zj}\hat{X}_j - \delta_Z\left(\hat{r} - \hat{w}\right) + \left(\sum_j \lambda_{Zj}\varepsilon_j - 1\right)\hat{Z} = 0. \tag{19}$$

The effects of quota in autarky Under quota regulation, the government determines emission quota to control the total amount of pollution discharge. Let Q denote the quota, then $Z = Q$ is exogenously given. Noting that $\hat{Z} = \hat{Q}$, rewrite (16) to (19) into

$$\begin{bmatrix} 1 & 0 & -1 & \theta_{ZA} \\ 0 & 1 & -1 & \theta_{ZM} \\ \lambda_{LA} & \lambda_{LM} & 0 & \delta_L \\ \lambda_{ZA} & \lambda_{ZM} & 0 & -\delta_Z \end{bmatrix} \begin{bmatrix} \hat{X}_A \\ \hat{X}_M \\ \hat{Y} - \hat{w} \\ \hat{r} - \hat{w} \end{bmatrix} = \begin{bmatrix} -\varepsilon_A \\ -\varepsilon_M \\ -\sum_j \lambda_{Lj}\varepsilon_j \\ 1 - \sum_j \lambda_{Zj}\varepsilon_j \end{bmatrix} \hat{Q}. \tag{20}$$

Solving (20) for $\hat{X}_A$, $\hat{X}_M$, and $\hat{r} - \hat{w}$ yields[5]

$$\frac{d \ln X_A}{d \ln Q} = -\varepsilon_A + a_{12} - \theta_{ZA} a_{22}, \tag{21}$$

$$\frac{d \ln X_M}{d \ln Q} = -\varepsilon_M + a_{12} - \theta_{ZM} a_{22}, \tag{22}$$

$$\frac{d \ln \left(\frac{r}{w}\right)}{d \ln Q} = a_{22}, \tag{23}$$

where

$$a_{12} = \frac{\lambda_{LM}\theta_{ZM}\left(\sigma_M - 1\right) + \lambda_{LA}\theta_{ZA}\left(\sigma_A - 1\right)}{\left(\lambda_{ZM}\theta_{LM} + \lambda_{LM}\theta_{ZM}\right)\left(\sigma_M - 1\right) + \left(\lambda_{ZA}\theta_{LA} + \lambda_{LA}\theta_{ZA}\right)\left(\sigma_A - 1\right) + 1}, \tag{24}$$

$$a_{22} = \frac{-1}{\left(\lambda_{ZM}\theta_{LM} + \lambda_{LM}\theta_{ZM}\right)\left(\sigma_M - 1\right) + \left(\lambda_{ZA}\theta_{LA} + \lambda_{LA}\theta_{ZA}\right)\left(\sigma_A - 1\right) + 1}. \tag{25}$$

[4] See Chang (1981) and Jones (1965) for more details.

[5] We do not give $\hat{Y} - \hat{w}$ since it is not our interest. It is easy to obtain that $\hat{Y} - \hat{w} = a_{12}\hat{Q}$.

We can decompose (21) and (22) and see the effects of quota on the outputs more intuitively:

$$\begin{aligned}\frac{d\ln X_j}{d\ln Q} &= \frac{d\ln G^j(Q)}{d\ln Q} + \theta_{Lj}\frac{d\ln L_j}{d\ln Q} + \theta_{Zj}\frac{d\ln Z_j}{d\ln Q} \\ &= \underbrace{-\varepsilon_j}_{\text{TFP}} + \theta_{Lj}\underbrace{\left[a_{12} + (\sigma_j - 1)\,\theta_{Zj}a_{22}\right]}_{\text{Labor reallocation}} + \theta_{Zj}\underbrace{\left[a_{12} - (\theta_{Lj}\sigma_j + \theta_{Zj})\,a_{22}\right]}_{\text{Permits reallocation}}.\end{aligned}$$

An increase in the emission quota has three effects on the outputs. First, it reduces the productivity, which is negative. Second, it induces reallocation of labor, which is necessarily positive for one good and negative for the other. Third, an increase in the emission quota induces an augment in the amount of emission permits as well as the reallocation of permits among two goods, which can be positive in both goods or positive for one good and negative for the other. The aggregate effects are then ambiguous. This is also true for the the price of permits in terms of the wage, i.e., r/w.

To see the effects on r/p_j, use $p_j = G^j(Z)\,c^j(r,w) = G^j(Q)\,c^j(r,w)$ to obtain

$$\frac{d\ln\left(\frac{r}{p_j}\right)}{d\ln Q} = \frac{d\ln\left(\frac{r}{G^j(Q)c^j(r,w)}\right)}{d\ln Q} = \frac{\hat{r} - \hat{c}^j(r,w)}{\hat{\bar{Z}}} + \varepsilon_j. \tag{26}$$

On the other hand, recall that $\hat{c}^j(r,w) = \theta_{Zj}\hat{r} + \theta_{Lj}\hat{w} = \hat{r} - \theta_{Lj}(\hat{r} - \hat{w})$, which together with (26) yields

$$\frac{d\ln\left(\frac{r}{p_j}\right)}{d\ln Q} = \theta_{Lj}\frac{\hat{r} - \hat{w}}{\hat{Q}} + \varepsilon_j. \tag{27}$$

