Microeconomic foundation for Phillips curve with three-periods overlapping generations model and negative real balance effect

Tanaka, Yasuhito

16 October 2020

Microeconomic foundation for Phillips curve with three-periods overlapping generations model and negative real balance effect

Yasuhito TANAKA
Faculty of Economics, Kamigyo-ku, Kyoto, 602-8580, Japan,

E-mail: yatanaka@mail.doshisha.ac.jp.
Abstract: We show a negative relation between the inflation rate and the unemployment rate , that is, the Phillips curve using a three-periods overlapping generations (OLG) model with childhood period and pay-as-you-go pension for older generation under monopolistic competition. We consider the effects of a change in nominal wage rate with negative real balance effect and the effects of an exogeneous change in labor productivity. In a three periods OLG model there may exist a negative real balance effect. A fall (or rise) in nominal wage rate induces a fall (or rise) in the price, then by negative real balance effect the unemployment rate rises (or falls), and we get a negative relation between the inflation rate and the unemployment rate. This conclusion is based on the premise of utility maximization of consumers and profit maximization of firms. Therefore, we presented a microeconomic foundation of the Phillips curve. About the effects of a change in labor productivity we obtain similar results. We also examine the effects of fiscal policy financed by seigniorage.

JEL classifications: E12, E24, E31

Keywords: Phillips Curve, Microeconomic foundation, Three-periods overlapping generations model, Monopolistic competition, Negative real balance effect.

1. Introduction

Otaki and Tamai (2011, 2012) presented a microeconomic foundation of the negative relation between the unemployment rate and the inflation rate, that is, the Phillips Curve (Phillips (1958)) using an overlapping generations model (OLG model) under monopolistic competition[1]. They have shown that, the lower the unemployment rate in a period (for example period $t-1$), the higher the inflation rate from period t to period $t+1$. Their logic is as follows. They assume that the low (or high) unemployment rate in period $t-1$ raises (or lowers) the labor productivity in period t by learning effect. If the unemployment rate in period $t-1$ increases, the labor productivity in period t falls. Then, by the behavior of firms in monopolistic competiton the price of the goods in period t rises given nominal wage rate, and the inflation rate from period t to period $t+1$ falls given the (expected) price of the goods in period $t+1$. Alternatively, a decrease in the unemployment rate in period $t-1$ raises the labor productivity in period t. Then, the price of the goods falls, and the infaltion rate from period t to period $t+1$ rises given the (expected) price of the goods in period $t+1$. However, we do not find their conclusion that the low unemployment rate in period $t-1$ explains the high inflation rate from period t to period $t+1$ to be satisfactory. A fall in the price in period t means that the inflation rate from period $t-1$ to period t falls, that is, the low unemployment rate in period $t-1$ explains the low (not high) inflation rate from period $t-1$ to period t.

Instead, in this paper we consider the effects of a change in nominal wage rate with negative real balance effect and the effects of an exogenous change in labor productivity. Changes in labor productivity may be due to a change in the unemployment rate in the previous period as assumed by Otaki and Tamai (2011, 2012). According to Tanaka (2020), we use a three-periods OLG model with childhood period, younger period and older period. Also we consider a pay-as-you-go pension system to bring about the negatiive real balance effect of a fall in nominal wage rate. The negative balance effect (or negative Pigou effect) means that by falls in nominal wage rate and the price the real asset of consumers (difference between savings and debts) decreases.

We will show the negative ralationship between the unemployment rate and the inflation rate in the same period. Our logic is as follows. If nominal wage rate in a period, for example, period t falls, the price of the goods falls. This means that the inflation rate from period $t-1$ to period t decreases. By the negative real balance effect aggregate demand for the goods and employment decrease and the unemployment rate increases in period t. Alternatively, if nominal wage rate in period t rises, the price of the goods rises, and the inflation rate from period $t-1$ to period t increases. By the negative real balance effect aggregate demand for the goods and employment increase and the unemployment rate decreases in period t. About the effects of an exogenous change in labor productivity we obtain similar results. For details please see Section 4.

Some other references about Phillips curve are Lucas (1972), Calvo (1983), Mankiw and Reis (2002) and Woodford (1996). According to Otaki and Tamai (2011, 2012), every work on the Phillips curve presumes some market imperfection, and it implies that if there does not exist some price stickiness assumption or imperfect information, the negative correlation between inflation and unemployment will disappear. This paper will show that it isn't.

In Section 2 we analyze behaviors of consumers and firms. In Section 3 we consider the equilibrium of the economy with involuntary unemployment. In Section 4 we show the main results about the negative relation between the unemployment rate and the inflation rate due to a change in nominal wage rate. We also examine the effects of fiscal policy financed by seigniorage and the Phillips curve due to an exogenous change in labor productivity. The effect of fiscal policy is represented as leftward shift of the Phillips curve.

2. Behaviors of consumers and firms

[1] We consider the relation between the unemployment rate and the inflation rate, that is, the rate of a change in the price of the goods give nominal wage rate.

