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Abstract

This paper examines the role of endogenous capital accumulation in the interaction between trade and the environment in a two-country, two-sector model. Atomic households follow a simple rule of saving: higher the real interest rate, higher the saving rate is. In autarky, the real interest rate depends only on the quality of the environment, and investment provides a channel allowing for the trade-off between the amount of factor of production and the quality of the environment. In free trade, the real interest rate depends on i) terms of trade if the environmentally sensitive sector (agriculture) is shut down, or ii) both terms of trade and the quality of the environment if agriculture is still active. In either case, the real interest rate jumps right after trade liberalization due to terms of trade improvement, which boosts investment and causes capital accumulation. This scale effect dominates in the long run, causing environmental degradation even if the economy specializes in the relatively clean sector. Trade improves the total world consumption and the country preserving agriculture has relatively better environment in the long run.

Keywords: Environment; intertemporal optimization; investment; environmentally sensitive

JEL classification: F18; Q20

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1 Introduction

The relationship between trade and the environment is a central topic in international economics and environmental economics. Does trade liberalization harm the environment? Empirical analyses provide no definitive answer. For instance, Lucas et al. (1992) suggest that free trade is bad for the environment, while Antweiler et al. (2001) and Frankel and Rose (2005) find no significant link between trade liberalization and environmental degradation. This is perhaps not surprising, because there are so many factors, including industrial structure, factor endowment, abatement technology and policy stringency, that can affect the environment, and trade is one source, but not the only, that can induce changes in these factors. There is little reason to expect a simple relationship between trade and the environment.

Theoretical analyses that attempt to understand the link between trade and the environment are conducted in a wide range of contexts. Grossman and Krueger (1993) suggests three channels—*scale*, *composition* and *technique* effects—by which trade can affect the environment. This criterion helps to characterize these analyses according to what channels (among the three) are formulated and how (interacting with trade or not).[1] Much of earlier work consider only composition effect since it is the direct consequence of trade. For instance, Markusen (1975) focuses on policy instruments that can alter composition effect and discusses how the optimal policy responds to trade. Later work incorporates also technique effect or both scale and technique effects. For instance, Copeland and Taylor (1994) and Ishikawa and Kiyono (2006) introduces the interaction between technique effect and trade (through endogenously determined policies) by allowing for choice of abatement efforts.

However, as Copeland and Taylor (2004) pointed out in their insightful review, most of the literature examines scale effect (if any) separately by conducting comparative statics. This absence of endogenous economic development stands out as a notable limitation in this literature. As Ayres and Kneese (1969) emphasized a half century ago, the significance of waste "tends to increase as economic development proceeds, and the ability of the ambient environment to receive and assimilate them is an important natural resource of increasing value." This limitation remains still in the current literature on trade and the environment, and there is little work available exploring the interaction between scale effect and trade through endogenous channels.

This paper develops a growth model to consider trade and the environment while formulating endogenous capital accumulation. This simple model for considering these issues

[1] Of course there are many other criteria, such as who takes the damage of environmental degradation (consumer or producer), the type of the model (static or dynamic), which aspect of the environment to focus (resource or pollution), and the characteristics of pollution (transboundary or local).

is a two-country model in which each country is similar to the small country considered in Li (2015). The model is based on Copeland and Taylor (1999) and extends theirs in two directions. First, households make intertemporally optimized investment decisions by deciding how much to invest at every point in time to maximize the discounted lifetime utility. Second, instead of being purely clean in Copeland and Taylor (1999), the environmentally sensitive sector—agriculture—can be polluting with the intensity measured by a parameter. This is motivated by the evidence that industries like agriculture are also pollution intensive.[2] Our analysis focuses on three fundamental effects of trade: specialization patterns, welfare effects, and environmental impacts.

The analysis of the model yields several interesting results. First, without investment, the qualities of the environment in closed countries depend on their own private capital and environmental endowments. With investment, however, this paper predicts that the qualities of the environment tend to converge (across closed countries). Although the convergence requires quite strict restrictions (identical technology, time preference rate, and depreciation rate across countries), it provides an insight about how investment and the environment interact with each other. On one hand, the interest rate in a closed country is positively related to its environmental quality. In the country with a better environment, households face higher interest rate and thus invest more. On the other hand, capital accumulation through investment raises pollution (ceteris paribus) and consequently harms the environment. This lowers the interest rate and discourages investment.

Second, moving from autarky to trade, without investment, whether free trade brings about dramatic changes in industrial structure depends on whether agriculture is clean.[3] In contrast, this papers shows that with investment, free trade gives rise to a strong tendency to specialization regardless of the type of agriculture. Third, as shown in Copeland and Taylor (1999), free trade enhances (harms) the environment in the country that specializes in the clean (dirty) sector, while in this paper, free trade increases the interest rate and encourages further investment. This scale effect dominates in the long run and harms the environment, even if the country specializes in the clean sector.

Moreover, in this two-country model, there may exist a set of steady-state equilibria. Among those, the complete specialization steady-state equilibrium yields the highest world total consumption. It is also shown that the country that preserves agriculture has its

[2] IPCC (2007) estimates that in 2004, agriculture contributes 14% of total GHG emissions, following 19% from industrial processes.

[3] This is because the type of agriculture determines the curvature of the steady-state production possibility frontier (PPF). If agriculture is clean, as shown in Copeland and Taylor (1999), the steady-state PPF is convex to the origin, implying a strong tendency to specialization under free trade. If agriculture is dirty, as shown later in this paper, the PPF is concave and thus the industrial structure will not change much if the world price is close to the autarky price.

environment less harmed. This is consistent with a recent view towards protectionism in terms of environmental roles of agriculture.

The introduction of dynamic optimization complicates the model, and the characterization of specialization patterns is challenging since specialization patterns depend on environmental capital stock, which is an endogenous variable related to another endogenous variable, private capital stock, and the world relative price of two intermediate goods is an endogenous variable in the two-country model. To cope with these complications, we list all possible specialization patterns, derive the constraints for each pattern, and then, using these constraints, divide the private-environmental capital space into regimes, each corresponding with a specialization pattern. In this way, the dynamics of private capital stock and environmental capital stock within a regime can be described by the same set of dynamic equations. This makes it possible to analyze the properties of steady states regime by regime. Finally, we combine the results from all regimes to construct a whole picture of the two-county world.

The structure of the paper is as follows. Section 2 describes the basic model. Section 3 considers the autarky case. Section 4 investigates the effects of free trade between two countries. Section 5 presents conclusions.

2 The model

The model setup closely follows those in Copeland and Taylor (1999), except that we introduce investment behavior and allow agriculture to be polluting. There are two factors of production (private capital and environmental capital), two tradable intermediate goods (manufacturing and agriculture), and a single nontradable final good.[4] Manufacturing needs only private capital, while agriculture needs both factors. The final good is produced from two intermediate goods, and can be either consumed or invested.[5] The households choose between consumption and investment to maximize their discounted lifetime utility.

Factors of production Private capital (K) is inelastically supplied, domestically freely, and instantaneously mobile. Thus, private capital must be fully employed and its rentals are equalized across active intermediate industries. The stock of private capital changes according to

$$\dot{K} = I - \delta K, \tag{1}$$

[4] Here the factor of production is defined in a broader sense as all factors that are crucial to the production. This definition, which can be found in environmental and resource economics, emphasizes the importance of environmental resources.