Using (23), a change in emission quota affects r/p_j as follows.

$$\frac{d\ln\left(\frac{r}{p_j}\right)}{d\ln Q} = \theta_{Lj}a_{22} + \varepsilon_j, \tag{28}$$

Clearly, the effects of quota on r/p_j are also ambiguous. It follows that

Proposition 1. *In autarky, the emission quota may have regular effects (i.e., an increase in quota decreases the price of permits) or irregular effects (an increase in quota increases the price of permits), depending on in terms of what the permits is measured and the signs of (25) and (28). Moreover, the quota is more likely to have regular effects on r/w compared to r/p_j.*

Consider the special case in which F^A and F^M are of Cobb-Douglas forms and thus $\sigma_A = \sigma_M = 1$. It follows directly from (25) and (28) that

Corollary 2. *In autarky, given Cobb-Douglas form of F^A and F^M, the emission quota has regular effects on r/w, but may have irregular effects on r/p_j (which arises if $\varepsilon_j > \theta_{Lj}$).*

These results highlight that, given a change in quota, the price of permits may change in different directions when measured in different references. This has an important implication for tax regulation that, as shown in coming analysis, the effects of tax may vary dramatically with the choice of numeraire.

The necessary condition for the optimal quota can be readily obtained using the results above. Note that $\hat{u} = b\hat{C}_A + (1-b)\,\hat{C}_M = b\hat{X}_A + (1-b)\,\hat{X}_M$ in autarky, which together with (21) and (22) gives

$$\frac{d\ln u}{d\ln Q} = -\left[b\varepsilon_A + (1-b)\,\varepsilon_M\right] + a_{12} - \left[b\theta_{ZA} + (1-b)\,\theta_{ZM}\right] a_{22}. \tag{29}$$

Therefore, the optimal quota, denoted Q^*, satisfies $Q^* \in (0, \gamma L)$ and $d\ln u/d\ln Q = 0$ in (29) (the interior solution), or $Q^* = \gamma L$ and $d\ln u/d\ln Q \geq 0$ in (29) (the corner solution).

The effects of tax in autarky Under tax regulation, the government determines the pollution tax in terms of certain other price. If the government imposes the pollution tax in terms of the wage, then r/w is exogenously given. Let τ_w denote the tax imposed in terms of the wage, then $r/w = \tau_w$ and $\hat{r} - \hat{w} = \hat{\tau}_w$. Rewrite (16) to (19) into

$$\begin{bmatrix} 1 & 0 & -1 & -\varepsilon_A \\ 0 & 1 & -1 & -\varepsilon_M \\ \lambda_{LA} & \lambda_{LM} & 0 & -\sum_j \lambda_{Lj}\varepsilon_j \\ \lambda_{ZA} & \lambda_{ZM} & 0 & 1-\sum_j \lambda_{Zj}\varepsilon_j \end{bmatrix} \begin{bmatrix} \hat{X}_A \\ \hat{X}_M \\ \hat{Y} - \hat{w} \\ \hat{Z} \end{bmatrix} = \begin{bmatrix} \theta_{ZA} \\ \theta_{ZM} \\ \delta_L \\ -\delta_Z \end{bmatrix} \hat{\tau}_w. \tag{30}$$

The results under quota regulation turn out to be very useful in deriving the effects of tax regulation.

Lemma 3. *In autarky, if the pollution tax is imposed in terms of the wage, i.e., the govern-*

ment determines $r/w = \tau_w$, a change in the pollution tax has effects:

$$\frac{d\ln X_A}{d\ln \tau_w} = \frac{d\ln X_A}{d\ln Q}\left(\frac{d\ln\left(\frac{r}{w}\right)}{d\ln Q}\right)^{-1} = \frac{-\varepsilon_A + a_{12}}{a_{22}} - \theta_{ZA}, \tag{31}$$

$$\frac{d\ln X_M}{d\ln \tau_w} = \frac{d\ln X_M}{d\ln Q}\left(\frac{d\ln\left(\frac{r}{w}\right)}{d\ln Q}\right)^{-1} = \frac{-\varepsilon_M + a_{12}}{a_{22}} - \theta_{ZM}, \tag{32}$$

$$\frac{d\ln Z}{d\ln \tau_w} = \left(\frac{d\ln\left(\frac{r}{w}\right)}{d\ln Q}\right)^{-1} = \frac{1}{a_{22}}. \tag{33}$$

Since the sign of a_{22} is ambiguous, an increase in the pollution tax imposed in terms of the wage does not necessarily reduce the total amount of pollution discharge.

If the government imposes the pollution tax in terms of good j, then r/p_j is exogenously determined. Let τ_{p_j} denote the tax imposed in terms of good j, then $r/p_j = \tau_{p_j}$ and $\hat{r} - \hat{p}_j = \hat{\tau}_{p_j}$. The following lemma follows directly from (21), (22), (23) and (28).

