Fixed Costs and the Division of Labor

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Abstract

How market size and the level of coordination costs determine the degree of specialization is studied in an infinite horizon model with the amount of capital determined endogenously. Firms producing the same intermediate good engage in oligopolistic competition and choose the degree of specialization of their technologies to maximize profits. A more specialized technology is a technology with a lower marginal cost, but a higher fixed cost. Interestingly, the relationship between the level of coordination costs and a firm's degree of specialization is ambiguous. A firm in a country with a larger market size, more patient citizens, or a higher amount of knowledge will choose more specialized technologies and this country will have a higher wage rate and a higher capital stock. If fixed costs decrease, firms will choose more flexible manufacturing.

Keywords: The division of labor, market size, fixed costs, flexible manufacturing, coordination costs

JEL Classification: D43, L13, O14

1. Introduction

To explain huge productivity differences among countries, Smith (1776) has argued that countries differ in their degrees of specializations in production and a country with a higher degree of the division of labor will enjoy a higher productivity. The adoption of machines plays an important role in affecting the division of labor (Smith, 1776, p. 11). Machines are capital and fixed costs of production. Stopford (2009) provides examples that higher degrees of specialization in production are associated with higher fixed costs and lower marginal costs: the transportation of some products requires specialized vessels such as oil tanks and liquified natural gas (LNG) carriers. Oil tanks and LNG carriers are fixed costs of production. While they are more expensive if compared with unspecialized vessels, they are more efficient in transporting oil and LNG and marginal costs are much lower. Large volumes of transportation make the adoption of specialized vessels profitable. Nowadays the service sector is the largest sector in developed economies. For some service sectors, higher physical capital reduces marginal cost of production. For example, compared with car transportation, high-speed trains require much higher fixed costs, but the marginal cost of transporting individuals among cities will be lower. Division of labor in the service economy is also frequently related to special skill training and high human capital investment. Research and development activities also come with a high fixed cost and bring down the marginal cost of production.[1]

As illustrated in Young (1928), when firms adopt more specialized technologies, it will be profitable only if firms produce extremely high levels of output. In a famous article, Young (1928, p. 530) argued the reason that productivity was higher in the US compared to the UK lied in differences in the adoption of more specialized technologies in the US and output difference led to this technology difference between the two countries: "It would be wasteful to make a hammer to drive a single nail: it would be better to use whatever awkward implement lies conveniently at hand. It would be wasteful to furnish a factory with an elaborate equipment of specially constructed jigs, gauges, lathes, drills, presses and conveyors to build a hundred automobiles; it would be better to rely mostly upon tools and machines of standard types, so as to make a relatively larger use of directly-applied and a relatively smaller use of indirectly-applied labor. Mr. Ford's methods would be absurdly uneconomical if his output were very small, and would be unprofitable even if his output were what many other manufacturers of automobiles would call large."

With high levels of output necessary to recover high level of fixed costs, larger firms will have cost advantages over smaller firms and perfect competition will not prevail. For example, only a small number of firms can incur the research and development costs to produce LNG boats and the type of market structure for this market is oligopoly. With increasing returns in production, management, and distribution, oligopoly became a vital type of market structure for countries experienced the Second Industrial Revolution between the end of the 19th century and the start of the 20th century (Chandler, 1990). In their textbook, Pindyck and Rubinfeld (2005, p. 441) write that "oligopoly is a prevalent form of market structure. Examples of oligopolistic industries include automobiles, steel, aluminum, petrochemicals, electrical equipment, and computers."

While Smith's emphasis on the division of labor has received much attention, the interaction among fixed costs of production, high levels of output, and the degree of specialization has not received much attention in formal models of specialization.[2] One exception is Rosen (1983) who emphasizes the increasing returns to utilization of human capital. Indivisibilities imply fixed elements of human capital investment that are independent of subsequent utilization. Thus, individuals have the incentives to specialize in one skill even though they are *ex ante* identical.

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[1] The author thanks an anonymous referee for suggesting this broader interpretation of capital and fixed costs.

[2] Yang and Ng (1998) provide a survey of the literature on the division of labor.

In this paper, we study how different factors such as market size, knowledge, and coordination costs affect a firm's degree of specialization in an infinite horizon model. This paper captures the role of machines in affecting specialization by specifying capital as fixed costs of production. Capital in this model should be interpreted broadly to include human capital, rather than restricted to physical capital. The amount of capital is endogenously determined by saving behavior of individuals. Individuals live forever and an individual's effective supply of labor in each period is augmented by the amount of knowledge. While firms producing the final good engage in perfect competition, firms producing intermediate goods engage in oligopolistic competition and choose outputs to maximize profits.[3]

In this model, firms producing intermediate goods choose the degree of specialization of their technologies. A more specialized technology has a higher fixed cost but a lower marginal cost of production. This specification that a higher level of fixed costs is associated with a higher degree of specialization is consistent with Smith (1776, p. 11) that "labor is facilitated and abridged by the application of proper machinery". Also, when an individual spends more time in acquiring more human capital by learning knowledges in given fields, this individual becomes more specialized. Thus, the degree of specialization of a worker increases with the level of human capital of this worker.

The combination of the specification of capital as fixed costs of production and the choice of technology in this model is consistent with presentation on short run and long run cost curves in standard microeconomics textbooks. Suppose output is produced by machines (capital) and labor. In the short run, the number of machines is fixed and the number of workers is adjustable. Fixed costs are associated with the usage of machines. In the long run, a firm can choose the number of machines. When the number of machines used by a firm increases, variable costs in terms of labor costs decrease. That is, a higher fixed costs shown as a higher level of capital leads to a lower marginal cost of production.