[5] That is, investment good and consumption good are produced with the same technology.

where I is the investment, and δ the depreciation rate.

The services of environmental capital (V) are freely used.[6] The stock of environmental capital is given at every point in time, and may evolve over time depending on the flow of pollution (Z), the current level of environmental capital (V), and the "natural" level of environmental capital $(\bar{V})$: $\dot{V} = E\left(Z, V, \bar{V}\right)$. Following Copeland and Taylor (1999), assume that

$$\dot{V} = g\left(\bar{V} - V\right) - Z, \tag{2}$$

where g is the recovery rate of the environment. In steady state $(\dot{V} = 0)$, V and Z satisfy $V = \bar{V} - Z/g$.

Intermediate good firms Manufacturing good (M) and agriculture good (A) are both tradable and under perfect competition. The representative firm in manufacturing employs only private capital and emits pollutants as a joint product. The units are normalized so that one unit of private capital produces one unit of manufacturing good and generates $\lambda_m > 0$ units of pollution:

$$M = K_m, \tag{3}$$

$$Z_m = \lambda_m K_m, \tag{4}$$

where M is the output of manufacturing good, and Z_m the flow of pollution from manufacturing. The representative firm in agriculture uses both private capital and environmental capital:

$$A = G\left(V\right) K_a, 0 < \varepsilon \leq 1, \tag{5}$$

where $G\left(V\right)$ is the flow of services from an environmental capital stock of V. Therefore, agriculture is environmentally sensitive in the sense that its productivity is affected by the stock of environmental capital. Following Copeland and Taylor (1999), let $G\left(V\right)$ take the simple form

$$G\left(V\right) = V^{\varepsilon}, 0 < \varepsilon \leq 1,$$

where ε can be thought of as the sensitivity of agriculture to the quality of environment. In contrast to Copeland and Taylor (1999), in which agriculture emits no pollutant, this model allows for agriculture that generates pollutants at the intensity of $\lambda_a > 0$:

$$Z_a = \lambda_a K_a. \tag{6}$$

[6] The implicit assumption is that there is no regulation, which may be because the management of the environment is too difficult or costly.

We say that agriculture is clean if $\lambda_a < \lambda_m$, and dirty if $\lambda_a > \lambda_m$.

MRT, short-run PPF, and steady-state PPF Manufacturing firms maximize profits under perfect competition. If manufacturing is operating, the interest rate (rental of private capital, r) and the price of manufacturing good (p_m) must satisfy

$$p_m = r. \tag{7}$$

Agriculture firms maximizes profits, treating the environmental capital stock as given. Therefore, if agriculture is operating, perfect competition requires

$$p_a V^\varepsilon = r. \tag{8}$$

Let MRT denote the (private) marginal rate of transformation of agriculture good for manufacturing good, then[7]

$$\text{MRT} = V^\varepsilon. \tag{9}$$

Clearly, a necessary condition for both intermediate industries to be active is

$$P = \text{MRT} = V^\varepsilon, \tag{10}$$

where P denotes the relative price of intermediate goods, i.e., $P \equiv p_m/p_a$.

Full employment of private capital requires, using (3) and (5), that

$$K = K_m + K_a = M + \frac{A}{V^\varepsilon}. \tag{11}$$

In the short run, the environmental capital stock does not change, nor does the productivity of agriculture. Hence, (11) is the expression for the short-run production possibility frontier (PPF), where M and A are linearly correlated, reflecting the short-run Ricardian structure of the model.

In the long run, however, production schedules along (11) are not necessarily sustainable, since the environmental capital stock may change over time. To obtain the (long-run) steady-state PPF, it is useful to express the flow of pollution in terms of the output of manufacturing good. For this purpose, substitute (3) into (4) for K_m and (5) into (6) for K_a, and obtain

[7] Each firm takes no account of the influence of its own behavior on others. This is one of the two sources in this model from which externalities arise. The term "private" is used to distinguish the MRT realized by private firms from the social MRT.

$Z = \lambda_m M + \lambda_a A/V^{\varepsilon}$. Substituting (11) for A/V^{ε} yields

$$Z = \lambda_a K + (\lambda_m - \lambda_a) M. \tag{12}$$

Using (12) and the relation in steady state $V = \bar{V} - Z/g$, we have

$$V = \bar{V} - \frac{1}{g} (\lambda_a K + (\lambda_m - \lambda_a) M). \tag{13}$$

To ensure specialization in either M or A will not destruct the environment, we need $\bar{V} > \max\{\lambda_m K/g, \lambda_a K/g\}$. Using (13) to eliminate V in (11), we can obtain the expression for the steady-state PPF

$$A = (K - M) \left[\bar{V} - \frac{1}{g} (\lambda_a K + (\lambda_m - \lambda_a) M) \right]^{\varepsilon}. \tag{14}$$

It can be shown that

Lemma 1. *Given that the stock of private capital K is fixed, then the steady-state PPF (14) is concave/convex if agriculture is dirty ($\lambda_a > \lambda_m$)/clean ($\lambda_a < \lambda_m$).*

Proof. See Appendix A.1. □

The curvature of the steady-state PPF is crucial in determining whether the openness of trade brings about dramatic changes in domestic industrial structure. If the steady-state PPF is convex as in Copeland and Taylor (1999), the country tends to completely specialize in one sector, even if the world price is very close to the autarky price.

Final good firms The final good (Q) is nontradable and is produced from two intermediate goods under constant returns to scale and perfect competition. For simplicity, assume the Cobb-Douglas technology:

$$Q = D_m^{b_m} D_a^{b_a}, \tag{15}$$

where $b_m + b_a = 1$, and D_m and D_a are the inputs of manufacturing good and agriculture good, respectively.

Final good firms maximize profits, treating the prices of intermediate goods as given. The zero profit condition gives the price (also the cost) of final good (p) in terms of the prices of two intermediate goods:

$$p = \frac{p_m^{b_m} p_a^{b_a}}{b}, \quad b \equiv b_m^{b_m} b_a^{b_a}. \tag{16}$$

Also, given p_m and p_a, D_m and D_a satisfy

$$\frac{b_m}{b_a} = \frac{p_m D_m}{p_a D_a}. \tag{17}$$

Households Households own private capital and receive interest revenue (rK). They choose between consumption (C) and investment (I) to maximize the discounted lifetime utility subject to the budget constraint $rK = pQ$. Formally, the representative household faces the following problem, using $Q = C + I = C + \dot{K} + \delta K$,

$$\max \int_0^\infty \ln C e^{-\rho t} dt, \text{ s.t. } rK = p\left(C + \dot{K} + \delta K\right). \tag{18}$$

Atomistic households treat the real interest r/p as given.[8] The first order condition gives the Euler equation $\dot{C}/C = r/p - \delta - \rho$, $\forall t\,[0,\infty)$, where r/p can be seen as the real interest rate. The transversality condition requires $\lim_{s\to\infty} \int_0^s \gamma K e^{-\rho t} dt = 0$, with γ the Lagrange multiplier in the Hamiltonian $H = \ln C + \gamma\left(rK/p - C - \delta K\right)$.