We consider a three-periods (childhood, young and old) OLG model under monopolistic competition. It is an extension and arrangement of the model in Otaki (2007, 2009, 2011, 2015 and 2016). There is one factor of production, labor, and there is a continuum of goods indexed by $z \in [0,1]$. Each good is monopolistically produced by Firm z. Consumers live over three periods, period 0 (childhood period), period 1 (young period) and period 2(old period).There are consumers of three generations, childhood, youger and older generations, at the same time. They can supply only one unit of labor when they are young (period 1).

2.1 Consumers

We use the following notations.

$c^i(z)$: consumption of good z in period i, $i=1,2$.
$p^i(z)$: price of good z in period i, $i=1,2$.
X^i: consumption basket in period i, $i=1,2$.

$$X^i=\left\{\int_0^1 c^i(z)^{1-\frac{1}{\eta}}dz\right\}^{\frac{1}{1-\frac{1}{\eta}}},\qquad i=1,2,\qquad \eta>1.$$

$c^0(z)$: consumption of good z in period 0. It is constant.
$p^0(z)$: price of good z in period 0. We assume $p^0(z)=1$.
$X^0=\left\{\int_0^1 c^0(z)^{1-\frac{1}{\eta}}dz\right\}^{\frac{1}{1-\frac{1}{\eta}}}$. It is constant.
$X^{0\prime}$: consumtion basket in the childhood period of a consumer of the next generation
β: disutility of labor, $\beta>0$.
W: nominal wage rate.
Π: profits of firms which are equally distributed to each consumer.
L: employment of each firm and the total employment.
L_f: population of labor or employment in the full-employment state.
$y(L)$: labor productivity. $y(L)\geq 1$.
R: unemployment benefit fo an unemployed consumer, $R=X^0$.
R': unemployment benefit fo an unemployed consumer in the next generation, $R'=X^{0\prime}$.
Θ: the tax for unemployment benefit.
Φ: pay-as-you-go pension for a consumer of the older generation.
Φ': pay-as-you-go pension for a consumer of the younger generation when he is retired.
Ψ: the tax for pay-as-you-go pension.

Consumers in period 0 consume the goods by borrowing money from consumers of the previous generation or the government (for example, scholarship). They must repay the debts when they are young. However, if they are unemployed, they can not repay the debts. Then, they receive the unemployment benefits which are coverred by taxes on employed younger generation consumers. Thus, employed younger genartion consumers must pay the taxes for unemployment benefit as well as they must repay their own debts. R and Θ satisfy the follwiing relation.

$$(L_f-L)R=L\Theta. \tag{1}$$

In addition we consider the existence of pay-as-you-go pension for consumers of the older generation. They are also coverred by taxes o employed consumers of the younger generation. Φ and Ψ satisfy

$$L_f\Phi=L\Psi. \tag{2}$$

Consumptions in period 0 of the consumers are constant, and in period 1 they determine their consumptions in periods 1 and 2, and their labor supply. δ is the definition function. If a consumer is employed, $\delta=1$; if he is not employed, $\delta=0$. The labor productivity is $y(L)$. We assume increasing or constant returns to scale technology. Thus, $y(L)$is increasing or constant with respect to the employment of a firm L. We define the employment elasticity of the labor productivity as follows.

$$\zeta = \frac{y'}{\frac{y(L)}{L}}.$$

We assume $0 \le \zeta < 1$. Increasing returns to scale means $\zeta > 0$. η is (the inverse of) the degree of differentiation of the goods. In the limit when $\eta \to +\infty$, the goods are homogeneous. We assume

$$\left(1 - \frac{1}{\eta}\right)(1 + \zeta) < 1$$

so that the profits of firms are positive.

The utility of consumers of one generation over two periods is

$$U(X^0, X^1, X^2, \delta, \beta) = u(X^0, X^1, X^2) - \delta\beta.$$

We assume that $u(X^0, X^1, X^2)$ is homogeneous of degree one (linearly homogeneous). Note that X^0 is constant. The budget constraint is

$$\int_0^1 p^1(z)c^1(z)dz + \int_0^1 p^2(z)c^2(z)dz = \delta W + \Pi + \Phi' - \delta R - \delta(\Theta + \Psi).$$

$p^2(z)$ is the expectation of the price of good z in period 2. The Lagrange function is

$$\mathcal{L} = u(X^0, X^1, X^2) - \delta\beta - \lambda\left(\int_0^1 p^1(z)c^1(z)dz + \int_0^1 p^2(z)c^2(z)dz - \delta W - \Pi - \Phi' + \delta R + \delta(\Theta + \Psi)\right).$$

λ is the Lagrange multiplier. The first order conditions are

$$\frac{\partial u}{\partial X^1}\left(\int_0^1 c^1(z)^{1-\frac{1}{\eta}}dz\right)^{\frac{\frac{1}{\eta}}{1-\frac{1}{\eta}}} c^1(z)^{-\frac{1}{\eta}} = \lambda p^1(z), \qquad (3)$$

and

$$\frac{\partial u}{\partial X^2}\left(\int_0^1 c^2(z)^{1-\frac{1}{\eta}}dz\right)^{\frac{\frac{1}{\eta}}{1-\frac{1}{\eta}}} c^2(z)^{-\frac{1}{\eta}} = \lambda p^2(z). \qquad (4)$$

