Persistent Ideology and the Determination of Public Policies over Time

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Abstract

This paper investigates how public policy responds to persistent ideological shifts in dynamic politico-economic equilibria. To this end, we develop a tractable model to analyze the dynamic interactions among public policy, individuals’ intertemporal choice and the evolution of political constituency. Analytical solutions are obtained to characterize Markov perfect equilibria. Our main finding is that a right-wing ideology may increase the size of government. Data from a panel of 18 OECD countries confirm that after controlling for the partisan effect, there is a positive relationship between the right-wing political constituency and the government size. This is consistent with our theoretical prediction, but hard to explain by existing theories.

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Key Words: Markov Equilibrium, Persistent Ideology, Political Economy, Public Policy, Repeated Voting

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1 Introduction

Modern political economy is designed to reveal the underlying mechanism of policy decision-making. A salient feature in real world democracies is that policy attitudes are often driven by motives that seem hard to reconcile with mere economic factors. The empirical literature has long documented that ideology plays a key role in shaping policy preferences.\footnote{For instance, Sears, Lau, Tyler and Allen (1980) show that symbolic attitude (mainly liberalism-conservatism ideology and party identification) far outstripped all self-interest variables in terms of predicting support for policies in the United States: the contribution of symbolic attitudes to $R^2$ ranged between 10% and 17%, while the contribution of self-interests never exceeds 4%. In addition, Levitin and Miller (1979), Knight (1985), Alvarez and Nagler (1995, 1998), among many others, show that ideology turns out to be a significant predictor for individuals' voting choice in the U.S. presidential elections.} Many theoretical frameworks like the probabilistic voting model (e.g. Lindbeck and Weibull, 1987) also incorporate ideology as an important factor for political decisions. An ignored fact by the existing theory, however, is the persistency of ideological shifts. For example, pro-redistribution "leftist" policies were highly popular in the 1950s and 1960s, while a rightist mood appeared to dominate in the late 1970s and 1980s.\footnote{See Robinson (1984), Robinson and Fleishman (1984) and Durr (1993) for discussions of the US survey data.} The impacts of such persistent ideological waves are far from trivial. In particular, they lead to prospective changes in the government type and the associated policy outcomes, which may influence private intertemporal choices and even the distribution of future voters.\footnote{Recent theoretical work has shown that the evolution of political constituency can be endogenously driven by private intertemporal choices in a full-fledged dynamic environment (e.g. Hassler, Rodriguez Mora, Storesletten and Zilibotti, 2003).} Such variations in response to ideological shifts naturally affect choices of the incumbent government, indicating a distinct role of ideology in the policy decision process.

This paper therefore aims to show explicitly how persistent ideology influences the determination of public policies. To this end, we construct a politico-economic model that has the ability of capturing rich dynamic interactions among policies, private decisions, and the evolution of the distribution of voters. Our main finding is that a right-wing ideology may increase the size of government. The underlying mechanism is two-fold. First, a persistent ideological shift towards the right implies that the right-wing be more likely to come to power in the future. Since the right-wing imposes lower taxes on average, the ideological shock encourages investment by reducing expected future tax rates. This makes the investment less elastic to the current tax rate and hence provides the incentive for the incumbent to increase taxes. Moreover, the shock generates a self-reinforcing process on the distribution of future voters.
voters. More investment results in a larger size of individuals in favor of the right-wing, which increases further future election probabilities of the right-wing. The impact of ideology can thus be amplified by this endogenous response of probabilities over future government types.

The model is primarily based on a tractable framework recently developed by Hassler, Storesletten and Zilibotti (2007). There are two types of individual economic status, the rich and poor. Individuals make human capital investment, which can increase their likelihood of being rich. Two political parties run electoral competition. The right-wing and left-wing party, modeled as citizen-candidates (Osborne and Slivinski, 1996, Besley and Coate, 1997), represent the rich and poor, respectively. To incorporate ideology, we assume that a proportion of the poor (rich) vote for the right-wing (left-wing) party. The discrepancy between individuals’ economic interests and their political preferences captures the impact of ideology on voting behavior. The election is thus codetermined by two fundamentals in the economy: the size of the rich (or poor) and the ideological state.

A distinctive feature of our model is that public policies, private investment and the distribution of future voters are mutually affected over time. To show how policies are determined in this environment, we focus on Markov perfect equilibria, where the dynamic interactions can are characterized by two fixed-points: the ideology-contingent distribution of future voters and the ideology-contingent policy rules. Under quasi-linear preferences and uniformly distributed ideological shocks, the equilibrium can be solved analytically.

The standard partisan model suggests that ideological shifts play no role in the policy decision process, as long as the current type of government remains unchanged. By contrast, our model implies a positive relationship between the government size and the right-wing ideology within each political regime. It is then left for empirical study whether the positive relationship holds in real democracies or is just a counterfactual result. We provide evidence from an OECD panel that a more right-wing political constituency indeed leads to a larger size of government, which is consistent with our prediction but hard to explain by existing theories. Specifically, we find that one percentage increase in the vote share of right-wingers is associated with an increase in the central government revenue GDP ratio of 0.17%. This result is statistically significant and quite stable to a number of control variables and estimation specifications.

There is a growing literature on the dynamics of government without commitment techniques (e.g., Besley and Coate, 1998, Hassler et al., 2003, Hassler et al., 2007).\textsuperscript{4} This strand of research, including the present paper, emphasizes the fact that in representative democracies,

\textsuperscript{4}See also Krusell, Quadrini and Rios-Rull (1997), Krusell and Rios-Rull (1999).
the incumbent government has limited abilities to commit to policies after the next election. An important finding is that the multiplicity of equilibrium may appear due to the lack of commitment. Although multiple equilibria *per se* are theoretically interesting, they are unable to provide sharp empirical predictions. Our paper contributes to the literature by providing a way of killing multiple equilibria. We show that sufficient ideological uncertainty rules out the indeterminacy of belief in future political outcomes, which can easily be self-fulfilled in the context without uncertainty. The unambiguous hypothesis on the relationship between ideology and public policies allows empirical study to test again the theory.

The effects of changes in the future government type on equilibrium outcomes have been studied by some recent work such as Amador (2003) and Song, Storesletten and Zilibotti (2007). However, much of the literature ignores a potential important channel that runs from current policies back to future election probabilities. Azzimonti Renzo (2005) extends the analysis by endogenizing the distribution of voters in a dynamic setup. Like Azzimonti Renzo, we also allow current political and private decisions to affect the evolution of political constituency. The focus of our paper, however, is fundamentally different. We are interested in how persistent ideological shifts change policies, while ideological shocks are purely iid in Azzimonti Renzo (2005), acting as a device of endogenizing election outcomes, and therefore play a similar role as in the partisan model.

Although this paper aims to understand the influence of persistent ideology on the determination of public policies, it is also relevant for a long-standing issue in political science and sociology concerning the causes of changes in political constituency. A sizable empirical literature shows that political identifications are related to lagged economic conditions. However, few works have been done for formalizing the dynamic interaction between macroeconomy and political cycles. Our model, based on rational choices of parties and individuals, contributes to the literature by building a theoretical framework of analyzing changes in political constituency in response to exogenous ideological shocks and endogenous public policies.

The rest of the paper is organized as follows. Section 2 describes the model and solves a static example. Section 3 gives the conditions for the existence and uniqueness of the Markov perfect equilibrium. In Section 4, we provide a closed-form solution when the ideological shock follows uniform distribution. Section 5 shows empirical evidence. Section 6 discusses the role of ideological uncertainty on the multiplicity of equilibria. Section 7 concludes.

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5 Earlier researches include Persson and Svensson (1989), Alesina and Tabellini (1990), among others, providing examples of strategic policy decision-making under future electoral uncertainty.

2 The Model

2.1 The Model Economy

The model economy is primarily based on a tractable framework recently developed by Hassler \textit{et al.} (2003) and Hassler \textit{et al.} (2007). The economy is inhabited by an infinite sequence of overlapping-generations. Each generation has a unit mass and lives two periods. There are two types of old individuals endowed with different productivity, referred to as the old poor and rich, respectively. The wage of the old rich is normalized to unity and the poor earn zero. The benefits from public good consumption $g$ are identical across old individuals. The government imposes a proportional income tax rate $\tau^o$ on the old. Let $u^{ou}$ and $u^{os}$ be the utilities of the old poor and rich, respectively. These are equal to

\begin{align}
  u^{ou}_t &= a^o g_t, \\
  u^{os}_t &= 1 - \tau^o_t + a^o g_t,
\end{align}

where $a^o \in (0, 2]$ is the constant marginal utility of public good for the old.\footnote{Assuming equal marginal utility of public spending across households is for notational convenience. It can be argued that the poor care public spending more than the rich. The following results carry over to the case in which $a^o$ is different between the poor and the rich.}

Young individuals are ex-ante homogenous. They make a human capital investment $h$, which increases the probability $p$ of being rich in their life time. Without loss of generality, let $p = h \in [0, 1]$. As old individuals, the wage of the young rich equals unity and the poor earn zero. $\tau^y$ is the proportional income tax rate for young individuals. Assuming a linear-quadratic preference over consumption and costs of human capital investment, the expected utility of a young household is

\begin{equation}
  u^y_t = h_t (1 - \tau^y_t) + a^y g_t - h_t^2 + \beta E[u^{ou}_{t+1}],
\end{equation}

where $E$ is the expectation operator and $\beta \in [0, 1]$ denotes the discount factor. $a^y$ is the marginal utility of public good for the young. Since the probability of being rich when old is equal to $h$, we have

\begin{equation}
  E[u^o_{t+1}] = h_t E[u^{ou}_{t+1}] + (1 - h_t) E[u^{os}_{t+1}].
\end{equation}

The age-dependent taxation has its realistic counterparts. Many public programs and tax policies have important age-dependent elements. In addition, the young and old may evaluate public goods, such as public health care, in quite different ways. Allowing for age-
dependent taxation also simplifies the analytical characterization, without making any fundamental change to the results.8

Through the wage structure, the old and young produce $h_{t-1}$ and $h_t$, respectively. Thus, the aggregate output $y_t$ equals

$$y_t = h_{t-1} + h_t.$$  

(5)

Total tax incomes and public spending amount to $\tau^o_t h_{t-1} + \tau^y_t h_t$ and $2g_t$, respectively. We assume that the government budget must be balanced in each period, which implies

$$g_t = \frac{\tau^o_t h_{t-1} + \tau^y_t h_t}{2}.$$  

(6)

2.2 The Political Decision Process

The sequence of tax rates is set through a repeated political decision process. We assume that only old individuals vote. This captures, in an extreme fashion, the phenomenon that the old are more influential in the determination of public policies.9 It would be observationally equivalent to assume that voting occurs at the end of each period. Old individuals have no interests at stake and thus, abstain from voting. For expositional ease, we keep the former interpretation throughout the paper. We will show in Section 4.2 that a relaxation of this assumption leads to no major changes in our main findings.

The left-wing and right-wing party, modeled as citizen-candidates, represent the old poor and rich, respectively. The party candidates cannot credibly commit to any policy other than that preferred by the group they represent. For simplicity, we assume zero entry cost.10 With absence of ideology, the majority rule implies that election outcomes are deterministic and solely depend on the distribution of old individuals’ economic situation. However, there are numerous papers providing convincing evidence that, besides economic reasons, electorates’ ideological label also plays a significant role in their policy preference and voting choice. To capture this phenomenon, we introduce another variable, namely the ideological state, to reflect the discrepancy between electorates’ economic interests and their political preference. As a consequence, election outcomes become codetermined by the distribution of old individuals’

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8See Section 4.2 for more details.

