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Inter-temporal Calculative Trust Design to Reduce Collateral Need for Business Credits

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Abstract

Credit rationing arising out of informational asymmetry and lack of collateral is a well-recognised economic constraint in the credit market. These constraints get magnified for small businesses. This paper attempts to capture the dimension of trustworthiness (calculative trust) by designing a multi-period, incentivised payment structure that will induce economic agents to reveal the existence of private information about any projects or true intentions of paying up the credit that is going to fund the project. The model dynamically estimates the collateral needed by taking into account the truthfulness of the borrower. The proposed design is compared with the benchmark model - credit scoring-based model. Randomised simulations are carried out for the ex ante solution for the borrower. We find that the proposed design outperforms from the perspective of lenders when the probability of default of any project is less than 80 per cent. Our simulation result also finds that building trust helps small business owner to significantly reduce the need for collateral.

Keywords: Calculative trust, collateral. **JEL Classification:** M21, R51, G21.

1 Introduction

Creation of jobs is one of the most important political economic issues developing economies are facing. Growth of small and medium enterprises (SMEs) is imperative for employment growth because across developing economies SME's contributes a larger share of total employable workforce (Chu et al., 2007; Lee, 1998; Lin, 1998). SME's will grow when the entrepreneur (business owner) invest in new projects. Retained earnings may not be sufficient to meet the growth capital needed and it may need additional capital in the form of debt to fund the growth. However, small businesses encounter financial constraints while raising capital to fund the growth (Panda & Dash, 2014; Thampy, 2010).

Constraints faced by small businesses can be explained by higher perceived credit risk associated with them. Credit risk can come from two sources - first, risk arising from the inability of the project to pay back the loan, and second, risk arising from unwillingness of the business owner to pay back the loan. The risk mitigation strategy for the project's inability to pay back the loan is well developed, and driven by intermediaries such as credit rating agencies, credit-bureau's, and internally developed scorecards. On the other hand, the risk mitigation strategy with respect to unwillingness of the business owner to pay back the credit is not well developed. This is the grey area in underwriting processes within a lending institution.

Willingness to pay can be traced to trustworthiness of the business owner. This kind of trustworthiness can be visualized as calculative trust as discussed in Lewicki et al. (2006). One can argue that any economic relationship will start with the principal (lending institution) calculating the trustworthiness of the agent (borrower) on the

The views expressed in the paper are those of the author(s) and not necessarily those of the institution to which they belong.

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basis of perceived credit risk associated with dealing with the agent. As the principal deals with the agent over time, information set of the principal expands, and the trustworthiness of the agent in the eyes of principal will change. These inter-temporal changes in calculative trust have not been adequately captured in the literature. Besides, it has not been modeled to apply in understanding risk arising out of unwillingness to pay back the loan.

The inter-temporal dimensions of perceived credit risk can also be looked at from a perspective of existence of private information with business owner. This asymmetry of information between borrower and lender will lead to adverse selection (ex-ante), and existence of moral hazard (ex-post) problem. To solve the asymmetric information problem lending institutions will demand collateral from all types of borrower (including the good borrower). Non-availability of sufficient collateral will result into non-availability of credit to the potential good quality borrower. This is the classical credit rationing problem put forwarded by Stiglitz & Weiss (1981). The asymmetry of information problem is high in case of SME's, and they also face lack of collateral. Therefore, one can argue that small business owner is likely to face acute credit rationing.

Present paper makes an effort to help trustworthy small business owner to avail credit with lower amount of collateral. This is done by designing an incentivized metrics, which induces the small business owner to reveal certain amount of private information, so that the lender can asses the risk associated with the borrower's unwillingness to pay back the loan. The paper does not address the credit risk arising out of genuine risk of failure of the project due to external socio economic and political circumstances. The paper assumes that these kinds of genuine risks can be taken care of by credit default scores, and pricing of credit. The paper is organized into following 5 sections. Section 2 will discuss the literature. Section 3 will discuss the model, ex-ante solution and the simulation results. In Section 4 we will discuss the ex-post solution of the borrower and section 5 will conclude the paper.

2 Literature Review

Non-availability of financial resources is a major constraint faced by start-ups and SMEs in developing countries (Cook, 2001; Gray et al., 1997; Levy, 1993; Peel & Wilson, 1996). It is perceived that SME's belong to high risk credit category; hence SME's find it difficult to raise debt capital to fund their business growth (Thampy, 2010). Stiglitz & Weiss (1981) showed that credit rationing will happen due to existence of information asymmetry. Principal and agents have different information about the project for which credit is needed. Both principal and agents as rational individuals and might not have sufficient incentives to work towards a common goal (Akerlof, 1978; Ross, 1973). Lender solves this problem of asymmetry of information through contract and monitoring Sahlman (1990). Propose of monitoring here is to make sure there is incentive compatibility between borrower and lender so that the borrower does not take undue risk. Success of contractual mechanism to reduce the agency risk will depend on the completeness of contract, and enforcement of contract. However, enforcement of contract is a challenge, and is costly (Lawton, 2002).