The logarithmic instantaneous utility yields a simple form of the consumption function on the optimal saddle path:

$$C = \rho K, \forall t\,[0,\infty). \tag{19}$$

Therefore, as long as households are optimally saving and investing, we can use the level of K to measure the instantaneous welfare level. Substituting (19) into the Euler equation yields

$$\frac{\dot{K}}{K} = \frac{\dot{C}}{C} = \frac{r}{p} - \delta - \rho, \forall t\,[0,\infty). \tag{20}$$

Thus, we have K and V ruled by (20) and (2) together. Note that ρ, δ, g, and $\bar{V}$ are exogenous parameters, while r/p and Z are endogenous variables. The two equations do not close the dynamics until r/p and Z are expressed in terms of K and V. This is one of our main tasks in the following sections.

[8] That is, for households, the marginal revenue of investment is r/p. This is another (possible) source of externalities in this model. To see this, note that as long as agriculture good is produced, r/p depends on environmental capital stock, which is, in the long run, determined by private capital stock and industrial structure. Therefore, instead of r/p, the marginal revenue of investment should be

$$\frac{d}{dK}\left(\frac{r}{p}K\right) = \frac{r}{p} + K\frac{d}{K}\left(\frac{r}{p}\right).$$

3 Autarky

This section analyzes the equilibrium, especially the steady-state equilibrium, in the case of autarky. In autarky equilibrium, the demand for intermediate goods is fulfilled by domestic firms. The market clearing condition for intermediate goods is

$$D_m = M, \;\; D_a = A.$$

It follows from (15) that both intermediate industries must be active and thus (10) holds. This implies that, using (3), (5), (17), the full employment condition $K = K_m + K_a$, and again the market clearing condition, that

$$K_m = b_m K, \;\; K_a = b_a K. \tag{21}$$

Therefore, in autarky equilibrium, the outputs of goods are

$$M = b_m K, \;\; A = b_a V^{\varepsilon} K, \;\; Q = b V^{\varepsilon b_a} K,$$

the flow of pollution is

$$Z = Z_m + Z_a = \lambda K, \;\; \lambda \equiv b_m \lambda_m + b_a \lambda_a, \tag{22}$$

and the real interest rate satisfies, using (7), (8) and (16),

$$\frac{r}{p} = b V^{\varepsilon b_a}. \tag{23}$$

The dynamic equations (20) and (2) become, using (23) and (22),

$$\frac{\dot{K}}{K} = b V^{\varepsilon b_a} - \delta - \rho, \tag{24}$$

$$\dot{V} = g\left(\bar{V} - V\right) - \lambda K, \tag{25}$$

where b and λ are defined in (16) and (22) respectively. The dynamics of K and V are now completely described by (24) and (25).

Figure 1 depicts the phase diagram, where the negatively sloped $\dot{V} = 0$ line represents the combination of (K, V) so that the environmental capital stock V will not change, while along the horizontal $\dot{K} = 0$ line, the private capital stock K will not change. The unique intersection point (K_0, V_0) is then the steady state. It can be shown that

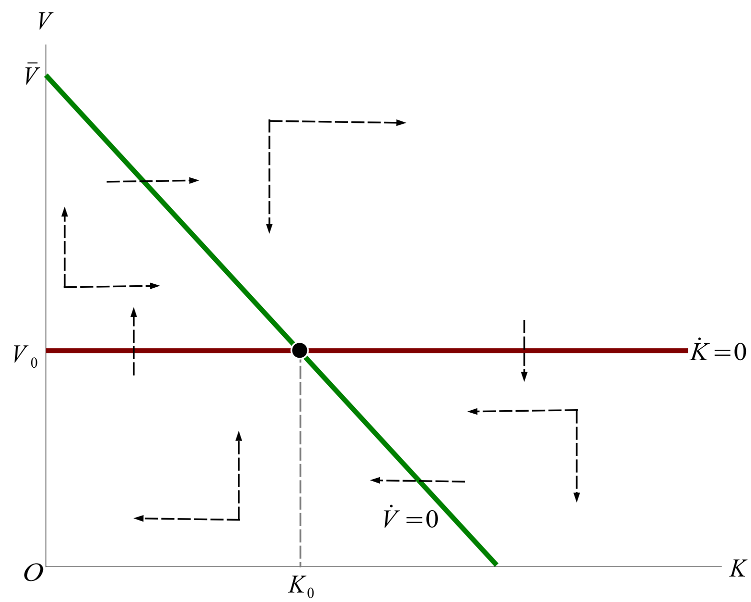


Figure 1: Dynamics of K and V in autarky

Proposition 2. *In autarky, there exists a unique, locally stable steady-state equilibrium* (K_0, V_0) *that satisfies*

$$K_0 = \frac{g}{\lambda}\left(\bar{V} - V_0\right), \quad V_0 = \left(\frac{\delta+\rho}{b}\right)^{\frac{1}{\varepsilon b_a}}. \tag{26}$$

Proof. See Appendix A.2 □

The autarky steady-state stock of environmental capital V_0 is positively related to depreciation rate δ and time preference rate ρ. Therefore, if households are less patient (larger ρ), the country is more protective of the environment. The intuition is straightforward. Households with less patience tend to invest less, and thus the country ends up with a lower level of private capital stock in steady state. This leads to less pollution and eventually a better environment. Although the result may be reversed if we drop the assumption that households do not really care about the environment, the implication remains here: investment introduces a channel to enjoy higher income by sacrificing the environment, and households with more patience tend to exploit this opportunity more.

On the other hand, V_0 is negatively related to b, which can be seen as an index that measures the difference between the shares of two intermediate goods in the final good.[9] The relationship between V_0 and ε depends on whether $(\delta+\rho)/b$ is greater or less than one. Since the possible minimum value of b is $1/2$ when $b_m = b_a = 1/2$, $(\delta+\rho)/b < 1$ as long as $\delta+\rho$ is less than $1/2$. Taking $(\delta+\rho)/b < 1$ as granted, V_0 is positively related to ε. This is

[9] It is easy to check that b is an increasing function of $|b_m - b_a|$.

intuitive because it says that if agriculture in a country is more sensitive to the environment than other countries, then the environment in this country should be better.

It is clear from (26) that V_0 has nothing to do with $\bar{V}$ (the natural level of environmental capital), nor λ_i $(i = m, a)$ (the emission intensities). This implies a convergence of steady-state environmental capital stocks across countries that have the same production technologies, depreciation rate, and time preference rate, regardless of the differences in environmental endowments. The intuition comes by realizing that, in the presence of investment, households in a country with higher $\bar{V}$ will exploit this advantage by investing more. In steady state, the advantage in environmental endowment is exploited to the point so that all countries have the same level of environmental capital stock. As a result, the country more abundant in environmental endowment will have higher private capital stock and consumption in steady state, which can be seen from the expression of K_0. Let P_0 and $(r/p)_0$ denote, respectively, the relative price of intermediate goods and the real rental in autarky steady state, then we have $P_0 = V_0^{\varepsilon}$ and$(r/p)_0 = bV_0^{\varepsilon b_a}$.