Lemma 4. *In autarky, if the pollution tax is imposed in terms of good j, i.e., the government determines r/p_j, a change in the pollution tax has effects:*

$$\frac{d\ln X_A}{d\ln \tau_{p_j}} = \frac{d\ln X_A}{d\ln Q}\left(\frac{d\ln\left(\frac{r}{p_j}\right)}{d\ln Q}\right)^{-1} = \frac{-\varepsilon_A + a_{12} - \theta_{ZA}a_{22}}{\theta_{Lj}a_{22} + \varepsilon_j}, \tag{34}$$

$$\frac{d\ln X_M}{d\ln \tau_{p_j}} = \frac{d\ln X_M}{d\ln Q}\left(\frac{d\ln\left(\frac{r}{p_j}\right)}{d\ln Q}\right)^{-1} = \frac{-\varepsilon_M + a_{12} - \theta_{ZM}a_{22}}{\theta_{Lj}a_{22} + \varepsilon_j}, \tag{35}$$

$$\frac{d\ln Z}{d\ln \tau_{p_j}} = \left(\frac{d\ln\left(\frac{r}{p_j}\right)}{d\ln Q}\right)^{-1} = \frac{1}{\theta_{Lj}a_{22} + \varepsilon_j}. \tag{36}$$

Since the sign of $\theta_{Lj}a_{22} + \varepsilon_j$ is ambiguous, an increase in the pollution tax imposed in terms of good j does not necessarily reduce the total pollution discharge. By (33) and (36), the proof of Proposition 1 also applies here and we can obtain the following proposition.

Proposition 5. *In autarky, the pollution tax may have regular effects (i.e., an increase in tax reduces the total pollution) or irregular effects (i.e., an increase in tax increases the total pollution), depending on in terms of what the tax is imposed and the signs of (25) and (28). The pollution tax in terms of the wage is more likely to have regular effects, compared to the pollution tax in terms of consumption goods. In other words, the pollution tax is more likely to have regular effects when choosing the wage rate as the numeraire.*

Intuitively, when imposed in terms of the wage, a change in pollution tax means a change in r/w, which takes effect through the substitution between labor input and pollution discharge. In contrast, when imposed in terms of good j, a change in pollution tax means a change in r/p_j, which takes effect through not only the substitution channel but also the negative impact of pollution on productivities.

Similarly, if F^A and F^M are of Cobb-Douglas type, it follows directly from (33) and (36) that

Corollary 6. *In autarky, given Cobb-Douglas form of F^A and F^M, the pollution tax has regular effects when imposed in terms of the wage, but may have irregular effects when imposed in terms of good j (which arises if $\varepsilon_j > \theta_{Lj}$).*

Stability under tax regulation Suppose that the economy was originally in equilibrium. Now consider a slight increase in pollution discharge due to some unexpected shocks. If the pollution tax has irregular effects, the increase in total pollution will raise the market evaluation (imputed price) of emission permits. At the same time, the explicit price of permits, i.e., the pollution tax, remains unchanged under tax regulation. So firms have the incentive to "use" more pollution (or, reduce the effort in abatement), which causes further increases in the total pollution discharge. This suggests that, if the pollution tax has irregular effects, an equilibrium under tax regulation would be unstable. Rigorously, suppose a Marshallian adjustment process in the pollution discharge:

$$\dot{Z} = \beta_\tau \left(\frac{r}{w} - \tau_w \right), \tag{37}$$

where $\beta_\tau > 0$ is the adjustment speed; τ_w is pollution tax in terms of the wage, determined by the government; r/w is the market evaluation (imputed price) of emission permits in terms of the wage, calculated by taking the total pollution discharge Z at the moment as given. If the pollution tax has irregular effects, by (33), $d\left(r/w\right)/dZ > 0$, which simply says that (37) is unstable.

If the government imposes the tax in terms of good j, we can assume the adjustment process

$$\dot{Z} = \beta_\tau \left(\frac{r}{p_j} - \tau_{p_j} \right),$$

and the similar argument applies. The discussion above can be summarized as follows.

Lemma 7. *Assume the Marshallian adjustment in pollution discharge, under tax regulation, an equilibrium is unstable if the pollution tax has irregular effects.*

4 Free Trade

In this section, I examine the effects of quota and tax regulations in free trade by focusing on a small open economy (SOE). I first consider the case in which the SOE remains diversified, and then, since the SOE may specialize, move on to the characterization of complete specialization.

4.1 Diversified Equilibrium

If the SOE remains diversified in equilibrium, both sectors are active and the minimized costs satisfy

$$\frac{G^M(Z)\,c^M(r,w)}{G^A(Z)\,c^A(r,w)} = \frac{p_M}{p_A}, \tag{38}$$

where p_M/p_A is given in SOE. The labor market clearing condition (8) and the total pollution discharge (9) still hold. Taking the logarithmic differential of (38) and using $d\,(p_M/p_A) = 0$, we have

$$(\varepsilon_M - \varepsilon_A)\,\hat{Z} + (\theta_{ZM} - \theta_{ZA})\,(\hat{r} - \hat{w}) = 0, \tag{39}$$

which together with (18) and (19) gives the system of comparative statics.