In addition to appropriate contract, and monitoring, the lender can ask for collateral to reduce the risk due to failure of funded project. Collateral as means to reduce credit risk depends on market structure of the credit market . Chen (2006) showed that the riskier borrowers pledge higher collateral than safe borrowers, and Jiménez & Saurina (2004) find evidence that highly collateralized loans have higher probability of default. Literature on relationship between ex-ante demand for collateral and ex-post risk associated with the loan is inconclusive. Therefore taking high amount of collateral may not reduce the credit default ex-post. Logically one can argue that willingness to pay will play an important role in assessing the risk associated with any borrower. A borrower who have low pledgeable collateral and is willing to pay back the loan is likely to be a safer borrower in comparison to the borrower with highly pledgeable collateral but unwillingness to pay back the loan. Therefore, the credit rationing will be high when a small business owner is willing to pay back the loan but does not have

enough pledge able collateral.

An alternative way to addresses the asymmetry of information problem has been looked from the prospective of trust. Das & Teng (1998), Shepherd & Zacharakis (2001), and Vosselman & der Meer-Kooistra (2009) argues that the agency risk can be reduced by employing both contractual mechanisms and trust building in a dyadic relationship. Literature on uses of contract and trust can be visualized from three prospective. First, presence of higher trust level will drive lower level of control and lesser stringent contract (Dyer & Singh, 1998). Second, contractual control increases the trust level; hence contractual control and trust are complementary to each other (Leifer & Mills, 1996; Poppo & Zenger, 2002). Third, trust itself is a form of control mechanism; hence both of them are substitutes (Bradach & Eccles, 1989).

Most of the literature on trust with respect to asymmetry information problem is discussed in the context of venture capitalist and entrepreneurship relationship where there is great uncertainty. Panda & Dash (2016) studied different stages of entrepreneurial venture and found evidence for trust based control by the Indian venture capitalists in early stage of the firm, they also found combination of trust and control based risk mitigation methods adopted by the venture capitalists in late stages of the firm. The role of trust in building mutually beneficial cooperation has been explored in case bank – entrepreneur relationships (Saparito et al., 2004). At present there is lack of literature on looking at trust in the context of lender and borrower prospective. In this paper, we make an attempt to bring trust building over time to calculate the need for collateral. This model takes the concept of calculative trust as discussed in Lewicki et al. (2006), to facilitate trust building between borrower and lender. This model has objective of reducing credit constraint for individuals (non-wilful defaulters) without hard information. Since many of the credit institutions demand collateral from borrower, the trustworthy borrowers face credit constraint in this regard. The model suggests one methodology to reduce this credit constraint. The proposed model is an attempt to mitigate the need of collateral by incentivizing the borrower to be trustworthy overtime. The model is discussed in the next section.

3 The Model

A design in the world of information asymmetry is a procedure to nudge the agent to behave in a pre-specified way, without cohesion. Importance of design can be seen in the VCG mechanism (Clarke, 1971; Groves, 1973; Vickrey, 1961), where the design help us to solve the following twin objectives: helps in efficient allocation of the public goods among agents and force the agents to reveal their true value for these public goods. This is possible by appropriately incentivizing the agent to reveal the truth. Our model gets inspirations from VCG mechanism to incentivize the borrower to reveal the private information about the project for which the loan is sought.

At the time of borrower seeking loan for any project, it pays for the borrower to communicate in a way that will positively influence the lender to decide on giving the credit. For example, the borrower may overestimate the project cash-flows to show a favorable picture in support of the project. Lender will suffer when a project with inflated cash-flows is provided credit, and ultimately the project fails and the lender suffer losses. The proposed model creates a design where the borrower will be dis-incentivized to inflate the cash-flow numbers, and makes an attempt to tell the truth about the cash-flow of the project (as per the information available with the borrower). Our design incentivizes the borrower who does not deviate from the ex-ante promises.

3.1 Parameters of the model

The model assumes a rational borrower who has *n* mutually exclusive projects. Each project has a probability of default, which is represented by $\theta_i \in (0, 1]$ for $i \in \{1, 2, 3, ..., n\}$. The θ_i is exogenous to our model, and is known to the lender. The agent approaches the lender. The lender does not have any creditworthiness information about

the borrower, and finances one project at a time. Funding of the project is done if previous loan is paid in full including interest rate as per the due date. Borrower's tuple is (B_i, T_i, W_i) for project $i \in \{1, 2, 3, ..., n\}$, where W_i is borrower's pledgeable asset, B_i is the amount of loan for project *i* for a time period T_i . Similarly lender's tuple is $(\bar{B}_i, \alpha_i, r_i)$, where \bar{B}_i represents the upper bound of the loan sanctioned for the project *i*, α_i represents fractions of the $B_i(1 + r_i)^{T_i}$ needed as collateral, and r_i is the interest rate on the loan. The interest rate for the project *i* depends upon the riskiness (θ_i) of the project *i*. Keeping in mind the credit risk, the sanctioned loan amount for each project is constrained by the expected cash-flows from the project and the wealth available at that period.

The model assumes;

$$r_i = r_f + r_p(\theta_i)$$
 , $r_p(\theta_i) \ge 0$, $\frac{dr_p(\theta_i)}{d\theta_i} > 0$

Where r_f is risk - free rate and $r_p(\theta_i)$ is risk premium, which increases with riskiness of the project.