It is also worth noting that the local stability is quite robust. Suppose the more general evolution function $\dot{V} = E\left(\bar{V}, V, Z\right)$ instead of (2), and $G(V)$ instead of V^{ε} in (5). The local asymptotic stability holds as long as $\partial E/\partial Z < 0$, $\partial E/\partial V < 0$ and $dG/dV > 0$ around the steady state. The global stability, however, is not necessarily true. The possibility cannot be excluded of the existence of a limit cycle where the pair (K, V) repeats the same pattern of evolution. The discussion on global stability and limit cycle is an interesting issue, but beyond the scope of this paper.

4 Trade equilibrium

This section considers a world in which there are two countries, Home and Foreign, with the asterisk superscript denoting Foreign-related variables. To neutralize other factors for trade, we focus on the symmetric case in which two countries share the same $\bar{V}$, b_m, ρ, and δ.[10] Let the world relative price of intermediate goods $P_w \equiv p_{wm}/p_{wa}$, where p_{wm} and p_{wa} denote, respectively, the world price of manufacturing good and that of agriculture good. In this two-country world, P_w is an endogenous variable, and thus specialization patterns depend on both the production side and the demand side, including incomes (rK and r^*K^*), levels of environmental capital (V and V^*), and the expenditure shares on intermediate goods (b_a and b_m). To tackle this problem, as a first step we examine how many possible specialization

[10] Environmental capital stocks in both countries are endogenously determined and not necessarily identical. It is possible for two countries to have different levels of environmental capital stock in equilibrium (thus obtaining comparative advantages) and trade with each other. Showing this possibility is actually the focus in what follows.

patterns there are and what range of K and V each pattern corresponds with.

4.1 Specialization patterns and regimes

Note that the relative magnitude of environmental capital stocks determines the comparative advantage. If $V > V^*$, then Home has a comparative advantage in agriculture and must produce it, while Foreign must produce manufacturing good. Clearly, there are three possible specialization patterns, which I denote Pattern (D, M^*), (A, M^*), and (A, D^*). In Pattern (D, M^*), Home produces both while Foreign completely specializes in manufacturing. In Pattern (A, M^*), Home completely specializes in agriculture while Foreign completely specializes in manufacturing. In Pattern (A, D^*), Home completely specializes in agriculture while Foreign produces both.

If $V < V^*$, there are also three possible specialization patterns. Since the two countries are symmetric, these patterns can be obtained by swapping "Home" and "Foreign" in the situation of $V > V^*$. Let (M^*, D), (M^*, A), and (D^*, A) refer to these symmetric specialization patterns. Finally, if $V = V^*$, Home and Foreign have the same MRT, and by (A1), there is no trade between two countries.

It is also useful to partition the factor space (K, K^*, V, V^*) into regimes, each of which corresponds with a specialization pattern. Let each regime be named in the same way. Table 1 summarizes all possible specialization patterns and the corresponding regimes.

	Range of V	Specialization pattern
Regime (D, M^*)		Home produces both, Foreign produces only M
Regime (A, M^*)	$V > V^*$	Home produces only A, Foreign produces only M
Regime (A, D^*)		Home produces only A, Foreign produces both
Boundary	$V = V^*$	as in autarky
Regime (M^*, D)		Home produces only M, Foreign produces both
Regime (M^*, A)	$V > V^*$	Home produces only M, Foreign produces only A
Regime (D^*, A)		Home produces both, Foreign produces only A

Table 1: Possible specialization patterns

Note that Table 1 does not provide a complete characterization of regimes, because in the two-country case, the range of a regime is not only related to V, but also related to K. The rest of this section accomplishes three main tasks. The first is to calculate the range of K that corresponds with each pattern.

After characterizing regimes, I investigate the existence of steady-state equilibria, their stability, and the effects of trade on welfare and the environment in each regime. Again, the analysis focuses on the situations $V > V^*$ and $V = V^*$, since the results of $V < V^*$ can be obtained simply by switching "Home" and "Foreign" in the results of $V > V^*$. The third task is to combine the results of all regimes to construct a whole picture of the two-country world.

4.2 Regime (D, M^*)

To characterize this regime, assume that the two-country world is experiencing Pattern (D, M^*) to derive the range of private capital stocks (K and K^*) that characterize Regime (D, M^*), the world relative price (P_w), the real interest rates (r/p and r^*/p^*), and the flows of pollution (Z and Z^*).

Since Home produces both intermediate goods in Pattern (D, M^*), the world relative price is determined by the environmental capital stock in Home: $P_w = V^\varepsilon$. Since both countries produce manufacturing good, the interest rates are equalized across countries: $r = r^* = p_{wm}$. It follows from (16) that the real interest rates are also equalized across countries:

$$\frac{r}{p} = \frac{r^*}{p^*} = p_{wm} \left(\frac{p_{wm}^{b_m} p_{wa}^{b_a}}{b} \right)^{-1} = bP_w^{b_a} = bV^{\varepsilon b_a}. \tag{27}$$

The Cobb-Douglas type of final good production function implies that the world demand for manufacturing good D_{wm} satisfies

$$D_{wm} = \frac{b_m \left(rK + r^* K^* \right)}{p_{wm}} = b_m \left(K + K^* \right). \tag{28}$$

Foreign supplies $M^* = K^*$ units of manufacturing good, thus the world market clearing condition requires

$$M = D_{wm} - K^* = b_m K - b_a K^*. \tag{29}$$

Since $M > 0$ in Pattern (D, M^*), (29) imposes a constraint on the relative magnitude of private capital stocks:

$$\frac{K}{K^*} > \frac{b_a}{b_m}. \tag{30}$$

Therefore, Regime (D, M^*) is characterized by both $V > V^*$ and (30). If (30) fails to hold, there is no positive solution for M, which means Pattern (D, M^*) does not exist.

Given M and M^*, the flows of pollution in both countries are, using (12),

$$Z = \lambda K - b_a \left(\lambda_m - \lambda_a \right) K^*, \ \ Z^* = \lambda_m K^*. \tag{31}$$

Substituting (27) and (31) into (20) and (2), as well as its Foreign counterpart, yields the dynamic system in Regime (D, M^*):

$$\begin{aligned}\frac{\dot{K}}{K} &= bV^{\varepsilon b_a} - \delta - \rho, \\ \dot{V} &= g\left(\bar{V} - V\right) - \lambda K + b_a\left(\lambda_m - \lambda_a\right) K^*, \\ \frac{\dot{K}^*}{K^*} &= bV^{\varepsilon b_a} - \delta - \rho, \\ \dot{V}^* &= g\left(\bar{V} - V^*\right) - \lambda_m K^*.\end{aligned}$$

Simple calculation gives the steady-state environmental capital stock in Home:

$$V_T = V_0 = \left(\frac{\delta + \rho}{b}\right)^{\frac{1}{\varepsilon b_a}},$$

and a linear relationship between the steady-state private capital stock in Home K_T and in Foreign K_T^*:

$$K_T = \frac{b_a\left(\lambda_m - \lambda_a\right)}{\lambda} K_T^* + \frac{g}{\lambda}\left(\bar{V} - V_0\right) = \frac{b_a\left(\lambda_m - \lambda_a\right)}{\lambda} K_T^* + K_0, \tag{32}$$

where K_0 and V_0 are the autarky steady-state stocks of private and environmental capital, respectively. The steady-state environmental capital stock in Foreign is also linearly related to K_T^*

$$V_T^* = \bar{V} - \frac{\lambda_m}{g} K_T^*. \tag{33}$$

Therefore, the steady-state equilibrium vector $\{V_T, K_T, K_T^*, V_T^*\}$ has one dimension of freedom.