9For instance, Mulligan and Sala-i-Martin (1999) argue that the old have more influence in the political decision process because they have a lower cost of time. Empirically, the voting turnout is indeed lower for younger households (e.g. Wollinger and Rosenstone, 1980). See Hassler et al. (2003) and Hassler et al. (2006) for more detailed discussions.

10For simplicity, we assume zero entry cost, which shuts down the entry game in the standard citizen-candidate model. Consequently, both the candidate representing the rich and that representing the poor will participate in the electoral competition. However, we still regard the two-party system as a simplified citizen-candidate model, since the party candidates cannot credibly commit to any policy platform other than their preferred policies, as in Osborne and Slivinski (1996) and Besley and Coate (1997).
economic situation and the ideological state. Specifically, we assume that an ideological shock can switch a proportion of the poor (rich) to the right-wing (left-wing) side in terms of voting choice. Define the left-wingers (right-wingers) as old households voting for the left-wing (right-wing) party. The election outcome is determined by the proportion of right-wingers $e_t$:

$$
e_t = \begin{cases} 
1 & s_t \geq 1 - h_{t-1} \\
h_{t-1} + s_t & s_t \in (-h_{t-1}, 1 - h_{t-1}) \\
0 & s_t \leq -h_{t-1}
\end{cases}, \quad (7)$$

where $s_t$ is the ideological state at time $t$ and $h_{t-1}$ is the population of the old rich, or equivalently, the human capital investment at time $t - 1$. A positive (negative) $s_t$ switches some of the poor (rich) to vote for the right-wing (left-wing) party. Thus, a high (low) $s_t$ refers to a more right-leaning (left-leaning) ideology. The right-wing party wins the election if $e_t > \frac{1}{2}$. Otherwise the left-wing is elected.\(^{11}\) Note that (7) ensures that $e_t \in [0, 1]$ always holds. When $s_t$ takes an extreme value (either very high or very low), the economic determinant $h_{t-1}$ is wiped off. Outside these "ages of extremes" (Hobsbawm, 1996), economic motives may sway voters. Since ideological movements tend to be persistent (e.g. Robinson and Fleishman, 1984), $s_t$ is assumed to follow a stationary AR(1) process, whose properties will be defined and discussed below.\(^{12}\)

It has been a long tradition in the literature of political economy that the poor (rich) is synonymous to the left (right). This receives some empirical supports from the finding that increased employment raises the popularity of the left government, while inflation reduces the popularity of the right via the wealth effect (e.g. Haynes and Jacob, 1994). However, it will be far-fetched to believe that political constituency is purely determined by economic factors.\(^{13}\) In fact, (7) can be thought of as a parsimonious way of capturing the influence of both economic and ideological factors on the formation of political constituency.

The timing of events in each period is described as follows. Citizen candidates announce their policy platforms at the beginning of each period. An ideological shock is realized afterwards. The elected party then implements her preferred tax rates and public spending. Given public policies, young individuals invest in human capital. Their being rich or poor is unfolded after the investment.

\(^{11}\)We assume that the left-wing comes into power if the proportion of left-wingers ties the right-wingers.

\(^{12}\)The existence and uniqueness of the dynamic politico-economic equilibrium can easily be extended to an AR($n$) process with $n > 1$.

\(^{13}\)Besides the extensive evidence provided by political scientists, it is worthy of mentioning a recent empirical study from Di Tella and MacCulloch (2005) suggesting the importance of ideology. Based on survey data from 10 OECD countries for 1975-1992, they find that "... respondents declare themselves to be happier when the party in power has a similar ideological position to themselves, even after we control for key performance indicators such as unemployment, inflation and income." (Di Tella and MacCulloch, 2005, pp.378)
2.3 Two Effects of Ideology

The right-wing party sets $\tau^o_t$ so as to maximize the utility of the rich $u^o_t$ in (2), subject to the balanced-budget constraint (6). The assumption that $a^o \leq 2$ is sufficient for the right-wing to set $\tau^o_t = 0$. The left-wing sets $\tau^l_t$ by maximizing $u^o_t$ in (1), which is equivalent to maximizing fiscal revenues $\tau^o_t h_{t-1} + \tau^l_t h_t$. Since $h_{t-1}$ is predetermined and $\tau^o_t$ does not distort young individuals' human capital investment, the left-wing will set $\tau^o_t = 1$. In other words, the left-wing government eliminates the income inequality of old individuals by imposing a 100% tax rate. To conclude, $\tau^o_t$ follows a binary rule

$$
\tau^o_t = \begin{cases} 
1 & \text{if } e_t \leq \frac{1}{2} \\
0 & \text{otherwise}
\end{cases},
$$

(8)

The disagreement on tax $\tau^o_t$ exhibits the feature of a two-party system. Despite the conflict of interest between left-wingers and right-wingers in terms of $\tau^o_t$, their preferences on $\tau^o_t$ are perfectly aligned: attaining the top of the Laffer curve to maximize taxes from young individuals. This is because citizen candidates only represent the interests of the old rich and poor. None of them care about the welfare of the young.\footnote{The indifference between $\tau^l_t$ preferred by the leftist and rightist seems reasonable since the young are homogenous when candidates implement their policies.} So $\tau^y_t$ solves

$$
\tau^y_t = \arg\max_{T_t} T_t,
$$

(9)

where $T_t \equiv \tau^y_t h_t$. Three remarks are in order. First, the incumbent at time $t$ would be better off if she could promise $\tau^{o+1}_t = 0$ to encourage human capital investment $h_t$. Without commitment techniques, however, the promise is not credible since future polices are repeatedly decided by the winners of future elections. So $\tau^{o+1}_t$ must follow the binary rule (8). Second, in the present setup, both parties would like to see the right-wing being elected in the next period, since $\tau^{o+1}_t = 0$ in the right-wing regime encourages investment and thus, enlarges the tax base. The lack of reelection concern is consistent with the "short-sighted" citizen candidates only living one period.\footnote{In an earlier version of this paper, we consider the case in which the incumbent has reelection concern by assuming politicians care about future policies. This gives rise to the "opportunist" motive and the "strategic" motive, which generate two opposite effects, as in Jonsson (1997). However, the two opposite effects largely cancel out with each other and do not affect our main results.} Finally, the political parties would disagree on the tax rate imposed on the young if the government were not allowed to adopt age-dependent taxation. As will be discussed later, the disagreement will not leave qualitative changes to our main results.

We now distinguish in our model two channels for ideology to affect policies. Ideology affects election and therefore the current government type. The effect of an ideological shock on policies through this channel is analogous to that in the standard partisan models and will
be referred to as the partisan effect. The second channel, the focus of the paper, governs how policies respond to endogenous changes in private decisions and the distribution of future voters driven by an ideological shock in equilibrium. The effect of ideology through this channel is referred to as the equilibrium effect. As a warm-up exercise to facilitate the intuition, let us first study a static example with no private intertemporal trade-off. This helps identify the partisan effect of ideology.

2.3.1 A Static Example: the Partisan Effect of Ideology

In this static example, we assume the probability of being rich in old age \( p \) to be exogenous. The corresponding politico-economic equilibrium is straightforward. The policy rule of \( \tau_y \) follows (8). According to (3), young individuals’ human capital investment solves

\[
h = \arg \max_{\hat{h} \in [0,1]} (1 - \tau_y) \hat{h} - \hat{h}^2,
\]

which yields

\[
h = \frac{1 - \tau_y}{2}.
\]

We will continue to drop the time subscript when it does not create any confusion. (11) shows that private choice is independent of ideology. Substituting (11) into (9), we obtain an equalized distorting tax rate and human capital investment under any ideological state.

\[
\begin{align*}
\tau_y &= \frac{1}{2}, \\
h &= \frac{1}{4}.
\end{align*}
\]

(8) and (12) give the policy rules. Assuming away intertemporal trade-offs shuts down the link between ideology and the current distorting policy \( \tau_y \). Since \( e = p + s \), combining (8) and (12), the share of government spending or revenue can be computed as a percentage of aggregate output \( \gamma \equiv 2g/y \):

\[
\gamma = \begin{cases} 
\text{1/8} + p & \text{if } s \leq \frac{1}{2} - p \\
\frac{1}{4} + p & \text{otherwise}
\end{cases},
\]

where \( \gamma \) measures the size of government. If the ideology shock \( s \leq 1/2 - p \), the left-wing party, representing the interests of the poor, wins the election and implements higher taxes and larger spending for redistributive reasons. Otherwise, the right-wing wins and imposes lower taxes for the sake of the rich. Since the impact of ideology is limited to election outcomes, the difference in \( \gamma \) reflects the partisan effect of ideology on policies. Particularly, a right-wing ideology may increase the government size by changing the political regime from the left-wing to the right-wing.\textsuperscript{16} The implication from the partisan effect is thus in accordance with the

\textsuperscript{16}The ratio of \( \gamma \) in the left-wing regime to \( \gamma \) in the right-wing regime is equal to \( 1 + 8p > 1 \).
3 The Politico-Economic Equilibrium

This section analyses the benchmark setup, where the probability of being rich in old age is endogenously determined by human capital investment, i.e., \( p = h \). Since the investment and the future political outcomes are mutually affected, the private intertemporal trade-off turns out to be much more complicated than (10) in the static example. Denote \( x \) and \( x' \) as the variable \( x \) in the current and following period, respectively. The expected utility \( u^y \) in (3) implies that \( h \) depends on \( E[\tau^o] \). According to the binary tax rule (8), \( E[\tau^o] \) is equal to \( 1 - \pi \), where \( \pi \equiv \Pr (e' > 1/2) \) denotes the right-wing’s probability of being elected in the next period. Alternatively, \( \pi \) can be considered as a variable characterizing the distribution of future voters. Plugging (8) into (3), young individuals solve

\[
h = \arg \max_{h \in [0,1]} (1 - \tau^y + \beta\pi) \hat{h} - \hat{h}^2.
\]

The utility from public good is irrelevant for the decision \( h \), due to the atomistic individuals taking \( g \) and \( g' \) as given. (15) yields

\[
h = \frac{1 - \tau^y + \beta\pi}{2}.
\]

In addition to the negative effect of \( \tau^y \) on \( h \) in (11), (16) says that \( h \) increases in \( \pi \), i.e. the probability for a right-wing government to be elected, since such a government will adopt the tax-free policy for old individuals. Moreover, (16) provides a way of pinning down \( \pi \) in equilibrium. Substituting (16) into (7) and recalling that \( e' = h + s' \), we obtain

\[
\pi = \Pr (e' > \frac{1}{2}) = \Pr \left( \frac{1 - \tau^y + \beta\pi}{2} + s' > \frac{1}{2} \right).
\]

The equilibrium probability \( \pi \) is thus a fixed point of equation (17). In particular, the link between \( h \) and \( \pi \) implies that an ideological shock generate a self-reinforcing process on the distribution of future voters. An increase of \( s \) (a right-wing ideology) leads to a high \( \pi \) and therefore a high \( h \). More human capital investment, in turn, increases \( \pi \) as more individuals will be rich and in favor of the right-wing in the next period. We shall see explicitly in Section 4 how the effect of ideology is amplified through this process.