The effects of quota in SOE (Diversified equilibrium) Under quota regulation, $Z = Q$ is exogenously given. Rewrite (18), (19), and (39) into

$$\begin{bmatrix} 0 & 0 & \theta_{ZM} - \theta_{ZA} \\ \lambda_{LA} & \lambda_{LM} & \delta_L \\ \lambda_{ZA} & \lambda_{ZM} & -\delta_Z \end{bmatrix} \begin{bmatrix} \hat{X}_A \\ \hat{X}_M \\ \hat{r} - \hat{w} \end{bmatrix} = \begin{bmatrix} \varepsilon_A - \varepsilon_M \\ -\sum_j \lambda_{Lj}\varepsilon_j \\ 1 - \sum_j \lambda_{Zj}\varepsilon_j \end{bmatrix} \hat{Q} \tag{40}$$

Some algebra yields

$$\frac{d\ln X_A}{d\ln Q} = -\varepsilon_A - \frac{\lambda_{LM}}{|\lambda|} + \frac{(\varepsilon_M - \varepsilon_A)\,(\lambda_{ZM}\delta_L + \lambda_{LM}\delta_Z)}{|\lambda|\,|\theta|}, \tag{41}$$

$$\frac{d\ln X_M}{d\ln Q} = -\varepsilon_M + \frac{\lambda_{LA}}{|\lambda|} - \frac{(\varepsilon_M - \varepsilon_A)\,(\lambda_{ZA}\delta_L + \lambda_{LA}\delta_Z)}{|\lambda|\,|\theta|}, \tag{42}$$

$$\frac{d\ln\left(\frac{r}{w}\right)}{d\ln Q} = -\frac{\varepsilon_M - \varepsilon_A}{|\theta|}, \tag{43}$$

where $|\lambda| = \lambda_{ZM} - \lambda_{LM} = \lambda_{LA} - \lambda_{ZA}$, $|\theta| = \theta_{ZM} - \theta_{ZA} = \theta_{LA} - \theta_{LM}$. As for the effects on r/p_j, note that (27) remains true. Substituting (43) into (27) gives

$$\frac{d\ln\left(\frac{r}{p_j}\right)}{d\ln Q} = -\frac{\theta_{Lj}\left(\varepsilon_M - \varepsilon_A\right)}{|\theta|} + \varepsilon_j, \tag{44}$$

Focusing on the effects of quota changes on the price of permits, we have the following proposition.

Proposition 8. *In a diversified SOE, the emission quota may have regular or irregular effects, depending on in terms of what the permits are measured and the signs of (43) and (44). If the pollution-intensive good is more (less) sensitive to pollution, the emission quota has regular (irregular) effects on r/w. Again, the emission quota is more likely to have regular effects on r/w, compared to r/p_j.*

The effects of tax in SOE (Diversified equilibrium) Under tax regulation, the pollution tax (in terms of certain price) is determined by the government. If the pollution tax is imposed in terms of the wage, $r/w = \tau_w$ is exogenously given. Similar with Lemma 3, the results under quota regulation can be used and we can obtain

$$\frac{d\ln X_A}{d\ln \tau_w} = \frac{d\ln X_A}{d\ln Q}\left(\frac{d\ln\left(\frac{r}{w}\right)}{d\ln Q}\right)^{-1} = \frac{\varepsilon_A|\lambda||\theta| + \lambda_{LM}|\theta| - \left(\varepsilon_M - \varepsilon_A\right)\left(\lambda_{ZM}\delta_L + \lambda_{LM}\delta_Z\right)}{|\lambda|\left(\varepsilon_M - \varepsilon_A\right)}, \tag{45}$$

$$\frac{d\ln X_M}{d\ln \tau_w} = \frac{d\ln X_M}{d\ln Q}\left(\frac{d\ln\left(\frac{r}{w}\right)}{d\ln Q}\right)^{-1} = \frac{\varepsilon_M|\lambda||\theta| - \lambda_{LA}|\theta| + \left(\varepsilon_M - \varepsilon_A\right)\left(\lambda_{ZA}\delta_L + \lambda_{LA}\delta_Z\right)}{|\lambda|\left(\varepsilon_M - \varepsilon_A\right)}, \tag{46}$$

$$\frac{d\ln Z}{d\ln \tau_w} = \left(\frac{d\ln\left(\frac{r}{w}\right)}{d\ln Q}\right)^{-1} = -\frac{|\theta|}{\varepsilon_M - \varepsilon_A}. \tag{47}$$

If the pollution tax is imposed in terms of good j, $r/p_j = \tau_{p_j}$ is given. Similar with Lemma 4, we have

$$\frac{d\ln X_A}{d\ln \tau_{p_j}} = \frac{d\ln X_A}{d\ln Q}\left(\frac{d\ln\left(\frac{r}{p_j}\right)}{d\ln Q}\right)^{-1} = \frac{\varepsilon_A |\lambda| |\theta| + \lambda_{LM} |\theta| - (\varepsilon_M - \varepsilon_A)(\lambda_{ZM}\delta_L + \lambda_{LM}\delta_Z)}{|\lambda| (\theta_{Lj}\varepsilon_M - \theta_{Lj}\varepsilon_A - |\theta| \varepsilon_j)}, \tag{48}$$

$$\frac{d\ln X_M}{d\ln \tau_{p_j}} = \frac{d\ln X_M}{d\ln Q}\left(\frac{d\ln\left(\frac{r}{p_j}\right)}{d\ln Q}\right)^{-1} = \frac{\varepsilon_M |\lambda| |\theta| - \lambda_{LA} |\theta| + (\varepsilon_M - \varepsilon_A)(\lambda_{ZA}\delta_L + \lambda_{LA}\delta_Z)}{|\lambda| (\theta_{Lj}\varepsilon_M - \theta_{Lj}\varepsilon_A - |\theta| \varepsilon_j)}, \tag{49}$$

$$\frac{d\ln Z}{d\ln \tau_{p_j}} = \left(\frac{d\ln\left(\frac{r}{p_j}\right)}{d\ln Q}\right)^{-1} = -\frac{|\theta|}{\theta_{Lj}\varepsilon_M - \theta_{Lj}\varepsilon_A - |\theta| \varepsilon_j}. \tag{50}$$

The follow proposition summarizes these results by focusing on the effects of tax on total pollution discharge.