They are rewritten as

$$\frac{\partial u}{\partial X^1}X^1\left(\int_0^1 c^1(z)^{1-\frac{1}{\eta}}dz\right)^{-1} c^1(z)^{1-\frac{1}{\eta}} = \lambda p^1(z)c^1(z), \qquad (5)$$

$$\frac{\partial u}{\partial X^2}X^2\left(\int_0^1 c^2(z)^{1-\frac{1}{\eta}}dz\right)^{-1} c^2(z)^{1-\frac{1}{\eta}} = \lambda p^2(z)c^2(z). \qquad (6)$$

Let

$$P^1 = \left(\int_0^1 p^1(z)^{1-\eta}dz\right)^{\frac{1}{1-\eta}},\ P^2 = \left(\int_0^1 p^2(z)^{1-\eta}dz\right)^{\frac{1}{1-\eta}}.$$

They are prices of the consumption baskets in period 1 and period 2. By some calculations we obtain (please see Appendix)

$$\begin{aligned} u(X^0, X^1, X^2) &= \lambda\left[\int_0^1 p^1(z)c^1(z)dz + \int_0^1 p^2(z)c^2(z)dz\right] \\ &= \lambda[\delta W + \Pi + \Phi' - \delta R - \delta(\Theta + \Psi)], \end{aligned} \qquad (7)$$

$$\frac{P^2}{P^1} = \frac{\frac{\partial u}{\partial X^2}}{\frac{\partial u}{\partial X^1}}, \qquad (8)$$

$$P^1X^1 + P^2X^2 = \delta W + \Pi + \Phi' - \delta R - \delta(\Theta + \Psi). \qquad (9)$$

The indirect utility of consumers is written as follows

$$V = \frac{1}{\varphi(P^1, P^2)}[\delta W + \Pi + \Phi' - \delta R - \delta(\Theta + \Psi)] - \delta\beta. \qquad (10)$$

$\varphi(P^1, P^2)$ is a function which is homogeneous of degree one. The reservation nominal wage rate W^R is a solution of the following equation.

$$\frac{1}{\varphi(P^1,P^2)}[W^R+\Pi+\Phi'-R-\Theta-\Psi]-\beta=\frac{1}{\varphi(P^1,P^2)}(\Pi+\Phi').$$

From this

$$W^R=\varphi(P^1,P^2)\beta+R+\Theta+\Psi.$$

The labor supply is indivisible. If $W>W^R$, the total labor supply is L_f. If $W<W^R$, it is zero. If $W=W^R$, employment and unemployment are indifferent for consumers, and there exists no involuntary unemployment even if $L<L_f$.

Indivisibility of labor supply may be due to the fact that there exists minimum standard of living even in the advanced economy (please see Otaki (2015)).

Let $\rho=\frac{P^2}{P^1}$. This is the expected inflation rate (plus one). Since $\varphi(P^1,P^2)$ is homogeneous of degree one, the reservation real wage rate is

$$\omega^R=\frac{W^R}{P^1}=\varphi(1,\rho)\beta+\frac{R+\Theta+\Psi}{P^1}.$$

If the value of ρ is given, ω^R is constant.

Otaki (2007) assumes that the wage rate is equal to the reservation wage rate in the equilibrium. However, there exists no mechanism to equalize them. We assume that β and ω^R are not so large.

2.2 Firms

Let

$$\alpha=\frac{P^1X^1}{P^1X^1+P^2X^2}=\frac{X^1}{X^1+\rho X^2},\qquad 0<\alpha<1.$$

From (5) ~ (9),

$$\alpha[\delta W+\Pi+\Phi+(1-\delta)R-\delta(\Theta+\Psi)]\left(\int_0^1 c^1(z)^{1-\frac{1}{\eta}}dz\right)^{-1}c^1(z)^{-\frac{1}{\eta}}=p^1(z).$$

Since

$$X^1=\frac{\alpha[\delta W+\Pi+\Phi'-\delta R-\delta(\Theta+\Psi)]}{P^1},$$

we have

$$(X^1)^{\frac{1}{\eta}-1}=\left(\int_0^1 c^1(z)^{1-\frac{1}{\eta}}dz\right)^{-1}=\left(\frac{\alpha[\delta W+\Pi+\Phi'-\delta R-\delta(\Theta+\Psi)]}{P^1}\right)^{\frac{1}{\eta}-1}.$$

Therefore,

$$\alpha(\delta W+\Pi)\left(\frac{\alpha[\delta W+\Pi+\Phi'-\delta R-\delta(\Theta+\Psi)]}{P^1}\right)^{\frac{1}{\eta}-1}c^1(z)^{-\frac{1}{\eta}}$$

$$=\left(\frac{\alpha[\delta W+\Pi+\Phi'-\delta R-\delta(\Theta+\Psi)]}{P^1}\right)^{\frac{1}{\eta}}P^1c^1(z)^{-\frac{1}{\eta}}=p^1(z).$$

Thus,

$$c^1(z)^{\frac{1}{\eta}}=\left(\frac{\alpha[\delta W+\Pi+\Phi'-\delta R-\delta(\Theta+\Psi)]}{P^1}\right)^{\frac{1}{\eta}}P^1(p^1(z))^{-1}.$$