3.1 The Ideology-Contingent Distribution of Future Voters

Before characterizing the fixed-point problem, we first specify the properties of the stochastic process of \( s \) as follows. The density function is defined by \( f : R^2 \rightarrow [0, \infty) \) with \( \int f (s', s) \, ds' = 1 \).
for any given $s$. By (17), we know that $\pi$ depends on $\tau^y$ and the probability of the future ideological state $s'$, which in turn is contingent on the current ideological state $s$. Hence, $\pi$ can be written as a function of $\tau^y$ and $s$, $\pi : [0,1] \times R \to [0,1]$, which solves the following functional equation implied by (17):

$$
\pi (\tau^y, s) = \int_{s' > \frac{\tau^y + \beta \pi (\tau^y, s)}{2}} f (s', s) \, ds'.
$$

(18)

The existence of the ideology-contingent probability $\pi (\tau^y, s)$ can easily be obtained by the following assumptions. Define $X \equiv [\underline{s}, \bar{s}]$, where $-\infty < \underline{s} < \bar{s} < \infty$. Assume

A1: $s'$ and $s \in X$.

A2: $f (s', s)$ is bounded and uniformly continuous.

Lemma 1 Assume A1 and A2. Then there exists a uniformly continuous function $\pi (\tau^y, s)$ that solves (18).

Proof: See the appendix.

A2 is only a sufficient condition for the existence. $\pi (\tau^y, s)$ can exist under discontinuous distributions, as shall be seen in Section 4. The following assumption gives the sufficient condition for the uniqueness of $\pi (\tau^y, s)$.

A3: $f (s', s) < 2/\beta$ for all $s'$ and $s \in X$.

Lemma 2 Assume A1 and A3. Then there exists a unique $\pi (\tau^y, s)$ that solves (18).

Proof: See the appendix.

Lemma 2 implies that sufficient ideological uncertainty can rule out the indeterminacy of beliefs, which features a number of recent researches on dynamic politico-economic equilibrium with endogenous identity of the policymaker (e.g. Hassler et al., 2003). We will relax assumption A3 and study the multiplicity of equilibria in Section 6.

Plugging the ideology-contingent probability $\pi (\tau^y, s)$ into (16) gives

$$
h (\tau^y, s) = \frac{1 - \tau^y + \beta \pi (\tau^y, s)}{2}.\ 
$$

(19)

By (7), the future political constituency $e'$ evolves according to

$$
e' (s', \tau^y, s) = \begin{cases} 
1 & \text{if } s' \geq 1 - h (\tau^y, s) \\
h (\tau^y, s) + s' & \text{if } s' \in (-h (\tau^y, s), 1 - h (\tau^y, s)) \\
0 & \text{if } s' \leq -h (\tau^y, s)
\end{cases}.\ 
$$

(20)
3.2 Markov Perfect Equilibrium

Given the ideology-contingent probability \( \pi(\tau_y, s) \) solved from (18) and the individual investment strategy (19), an incumbent sets \( \tau_y \) according to (9). Let \( T(\tau_y, s) \equiv h(\tau_y, s) \tau_y \). The problem is

\[
\tau_y(s) = \arg \max_{\tau_y \in [0,1]} T(\tau_y, s).
\] (21)

Two remarks are in order. First, the theorem of maximum implies that \( \tau_y : \mathbb{R} \rightarrow [0,1] \) be an upper hemi-continuous mapping. Second, \( \tau_y \) only depends on the current ideological state \( s \). One may guess that \( \tau_y \) should also depend on the state of political constituency \( e = s + h_{-1} \), as \( \tau^0 \) in the Markovian tax rule (8). In fact, \( e \) or the identity of the incumbent has no influence on \( \tau_y \), since the objectives of two parties over \( \tau_y \) are perfectly aligned: maximizing tax revenue from the young.

Compared with the ideology-independent policy rule (12) in the static example, it can be found that \( \tau_y(s) \) reflects the equilibrium effect of ideology, i.e., the impact of ideology on the policy decision-making via endogenous changes of private choices and political constituency in equilibrium. The equilibrium effect appears when individuals condition human capital investment on ideology-contingent probabilities over future political outcomes. More specifically, ideological movements may affect policy decision-making via the ideology-contingent elasticity of tax base \( h \):

\[
\epsilon(\tau_y, s) = \frac{\tau_y - \beta \tau_y \partial \pi(\tau_y, s) / \partial \tau_y}{1 - \tau_y + \beta \pi(\tau_y, s)},
\] (22)

where \( \epsilon \) denotes the absolute value of the elasticity of the tax base with respect to \( \tau_y \). An immediate observation is that, given \( \partial \pi(\tau_y, s) / \partial \tau_y \), \( \epsilon \) is decreasing in \( \pi \). That is to say, the current tax base tends to be less elastic when the future voting is more favorable for the right-wing. The intuition is straightforward. Since \( \tau^0 \) in the right-wing regime is lower than in the left-wing regime, a more rightist future political constituency implies a low expected \( \tau^0 \) and hence encourages the current investment. This leads to a less elastic tax base.

We focus on Markov perfect equilibria, in which private and public choices are conditioned to payoff-relevant state variables.17 There are two state variables in our model: the ideological state \( s \) and the proportion of right-wingers \( e = s + h_{-1} \). These two state variables are payoff-relevant since they determine the current election and thus, policy outcomes. So the Markovian political equilibrium can be defined as follows.

**Definition 1** A (Markov perfect) political equilibrium is a set of mappings \( \tau^0(e) \), \( \tau_y(s) \), \( \pi(\tau_y(s), s) \), and \( h(\tau_y(s), s) \) such that:

17The dynamic game in our model also allows for equilibria with trigger strategies.
(1) $\tau^0 (e)$ follows (8);
(2) given $\tau^y (s)$, the probability of election $\pi (\tau^y (s), s)$ solves (18);
(3) given $\pi (\tau^y (s), s)$, the human capital investment $h (\tau^y (s), s)$ follows (19);
(4) given $h (\tau^y (s), s)$, the incumbent solves $\tau^y (s)$ by (21).

4 An Analytical Solution

In this section, we provide a closed-form solution of the Markov perfect equilibrium. The complete characterization of the equilibrium reveals the dynamic interactions among political constituency, distortionary policy decision-making and individuals’ intertemporal choice. We assume that $s'$ follows an AR(1) process with a symmetric uniformly distributed innovation

$$s' = \rho s + \varepsilon'.$$

The ideological shock is stationary and persistent, i.e., $\rho \in (0, 1)$. The density of $\varepsilon$ equals $1/(2z)$ if $\varepsilon \in (-z, z)$ and 0 otherwise. So the conditional density function of $s'$ is

$$f (s', s) = \begin{cases} \frac{1}{2z} & \text{if } s' \in (\rho s - z, \rho s + z) \\ 0 & \text{otherwise} \end{cases}.
$$

Now the functional equation (18) becomes

$$\pi (\tau^y, s) = \begin{cases} \frac{1}{2z} (\rho s + z - \frac{\tau^y - \beta \pi (\tau^y, s)}{2}) & \text{if } \frac{\tau^y - \beta \pi (\tau^y, s)}{2} \leq \rho s - z \\ 0 & \text{if } \frac{\tau^y - \beta \pi (\tau^y, s)}{2} \in (\rho s - z, \rho s + z) \\ \frac{1}{2z} (\rho s + z - \frac{\tau^y - \beta \pi (\tau^y, s)}{2}) & \text{if } \frac{\tau^y - \beta \pi (\tau^y, s)}{2} \geq \rho s + z \end{cases}.
$$

The linearity makes the analytical solution straightforward. Assumption A3 implies that $z > \beta/4$, which gives the sufficient condition for the uniqueness of $\pi (\tau^y, s)$ under the uniform distribution (24). In this section, we assume that $z > \beta/4$. It can be shown that $z > \beta/4$ is also necessary. The opposite case $z < \beta/4$, which produces multiple equilibria, will be studied in Section 6. Solving (25) yields:

$$\pi (\tau^y, s) = \begin{cases} \frac{1}{2(\rho s + z - \tau^y)} & \text{if } \tau^y \leq \lambda^- (s) \\ \frac{2(\rho s + z - \tau^y)}{4z - \beta} & \text{if } \tau^y \in (\lambda^- (s), \lambda^+ (s)) \\ \frac{1}{2(\rho s + z - \tau^y)} & \text{if } \tau^y \geq \lambda^+ (s) \end{cases},
$$

where $\lambda^- (s) \equiv 2 (\rho s - z) + \beta$ and $\lambda^+ (s) \equiv 2 (\rho s + z)$. Note that $\lambda^+ (s) > \lambda^- (s)$ as long as $z > \beta/4$. For notational convenience, we refer to $\lambda^+ (s) < 0$, or equivalently $s \leq -z/\rho$, as

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18 Analytical solution is also available, though much more tedious, under more general setups. For example, $s'$ follows an AR(n) process with the innovation that has a piecewise linear cumulative distribution function.

19 $\pi (\tau^y, s)$ does not exist if $z = \beta/4$. The non-existence of $\pi (\tau^y, s)$ is due to the fact that the uniform distribution (24) is not continuous and thus, does not satisfy assumption A2.
the left-dominating region, where the left-wing will be elected with probability one in the next period, irrespective of \( \tau^y \). Symmetrically, \( \lambda^- (s) \geq 1 \), or equivalently \( s \geq ((1 - \beta) / 2 + z) / \rho \), is referred to as the right-dominating region, where the right-wing will be elected with probability one under any \( \tau^y \).

It immediately follows that \( \partial \pi (\tau^y, s) / \partial s \geq 0 \) by (26). A higher \( s \) leads to a higher expectation of \( s_0 \), which tends to increase the probability of the right-wing being elected in the next period. Such an effect is amplified through the self-reinforcing process on \( \pi \) via private investment \( h \) as discussed above. To see this, let us assume that voting is completely determined by ideology as in the static example. Then \( \pi \) becomes purely exogenous and (26) reduces to

\[
\pi (\tau^y, s) = \begin{cases} 
1 & \text{if } s \leq \frac{z}{\rho} \\
\frac{\rho s + z}{22} & \text{if } s \in \left( -\frac{z}{\rho}, \frac{z}{\rho} \right) \\
0 & \text{if } s \leq -\frac{z}{\rho}.
\end{cases}
\]

A comparison between (26) and (27) shows that the marginal effect of ideology on \( \pi \) is increased from \( \frac{\rho}{2} z \) to \( \frac{2 \rho}{4 z - \beta} \).

Introducing private investment as a determinant of future political constituency also allows distorting taxes to affect \( \pi \).

\[
\frac{\partial \pi (\tau^y, s)}{\partial \tau^y} = \begin{cases} 
-\frac{1}{1 - \tau^y} & \text{if } \tau^y \in (\lambda^- (s) , \lambda^+ (s)) \\
0 & \text{otherwise}
\end{cases}
\]

Thus, \( \partial \pi (\tau^y, s) / \partial \tau^y \leq 0 \). Intuitively, a low \( \tau^y \) encourages human capital investment and increases the size of rich individuals in the next period. This makes the right-wing more likely to win the next election. In the left-dominating (right-dominating) region with \( \lambda^+ (s) \leq 0 \) (\( \lambda^- (s) \geq 1 \)), however, \( \tau^y \) cannot affect the policymaker’s identity in the next period and hence, the probability \( \pi \) is independent of \( \tau^y \).