3.2 The Design

3.2.1 Project - i

Loan for project *i* is sanctioned if all loans availed previously are paid fully. The borrower has pledgeable asset W_i . The project cashflow is random and distributed uniformly in $[\underline{y}_i, \overline{y}_i]$. The probability of default θ_i for the project *i* is determined by the lender from its past experience of the similar projects. For this project the lender charges interest rate r_i such that

$$r_i = r_f + r_p(\theta_i)$$
 , $r_p(\theta_i) \ge 0$, $\frac{dr_p(\theta_i)}{d\theta_i} > 0$

where r_f is risk free rate and $r_p(\theta_i)$ is risk premium which depends upon probability of the default of the project *i*. The upper bound for loan amount (\bar{B}_i) is decided by the lender for time period T_i by

$$B_i \le \min\left\{\frac{1}{\alpha_i} \left(\frac{y_i + \bar{y_i}}{2} - \bar{C}\right) \sum_{t=1}^{T_i} \frac{1}{(1+r_i)^t} , \frac{1}{\alpha_i} \frac{W_i}{(1+r_i)^{T_i}}\right\} = \bar{B}_i$$

 $\alpha_i \in [0, 1]$ is percentage point of $B_i(1 + r_i)^{T_i}$ the lender need to keep as collateral for the project *i*, \bar{C} is subsistence consumption of a borrower. For project 1 it has been assumed to be equal to 1, because the lender wants to mitigate all the risk by demanding 100 percent of the loan amount including interest rate as collateral. Besides, the binding collateral requirement exists because the lender does not have any prior information of trust worthiness of the borrower. This construct is contrary to Bester (1985), which finds higher collateral requirement will attract high risk borrowers. Bester (1985) finding is static, and our model takes an inter-temporal approach. Therefore, in the second period (when the borrower comes for funding for project -2), the collateral needed will be dynamically determined, and will come down if borrower honors the reported payment schedule in period 1.

- The borrower pays the collateral $\alpha_i B_i (1+r_i)^{T_i}$ for the project *i*.
- The borrower is asked to reports a payment schedule

$$P_{i} = \left\{ (P_{i1}, P_{i2}, \cdots, P_{iT_{i}-1}, P_{iT_{i}}) \in \mathbb{R}_{+}^{T_{i}} : \sum_{t=1}^{T_{i}} \frac{P_{it}}{(1+r_{i})^{t}} = B_{i} \right\}$$

for the project *i*. Here P_{it} represents reported payment for the time period *t* for the project *i*, \mathbb{R}_+ is the set of non-negative real numbers.

- The borrower pays P'_{i1} ≥ 0, and this payment is independent of P_{i1} in time 1, pays P'_{i2} ≥ 0 in time 2, and so on.
- If

$$\sum_{t=1}^{T_i} \frac{P'_{it}}{(1+r_i)^t} < B_i$$

lender liquidates the collateral and retains an amount (M_i) of the proceeds from liquidation of collateral such that

$$\sum_{t=1}^{T_i} \frac{P'_{it}}{(1+r_i)^t} + \frac{M_i}{(1+r_i)^{T_i}} = B_i$$

and returns back rest of the amount to the borrower. In case the liquidated collateral value is less than M_i , the lender retains all the proceeds. In this case the lender suffers a loss.

• Let T'_i be the time period in which the loan amount is fully paid. So T'_i can be less than or equal to T_i

Taking these into account, we define a term c_{it} that captures the trust building process between borrower and lender in time *t* for the project *i*.

$$c_{it} = \begin{cases} 0 & : P'_{it} = P_{it} = 0\\ \frac{P'_{it}(P'_{it} - P_{it})}{P'_{it} + P_{it}} & : \text{otherwise} \end{cases}$$

 $\forall t \in \{1, 2, 3, \dots, T'_i\}$. As c_{it} increases over the period, the trustworthiness increases. We use this to incentivize or dis-incentivize the borrower depending on the behavior of the borrower over the time. The construct of incentivizing scheme is provided below. Let n_{it} is the cardinality of the set $S = \{c_{ij} : j = 1, 2, 3, ..., t - 1 \& c_{ij} \geq 0\}$

0 $\}$. The n_{it} is the number of periods in which the borrower had kept its promise before time *t* for the project *i*. Let's define the following *reward-penalty function*:

$$\beta_{it} = \begin{cases} c_{it}^{2(t-n_{it})-1} & :c_{it} < 0, n_{it} \ge 0\\ c_{it}^{\frac{1}{1+n_{it}}} & :c_{it} \ge 0 \end{cases}$$
(1)

For $t = 2, 3, 4, ..., T'_i$ and $\beta_{i1} = c_{i1}$.

The variable n_{it} plays very important role in *reward - penalty function* β_{it} . It controls the magnitude of penalty or reward depending on the promised payment schedule and the actual payment schedule. The complexity arises when $n_{it} = 0$, because n_{it} is 0 when the borrower pays everything in first payment or when the borrower has not kept any promises. Former is a trustworthy behavior which is rewarded in our design and later is not a trustworthy behavior and penalized in our design.