Proposition 9. *In a diversified SOE, the pollution tax may have regular or irregular effects, depending on in terms of what the tax is imposed and the signs of (43) and (44). If the pollution-intensive good is more (less) sensitive to pollution, the pollution tax in terms of the wage has regular (irregular) effects. Moreover, the pollution tax in terms of the wage is more likely to have regular effects, compared to the pollution tax in terms of consumption goods.*

By Lemma 7, the equilibrium is unstable if the pollution tax has irregular effects, implying that tax cannot achieve some environmental goals that quota can. In this sense, tax is not equivalent to quota. It follows directly from Proposition 9 that

Corollary 10. *If the pollution-intensive good is less sensitive to the pollution in an SOE, the diversified equilibrium under tax regulation is unstable.*

4.2 Specialized Equilibrium

So far we consider the case of diversified equilibrium in an SOE. However, it is possible for the economy to specialize completely in free trade. If the economy specializes in manufacturing, we have

$$X_M G^M(Z)\, a_M(r, w) = L, \tag{51}$$

$$X_M G^M(Z)\, e_M(r, w) = Z. \tag{52}$$

Take the logarithmic differential and obtain

$$\hat{X}_M + \theta_{ZM}\sigma_M(\hat{r} - \hat{w}) + \varepsilon_M \hat{Z} = 0, \tag{53}$$

$$\hat{X}_M - \theta_{LM}\sigma_M(\hat{r} - \hat{w}) + (\varepsilon_M - 1)\hat{Z} = 0. \tag{54}$$

In contrast, if the economy specializes in agriculture, we have

$$X_A G^A(Z)\, a_A(r, w) = L, \tag{55}$$

$$X_A G^A(Z)\, e_A(r, w) = Z. \tag{56}$$

Take the logarithmic differential and obtain

$$\hat{X}_A + \theta_{ZA}\sigma_A(\hat{r} - \hat{w}) + \varepsilon_A \hat{Z} = 0, \tag{57}$$

$$\hat{X}_A - \theta_{LA}\sigma_A(\hat{r} - \hat{w}) + (\varepsilon_A - 1)\hat{Z} = 0. \tag{58}$$

The effects of quota in SOE (Specialized equilibrium) Under quota regulation, $Z = Q$ is given. The effects of quota depend on in which sector the economy specializes. If the economy specializes in manufacturing, (53) and (54) imply that

$$\frac{d\ln X_M}{d\ln Q} = (\theta_{ZM} - \varepsilon_M), \tag{59}$$

$$\frac{d\ln\left(\frac{r}{w}\right)}{d\ln Q} = -\frac{1}{\sigma_M}. \tag{60}$$

As for the effects on r/p_j, it follows from (27) that

$$\frac{d\ln\left(\frac{r}{p_j}\right)}{d\ln Q} = -\frac{\theta_{Lj}}{\sigma_M} + \varepsilon_j. \tag{61}$$

If the economy specializes in agriculture, (57) and (58) imply that

$$\frac{d\ln X_A}{d\ln Q} = (\theta_{ZA} - \varepsilon_A), \tag{62}$$

$$\frac{d\ln\left(\frac{r}{w}\right)}{d\ln Q} = -\frac{1}{\sigma_A}. \tag{63}$$

Again, it follows from (27) that

$$\frac{d\ln\left(\frac{r}{p_j}\right)}{d\ln Q} = -\frac{\theta_{Lj}}{\sigma_A} + \varepsilon_j. \tag{64}$$

Focusing on the effect on the price of permits, we have the following proposition.

Proposition 11. *If the SOE specializes completely, the quota has regular effects on r/w, but may have irregular effects on r/p_j (which arises if $\sigma_M \varepsilon_j - \theta_{Lj} > 0$ when specializing in manufacturing or $\sigma_A \varepsilon_j - \theta_{Lj} > 0$ when specializing in agriculture).*

The effects of tax in SOE (Specialized equilibrium) Under tax regulation, the pollution tax is determined by the government. If the pollution tax is imposed in terms of the wage, $r/w = \tau_w$ is exogenously given. If the economy specializes in manufacturing, from (53) and (54) we have

$$\frac{d\ln X_M}{d\ln \tau_w} = \frac{d\ln X_M}{d\ln Q}\left(\frac{d\ln\left(\frac{r}{w}\right)}{d\ln Q}\right)^{-1} = -\sigma_M\left(\theta_{ZM} - \varepsilon_M\right), \tag{65}$$

$$\frac{d\ln Z}{d\ln \tau_w} = \left(\frac{d\ln\left(\frac{r}{w}\right)}{d\ln Q}\right)^{-1} = -\sigma_M. \tag{66}$$