The ideology-contingent probability over future political outcomes gives the ideology-contingent elasticity of tax base, which plays a central role in the equilibrium effect of ideology. Using (26) and (28), (22) yields

\[
\epsilon (\tau^y, s) = \begin{cases} 
\frac{\tau^y}{1 + \beta - \tau^y} & \text{if } \tau^y \leq \lambda^- (s) \\
\kappa (\tau^y, s) & \text{if } \tau^y \in (\lambda^- (s) , \lambda^+ (s)) \\
\frac{\rho s + z}{4 \tau^y - \beta} & \text{if } \tau^y \geq \lambda^+ (s)
\end{cases}
\]

where \( \kappa (\tau^y, s) \equiv (1 + \beta / (4 z - \beta)) \tau^y / (1 - \tau^y + \beta (2 \rho s + z) - \tau^y) / (4 z - \beta) \). (29) illustrates how ideology affects the elasticity of tax base \( h \). First, the tax base is more elastic in the left-dominating region (\( \epsilon = \tau^y / (1 - \tau^y) \)) than in the right-dominating region (\( \epsilon = \tau^y / (1 + \beta - \tau^y) \)). In the left-dominating (right-dominating) region, the expectation on \( \tau^o = 1 \) (\( \tau^o = 0 \)) discourages (encourages) human capital investment and hence, makes the tax base more (less) elastic.
The same mechanism holds outside these extreme cases; it is straightforward that \( \kappa(\tau_y, s) \) is decreasing in \( s \). To conclude, a right-wing ideology tends to make the tax base \( h \) less elastic. As shall be seen below, the ideology-contingent elasticity plays a key role in the determination of taxes.

Now we are well-equipped to solve for \( \tau_y(s) \). By (19) and (26), tax revenues from young individuals are

\[
T(\tau_y, s) = \begin{cases} 
    T^R(\tau_y, s) = \frac{1}{2} (1 - \tau_y + \beta) \tau_y & \text{if } \tau_y \in [0, \lambda^- (s)] \\
    T^M(\tau_y, s) = \frac{1}{2} (1 - \tau_y + \beta \frac{2(\rho s + z) - \tau_y}{4z - \beta}) \tau_y & \text{if } \tau_y \in (\lambda^- (s), \lambda^+ (s)) \\
    T^L(\tau_y, s) = \frac{1}{2} (1 - \tau_y) \tau_y & \text{if } \tau_y \in [\lambda^+ (s), 1] 
\end{cases}
\]

Taking \( s \) as the state variable, \( \tau_y \) can be pinned down by maximizing the piecewise quadratic function \( T(\tau_y, s) \). A characterization of the ideology-contingent policy rule \( \tau_y(s) \) is given by

**Proposition 1** Assume that (24) and \( z \geq \hat{z} \), where

\[
\hat{z} \equiv \frac{\beta}{8 \left( - (\beta^2 + 2 \beta) + (1 + \beta) \sqrt{\beta^2 + 2 \beta} \right)}.
\]

Then, the Markov perfect equilibrium is such that

\[
\tau_y(s) = \begin{cases} 
    \frac{1}{2} \phi(s) & \text{if } s \leq s^1 \\
    \lambda^- (s) & \text{if } s \in [s^1, s^M_H] \\
    \frac{1 + \beta}{2} & \text{if } s > s^R
\end{cases}
\]

where

\[
\phi(s) \equiv (2\beta (\rho s + z) + 4z - \beta) / 8z, \quad s^1 = \frac{\sqrt{2} (4z - \beta) - (4z - \beta) / 2 - \beta z}{\beta \rho},
\]

\[
s^M_H = \frac{16z^2 - 6\beta z + 4z - \beta}{\rho (16z - 2\beta)}, \quad s^R = \frac{(1 - \beta) / 4 + z}{\rho}.
\]

**Proof:** See the appendix.

To simplify the statement in the paper, we assume that \( z \geq \hat{z} \). The other case where \( z \in (\beta / 4, \hat{z}) \) will be investigated in the appendix, which delivers qualitatively similar results.\(^{20}\)

Panel A and B in Figure 1 plot the policy rule \( \tau_y(s) \) and the ideology-contingent probability \( \pi(\tau_y(s), s) \) under the benchmark parameter values, which are set to \( z = 0.50, \rho = 0.50 \) and \( \beta = 1.00 \).\(^{21}\)

\(^{20}\)However, there will be no electoral uncertainty if \( z < \hat{z} \).

\(^{21}\)Given \( z \) and \( \rho \), we calibrate \( \beta \) such that the long-run average \( h \) is equal to 0.5, which gives the symmetric support for the left- and right-wing regimes in the long run.
In the left-dominating region, $\partial \pi (\tau, s) / \partial \tau = 0$, private investment and distortionary taxes have no effect on future election outcomes. (30) reduces to a quadratic function $(1 - \tau) \tau^2 / 2$ and the incumbent sets $\tau = 1/2$.

For $\lambda^+ (s) > 0$ or equivalently $s > -z/\rho$, ideology becomes less hospitable for the left-wing and private investment plays a role in probabilities over future political outcomes. Taxes need to be adjusted accordingly. Specifically, a low $\tau$ can increase investment $h$ and thus, the probability for the rightist of being elected in the next period. In this case, the future policymaker’s identity becomes endogenous and dependent on $\tau$. Then, the corresponding objective function $T$ is composed of two different quadratic functions, $T = T^M$ for low $\tau$ and $T = T^L$ for high $\tau$. The emergence of the effect of $\tau$ on the future election probability $\pi$ makes the tax base $h$ more elastic, as shown by (29). This provides the incentive for an incumbent to cut the tax rate. However, if $\lambda^+ (s)$ is close to zero, the incumbent needs to cut $\tau$ substantially to affect $\pi$ and $h$. Proposition 1 shows that for $s < s^1$, it is still optimal to set $\tau = 1/2$ (see the upper panel of Figure 2).

As $s$ moves rightward, the future election outcome becomes more easily affected by tax-cutting. Particularly, when $s$ reaches the threshold point $s^1$, we have an equalized maximum of the two quadratic functions in $T$. The incumbent becomes indifferent between $\tau = 1/2$ and $\tau = \phi (s^1)$, where

$$\phi (s^1) = \sqrt{\frac{4 - \beta z}{4}}.$$  \hspace{1cm} (32)

This can directly be seen from the lower panel in Figure 2. The indifference produces multiplicity and discontinuity of $\tau$ at $s^1$. For a small increment $\xi$ in $s$, the incumbent will cut $\tau$ from $1/2$ to $\phi (s^1 + \xi)$, to attain the top of the Laffer curve.

We next investigate a more realistic region $[s^1, s^M_H]$, where both parties have positive probabilities to win the next election. This is referred to as "the competitive political region". In this region, $\tau$ is linearly increasing in $s$. That is to say, a more right-leaning ideology is associated with a higher distortionary tax rate. The somewhat surprising result is due to the decreasing elasticity of $h$ with respect to $s$, as shown by $\kappa (\tau, s)$ in (29). A high $s$ increases the probability for a right-wing government of winning the next election, via its persistent impact.

\footnote{More specifically, $\tau (s)$ is not lower hemi-continuous. The theorem of maximum (e.g. Stokey and Lucas, 1989, pp. 62) only ensures that $\tau (s)$ is upper hemi-continuous.}
on the future ideological state $s'$. This reduces the expected taxes and makes the current tax base less elastic. The lower elasticity provides the incentive for an incumbent to raise $\tau^y$.

Ideology has an even greater impact on $\tau^y$ for $s \in (s^R_H, s^L_H)$ than for $s \in [s^1, s^M_H]$. Under a modest $s$, $\tau^y$ has a negative effect on $\pi$. When $s$ is sufficiently right-leaning, however, reducing $\tau^y$ has no effect on $\pi$ since $\pi$ has reached its upper boundary. The silence of tax-cutting on $\pi$ amplifies the impact of $s$ on $\tau^y$.

In the right-dominating region, young individuals rationally expect the right-wing to win the next election and thus raises $\tau$.

To conclude, the equilibrium effect of ideology on public policy, reflected by $\tau^y(s)$, turns out to be very different from the partisan effect in the static example. Particularly, if we focus on the competitive political region $[s^1, s^M_H]$, the equilibrium effect induces a positive relationship between the distortionary tax rate and the right-wing ideology. The positive relationship boils down to the ideology-contingent elasticity of the tax base: investment responses less sensitively to the ideological state $s$ than to the ideology-contingent elasticity of the tax base: investment responses less sensitively to the current tax rate when a persistent right-wing ideology leads to lower expected future taxes.

Given the distortionary tax rule $\tau^y(s)$ and the ideology-contingent probability $\pi(\tau^y(s), s)$, human capital investment $h$ follows

**Proposition 2** Assume that (24) and $z \geq \hat{z}$. Then, the Markov perfect equilibrium is such that

$$ h = \begin{cases} \frac{1}{4} + \frac{\beta (ps + z)}{2(4z - \beta)} & \text{if } s \leq s^1 \\ \frac{1}{4} - \frac{(ps - z)}{4} & \text{if } s \in [s^1, s^M_H] \\ \frac{1}{4} & \text{if } s \in (s^M_H, s^R] \\ \frac{1}{4} + \frac{\beta (ps + z)}{2(4z - \beta)} & \text{if } s > s^R \end{cases} $$

(33)

The proof is straightforward and immediately follows from (19), (26) and (31). $h$ increases in $s$ for $s \in [s^1, s^M_H]$ and peaks at $s = s^M_H$. Then, $h$ decreases in $s$ for $s \in (s^M_H, s^R]$. Panel C in Figure 1 plots the inverted U-shaped $h$. The non-monotonicity of $h$ is due to the fact that a rightward ideological shift has two opposite effects on $h$. First, it helps the right-wing win the next election and thus raises $\pi$, which has a positive impact on $h$. However, a high $\pi$ makes $h$ less elastic and thus, induces the incumbent to raise $\tau^y$, which has a negative impact on $h$.

For $s \in [s^1, s^M_H]$, the positive effect dominates the negative effect and $h$ increases in $s$. For

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23 According to Proposition 1, $d\tau^y(s)/ds = \rho/4z$ for $s \in [s^1, s^M_H]$ and $d\tau^y(s)/ds = 2\rho$ for $s \in (s^M_H, s^R]$. The latter is greater than the former since $z > \beta/4$ and $\beta \leq 1$.

24 For $s \in [s^1, s^M_H]$, $\partial \tau^y(s)/\partial \tau^y = -1/16 < 0$. 


$s \in (s_M^H, s^R]$, however, the positive effect disappears since $\pi$ has reached its upper boundary. The remaining negative effect produces a decreasing $h$.