Explanation for β_{it}

Value of β_{it} is directly related to the trustworthiness of a borrower. Calculation of β_{it} depends on the behavior of borrower with respect to the promise that is made ex-ante. While calculating β_{it} we have separated three type of borrowers for rewarding and penalizing.

 $c_{it} < 0, n_{it} = 0$: These are types of borrower who historically have not kept any promises, and is not keeping the promise in time period *t* as well. Borrowers of these kinds are not trusted, and are asked to provide 100 percentage point of the loan amount as collateral.

 $c_{it} < 0, n_{it} > 0$: These are types of borrower who historically have kept promises at least once, but is not

keeping the promise in time period *t*. Borrowers of these kinds are not trusted fully and penalized by asking for higher amount of collateral. The degree of penalty depends on the number of times promises are kept.

 $c_{it} \ge 0$: This is a type of borrower who is keeping the promises at time period *t*. In this case n_{it} negatively affects the magnitude of reward, and the design incentives the borrower to pay back the loan as early as possible.

If $c_{it} = 0$ for all $t = 1, 2, 3, ..., T'_i$, the borrower kept the promise hence has to be rewarded. Then the lender asks for collateral of $\alpha_i \sim U(0, 1)$, i.e. distributed uniformly over (0,1), as a percent of the amount $B_{i+1}(1 + r_{i+1})^{T_{i+1}}$ for project i + 1. However, our objective is to reduce the collateral ratio by incentivising the borrower to prepay the credit by deviating from the promised payment positively. Therefore we have taken $E(\hat{\alpha}) = 0.5$ as the collateral requirement ratio.

In case $c_{it} \neq 0$ for some *t*, we define

$$lpha_{i+1} = rac{1}{\max \Big\{ 1 \ , \ 1 + \sum_{t=1}^{T_i'} eta_{it} \Big\}}$$

For the next project i + 1 lender asks for $\alpha_{i+1}B_{i+1}(1 + r_{i+1})^{T_{i+1}}$, amount of collateral.

3.3 Implications of the Design

Proposition 3.1. It is optimal strategy for borrower to report payment scheme $(0,0,0,...,0,B_i(1+r_i)^{T_i})$ and pay within time period $T'_i < T_i$.

Proof. Let

$$\Delta_{it} = P'_{it} - P_{it}$$

If the borrower pays the loan amount (no default case), then

$$\sum_{t=1}^{T_i} \frac{\Delta_{it}}{(1+r_i)^t} = 0$$
(2)

And

$$c_{it} = \frac{P'_{it}\Delta_{it}}{P'_{it} + P_{it}} \quad , \quad \forall t \in \{1, 2, 3, \cdots, T_i\}$$

One of the trivial solution that satisfy Eq. (2), $\Delta_{it} = 0$ for all t. If $\Delta_{ij} < 0$ for some time period $j \in \{1, 2, 3, ..., T_i\}$ then it will increase α_{i+1} . To have each $\Delta_{it} \ge 0$ and to minimize α_{i+1} , the borrower should report to pay 0 till $T_i - 1$. Since $\Delta_{it} \ge 0$ for all $t < T_i - 1$ then $\Delta_{iT_i} < 0$. This will reduce the value of α_{i+1} . Now the borrower thinks to pay the entire credit before T_i , so that the bank will not take $\Delta_{iT_i} < 0$ into consideration for calculation of α_{i+1} . Therefore, the optimal strategy will be to report $(0, 0, 0, ..., 0, B_i(1 + r_i)^{T_i})$ and pay within time period $T'_i < T_i$.

Taking optimal reporting from Proposition-I into account $P_{it} = 0, \forall t < T_i - 1$, therefore

$$c_{it} = \frac{P_{it}'(P_{it}')}{P_{it}'} = P_{it}'$$

and

 $\beta_{it} = c_{it}^{\frac{1}{t}}$

3.3.1 Ex-ante Solution for the Borrower

The Proposition-I helps us to know the ex - ante payment schedule of a rational borrower. However the same rational borrower will minimize ex - post α_{i+1} . The ex-ante optimization problem for the borrower will be

$$\min_{c_{i1}, c_{i2}, \dots, c_{iT_i-1}} \frac{1}{\max\left\{1 \ , \ 1 + \sum_{t=1}^{T'_i} c_{it}^{\frac{1}{t}}\right\}}$$

such that

$$\sum_{t=1}^{T_i-1} \frac{c_{it}}{(1+r_i)^t} = B_i$$
$$c_{it} \ge 0$$

Equivalently

$$\max_{c_{i1}, c_{i2}, \dots, c_{iT_i-1}} \sum_{t=1}^{I_i-1} c_{it}^{\frac{1}{t}}$$

such that

$$\sum_{t=1}^{T_i-1} \frac{c_{it}}{(1+r_i)^t} = B_i$$
$$c_{it} > 0$$

Clearly the function $\sum_{t=1}^{T_i-1} c_{it}^{\frac{1}{t}}$ is a strictly concave function, hence the first order conditions solution will be sufficient for the optimal solution and it will be unique.

Using Lagrangian multiplier method we can solve the above problem.