The modeling of specialization here is consistent with an alternative approach in which an increase in the degree of specialization is captured by an increase in the number of intermediate inputs used in producing a final good, as employed in Ethier (1982) and Rodriguez-Clare (1996).

[3] If firms engage in monopolistic competition and the elasticity of demand is constant, a firm's output will be constant. In this model a firm's optimal choice of specialization depends on its output level. This motivates the choice of oligopoly as the type of market structure because a firm's output will change with fundamentals such as population size under oligopoly even with constant elasticity of demand.

In their models, when a higher number of intermediate inputs are used, marginal cost of production decreases. Since the production of each intermediate input requires fixed costs, an increase in the number of intermediate inputs indicates an increase in total fixed costs. Thus, their approach can also be interpreted as there is a tradeoff between fixed costs and marginal cost of production.

Scholars have emphasized two factors affecting the degree of specialization: market size and coordination costs. First, for the importance of market size, Smith (1776) describes that the division of labor depends on the extent of the market, while the extent of the market itself depends on the division of labor.[4] Kim (1989) presents a formal model in which a worker chooses the depth and breadth of her skills. A more specialized skill increases productivity of a worker but makes matching with a firm less likely. He demonstrates that a larger market size induces workers to choose more specialized human capital. Yang and Borland (1991) study a dynamic model in which an increase in the division of labor is captured by an increase in the proportion of output that is sold to other consumer-producers. With learning by doing, they prove that the range of goods produced by a consumer-producer decreases over time. Learning by doing in their model increases factor supply and can be interpreted as an increase in market size in essence. To formalize Smith's insight, Zhou (2004) establishes the mutual dependence between the division of labor and the extent of the market in a general equilibrium model. There are some significant differences between this model and Zhou (2004). First, Zhou (2004) is a one-period model with labor as the only factor of production while this is an infinite horizon model with capital as an additional factor of production. Second, the impact of coordination costs on specialization is not addressed in Zhou (2004).

Second, Becker and Murphy (1992) have stressed the importance of coordination costs rather than market size in affecting specialization. In their model, market size is measured by team size. They show that coordination costs and the accumulation of knowledge play important roles in determining the degree of specialization. Kikuchi, Nishimura, and Stachurski (2018) have studied a model in which coordination costs and transaction costs determine a firm's level of specialization when firms engage in perfect competition.

The tradeoff between fixed cost and marginal cost in this model is employed by Zhou (2019a, 2019b). Zhou (2019a) has addressed the impact of coordination costs and market size in affecting the degree of specialization of firms in a one-period general equilibrium model. There

[4] Stigler (1951) emphasizes that the division of labor is determined by the extent of the market.

are two important differences between this paper and Zhou (2019a). First, capital is not a factor of production in Zhou (2019a) while capital is a factor of production in this model and the amount of capital is endogenously determined. Second, the specifications of coordination costs are different. In Zhou (2019a), coordination is conducted by a hierarchy while in this model coordination costs are specified directly as a function of the degree of specialization, consistent with Becker and Murphy (1992). Zhou (2019b) has studied how resource abundance and market size affect a firm's technology choice in which the amount of capital is endogenously determined. Like a specific-factors model in international trade, labor, capital, and land are the three factors of production. While Zhou (2019b) is an overlapping generations model, this is an infinite horizon model which is convenient in addressing stability of the steady state. Also, the impact of knowledge and coordination costs on specialization is not considered in Zhou (2019b).

The plan of the paper is as follows. Section 2 specifies the model. A representative consumer's utility maximization, an intermediate good producer's profit maximization, and market-clearing conditions are studied. Section 3 addresses the stability and properties of the steady state. Section 4 concludes.

2. The model

Time is continuous and subscripts are used to denote time periods. Frequently subscripts may not be used if there is no confusion from doing this. There are two sectors of production: the sector producing the final good and the sector producing intermediate goods. Capital and labor are the two factors of production. The amount of capital in period t is K_t. For simplicity, it is assumed that capital does not depreciate. The interest rate in period t is r_t. The size of the population is L. Individuals live forever and the size of population does not change over time. Each individual supplies one unit of labor inelastically in a period. The amount of knowledge for this economy is z, which is assumed to be exogenously given. An increase in knowledge increases effective labor supply of an individual and each individual's effective supply of labor in a period is z. The wage rate for each effective unit of labor is w_t.

2.1. Utility maximization

An individual's discount rate is ρ and her consumption of the final good is c_t. Her utility function is specified as

$$U = \int_0^{\infty} e^{-\rho t} ln c_t dt. \tag{1}$$

The price of the final good is P_t. A consumer's expenditure in a period is

$$E_t = P_t c_t. \tag{2}$$

Individuals may own capital as assets. The amount of assets owned by an individual is a_t. An individual's income in a period is the sum of income from owning assets $r_t a_t$ and the wage income zw_t. Let a dot over a variable denote its time derivative. The evolution of asset for an individual is

$$\dot{a}_t = r_t a_t + zw_t - E_t. \tag{3}$$

An individual takes the wage rate, the interest rate, and the price of the final good as given, and chooses expenditure over time to maximize (1), subject to the constraint (3). A consumer's utility maximization yields the following familiar result:

$$\frac{\dot{E}_t}{E_t} = r_t - \rho. \tag{4}$$

The no-Ponzi-game condition is

$$\lim_{t \to \infty} a_t e^{-\int_0^t r(s) ds} \geq 0.$$

2.2. The final good sector

Firms producing the final good engage in perfect competition. The final good is produced by combining intermediate goods without incurring additional costs. There is a continuum of intermediate goods indexed by a number $\varpi \in [0, 1]$.[5] The final good can be used either for consumption or investment.