If the pollution tax is imposed in terms of good j, $r/p_j = \tau_{p_j}$ is given. We have

$$\frac{d\ln X_M}{d\ln \tau_{p_j}} = \frac{d\ln X_M}{d\ln Q}\left(\frac{d\ln\left(\frac{r}{p_j}\right)}{d\ln Q}\right)^{-1} = \frac{\sigma_M\left(\theta_{ZM} - \varepsilon_M\right)}{-\theta_{Lj} + \sigma_M\varepsilon_j} \tag{67}$$

$$\frac{d\ln Z}{d\ln \tau_{p_j}} = \left(\frac{d\ln\left(\frac{r}{p_j}\right)}{d\ln Q}\right)^{-1} = \frac{\sigma_M}{-\theta_{Lj} + \sigma_M\varepsilon_j}. \tag{68}$$

If the economy specializes in agriculture, we have

$$\frac{d\ln X_A}{d\ln \tau_w} = \frac{d\ln X_A}{d\ln Q}\left(\frac{d\ln\left(\frac{r}{w}\right)}{d\ln Q}\right)^{-1} = -\sigma_A\left(\theta_{ZA} - \varepsilon_A\right),$$

$$\frac{d\ln Z}{d\ln \tau_w} = \left(\frac{d\ln\left(\frac{r}{w}\right)}{d\ln Q}\right)^{-1} = -\sigma_A.$$

Similarly, if the pollution tax is imposed in terms of good j,

$$\frac{d\ln X_A}{d\ln \tau_{p_j}} = \frac{d\ln X_A}{d\ln Q}\left(\frac{d\ln\left(\frac{r}{p_j}\right)}{d\ln Q}\right)^{-1} = \frac{\sigma_A\left(\theta_{ZA}-\varepsilon_A\right)}{-\theta_{Lj}+\sigma_A\varepsilon_j}$$

$$\frac{d\ln Z}{d\ln \tau_{p_j}} = \left(\frac{d\ln\left(\frac{r}{p_j}\right)}{d\ln Q}\right)^{-1} = \frac{\sigma_A}{-\theta_{Lj}+\sigma_A\varepsilon_j}.$$

The effects of tax on the total pollution discharge can be summarized as follows.

Proposition 12. *If the SOE specializes completely, the pollution tax in terms of wage has regular effects, but the pollution tax in terms of good j may have irregular effects (which arises if $\sigma_M\varepsilon_j-\theta_{Lj}>0$ when specializing in manufacturing or $\sigma_A\varepsilon_j-\theta_{Lj}>0$ when specializing in agriculture). With irregular effects, the equilibrium is unstable.*

4.3 The Whole Picture

The specialization pattern is not independent of the environmental regulations. An SOE may go through both diversified and specialized equilibria as the stringency of environmental regulation changes. To build up the whole picture for free trade, we need to know under what condition the economy specializes. For the purpose, it is convenient to focus on quota regulation first. Define

$$\kappa_j\left(\frac{r}{w}\right) \equiv \frac{e_j\left(r,w\right)}{a_j\left(r,w\right)} = \frac{e_j\left(\frac{r}{w},1\right)}{a_j\left(\frac{r}{w},1\right)}, \quad \kappa \equiv \frac{Q}{L},$$

where Q is the quota determined by the government.

Suppose without loss of generality that the production of manufacturing good is pollution-intensive, which implies that $\kappa_M\left(r/w\right) > \kappa_A\left(r/w\right)$. The sufficient and necessary condition for both goods to be produced is then $\kappa_M\left(r/w\right) > \kappa > \kappa_A\left(r/w\right)$. Otherwise, if $\kappa = \kappa_M\left(r/w\right)$, the economy specializes in manufacturing (i.e. $X_A = 0$); if $\kappa = \kappa_A\left(r/w\right)$, the economy specializes in labor-intensive agriculture (i.e. $X_M = 0$).

Suppose that the SOE was originally diversified. Given the world relative price p_M/p_A and the quota Q, we can derive r/w from (38). It follows from (43) that

$$\frac{d\ln\kappa_j\left(\frac{r}{w}\right)}{d\ln Q} = \frac{d\ln\kappa_j\left(\frac{r}{w}\right)}{d\ln\left(\frac{r}{w}\right)}\frac{d\ln\left(\frac{r}{w}\right)}{d\ln Q} = \frac{\sigma_j\left(\varepsilon_M-\varepsilon_A\right)}{|\theta|}, \tag{69}$$

where $d\ln\kappa_j\left(\frac{r}{w}\right)/d\ln\left(\frac{r}{w}\right) = -\sigma_j$ is used. By (69), if manufacturing good is less sensitive

$(\varepsilon_M - \varepsilon_A < 0)$, $\kappa_j(r/w)$ decreases with quota Q . On the other hand, $\kappa = Q/L$ increases with Q. Therefore, if we keep raising Q, eventually $\kappa_M(r/w) = \kappa$; and if we keep reducing Q, eventually $\kappa_A(r/w) = \kappa$. That is, there exists $Q_l < Q_h$ such that the economy (i) remains diversified if $Q \in (Q_l, Q_h)$; (ii) completely specializes in the labor-intensive good if $Q \leq Q_l$; (iii) completely specializes in the pollution-intensive good if $Q \geq Q_h$.[6]

In contrast, if manufacturing good is more sensitive $(\varepsilon_M - \varepsilon_A > 0)$, both $\kappa_j(r/w)$ and κ increase with Q. In this case, specialization does not necessarily arise or there may be multiple intervals of quota where the SOE remains diversified.