Hence,

$$c^1(z)=\frac{\alpha[\delta W+\Pi+\Phi'-\delta R-\delta(\Theta+\Psi)]}{P^1}\left(\frac{p^1(z)}{P^1}\right)^{-\eta}.$$

This is demand for good z of an individual of younger generation. Similarly, his demand for good z in period 2 is

$$c^2(z)=\frac{(1-\alpha)[\delta W+\Pi+\Phi'-\delta R-\delta(\Theta+\Psi)]}{P^2}\left(\frac{p^2(z)}{P^2}\right)^{-\eta}.$$

Let M be the total savings of consumers of the older generation carried over from their period 1. It is written as

$$M=(1-\alpha)[\overline{W}\overline{L}+L_f\overline{\Pi}+L_f\Phi-\overline{L}\overline{R}-\overline{L}(\overline{\Theta}+\overline{\Psi})].$$

$\overline{W}$, $\overline{L}$ and $\overline{\Pi}$ are the nominal wage rate, the employment and the profit in the previous period. $\overline{R},\overline{\Theta}$ and $\overline{\Psi}$ are the unempoymemt benefit, the tax for the unempoymemt benefit and the tax for the pay-as-you-go pension in the previous period. Note that Φ is the pay-as-you-go pension for a consumer of the older generation. M is the total savings or the total consumption of the older generation consumers including the pay-as-you-go pensions they receive in their period 2. It is the planned consumption that is determined in period 1 of the older generation consumers. Net savings is the difference between M and the pay-as-you-go pensions in their period 2, as follows:

$$M-L_f\Phi=(1-\alpha)\left[\overline{W}\overline{L}+L_f\overline{\Pi}-\overline{L}\overline{R}-\overline{L}(\overline{\Theta}+\overline{\Psi})\right]-\alpha L_f\Phi.$$

With this M their demand for good z is

$$\frac{M}{P^1}\left(\frac{p^1(z)}{P^1}\right)^{-\eta}.$$

The government expenditure constitutes the national income as well as consumptions of younger and older generations. The total demand for good z is written as

$$c(z)=\frac{Y}{P^1}\left(\frac{p^1(z)}{P^1}\right)^{-\eta}.$$

Y is the effective demand defined by

$$Y=\alpha\left[WL+L_f\Pi+L_f\Phi'-LR-L(\Theta+\Psi)\right]+R'+G+M.$$

G is the government expenditure other than the pay-as-yu-go pensions and the unemployment benefits, and R' is the consumption in the childhood period of consumers of the next generation (about this demand function please see Otaki (2007), (2009)). The total employment, the total profits and the total government expenditure are

$$\int_0^1 Ldz=L,\qquad \int_0^1 \Pi dz=\Pi,\qquad \int_0^1 Gdz=G.$$

We have

$$\frac{\partial c(z)}{\partial p^1(z)}=-\eta\frac{Y}{P^1}\frac{p^1(z)^{-1-\eta}}{(P^1)^{-\eta}}=-\eta\frac{c(z)}{p^1(z)}.$$

From $c(z)=Ly(L)$,

$$\frac{\partial L}{\partial p^1(z)}=\frac{1}{y(L)+Ly'}\frac{\partial c(z)}{\partial p^1(z)}.$$

The profit of Firm z is

$$\pi(z)=p^1(z)c(z)-\frac{W}{y(L)}c(z).$$

P^1 is given for Firm z. Note that the employment elasticity of the labor productivity is

$$\zeta=\frac{y'}{\frac{y(L)}{L}}.$$

The condition for profit maximization with respect to $p^1(z)$ is

$$c(z)+\left[p^1(z)-\frac{y(L)-c(z)y'\frac{1}{y(L)+Ly'}}{y(L)^2}W\right]\frac{\partial c(z)}{\partial p^1(z)}$$

$$=c(z)+\left[p^1(z)-\frac{1-Ly'\frac{1}{y(L)+Ly'}}{y(L)}W\right]\frac{\partial c(z)}{\partial p^1(z)}$$

$$=c(z)+\left[p^1(z)-\frac{W}{y(L)+Ly'}\right]\frac{\partial c(z)}{\partial p^1(z)}=0.$$

From this

$$p^1(z) = \frac{W}{y(L) + Ly'} - \frac{c(z)}{\frac{\partial c(z)}{\partial p^1(z)}} = \frac{W}{(1+\zeta)y(L)} + \frac{1}{\eta}p^1(z).$$

Therefore, we obtain

$$p^1(z) = \frac{W}{\left(1 - \frac{1}{\eta}\right)(1+\zeta)y(L)}.$$

With increasing returns to scale, since $\zeta > 0$, $p^1(z)$ is lower than that in a case without increasing returns to scale given the value of W.

3. The market equilibrium

3.1 The equilibrium with involuntary unemployment

Since the model is symmetric, the prices of all goods are equal. Then,

$$P^1 = p^1(z).$$

Hence

$$P^1 = \frac{W}{\left(1-\frac{1}{\eta}\right)(1+\zeta)y(L)}. \tag{11}$$

The real wage rate is

$$\omega = \frac{W}{P^1} = \left(1 - \frac{1}{\eta}\right)(1+\zeta)y(L).$$

If ζ is constant, this is increasing with respect to L.