4.1 The Size of Government

Policy rules (8) and (31) capture the partisan and equilibrium effects of ideology on public policy, respectively. Now these two effects are aggregated. The size of government can be written as

$$
\gamma(s, s-1) = \begin{cases} 
\frac{h(s-1) + \tau^y(s)h(s)}{h(s-1) + h(s)} & \text{if } s \leq \frac{1}{2} - h(s-1) \\
\frac{\tau^y(s)h(s)}{h(s-1) + h(s)} & \text{otherwise}
\end{cases}
$$

(34)

where $\tau^y(s)$ and $h(s)$ follow (31) and (33), respectively. Note that $\gamma$ is not only affected by the current ideological state $s$, but also dependent of the past ideological state $s-1$, which determines the size of the rich. Figure 3 plots the government size $\gamma$ with different $s-1$. An immediate observation is that, driven by the equilibrium effect of ideology, a rightward ideological shift increases $\gamma$ within each political regime.

[Insert Figure 3]

To investigate in a straightforward way the implications of ideology, we simulate the model and use simulated data to estimate the following linear equation.

$$
\gamma = a_0 + a_1 R + a_2 s + a_3 s-1 + \zeta,
$$

(35)

where $R$ is a dummy variable which equals zero and one for the left- and right-wing regime, respectively. $a_0$ is a constant and $\zeta$ is an error term. Although simple, (35) reveals two effects of ideology, the main prediction of our theory. The coefficients $a_1$ and $a_2$ capture the partisan effect and the equilibrium effect of ideology, respectively.

We run 1100 simulations repeatedly for 50 times with the benchmark parameterization. The estimated results are reported in column 1 of Table 1. $R^2$ amounts to 0.98, indicating a high degree of fitness of the linear specification. As expected, $a_1$ is negative and $a_2$ is positive. $a_3$ turns out to be statistically insignificant. (33) shows that $h_{-1}$ is a non-monotonic function of $s_{-1}$. The resulting ambiguous correlation between $s_{-1}$ and the identity of political regime makes $a_3$ insignificant. In fact, dropping $s_{-1}$ from the regression has little effect on the estimates of $a_1$ and $a_2$, as stated in column 2 of Table 1.

[Insert Table 1]

25 The first 100 observations are discarded to eliminate the effect of the initial ideological state.
Our theory also has implication on the relationship between government sizes and political constituency. This can directly be seen by estimating

$$\gamma = b_0 + b_1 R + b_2 e + \varepsilon,$$

(36)

where $e$ is the proportion of right-wingers and $\varepsilon$ is an error term. Column 3 of Table 1 reports the result. Since $e$ is linear in $s$ outside "ages of extremes" (see equation 7), it should not be surprising that (35) and (36) give similar results. In particular, $b_2$ is positive. In words, an increase in the right-wing voter share leads to a larger size of government within each political regime.

4.1.1 Sensitivity to Model Parameters

Now we check the parameter sensitivity of the coefficients of interests $a^2$ and $b^2$. Specifically, we consider sensitivity analysis to two key model parameters: $z$ and $\rho$. Panel A of Figure 4 shows that an increase in $z$, implying more volatile ideology, tends to reduce $a^2$ and $b^2$. This is consistent with (31); the equilibrium effect of ideology for $s \in [s^1, s^M_H]$, captured by $d\tau_y(s)/ds = \beta \rho / 4 z$, mitigates as $z$ increases. Note that $a^2 > 0$ and $b^2 > 0$ hold even in the limit as $z$ approaches infinity. Though $\lim_{z \to \infty} d\tau_y(s)/ds = 0$ for $s \in [s^1, s^M_H]$, there always exist a positive equilibrium effect of ideology: $d\tau_y(s)/ds = 2 \rho$ for $s \in (s^M_H, s^R]$.

[Insert Figure 4]

Panel B shows the impact of changing $\rho$. When $\rho = 0$, the equilibrium effect of ideology goes away. This implies $a^2 = b^2 = 0$. Increasing $\rho$ from zero leads to positive $a^2$ and $b^2$, due to the rising equilibrium effect of ideology. However, the effect of $\rho$ on $a^2$ is non-monotonic. On the one hand, $\rho$ increases the magnitude of $d\tau_y(s)/ds$ for $s \in (s^1, s^R]$. On the other hand, the region $(s^1, s^R)$ shrinks with larger $\rho$, suggesting a lower probability for the equilibrium effect of ideology to be functioning. This reduces the estimate of $a^2$. In the limiting case as $\rho$ approaches unity, the probability for $s \in (s^1, s^R)$ goes to zero since $\underline{s} \to -\infty$ and $\bar{s} \to \infty$. The estimate of $a^2$ is then completely determined by policy outcomes in the polar cases featuring no electoral uncertainty, which implies a zero $a^2$. These two opposite effects result in an inverted-U shape of $a^2$; the first effect is dominating for small $\rho$ and the second effect is dominating for large $\rho$.

Although there are similar opposite effects of $\rho$ on $b^2$, the aggregate impact turns out to be monotonic. Different from the ideological state $s$, the proportion of right-wingers $e$ is bounded between 0 and 1. Consequently, the estimate of $b^2$ is much less affected by policy outcomes in the non-competitive political regimes.
To conclude, the result that $a^2 > 0$ and $b^2 > 0$ is robust to a wide range of parameter values. This allows us to test our theory against the standard partisan theory predicting zero $a^2$ and $b^2$.

### 4.2 Robustness

We have analytically characterized the Markov perfect equilibrium under two important assumptions, i.e., old politicians and age-dependent taxation. In this subsection, we discuss in order whether our main findings are robust to changes in these assumptions.

The first assumption seems too extreme as old politicians completely ignore welfare of the young in our model. A natural extension is to assume that politicians are altruistic towards the young. The political objective function (21) can therefore be written as

$$
\tau^y(s) = \arg \max_{\tau^y \in [0,1]} T(\tau^y, s) + \omega u^y(\tau^y, s),
$$

(37)

where $\omega \geq 0$ stands for the intensity of altruism. Although the analysis becomes much more complicated, the results remain qualitatively similar. Nevertheless, two quantitative changes are worthy of being mentioned. First, $\tau^y$ in the extended model is lower than that in the benchmark model. Second, the equilibrium effect of ideology, $\partial \tau^y / \partial s$, turns out to be larger.

The underlying intuitions are rather straightforward. A positive $\omega$ introduces a new ingredient in the policy decision process: the impact of $\tau^y$ on welfare of the young. This additional marginal disutility leads to lower $\tau^y$. Moreover, since a right-wing ideology encourages human capital investment by reducing expected future taxes, the marginal disutility of $\tau^y$ for the young is actually decreasing in $s$. This provides the incentive for politicians to increase further $\tau^y$ in response to a right-wing ideological shock.

Throughout the paper, we maintain the assumption that the government can condition taxes on age. Although age-dependent taxation has its realistic counterpart and substantially simplifies the analysis, this assumption is not innocuous. Since both parties are perfectly aligned with $\tau^y$, the partisan effect only works on the non-distortionary tax rate $\tau^o$. More crucially, one may wonder whether the binary taxation (8), which obviously overstates the partisan effect of ideology, is crucial for the positive relationship between the distortionary tax rate and the right-wing ideology. In an earlier version of this paper (Song, 2005), we assess the robustness of the main result under age-independent taxation and find that imposing the weaker policy instrument does not lead to any major change. The intuition is again simple. Age-independent tax rates in the right-wing regime are on average lower than those in the

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26 We solve the extended model numerically. Details are available upon request.
left-wing regime. A rightward ideological shift, as in the case with age-dependent taxation, reduces the expected tax rate by increasing the probability for the right-wing to win the next election. This encourages investment and induces an incumbent to behave in a similar way as described above.

5 Empirical Evidence

In this section, we test the main prediction of our theory concerning the effects of ideology on the determination of public policies. A novel prediction of the theory is that after controlling for the partisan effect, a more right-leaning ideology leads to a larger size of government due to the equilibrium effect of ideology. One of the major difficulties in testing the prediction is how to measure ideology. A commonly used measure of ideology in the literature of political science is self-placement scores of the left-right position from opinion polls or survey data (Inglehart, 1990). This approach obviously suffers from limited comparable observations across countries and time.27 Our estimation strategy takes an alternative approach. The idea is based on regression equation (36); the equilibrium effect of ideology can be inferred from its relationship to the proportion of right-wingers for which data exist.

5.1 Data and Specification

We use the Comparative Welfare States Data Set, assembled by Huber et al. (1997) and updated by Brady et al. (2004). The sample consists of at most 41 years of observations (1960-2000) from 18 democracies, including Australia, Austria, Belgium, Canada, Denmark, Finland, France, Germany, Ireland, Italy, Japan, Netherlands, New Zealand, Norway, Sweden, Switzerland, United Kingdom and United States.28

Following Persson (2002), Persson and Tabellini (2004), among many others, we use central government expenditure (or revenue) as a percentage of GDP, denoted by $CGEXP$ (or $CGREV$), to measure the size of government. We use the percentage of vote for the right parties ($RVOT$) as the empirical counterpart of the proportion of right-wingers in the model. A dummy variable, $R$, is created for controlling the partisan effect. We let $R = 1$ if both shares of seats of the right and center parties in parliament and government are larger than 66.6%. $R = 0$ otherwise. In all 738 observations, 201 are associated with $R = 1$. In words, about 27%

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27 Moreover, it has long been questioned whether all respondents have consistent views on the location of the "left" and "right" (e.g. Levitin and Miller, 1979).

28 All the data used in this subsection are from the Comparative Welfare States Data set, which is available at http://www.lisproject.org/publications/welfaredata/welfareaccess.htm.
governments in the sample are labeled as the right-wing.\textsuperscript{29}

Based on (36), we estimate regressions in which policy outcomes $Y_{it}$ are linear functions of $R$ and $RVOT$.

$$Y_{it} = b^0 + b^1 R_{it} + b^2 RVOT_{it} + \phi X_t + \varepsilon_{it},$$

where $b^0_i$ is a country-specific effect and $X_t$ is a set of year dummies to control for the unobserved common shocks. We also run this regression with some additional explanatory variables containing country-specific factors. Specifically, we use the log of real GDP per capital ($YPC$) to control the potential impact of Wagner’s Law, i.e., the size of government will rise as income rises. Deviation of $YPC$ from its trend (obtained by the HP filter), denoted by $YGAP$, is added, as government tends to implement countercyclical policies to smooth economic fluctuations. Other control variables include the unemployment rate ($UNEMPL$), export plus import share of GDP ($OPEN$), proportion of population over 65 ($POP_{65}$) and below 14 ($POP_{14}$), which are constantly adopted in recent empirical studies (e.g. Razin \textit{et al.}, 2004, Persson and Tabellini, 2005).

5.2 Results

Table 2 gives estimation results from fixed effects regressions. We start with column 1, where the equilibrium effect of ideology is shut down by excluding $RVOT$ in (38). This parallels the standard approach for testing the partisan effect. The baseline regression (column 1) shows that $b^1$ is negative, but statistically insignificant. When additional explanatory variables are included (column 2), $|b^1|$ increases from 0.82 to 1.05 and becomes significant at 10\% level. According to the point estimation, switching from the right-wing regime to the left-wing causes government expenditure to increase by about one percent of GDP. The magnitude of the partisan effect in OECD countries is roughly in line with previous findings.\textsuperscript{30} Column 3 and 4 display the same regressions on $CGREV$. There is a much stronger partisan effect for government revenue: the estimated $|b^0|$ amounts to 1.8 and is significant at 1\% level. This result is rather stable, irrespective of whether additional explanatory variables are included.