The similar argument applies to the case in which agriculture is pollution-intensive. To summarize, we have the following lemma.

Lemma 13. *Under quota regulation, if the pollution-intensive good is less sensitive to the pollution, as the quota increases, the SOE first specializes in the labor-intensive good, then becomes diversified, and finally specializes in the pollution-intensive good.*

By Corollary 10, if the pollution-intensive good is less sensitive to the pollution, a diversified equilibrium under tax regulation is unstable. By Lemma 13, this implies that tax regulation cannot achieve the environmental goals between the range (Q_l, Q_h), which quota regulation can. In this sense, tax is not equivalent to quota. In contrast, if the pollution-intensive good is more sensitive, quota and tax (in terms of the wage) are negatively related, and the equilibrium is stable. In this sense, the two regulations are equivalent.

Figure 1 draws the nonequivalence case in which manufacturing is pollution-intensive but less sensitive to pollution. The figure shows how the specialization pattern and the price of permits (in terms of the wage) change with the quota. As illustrated in the figure, a pollution tax $\tau_w \in [(r/w)_l, (r/w)_h]$ corresponds with three levels of total pollution discharge: Z_1, Z_2, and Z_3. Among those, Z_1 and Z_3 correspond with specialization equilibria in agriculture and manufacturing, respectively, whereas Z_2 corresponds with a diversified equilibrium. According to Lemma 7 and Proposition 9, the diversified equilibrium is unstable. Therefore, any small shock will lead to complete specialization equilibrium (τ_w, Z_1) or (τ_w, Z_3).

5 Conclusion

Previous literature told us that tax and quota may not be equivalent if there is uncertainty or incomplete information. In this paper, I show that the nonequivalence between tax and quota can arise in a deterministic model with externalities on the production side. The

[6]For simplicity, we have implicitly assumed that $Q_h < \gamma L$. That is, the quota takes effect when specializing in agriculture or remaining diversified, and takes effect in part of range when specializing in manufacturing.

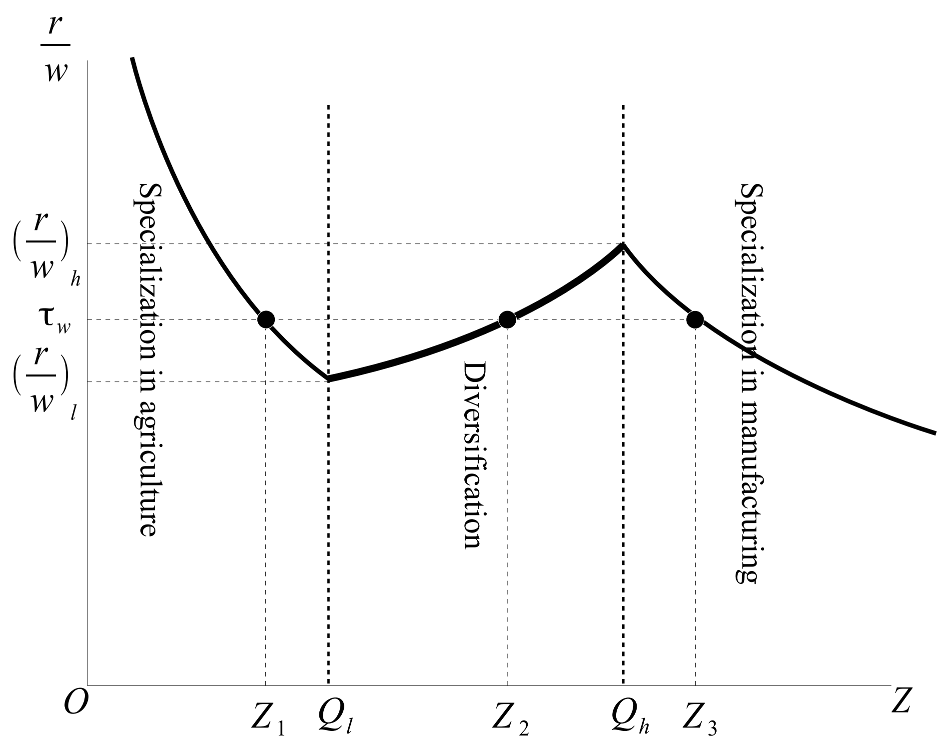

Figure 1: Nonequivalence between quota and tax ($\theta_{ZM} > \theta_{ZA}$ and $\varepsilon_M < \varepsilon_A$)

analysis of the model suggests that, without any cost of implementation, quota is superior to tax since tax may not be able to achieve the environmental goals that quota can. This holds in autarky and in free trade, although the conditions for such nonequivalence are different.

In this paper, I consider a small open economy and a local type of pollution. We can extend the model to a large country or a two-country model. We can also consider a trans-boundary pollution. Both extensions are especially interesting if one wants to examine global environmental issues such as greenhouse gases.