The nominal aggregate supply of the goods is equal to

$$WL + L_f\Pi = P^1Ly(L).$$

The nominal aggregate demand is

$$\alpha\left[WL + L_f\Pi + L_f\Phi' - LR - L(\Theta + \Psi)\right] + R' + G + M$$
$$= \alpha P^1Ly(L) + \alpha\left[L_f\Phi' - LR - L(\Theta+\Psi)\right] + R' + G + M$$

Since they are equal,

$$P^1Ly(L) = \alpha P^1Ly(L) + \alpha\left[L_f\Phi' - LR - L(\Theta+\Psi)\right] + R' + G + M.$$

From (1) and (2) we have

$$(L_f - L)R = L\Theta,$$

and

$$L_f\Phi = L\Psi.$$

Therefore, we get

$$P^1Ly(L) = \alpha P^1Ly(L) + \alpha(L_f\Phi' - L_f\Phi - L_fR) + R' + G + M.$$

This means

$$P^1Ly(L) = \frac{\alpha(L_f\Phi' - L_f\Phi - L_fR) + R' + G + M}{1-\alpha} \tag{12}$$

In real terms

$$Ly(L) = \frac{\alpha\left(L_f\Phi' - L_f\Phi - L_fR\right) + R' + G + M}{P^1(1-\alpha)}, \tag{13}$$

or

$$L = \frac{\alpha\left(L_f\Phi' - L_f\Phi - L_fR\right) + R' + G + M}{P^1(1-\alpha)y(L)}. \tag{14}$$

$\frac{1}{1-\alpha}$ is a multiplier. (13) and (14) mean that the employment L is determined by $g + m$. It can not be larger than L_f. However, it may be strictly smaller than L_f $(L < L_f)$. Then, there exists involuntary umemployment. Since the real wage rate $\omega=\left(1-\frac{1}{\eta}\right)(1+\zeta)y(L)$ is increasing with respect to L, and the reservation real wage rate ω^R is constant, if $\omega>\omega^R$ there exists no mechanism to reduce the difference between them.

3.2 Negative real balance effect

If the nominal wage rate falls, the price of the goods (price of consumption basket) proportionately falls. If the employment changes, the rate of a fall in the nominal wage rate and that of the price may be different in the case of increasing or decreasing returns to scale. We assume that the difference is small. We can suppose that the real values of $G,\ \Phi,\ \Phi',\ R'$ do not change even when the price of the goods falls. On the other hand, the nominal value of R and
Then, whether the aggregate demand increases or decreases when the nominal wage rate and the price of the goods fall depend on whether

$$M-L_f\Phi-\alpha L_f R$$

Is positive or negative. If $M-L_f\Phi-\alpha L_f R<0$, there is a negative real balance effect (Pigou effect)[2].

4. Phillips Curve and fiscal policy

4. 1 A chgange in nominal wage rate with begative real balance effect

Suppse that the nominal wage rate falls. From (11) the price of the goods also falls. If the negative real balance effect works, the real aggregate demand decreases. Then, the output and the employment decrease. Therefore, the lower price is accompanied by employment loss. Alternatively, suppose that the nominal wage rate rises. The price of the goods also rises. If the negative real balance effect works, the real aggregate demand increases. Then, the output and the employment increase. Therefore, higher price is accompanied by an increase in employment. Thus, we obtain a negative relationship between the price and the unemployment rate (positive relationship between the price and employment) as represented by the Phillips curve.

Therefore, we obtain the negative ralationship between the unemployment rate and the inflation rate in the same period. Figure 1 depicts an example the Phillips Curve. U_t denotes the unemployment rate in period t.

[2] About the discussion of real balance effect please see Pigou (1934) and Kalecki (1940).

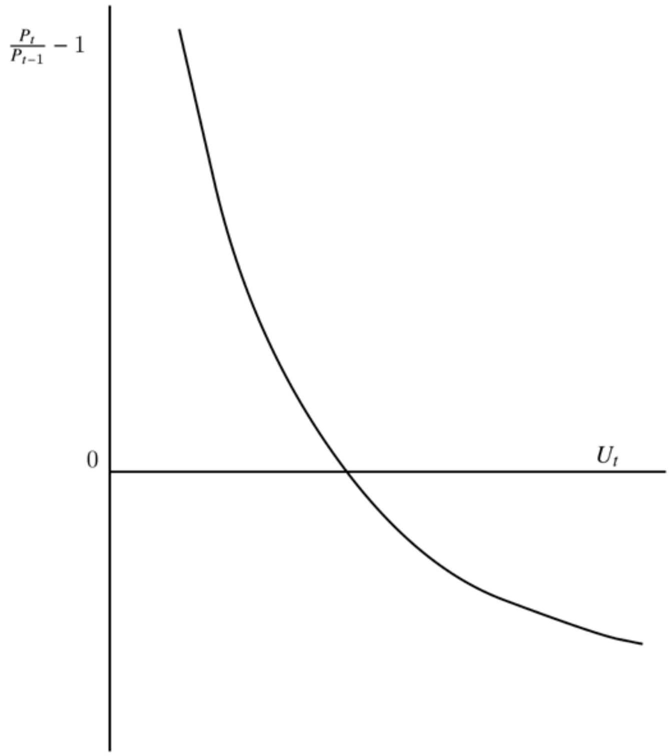

Figure 1: Phillips Curve

4. 2 Fiscal Policy

Let T be the tax revenue other than the taxes for the pay-as-you-go pension system and the unemployment benefits. Then, the budget constraint of the government is $G = T$, and the aggregate demand is

$$\alpha P^1 Ly(L) + \alpha\left[L_f\Phi' - LR - L(\Theta + \Psi) - T\right] + R' + G + M \text{ with } G = T. \tag{15}$$