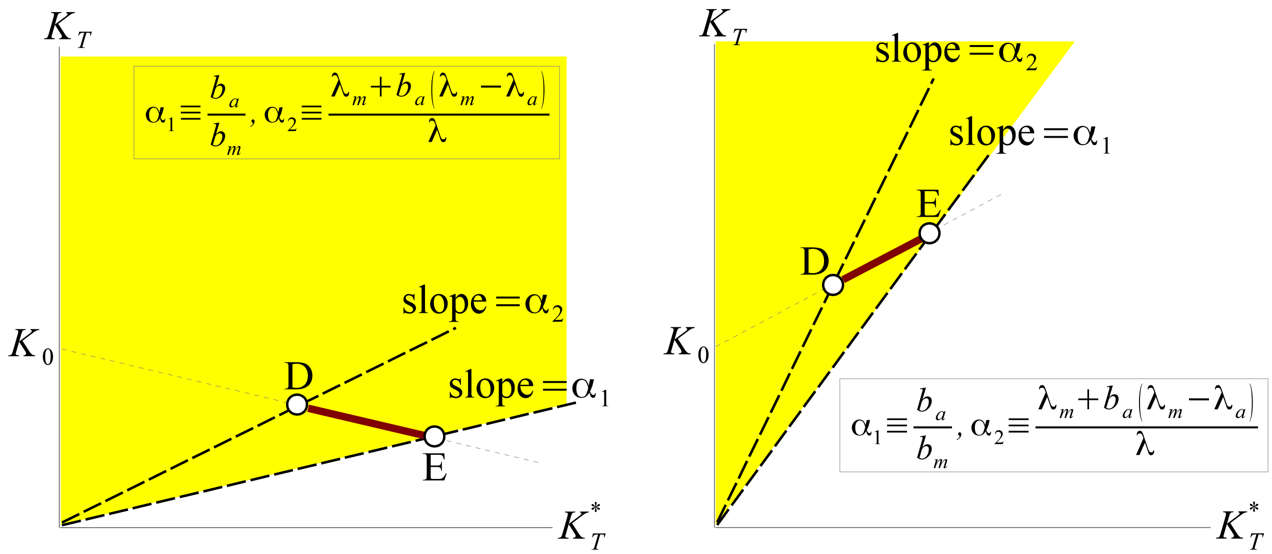
Note that the condition $V > V^*$ in Pattern (D, M^*) implies another constraint on the relative magnitude of K and K^* in steady state:[11]

$$\frac{K_T}{K_T^*} < \frac{\lambda_m + b_a\left(\lambda_m - \lambda_a\right)}{\lambda}. \tag{34}$$

Together with (30), the condition for the existence of steady-state equilibrium in Pattern (D, M^*) is $b_a/b_m < \left[\lambda_m + b_a\left(\lambda_m - \lambda_a\right)\right]/\lambda$, which can be simplified into

$$b_m \lambda_m > b_a \lambda_a. \tag{35}$$

[11]Since $V > V^*$, we must have $V_T > V_T^*$. By substituting V_T and V_T^* into $\dot{V} = \dot{V}^* = 0$ for V and V^*, and using $V_T > V_T^*$, we can obtain $-\lambda K + b_a\left(\lambda_m - \lambda_a\right) K^* > -\lambda_m K^*$ in steady state. This directly gives (34).



(a) Dirty agriculture ($\lambda_a > \lambda_m$) (b) Clean agriculture ($\lambda_a < \lambda_m$)

Figure 2: Steady-state private capital stocks in Regime (D, M^*)

It follows directly that

Lemma 3. *There exists steady-state equilibrium satisfying Pattern (D, M^*) if and only if* $b_m\lambda_m > b_a\lambda_a$.

Figure 2 illustrates the linear relationship between K_T and K_T^*. In the figure, Regime (D, M^*) is the area that lies above the ray with slope b_a/b_m. In Regime (D, M^*), the area allowing for the existence of steady-state equilibrium lies below the ray with slope $(\lambda_m + b_a(\lambda_m - \lambda_a))/\lambda$. The line passing through D and E satisfies (32). Hence the line segment DE (except for the two end points D and E) represents the set of the steady-state equilibria that have specialization pattern (D, M^*).

According to (33), larger K_T^* means smaller V_T^*, thus V_T^* declines when moving along DE towards E. Moreover, if agriculture is dirty (see Figure 2a), the slope of DE is negative.[12] This implies that if one country becomes better off (higher private capital stock and thus consumption), the other country must be worse off. In contrast, if agriculture is clean (see Figure 2b), DE is positively sloped. An increase in private capital stock in one country is a win-win adjustment. The following proposition summarizes the results.

[12] It is easy to show that $b_a(\lambda_m - \lambda_a)/\lambda < b_a/b_m$ always holds, thus DE must intersect the two rays with slope b_a/b_m and slope $(\lambda_m + b_a(\lambda_m - \lambda_a))/\lambda$, respectively. Also, according to the condition for the existence of steady-state equilibrium (35), we can obtain that, if $\lambda_a > \lambda_m$ then $1 > (\lambda_m + b_a(\lambda_m - \lambda_a))/\lambda > b_a/b_m$, and if $\lambda_a < \lambda_m$ then $(\lambda_m + b_a(\lambda_m - \lambda_a))/\lambda > b_a/b_m > 1$.

Proposition 4. *Given that $b_m\lambda_m > b_a\lambda_a$ holds, a two-country world adopts these characteristics.*

SP (steady state): There is a set of steady-state equilibria satisfying Pattern (D, M^*). Each steady-state equilibrium $\{V_T, K_T, K_T^*, V_T^*\}$ is characterized by (32), (33), (30) and (34). Moreover, each steady-state equilibrium is stable if and only if $\lambda + b_a(\lambda_a - \lambda_m) > 0$. But no steady-state equilibrium is asymptotically stable.

WE (steady state): In Home, if agriculture is dirty/clean, then steady-state consumption is necessarily lower/higher than in autarky. In Foreign, if agriculture is dirty, then steady-state consumption is necessarily higher than in autarky.

EI (steady state): The environment in Home remains the same as in autarky, while it deteriorates in Foreign.

Proof. See Appendix A.3. □

4.3 Regime (A, M^*)

We now consider Regime (A, M^*). Similarly, we derive related results by assuming the two-country world is experiencing Pattern (A, M^*). Since Home produces only agriculture good and Foreign produces only manufacturing good in Pattern (A, M^*), the interest rates in both countries are, respectively,

$$r = p_{wa}V^{\varepsilon}, \quad r^* = p_{wm}. \tag{36}$$

The world supply of manufacturing good is provided only by Foreign:

$$M^* = K^*.$$

On the other hand, the world demand is

$$D_{wm} = \frac{b_m(rK + r^*K^*)}{p_{wm}} = b_m\left(\frac{V^{\varepsilon}}{P_w}K + K^*\right).$$

The world market clearing condition determines the world relative price as follows:

$$P_w = \frac{b_m K}{b_a K^*}V^{\varepsilon}. \tag{37}$$

Note that in Pattern (A, M^*), we have $V^{*\varepsilon} \le P_w \le V^{\varepsilon}$, which imposes a constraint on the relative magnitude of K and K^*:

$$\frac{b_a}{b_m}\left(\frac{V^*}{V}\right)^{\varepsilon} \le \frac{K}{K^*} \le \frac{b_a}{b_m}. \tag{38}$$

Therefore, Regime (A, M^*) is characterized by both $V > V^*$ and (38).