If the amount of intermediate good ϖ used for producing the final good is $q_t(\varpi)$, for a constant $\sigma > 1$, output of the final good Q_t is determined in the following way:

$$Q_t = \left[\int_0^1 q_t^{\frac{\sigma-1}{\sigma}} (\varpi) d\varpi \right]^{\frac{\sigma}{\sigma-1}}. \tag{5}$$

[5] One advantage of this assumption of a continuum of intermediate goods is the elimination of an intermediate good producer's potential market power in the labor market (Neary, 2016). While a firm has market power in the market of the intermediate good it produces because there is only a small number of firms producing each intermediate good, a firm does not have market power in the labor market because there is a continuum of firms demanding labor in the labor market.

A firm producing the final good takes the prices of the final good and intermediate goods as given and chooses quantities of intermediate goods to maximize profit $P\left[\int_0^1 q^{\frac{\sigma-1}{\sigma}}(\varpi)d\varpi\right]^{\frac{\sigma}{\sigma-1}} - \int_0^1 p(\varpi)q(\varpi)d\varpi$. From a final good producer's profit maximization, the elasticity of demand for an intermediate input is

$$\frac{\partial q}{\partial p}\frac{p}{q} = -\sigma. \tag{6}$$

The relationship between the price of the final good and prices of intermediate goods is given by

$$P = [\int_0^1 p(\varpi)^{1-\sigma}d\varpi]^{\frac{1}{1-\sigma}}. \tag{7}$$

2.3. Intermediate goods sector

For an intermediate good to reach customs, coordination is needed. All intermediate goods are assumed to have the same costs of production and coordination. For each unit of output, coordination requires $\theta s(n)$ units of labor, where θ is an exogenously given positive number and the function $s(n)$ is assumed to be twice continuously differentiable with $s'(n) > 0$.[6] That is, a higher level of specialization increases the level of coordination costs. Other things equal, an increase in θ means coordination cost is higher. Coordination of an intermediate good is conducted by the firm producing it.

The number of identical firms producing intermediate good ϖ is $m(\varpi)$, and $m \in R^1$. Firms producing the same intermediate good engage in Cournot competition. Like Zhou (2004, 2009, 2011, 2013), to produce an intermediate good, there is a continuum of technologies indexed by $n \in R^+$. A higher value of n indicates a more specialized technology. For a positive constant b, the fixed costs in terms of capital used with technology n is $bf(n)$, while the marginal cost in terms of labor used is $\beta(n)$. When b decreases, fixed costs are lower. We assume that functions f and β are twice continuously differentiable, or C^2. To capture the substitution between capital and

[6] There are some alternative ways to model coordination costs. First, coordination can be performed by a separate sector like the transportation sector in Zhou (2014). In Zhou (2014), the transportation sector chooses technologies with different degrees of specialization. A more specialized technology in the manufacturing sector will induce the transportation sector to choose a more specialized technology, and vice versa. Second, the level of coordination costs can be specified as an exogenous parameter independent of the level of specialization. The results with exogenous coordination costs are similar to those here.

labor in production, we assume that fixed costs increase while marginal cost decreases with the level of technology: $f'(n) > 0$ and $\beta'(n) < 0$.[7] One example of this tradeoff between fixed cost and marginal cost is as follows: container ports are more specialized than traditional ports. Compared with traditional terminals, container terminals can handle higher volumes of trade (more than twenty times higher) even though they are ten times costlier to build (Levinson, 2006). Another example is the grading of multiple-choice exams. Compared with hand-grading of exams, machine-grading requires fixed costs, but marginal cost is lower. To make a more specialized technology feasible, we also assume that $\beta'(n) + \theta s'(n) < 0$. That is, when a firm chooses a more specialized technology, even though coordination costs increase, the sum of marginal cost and coordination costs decreases.

Since the price charged by a firm producing intermediate good ϖ is $p(\varpi)$ and this firm's level of output is $x(\varpi)$, its total revenue is $p(\varpi)x(\varpi)$. This firm's cost has three components: fixed costs bfr, coordination costs θsxw, and production costs βxw. Thus, this firm's profit is

$$\pi = p(\varpi)x(\varpi) - bf(n)r - [\beta(n) + \theta s(n)]xw. \tag{8}$$

In each period, a firm producing an intermediate input takes the wage rate, the interest rate, and outputs and technologies of other firms as given and chooses its output and technology to maximize profit. This firm's optimal choice of output yields

$$p\left(1 + \frac{x}{p}\frac{\partial p}{\partial x}\right) = (\beta + \theta s)w. \tag{9}$$

This firm's optimal choice of technology yields the following first order condition with respect to n:

$$-bf'(n)r - x[\beta'(n) + \theta s'(n)]w = 0. \tag{10}$$

Other things equal, equation (10) shows that a firm will choose a more specialized technology when its output is larger. This direct relationship between output and specialization is consistent with the argument in Young (1928).

2.4. Market-clearing conditions

[7] To make sure that the second order condition for a firm's optimal choice of technology is satisfied, it is also assumed that $f''(n) \geq 0$ and $\beta''(n) \geq 0$. Those assumptions mean that fixed costs increase with the level of technology at a nondecreasing rate and marginal cost decreases at a nonincreasing rate. Those second order conditions are used to sign comparative static studies.