With this aggregate demand (14) is

$$L = \frac{\alpha\left(L_f\Phi' - L_f\Phi - L_f R - T\right) + R' + G + M}{P^1(1-\alpha)y(L)} \text{ with } G = T. \tag{16}$$

This implies that the balanced budget multiplier is one.

Given labor productivity $y(L)$ and nominal wage rate W the price of the goods P^1 is determined by (11). If the government expenditure G increases given T, that is, an increment of the government expenditure is financed by seigniorage, from (16) employment L increases given the price P^1. Then, the Phillips curve in Figure 1 shifts to the left as in Figure 2, the employment increases and the unemployment rate decreases given the inflation rate. With increasing returns to scale $y(L)$ is increasing with respect to L. However, employment will still increase in that case. From (11) and (16) we obtain

$$\frac{dL}{dG} = \frac{1}{(1-\alpha)W} > 0,$$

$$\frac{dP^1}{dG} = -\frac{1}{(1-\alpha)W}\frac{P^1 y\prime(L)}{y(L)} < 0.$$

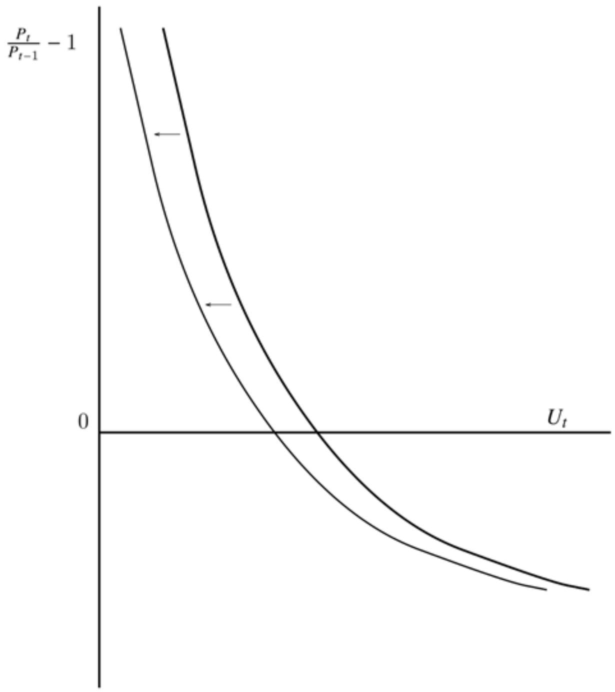

Figure 2 (a)

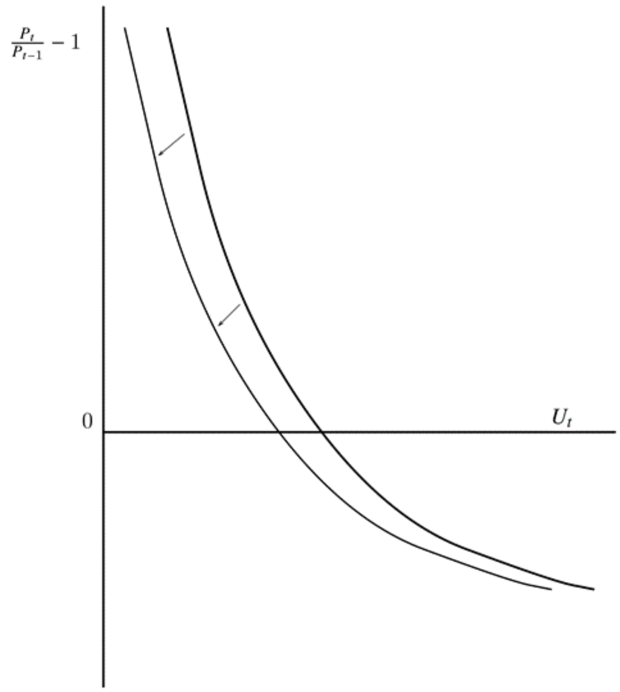

Figure 2 (b)

Figure 2 (a) depicts a case of $y(L) = 0$ (constant returns to scale), and Figure 2 (b) depicts a case of $y(L) > 0$ (increasing returns to scale).

4.3 Fiscal policy is a fiscal-monetary policy

The money suppy equals the sum of

1) Consumptions of the older generation consumers － pay-as-you-go pensions,
2) Government expenditure － taxes.

If the increase in the government expenditure is financed by seigniorage, it equals the increase in money supply. Therefore, the fiscal policy is also a fiscal-monetary policy. The increase in money supply does not raise the price. Thus, money is not neutral.