It follows from (36)and (16) that in Home

$$\frac{r}{p} = p_{wa} V^{\varepsilon} \left(\frac{p_{wm}^{b_m} p_{wa}^{b_a}}{b} \right)^{-1} = \frac{bV^{\varepsilon}}{P_w^{b_m}}, \tag{39}$$

which together with (37) yields the real interest rate in Home:

$$\frac{r}{p} = b_a \left(\frac{K^*}{K} \right)^{b_m} V^{\varepsilon b_a}. \tag{40}$$

From (36), (16) and (37), we can also obtain the real interest rate in Foreign:

$$\frac{r^*}{p^*} = bP_w^{b_a} = b_m \left(\frac{K}{K^*} \right)^{b_a} V^{\varepsilon b_a}. \tag{41}$$

The flows of pollution are simply determined by

$$Z = \lambda_a K, \;\; Z^* = \lambda_m K^*. \tag{42}$$

Substituting (40), (41) and (42) into (20) and (2) for r/p and Z, and doing the same for Foreign, we can obtain the dynamic system in Regime (A, M^*):

$$\begin{aligned}
\frac{\dot{K}}{K} &= b_a \left(\frac{K^*}{K} \right)^{b_m} V^{\varepsilon b_a} - \delta - \rho, \\
\dot{V} &= g\left(\bar{V} - V\right) - \lambda_a K, \\
\frac{\dot{K}^*}{K^*} &= b_m \left(\frac{K}{K^*} \right)^{b_a} V^{\varepsilon b_a} - \delta - \rho, \\
\dot{V}^* &= g\left(\bar{V} - V^*\right) - \lambda_m K^*.
\end{aligned}$$

In steady state, the stock of private capital must satisfy

$$\frac{K_T}{K_T^*} = \frac{b_a}{b_m}. \tag{43}$$

Substituting into the dynamic system yields the steady-state environmental capital stock in Home:

$$V_T = V_0 = \left(\frac{\delta + \rho}{b} \right)^{\frac{1}{\varepsilon b_a}},$$

which is the same as in Pattern (D, M^*), as well as in autarky. The steady-state private capital stock in Home is then

$$K_T = \frac{g\left(\bar{V} - V_0\right)}{\lambda_a}, \tag{44}$$

while in Foreign, it is

$$K_T^* = \frac{b_m g\left(\bar{V} - V_0\right)}{b_a \lambda_a}, \;\; V_T^* = \bar{V} - \frac{b_m \lambda_m}{b_a \lambda_a}\left(\bar{V} - V_0\right). \tag{45}$$

Since Pattern (A, M^*) requires $V_T > V_T^*$, it leads to the same condition as in (35). Therefore,

Lemma 5. *There exists steady-state equilibrium satisfying Pattern (A, M^*) if and only if $b_m\lambda_m > b_a\lambda_a$.*

These results can be summarized as follows.

Proposition 6. *Given that $b_m\lambda_m > b_a\lambda_a$ holds, a two-country world adopts these characteristics.*

SP (steady state): There is a unique, locally half-stable (stable in the half space: $K/K^* \leq b_a/b_m$), steady-state equilibrium satisfying Pattern (A, M^*). The steady-state equilibrium $\{V_T, K_T, K_T^*, V_T^*\}$ is described by (33), (44), and (45).

WE (steady state): In Home, if agriculture is dirty/clean, then steady-state consumption is necessarily lower/higher than in autarky. In Foreign, if agriculture is dirty, then steady-state consumption is necessarily higher than in autarky.

EI (steady state): The environment in Home remains the same as in autarky, while it deteriorates in Foreign.

Proof. See Appendix A.4. □

4.4 Regime (A, D^*)

We now characterize Regime (A, D^*), in which the two-country world has Pattern (A, D^*). Since Home produces only agriculture good and Foreign produces both, the world relative price is determined by the Foreign environment: $P_w = V^{*\varepsilon}$. The real interest rate in Home is, using (39),

$$\frac{r}{p} = \frac{bV^{\varepsilon}}{P_w^{b_m}} = bV^{*\varepsilon b_a}\left(\frac{V}{V^*}\right)^{\varepsilon}, \tag{46}$$

and the real interest rate in Foreign is, using (27),

$$\frac{r^*}{p^*} = bP_w^{b_a} = bV^{*\varepsilon b_a}. \tag{47}$$

We have two observations for Pattern (A, D^*). First, as with the constraint (30) for Pattern (D, M^*) and (38) for Pattern (A, M^*), the constraint for Pattern (A, D^*) is

$$\frac{K}{K^*} < \frac{b_a}{b_m}\left(\frac{V^*}{V}\right)^{\varepsilon}, \tag{48}$$

which, together with $V > V^*$, characterize Regime (A, D^*). Second, we have $r/p > r^*/p^*$ for $V > V^*$. Hence, the growth rate of private capital stock in Home is higher than in Foreign, implying that the ratio K/K^* increases over time. Sooner or later, (48) will break down and the two-country world leaves Regime (A, D^*). This argument is summarized in the following proposition.

Proposition 7. *There is no steady-state equilibrium satisfying Pattern (A, D^*).*

4.5 Boundary

The range of boundary is simply the subspace satisfying $V = V^*$. On the boundary, there is no trade between Home and Foreign according to assumption (A1). We can treat them as closed countries and calculate the steady-state results. Since Home and Foreign are symmetric, we have

$$K_T = K_T^* = K_0. \tag{49}$$

$$V_T = V_T^* = V_0 \tag{50}$$

However, these steady states is unstable, because any shock, as long as the shock does not result in $K = K^*$ and $V = V^*$, will generate a difference in V and V^* or a tendency for V and V^* becoming different. This causes the two-country world to leave the boundary.

4.6 The whole picture

For the two-country world, we have analyzed the relevant steady-state properties in regimes (D, M^*), (A, M^*), (A, D^*) and on the boundary. With these results in hand, we can see how the steady-state equilibria in one regime are related to those in another regime.

First, we note that values of b_m, λ_m and λ_a are crucial for the existence of the steady-state equilibrium in which two countries are trading with each other. It follows from Lemma 3, Lemma 5, and Proposition 7 that

Proposition 8. *There exists steady-state trade equilibrium if and only if $b_m\lambda_m > b_a\lambda_a$.*

Second, the unique steady-state equilibrium in Regime (A, M^*) can be arrived at by letting K_T/K_T^* in Regime (D, M^*) approach b_a/b_m. This can be verified by letting $K_T/K_T^* = b_a/b_m$ in (29) to obtain $M = 0$. Therefore, point E in Figure 2b is actually the unique equilibrium in Regime(A, M^*).