For the labor market, to produce intermediate good ϖ, each of the $m(\varpi)$ firms needs $\beta(\varpi)x(\varpi)$ units of labor for production and θsx units of labor for coordination. Thus, total demand for labor from the production of intermediate good ϖ is $m(\beta + \theta s)x$. Integrating over intermediate goods, total demand for labor in a period is $\int_0^1 m\,(\beta + \theta s)x d\varpi$. Total effective supply of labor in a period is zL. The clearance of the labor market requires

$$\int_0^1 m\,(\beta + \theta s)x d\varpi = zL. \tag{11}$$

For the market for capital, each of the $m(\varpi)$ firms demands $bf(\varpi)$ units of capital. Integrating over intermediate goods, total demand for capital in a period is $\int_0^1 m(\varpi)\, bf(\varpi)d\varpi$. Total supply of capital is K_t. The clearance of the market for capital requires

$$\int_0^1 m(\varpi)b\,f(\varpi)d\varpi = K_t. \tag{12}$$

Since the number of firms producing an intermediate good is a real number rather than restricted to be an integer, firms will enter until the level of profit is zero.[8] The zero-profit condition for an intermediate good producer requires

$$\pi = px - bfr - (\beta + \theta s)xw = 0. \tag{13}$$

Demand for the final good is the sum of demand for consumption and demand for investment. Each of the L individuals demand c_t units of the final good for consumption and total demand of the final good for consumption is Lc_t. The amount of the final good used for investment is I_t. The supply of the final good in a period is Q_t, which in equilibrium equals the sum of output of intermediate goods producers $m_t x_t$. The clearance of the market for an intermediate good requires

$$Lc_t + I_t = Q_t = m_t x_t. \tag{14}$$

Let x^i denote the level of output of a representative intermediate producer i and x^{-i} denote the sum of output of other intermediate good producers producing the same intermediate good: $m_t x_t = x^i + x^{-i}$. In a Cournot-Nash equilibrium, when an intermediate good producer chooses its output x^i, it takes output of other intermediate good producers x^{-i} as given. Differentiating equation (14) and combining the result with equation (6) yields[9]

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[8] Mankiw and Whinston (1986) and Liu and Wang (2010) provide examples of models in which firms engage in Cournot competition with free entry.

[9] The derivation of the elasticity of demand for an intermediate good producer here is like Heijdra (1998, p. 666). One difference between his paper and this one is that in his model firms engage in monopolistic competition while firms engage in oligopolistic competition in this model.

$$\frac{\partial x^i}{\partial p} = \frac{\partial (x^i + x^{-i})}{\partial p} = \frac{\partial Q}{\partial p} = \frac{\partial Q}{\partial p}\frac{p}{Q}\frac{Q}{p} = \frac{\partial q}{\partial p}\frac{p}{q}\frac{Q}{p} = -\sigma\frac{Q}{p}. \tag{15}$$

From equations (14) and (15), $\frac{\partial x^i}{\partial p}\frac{p}{x^i} = -m\sigma$. Plugging the result into equation (9) yields

$$p\left(1 - \frac{1}{m\sigma}\right) = (\beta + \theta s)w. \tag{16}$$

Equation (16) is the familiar condition that a firm's price is a markup over its marginal cost of production and the markup factor depends on the number of firms in the industry and the elasticity of substitution among goods.

Total capital stock is equal to the total amount of assets owned by individuals: $K_t = La_t$. Timing both sides of equation (3) by L yields

$$\dot{K}_t = r_t K_t + Lzw_t - LE_t. \tag{17}$$

From equation (4), the interest rate equals the discount rate in the steady state:

$$r = \rho. \tag{18}$$

In the steady state, the amount of capital does not change. From (14), the clearance of goods market requires

$$\int_0^1 p\, m\, x d\varpi = wzL + rK. \tag{19}$$

In a symmetric equilibrium, all intermediate goods have the same number of firms producing it, the same level of output, and the same price. Since the measure of intermediate goods is one and all intermediate goods are symmetric, for simplicity, we drop the integration operator in a symmetric equilibrium. For the rest of the paper, the price of an intermediate good is normalized to one: $p \equiv 1$. Thus, from equation (7), $P = 1$.

In the steady state, for an initial amount of capital K_0, equations (10)-(13), (16), and (18)-(19) form a system of seven equations defining a set of seven variables p, m, x, w, r, K, and n as functions of exogenous parameters.

3. The steady state

In the steady state, variables do not change over time. We drop time subscripts for variables in the steady state. In this section, first we establish that the steady state is a saddle path. Second, we conduct comparative statics to explore the properties of the steady state.

3.1. Stability of the steady state

With the price of a representative intermediate good normalized to one, $E = c$. Plugging the value of x from equation (11) and the value of w from equation (9) in the steady state into equation (13) yields $r = \frac{bfzL}{K^2\sigma(\beta+\theta\ )}$. Plugging this interest rate into equations (4) and (17), the evolution of per capita consumption and capital stock is defined by the following system of two equations:

$$\Lambda_1 \equiv \dot{c} = c\left\{\frac{bfzL}{K^2\sigma(\beta+\theta s)} - \rho\right\}, \tag{20a}$$

$$\Lambda_2 \equiv \dot{K} = \frac{L}{\beta+\theta s} - Lc. \tag{20b}$$

Plugging the value of x from equation (13) into equation (10), the degree of specialization of a firm is defined implicitly by the following equation

$$-f'b(\beta+\theta s) - (\beta'+\theta s')(\sigma K - bf) = 0. \tag{21}$$

Let $\bar{c}$ and $\bar{K}$ denote the amount of consumption and capital stock in the steady state respectively. Linearizing equations (20a) and (20b) around the steady state yields

$$\begin{pmatrix}\dot{c}\\ \dot{K}\end{pmatrix} = \begin{pmatrix}\frac{\partial\Lambda_1}{\partial c} & \frac{\partial\Lambda_1}{\partial K}\\ \frac{\partial\Lambda_2}{\partial c} & \frac{\partial\Lambda_2}{\partial K}\end{pmatrix}\begin{pmatrix}c-\bar{c}\\ K-\bar{K}\end{pmatrix}. \tag{22}$$