Lagrangian of the optimization problem is given by

$$\mathscr{L}_{1} = \sum_{t=1}^{T_{i}-1} c_{it}^{\frac{1}{t}} - \lambda_{1} \Big(\sum_{t=1}^{T_{i}-1} \frac{c_{it}}{(1+r_{i})^{t}} - B_{i} \Big)$$

Here ' λ_1 ' is Lagrange multiplier. From the first order conditions, the solution of the above problem will be

$$\lambda_1 = 1 + r_i \tag{3}$$

The optimal payment scheme for borrower $P_{it}^{*'}$ is given by

$$P_{it}^{*'} = c_{it}^{*} = \frac{(1+r_i)^t}{t^{\frac{t}{t-1}}} , \quad \forall \ t = 2, 3, 4, \cdots, T_i - 1.$$
(4)

$$P_{i1}^{*'} = c_{i1}^* = B_i - \left(\sum_{t=2}^{T_1 - 1} \frac{c_{it}^*}{(1 + r_i)^t}\right)(1 + r_i)$$
(5)

The above optimal solution from our design incentivises the individual to create a payment schedule that decreases over time.

3.4 Comparing with a Benchmark Model

3.4.1 Benchmark Model

To showcase the value of the proposed design we have compared the results of our model with a benchmark model. The benchmark case is where a lender provides loan to those projects with probability of default less than k. The probability of default is predetermined exogenously. The lender takes collateral before providing the

loan. Let the collateral amount be $\phi B_i(1+r_i)^{T_i}$ for project *i*, where $\phi \in [0,1]$ which is a policy parameter that decides collateral amount. In case of default the lender recovers some portion of the loan amount by liquidating the collateral. The liquidating factor is δ_i , which lies in [0,1].

Then expected profit π for the risk-neutral lender when the benchmark model is used can be written as;

$$E(\pi_{\text{Benchmark Model}}) = \sum_{i \in \{j: \theta_j \le k\}} \left[(1 - \theta_i) B_i (1 + r_i)^{T_i} + \theta_i \delta_i \phi B_i (1 + r_i)^{T_i} \right]$$

Similarly, in the proposed model we keep assets as collateral, which has liquidation factor $\gamma_i \in [0, 1]$ for the project *i*.

The expected profit for the risk-neutral lender when the proposed model is used can be written as;

$$E(\pi_{\text{Model}}) = \sum_{i=1}^{n} \left[(1 - \theta_i) B_i (1 + r_i)^{T_i} + \theta_i \gamma_i \alpha_i B_i (1 + r_i)^{T_i} \right]$$