4.4 Exogenous changes in labor productivity

We consider exogenous changes in labor productivity given nominal wage rate. It may be due to a change in the unemployment rate in the previous period as assumed by Otaki and Tamai (2011, 2012). Suppose that the labor productivity $y(L)$ in a period, for example, period t increases to $\theta y(L)$ with a constant $\theta > 1$ given L. From (13) if g and m are constant, employment L decreases, that is, the unemployment rate in period t increases. (11) means that the price of the goods in period t given W falls because η an ζ are constant. Let P_t and P_{t-1} be the price of the goods (price of the concumption basket) in period t and that in period $t-1$. Then, the inflation rate from period $t-1$ to t, $\frac{P_t}{P_{t-1}} - 1$, falls given P_{t-1}.

Alternatively, a decrease in the labor productivity ($\theta < 1$) increases employment, decreases the unemployment rate, and raises the price of the goods and the inflation rate from period $t-1$ to t.

4.5 Analysis by Otaki and Tamai (2011, 2012)

Otaki and Tamai (2011, 2012) suppose that the low (or high) unemployment rate in a period, for example, period $t-1$ raises (or lowers) the labor productivity in period t by learning effect. If the unemployment rate in period $t-1$ increases, the labor productivity in period t falls. Then, from (11) the price of the goods rises, and the inflation rate from period t to period $t+1$ falls given the (expected) price of the goods in period $t+1$. Alternatively, a decrease in the unemployment rate in period $t-1$ rases the labor productivity in period t. Then, the price of the goods falls, and the infaltion rate from period t to period $t+1$ rises given the (expected) price of the goods in period $t+1$. Thus, they have shown the negative relation between the unemployment rate in period $t-1$ and the inflation rate from period t to period $t+1$, $\frac{P_{t+1}}{P_t}-1$. On the other hand, a fall in the price in period t means that the inflation rate from period $t-1$ to period t falls, that is, the low unemployment rate in period $t-1$ explains the low (not high) inflation rate from period $t-1$ to period t, $\frac{P_t}{P_{t-1}}-1$.

Their Phillips curve is depicted in Figure 3. U_{t-1} denotes the unemployment rate in period $t-1$.

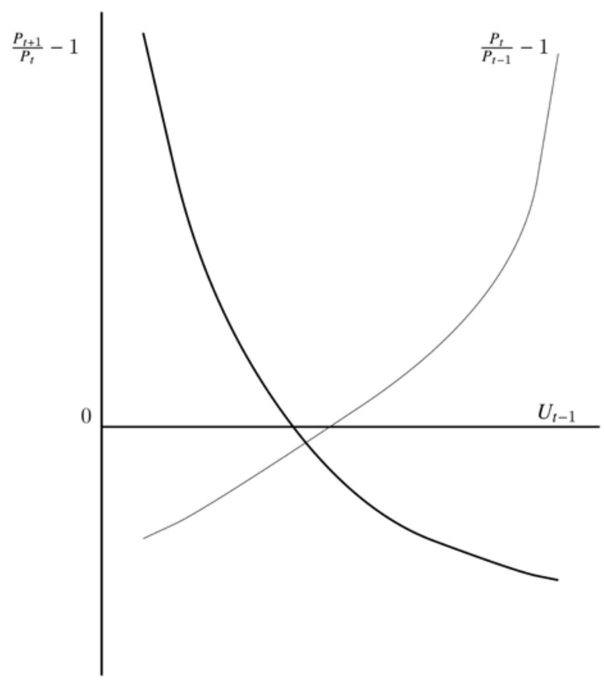

Figure 3: Phillips curve by Otaki and Tamai (2011, 2012)

5. Conclusion

We have shown that in a three-periods overlapping generations model under monopolistic competiton changes in labor prodictivity bring about the negative relation between the unemployment rate and the inflation rate in the same period. This conclusion is based on the premise of utility maximization of consumers and profit maximization of firms. Therefore, we have presented a microeconomic foundation of the Phillips curve.

Appendix: Derivations of (7), (8), (9) and (10)

From (5) and (6)

$$\frac{\partial u}{\partial X^1}X^1\left(\int_0^1 c^1(z)^{1-\frac{1}{\eta}}dz\right)^{-1}\int_0^1 c^1(z)^{1-\frac{1}{\eta}}dz=\frac{\partial u}{\partial X^1}X^1=\lambda\int_0^1 p^1(z)c^1(z)dz,$$

$$\frac{\partial u}{\partial X^2}X^2\left(\int_0^1 c^2(z)^{1-\frac{1}{\eta}}dz\right)^{-1}\int_0^1 c^2(z)^{1-\frac{1}{\eta}}dz=\frac{\partial u}{\partial X^2}X^2=\lambda\int_0^1 p^2(z)c^2(z)dz.$$

Since $u(X^1,X^2)$ is homogeneous of degree one,

$$u(X^1,X^2)=\frac{\partial u}{\partial X^1}X^1+\frac{\partial u}{\partial X^2}X^2.$$

Thus, we obtain

$$\frac{\int_0^1 p^1(z)c^1(z)dz}{\int_0^1 p^2(z)c^2(z)dz}=\frac{\frac{\partial u}{\partial X^1}X^1}{\frac{\partial u}{\partial X^2}X^2},$$

and

$$u(X^1,X^2)=\lambda\left[\int_0^1 p^1(z)c^1(z)dz+\int_0^1 p^2(z)c^2(z)dz\right]=\lambda[\delta W+\Pi+\Phi+(1-\delta)R-\delta(\Theta+\Psi)].$$