From (20a), $\frac{\partial\Lambda_1}{\partial c} = 0$. From (20b), $\frac{\partial\Lambda_2}{\partial c} < 0$. In general, we may not be able to determine the sign of $\frac{\partial\Lambda_1}{\partial K}$ because $f(n)$ and $\beta(n)$ depend on K through equation (21). However, there are special cases that we can determine the sign of $\frac{\partial\Lambda_1}{\partial K}$. (i) In the first case, suppose that the degree of specialization is exogenously given (f, β, and s do not depend on n). For the system of equations (20a) and (20b) with $\frac{\partial\Lambda_1}{\partial K} < 0$, the determinant of the coefficient matrix of (22) is negative. With $\frac{\partial\Lambda_2}{\partial K} = 0$, there is a positive characteristic root and a negative characteristic root. Thus, the steady state for the system of equations (20a) and (20b) is a saddle path. (ii) In the second case if the degree of specialization is endogenously chosen, for special fixed costs, marginal cost, and coordination costs, we can sign $\frac{\partial\Lambda_1}{\partial K}$. One example is the following. Suppose fixed costs are specified as $b = 1$ and $f(n) = n$, marginal costs are specified as $\beta(n) = 1/n$, and coordination costs are specified as $s(n) = 1$. Plugging those specifications into equation (21) yields $n = \frac{1}{\theta}\left(\sqrt{1+\theta\sigma K} - 1\right)$. Plugging this value of n into equations (20a) and (20b), the dynamics of the system is defined by

$$\Phi_1 \equiv \dot{c} = c\left\{\frac{L}{\theta\sqrt{1+\theta\sigma K}} - \rho\right\}, \tag{23a}$$

$$\Phi_2 \equiv \dot{K} = \frac{L}{\theta}\left(1 - \frac{1}{\sqrt{1+\theta\sigma K}}\right) - Lc. \tag{23b}$$

With $\frac{\partial \Phi_1}{\partial c} = 0, \frac{\partial \Phi_2}{\partial c} < 0$, and $\frac{\partial \Phi_1}{\partial K} < 0$, the determinant of the coefficient matrix of the linearized system of (23a) and (23b) is negative. That is, when the degree of specialization is optimally chosen, for some kinds of fixed costs, marginal cost, and coordination costs, the steady state for equations (23a) and (23b) is also a saddle path.

3.2. Properties of the steady state

We now reduce the set of seven equations defining the steady state into the following system of three equations defining three variables n, x, and w as functions of exogenous parameters so that it is more manageable:[10]

$$\Gamma_1 \equiv -f'[1-(\beta+\theta s)w] - w(\beta'+\theta s')f = 0, \tag{24a}$$

$$\Gamma_2 \equiv [1-(\beta+\theta s)w]x - b\rho f = 0, \tag{24b}$$

$$\Gamma_3 \equiv (\beta+\theta)b\rho f - \sigma L[1-(\beta+\theta)w]^2 = 0. \tag{24c}$$

Total differentiating equations (24a) - (24c) with respect to $n, x, w, \rho, L, \sigma, \theta, b$, and z yields[11]

$$\begin{pmatrix} \frac{\partial \Gamma_1}{\partial n} & 0 & \frac{\partial \Gamma_1}{\partial w} \\ 0 & \frac{\partial \Gamma_2}{\partial x} & \frac{\partial \Gamma_2}{\partial w} \\ \frac{\partial \Gamma_3}{\partial n} & 0 & \frac{\partial \Gamma_3}{\partial w} \end{pmatrix} \begin{pmatrix} dn \\ dx \\ dw \end{pmatrix} = -\begin{pmatrix} 0 \\ \frac{\partial \Gamma_2}{\partial \rho} \\ \frac{\partial \Gamma_3}{\partial \rho} \end{pmatrix} d\rho - \begin{pmatrix} 0 \\ 0 \\ \frac{\partial \Gamma_3}{\partial L} \end{pmatrix} dL - \begin{pmatrix} 0 \\ 0 \\ \frac{\partial \Gamma_3}{\partial \sigma} \end{pmatrix} d\sigma$$

$$-\begin{pmatrix} 0 \\ \frac{\partial \Gamma_2}{\partial b} \\ \frac{\partial \Gamma_3}{\partial b} \end{pmatrix} db - \begin{pmatrix} 0 \\ 0 \\ \frac{\partial \Gamma_3}{\partial z} \end{pmatrix} dz - \begin{pmatrix} \frac{\partial \Gamma_1}{\partial \theta} \\ \frac{\partial \Gamma_2}{\partial \theta} \\ \frac{\partial \Gamma_3}{\partial \theta} \end{pmatrix} d\theta. \tag{25}$$

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[10] The derivation of equations (24a) - (24c) is as follows. First, equation (24a) comes from plugging the value of x from equation (11), the value of w from equation (16), and the value of r from equation (18) into equation (10). Second, equation (24b) comes from plugging the value of r from equation (18) into equation (13). Third, equation (24c) comes from plugging the value of m from equation (16) into equation (11) and replacing the value of x by using (24b).