Our main finding is in column 5 to 8. The baseline regression (column 5) shows that $b^2$, the coefficient on $RVOT$, is positive and significant at the level of 1\%. Including additional

\textsuperscript{29}A similar criteria is adopted by Woldendorp \textit{et al.} (1993, 1998), where $R = 1$ is referred to as the "right-wing dominance" regime.

\textsuperscript{30}In Blais \textit{et al.} (1993), such a shift from the right to the left will increase government spending by 0.7 percentage point. In Perotti and Kontopoulos (2002), the increase by a shift from a modest right government to a modest left government amounts to 1.6 percentage point in the steady state.
explanatory variables reduces the estimate of $b^2$ substantially (column 6). The statistical
significance, however, is at the same level. The point estimation in column 6 implies that given
the incumbent’s political identity, one percentage increase in the vote share for right parties
will on average increase government expenditure by 0.06 percent of GDP. The equilibrium effect
of ideology is more evident in column 7 and 8 where we use central government revenue as an
indicator of government size. One percentage increase in $RVOT$ will lead to 0.17 percentage
increase in $CGREV$ when additional explanatory variables are added. These results provide
statistical evidence in favor of the equilibrium effect of ideology.

The positive $b^2$ does not suggest right-wing governments to be larger. In fact, controlling
for political constituency reveals a larger partisan effect, and $b^1$ that is significant in column
1 to 4 remains significant in column 5 to 8. This suggests that left-wing governments indeed
have a greater preference over public spending. The co-existence of the two effects is essential
for the theory, since the equilibrium effect cannot exist without the partisan effect.

5.3 Robustness

A key issue is whether $RVOT$ is an appropriate proxy of political constituency. Party ideology,
on which $RVOT$ is completely silent, is clearly an important dimension of ideology. Our model
adopts a two-party system for simplicity. However, real democracies have much more complex
political systems. One country may have several left (right) parties with different ideological
positions from each other. Taken into consideration this issue, we use the index developed
by Kim and Fording (1998, 2003) as an alternative proxy of ideology. The advantage of the
index is that it reflects not only changes of vote, but also changes of party ideology.\(^{31}\) Using
the Kim-Fording index gives no qualitative changes to the results. The coefficients that are
significant remain significant.\(^{32}\)

Our panel regressions contain 18 countries. It is important to check the sensitivity of the
results to individual countries. To this end, we run all the regressions in Table 2 excluding one
country at a time. In every regression, the coefficients that are significant at the 5\% level are
still significant at the same level no matter which country is excluded.

Some evidence shows that fiscal policies legislated in the states typically take one year to
be effective (Poterba, 1994, Gilligan and Matsusaka, 1995). It is not clear whether the same
mechanism carries over to the sample of OECD countries. Be as it may, we replace $R_{it}$ in the
regressions with one-year lagged variable $R_{it-1}$. The results change only marginally and the

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\(^{31}\)Kim and Fording first estimate party ideology, based on party manifesto statements, and then use the
percentage of the vote received by each party to construct an adjusted index for the median ideological position.

\(^{32}\)The results are available upon request.
statistical significance for the coefficients of interest is never affected.

Finally, it has not yet been addressed that there might be omitted variables affecting political variables and fiscal outcomes simultaneously. Taking this concern, we choose one-year lagged political variables $R_{it-1}$ and $RVOT_{it-1}$ to instrument the current political variables $R_{it}$ and $RVOT_{it}$. These lagged variables are highly correlated with the respective current values, and expected to be independent of current policy outcomes. Table 3 reports the results of reestimating (38) using Two-State-Least Squares (2SLS). First note that the 2SLS estimates of $b^1$ are significant in all cases. There is some evidence for the endogeneity of $R$ since a Hausman test rejects the null hypothesis in column 3 and 4 at the level of 10%. Second, though the endogeneity leads to a significant underestimation of the partisan effect, the endogeneity of $RVOT$ is much less severe. Application of the Hausman test cannot reject the null hypothesis. The 2SLS estimate of $b^2$ becomes statistically insignificant in column 2, but remains highly significant in all other cases.

[Insert Table 3]

To conclude, we find a significantly positive relationship between government revenue and the right-wing vote share, after controlling for the partisan effect. The point estimate is quite stable to a number of control variables and specifications. The empirical finding is in line with our theoretical prediction, but hard to explain by the existing literature. An interesting question left is why the positive relationship is less evident for government expenditure? Recall that in the model, the equilibrium effect of ideology is mainly driven by the government’s target of tax revenue maximization. Therefore, as an indicator of tax policy, $CGREV$ is perhaps a better measure of $\gamma$ for identifying the equilibrium effect of ideology.\footnote{Although government expenditures and revenues usually move in tandem, imbalances between expenditures and revenues occur occasionally since governments may run deficits, which is totally assumed away in the theory.}

6 Multiple Equilibria

In the previous sections, equilibria have been shown to be determinate. We regard this as an important progress, since much of the previous related literature (including Hassler et al., 2003) is plagued by multiple equilibria and indeterminacies, which undermine the ability of the theory to provide sharp empirical implications. The multiplicity should not be surprising. When the government cannot make a commitment on future policies, individuals must condition their choice on self-fulfilled expectations, which are usually not unique. Introducing ideology has
been proved useful to eliminate multiplicity. In this section, we show that enough ideological uncertainty is indeed needed. Else, multiple Markov equilibria reemerge.

If \( z < \beta / 4 \), (26) no longer holds. Instead, (25) solves

\[
\pi (\tau_y, s) = \begin{cases} 
\frac{1}{\tau_y - 2(\rho s + z)} & \text{if } \tau_y \leq \lambda^- (s) \\
0 & \text{if } \tau_y \in (\lambda^+ (s), \lambda^- (s)) \\
\frac{\beta - 4z}{\beta - 4z} & \text{if } \tau_y \geq \lambda^+ (s)
\end{cases}
\]  

(39)

where \( \lambda^+ (s) < \lambda^- (s) \) under \( z < \beta / 4 \). (39) implies that the ideology-contingent \( \pi (\tau_y, s) \) is not unique for \( \tau_y \in [\lambda^+ (s), \lambda^- (s)] \). The expectation \( (\tau_y - 2(\rho s + z)) / (\beta - 4z) \) seems bizarre since it implies that \( \partial \pi / \partial \tau_y > 0 \) and \( \partial \pi / \partial s < 0 \). Both signs are counter-intuitive and hard to explain. However, the indeterminacy of \( \pi (\tau_y, s) \) still remains even if we rule out the counter-intuitive self-fulfilled expectation \( (\tau_y - 2(\rho s + z)) / (\beta - 4z) \) by requiring the expectation to be monotonic:

\[
\pi (\tau_y, s) = \begin{cases} 
1 & \text{if } \tau_y \leq \lambda^- (s) \\
0 & \text{if } \tau_y \geq \lambda^+ (s)
\end{cases}
\]  

(40)

Since \( \lambda^+ (s) < \lambda^- (s) \), \( \pi \) is indeterminate for \( \tau_y \in [\lambda^+ (s), \lambda^- (s)] \).

The indeterminacy of expectations opens the door to multiple Markov perfect equilibria. To see this, let us pick up a particular expectation rule satisfying (40)

\[
\pi (\tau_y, s) = \begin{cases} 
1 & \text{if } \tau_y < \lambda^+ (s) + \psi \\
0 & \text{if } \tau_y \geq \lambda^+ (s) + \psi
\end{cases}
\]

where \( \psi \in [0, \beta - 4z] \). Then, the human capital investment follows

\[
h (\tau_y, s) = \begin{cases} 
\frac{1 + \beta - \tau_y}{2} & \text{if } \tau_y < \lambda^+ (s) + \psi \\
\frac{1}{\tau_y} & \text{if } \tau_y \geq \lambda^+ (s) + \psi
\end{cases}
\]

Correspondingly, the incumbent sets \( \tau_y (s) \) by maximizing \( T (\tau_y, s) \). Some algebra establishes

\[
\tau_y (s) = \begin{cases} 
\frac{1 + \beta}{2} \lambda^+ (s) + \psi & \text{if } s \geq \frac{(1 + \beta - 2\psi)(1 + \beta - 2\psi)}{2(1 + \beta - 2\psi)} \\
\frac{1 + \beta}{2} \lambda^+ (s) + \psi & \text{if } s \in ((\eta - \psi)/2 - z)/\rho, (1 + \beta - 2\psi)(1 + \beta - 2\psi)/\rho \}
\end{cases}
\]  

(41)

where \( \eta \equiv \left(1 + \beta - \sqrt{\beta (2 + \beta)}\right) / 2 \). This leads to

**Proposition 3** Assume that (24) and \( z < \beta / 4 \). There are multiple Markov perfect equilibria, where \( \tau_y (s) \) follows (41) for any \( \psi \in [0, \beta - 4z] \).

### 6.1 Discussion

The above analyses suggest that sufficient uncertainty helps pin down a unique belief on future political constituency. If \( z < \beta / 4 \), \( \tau_y \in [\lambda^+ (s), \lambda^- (s)] \) may induce two self-fulfilled beliefs,
Given any of the self-fulfilled beliefs, the tax rate $\tau^y \in [\lambda^+(s), \lambda^-(s)]$ can be utilized to eliminate uncertainty. Specifically, given the belief that $\pi = 1$ (0) for $\tau^y \leq \lambda^-(s)$ ($\geq \lambda^+(s)$), any $\tau^y \in [\lambda^+(s), \lambda^-(s)]$ can lead to a rightist (leftist) government in the next period, irrespective of the current ideological state $s$. If $z > \beta / 4$, large uncertainty weakens the link between $\tau^y$ and the future actual political outcomes. Given any $\tau^y \in [\lambda^+(s), \lambda^-(s)]$, neither the left-wing nor the right-wing can be elected with probability one. Consequently, individuals must use the current ideological state $s$ as useful information to figure out all possibilities of future political constituency and the corresponding policy outcomes. This highlights the role of information which dictates an unique belief on $\pi$.

Morris and Shin (1998, 2000) applied a similar methodology to pin down a unique belief in the financial market. They assume that agents receive differential information on fundamentals. Noisy information destroys the common knowledge. An agent must thus consider all possible strategies of others based on its received information. The unique belief in our model does not rely on differential information. Alternatively, it has roots in the imperfect information about the evolution of political constituency.

There is another advantage of introducing ideological uncertainty. The indeterminacy in the context without uncertainty has little to say about the shift in beliefs and the corresponding switches between different political regimes. Take Hassler et al. (2007) as an example. If $\pi = 0$ (1), the left-wing (right-wing) will be the incumbent forever in all periods except for the first election. This is apparently inconsistent with the observed political cycles. Introducing electoral uncertainty provides a straightforward mechanism that switches beliefs and the corresponding politico-economic equilibrium outcomes over time.

### 7 Conclusion

In spite of the growing literature on public policy decision-making in dynamic politico-economic equilibrium, most works are silent on the role of ideological shifts, which tend to be persistent and have a nontrivial influence on political outcomes. To explore the underlying mechanism of policy decision-making under stochastic ideological movements, we develop a tractable model to investigate the dynamic interactions among public policy, individuals’ intertemporal choice and the evolution of political constituency. Our main finding is that the relationship between the right-wing ideology and the size of government is positive within each political regime.

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34 The technique used in Herrendorf et al. (2000) is also related to ours. They find that sufficient heterogeneity can rule out the multiplicity of equilibria in a two-sector growth model.