Taking both expressions into consideration

$$\begin{split} E(\pi_{\text{Model}}) &- E(\pi_{\text{Benchmark Model}}) \\ &= \sum_{i \in \{j:\theta_j > k\}} (1-\theta_i) B_i (1+r_i)^{T_i} + \sum_{i=1}^n \theta_i \gamma_i \alpha_i B_i (1+r_i)^{T_i} - \sum_{i \in \{j:\theta_j \le k\}} \theta_i \phi \, \delta_i B_i (1+r_i)^{T_i} \\ &= \sum_{i \in \{j:\theta_j > k\}} (1-\theta_i) B_i (1+r_i)^{T_i} + \sum_{i \in \{j:\theta_j > k\}} \theta_i \gamma_i \alpha_i B_i (1+r_i)^{T_i} + \sum_{i \in \{j:\theta_j \le k\}} \theta_i \gamma_i \alpha_i B_i (1+r_i)^{T_i} - \sum_{i \in \{j:\theta_j \le k\}} \theta_i \phi \, \delta_i B_i (1+r_i)^{T_i} \\ &= \sum_{i \in \{j:\theta_j > k\}} (1-\theta_i) B_i (1+r_i)^{T_i} + \sum_{i \in \{j:\theta_j > k\}} \theta_i \gamma_i \alpha_i B_i (1+r_i)^{T_i} + \sum_{i \in \{j:\theta_j \le k\}} \theta_i \phi \, \delta_i B_i (1+r_i)^{T_i} \\ &= \sum_{i \in \{j:\theta_j > k\}} (1-\theta_i) B_i (1+r_i)^{T_i} + \sum_{i \in \{j:\theta_j > k\}} \theta_i \gamma_i \alpha_i B_i (1+r_i)^{T_i} + \sum_{i \in \{j:\theta_j \le k\}} \theta_i \phi \, \delta_i B_i (1+r_i)^{T_i} \\ &= \sum_{i \in \{j:\theta_j > k\}} (1-\theta_i) B_i (1+r_i)^{T_i} + \sum_{i \in \{j:\theta_j > k\}} \theta_i \gamma_i \alpha_i B_i (1+r_i)^{T_i} + \sum_{i \in \{j:\theta_j > k\}} \theta_i \phi \, \delta_i B_i (1+r_i)^{T_i} \\ &= \sum_{i \in \{j:\theta_j > k\}} (1-\theta_i) B_i (1+r_i)^{T_i} + \sum_{i \in \{j:\theta_j > k\}} \theta_i \gamma_i \alpha_i B_i (1+r_i)^{T_i} \\ &= \sum_{i \in \{j:\theta_j > k\}} (1-\theta_i) B_i (1+r_i)^{T_i} + \sum_{i \in \{j:\theta_j > k\}} \theta_i \gamma_i \alpha_i B_i (1+r_i)^{T_i} \\ &= \sum_{i \in \{j:\theta_j > k\}} (1-\theta_i) B_i (1+r_i)^{T_i} + \sum_{i \in \{j:\theta_j > k\}} \theta_i \gamma_i \alpha_i B_i (1+r_i)^{T_i} \\ &= \sum_{i \in \{j:\theta_j > k\}} (1-\theta_i) B_i (1+r_i)^{T_i} + \sum_{i \in \{j:\theta_j > k\}} \theta_i \gamma_i \alpha_i B_i (1+r_i)^{T_i} \\ &= \sum_{i \in \{j:\theta_j > k\}} (1-\theta_i) B_i (1+r_i)^{T_i} + \sum_{i \in \{j:\theta_j > k\}} \theta_i \gamma_i \alpha_i B_i (1+r_i)^{T_i} \\ &= \sum_{i \in \{j:\theta_j > k\}} (1-\theta_i) B_i (1+r_i)^{T_i} + \sum_{i \in \{j:\theta_j > k\}} \theta_i \gamma_i \alpha_i B_i (1+r_i)^{T_i} \\ &= \sum_{i \in \{j:\theta_j > k\}} (1-\theta_i) B_i (1+r_i)^{T_i} + \sum_{i \in \{j:\theta_j > k\}} \theta_i \gamma_i \alpha_i B_i (1+r_i)^{T_i} \\ &= \sum_{i \in \{0, 1 \le k\}} (1-\theta_i) B_i (1+r_i)^{T_i} + \sum_{i \in \{j:\theta_j > k\}} \theta_i \gamma_i \alpha_i B_i (1+r_i)^{T_i} \\ &= \sum_{i \in \{0, 1 \le k\}} (1-\theta_i) B_i (1+r_i)^{T_i} + \sum_{i \in \{0, 1 \le k\}} (1+r_i)^{T_i} \\ &= \sum_{i \in \{0, 1 \le k\}} (1-\theta_i) B_i (1+r_i)^{T_i} \\ &= \sum_{i \in \{0, 1 \le k\}} (1-\theta_i) B_i (1+r_i)^{T_i} \\ &= \sum_{i \in \{0, 1 \le k\}} (1-\theta_i) B_i (1+r_i)^{T_i} \\ &= \sum_{i \in \{0, 1 \le k\}} (1-\theta_i) B_i (1+r_i)^{T_i} \\ &= \sum_{i \in \{0, 1 \le k\}} (1-\theta_i) B_i (1+r_i)^{T_i} \\ &= \sum_{i \in \{0, 1 \le k\}$$

$$=\sum_{i\in\{j:\theta_i>k\}}[(1-\theta_i)B_i(1+r_i)^{T_i}+\theta_i\gamma_i\alpha_iB_i(1+r_i)^{T_i}]+\sum_{i\in\{j:\theta_i\leq k\}}\theta_iB_i(1+r_i)^{T_i}(\gamma_i\alpha_i-\phi\delta_i)$$

By using these equation we can put down the following proposion.

Proposition 3.2. The welfare of a lender (expected profit) will always be higher in the designed model when $\gamma_i \geq \frac{\phi \delta_i}{\alpha_i}$.

3.5 Simulation Result

Objective of the simulation is to compare the performance of the proposed model vis-á-vis the benchmark model. The performances are measured in terms of two outcomes. First, profit generated by the lender. Second, total amount of collateral needed as a percentage point of total loans given. The simulation is done by coding the model in R software, and have been done for 1000 (n = 1000) projects. Each project has a probability of default, which has been generated randomly from a uniform distribution $\theta_i \sim U[0, 1]$. The proposed model is applied to generate designed collateral metrics for different projects. The result of the same has been plotted in Figure 1 and Figure 2.



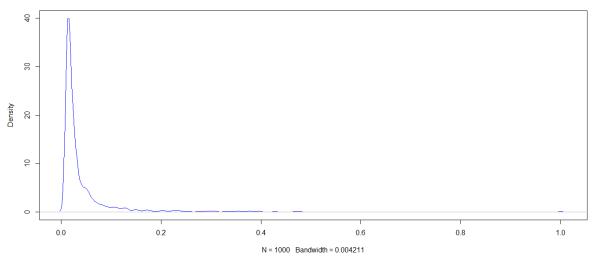


Figure 1: Distribution of Alpha

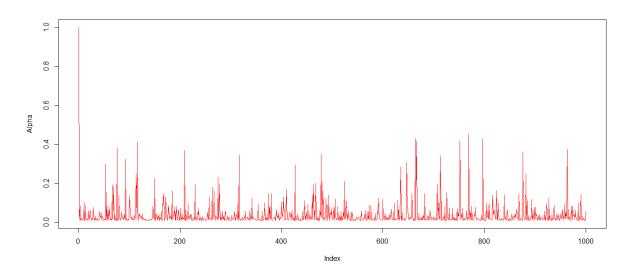


Figure 2: Value of α for Different Projects

In case of the benchmark model we have taken two cases. In one case the collateral requirement depends upon the values of $\theta_i s$, and the amount of collateral required is $\theta_i B_i (1 + r_i)^{T_i}$ where the amount borrowed is B_i (The result has been shown in the Table 1). In another case collateral needed is independent of default risk of the project (The has been shown in Table 2) and depends up on ϕ , which is a policy parameter as defined before in the model. Liquidating factor δ_i for collateral of the *ith* project is generated randomly from a uniform distribution over support [0, 1]. For the designed model we have assumed a random liquidating factor γ_i which is distributed uniformly over [0, 1]