[11] Equation (26a) is used to show $\frac{\partial \Gamma_2}{\partial n} = 0$.

Let Δ denote the determinant of the coefficient matrix of endogenous variables of (25). According to the correspondence principle (Samuelson, 1983, chap. 9), stability requires that $\Delta < 0$. When Δ is nonsingular, a unique steady state exists.

Individuals differ in their degrees of patience. Psychological studies have shown that people with higher degree of patience are more successful (Duckworth, 2016). The following proposition studies the impact of a change in the discount rate on endogenous variables.

Proposition 1: In the steady state, an increase in the discount rate leads to a lower capital stock and wage rate. Firms producing intermediate goods choose less specialized technologies.

Proof: Applying Cramer's rule on (25) yields

$$\frac{dn}{d\rho} = \frac{\partial \Gamma_1}{\partial w}\frac{\partial \Gamma_2}{\partial x}\frac{\partial \Gamma_3}{\partial \rho}/\Delta < 0,$$

$$\frac{dw}{d\rho} = -\frac{\partial \Gamma_1}{\partial n}\frac{\partial \Gamma_2}{\partial x}\frac{\partial \Gamma_3}{\partial \rho}/\Delta < 0.$$

Plugging the value of m from equation (16) into equation (12) yields

$$K = \frac{bf(n)}{\sigma(1-\beta(n)w-\theta w)}. \tag{26}$$

From equation (26), $\frac{dK}{d\rho} = \frac{dK}{dn}\frac{dn}{d\rho} + \frac{dK}{dw}\frac{dw}{d\rho}$. Since $\frac{dK}{dn} = 0$ from equation (24a), $\frac{dK}{dw} > 0$, and $\frac{dw}{d\rho} < 0$, it is clear that $\frac{dK}{d\rho} < 0$. ■

Proposition 1 shows that a country with more patient citizens will have a higher degree of specialization. It is intuitive that a country with more patient citizens will have a higher steady state capital stock because citizens care more about the future and are willing to save more. With more capital, firms producing intermediate goods choose more specialized technologies. Proposition 1 is consistent with the observation that the fast-growing economies in East Asia have high saving rates.

A country's market size is the product of population size and the wage rate (Smith, 1776, p. 23), which is endogenously determined in this model. Other things equal, an increase in population size means an increase in market size. The following proposition studies the impact of a change in population size on endogenous variables.

Proposition 2: In the steady state, an increase in population size leads firms producing intermediate goods to choose more specialized technologies. The level of output, the equilibrium wage rate, and the amount of capital increase.

Proof: Applying Cramer's rule on (25) yields

$$\frac{dn}{dL} = \frac{\partial \Gamma_1}{\partial w}\frac{\partial \Gamma_2}{\partial x}\frac{\partial \Gamma_3}{\partial L}/\Delta > 0,$$

$$\frac{dx}{dL} = \frac{\partial \Gamma_1}{\partial n}\frac{\partial \Gamma_2}{\partial w}\frac{\partial \Gamma_3}{\partial L}/\Delta > 0,$$

$$\frac{dw}{dL} = -\frac{\partial \Gamma_1}{\partial n}\frac{\partial \Gamma_2}{\partial x}\frac{\partial \Gamma_3}{\partial L}/\Delta > 0.$$

From equation (26), $\frac{dK}{dL} = \frac{dK}{dn}\frac{dn}{dL} + \frac{dK}{dw}\frac{dw}{dL}$. Since $\frac{dK}{dn} = 0$, $\frac{dK}{dw} > 0$, and $\frac{dw}{dL} > 0$, it is clear that $\frac{dK}{dL} > 0$. ■

Proposition 2 shows that the result in Stigler (1951), Kim (1989), and Zhou (2004, 2019a) that an increase in market size induces firms to choose more specialized technologies is robust to this model with capital as a factor of production. The intuition behind Proposition 2 is as follows. Other things equal, an increase in population size means a higher level of output. A higher level of output makes the adoption of more specialized technologies profitable because the higher fixed costs can be spread over a higher level of output. A more specialized technology is associated with a lower average cost of production. Since firms producing intermediate inputs earn a profit of zero, a lower average cost shows up as a lower price charged by an intermediate good producer. With the price of an intermediate good normalized to one, a lower average cost means a higher real wage rate.

For empirical research on the impact of market size on a firm's degree of specialization, Garicano and Hubbard (2007) have shown that an increase in market size leads to an increase in the share of lawyers working in field-specialized firms.

When σ increases, the absolute value of the elasticity of substitution among intermediate goods is higher. The following proposition studies the impact of a change in the elasticity of demand among intermediate goods on endogenous variables.

Proposition 3: In the steady state, an increase in the elasticity of substitution among intermediate goods leads intermediate goods producers to choose more specialized technologies.

While the level of output and the equilibrium wage rate increase, the impact on the capital stock is ambiguous.