35 If the proportion of old rich exceeds $1/2$, the right-wing wins the first election. Otherwise, the left-wing wins.
This distinguishes the literature of partisan politics predicting that ideology has no effect on public policies if the political regime remains unchanged. We document empirical evidence from an OECD panel that supports our theory.

Our analysis is subject to a number of caveats. For instance, our theory is completely silent on the determination of public policies under coalition government. Moreover, the policy-decision through the paper is static. When the government is allowed to borrow and agents are altruistic towards their children, however, public choices may appear to be dynamic. In a related work, Song, Storesletten and Zilibotti (2007) introduce altruism and analyze the determination of public debt in a stochastic ideological environment. In that model, however, private choices are irrelevant for the evolution of political constituency. It will be interesting for future research to incorporate altruism and public debt in the current setup, to see how public intertemporal trade-off is interacted with private intertemporal choices.

References


8 Appendix

8.1 Proof of Lemma 1

Apply the Schauder fixed point theorem. Let $C$ be a set of bounded and uniformly continuous functions mapping from $[0,1] \times X$ to $[0,1]$. Define $F = \int_{s' \geq \tau y-\beta_{\theta} (\tau y, s)} f (s', s) \, ds'$, where $\alpha (\tau y, s) \in C$. We need to prove that the mapping $F$ has a fixed point.

Let $\Omega = \{ F (\alpha), \alpha \in C \}$. We first claim that $\Omega$ is equicontinuous, i.e., $F (\alpha)$ is bounded and uniformly continuous for any $\alpha \in C$. The boundedness is trivial since $F (\alpha) (\tau y, s) \leq \int f (s', s) \, ds' = 1$. To prove that $F (\alpha)$ is uniformly continuous, we pick up any two vectors $x = (\tau y_1, s_1)$ and $y = (\tau y_2, s_2)$ from $[0,1] \times X$. It is straightforward to show that

$$|F (\alpha) (\tau y_1, s_1) - F (\alpha) (\tau y_2, s_2)|$$

$$= \left| \int_{s' \geq \tau y_1-\beta_{\theta} (\tau y_1, s_1)} f (s', s_1) \, ds' - \int_{s' \geq \tau y_2-\beta_{\theta} (\tau y_2, s_2)} f (s', s_1) \, ds' \right|$$

$$\leq \left| \int_{s' \geq \tau y_1-\beta_{\theta} (\tau y_1, s_1)} f (s', s_1) \, ds' - \int_{s' \geq \tau y_2-\beta_{\theta} (\tau y_2, s_2)} f (s', s_1) \, ds' \right|$$

$$+ \left| \int_{s' \geq \tau y_2-\beta_{\theta} (\tau y_2, s_2)} f (s', s_1) \, ds' - \int_{s' \geq \tau y_2-\beta_{\theta} (\tau y_2, s_2)} f (s', s_1) \, ds' \right|$$

$$\leq \frac{\tau y_1 - \tau y_2 - \beta (\alpha (\tau y_1, s_1) - \alpha (\tau y_2, s_2))}{2} \sup_{s' \geq \tau y_1-\beta_{\theta} (\tau y_1, s_1)} f (s', s_1)$$

$$+ \left| \sup_{s' \geq \tau y_2-\beta_{\theta} (\tau y_2, s_2)} f (s', s_1) - f (s', s_2) \right|.$$ 

As $\| x - y \| \to 0$, we have $|(\tau y_1 - \tau y_2 - \beta (\alpha (\tau y_1, s_1) - \alpha (\tau y_2, s_2))) / 2 \| f (s', s_1) \|_{\sup} \to 0$ by the uniform continuity of $\alpha$ and $\| f (s', s) \|_{\sup} < \infty$ (A2). Moreover, from A1 and the boundedness of $\alpha$, it immediately follows that $\sup_{s' \geq \tau y_2-\beta_{\theta} (\tau y_2, s_2)} | f (s', s_1) - f (s', s_2) | \to 0$ as $\| x - y \| \to 0$. Therefore, we have $|F (\alpha) (x) - F (\alpha) (y)| \to 0$ as $\| x - y \| \to 0$.

Next we check the conditions of the Schauder fixed point theorem (Theorem 17.4, Stokey and Lucas, 1989). $\Omega$ has been to proved equicontinuous. And it is easily shown that $C$ is nonempty, closed and convex and $F$ is continuous. Thus, all conditions are satisfied. □

8.2 Proof of Lemma 2

We only need to prove that, given any $(\tau y, s)$, the following equation has a unique solution

$$x = G (x) \equiv \int_{s' \geq \tau y-\beta_{\theta} (\tau y, s)} f (s', s) \, ds'.$$  \hspace{1cm} (42)

A3 implies that $dG (x) / dx = \beta f (s', s) / 2 < 1$. The proof is complete by applying the contraction mapping theorem. □
8.3 Proof of Proposition 1

The proof is based on the following lemma.

**Lemma 3** Assume that (24) and \( z > \beta/4 \).

(i) If \( z \geq \hat{z} \),

\[
\tau^y(s) = \begin{cases} 
\frac{1}{2} & \text{if } s \leq s^1 \\
\phi(s) & \text{if } s \in \left[ s^1, s^M \right] \\
\lambda^-(s) & \text{if } s \in \left( s^M, s^R \right] \\
\frac{1+\beta}{2} & \text{if } s > s^R 
\end{cases},
\]  

(ii) If \( z < \hat{z} \),

\[
\tau^y(s) = \begin{cases} 
\frac{1}{2} & \text{if } s \leq s^2 \\
\lambda^-(s) & \text{if } s \in \left[ s^2, s^R \right] \\
\frac{1+\beta}{2} & \text{if } s > s^R 
\end{cases}.
\]

where

\[
\hat{z} \equiv \frac{\beta}{8 \left( - (\beta^2 + 2\beta) + (1 + \beta) \sqrt{\beta^2 + 2\beta} \right)}, \\
s^1 \equiv \frac{\sqrt{z(4z-\beta) - (4z - \beta) / 2 - \beta z}}{\beta \rho}, \\
s^2 \equiv \frac{1 - \beta - \sqrt{\beta^2 + 2\beta + 4z}}{4 \rho}, \\
s^M_H \equiv \frac{16z^2 - 6\beta z + 4z - \beta}{\rho (16z - 2\beta)}, \\
s^R \equiv \frac{(1 - \beta) / 4 + z}{\rho}.
\]

8.3.1 Proof of Lemma 3

The solution of maximizing (30) is straightforward under two polarized cases, i.e., \( s \geq ((1 - \beta) / 2 + z) / \rho \) and \( s \leq -z / \rho \). Thus, we need only to focus on \( s \in (-z / \rho, ((1 - \beta) / 2 + z) / \rho) \).

For notational convenience, we define

\[
L(s) = \max_{\tau^y \in \left[ \lambda^+(s), 1 \right]} T^L(\tau^y, s) \quad \tau^L(s) = \arg\max_{\tau^y \in \left[ \lambda^+(s), 1 \right]} T^L(\tau^y, s) \\
M(s) = \max_{\tau^y \in \left( \lambda^-(s), \lambda^+ \right)} T^M(\tau^y, s) \quad \tau^M(s) = \arg\max_{\tau^y \in \left( \lambda^-(s), \lambda^+ \right)} T^M(\tau^y, s) \\
R(s) = \max_{\tau^y \in [0, \lambda^-(s)]} T^R(\tau^y, s) \quad \tau^R(s) = \arg\max_{\tau^y \in [0, \lambda^-(s)]} T^R(\tau^y, s)
\]
It is also convenient to classify the regions where interior solutions hold.

\[ \tau^L (s) = \begin{cases} \frac{1}{2} & \text{if } s \leq s^L \\ \lambda^+ (s) & \text{if } s > s^L \end{cases}, \quad (45) \]

\[ \tau^M (s) = \begin{cases} \phi (s) & \text{if } s \in [s^M_L, s^M_H] \\ \lambda^- (s) & \text{if } s > s^M_H \\ \lambda^+ (s) & \text{if } s < s^M_L \end{cases}, \quad (46) \]

\[ \tau^R (s) = \begin{cases} \frac{1 + \beta}{2} & \text{if } s \geq s^R \\ \lambda^- (s) & \text{if } s < s^R \end{cases}, \quad (47) \]

where

\[ s^L \equiv \frac{1}{4} - \frac{z}{\rho}, \]

\[ s^M_{L} \equiv \frac{-16z^2 + 2\beta z + 4z - \beta}{\rho (16z - 2\beta)}, \]

and

\[ s^R > s^M_H > s^M_L, \quad (48) \]

\[ s^R > s^L > s^M_L. \quad (49) \]

We proceed by classifying the following six cases (see Table A-1), according to the conditions in (45) and (46). Some results are immediate. By (49), the sixth case is empty. In Case 1-3, \( s \leq s^L \). So, \( \tau^L (s) = 1/2 \). In Case 4 and 5, \( \tau^L (s) = \lambda^+ (s) \). The continuity of \( T^L (s) \) implies that \( M^L (s) = L^L (s) \). So we are left to compare \( M^L (s) \) and \( R^L (s) \) in Case 4 and 5.

<table>
<thead>
<tr>
<th>Case 1: if</th>
<th>Case 2: if</th>
<th>Case 3: if</th>
<th>Case 4: if</th>
<th>Case 5: if</th>
<th>Case 6: if</th>
</tr>
</thead>
<tbody>
<tr>
<td>( s \leq s^L ) and ( s \in [s^M_L, s^M_H] )</td>
<td>( s \leq s^L ) and ( s &gt; s^L ) and ( s &lt; s^L ) and ( s \in [s^M_L, s^M_H] )</td>
<td>( s &gt; s^M_H ) and ( s &lt; s^L ) and ( s \in [s^M_L, s^M_H] )</td>
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Now let us consider the five cases in order. In the first case, \( \tau^M (s) = \phi (s) \). Moreover, (48) and (47) give that \( s < s^R \) and \( \tau^R (s) = \lambda^- (s) \). The continuity of \( T^L (s) \) implies that \( M^L (s) > L^L (s) \). So we only need to compare \( L^L (s) \) and \( M^L (s) \). It immediately follows that \( \tau^L (s) = 1/2 \) if \( s \leq s^1 \) and \( \tau^L (s) = \phi (s) \) if \( s \geq s^1 \), where \( s^1 \) solves

\[ T^M (\phi (s^1), s^1) = T^L (1/2, s^1). \]

This yields

\[ s^1 = \frac{\sqrt{z (4z - \beta) - (4z - \beta) / 2 - \beta z}}{\beta \rho}. \quad (50) \]

The other root is omitted since \( s^1 > -z/\rho \). It is easy to see that

\[ s^1 < s^L. \quad (51) \]

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Moreover, by the assumption that $4z > \beta$, one can show that
\[
s^1 > -\frac{z}{\rho} + \frac{\beta}{4\rho} > s^M_L. \tag{52}
\]
(51) and (52) will be used in obtaining (55).