A Appendix A

A.1 Proof of Proposition 1

By (28), the sign of $d(r/w)/dQ$ depends on the sign of a_{22}, which can be positive or negative. Similarly, the sign of $d(r/p_j)/dQ$ depends on that of (25), which is also ambiguous. Note that $d(r/w)/dQ > 0$, i.e., $a_{22} > 0$, which implies $d(r/p_j)/dQ > 0$. On the other hand, $d(r/p_j)/dQ > 0$ does not necessarily mean $d(r/w)/dQ > 0$. The space satisfying $d(r/w)/dQ > 0$ is smaller than that satisfying $d(r/p_j)/dQ > 0$. In such sense, we say that the latter is more likely to arise.

A.2 Proof of Lemma 3

It follows from (30) that, using the Cramer's rule,

$$\frac{d\ln Z}{d\ln \tau_w} = \frac{\begin{vmatrix} 1 & 0 & -1 & -\theta_{ZA} \\ 0 & 1 & -1 & -\theta_{ZM} \\ \lambda_{LA} & \lambda_{LM} & 0 & -\delta_L \\ \lambda_{ZA} & \lambda_{ZM} & 0 & \delta_Z \end{vmatrix}}{\begin{vmatrix} 1 & 0 & -1 & \varepsilon_A \\ 0 & 1 & -1 & \varepsilon_M \\ \lambda_{LA} & \lambda_{LM} & 0 & \sum_j \lambda_{Lj}\varepsilon_j \\ \lambda_{ZA} & \lambda_{ZM} & 0 & \sum_j \lambda_{Zj}\varepsilon_j - 1 \end{vmatrix}} = \left(\frac{d\ln\left(\frac{r}{w}\right)}{d\ln Q}\right)^{-1},$$

which gives (33). Moreover,

$$\begin{aligned}\frac{d\ln X_A}{d\ln \tau_w} &= \frac{\begin{vmatrix} -\theta_{ZA} & 0 & -1 & \varepsilon_A \\ -\theta_{ZM} & 1 & -1 & \varepsilon_M \\ -\delta_L & \lambda_{LM} & 0 & \sum_j \lambda_{Lj}\varepsilon_j \\ \delta_Z & \lambda_{ZM} & 0 & \sum_j \lambda_{Zj}\varepsilon_j - 1 \end{vmatrix}}{\begin{vmatrix} 1 & 0 & -1 & \varepsilon_A \\ 0 & 1 & -1 & \varepsilon_M \\ \lambda_{LA} & \lambda_{LM} & 0 & \sum_j \lambda_{Lj}\varepsilon_j \\ \lambda_{ZA} & \lambda_{ZM} & 0 & \sum_j \lambda_{Zj}\varepsilon_j - 1 \end{vmatrix}} \\ &= -\frac{\begin{vmatrix} -\varepsilon_A & 0 & -1 & \theta_{ZA} \\ -\varepsilon_M & 1 & -1 & \theta_{ZM} \\ -\sum_j \lambda_{Lj}\varepsilon_j & \lambda_{LM} & 0 & \delta_L \\ 1-\sum_j \lambda_{Zj}\varepsilon_j & \lambda_{ZM} & 0 & -\delta_Z \end{vmatrix}}{\begin{vmatrix} 1 & 0 & -1 & \theta_{ZA} \\ 0 & 1 & -1 & \theta_{ZM} \\ \lambda_{LA} & \lambda_{LM} & 0 & \delta_L \\ \lambda_{ZA} & \lambda_{ZM} & 0 & -\delta_Z \end{vmatrix}} \frac{\begin{vmatrix} 1 & 0 & -1 & \theta_{ZA} \\ 0 & 1 & -1 & \theta_{ZM} \\ \lambda_{LA} & \lambda_{LM} & 0 & \delta_L \\ \lambda_{ZA} & \lambda_{ZM} & 0 & -\delta_Z \end{vmatrix}}{\begin{vmatrix} 1 & 0 & -1 & \varepsilon_A \\ 0 & 1 & -1 & \varepsilon_M \\ \lambda_{LA} & \lambda_{LM} & 0 & \sum_j \lambda_{Lj}\varepsilon_j \\ \lambda_{ZA} & \lambda_{ZM} & 0 & \sum_j \lambda_{Zj}\varepsilon_j - 1 \end{vmatrix}} \\ &= \frac{d\ln X_A}{d\ln Q}\left(\frac{d\ln\left(\frac{r}{w}\right)}{d\ln Q}\right)^{-1},\end{aligned}$$

which gives (31). Similarly, we can obtain

$$\frac{d\ln X_M}{d\ln \tau_w} = \frac{d\ln X_M}{d\ln Q}\left(\frac{d\ln\left(\frac{r}{w}\right)}{d\ln Q}\right)^{-1},$$

which gives (32).

A.3 Proof of Proposition 8

By (43), the sign of $d\left(r/w\right)/dQ$ is ambiguous. This is also true for $d\left(r/p_j\right)/dQ$, the sign of which depends on that of (44). Suppose that good j is pollution-intensive compared to good j', i.e., $\theta_{Zj}-\theta_{Zj'} > 0$. By (43), $d\left(r/w\right)/dQ < 0$ ($d\left(r/w\right)/dQ > 0$) if $\varepsilon_j - \varepsilon_{j'} > 0$ ($\varepsilon_j - \varepsilon_{j'} < 0$), i.e., good j is more (less) sensitive than good j'. Again, by (43), $d\left(r/p_j\right)/dQ < 0$ implies $d\left(r/w\right)/dQ < 0$, but the opposite does not necessarily hold.

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