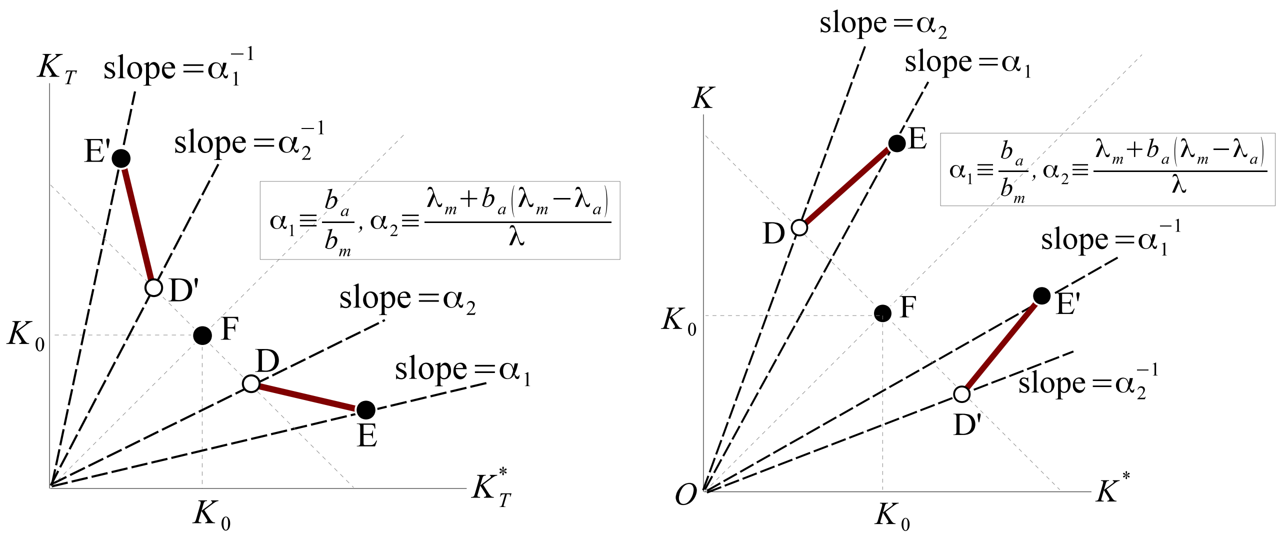
Third, if we let K_T/K_T^* in Regime (D, M^*) go in another direction to $\left[\lambda_m + b_a\left(\lambda_m - \lambda_a\right)\right]/\lambda$, we will arrive at the boundary. To see this, we substitute $K_T/K_T^* = \left[\lambda_m + b_a\left(\lambda_m - \lambda_a\right)\right]/\lambda$ into the dynamic system in Regime (D, M^*) and let $\dot{K} = \dot{K}^* = \dot{V} = \dot{V}^* = 0$, to obtain $V_T^* = V_T = V_0$ and $K_T + K_T^* = 2K_0$.

Finally, in the K^*-K plane, the unique steady-state (non-trade) equilibrium on the boundary is the intersection of $K_T + K_T^* = 2K_0$ and $K_T = K_T^*$. The steady-state equilibria in Regime (D, M^*) lie on the line segment with slope $b_a\left(\lambda_m - \lambda_a\right)/\lambda$. It is easy to check that $b_a\left(\lambda_m - \lambda_a\right)/\lambda > -1$. Thus, the steady-state private capital stocks in Regime (D, M^*) satisfy $K_T + K_T^* > 2K_0$, which means that the world total steady-state consumption in Pattern (D, M^*) is higher than in autarky.

These observations are illustrated in Figure 3, with 3a depicting the dirty agriculture case and 3b the clean agriculture case. In both figures, the line segment DE (except for the end points D and E) contains the steady-state equilibria satisfying Pattern (D, M^*), point E is the unique steady-state equilibrium satisfying Pattern (A, M^*), and point F is the unique steady-state equilibrium on the boundary in which there is no trade between two countries. The line passing through DD' satisfies $K_T + K_T^* = 2K_0$. Similarly, the line segment D'E' and point E' correspond, respectively, to the steady-state equilibria satisfying Pattern (M^*, D) (the symmetric counterpart of Pattern (D, M^*) satisfying $V < V^*$), and the unique steady-state equilibrium satisfying Pattern (M^*, A) (the symmetric counterpart of Regime (A, M^*) satisfying $V < V^*$). We can see very clearly from Figure 3 that the world total steady-state consumption at point E (also E') attains the highest level among all steady states. The above arguments can be summarized as follows.

Proposition 9. *Free trade increases the world total steady-state consumption, which attains the highest level when both countries completely specialize.*

Recalling the results associated with patterns (D, M^*) and (A, M^*), we notice that the two patterns share the same feature that Foreign completely specializes in manufacturing good. Also, the steady-state equilibria in the corresponding regimes are stable, though not asymptotically stable for those equilibria satisfying Pattern (D, M^*). Moreover, since there is no steady state in Regime (A, D^*), no country will completely specialize in agriculture in steady state. Finally, the knife-edge steady-state equilibrium on the boundary is unstable. These points leads to the following implication.



(a) Dirty agriculture ($\lambda_a > \lambda_m$) (b) Clean agriculture ($\lambda_a < \lambda_m$)

Figure 3: Steady-state private capital stocks in two-country world

Remark 10. In a two-country world, there is a strong tendency for one country to completely specialize in manufacturing, while the other country may either completely specialize in agriculture or remain diversified.

5 Conclusion

In a model without investment, which sector—clean or dirty—the country specializes in when opened to trade produces the reverse environmental consequence of trade. In contrast, in a world with endogenous investment, free trade always harms the environment in the long run, even if the country specializes in the clean sector. Moreover, in a world without investment, the environmentally sensitive sector—agriculture—being clean or dirty matters since it determines the shape of the steady-state PPF, which in turn determines specialization patterns and amplifies other effects of trade. However, in a world where investment is available, regardless of whether agriculture is clean or dirty, there always exists a strong tendency towards specialization. The intuition here bears some resemblance with the dynamic Heckscher-Ohlin model (see, e.g. Stiglitz, 1970; Baxter, 1992; Brecher et al., 2005). That is, when private capital can be produced and invested, it is actually, in the long run, an intermediate good rather than a primary factor. As shown in Samuelson (1951), if there is only one primary factor and no externality, the PPF must be linear (or contain a hyperplane). The new ingredient in our model is that the only “real” primary factor—environmental capital—is subject to production externalities.

Although this paper develops a framework allowing trade, investment, and the environment to be discussed together, it neglects many important elements. First, the model implicitly assumes that the production of capital good uses the same technology as consumption good. The assumption is restrictive and it is interesting to see what would happen by relaxing the assumption. Actually, if manufacturing good can be used as capital good, and if there is no capital income tax, then households will keep investing, even in autarky, because investment is always profitable for them. This eventually destroys the environment, and consequently, their own welfare.