The difference $E(\pi_{\text{Model}}) - E(\pi_{\text{Benchmark Model}})$ has been calculated and the difference between expected profits is tested (one tailed t-test).

$$H_0: E(\pi_{\text{Model}}) - E(\pi_{\text{Benchmark Model}}) = 0$$

 $H_1: E(\pi_{\text{Model}}) - E(\pi_{\text{Benchmark Model}}) > 0$

	$E(\pi_{\text{Model}}) - E(\pi_{\text{Benchmark}})$ for 1000 Replications (with t-test)							
Loan Amount	k = 0.05	k = 0.30	k = 0.5	k = 0.8	k = 0.9	k = 0.99		
0 - 25 lakhs	12493.85***	11348.91***	9155.421***	2528.791***	948.5928***	-1820.46***		
0 - 50 lakhs	24982.67***	22667.89***	18299.05***	5220.697***	1963.961***	-3713.25***		
0 - 100 lakhs	49872.13***	45475.22***	36589.55***	10450.63***	3784.372***	-734.234***		
*p - value <0.1; **p - value <0.05; ***p - value<0.01								

Table 1: Simulation Result

Table 1, shows that the proposed model does better when projects having probability of default less than 0.9 is accepted by the benchmark model, and the benchmark model will do better for projects with probability of default greater than 0.9 (which is an unlikely event). Table 2 shows that the proposed model does better than the benchmark model for all values of ϕ where k < 0.8. The benchmark model does better when the lender provides loan to very high risk projects, and demands very high percentage point of collateral. The previous

	$E(\pi_{\text{Model}}) - E(\pi_{\text{Benchmark}})$ for 1000 Replications (with t-test)							
Policy Parameter	k = 0.05	k = 0.30	k = 0.5	k = 0.8	k = 0.9	k = 0.99		
<i>φ</i> =0.01	49748.63***	45647.11***	37804.07***	18457.9***	9851.067***	365.75 ***		
<i>φ</i> =0.02	50159.46***	45590.56***	37611.09***	18605.34***	9689.724***	256.7251***		
<i>φ</i> =0.03	49787.31***	45720.78***	37709.135***	18214.72***	9706.169***	134.7639***		
<i>φ</i> =0.04	49885.81***	45812.82***	37538.95***	18109.82***	9434.034***	-15.3021**		
<i>φ</i> =0.05	50231.55***	45566.65***	37424.77***	17930.45***	9331.923***	-152.7418***		
<i>φ</i> =0.06	50102.85***	45813.46***	37370.11***	17703.19***	9204.735***	-234.5623***		
<i>φ</i> =0.07	50000.21***	45335.13***	37255.71***	17635.43***	9143.517***	-380.4298***		
$\phi = 0.08$	49958.29***	45517.59***	37343.81***	17412.35***	9040.467***	-500.2361***		
<i>φ</i> =0.09	49965.63***	45271.22***	37458.35***	17120.41***	8971.95***	-651.2846***		
φ =0.1	50124.56***	45750.08***	37210.02***	15675.07***	8998.338***	-754.1952***		
φ =0.2	50278.28***	45785.05***	36751.19***	14185.7***	6507.988***	-39672.2143***		
<i>φ</i> =0.3	50114.05***	45476.9***	36716.65***	12611.07***	4624.84***	-6356.8924***		
<i>φ</i> =0.4	50672.44***	44952.48***	35959.41***	10986.27***	2470.28***	-9035.6321***		
<i>φ</i> =0.5	50057.41***	44952.46***	35220.93***	9489.678***	497.98***	-10476.3020***		
<i>φ</i> =0.6	50454.53***	44770.08***	34486.6***	7495.718***	-1731.09***	-13562.1655***		
φ =0.7	50409.29***	44928.74***	33902.88***	6289.842***	-3598.10***	-16547.2541***		
<i>φ</i> =0.8	50129.72***	44042.05***	33995.35***	4643.144***	-5497.62***	-19042.2145***		
<i>φ</i> =0.9	50112.1***	44062.76***	32348.97***	3976.87***	-7861.64***	-21473.0224***		
$\phi = 1$	50529.24***	43792.57***	31993.65***	3071.992***	-9744.46***	-23782.3201***		

Table 2: Simulation Result with Policy Parameter ϕ

*p - value <0.1; **p - value <0.05; ***p - value<0.01

two simulations have shown in Table 1 and Table 2 finds the dominance of proposed model for all projects with probability of default 0.8. The Table 3, shows the calculated values of collateral needed as a percentage point of total loan amount for the proposed model for different cut-off values of k. We can see that there exists very little differences across the different value of k, which means that the proposed design is independent of the default probability of projects. However, lender can decide on a cut-off level of k, and after that decide on the collateral need using the proposed model.