Turn to the second case where $\tau^M(s) = \lambda^-(s)$. Since $s^L < s^R$, $\tau^R(s) = \lambda^-(s)$ and $M(s) = R(s)$. So we have that $\tau^y(s) = 1/2$ if $s \leq s^2$ and $\tau^y(s) = \lambda^-(s)$ if $s \geq s^2$, where $s^2$ solves
\[
T^R(\lambda^-(s^2), s^2) = T^L(1/2, s^2).
\]
This yields
\[
s^2 = \frac{1 - \beta - \sqrt{\beta^2 + 2\beta + 4z}}{4\rho} \tag{53}
\]
Since $L(s^1) > R(s^1)$, $L(s^1) = L(s^2)$ and $R(s)$ is increasing in $s$, $R(s^2) = L(s^2)$ implies that
\[
s^2 \geq s^1, \tag{54}
\]
Similarly, since $s^M_L < s^R$, $\tau^R(s) = \tau^M(s) = \lambda^-(s)$ and $M(s) = R(s)$ in the third case. So $\tau^y(s)$ follows the same rule as in the second case.

In the fourth case, $\tau^M(s) = \phi(s)$. Moreover, (48) and (47) establish that $s < s^R$ and $\tau^R(s) = \lambda^-(s)$. Hence, $M(s) > R(s)$ and $\tau^y(s) = \phi(s)$.

Finally, the fifth case gives that $\tau^M(s) = \lambda^-(s)$. If $s \leq s^R$, $\tau^R(s)$ is also equal to $\lambda^-(s)$ and $R(s) = M(s)$. On the other hand, if $s > s^R$, $\tau^R(s) = (1 + \beta)/2$, $R(s) > M(s)$. So, $\tau^y(s) = \lambda^-(s)$ if $s \leq s^R$ and $\tau^y(s) = (1 + \beta)/2$ otherwise.

To conclude, we have
\[
\tau^y = \begin{cases}
\frac{1}{2} & \text{if } s \in ([s, s^2] \cap [s^1, s^1]) \cup ([s, \min\{s^L, s^2\}] \cap (s^M_H, s^1]) \\
\phi(s) & \text{if } s \in ([s, s^L] \cap [s^1, s^1] \cap [s^M_L, s^M_H]) \cup ((s^L, s^1] \cap [s^M_L, s^M_H]) \\
\lambda^-(s) & \text{if } s \in ([s^2, s^2] \cap [s, s^L] \cap (s^M_H, s^1]) \cup ([s^2, s^2] \cap [s^L, s^L] \cap (s^M_H, s^1]) \\
\frac{1 + \beta}{2} & \text{if } s \in (s^L, s^L] \cap (s^R, s^1] \cap (s^M_H, s^1])
\end{cases}. \tag{55}
\]
The first line on the RHS of (55) comes from the results in Case 1, 2 and 3. The second line follows the results in Case 1 and 4. Case 2, 3, and 5 give the third line and the last line collects the result from Case 5.
To simplify (55), now we need to further classify two cases: $s^M_H < s^1$ and $s^M_H \geq s^1$. $s^M_H < s^1$ is equivalent to

$$2 - \frac{1}{4} \frac{\beta}{z} > (1 + \beta) \sqrt{(1 - \beta/z)}$$

after some algebra. It follows that $s^M_H < s^1$ if and only if

$$z < \hat{z}.$$  

When $z < \hat{z}$, (54) establishes that $s^M_H < s^1 \leq s^2$. Moreover, when $z < \hat{z}$, one can show that $s^L > s^2$ must hold. Together with (48), (49), (51), (52) and (54), (55) can be reduced to (44). Finally, since $-(\beta^2 + 2\beta) + (1 + \beta) \sqrt{\beta^2 + 2\beta} < 1/2$ always holds, $z \in (\beta/4, \hat{z})$ is not an empty set. Similarly, when $z \geq \hat{z}$, (54) establishes that $s^M_H \geq s^2 \geq s^1$. Then (55) can be written as (43). \]

8.4 $z \in (\beta/4, \hat{z})$

**Proposition 4** Assume that (24) and $z \in (\beta/4, \hat{z})$. Then, the Markov perfect equilibrium is such that

$$\tau^y(s) = \begin{cases} \frac{1}{\tau} & \text{if } s \leq s^2 \\ \lambda^{-}(s) & \text{if } s \in [s^2, s^R] \\ \frac{1 + \beta}{2} & \text{if } s > s^R \end{cases}.$$  

(56)

**Proposition 5** Assume that (24) and $z \in (\beta/4, \hat{z})$. Then, the Markov perfect equilibrium is such that

$$h = \begin{cases} \frac{1}{\tau} & \text{if } s \leq s^2 \\ \frac{1}{\tau} - \rho s - z & \text{if } s \in [s^2, s^R] \\ \frac{1 + \beta}{4} & \text{if } s > s^R \end{cases}.$$  

(57)

The proof is straightforward and immediately follows from Lemma 3, (19), (26) and (31). One can see that the implications of Proposition 4 and 5 are qualitatively the same as Proposition 1 and 2. It is worthy of noting that $\pi(\tau^y(s), s) = 0$ or 1. That is to say, there is no electoral uncertainty under a small $z$. 

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Figure 1: Equilibrium Results

Figure 1: Panel A represents the equilibrium policy rule $\tau^\pi(s)$. The probability for the right-wing to be elected, $\pi(\tau^\pi(s), s)$, is plotted in Panel B. Panel C corresponds to the equilibrium investment rule $h(\tau^\pi(s), s)$. The parameter values are set equal to the benchmark case: $\pi = 0.5$, $\rho = 0.5$, $\beta = 1.0$. 
Figure 2: Laffer Curves

Panel A: $s < s^1$

Panel B: $s = s^1$

Figure 2: Panel A and B plot $T(\tau^y, s)$ with respect to $\tau^y$ under different ideological states. The parameter values are set equal to the benchmark case as in Figure 1.
Figure 3: Sizes of Government

Panel A: $s_{-1} = -0.25$

Panel B: $s_{-1} = 0.25$

Figure 3: Panel A and B plot equilibrium sizes of government with respect to all possible ideological states, $s \in [\rho s_{-1} - z, \rho s_{-1} + z]$, given different $s_{-1}$. The parameter values are set equal to the benchmark case.
Figure 4: Sensitivity Analysis

Figure 4: Panel A and B plot the estimated $a^2$ (solid line) and $b^2$ (dotted line) with respect to $z$ and $\rho$, respectively. The other parameter values are held constant as in the benchmark case.
Table 1: OLS Estimation of the Determinants of Government Sizes (Simulated Data)

<table>
<thead>
<tr>
<th>Dep. Variable</th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>R</td>
<td>-0.4794***</td>
<td>-0.4792***</td>
<td>-0.4950***</td>
</tr>
<tr>
<td></td>
<td>(-430.67)</td>
<td>(-438.18)</td>
<td>(-410.43)</td>
</tr>
<tr>
<td>s</td>
<td>0.0759***</td>
<td>0.0761***</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>(45.66)</td>
<td>(46.60)</td>
<td></td>
</tr>
<tr>
<td>s-1</td>
<td>0.0007</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>(0.66)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>e</td>
<td>-</td>
<td>-</td>
<td>0.1010***</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(55.16)</td>
</tr>
<tr>
<td>R²</td>
<td>0.9792</td>
<td>0.9792</td>
<td>0.9805</td>
</tr>
</tbody>
</table>

Notes: $t$ statistics is in brackets. ***, ** and * is significant at 1%, 5% and 10%, respectively.
<table>
<thead>
<tr>
<th>Dep. Variable</th>
<th>CGEXP</th>
<th>CGREV</th>
<th>CGEXP</th>
<th>CGREV</th>
<th>CGEXP</th>
<th>CGREV</th>
<th>CGEXP</th>
<th>CGREV</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(1)</td>
<td>(2)</td>
<td>(3)</td>
<td>(4)</td>
<td>(5)</td>
<td>(6)</td>
<td>(7)</td>
<td>(8)</td>
</tr>
<tr>
<td><strong>R</strong></td>
<td>-0.8150</td>
<td>-1.0462*</td>
<td>-1.8006***</td>
<td>-1.8270***</td>
<td>-1.4589</td>
<td>-1.2728**</td>
<td>-2.3819***</td>
<td>-2.2232***</td>
</tr>
<tr>
<td></td>
<td>(-0.84)</td>
<td>(-1.65)</td>
<td>(-2.78)</td>
<td>(-3.10)</td>
<td>(-1.51)</td>
<td>(-1.96)</td>
<td>(-3.43)</td>
<td>(-3.49)</td>
</tr>
<tr>
<td><strong>RVOT</strong></td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>0.1621***</td>
<td>0.0598***</td>
<td>0.2001***</td>
<td>0.1651***</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>(6.69)</td>
<td>(2.62)</td>
<td>(8.72)</td>
<td>(7.06)</td>
</tr>
<tr>
<td>Control</td>
<td>No</td>
<td>Yes</td>
<td>No</td>
<td>Yes</td>
<td>No</td>
<td>Yes</td>
<td>No</td>
<td>Yes</td>
</tr>
<tr>
<td>Obs.</td>
<td>673</td>
<td>663</td>
<td>718</td>
<td>708</td>
<td>673</td>
<td>663</td>
<td>718</td>
<td>708</td>
</tr>
<tr>
<td>Adj. R²</td>
<td>0.832</td>
<td>0.881</td>
<td>0.812</td>
<td>0.833</td>
<td>0.842</td>
<td>0.882</td>
<td>0.829</td>
<td>0.842</td>
</tr>
</tbody>
</table>

Notes: Country dummies and year dummies are included to control for the fixed effects and time effects. Control variables are YPC (the log of real GDP per capita), YGAP (HP filtered YPC), openness, unemployment rate, and the sizes of population over 65 and below 14. Robust t statistics is in brackets. ***, ** and * is significant at 1%, 5% and 10%, respectively.
Table 3: 2SLS Estimation of the Determinants of Government Sizes

<table>
<thead>
<tr>
<th>Dep. Variable</th>
<th>CGEXP</th>
<th>CGREV</th>
<th>CGEXP</th>
<th>CGREV</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(1)</td>
<td>(2)</td>
<td>(3)</td>
<td>(4)</td>
</tr>
<tr>
<td>R</td>
<td>-2.4844*</td>
<td>-2.3420**</td>
<td>-3.8392***</td>
<td>-3.7943***</td>
</tr>
<tr>
<td></td>
<td>(-1.95)</td>
<td>(-2.13)</td>
<td>(-3.36)</td>
<td>(-3.44)</td>
</tr>
<tr>
<td>RVOT</td>
<td>0.1632***</td>
<td>0.0380</td>
<td>0.2156***</td>
<td>0.1728***</td>
</tr>
<tr>
<td></td>
<td>(5.55)</td>
<td>(1.34)</td>
<td>(7.64)</td>
<td>(5.87)</td>
</tr>
<tr>
<td>Control Variables</td>
<td>No</td>
<td>Yes</td>
<td>No</td>
<td>Yes</td>
</tr>
<tr>
<td>Obs.</td>
<td>657</td>
<td>648</td>
<td>700</td>
<td>708</td>
</tr>
</tbody>
</table>

Notes: Country dummies and year dummies are included to control for the fixed effects and time effects. R and RVOT are identified as endogenous. The corresponding instruments are the same variables, but one-year lagged. Control variables are YPC (the log of real GDP per capita), YGAP (HP filtered YPC), openness, unemployment rate, and the sizes of population over 65 and below 14. $t$ statistics is in brackets. ***, ** and * is significant at 1%, 5% and 10%, respectively.