Proof: Applying Cramer's rule on (25) yields

$$\frac{dn}{d\sigma} = \frac{\partial \Gamma_1}{\partial w}\frac{\partial \Gamma_2}{\partial x}\frac{\partial \Gamma_3}{\partial \sigma}/\Delta > 0,$$

$$\frac{dx}{d\sigma} = \frac{\partial \Gamma_1}{\partial n}\frac{\partial \Gamma_2}{\partial w}\frac{\partial \Gamma_3}{\partial \sigma}/\Delta > 0,$$

$$\frac{dw}{d\sigma} = -\frac{\partial \Gamma_1}{\partial n}\frac{\partial \Gamma_2}{\partial x}\frac{\partial \Gamma_3}{\partial \sigma}/\Delta > 0.$$

From equation (26), $\frac{dK}{d\sigma} = \frac{dK}{dn}\frac{dn}{d\sigma} + \frac{dK}{dw}\frac{dw}{d\sigma} + \frac{\partial K}{\partial \sigma}$. Since $\frac{dK}{dn} = 0$, $\frac{dK}{dw} > 0$, $\frac{dw}{d\sigma} > 0$, and $\frac{\partial K}{\partial \sigma} < 0$, the sign of $\frac{dK}{d\sigma}$ is ambiguous. ■

The intuition behind Proposition 3 is as follows. With a higher elasticity of demand among intermediate goods, a price decline will lead to a larger increase in demand for an intermediate good producer. A higher level of output makes the adoption of more specialized technologies profitable and tends to increase capital stock. However, from equation (26), an increase in the elasticity of substitution among intermediate goods has a direct effect of reducing the capital stock through σ. Since the two effects work in opposite directions, the overall impact of a change in the elasticity of demand on capital stock is ambiguous.

In a modern society, knowledge accumulation is significant, and individuals are relatively specialized. An increase in knowledge can be captured by an increase in the parameter z. The following proposition studies the impact of an increase in knowledge.

Proposition 4: In the steady state, an increase in the amount of knowledge leads firms producing intermediate goods to choose more specialized technologies. The level of output, the equilibrium wage rate, and capital stock increase.

Proof: Applying Cramer's rule on (25) yields

$$\frac{dn}{dz} = \frac{\partial \Gamma_1}{\partial w}\frac{\partial \Gamma_2}{\partial x}\frac{\partial \Gamma_3}{\partial z}/\Delta > 0,$$

$$\frac{dx}{dz} = \frac{\partial \Gamma_1}{\partial n}\frac{\partial \Gamma_2}{\partial w}\frac{\partial \Gamma_3}{\partial z}/\Delta > 0,$$

$$\frac{dw}{dz} = -\frac{\partial \Gamma_1}{\partial n}\frac{\partial \Gamma_2}{\partial x}\frac{\partial \Gamma_3}{\partial z}/\Delta > 0.$$

From equation (26), $\frac{dK}{dz} = \frac{dK}{dn}\frac{dn}{dz} + \frac{dK}{dw}\frac{dw}{dz}$. Since $\frac{dK}{dn} = 0$, $\frac{dK}{dw} > 0$, and $\frac{dw}{dz} > 0$, it is clear that $\frac{dK}{dz} > 0$. ■

Our result that a higher amount of knowledge leads to a higher level of specialization is consistent with Becker and Murphy (1992). One difference between their model and this one is that knowledge enters the benefit of division of labor directly in their model while knowledge augments factor of production in this model.

Changes from mass production with specialized equipment and extremely large quantities of output to flexible manufacturing with small batches of output in some industries are discussed in Milgrom and Roberts (1990) and Eaton and Schmitt (1994). The following proposition shows that this kind of changes are rational when there is a decrease in the level of fixed costs.

Proposition 5: In the steady state, a decrease in fixed costs leads to a higher capital stock and wage rate. Firms producing intermediate goods choose more specialized technologies.

Proof: Applying Cramer's rule on (25) yields

$$\frac{dn}{db} = \frac{\partial \Gamma_1}{\partial w}\frac{\partial \Gamma_2}{\partial x}\frac{\partial \Gamma_3}{\partial b}/\Delta < 0,$$

$$\frac{dw}{db} = -\frac{\partial \Gamma_1}{\partial n}\frac{\partial \Gamma_2}{\partial x}\frac{\partial \Gamma_3}{\partial b}/\Delta < 0.$$

From equation (26), $\frac{dK}{db} = \frac{dK}{dn}\frac{dn}{db} + \frac{dK}{dw}\frac{dw}{db}$. Since $\frac{dK}{dn} = 0$ from equation (24a), $\frac{dK}{dw} > 0$, and $\frac{dw}{db} < 0$, it is clear that $\frac{dK}{db} < 0$. ■

Using the method in proving Propositions 1-5, it can be shown that the impact of a change in the level of coordination costs on an intermediate good producer's degree of specialization and the wage rate is ambiguous. The reasoning is as follows. Equation (10) is the first order condition for a firm's optimal choice of specialization. From this equation, $f'r$ is the marginal cost from choosing a more advanced technology and $-x(\beta' + \theta s')w$ is the marginal benefit. With the interest rate equals the discount rate in the steady state, an increase in the level of coordination costs does not affect the marginal cost of specialization. However, an increase in the level of coordination costs affects marginal benefit directly through θ and through its influence on output and the wage rate indirectly because output and wage rate are functions of θ. The direct effect is that marginal benefit of specialization is lower. For the two indirect effects, first, from equation (16), other things equal, an increase in the level of coordination costs will increase the number of firms producing an intermediate good and a firm's level of output. This tends to increase the degree of specialization of a firm. Second, an increase in the level of coordination costs can have an impact like a decrease in population size, leading to a lower wage rate. The latter effect tends to decrease the degree of specialization of an intermediate good producer. Without adding additional structure, it is not clear which indirect effect will dominate. This result that a change in coordination costs has an ambiguous effect on the degree of specialization is different from that in Becker and Murphy (1992). In their model, the type of market structure is perfect competition and an increase in coordination cost will increase the marginal cost of specialization. Thus, the first indirect effect is absent in their model and they show that a higher coordination costs leads to a lower degree of specialization. Even if we had specified that coordination costs would not change with the degree of specialization, with the ambiguous relationship between marginal benefit of specialization and coordination costs in our model, our result that the relationship between coordination costs and specialization is ambiguous will be robust.