From (3) and (4), we have

$$\left(\frac{\partial u}{\partial X^1}\right)^{1-\eta}\left(\int_0^1 c^1(z)^{1-\frac{1}{\eta}}dz\right)^{-1}c^1(z)^{1-\frac{1}{\eta}}=\lambda^{1-\eta}p^1(z)^{1-\eta},$$

and

$$\left(\frac{\partial u}{\partial X^2}\right)^{1-\eta}\left(\int_0^1 c^2(z)^{1-\frac{1}{\eta}}dz\right)^{-1}c^2(z)^{1-\frac{1}{\eta}}=\lambda^{1-\eta}p^2(z)^{1-\eta}.$$

They mean

$$\left(\frac{\partial u}{\partial X^1}\right)^{1-\eta}\left(\int_0^1 c^1(z)^{1-\frac{1}{\eta}}dz\right)^{-1}\int_0^1 c^1(z)^{1-\frac{1}{\eta}}dz=\lambda^{1-\eta}\int_0^1 p^1(z)^{1-\eta}dz,$$

and

$$\left(\frac{\partial u}{\partial X^2}\right)^{1-\eta}\left(\int_0^1 c^2(z)^{1-\frac{1}{\eta}}dz\right)^{-1}\int_0^1 c^2(z)^{1-\frac{1}{\eta}}dz=\lambda^{1-\eta}\int_0^1 p^2(z)^{1-\eta}dz.$$

Then, we obtain

$$\frac{\partial u}{\partial X^1}=\lambda\left(\int_0^1 p^1(z)^{1-\eta}dz\right)^{\frac{1}{1-\eta}}=\lambda P^1,$$

and

$$\frac{\partial u}{\partial X^2}=\lambda\left(\int_0^1 p^2(z)^{1-\eta}dz\right)^{\frac{1}{1-\eta}}=\lambda P^2.$$

From them we get

$$u(X^1,X^2)=\lambda(P^1X^1+P^2X^2),$$

$$\frac{P^2}{P^1}=\frac{\frac{\partial u}{\partial X^2}}{\frac{\partial u}{\partial X^1}},$$

and

$$P^1X^1 + P^2X^2 = \delta W + \Pi + \Phi + (1-\delta)R - \delta(\Theta + \Psi).$$

Since $u(X^1, X^2)$ is homogeneous of degree one, λ is a function of P^1 and P^2, and $\frac{1}{\lambda}$ is homogeneous of degree one because proportional increases in P^1 and P^2 reduce X^1 and X^2 at the same rate given $\delta W + \Pi$. We obtain the following indirect utility function.

$$V = \frac{1}{\varphi(P^1, P^2)}[\delta W + \Pi + \Phi + (1-\delta)R - \delta(\Theta + \Psi)] - \delta\beta.$$

$\varphi(P^1, P^2)$ is a function which is homogenous of degree one.

Acknowledgment

This work was supported by the Japan Society for the Promotion of Science KAKENHI Grant Number 18K01594.

References

Calvo, G. A. (1983) Staggered prices in a utility-maximizing framework. Journal of Monetary Economics, 12, 383-398.

Kalecki, M. (1944) “The classical stationary state” A comment. Economic Journal, 54, 131-132.

Lucas, R.E. Jr. (1972). Expectations and the neutrality of money. Journal of Economic Theory, 4, 103-124.

Mankiw, N. G. and Reis, R. (2002). Sticky information versus sticky prices: a proposal to replace the new Keynesian Phillips curve, Quarterly Journal of Economics, 117, 1295-1328.

Otaki, M. (2007) The Dynamically Extended Keynesian Cross and the Welfare-Improving Fiscal Policy. *Economics Letters*, 96, 23–29.

Otaki, M. (2009) A Welfare Economics Foundation for the Full-Employment Policy. *Economics Letters*, 102, 1–3.

Otaki, M. (2011) *Fundamentals of the Theory of Money and Employment (Kahei-Koyo Riron no Kiso (in Japanese))*. Keiso Shobo.

Otaki, M. (2015) *Keynsian Economics and Price Theory: Re-orientation of a Theory of Monetary Economy,* Springer.

Otaki, M. (2016) *Keynes’s General Theory Reconsidered in the Context of the Japanese Economy*. Springer.

Otaki, M and M. Tamai (2011) Exact Microeconomic Foundation for the Phillips Curve under Complete Markets: A Keynesian View. DBJ Discussion Paper Series, No.1005, Development Bank of Japan.

Otaki, M and M. Tamai (2012) A Microeconomic Foundation for the Phillips Curve under Complete Markets without any Exogenous Price Stickiness: A Keynesian View. Theoretical Economics Letters, 2, 482-486.

Phillips, A. W. (1958). The relation between unemployment and the rate of change of money wage rates in the United Kingdom, 1861-1957. *Economica*, 25, 283-299.

Pigou, A. C. (1943) The classical stationary state. Economic Journal, 53, 343–351.

Woodford, M. (1996) Control of the public debt: A requirement for price stability? NBER Working Papers: 5684.