4 Ex-post Solution for the Borrower

Our ex-post solutions assumes Proposition-I to hold. Suppose in the first period borrower made cashflow be ψ .

Table 3: Collateral Need as percentage point of Total Borrowing for the Proposed Model

	Collateral Ratio=Collateral Amount Borrowing Amount						
Loan Amount Limits (in lakh)	k = 0.05	k = 0.30	k = 0.5	k = 0.8	k = 0.9	k = 0.99	
0 - 25	8.5%	8.1%	7.9%	8.5%	8%	8.3%	
0 - 50	7.46%	7 %	7.5%	8.4%	7.4%	8.1%	
0 - 100	8.07%	7.9%	7.4%	7.4%	8%	7.8%	

In case $\psi \ge \overline{C} + P_{i1}^{*'}$, the borrower achieves the first best solution as discussed in ex-ante solution for the borrower.

If not (i.e. $\psi < \bar{C} + c_{i1}^*$), then the borrower pays $P_{i1}^{*'} = c_{i1}^* = \psi - \bar{C}$ and consumes the subsistence amount. In this case the optimization problem becomes

$$\max_{c_{i2}, c_{i3}, \dots, c_{iT_{i-1}}} \sum_{t=2}^{T_{i-1}} c_{it}^{\frac{1}{t}}$$

such that

$$\sum_{t=2}^{T_i - 1} \frac{c_{it}}{(1 + r_i)^t} = B_i - \frac{P'_{i1}}{1 + r_i}$$
$$c_t \ge 0$$

The Lagrangian of the above problem becomes

$$\mathscr{L}_{2} = \sum_{t=2}^{T_{i}-1} c_{it}^{\frac{1}{t}} - \lambda_{2} \Big(\sum_{t=2}^{T_{i}-1} \frac{c_{it}}{(1+r_{i})^{t}} - B_{i} + \frac{P_{i1}'}{1+r_{i}} \Big)$$

Solving the first order condition, we have

$$P_{it}^{*'} = c_{it}^* = \frac{(1+r_i)^t}{(\lambda_2 t)^{\frac{t}{t-1}}}$$
(6)

Putting it in the constraint, we have

$$Z(\lambda_2) = \sum_{t=2}^{T_i - 1} \frac{1}{(\lambda_2 t)^{\frac{t}{t-1}}} - B_i + \frac{P'_{i1}}{1 + r_i} = 0$$
(7)

The function Z(.) has following characteristics

$$\frac{\partial Z}{\partial \lambda_2} < 0 \ , \ \lim_{\lambda_2 \to 0} Z(\lambda_2) = \infty > 0 \ , \ \lim_{\lambda_2 \to \infty} Z(\lambda_2) = -B_i + \frac{P'_{i1}}{1 + r_i} < 0$$

Thus there exist a unique positive solution for λ_2 of Eq.7.

Since Eq.7 is a non linear equation, we need computational capabilities to find the optimal payment schedule. We have designed a program in R to solve for the optimal schedule of the borrower, and the result is shown in Figure 3.

Ex-ante and Ex-post Payment Schedule

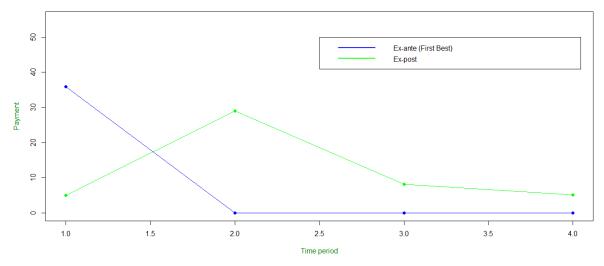


Figure 3: Ex - ante and Ex - post Payment Schedule

As we can see from the graph (Figure 3), in case the borrower is unable to pay the first best optimal payment, then the borrower can find the optimal solution for remaining periods. The optimal schedule shows the borrower has incentive to pay more in the next period in case missed the payment in previous period, which will depend upon the cash flow from the project in the previous period. Thus, the designed model incentivizes the borrower not to default willingly, and pay back the loan as early as possible.

5 Conclusion

The paper makes an attempt to link the main stream literature on need of collateral and credit rationing with the management literature on trust building over time. The model compares the ex-ante payment promise with ex-post payment structure for trust building. This is done by creating a design, which incentivizes the behavior of honoring commitment to proposed ex-ante payment schedule. In the proposed design a small business owner can improve the creditworthiness overtime, and can avail higher amounts of credit with smaller amount of collateral.

The simulation result shows that the lending institutions will able to increase their profit by using the proposed model vis-á-vis the benchmark model. The proposed model will always out-perform the benchmark model when probability of default of a project is less than 80 percentage point. Besides, with the help of trust building a small business owner can bring down collateral requirements to as low as 10 percentage point of the total borrowing.

The model can be improved by bring the probability of default into the design, and simulation can be done on real life data of a lending institution.

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