Therefore, the optimal policies that can correct the resource misallocation due to externalities (see footnotes 7 and 8), as well as the influences on trade and the environment, are another interesting topic. Also, the model considers only "damage on production". It should yield richer results by incorporating "damage on utility" into the model. In addition, the pollution in the model is non-transboundary. It is meaningful to consider the pollution that can cross the border. Moreover, by assuming constant pollution intensities, the model excludes the abatement behavior, which is another significant aspect in reality. Finally, the model considers two symmetric countries to neutralize other forces for trade. It may give rise to some interesting results if one allows for differences in time preference rates or pollution intensities between countries.

A Appendix

A.1 Proof of Lemma 1

Differentiate A with respect to M in (14) and obtain

$$\frac{dA}{dM} = -V^{\varepsilon-1}\left[\varepsilon\left(\mu_m - \mu_a\right)(K - M) + V\right],$$

where $\mu_m \equiv \lambda_m/g$ and $\mu_a \equiv \lambda_a/g$. Differentiating dA/dM again gives

$$\begin{aligned}\frac{d^2A}{dM^2} &= \varepsilon\left(\mu_m - \mu_a\right)V^{\varepsilon-2}\left[2V - (1-\varepsilon)\left(\mu_m - \mu_a\right)(K - M)\right] \\ &\equiv \varepsilon\left(\mu_m - \mu_a\right)V^{\varepsilon-2}\triangle. \end{aligned} \tag{51}$$

Since $M \leq K$, $0 < \varepsilon \leq 1$, we have $\triangle > 0$ when $\mu_m < \mu_a$. To obtain the sign of $\triangle$ when $\mu_m > \mu_a$, rewrite $\triangle$ into

$$\triangle = \left[\varepsilon\left(\mu_m - \mu_a\right)(K - M) + V\right] + \left[V - \left(\mu_m - \mu_a\right)(K - M)\right],$$

Using (13) gives

$$\triangle = [\varepsilon(\mu_m - \mu_a)(K - M) + V] + [\bar{V} - \mu_m K].$$

Since $\bar{V} > \max\{\mu_m K, \mu_a K\}$, we have $\triangle > 0$ when $\mu_m > \mu_a$. Hence, $\triangle > 0$ holds for both $\mu_m < \mu_a$ and $\mu_m > \mu_a$. The sign of (51) is the same as $\mu_m - \mu_a$.

A.2 Proof of Proposition 2

The uniqueness is obvious from (24) and (25). The Jacobian around the steady-state point is

$$J \equiv \frac{\partial\left(\dot{K}, \dot{V}\right)}{\partial(K, V)} = \begin{bmatrix} 0 & b\varepsilon b_a V^{\varepsilon b_a - 1} \\ -\lambda & -g \end{bmatrix}.$$

The local stability directly follows $\det J = \lambda b\varepsilon b_a V^{\varepsilon b_a - 1} > 0$ and $\mathrm{tr}J = -g < 0$.

A.3 Proof of Proposition 4

(SP) Since some results follow directly from the discussion prior to the proposition, we need only prove the stability. The Jacobian around the steady-state points is

$$J \equiv \frac{\partial\left(\dot{K}, \dot{V}, \dot{K}^*, \dot{V}^*\right)}{\partial(K, V, K^*, V^*)} = \begin{bmatrix} 0 & b\varepsilon b_a V^{\varepsilon b_a - 1} & 0 & 0 \\ -\lambda & -g & -b_a(\lambda_a - \lambda_m) & 0 \\ 0 & b\varepsilon b_a V^{\varepsilon b_a - 1} & 0 & 0 \\ 0 & 0 & -\lambda_m & -g \end{bmatrix}.$$

Routine calculation gives the characteristic equation

$$|J - \sigma I| = \sigma^4 + 2g\sigma^3 + \left(B + g^2\right)\sigma^2 + Bg\sigma = 0,$$

where $B \equiv b\varepsilon b_a V^{\varepsilon b_a - 1}[\lambda + b_a(\lambda_a - \lambda_m)]$. Factorization gives

$$|J - \sigma I| = \sigma(\sigma + g)\left(\sigma^2 + g\sigma + B\right).$$

Note $|J - \sigma I| = 0$ has solutions $\sigma_1 = 0$, $\sigma_2 = -g < 0$. Hence, the stability is equivalent to that the remaining two solutions are negative, which means $B > 0$, and thus $\lambda + b_a(\lambda_a - \lambda_m) > 0$. Because $\sigma_1 = 0$, the system is not asymptotically stable.

(WE) By (32) and noting that $K_T^* > 0$, we have $K_T \lesseqgtr K_0$ if $\lambda_a \gtreqless \lambda_m$. By the consumption function (19), the steady-state consumption in Home C_T satisfies that $C_T \lesseqgtr C_0$ if $\lambda_a \gtreqless \lambda_m$.

Similarly, according to (33) and $V_T^* < V_T = V_0$, $K_T^* > K_0$ and thus $C_T^* > C_0$.

As for the world total steady-state consumption, $C_T + C_T^* = \rho (K_T + K_T^*)$. By (32), $K_T + K_T^* = \lambda_m K_T^* / \lambda + K_0$. Substituting (33) for K_T^* yields $K_T + K_T^* = K_0 + g\left(\bar{V} - V_T^*\right)/\lambda$. Given that $V_T^* < V_T = V_0$ in Pattern I, $g\left(\bar{V} - V_T^*\right)/\lambda > K_0$ and thus $K_T + K_T^* > 2K_0$.

(EI) The result can be directly obtained from the expression of V_T and the condition $V_T > V_T^*$.

A.4 Proof of Proposition 6

For the same reason, we need only prove the stability. The Jacobian around the steady-state point is

$$J = \begin{bmatrix} -b_a b_m \left(\frac{K^*}{K}\right)^{b_m} V^{\varepsilon b_a} & b_a \left(\frac{K^*}{K}\right)^{b_m} \varepsilon b_a V^{\varepsilon b_a - 1} & b_a b_m \left(\frac{K}{K^*}\right)^{b_a} V^{\varepsilon b_a} & 0 \\ -\lambda & -g & 0 & 0 \\ b_a b_m \left(\frac{K^*}{K}\right)^{b_m} V^{\varepsilon b_a} & b_m \left(\frac{K}{K^*}\right)^{b_a} \varepsilon b_a V^{\varepsilon b_a - 1} & -b_a b_m \left(\frac{K}{K^*}\right)^{b_a} V^{\varepsilon b_a} & 0 \\ 0 & 0 & -\lambda_m & -g \end{bmatrix}.$$

Simple calculation gives

$$\begin{aligned} |J_1| &= -B < 0, \\ |J_2| &= Bg + D\lambda_a > 0, \\ |J_3| &= -2\lambda_a \frac{b_a}{b_m} BD < 0, \\ |J| &= -g\,|J_3| > 0, \end{aligned}$$

where $B \equiv b_a b_m \left(\frac{K^*}{K}\right)^{b_m} V^{\varepsilon b_a} > 0$, and $D \equiv b_a \left(\frac{K^*}{K}\right)^{b_m} \varepsilon b_a V^{\varepsilon b_a - 1} > 0$. Hence, J is negative definite, which proves the asymptotical stability. Note that this holds around only one side of the equilibrium point, namely $K/K^* \leq b_a/b_m$. On the other side, namely $K/K^* > b_a/b_m$, the condition required for Pattern (A, M^*) fails.

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