4. Conclusion

In this paper, we have studied how market size and coordination costs affect a firm's choice of specialization in an infinite horizon model in which firms engage in oligopolistic competition and capital stock is endogenously determined by saving behavior of individuals. The model is tractable, and we have established the following analytical results for the steady state. First, the impact of an increase in coordination costs on a firm's degree of specialization is ambiguous. Second, an increase in market size, the degree of patience of citizens, or the amount of knowledge increases a firm's degree of specialization, the equilibrium wage rate, and the amount of capital in the steady state. Finally, firms will choose more flexible manufacturing when fixed costs of technologies decrease.

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References

Becker, Gary, and Kevin Murphy. 1992. The division of labor, coordination costs, and knowledge. *Quarterly Journal of Economics* 107, 1137-1160.

Chandler, Alfred. 1990. *Scale and Scope: The Dynamics of Industrial Capitalism*. Cambridge, MA: Harvard University Press.

Duckworth, Angela. 2016. *Grit: The Power of Passion and Perseverance*. New York, NY: Scribner.

Eaton, B. Curtis, and Nicolas Schmitt. 1994. Flexible manufacturing and market structure. *American Economic Review* 84, 875-888.

Ethier, Wilfred. 1982. National and international returns to scale in the modern theory of international trade. *American Economic Review* 72, 389-405.

Garicano, Luis and Thomas Hubbard. 2007. Managerial leverage is limited by the extent of the markets: hierarchies, specialization, and the utilization of lawyers' human capital. *Journal of Law and Economics* 50, 1-45.

Heijdra, Ben. 1998. Fiscal policy multipliers: the role of monopolistic competition, scale economies, and intertemporal substitution in labor supply. *International Economic Review* 39, 659-696.

Kikuchi, Tomoo, Kazuo Nishimura, and John Stachurski. 2018. Span of control, transaction costs, and the structure of production chains. *Theoretical Economics* 13, 729-760.

Kim, Sunwong. 1989. Labor specialization and the extent of the market. *Journal of Political Economy* 97, 692-705.

Levinson, Marc. 2006. *The Box: How the Shipping Container Made the World Smaller and the World Economy Bigger*. Princeton, NJ: Princeton University Press.

Liu, Lin, and X. Henry Wang. 2010. Free entry in a Cournot market with imperfectly substituting goods. *Economics Bulletin* 30, 1935-1941.

Mankiw, N. Gregory, and Michael Whinston. 1986. Free entry and social inefficiency. *RAND Journal of Economics* 17, 48-58.

Milgrom, Paul, and John Roberts. 1990. The economics of modern manufacturing: technology, strategy, and organization. *American Economic Review* 80, 511-528.

Neary, J. Peter. 2016. International trade in general oligopolistic equilibrium. *Review of International Economics* 24, 669-698.

Pindyck, Robert, and Rubinfeld, Daniel. 2005. *Microeconomics*, sixth edition, Upper Saddle River, New Jersey: Pearson Education.

Rodriguez-Clare, Andres. 1996. The division of labor and economic development. *Journal of Development Economics* 49, 3-32.

Rosen, Sherwin. 1983. Specialization and human capital. *Journal of Labor Economics* 1, 43-49.

Samuelson, Paul. 1983. *Foundations of Economic Analysis*. Enlarged edition, Cambridge, MA: Harvard University Press.

Smith, Adam. 1776 (republished in 1976). *An Inquiry into the Nature and Causes of the Wealth of Nations*. Chicago, IL: University of Chicago Press.

Stigler, George. 1951. The division of labor is limited by the extent of the market. *Journal of Political Economy* 59, 185-193.

Stopford, Martin. 2009. *Maritime Economics*, 3rd edition, New York, NY: Routledge.

Yang, Xiaokai, and Jeff Borland. 1991. A microeconomic mechanism for economic growth. *Journal of Political Economy* 99, 460-482.

Yang, Xiaokai, and Siang Ng. 1998. Specialization and division of labor: a survey. in *Increasing Returns and Economic Analysis*, edited by Kenneth Arrow, Ng, K., and Xiaokai Yang, New York: St. Martin Press.

Young, Allyn, 1928. Increasing returns and economic progress. *Economic Journal* 38, 527-542.

Zhou, Haiwen. 2004. The division of labor and the extent of the market. *Economic Theory* 24, 195-209.

Zhou, Haiwen. 2009. Population growth and industrialization. *Economic Inquiry* 47, 249-265.

Zhou, Haiwen. 2011. Economic systems and economic growth. *Atlantic Economic Journal* 39, 217-229.

Zhou, Haiwen. 2013. The choice of technology and rural-urban migration in economic development. *Frontiers of Economics in China* 8, 337-361.

Zhou, Haiwen. 2014. International trade with increasing returns in the transportation sector. *Frontiers of Economics in China* 9, 606-633.

Zhou, Haiwen. 2019a. Coordination costs, market size, and the choice of technology. *Frontiers of Economics in China* 14, 131-148.

Zhou, Haiwen. 2019b. Resource abundance, market size, and the choice of technology. *Bulletin of Economic Research* 71, 641-656.